Applications of Mathematics in Computer Science (MACS)

LECTURE #5:
FOURIER
TRANSFORM

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The CS Problem:

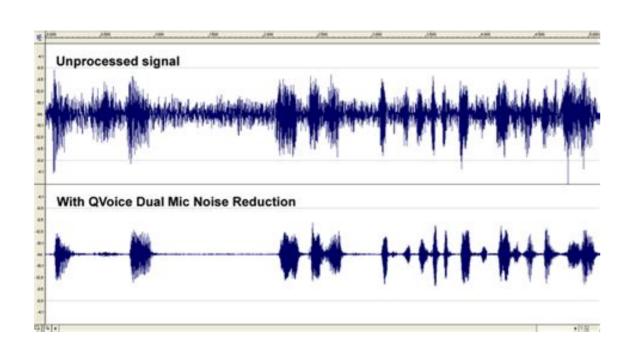
frequency filtering

- Time-series search and comparison (Shazam)
- Information compression (Zoom, Skype)
- Image processing

Frequency filtering

Sound

Image



Noise reduction

Original Image



Modified Image



Types of filters

Low-pass filter





High-pass filter

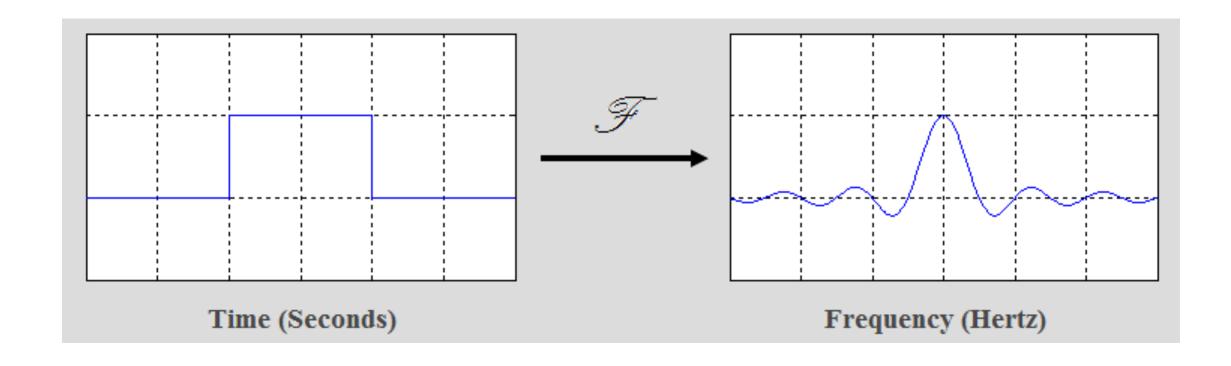


Band-pass filter

Method: Fourier transform

spacial/temporal domain y = f(x)

frequency domain y' = f(frequency)



But how?

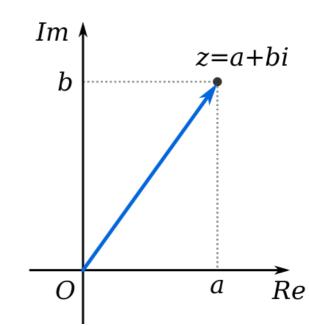
The math technique: Complex Numbers & Fourier Transform

Complex Numbers

$$x^2 + 1 = 0$$

$$x = ?$$

$$x = \sqrt{-1} = i$$



Complex numbers:

$$x = a + bi$$

Re(x) real Im(x)
imaginary

Complex Numbers: Operations

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a+bi)\cdot(c+di) = (ac-bd) + (ad+bc)i$$

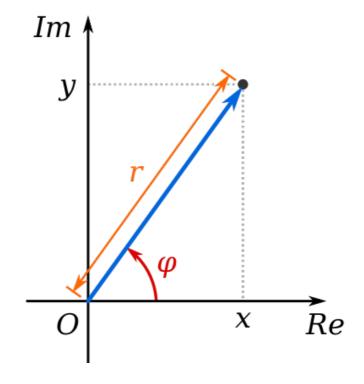
$$\frac{1}{a+bi} = \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i$$

Complex Numbers: Polar Form

$$x = a + bi = r \cdot e^{\varphi i}$$

$$r = \sqrt{a^2 + b^2}$$

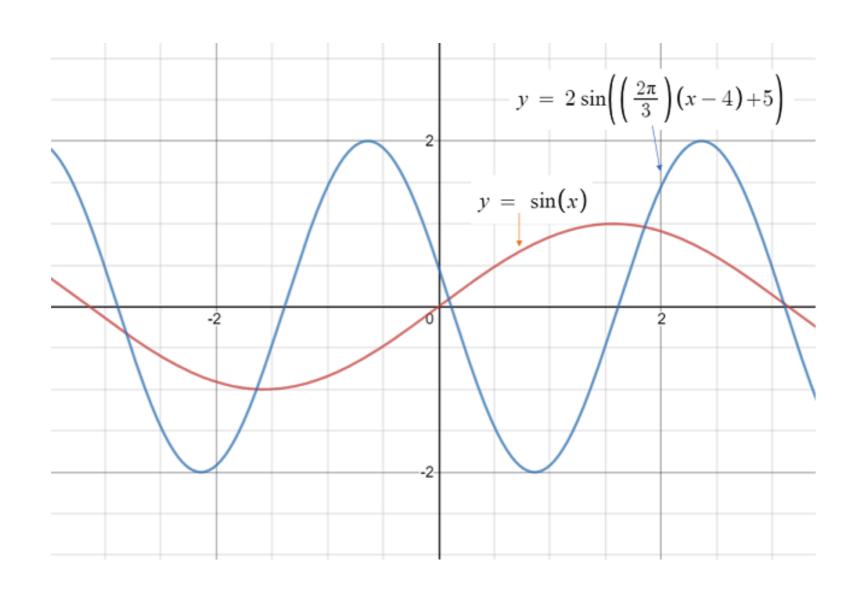
$$\varphi = \arctan\left(\frac{b}{a}\right)$$

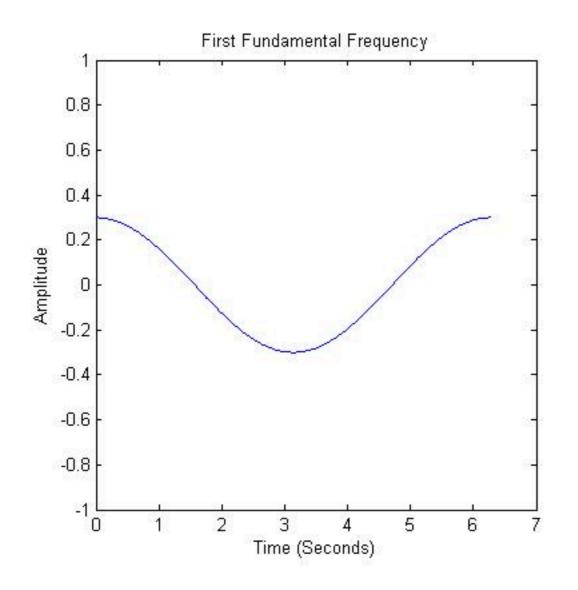


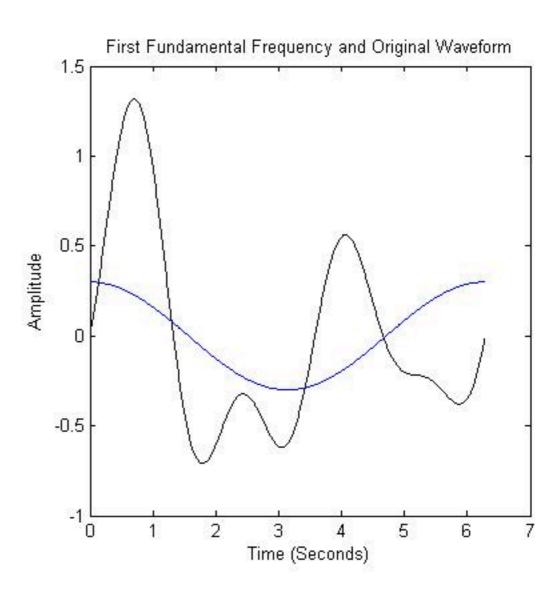
Euler's formula

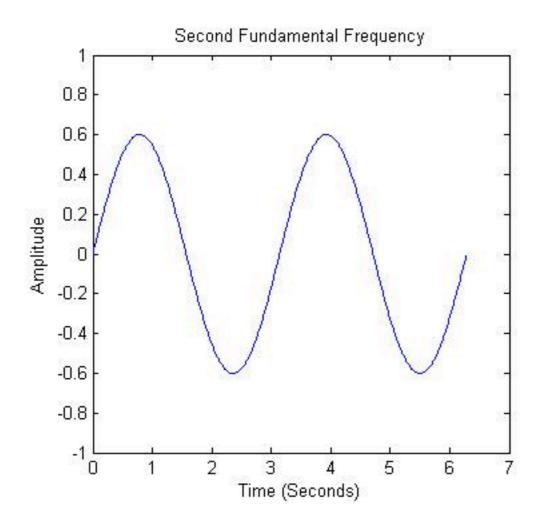
$$e^{a+bi} = e^a \cdot (\cos b + i \sin b)$$

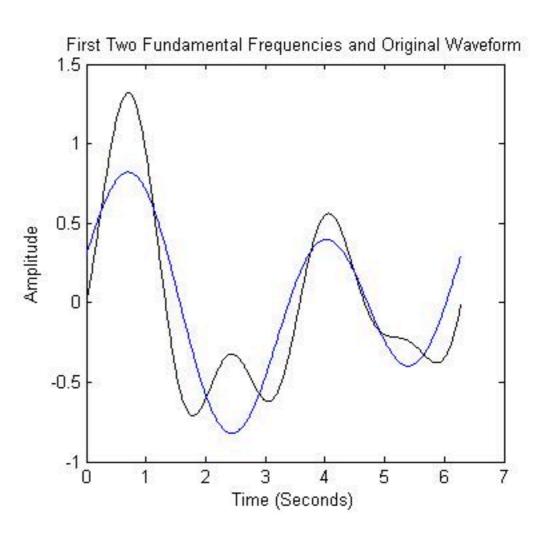
$$y = a \sin\left(\frac{2\pi}{b}(x - c)\right) + d$$

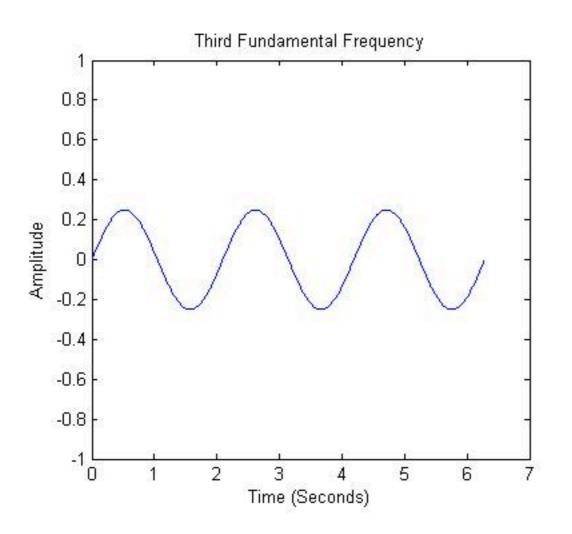


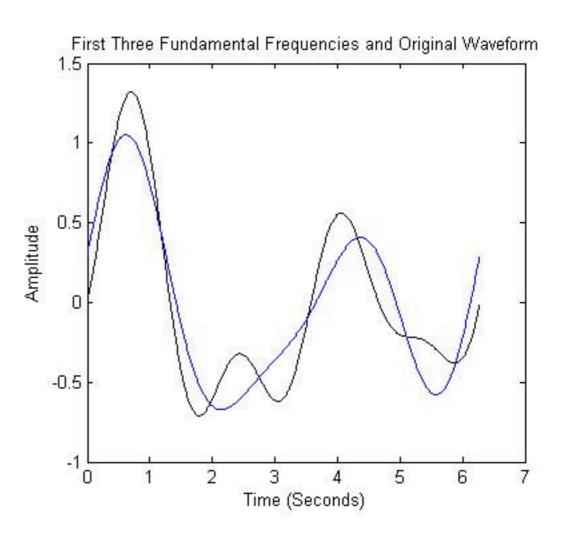












Fourier transform

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e^{\frac{2\pi n}{T}ti}$$

$$c_n = \frac{1}{T} \int_0^T f(t) \cdot e^{-\frac{2\pi n}{T}ti} dt$$

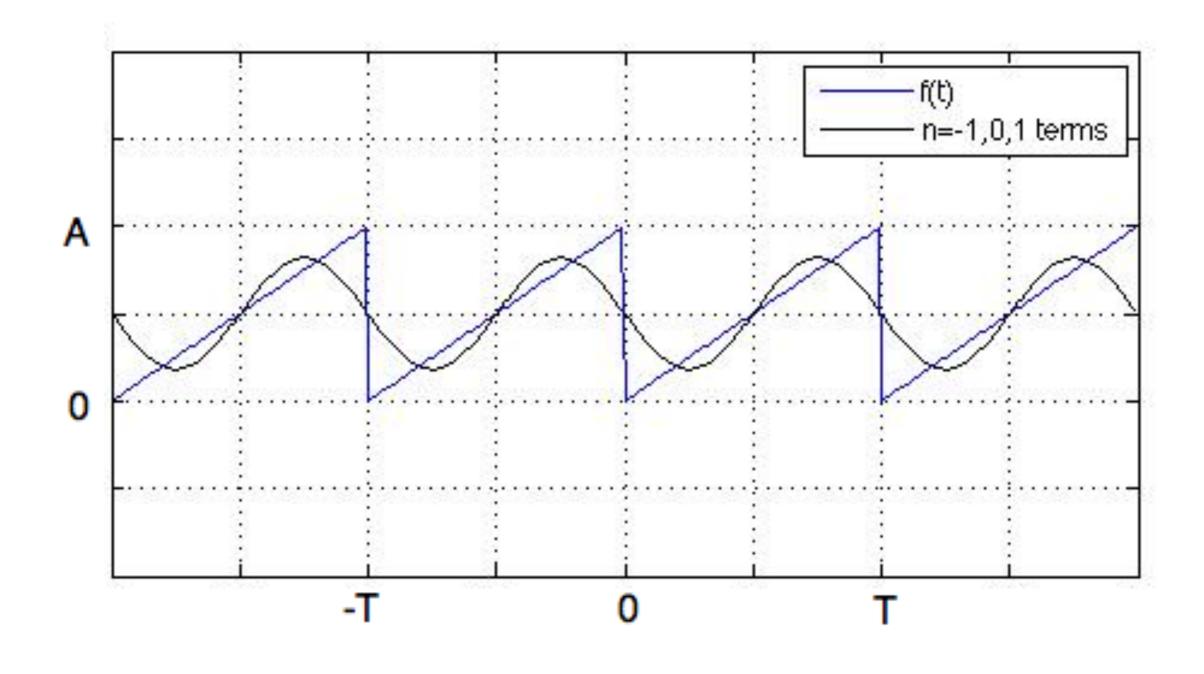
$$f(t) = A \cdot \frac{t}{T} \quad , \quad 0 \le t < T$$

Coefficients:

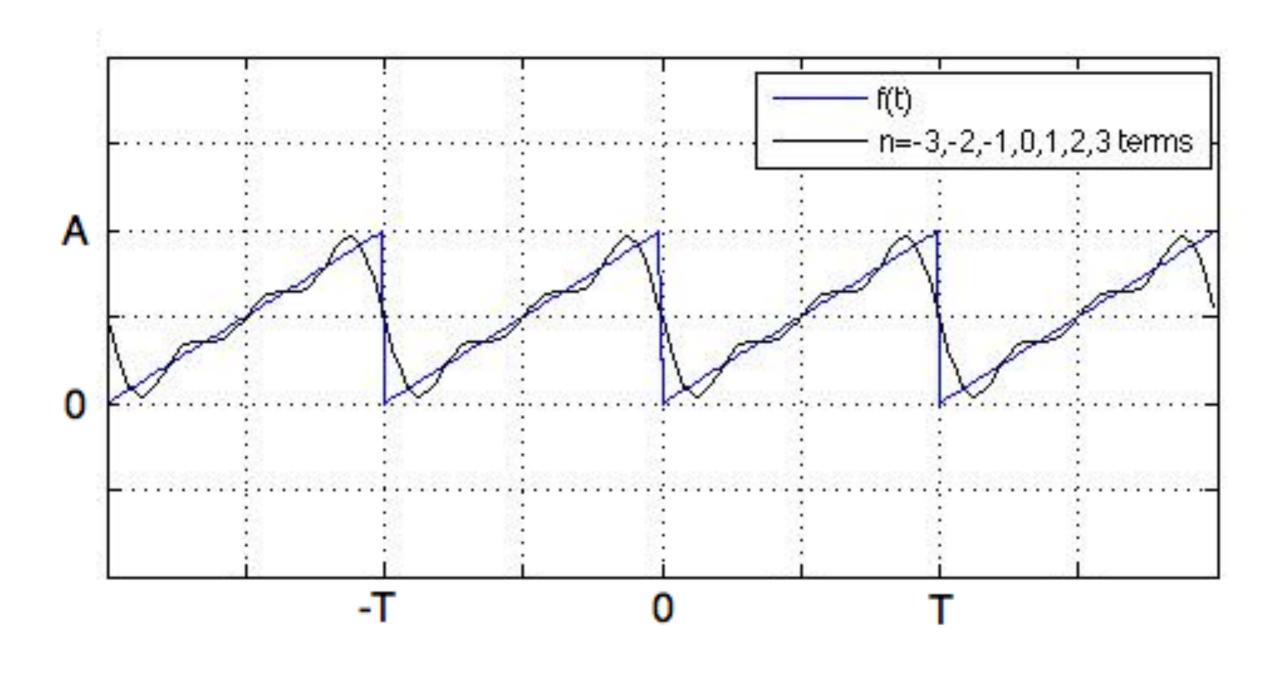
$$c_0 = \frac{A}{2} \qquad c_n = \frac{A}{2\pi n}i \qquad c_n = \frac{1}{T} \int_0^T f(t) \cdot e^{-\frac{2\pi n}{T}ti} dt$$

$$c_n = \frac{1}{T} \int_0^T f(t) \cdot e^{-\frac{2\pi n}{T}ti} dt$$

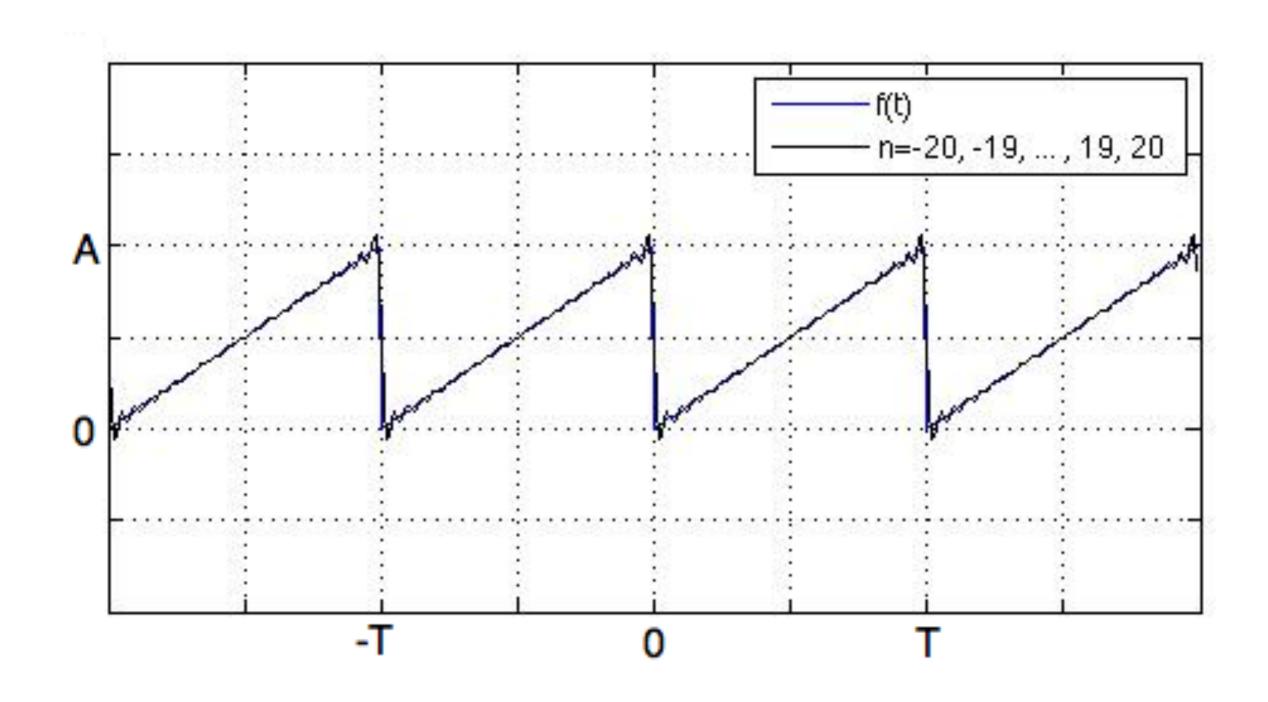
2 Terms



4 Terms



20 Terms



Discrete Fourier transform

When the data is discrete (pixels, measurements)

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-\frac{2\pi i}{N}kn}$$

Fast Fourier Transform (FFT) computes in $O(N \log N)$ instead of $O(N^2)$

Let's try...