EEE5108/ETI5103 Digital Signal Processing.

Prof. Ciira Maina
ciira.maina@dkut.ac.ke

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Today's Lecture

1. Discrete Time Fourier Transform

Complex Exponentials

- The response of an LTI system to an input $x[n] = e^{j\omega_0 n}$ is given by

$$y[n] = H(\omega_0)e^{j\omega_0 n}$$

  where

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

- $H(\omega)$ is the system function or frequency response.
- $H(\omega)$ is a complex function of the real variable ω

Examples

- Consider an LTI system whose impulse response is $h[n] = \delta[n] - \delta[n-1]$
  1. Determine $H(\omega)$.
  2. Compute the output of the system $y[n]$ when the input to the system is $x[n] = \cos(\pi n/2)$.

Orthogonality of complex exponential sequences

- We have

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega n} e^{-j\omega k} d\omega = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$

The Discrete Time Fourier Transform (DTFT)

- We can show that

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$$

where

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$X(\omega)$ is the Discrete-Time Fourier Transform (DTFT) of $x[n]$

The DTFT

- If a sequence is absolutely summable, its DTFT exists.
- The DTFT of the impulse response of an LTI system is its frequency response
- The DTFT is periodic with a period of 2π
- The DTFT a complex function and we often plot its magnitude and phase.

Examples

- Determine and plot the DTFT of
  - $x[n] = \delta[n]$
  - $x[n] = \delta[n-1]$
  - $x[n] = \delta[n-2]$
  - $x[n] = \delta[n+1] - \delta[n-1]$