Signals and Communication Lecture 5

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1 Summary

This lecture will focus on:

1. Introduction to Amplitude Modulation (AM)
2. Conventional AM
3. Demodulating conventional AM, the envelope detector
4. Double side band suppressed carrier AM (DSB-SC AM)
5. Demodulating DSB-SC AM

2 Introduction to Amplitude Modulation

Communication involves the transmission of information from one place to another. According to Shannon, "The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point."[1] The information is often contained in a baseband signal $m(t)$ with a frequency content ranging from 0 Hz to some maximum value W Hz. That is the bandwidth of the signal is W Hz. It is often the case that in order to transmit the signal over a channel, the message signal must be frequency shifted so that it lies within the passband of the channel. This frequency shift is achieved by *modulation*. Modulation involves using the message signal $m(t)$ to modify some characteristic of a carrier signal $c(t)$. We have

$$c(t) = A_c \cos(2\pi f_c t + \phi_c)$$

and we can use the message signal to modify either the amplitude A_c, frequency f_c or phase ϕ_c of the carrier signal. Each choice of a property to modify corresponds to a different modulation scheme.

3 Conventional AM

In conventional amplitude modulation (AM), the message $m(t)$ is used to modify the amplitude of the carrier. The AM signal is given as

$$s_{AM}(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

[1] Shannon, C. E. (1948). A mathematical theory of communication. *The Bell system technical journal*, 27(3), 379-423.

where k_a is a constant known as the amplitude sensitivity of the AM modulator. This value is chosen to ensure that

$$|k_a m(t)| < 1$$

When this condition holds, $1 + k_a m(t)$ is always positive and the envelope of the AM signal follows the message signal. See Figure 1. If k_a is large enough to make $|k_a m(t)| > 1$ for some

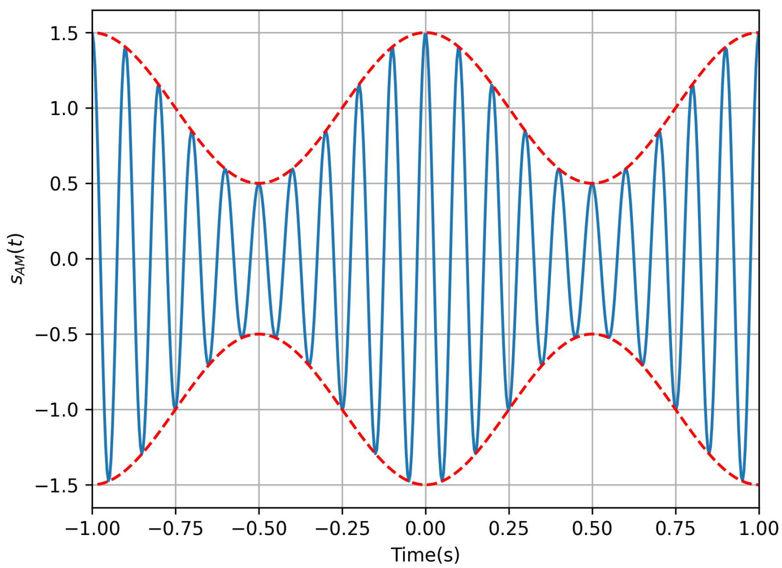

Figure 1: A conventional AM signal.

values of t then we say the carrier is *overmodulated* and a carrier phase reversal occurs whenever $1 + k_a m(t)$ changes sign. See Figure 2.

We must also ensure that $f_c >> W$.

Taking the Fourier transform of $s_{AM}(t)$ we get

$$S_{AM}(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

where $M(f)$ is the Fourier transform of $m(t)$ and is non-zero for $|f| \leq W$. We see that the negative frequency spectrum of $M(f)$ is now present at positive frequencies for $S_{AM}(f)$. The frequency spectrum of $S_{AM}(f)$ lying above f_c is known as the *upper sideband* and the frequency spectrum between $f_c - W$ and f_c is known as the *lower sideband*. The bandwidth of $s_{AM}(t)$ is $2W$, twice the bandwidth of the baseband signal.

3.1 Power of conventional AM signal

Consider the case where

$$m(t) = A_m \cos(2\pi f_m t).$$

We have

$$s_{AM}(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

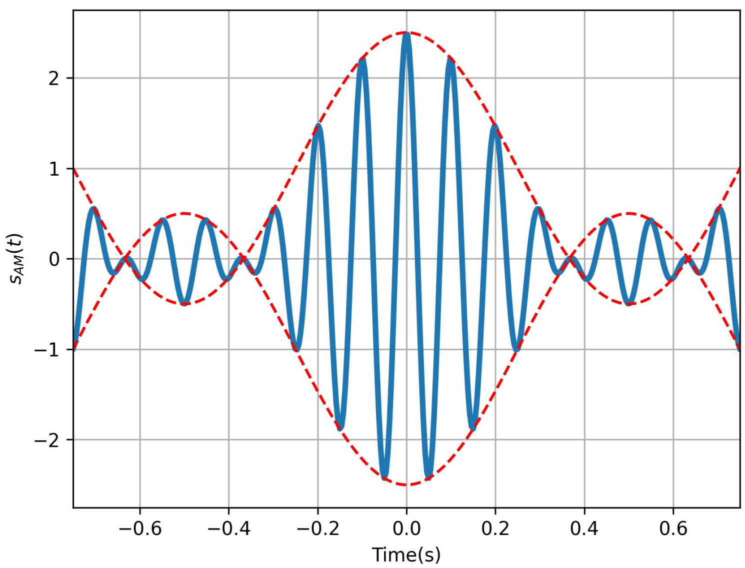

Figure 2: An overmodulated conventional AM signal.

and

$$S_{AM}(f) = \frac{A_c}{2}[\delta(f-f_c)+\delta(f+f_c)]+\frac{k_a A_m A_c}{4}[\delta(f-f_c-f_m)+\delta(f-f_c+f_m)+\delta(f-f_c+f_m)+\delta(f+f_c-f_m)]$$

The power in the carrier is

$$P_c = \frac{A_c^2}{2}$$

while the power in each of the sidebands is

$$P_{sb} = \frac{k_a^2 A_m^2 A_c^2}{8}$$

giving a total power of $\frac{k_b^2 A_m^2 A_c^2}{4}$ that conveys information. In general we can show that the power in the conventional AM signal P_{AM} is given by

$$P_{AM} = \frac{A_c^2}{2} + \frac{k_a^2 A_c^2 P_m}{2}$$

where P_m is the power of the message signal.

Putting $\mu = k_a A_m$, we see that the ratio of the power in the sidebands to the total signal power is $\frac{\mu^2}{2+\mu^2}$. If $\mu = 1$ only one third of the total signal power is used to transmit information. This is one of the disadvantages of conventional AM since a significant amount of power is used to transmit the carrier component even though it conveys no information. The other disadvantage is the doubling of the bandwidth requirement. We will consider other AM schemes such as Double side band suppressed carrier AM and Single sideband AM which aim to address these concerns.

3.2 Modulating conventional AM

3.2.1 Power law modulator

Consider a nonlinear device with the a voltage input-output characteristic given by

$$\upsilon_o(t) = a_1\upsilon_i(t) + a_2\upsilon_i^2(t).$$

If $\upsilon_i(t) = m(t) + A_c\cos(2\pi f_c t)$ then

$$\begin{aligned}\upsilon_o(t) &= a_1[m(t) + A_c\cos(2\pi f_c t)] + a_2[m(t) + A_c\cos(2\pi f_c t)]^2. \\ &= a_1 m(t) + a_2 m^2(t) + a_1 A_c[1 + \frac{2a_2}{a_1}m(t)]\cos(2\pi f_c t) + a_2 A_c^2\cos^2(2\pi f_c t)\end{aligned}$$

If we pass $\upsilon_o(t)$ through an ideal bandpass filter centered at f_c with bandwidth $2W$, then we obtain the conventional AM signal

$$a_1 A_c[1 + \frac{2a_2}{a_1}m(t)]\cos(2\pi f_c t)$$

where we ensure $|\frac{2a_2}{a_1}m(t)| < 1$ by design.

3.2.2 Switching modulator

Consider the input $\upsilon_i(t) = m(t) + \underbrace{A_c\cos(2\pi f_c t)}_{c(t)}$ applied across a diode and resistor circuit with the diode acting as an ideal switch. We assume that $A_c >> |m(t)|$. The voltage across the resistor is

$$\upsilon_r(t) \approx \begin{cases} \upsilon_i(t) & c(t) > 0 \\ 0 & c(t) < 0 \end{cases}$$

We have

$$\upsilon_r(t) \approx [m(t) + A_c\cos(2\pi f_c t)]g_{T_0}(t)$$

where $g_{T_0}(t)$ is the periodic pulse train with period $T_0 = \frac{1}{f_c}$ defined by

$$g_{T_0}(t) = \begin{cases} 1 & c(t) > 0 \\ 0 & c(t) < 0 \end{cases}$$

We can show that

$$\begin{aligned} g_{T_0}(t) &= \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{\pi n}{2}\right)\cos\left(\frac{2\pi n t}{T_0}\right) \\ &= \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c(2n-1)t] \end{aligned}$$

Now

$$\upsilon_r(t) \approx \frac{A_c}{2}[1 + \frac{4}{\pi A_c}m(t)]\cos(2\pi f_c t) + \text{plus other terms}$$

If we pass $\upsilon_r(t)$ through an ideal bandpass filter centered at f_c with bandwidth $2W$, then we obtain the conventional AM signal

$$\frac{A_c}{2}[1 + \frac{4}{\pi A_c}m(t)]\cos(2\pi f_c t)$$

3.3 Demodulating conventional AM

The demodulation of conventional AM signals is achieved using a circuit known as an envelope detector. During the positive cycle of the input signal, the capacitor charges to the maximum value of the input. When the input falls below this maximum value, the capacitor begins to discharge through the resistor with a time constant RC.

The value of RC is chosen to ensure that the output voltage follows the envelope of the input signal. If RC is too small the capacitor discharges too quickly. While if it is too large the capacitor doesn't discharge fast enough to track the message. As a rule of thumb we set

$$\frac{1}{f_c} << RC << \frac{1}{W}$$

Example: Consider a speech signal on bandwidth 5kHz modulated by a carrier of 1MHz. We have

$$10^{-6} << RC << 2 \times 10^{-4}$$

and setting $RC = 10^{-5}$ is a good choice.

4 Double side band suppressed carrier AM

We have seen that conventional AM is wasteful of both power and bandwidth. DSB-SC AM helps to prevent wasting power transmitting a carrier with no information. The DSB-SC AM signal is given by

$$s(t) = A_c \cos(2\pi f_c t) m(t).$$

Where $m(t)$ is the bandlimited message signal with nonzero frequency content in the interval $-W \leq f \leq W$. The spectrum of the DSB-SC AM signal is given by

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

Modulation translates that baseband signal to a bandpass signal with frequency content centered at $\pm f_c$ and bandwidth $2W$. From the expression for $S(f)$, we see that no carrier is present in the DSB-SC AM signal. Presence of a carrier would be indicated by presence of impulses at $\pm f_c$. We say that the carrier is suppressed.

4.1 Power of DSB-SC signals

It can be shown that the power of DSB-SC AM signals is given by

$$\frac{A_c^2}{2} P_m$$

where P_m is the power of the message signal

4.2 Modulation of DSB-SC signals

Modulation of DSB-SC signals is achieved via:

1. The balanced modulator

2. The ring modulator

4.3 Demodulation of DSB-SC signals

4.3.1 Coherent detection

If we assume the DSB-SC AM signal is transmitted through an ideal channel and the received signal $r(t)$ is the same as the transmitted signal. Then

$$r(t) = A_c \cos(2\pi f_c t) m(t)$$

The received signal can be demodulated by first multiplying the received signal by a locally generated sinusoid of the same frequency $\cos(2\pi f_c t + \phi)$ and the low pass filtering the result. The input to the low pass filter $v_i(t)$ is given by

$$\begin{aligned}
v_i(t) &= A_c \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) m(t) \\
&= A_c \cos(2\pi f_c t)[\cos(2\pi f_c t)\cos(\phi) - \sin(2\pi f_c t)\sin(\phi)] m(t) \\
&= A_c m(t)\cos(\phi)[\frac{1}{2} + \frac{1}{2}\cos(4\pi f_c t)] - A_c m(t)\frac{1}{2}\sin(4\pi f_c t)\sin(\phi) \\
&= \frac{A_c m(t)}{2}\cos(\phi) + \frac{A_c m(t)}{2}\cos(4\pi f_c t + \phi)
\end{aligned}$$

The output of the LPF is

$$\frac{A_c m(t)}{2}\cos(\phi).$$

We see that if the phase error ϕ is zero then then $m(t)$ is recovered. If $\phi = \pm\frac{\pi}{2}$ then the output of the filter is zero. This is known as the quadrature null effect and illustrates the need the maintain phase synchrony between the transmitter and the receiver.