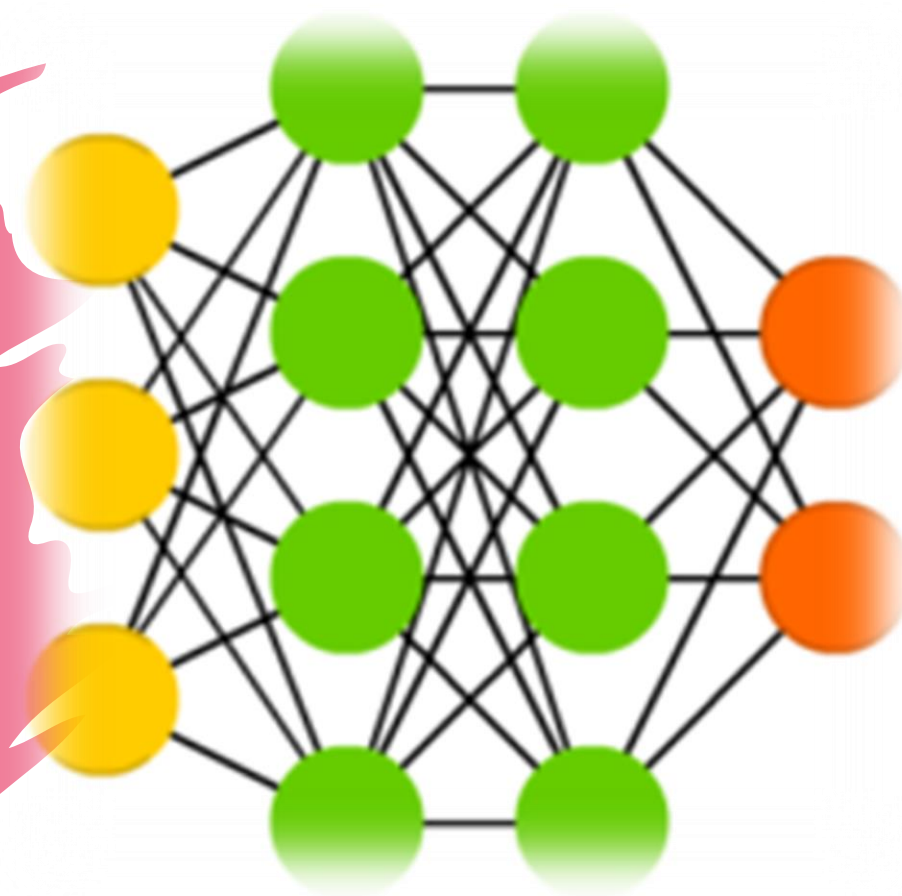


手計算 ニューラルネットワーク 入門 ②逆伝播編





予定

第1回 理論：順伝播

第2回 理論：逆伝播

第3回 実装

第4回 実験

第5回

ニューラルネットワーク
を完全に理解したい



pythonのライブラリ使うだけ

なんとなく理論は知ってるけど
結局呪文を呪文になってしまってる

引数やパラメータの説明を聞いても
多分理解できない（自分が）

```
from tensorflow.python.keras.models import Sequential
from tensorflow.python.keras.layers import Dense

model = Sequential()
model.add(Dense(...))
```



手計算ニューラルネットワーク

完全に理解したい

⇔手計算できるレベルで理解する

⇔for文if文で実装ができる

最終目標（最低限）

ニューラルネットワークを完全に理解する

C++でニューラルネットワークを実装し
何かしらの分類問題を解く

C++でニューラルネットワークを実装し
何かしらの回帰問題を解く

最終目標（理想）

Kaggle Titanic in C++



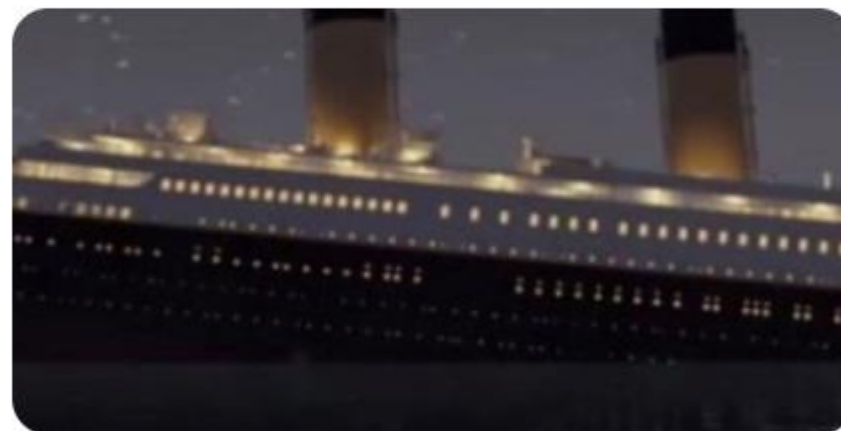
KAGGLE · GETTING STARTED PREDICTION COMPETITION · ONGOING

Submit Prediction



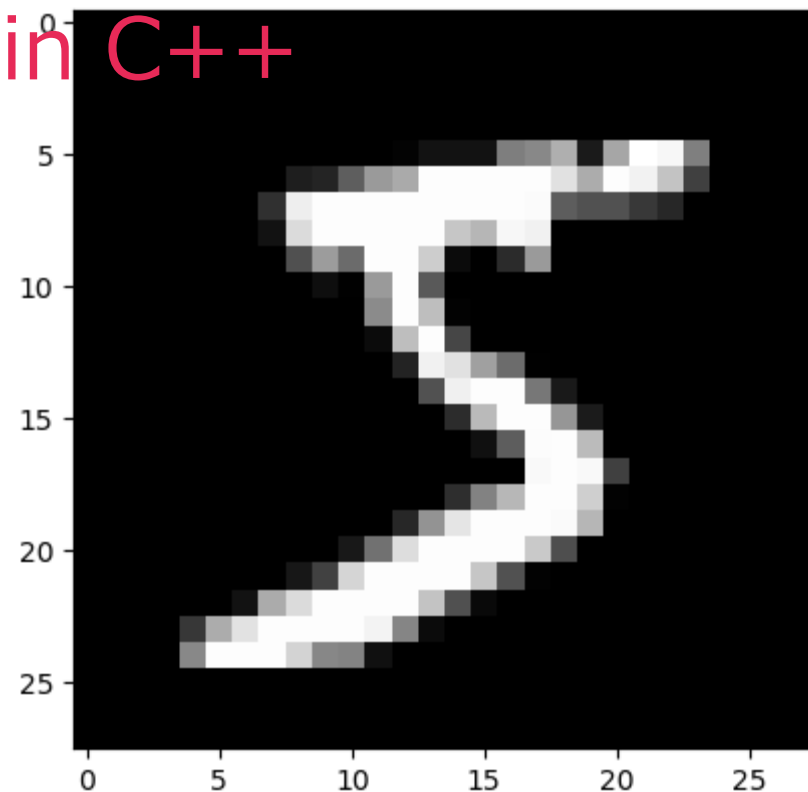
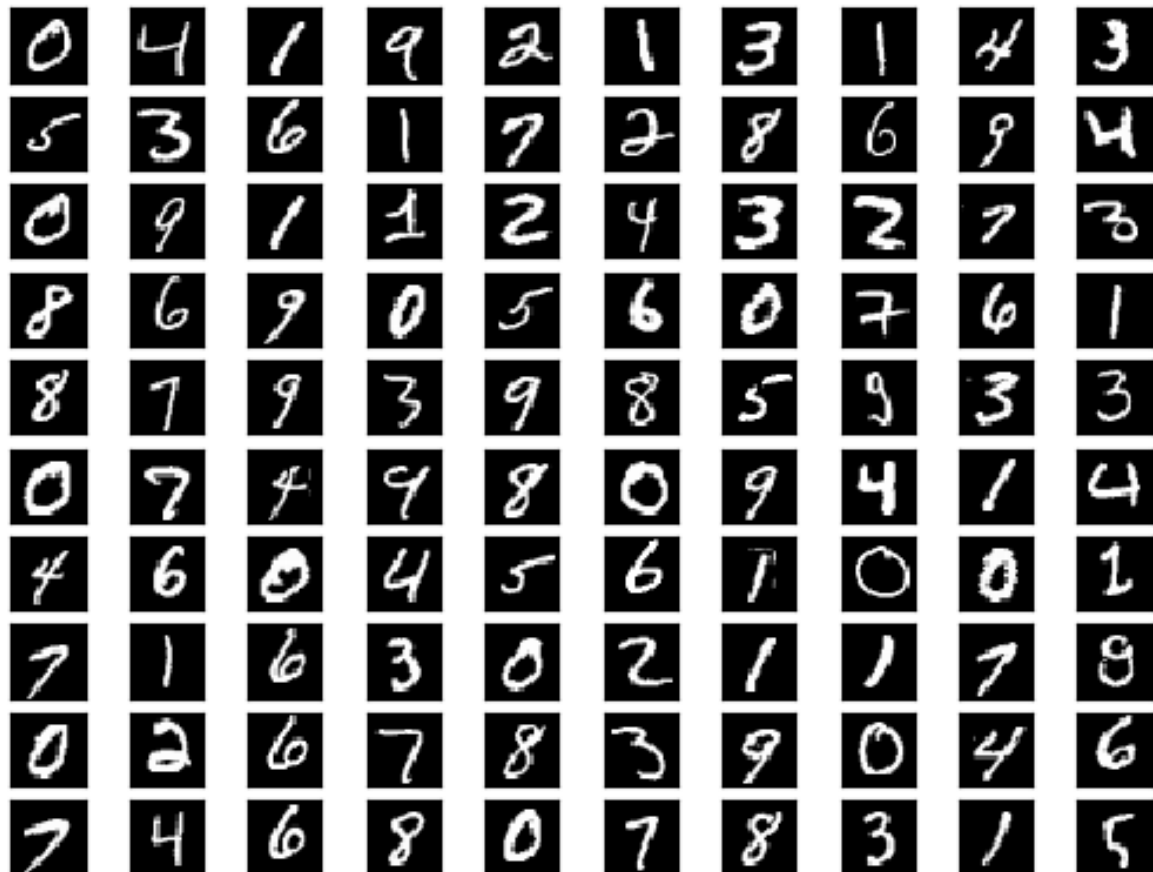
Titanic - Machine Learning from Disaster

Start here! Predict survival on the Titanic and get familiar with ML basics

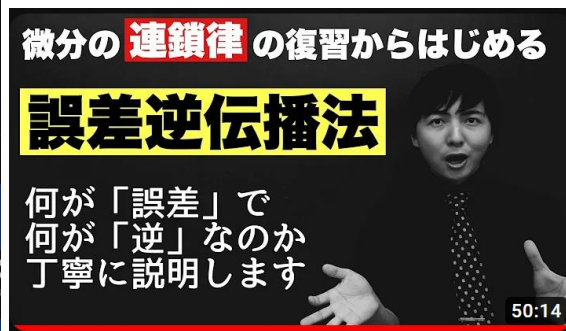


最終目標（理想）

MNISTデータセットの手書き文字認識 in C++



28×28×256



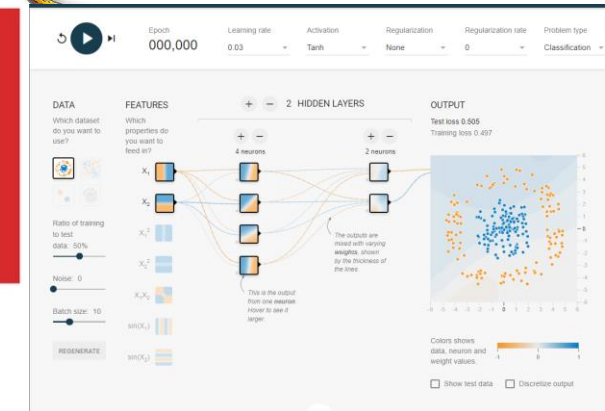
参考文献



ゼロから作る

Deep Learning

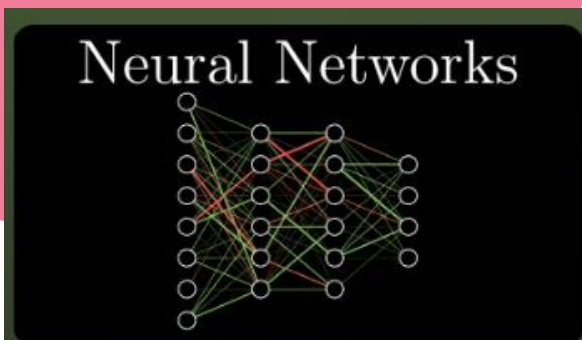
Pythonで学ぶディープラーニングの理論と実装



ディープラーニング入門 : 仕組み理解から実装まで

某处生活_LiveSomewhere

5本の動画 996回視聴 最終更新日: 2021/02/28



Neural networks















3Blue1Brown · コース

4本の動画 本日更新

全体の流れ

A mostly complete chart of Neural Networks

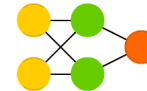
©2019 Fjodor van Veen & Stefan Leijnen asimovinstitute.org

-  Input Cell
-  Backfed Input Cell
-  Noisy Input Cell
-  Hidden Cell
-  Probablistic Hidden Cell
-  Spiking Hidden Cell
-  Capsule Cell
-  Output Cell
-  Match Input Output Cell
-  Recurrent Cell
-  Memory Cell
-  Gated Memory Cell
-  Kernel
-  Convolution or Pool

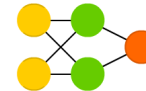
Perceptron (P)



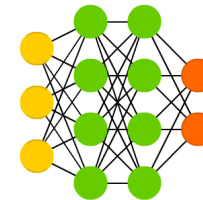
Feed Forward (FF)



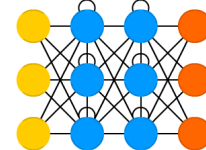
Radial Basis Network (RBF)



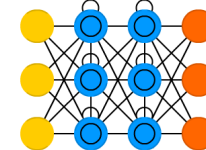
Deep Feed Forward (DFF)



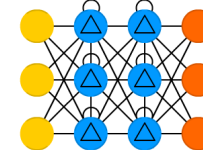
Recurrent Neural Network (RNN)



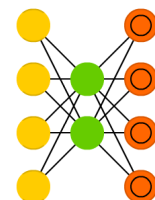
Long / Short Term Memory (LSTM)



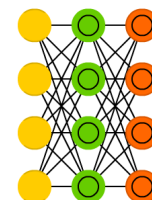
Gated Recurrent Unit (GRU)



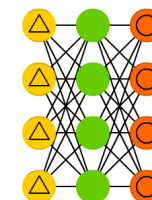
Auto Encoder (AE)



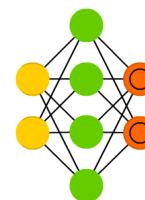
Variational AE (VAE)



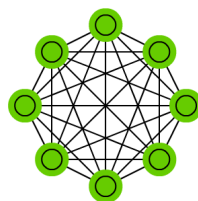
Denoising AE (DAE)



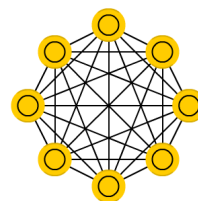
Sparse AE (SAE)



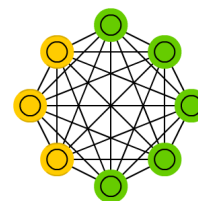
Markov Chain (MC)



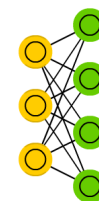
Hopfield Network (HN)



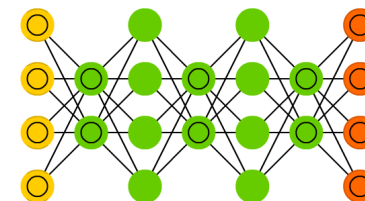
Boltzmann Machine (BM)



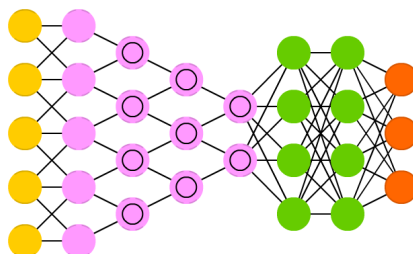
Restricted BM (RBM)



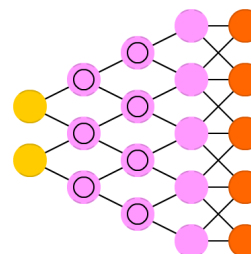
Deep Belief Network (DBN)



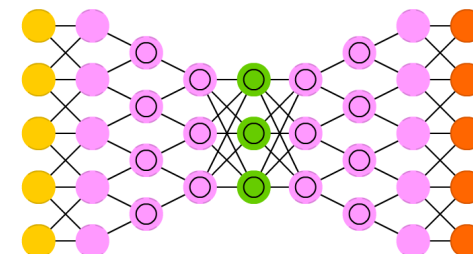
Deep Convolutional Network (DCN)



Deconvolutional Network (DN)

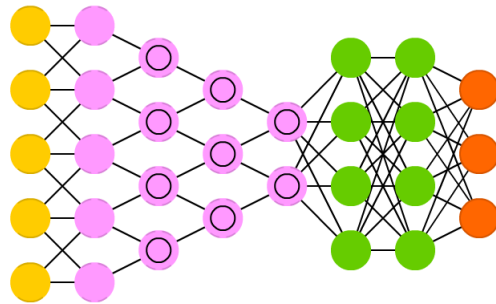


Deep Convolutional Inverse Graphics Network (DCIGN)

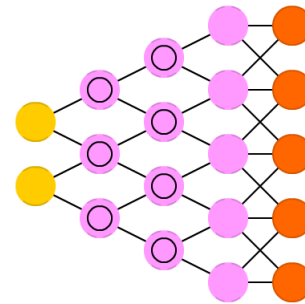


全体の流れ

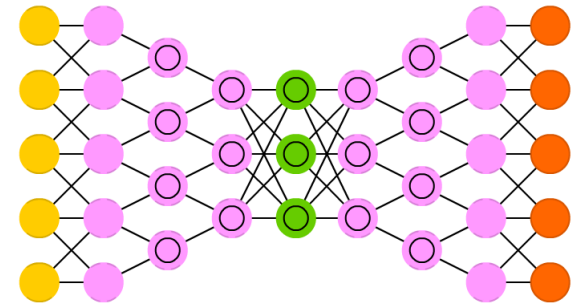
Deep Convolutional Network (DCN)



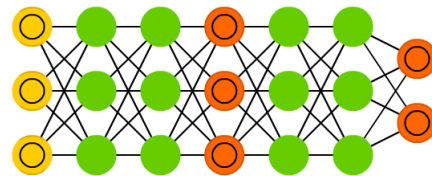
Deconvolutional Network (DN)



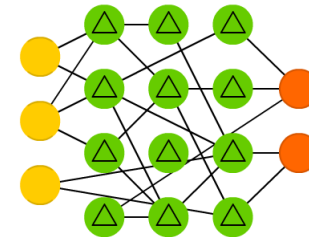
Deep Convolutional Inverse Graphics Network (DCIGN)



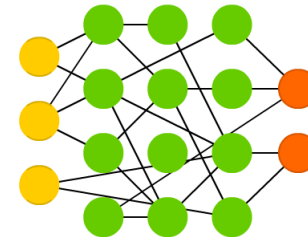
Generative Adversarial Network (GAN)



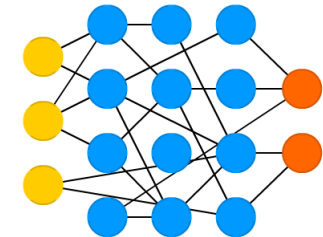
Liquid State Machine (LSM)



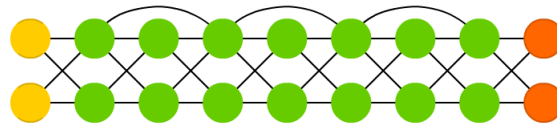
Extreme Learning Machine (ELM)



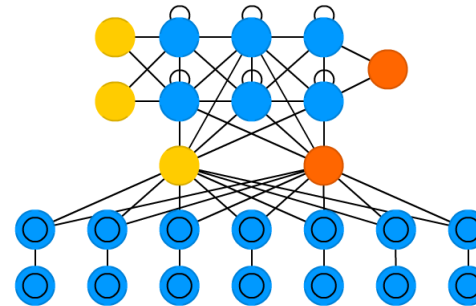
Echo State Network (ESN)



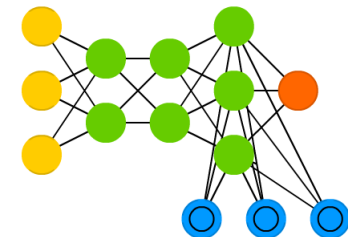
Deep Residual Network (DRN)



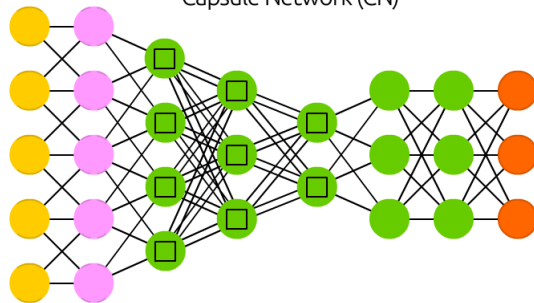
Differentiable Neural Computer (DNC)



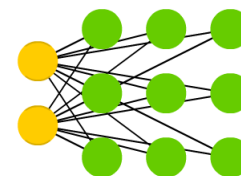
Neural Turing Machine (NTM)



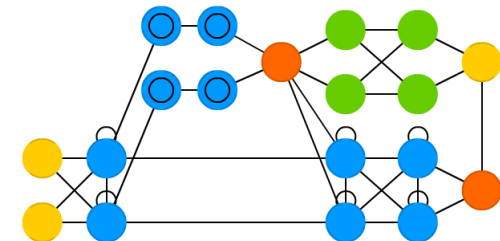
Capsule Network (CN)



Kohonen Network (KN)

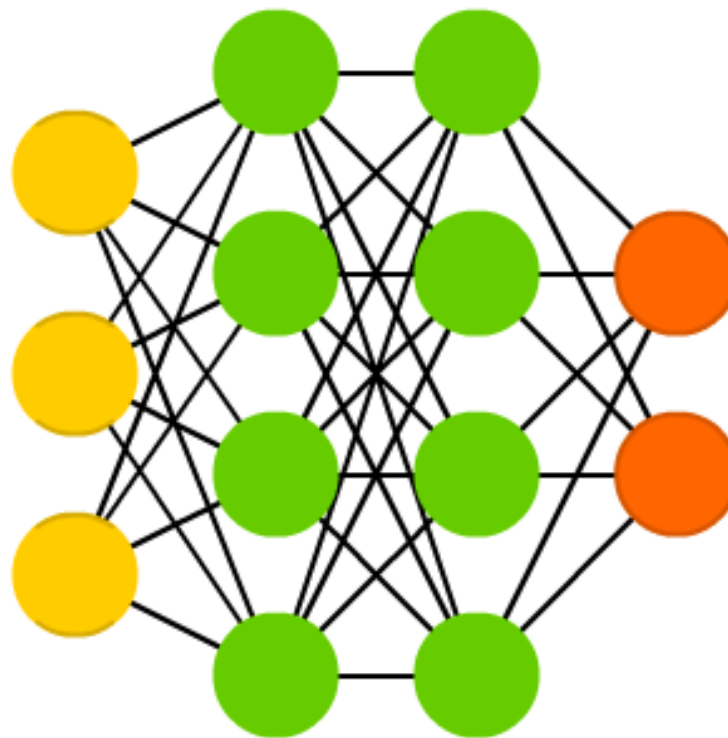


Attention Network (AN)



全体の流れ-全結合ニューラルネットワーク

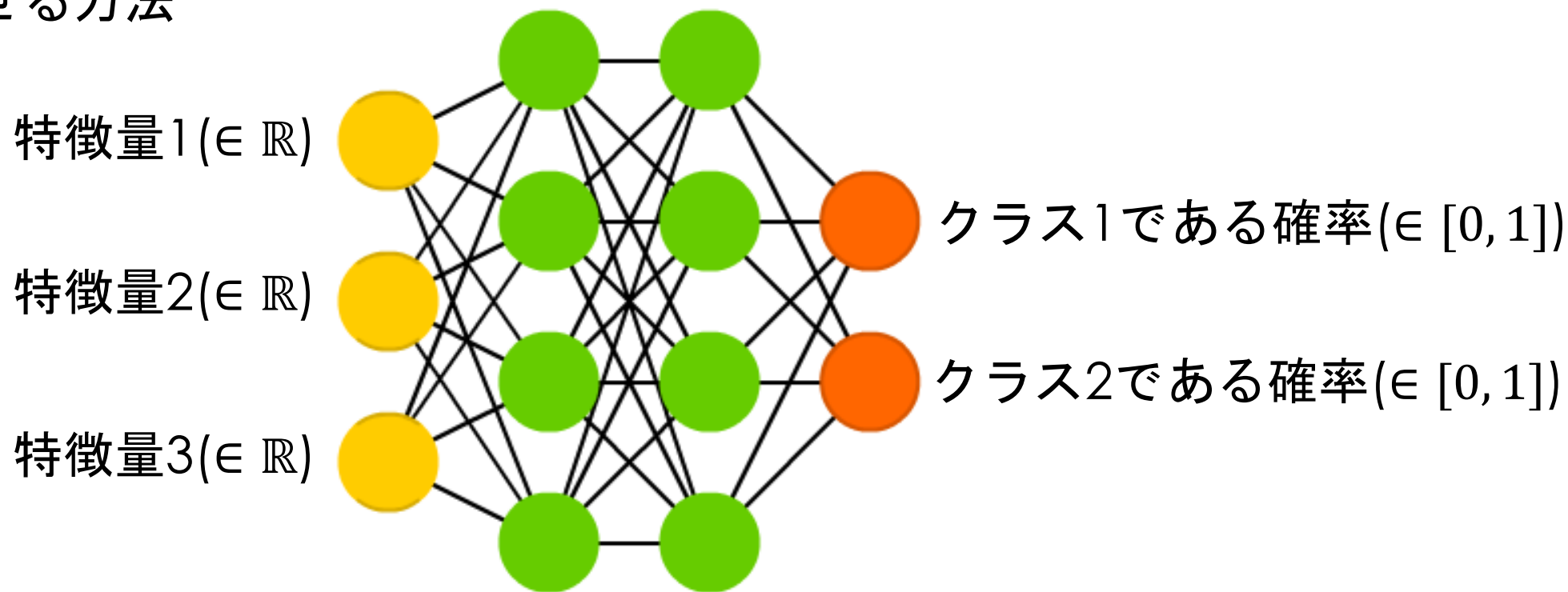
Deep Feed Forward (DFF)



全体の流れ-前回

適切な重みが既知のとき
データを伝搬させる方法
を学んだ

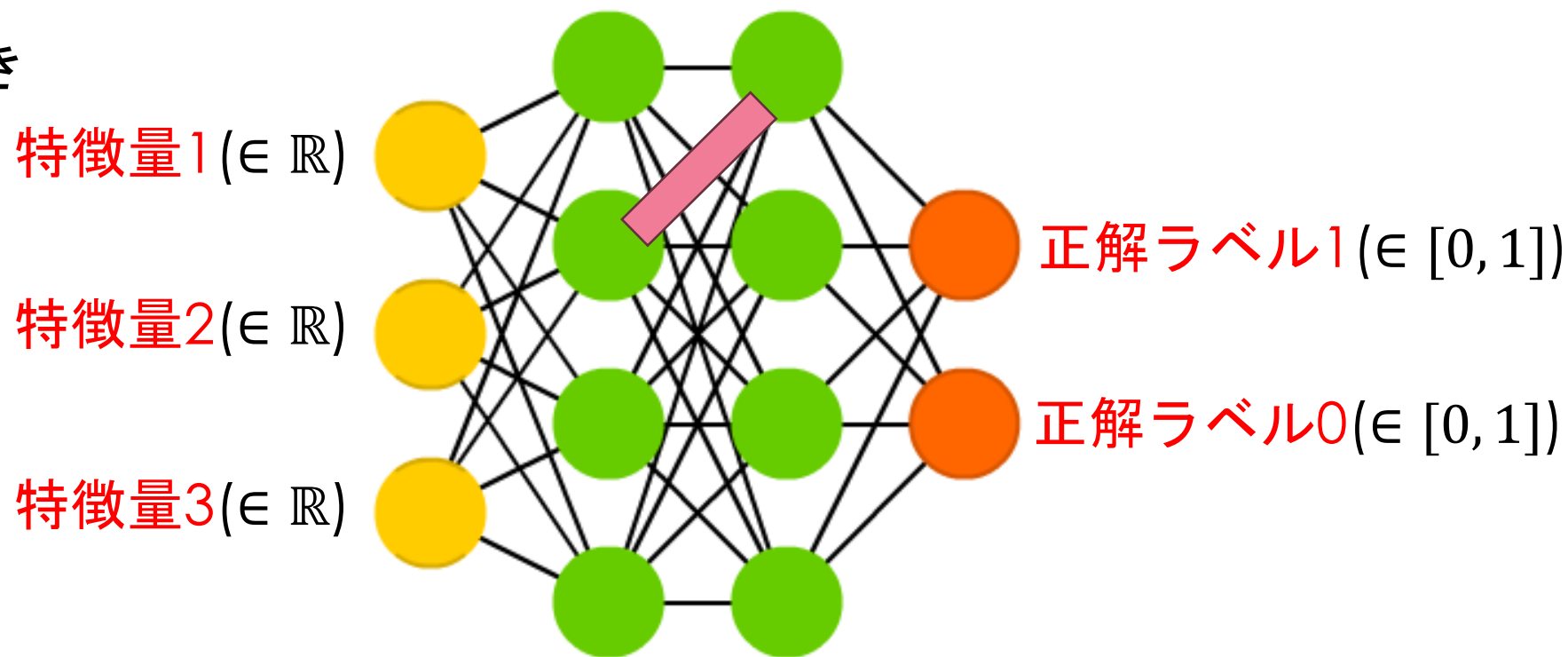
Deep Feed Forward (DFF)



全体の流れ-今回

特徴量に対して
何が正解か
わかっているとき
重みを求める

Deep Feed Forward (DFF)



偏微分復習

iPadのご準備を



偏微分-演習(ちょいムズ)

$$z = ax + by,$$

$$\frac{\partial z}{\partial x} = ?$$

偏微分-演習(ちょいムズ)

$$z = ax + by,$$

$$\frac{\partial z}{\partial x} = ?$$



冗談

偏微分-連鎖律 1変数

$$y = f(x), z = g(y)$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \frac{\partial y}{\partial x}$$

偏微分-連鎖律 2変数

$$z = z(u, v),$$
$$u = u(x, y), v = v(x, y)$$

$$\frac{\partial z}{\partial x} = ?$$

偏微分-連鎖律 2変数

$$z = z(u, v),$$
$$u = u(x, y), v = v(x, y)$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial x}$$

偏微分-連鎖律 多変数

$$\begin{aligned} z &= z(u_1, u_2, \dots, u_n), \\ u_1 &= u_1(x_1, x_2, \dots, x_m), \\ &\dots \\ u_n &= u_n(x_1, x_2, \dots, x_m) \end{aligned}$$

$$\frac{\partial z}{\partial x_k} = ?$$

偏微分-連鎖律 多變數

$$\begin{aligned}z &= z(u_1, u_2, \dots, u_n), \\u_1 &= u_1(x_1, x_2, \dots, x_m), \\&\dots \\u_n &= u_n(x_1, x_2, \dots, x_m)\end{aligned}$$

$$\frac{\partial z}{\partial x_k} = \frac{\partial z}{\partial u_1} \frac{\partial u_1}{\partial x_k} + \dots + \frac{\partial z}{\partial u_n} \frac{\partial u_n}{\partial x_k} = \sum_{i=1}^n \frac{\partial z}{\partial u_i} \frac{\partial u_i}{\partial x_k}$$

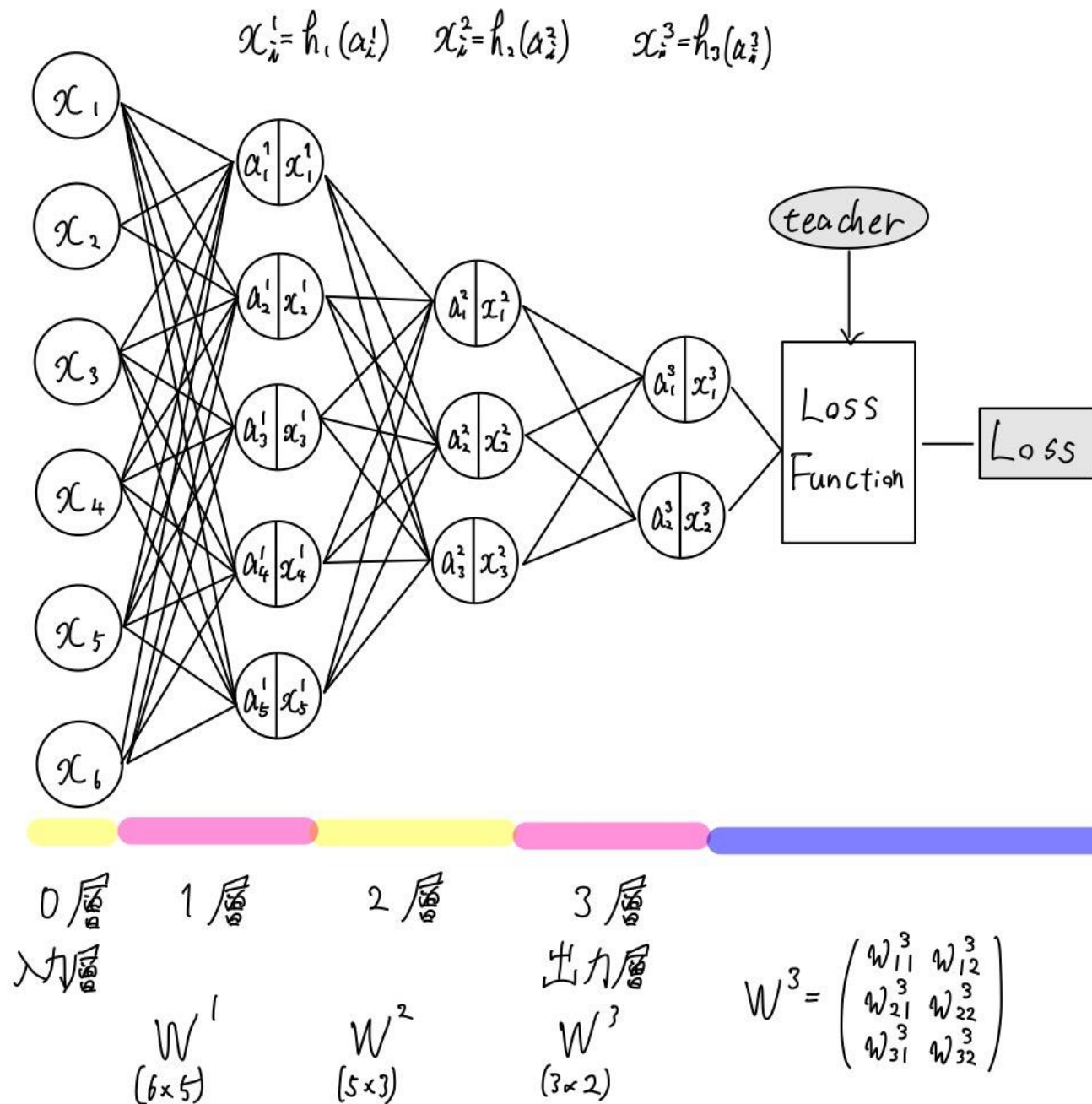
偏微分復習完

A large, irregular pink brushstroke shape on a white background, serving as a background for the title text.

誤差逆伝播法 オンライン

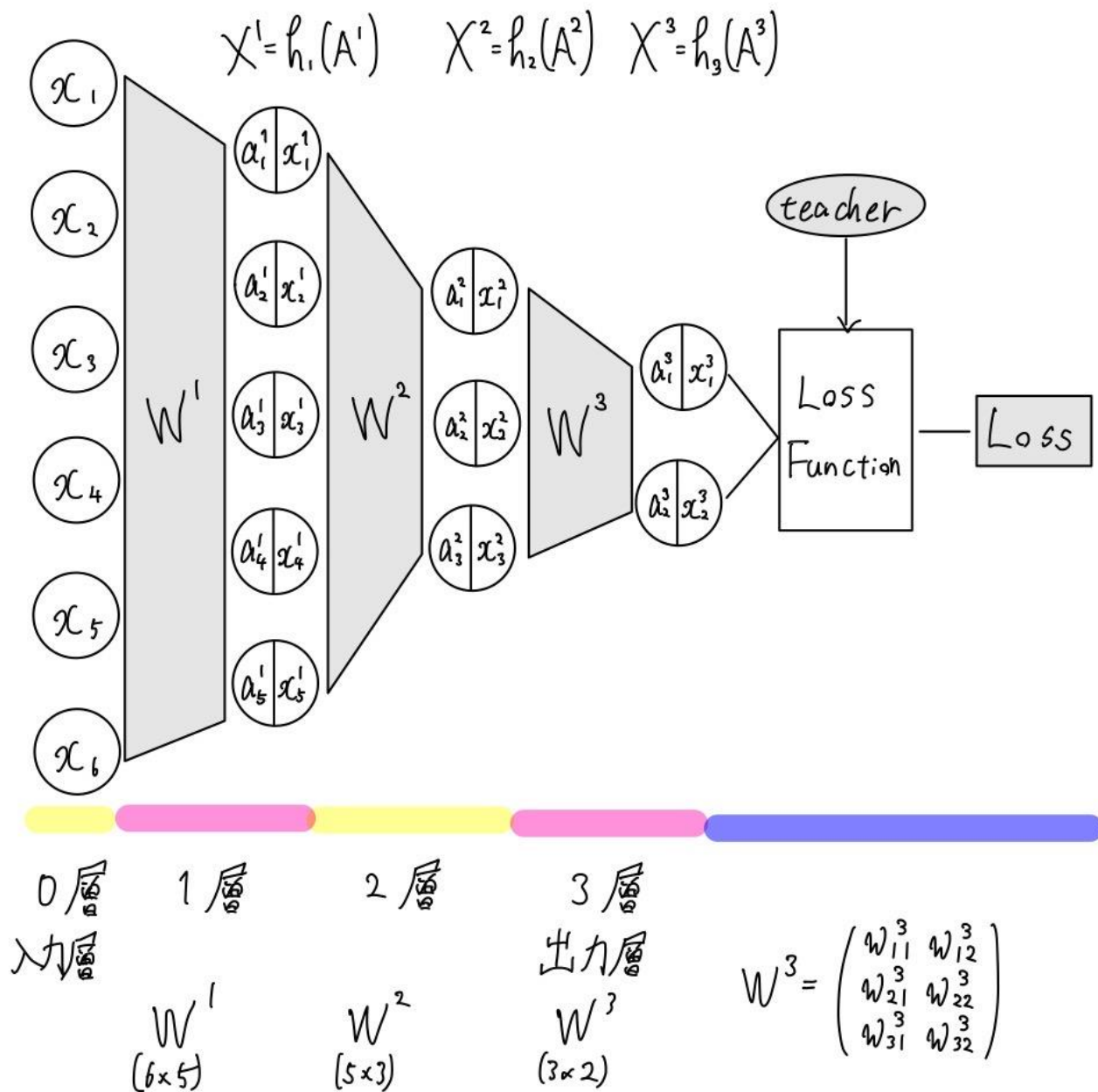
誤差逆伝播- 記号導入

k層NN

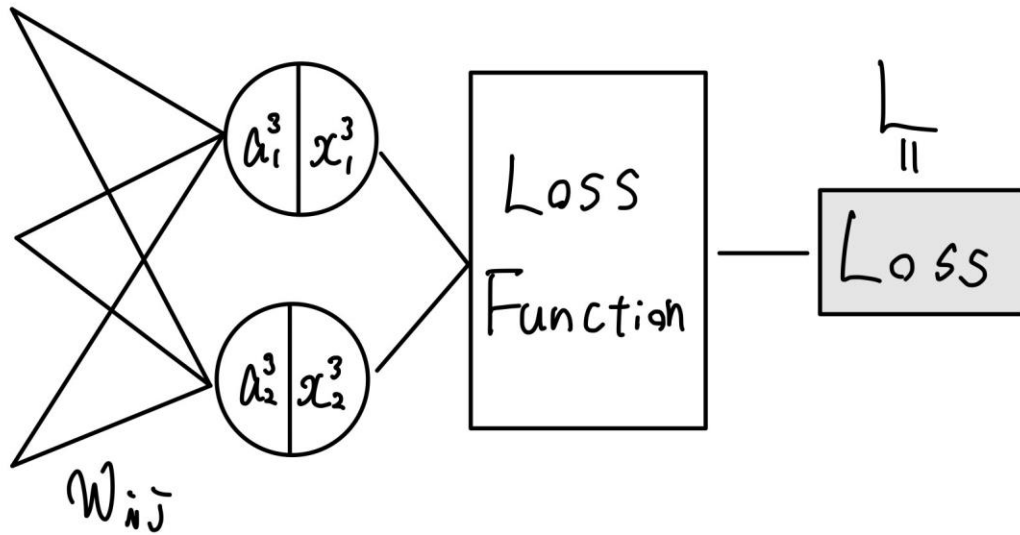


誤差逆伝播- 記号導入

k層NN



誤差逆伝播-出力層の重みの更新

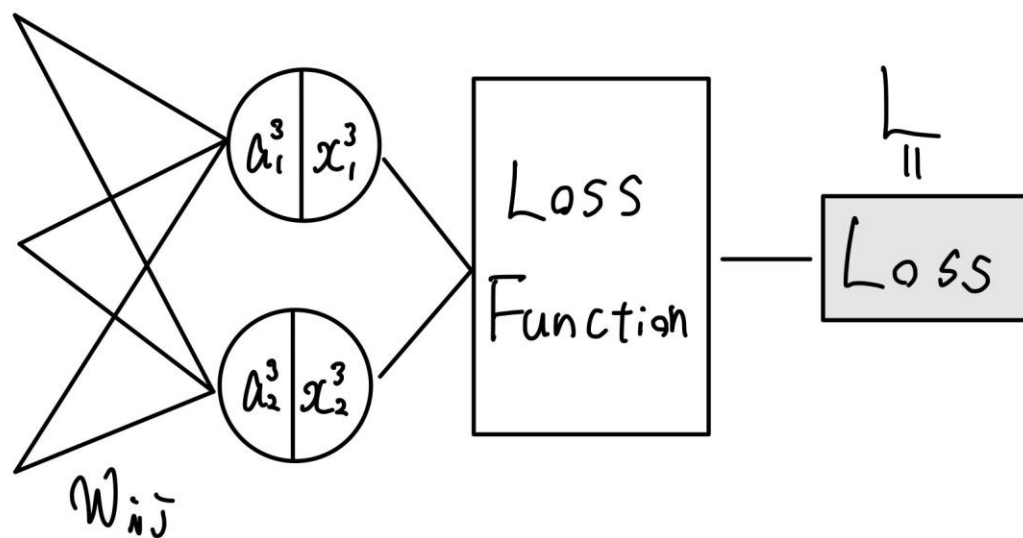


$$\frac{\partial L}{\partial w_{ij}^K} = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \frac{\partial a_j^K}{\partial w_{ij}^K}$$

さっきのシグマは？

$$\frac{\partial z}{\partial x_k} = \sum_{i=1}^n \frac{\partial z}{\partial u_i} \frac{\partial u_i}{\partial x_k}$$

誤差逆伝播-出力層の重みの更新



$$\frac{\partial L}{\partial w_{ij}^K} = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \frac{\partial a_j^K}{\partial w_{ij}^K}$$

$$L = L(\dots, a_{j-1}^K, a_j^K, a_{j+1}^K, \dots)$$

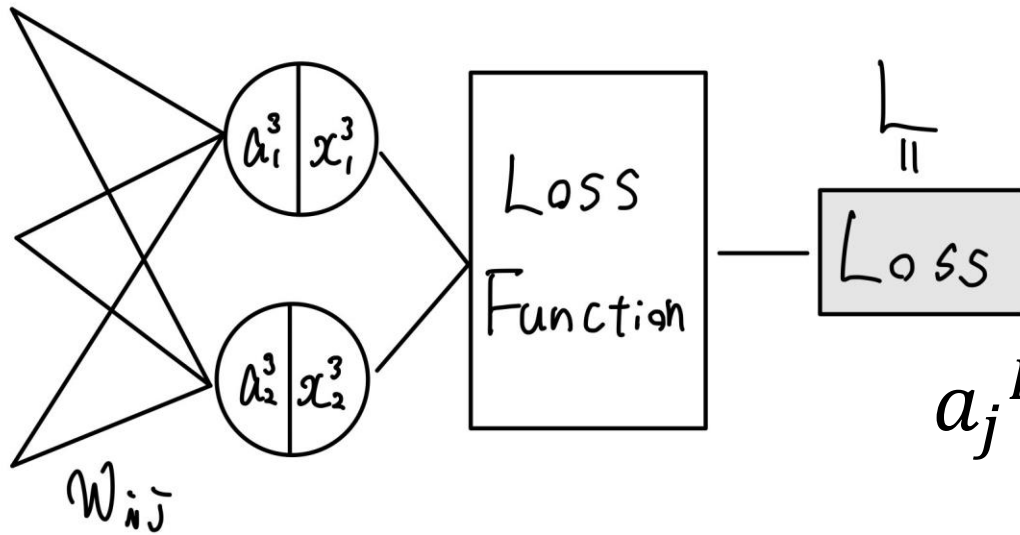
$$a_j^K = a_j^K(w_{1j}^K \dots, w_{i-1,j}^K, w_{ij}^K, \dots)$$

$$(l \neq j), a_l^K = a_l^K(w_{1l}^K \dots, w_{i-1,l}^K, w_{il}^K, \dots)$$

というように w_{ij}^K が引数にない

$\therefore w_{ij}^K$ での偏微分が0

誤差逆伝播-出力層の重みの更新

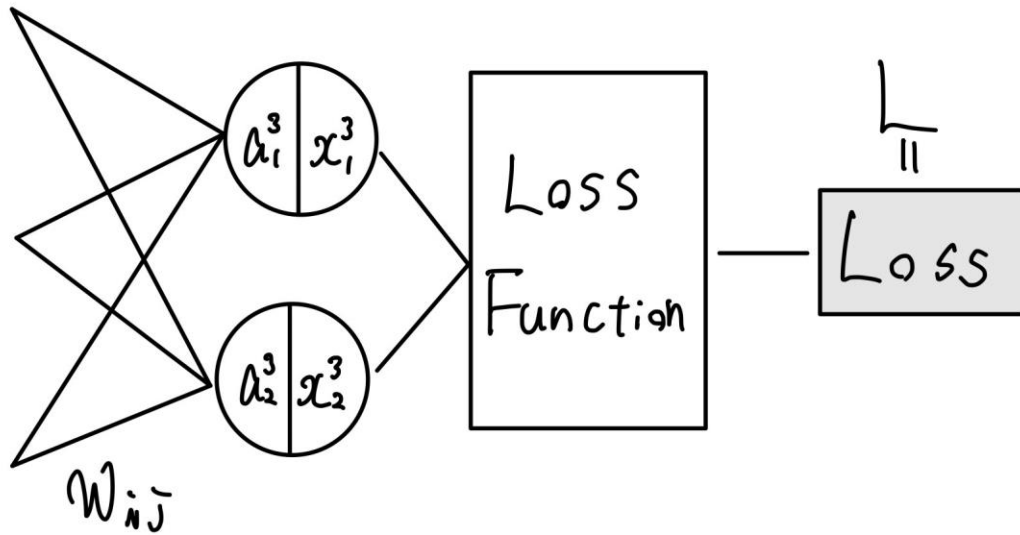


$$\frac{\partial L}{\partial w_{ij}^K} = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \frac{\partial a_j^K}{\partial w_{ij}^K}$$

$$a_j^K = w_{1j}^K x_1^{K-1} + \dots + w_{ij}^K x_i^{K-1} + \dots$$

$$\frac{\partial a_j^K}{\partial w_{ij}^K} = x_i^{K-1} \leftarrow \text{データが入れば既知}$$

誤差逆伝播-出力層の重みの更新

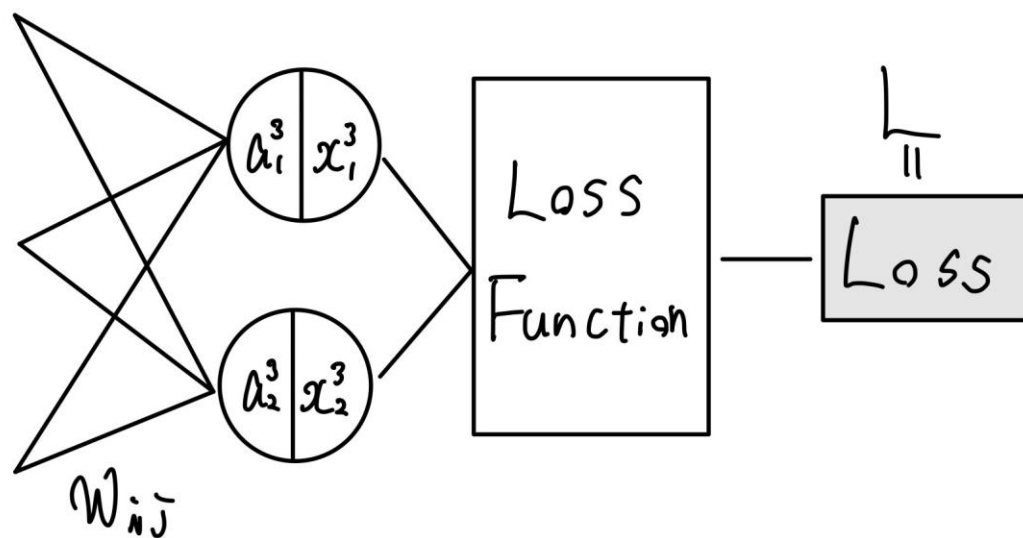


$$\frac{\partial L}{\partial w_{ij}^K} = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \frac{\partial a_j^K}{\partial w_{ij}^K}$$

$x_j^K = h_K(a_j^K)$: h は活性化関数
出力層の活性化関数は
softmax関数

$$x_j^K = \frac{e^{a_j^K}}{\sum_{i=1}^n e^{a_i^K}}$$

誤差逆伝播-出力層の重みの更新



$$\frac{\partial x_j^K}{\partial a_j^K}$$

$$x_j^K = \frac{e^{a_j^K}}{\sum_{i=1}^n e^{a_i^K}}$$

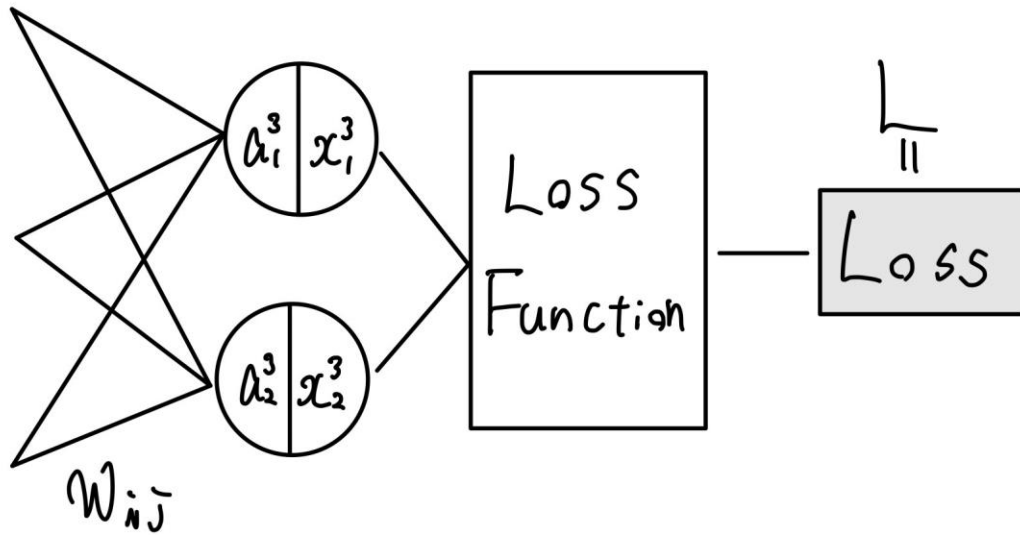
$$\frac{\partial x_j^K}{\partial a_j^K} = \frac{e^{a_j^K} \sum_{i=1}^n e^{a_i^K} - e^{a_j^K} e^{a_j^K}}{(\sum_{i=1}^n e^{a_i^K})^2}$$

$$= e^{a_j^K} \frac{\sum_{i=1}^n e^{a_i^K} - e^{a_j^K}}{(\sum_{i=1}^n e^{a_i^K})^2}$$

$$= \frac{e^{a_j^K}}{\sum_{i=1}^n e^{a_i^K}} \frac{\sum_{i=1}^n e^{a_i^K} - e^{a_j^K}}{\sum_{i=1}^n e^{a_i^K}}$$

$$= x_j^K (1 - x_j^K) \leftarrow \text{データが入れば既知}$$

誤差逆伝播-出力層の重みの更新

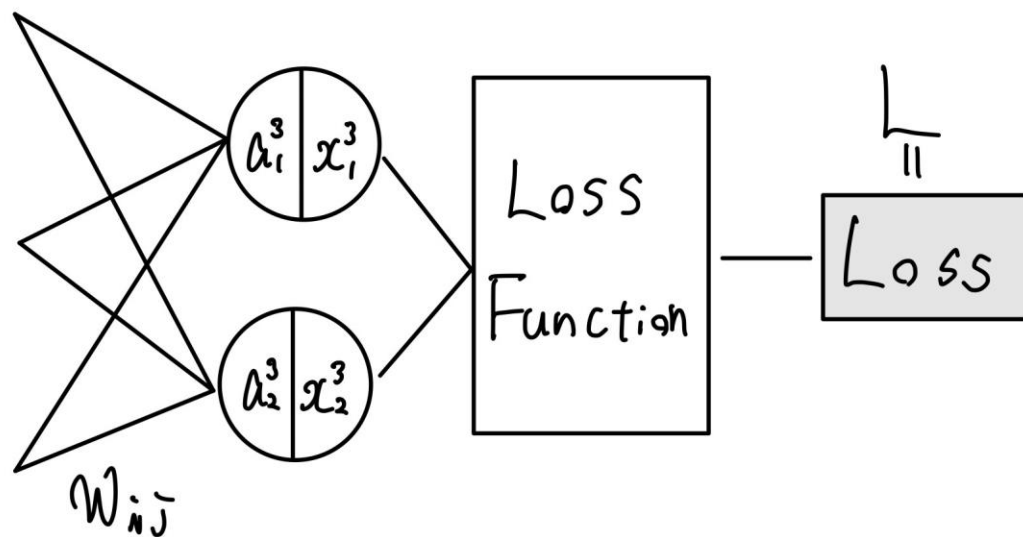


$$\frac{\partial L}{\partial w_{ij}^K} = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \frac{\partial a_j^K}{\partial w_{ij}^K}$$

Loss Functionは今はCross Entropy
(平均二乗誤差MSEやRMSEなど様々)

$$L = - \sum_{i=1}^n t_i \ln x_i^K$$

誤差逆伝播-出力層の重みの更新



$$\frac{\partial L}{\partial x_j^K} \quad L = - \sum_{i=1}^n t_i \ln x_i^K$$

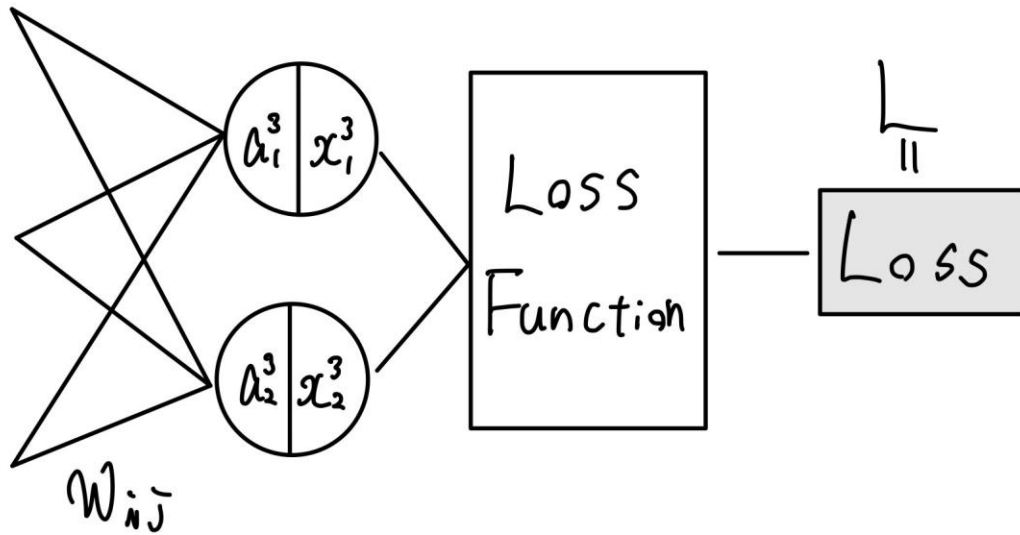
$$\begin{aligned} \frac{\partial L}{\partial x_j^K} &= - \sum_{i=1}^n t_i \frac{\partial}{\partial x_j^K} (\ln x_i^K) \\ &= - \sum_{i=1}^n t_i \frac{\partial}{\partial x_i^K} (\ln x_i^K) \frac{\partial x_i^K}{\partial x_j^K} \\ &= - \sum_{i=1}^n t_i \frac{1}{x_i^K} \frac{\partial x_i^K}{\partial x_j^K} \end{aligned}$$

$$\begin{aligned} (x_1^K + x_2^K + \dots + x_n^K &= 1 \\ \Leftrightarrow x_j^K &= 1 - x_i^K - \dots) \end{aligned}$$

$$\frac{\partial x_i^K}{\partial x_j^K} = \begin{cases} 1 (i = j) \\ -1 (i \neq j) \end{cases}$$

$$= - \frac{t_j}{x_j^K} + \sum_{i=1 \wedge i \neq j}^n \frac{t_i}{x_i^K} \quad \leftarrow \text{データが入れば既知}$$

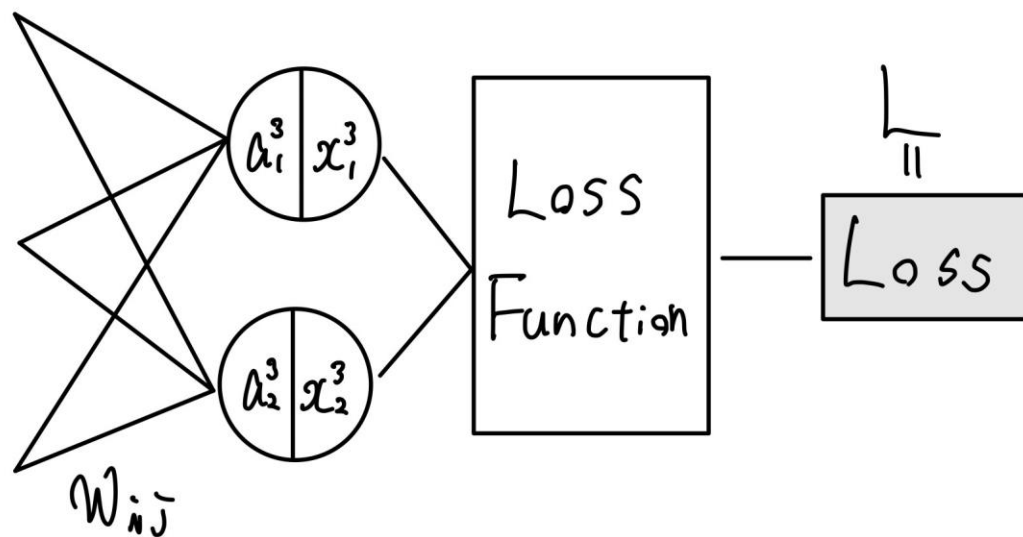
誤差逆伝播-出力層の重みの更新



$$\frac{\partial L}{\partial w_{ij}^K} = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \frac{\partial a_j^K}{\partial w_{ij}^K}$$

$$\frac{\partial L}{\partial w_{ij}^K} = \left(-\frac{t_j}{x_j^K} + \sum_{i=1 \wedge i \neq j}^n \frac{t_i}{x_i^K} \right) x_j^K (1 - x_j^K) x_i^{K-1}$$

誤差逆伝播-演習①出力層のバイアスの更新



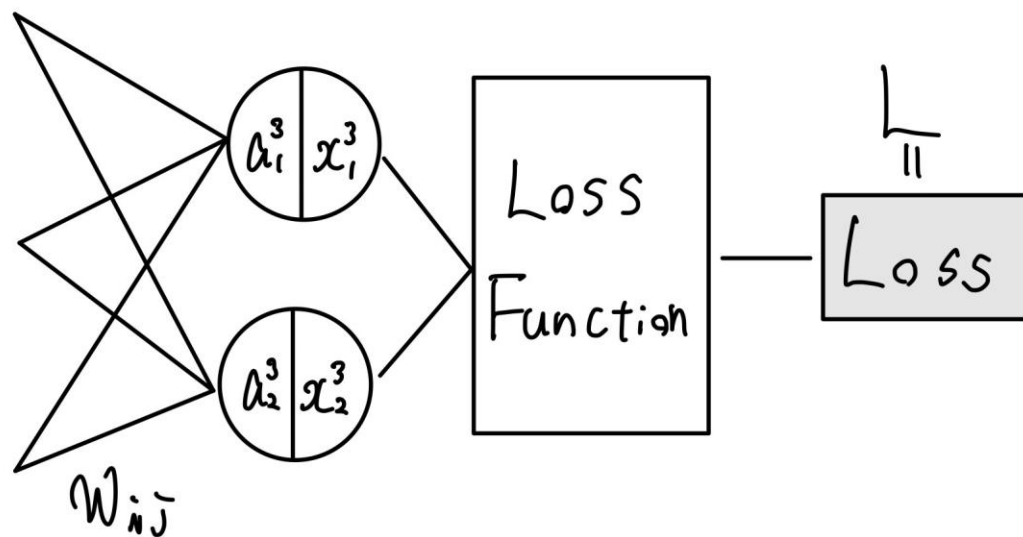
$\frac{\partial L}{\partial b_j^K}$ を求めよ

既知な x, t のみを用いて表すこと

$$\frac{\partial L}{\partial w_{ij}^K} = \left(-\frac{t_j}{x_j^K} + \sum_{i=1 \wedge i \neq j}^n \frac{t_i}{x_i^K} \right) x_j^K (1 - x_j^K) x_i^{K-1}$$



誤差逆伝播-演習①出力層のバイアスの更新

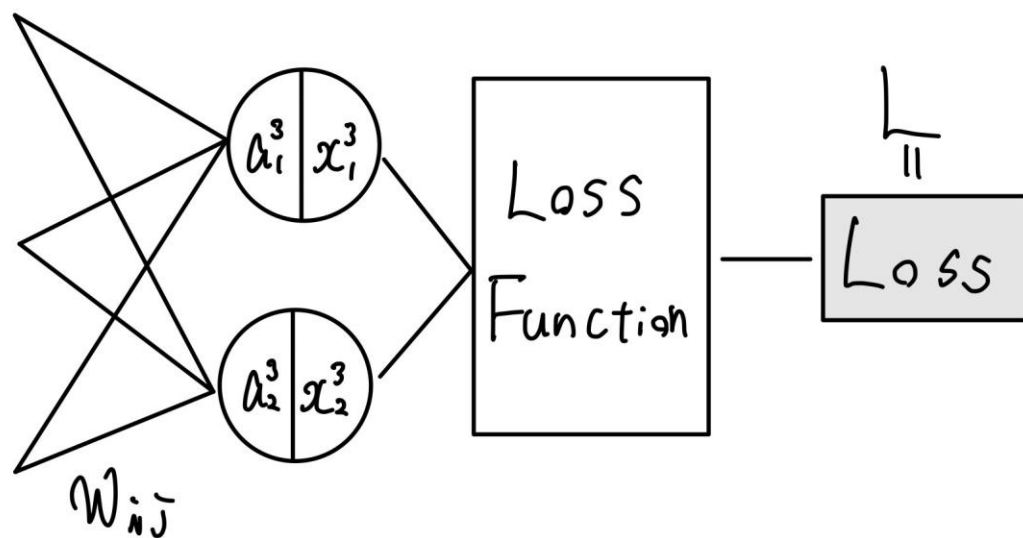


$\frac{\partial L}{\partial b_j^K}$ を求めよ

$$\frac{\partial L}{\partial b_j^K} = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \frac{\partial a_j^K}{\partial b_j^K}$$

$$\frac{\partial L}{\partial b_j^K} = \left(-\frac{t_j}{x_j^K} + \sum_{i=1 \wedge i \neq j}^n \frac{t_i}{x_i^K} \right) x_j^K (1 - x_j^K) 1$$

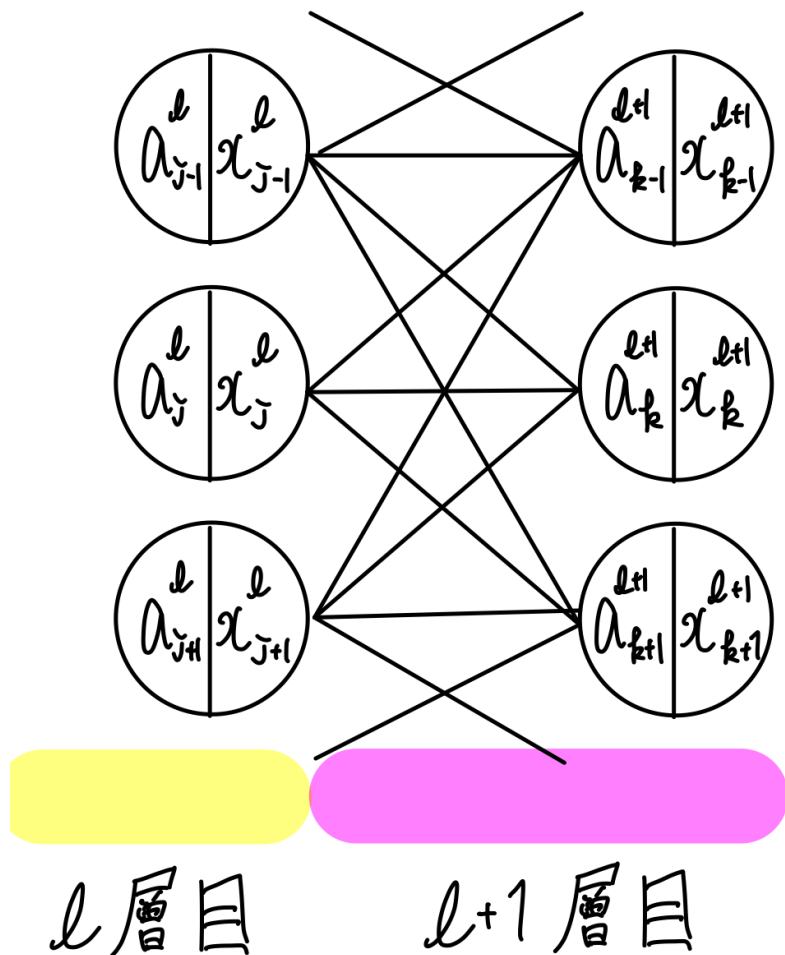
誤差逆伝播-出力層の重みの更新



$$\frac{\partial L}{\partial w_{ij}^K} = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \frac{\partial a_j^K}{\partial w_{ij}^K}$$

$$\delta_j^K := \frac{\partial L}{\partial a_j^K} \quad \text{誤差}$$

誤差逆伝播-中間層の重みの更新

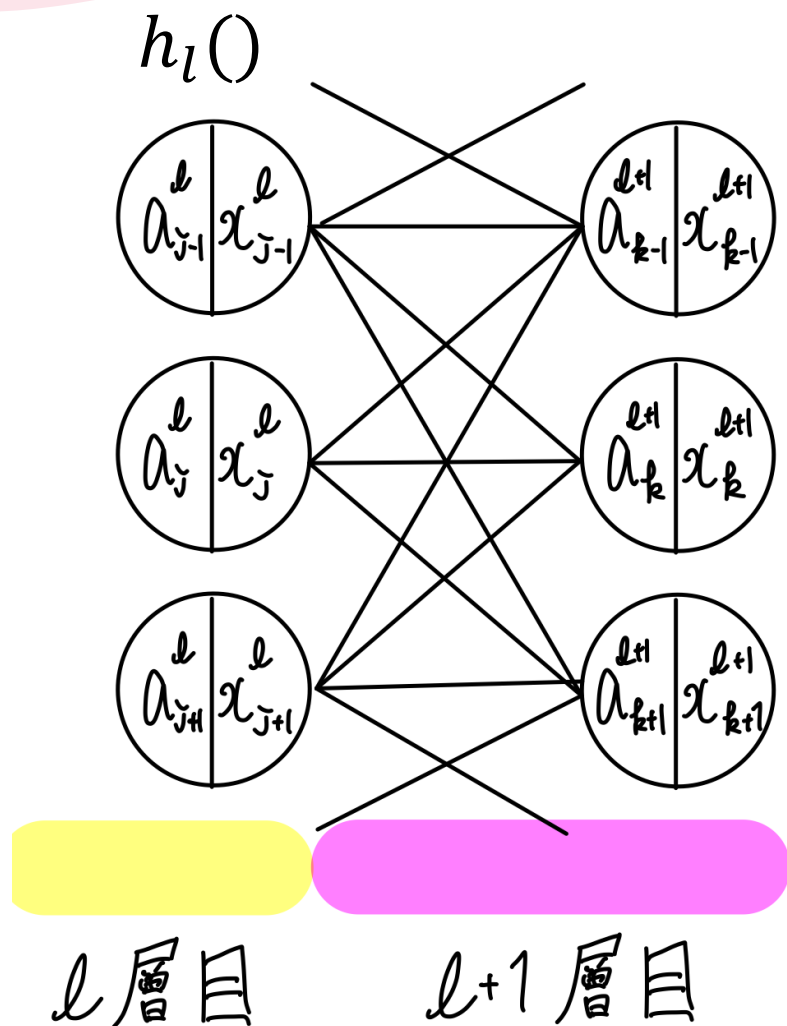


$$\frac{\partial L}{\partial w_{ij}^l} = \delta_j^l \frac{\partial a_j^l}{\partial w_{ij}^l}$$

$$a_j^l = w_{1j}^l x_1^{l-1} + \dots + w_{ij}^l x_i^{l-1} + \dots$$

$$\frac{\partial a_j^l}{\partial w_{ij}^l} = x_i^{l-1}$$

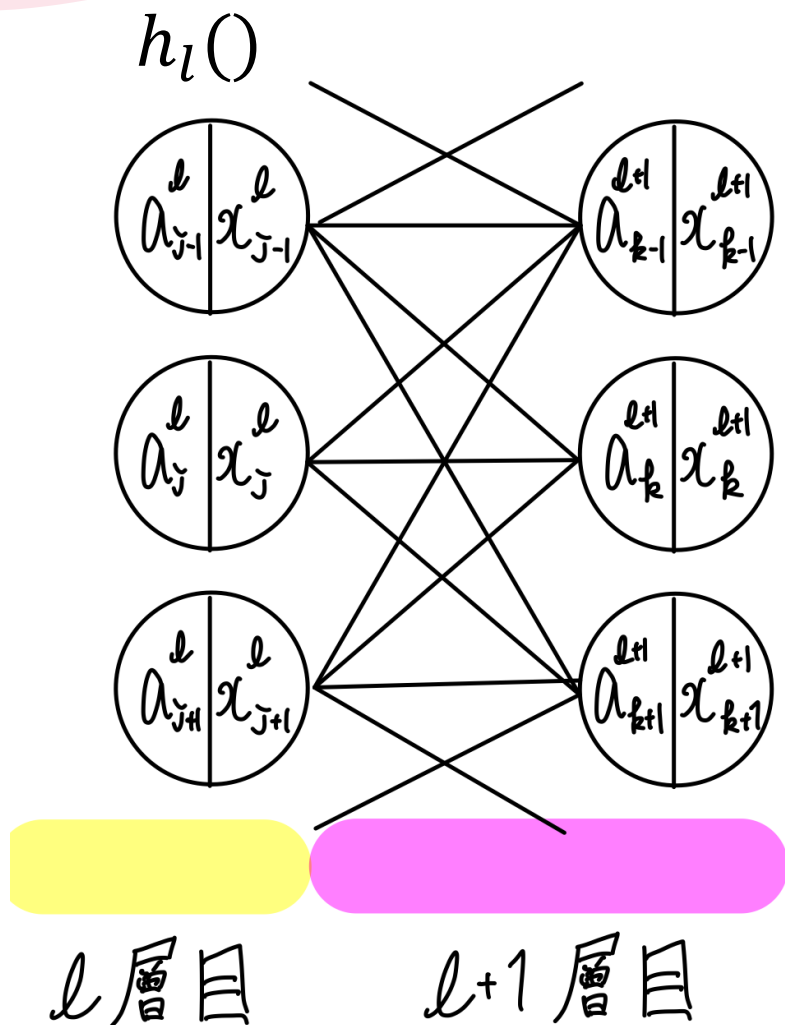
誤差逆伝播-中間層の重みの更新



$$\frac{\partial L}{\partial w_{ij}^l} = \delta_j^l \frac{\partial a_j^l}{\partial w_{ij}^l}$$

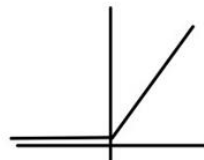
$$\begin{aligned} \delta_j^l &= \frac{\partial L}{\partial a_j^l} = \sum_{k=1}^n \frac{\partial L}{\partial a_k^{l+1}} \frac{\partial a_k^{l+1}}{\partial a_j^l} \\ &= \sum_{k=1}^n \frac{\partial L}{\partial a_k^{l+1}} \frac{\partial a_k^{l+1}}{\partial x_j^l} \frac{\partial x_j^l}{\partial a_j^l} \\ &= \sum_{k=1}^n \delta_k^{l+1} \frac{\partial a_k^{l+1}}{\partial x_j^l} \frac{\partial x_j^l}{\partial a_j^l} \end{aligned}$$

誤差逆伝播-中間層の重みの更新



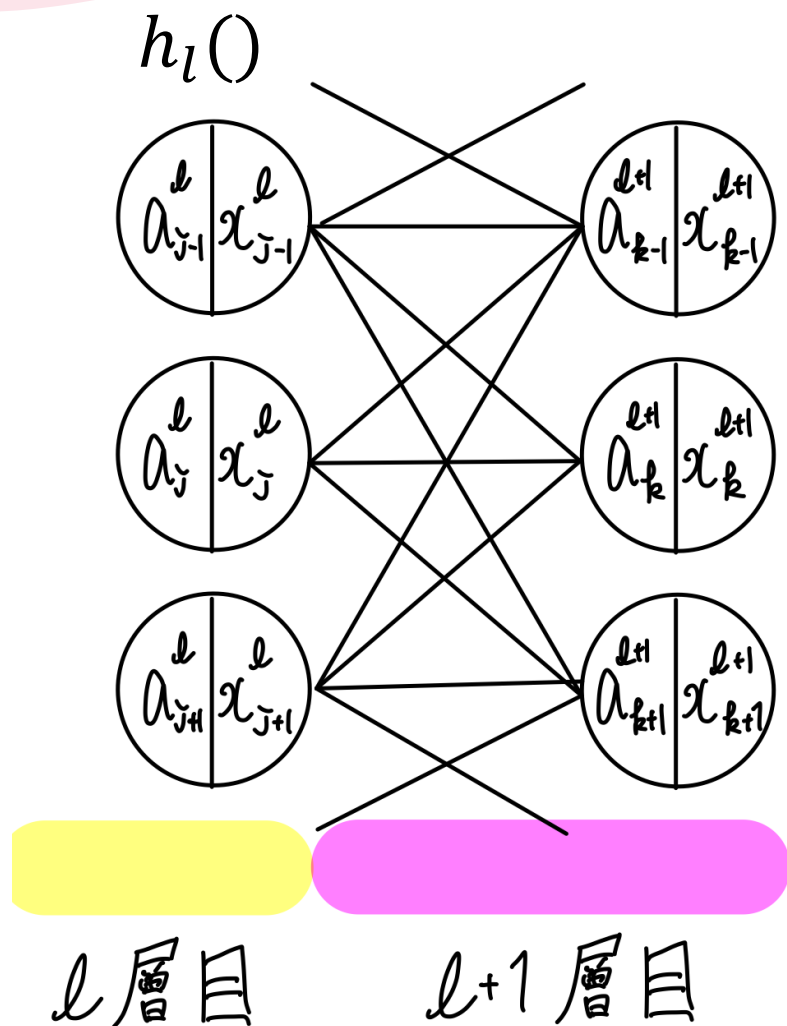
$$\frac{\partial L}{\partial w_{ij}^l} = \delta_j^l \frac{\partial a_j^l}{\partial w_{ij}^l}$$

$$\begin{aligned} \delta_j^l &= \frac{\partial L}{\partial a_j^l} = \sum_{k=1}^n \frac{\partial L}{\partial a_k^{l+1}} \frac{\partial a_k^{l+1}}{\partial x_j^l} \frac{\partial x_j^l}{\partial a_j^l} \\ &= \sum_{k=1}^n \delta_k^{l+1} \frac{\partial a_k^{l+1}}{\partial x_j^l} h_l'(a_j^l) \end{aligned}$$

ReLU関数 $f(x) = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}$ 

主にこれを使っていく

誤差逆伝播-中間層の重みの更新

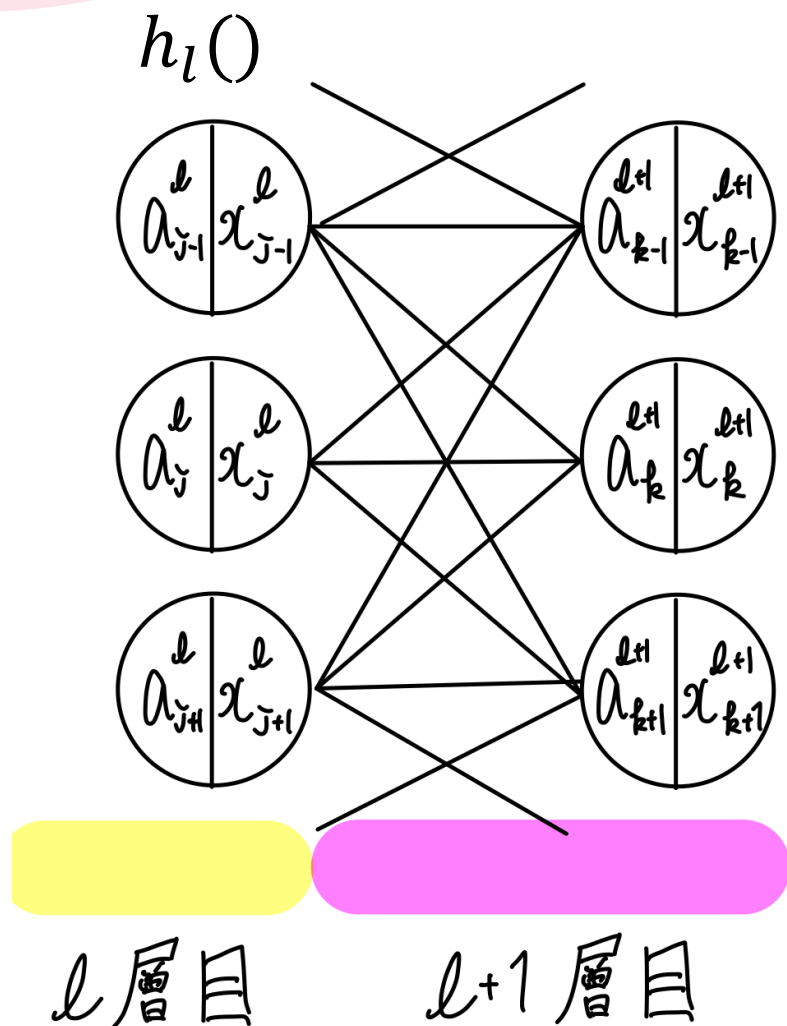


$$\frac{\partial L}{\partial w_{ij}^l} = \delta_j^l \frac{\partial a_j^l}{\partial w_{ij}^l}$$

$$\begin{aligned} \delta_j^l &= \frac{\partial L}{\partial a_j^l} = \sum_{k=1}^n \frac{\partial L}{\partial a_k^{l+1}} \frac{\partial a_k^{l+1}}{\partial x_j^l} \frac{\partial x_j^l}{\partial a_j^l} \\ &= \sum_{k=1}^n \delta_k^{l+1} w_{jk}^{l+1} h_l'(a_j^l) \end{aligned}$$

更新前の重み

誤差逆伝播-中間層の重みの更新



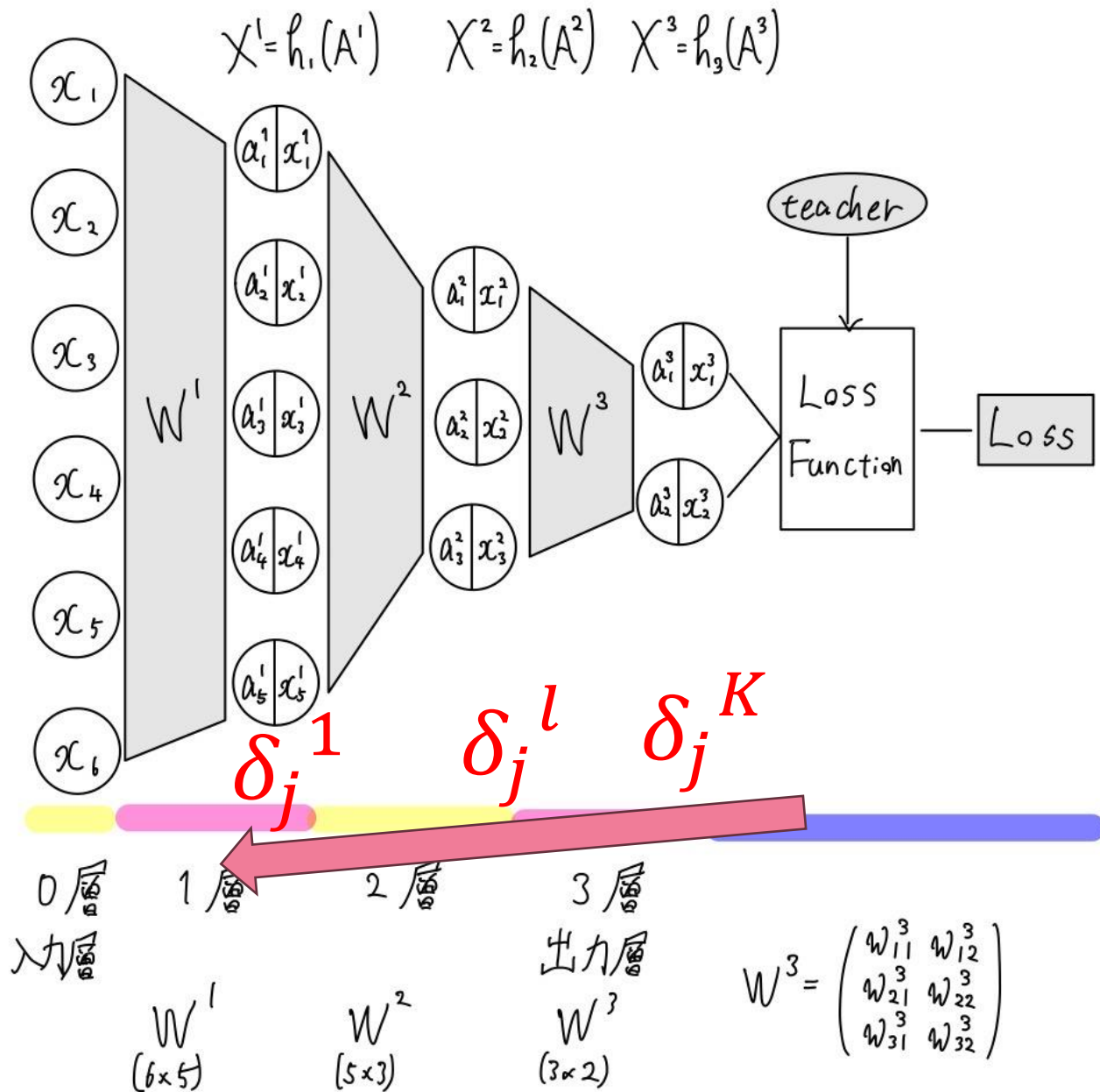
$$\frac{\partial L}{\partial w_{ij}^l} = \delta_j^l \frac{\partial a_j^l}{\partial w_{ij}^l}$$

$$\delta_j^l = \sum_{k=1}^n \delta_k^{l+1} w_{jk}^{l+1} h_l'(a_j^l)$$

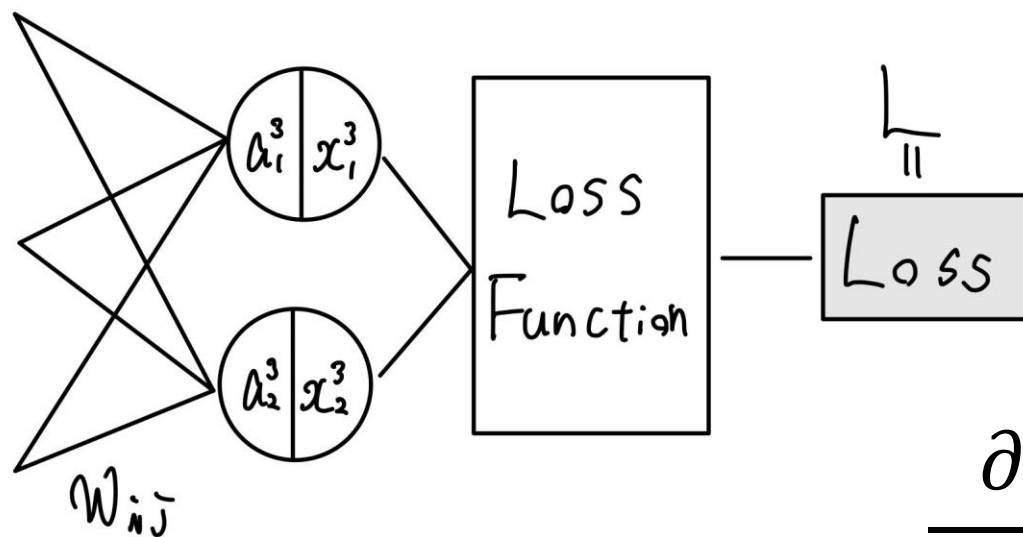
これって誤差の漸化式
後ろの層の誤差がわかればよい

誤差逆伝播

誤差 δ_j^l が
逆方向に
伝播してる



誤差逆伝播-演習②中間層のバイアスの更新



$\frac{\partial L}{\partial b_j^l}$ を求めよ

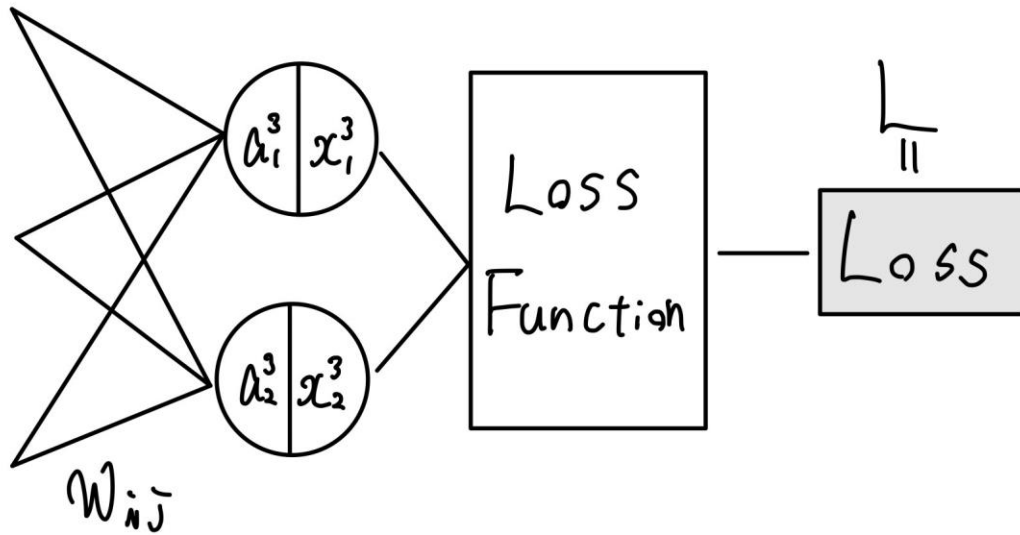
既知な a, x, w, δ^{l+1} などを用いて表すこと

$$\frac{\partial L}{\partial w_{ij}^l} = \delta_j^l x_i^{l-1}$$

$$\delta_j^l = \sum_{k=1}^n \delta_k^{l+1} w_{jk}^{l+1} h_l'(a_j^l)$$



誤差逆伝播-演習②中間層のバイアスの更新

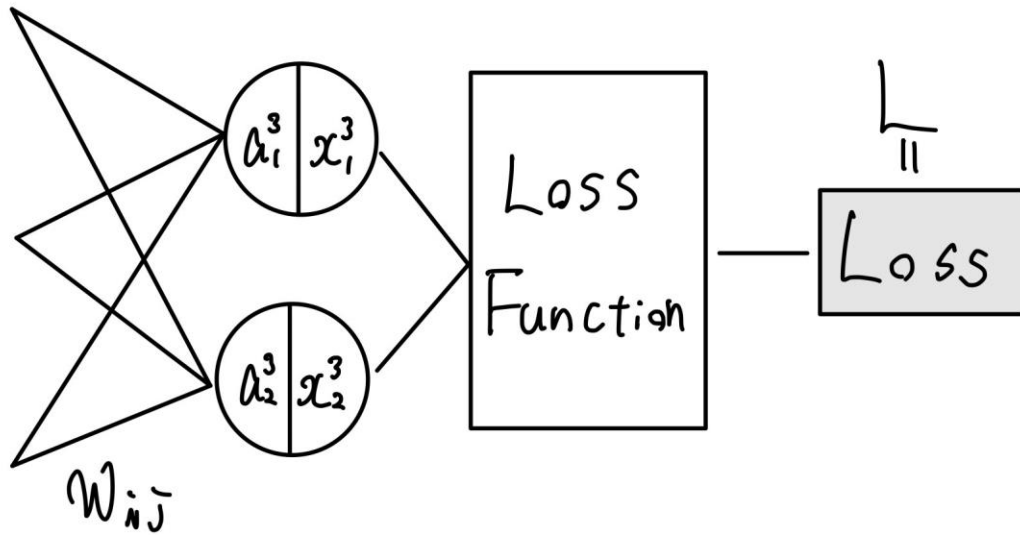


$\frac{\partial L}{\partial b_j^l}$ を求めよ

既知な a, x, w, δ^{l+1} のみを用いて表すこと

$$\frac{\partial L}{\partial b_j^l} = \delta_j^l \frac{\partial a_j^l}{\partial b_j^l}$$

誤差逆伝播-演習②中間層のバイアスの更新



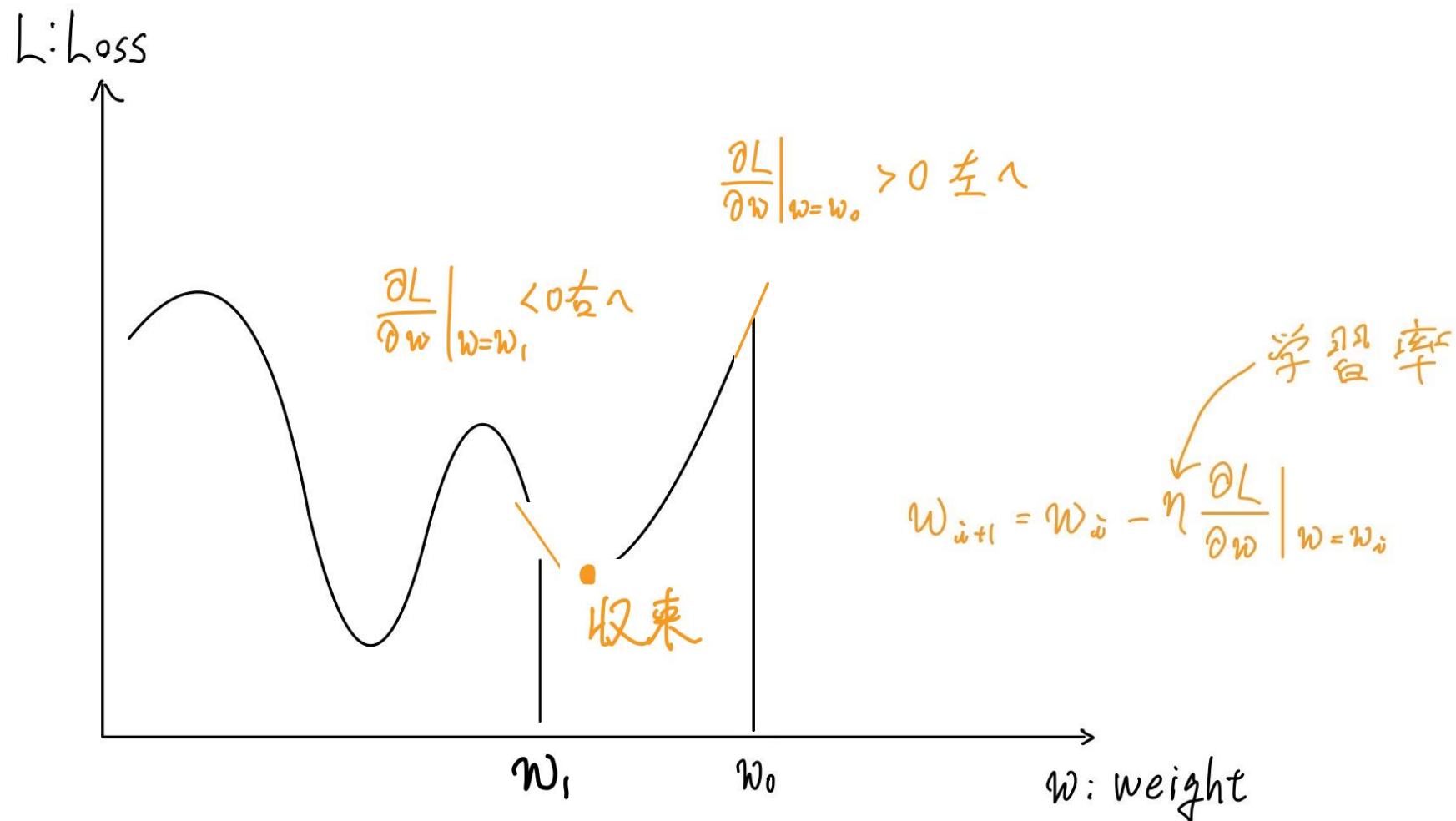
$\frac{\partial L}{\partial b_j^l}$ を求めよ

既知な a, x, w, δ^{l+1} のみを用いて表すこと

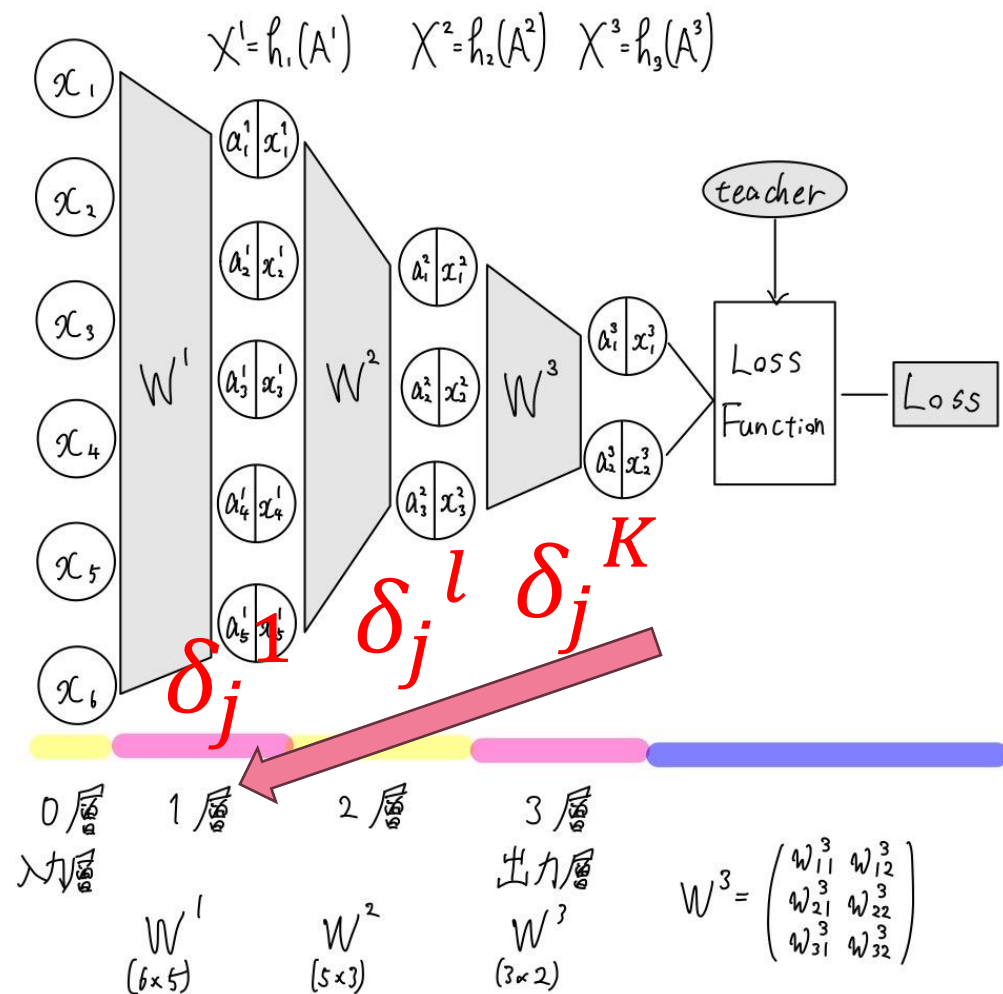
$$\frac{\partial L}{\partial b_j^l} = \delta_j^l \mathbf{1}$$

$$\delta_j^l = \sum_{k=1}^n \delta_k^{l+1} w_{jk}^{l+1} h_l'(a_j^l)$$

ニューラルネットワーク-勾配法 **再掲**

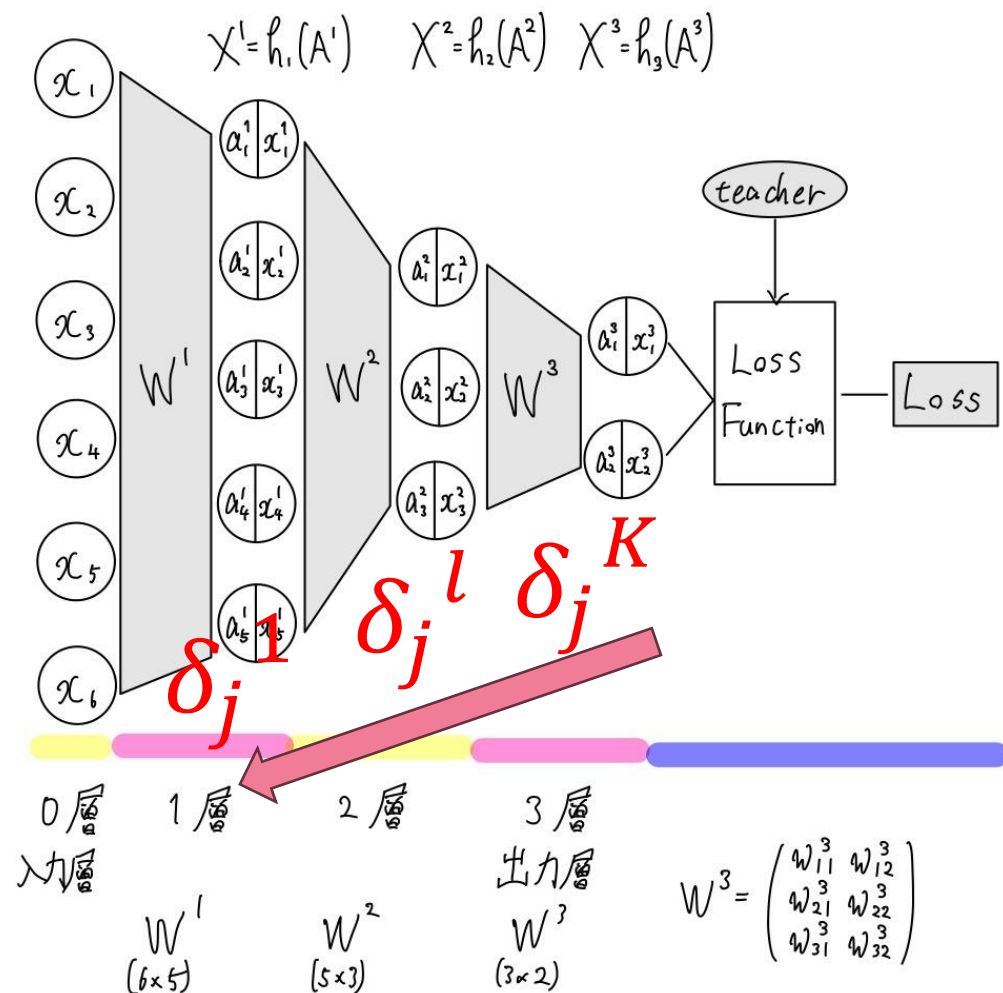


誤差逆伝播-中間層の重みの更新



$$w_{ij}^l = w_{ij}^l - \eta \frac{\partial L}{\partial w_{ij}^l}$$

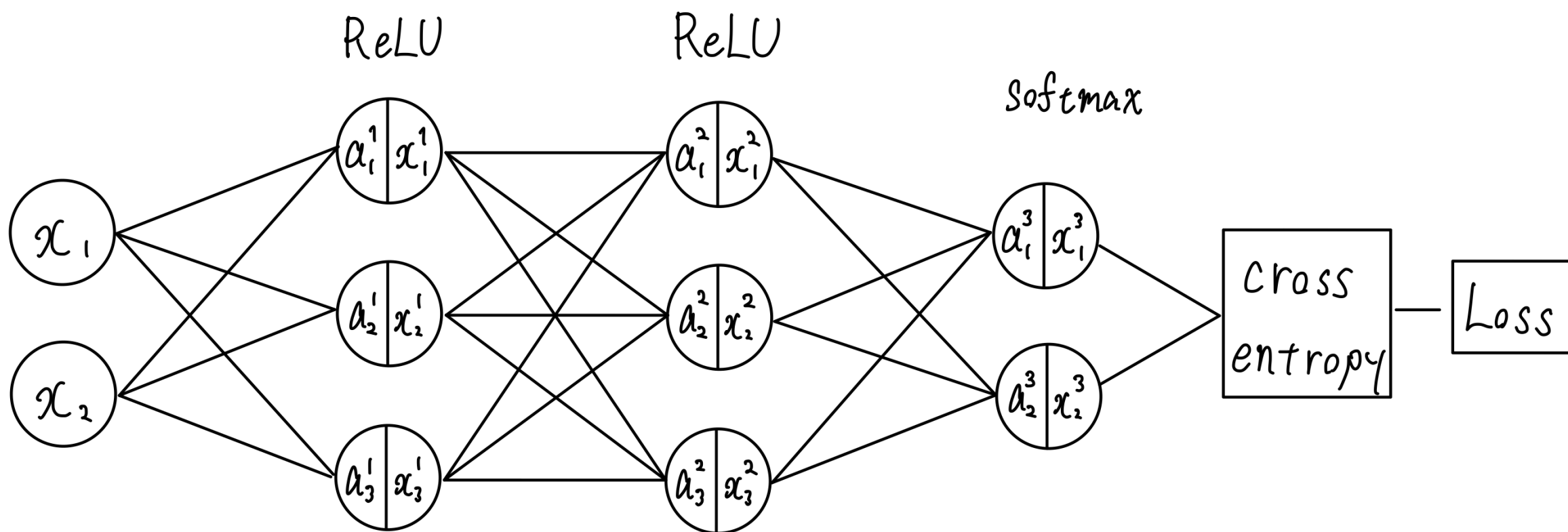
誤差逆伝播-中間層の重みの更新



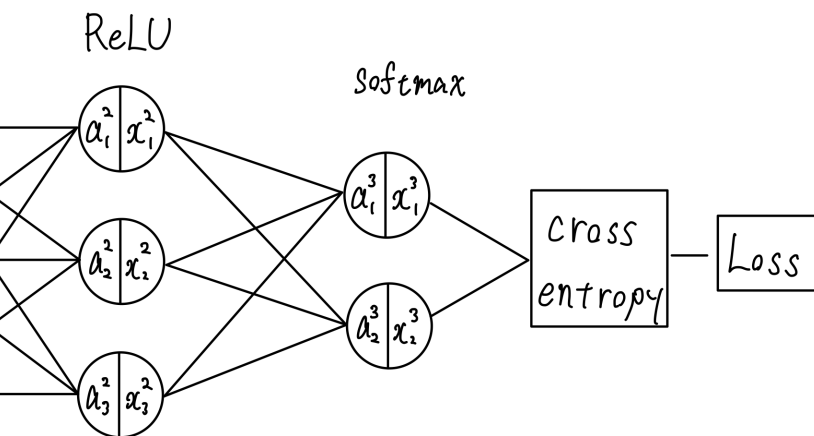
$$w_{ij}^l = w_{ij}^l - \eta \frac{\partial L}{\partial w_{ij}^l}$$

ニューラルネットワーク
の神髄を完全に理解した

誤差逆伝播-パラメータの更新 行列表現



誤差逆伝播-行列表現 出力層 重み

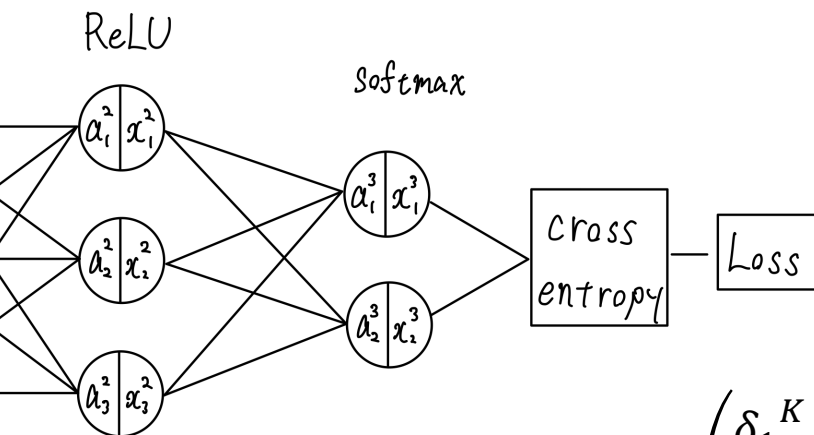


$$\frac{\partial L}{\partial w_{ij}^K} = \delta_j^K \frac{\partial a_j^K}{\partial w_{ij}^K} = \delta_j^K x_i^{K-1}$$

$$W^K = \begin{pmatrix} w_{11}^K & w_{12}^K \\ w_{21}^K & w_{22}^K \\ w_{31}^K & w_{32}^K \end{pmatrix}$$

$$\frac{\partial L}{\partial W^K} = \begin{pmatrix} \frac{\partial L}{\partial w_{11}^K} & \frac{\partial L}{\partial w_{12}^K} \\ \frac{\partial L}{\partial w_{21}^K} & \frac{\partial L}{\partial w_{22}^K} \\ \frac{\partial L}{\partial w_{31}^K} & \frac{\partial L}{\partial w_{32}^K} \end{pmatrix}$$

誤差逆伝播-行列表現 出力層 重み

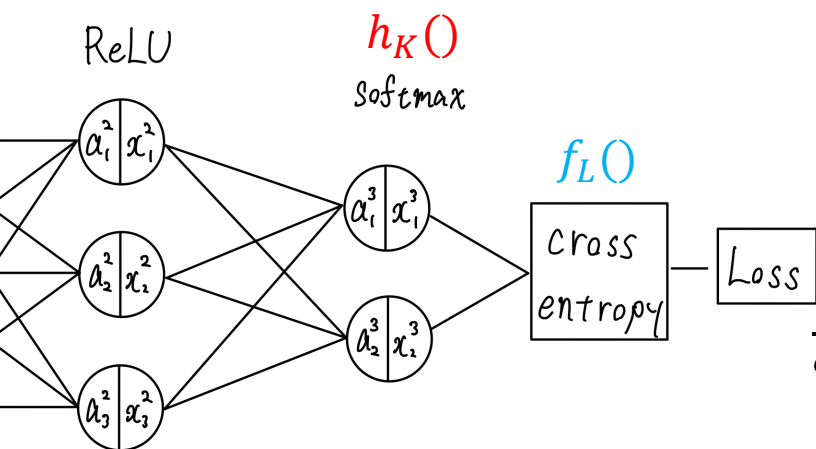


$$\frac{\partial L}{\partial w_{ij}^K} = \delta_j^K \frac{\partial a_j^K}{\partial w_{ij}^K} = \delta_j^K x_i^{K-1}$$

$$\begin{aligned} \frac{\partial L}{\partial W^K} &= \begin{pmatrix} \delta_1^K \frac{\partial a_1^K}{\partial w_{11}^K} & \delta_2^K \frac{\partial a_2^K}{\partial w_{12}^K} \\ \delta_1^K \frac{\partial a_1^K}{\partial w_{21}^K} & \delta_2^K \frac{\partial a_2^K}{\partial w_{22}^K} \\ \delta_1^K \frac{\partial a_1^K}{\partial w_{31}^K} & \delta_2^K \frac{\partial a_2^K}{\partial w_{32}^K} \end{pmatrix} = \begin{pmatrix} \delta_1^K x_1^{K-1} & \delta_2^K x_1^{K-1} \\ \delta_1^K x_2^{K-1} & \delta_2^K x_2^{K-1} \\ \delta_1^K x_3^{K-1} & \delta_2^K x_3^{K-1} \end{pmatrix} = \begin{pmatrix} x_1^{K-1} \\ x_2^{K-1} \\ x_3^{K-1} \end{pmatrix} (\delta_1^K \quad \delta_2^K) \\ &= X^{2T} \Delta^3 \end{aligned}$$

$$\Delta^3 = (\delta_1^K \quad \delta_2^K) = \left(\frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} \right)$$

誤差逆伝播-行列表現 出力層 重み



$$\frac{\partial L}{\partial w_{ij}^K} = \delta_j^K \frac{\partial a_j^K}{\partial w_{ij}^K} = \delta_j^K x_i^{K-1}$$

$$\frac{\partial L}{\partial W^K} = \begin{pmatrix} \delta_1^K \frac{\partial a_1^K}{\partial w_{11}^K} & \delta_2^K \frac{\partial a_2^K}{\partial w_{12}^K} \\ \delta_1^K \frac{\partial a_1^K}{\partial w_{21}^K} & \delta_2^K \frac{\partial a_2^K}{\partial w_{22}^K} \\ \delta_1^K \frac{\partial a_1^K}{\partial w_{31}^K} & \delta_2^K \frac{\partial a_2^K}{\partial w_{32}^K} \end{pmatrix} = \begin{pmatrix} \delta_1^K x_1^{K-1} & \delta_2^K x_1^{K-1} \\ \delta_1^K x_2^{K-1} & \delta_2^K x_2^{K-1} \\ \delta_1^K x_3^{K-1} & \delta_2^K x_3^{K-1} \end{pmatrix} = \begin{pmatrix} x_1^{K-1} \\ x_2^{K-1} \\ x_3^{K-1} \end{pmatrix} (\delta_1^K \quad \delta_2^K)$$

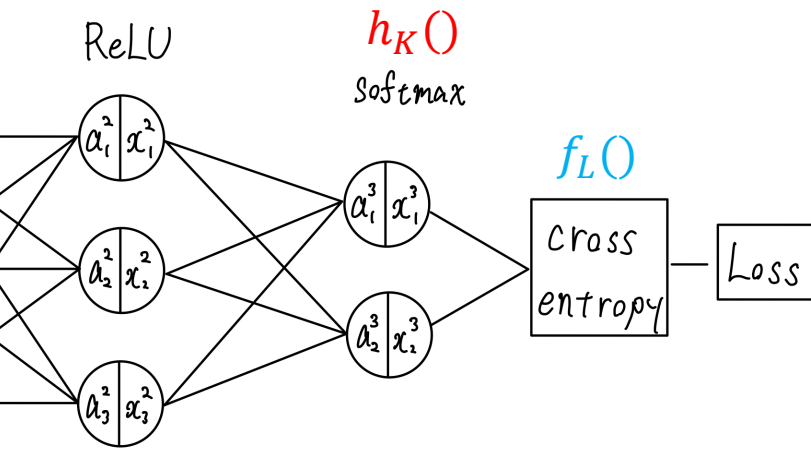
$$= X^{2T} \Delta^3$$

$$\delta_j^K = \frac{\partial L}{\partial x_j^K} \frac{\partial x_j^K}{\partial a_j^K} = \frac{\partial f_L}{\partial x_j^K} \frac{\partial h_K}{\partial a_j^K}$$

$$\begin{aligned} \Delta^3 &= (\delta_1^K \quad \delta_2^K) = \left(\frac{\partial f_L}{\partial x_1^K} \frac{\partial h_K}{\partial a_1^K} \quad \frac{\partial f_L}{\partial x_2^K} \frac{\partial h_K}{\partial a_2^K} \right) \\ &= \left(\frac{\partial f_L}{\partial x_1^K} \quad \frac{\partial f_L}{\partial x_2^K} \right) \odot \left(\frac{\partial h_K}{\partial a_1^K} \quad \frac{\partial h_K}{\partial a_2^K} \right) \\ &= \frac{\partial f_L}{\partial X^K} \odot \frac{\partial h_K}{\partial A^K} \end{aligned}$$

\odot : アダマール積
サイズが等しい行列の各要素の積

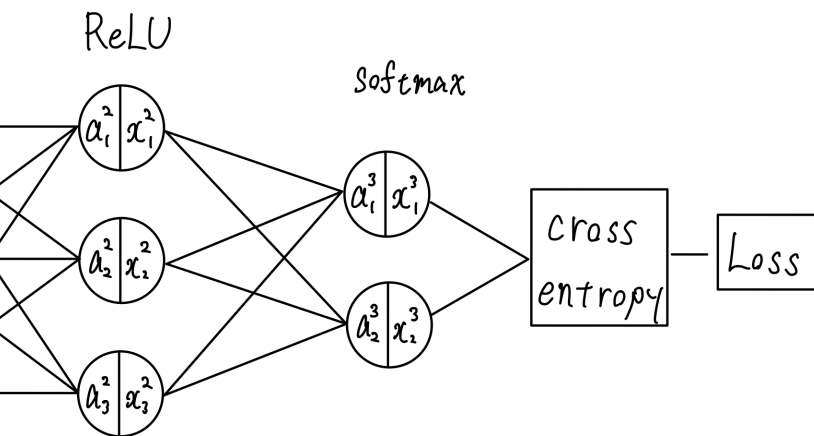
誤差逆伝播-行列表現 出力層 重み 公式まとめ



$$\frac{\partial L}{\partial W^K} = X^{(K-1)T} \Delta^K$$

$$\Delta^K = \frac{\partial f_L}{\partial X^K} \odot \frac{\partial h_K}{\partial A^K}$$

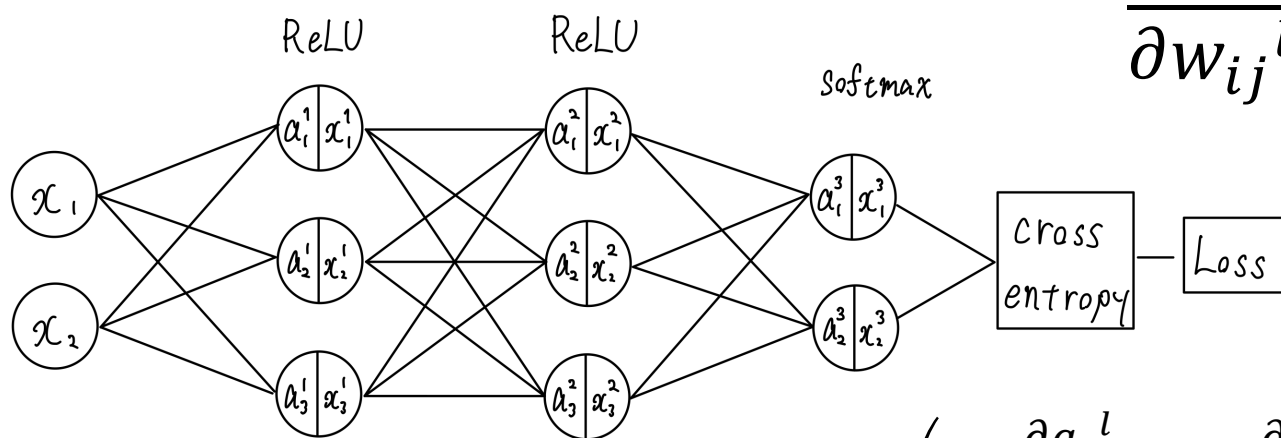
誤差逆伝播-行列表現 出力層 バイアス 公式



$$\frac{\partial L}{\partial b_j^K} = \delta_j^K \frac{\partial a_j^K}{\partial b_j^K} = \delta_j^K$$

$$\frac{\partial L}{\partial B^K} = \Delta^K$$

誤差逆伝播-行列表現 中間層 重み

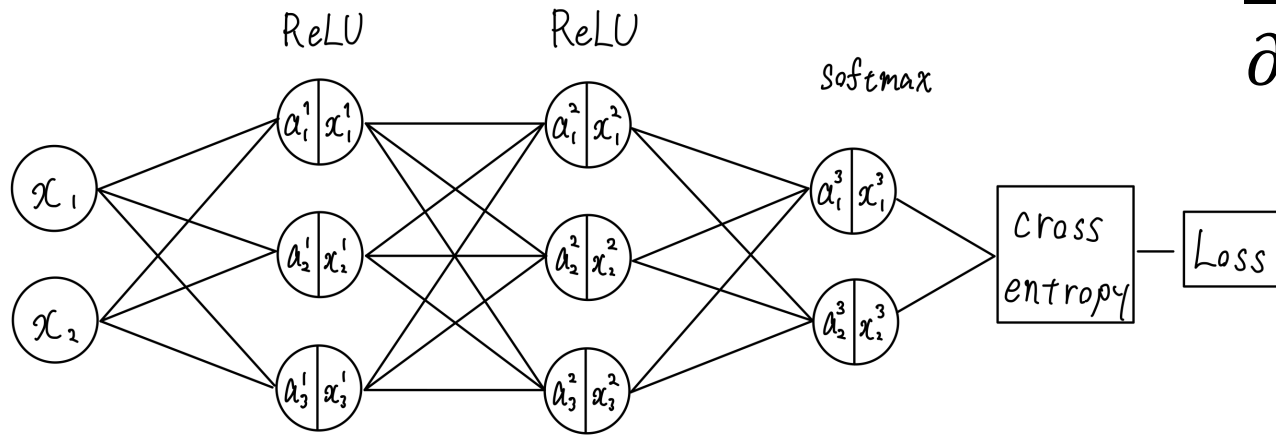


$$\frac{\partial L}{\partial w_{ij}^l} = \delta_j^l \frac{\partial a_j^l}{\partial w_{ij}^l} = \delta_j^l x_i^{l-1}$$

$$\delta_j^l = \sum_{k=1}^n \delta_k^{l+1} w_{jk}^{l+1} h'_l(a_j^l)$$

$$\begin{aligned} \frac{\partial L}{\partial W^{l=2}} &= \begin{pmatrix} \delta_1^l \frac{\partial a_1^l}{\partial w_{11}^l} & \delta_2^l \frac{\partial a_2^l}{\partial w_{12}^l} & \delta_3^l \frac{\partial a_3^l}{\partial w_{13}^l} \\ \delta_1^l \frac{\partial a_1^l}{\partial w_{21}^l} & \delta_2^l \frac{\partial a_2^l}{\partial w_{22}^l} & \delta_3^l \frac{\partial a_3^l}{\partial w_{23}^l} \\ \delta_1^l \frac{\partial a_1^l}{\partial w_{31}^l} & \delta_2^l \frac{\partial a_2^l}{\partial w_{32}^l} & \delta_3^l \frac{\partial a_3^l}{\partial w_{33}^l} \end{pmatrix} = \begin{pmatrix} \delta_1^l x_1^{l-1} & \delta_2^l x_1^{l-1} & \delta_3^l x_1^{l-1} \\ \delta_1^l x_2^{l-1} & \delta_2^l x_2^{l-1} & \delta_3^l x_2^{l-1} \\ \delta_1^l x_3^{l-1} & \delta_2^l x_3^{l-1} & \delta_3^l x_3^{l-1} \end{pmatrix} \\ &= \begin{pmatrix} x_1^{l-1} \\ x_2^{l-1} \\ x_3^{l-1} \end{pmatrix} (\delta_1^l \quad \delta_2^l \quad \delta_3^l) = X^{(l-1)T} \Delta^l \end{aligned}$$

誤差逆伝播-行列表現 中間層 バイアス

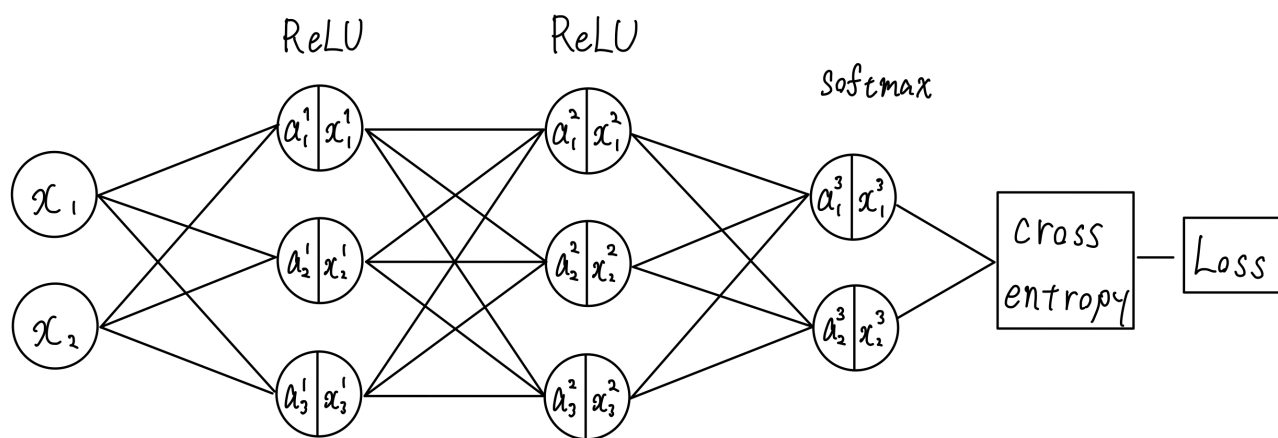


$$\frac{\partial L}{\partial b_j^l} = \delta_j^l \frac{\partial a_j^l}{\partial b_j^l} = \delta_j^l$$

$$\delta_j^l = \sum_{k=1}^n \delta_k^{l+1} w_{jk}^{l+1} h_k'(a_j^l)$$

$$\frac{\partial L}{\partial B^{l=2}} = \mathbf{X}^{(l-1)T} \Delta^l = \Delta^l$$

誤差逆伝播-行列表現 中間層 誤差



$$\delta_j^l = \sum_{k=1}^n \frac{\partial L}{\partial a_k^{l+1}} \frac{\partial a_k^{l+1}}{\partial x_j^l} \frac{\partial x_j^l}{\partial a_j^l}$$

$$= \sum_{k=1}^n \delta_k^{l+1} w_{jk}^{l+1} h_l'(a_j^l)$$

$$\Delta^l = (\delta_1^l \quad \delta_2^l \quad \delta_3^l) = ?$$

誤差逆伝播-行列表現 中間層 誤差

$$\begin{aligned}\Delta^l &= (\delta_1^l \quad \delta_2^l \quad \delta_3^l) \\ &= \left(\sum_{k=1}^{n=2} \delta_k^{l+1} w_{1k}^{l+1} h_l'(a_1^l) \quad \sum_{k=1}^n \delta_k^{l+1} w_{2k}^{l+1} h_l'(a_2^l) \quad \sum_{k=1}^n \delta_k^{l+1} w_{3k}^{l+1} h_l'(a_3^l) \right)\end{aligned}$$

$$\Delta^{l+1} = (\delta_1^{l+1} \quad \delta_2^{l+1}) \quad W^{l+1} = \begin{pmatrix} w_{11}^{l+1} & w_{12}^{l+1} \\ w_{21}^{l+1} & w_{22}^{l+1} \\ w_{31}^{l+1} & w_{32}^{l+1} \end{pmatrix} \quad \frac{\partial h_l}{\partial A^l} = \begin{pmatrix} \frac{\partial h_l}{\partial a_1^K} & \frac{\partial h_l}{\partial a_2^K} & \frac{\partial h_l}{\partial a_3^K} \end{pmatrix}$$

を使ってどう表せるか

誤差逆傳播-行列表現 中間層 誤差

$$\Delta^l = (\delta_1^l \quad \delta_2^l \quad \delta_3^l)$$

$$= \left(\sum_{k=1}^{n=2} \delta_k^{l+1} w_{1k}^{l+1} h_l'(a_1^l) \quad \sum_{k=1}^n \delta_k^{l+1} w_{2k}^{l+1} h_l'(a_2^l) \quad \sum_{k=1}^n \delta_k^{l+1} w_{3k}^{l+1} h_l'(a_3^l) \right)$$

$$\Delta^{l+1} = (\delta_1^{l+1} \quad \delta_2^{l+1}) \quad W^{l+1} = \begin{pmatrix} w_{11}^{l+1} & w_{12}^{l+1} \\ w_{21}^{l+1} & w_{22}^{l+1} \\ w_{31}^{l+1} & w_{32}^{l+1} \end{pmatrix} \quad \frac{\partial h_l}{\partial A^l} = \begin{pmatrix} \frac{\partial h_l}{\partial a_1^l} & \frac{\partial h_l}{\partial a_2^l} & \frac{\partial h_l}{\partial a_3^l} \end{pmatrix}$$

$$\Delta^l = \Delta^{l+1} W^{(l+1)T} \odot \frac{\partial h_l}{\partial A^l}$$

$$= (\delta_1^{l+1} \quad \delta_2^{l+1}) \begin{pmatrix} w_{11}^{l+1} & w_{21}^{l+1} & w_{31}^{l+1} \\ w_{12}^{l+1} & w_{22}^{l+1} & w_{32}^{l+1} \end{pmatrix} \odot \begin{pmatrix} \frac{\partial h_l}{\partial a_1^l} & \frac{\partial h_l}{\partial a_2^l} & \frac{\partial h_l}{\partial a_3^l} \end{pmatrix}$$

誤差逆伝播-公式 まとめ 行列表現

出力層

$$\frac{\partial L}{\partial W^K} = X^{(K-1)T} \Delta^K$$

$$\frac{\partial L}{\partial B^K} = \Delta^K$$

$$\Delta^K = \frac{\partial f_L}{\partial X^K} \odot \frac{\partial h_K}{\partial A^K}$$

中間層

$$\frac{\partial L}{\partial W^l} = X^{(l-1)T} \Delta^l$$

$$\frac{\partial L}{\partial B^l} = \Delta^l$$

$$\Delta^l = \Delta^{l+1} W^{(l+1)T} \odot \frac{\partial h_l}{\partial A^l}$$

誤差逆伝播-公式 まとめ 誤差/活性化関数の微分

誤差関数(cross-entropy)

$$\frac{\partial f_L}{\partial x_j^K} = \left(-\frac{t_j}{x_j^K} + \sum_{i=1 \wedge i \neq j}^n \frac{t_i}{x_i^K} \right)$$

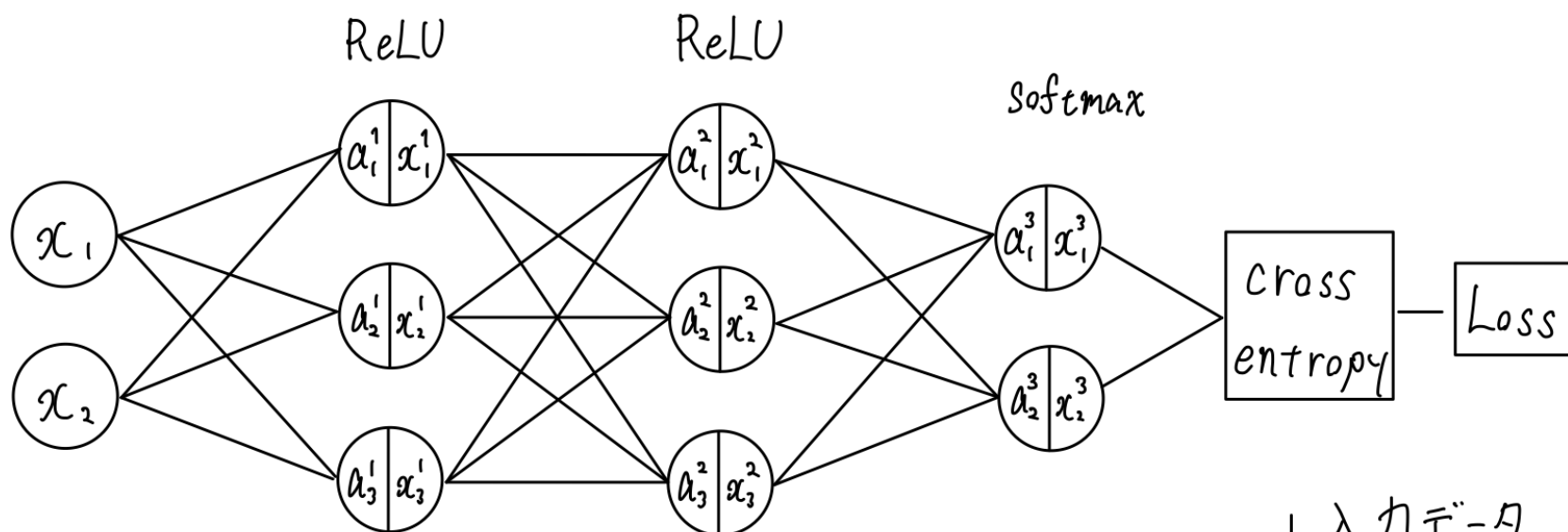
活性化関数(ReLU)

$$\frac{\partial h_l}{\partial a_j^l} = \begin{cases} 1(a_j^l > 0) \\ 0(a_j^l \leq 0) \end{cases}$$

活性化関数(soft-max)

$$\frac{\partial h_K}{\partial a_j^K} = x_j^K (1 - x_j^K)$$

誤差逆伝播-演習③手計算でNNを学習せよ



左のNNを学習せよ

=すべてのパラメータ
を1回更新せよ

Lossはいくら
改善するか？

$$W^1 = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & -3 \end{pmatrix}, W^2 = \begin{pmatrix} -1 & 2 & -3 \\ 1 & -2 & 3 \\ -1 & 2 & -3 \end{pmatrix}, W^3 = \begin{pmatrix} -1 & 2 \\ -3 & 1 \\ -2 & 3 \end{pmatrix}$$

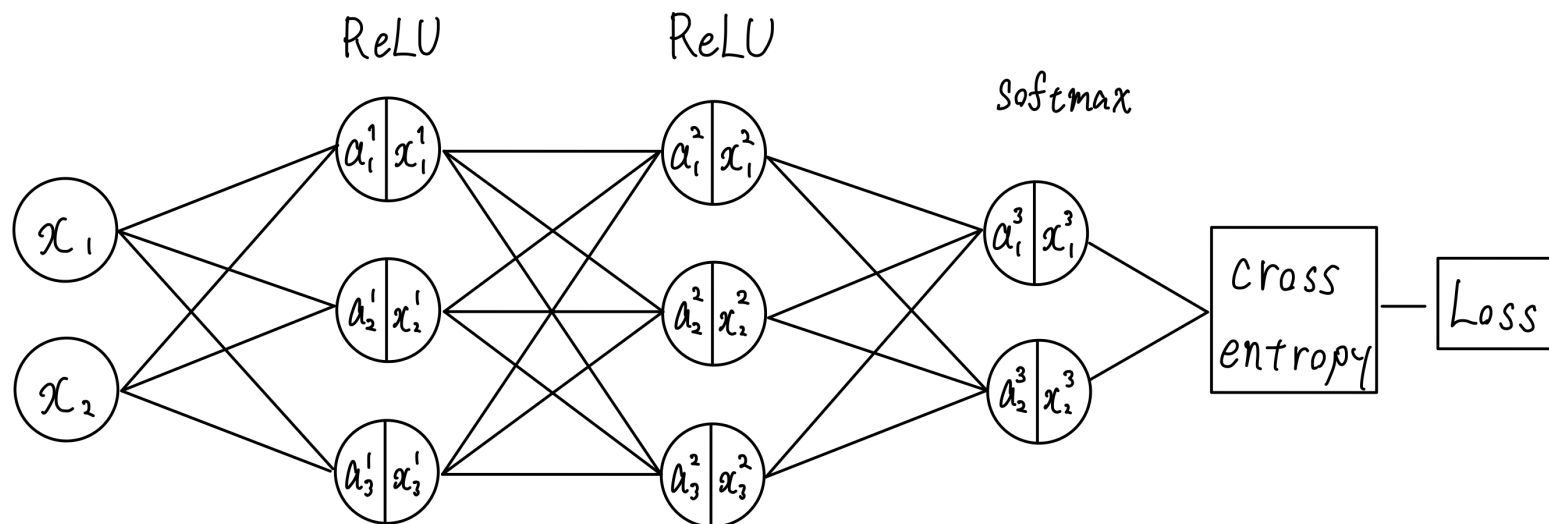
$$B^1 = \begin{pmatrix} 1 & -2 & 3 \end{pmatrix}, B^2 = \begin{pmatrix} 1 & -2 & 3 \end{pmatrix}, B^3 = \begin{pmatrix} 34 & -54 \end{pmatrix}$$

入力データ
 $X = \begin{pmatrix} -1 & 2 \end{pmatrix}$
 教師ラベル
 $T = \begin{pmatrix} 1 & 0 \end{pmatrix}$
 学習率
 $\eta = 0.1$

A large, irregular pink brushstroke shape on a white background, containing the text.

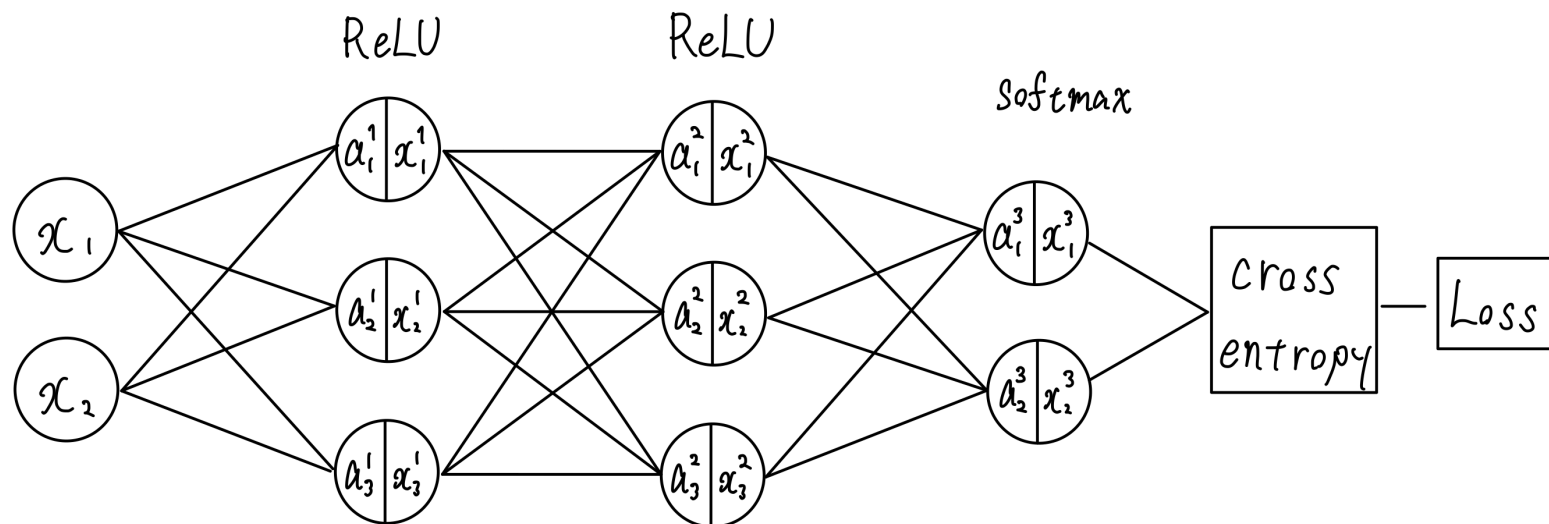
誤差逆伝播法
オンライン完

宿題



このNNを学習するプログラムを実装せよ。

宿題



冗談

NNを学習するプログラムを実装せよ。

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次回
誤差逆伝播法
ミニバッチor実装