EEE 6110 Speech Processing.

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Short Time Fourier Transform (STFT)

- The STFT is defined as

$$X_{\hat{n}}(e^{j\omega}) = \sum_{m=-\infty}^{\infty} x[m]w[\hat{n} - m]e^{-j\omega m} \tag{1}$$

- Also, the STFT can be defined alternatively as

$$\tilde{X}_{\hat{n}}(e^{j\omega}) = \sum_{m=-\infty}^{\infty} x[\hat{n} + m]w[m]e^{-j\omega m} \tag{2}$$

- We can show that

$$X_{\hat{n}}(e^{j\omega}) = \tilde{X}_{\hat{n}}(e^{j\omega})e^{-j\omega\hat{n}} \tag{3}$$

Short Time Fourier Transform (STFT)

- The STFT is the DTFT of $x[m]w[\hat{n} - m]$
- This is a function of $\hat{n}$ and the radian frequency ω
- As $\hat{n}$ varies we get a sequence of DTFTs that are periodic in ω with frequency 2π

Sampling the STFT

- To proceed we make use of

$$\tilde{X}_{\hat{n}}(e^{j\omega}) = \sum_{m=-\infty}^{\infty} x[\hat{n}+m]w[m]e^{-j\omega m} \tag{4}$$

- We assume $w[m]$ is causal and non-zero only in the range $0 \leq m \leq L-1$
- We evaluate the STFT at a discrete set of frequencies using a finite duration window and move in steps of $R > 1$. That is we set

$$\hat{n} = rR \quad -\infty < r < \infty \tag{5}$$

Sampling the STFT

- We have

$$\tilde{X}[\hat{n}, k] = \tilde{X}_{\hat{n}}(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{N}} \tag{6}$$

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$$\tilde{X}[rR, k] = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j\frac{2\pi k}{N}m} \tag{7}$$

- $\tilde{X}[rR, k]$ is the DFT of $x[rR + m]w[m]$

Sampling the STFT

- The following constraints are imposed on R and N
  - $R \leq L/(2C)$, C is a constant that depends on the window. $C = 1$ for a rectangular window and 2 for the Hamming window
  - $N \geq L$

Spectrograms

- Spectrograms are a display of the magnitude of the STFT
- We form an image from a matrix whose elements are

$$S(t_r, f_k) = 20 \log_{10} |\tilde{X}[rR, k]| \tag{8}$$

where $t_r = rRT$ and $f_k = k/(NT)$

Spectrograms

- If the analysis window is short we get wideband spectrograms. With good time resolution but poor frequency resolution
- When the widow is long we have narrowband spectrograms with good frequency resolution and poor time resolution

Readings

- HAH - Chapter 5-6
- RS - Chapter 5-6