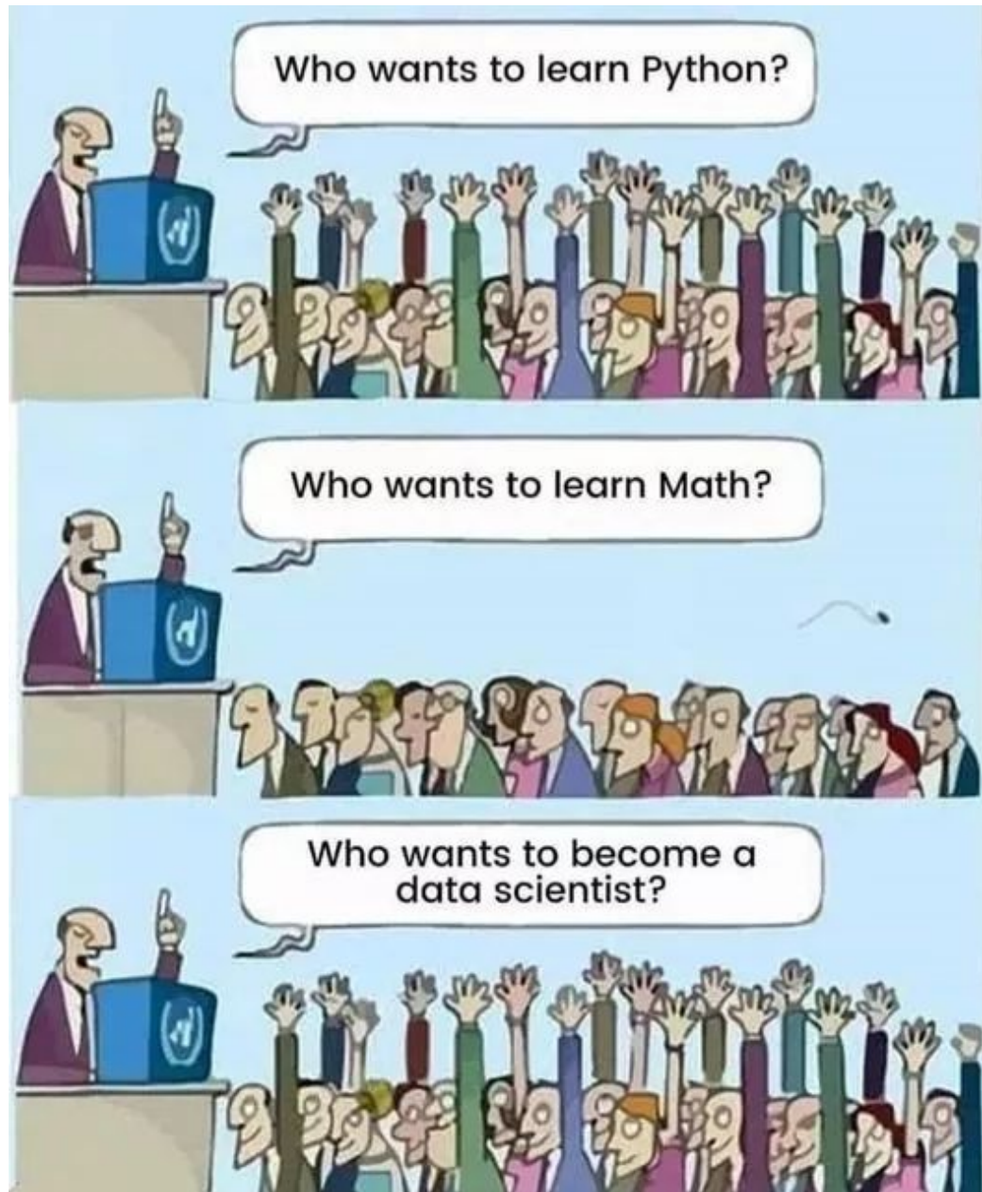
Applications of Mathematics in Computer Science (MACS)

LECTURE #0:

INTRODUCTION

+

COMPUTING WITH LARGE/SMALL NUMBERS



About the Course

Course Structure

- The course will consist of 6 main applications of mathematics in different areas in computer science

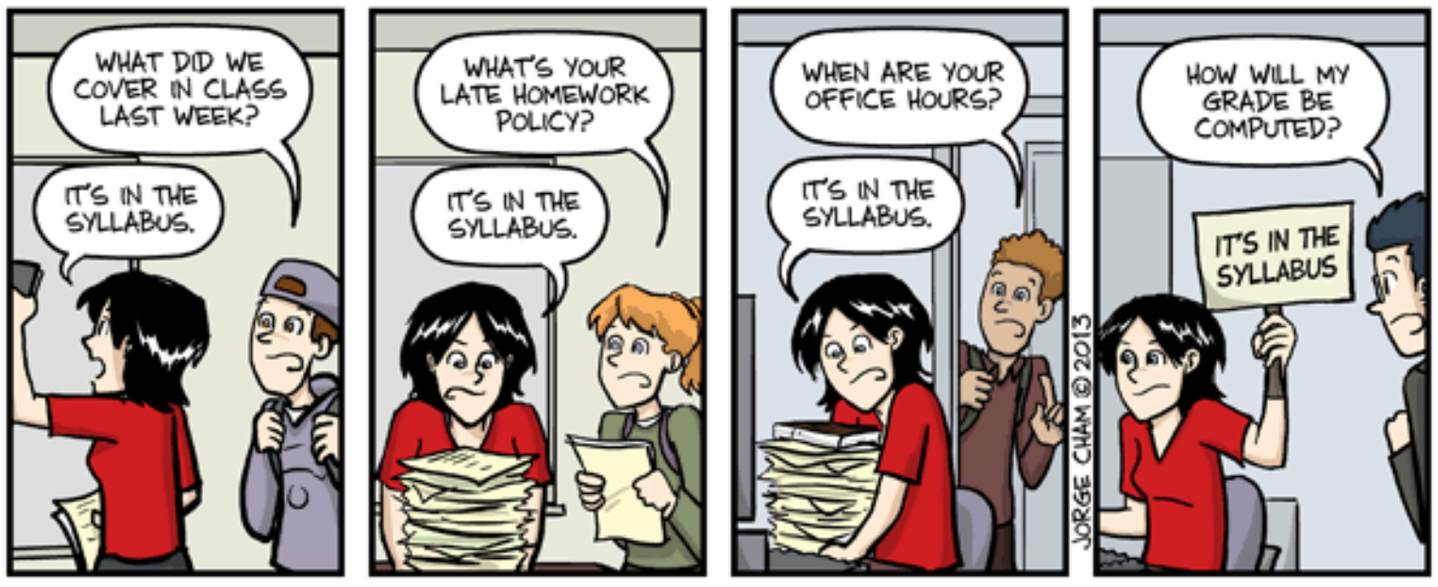
- Each application will consist of:
  - Presentation of the CS problem
  - Presentation of the mathematical techniques
  - Hands-on experiments in Python

This is NOT a course in Python!

- Final Grade = Average of 5 HW assignments

Course Material

- Moodle:
  - includes all instructions & material
  - where you submit assignments
  - Forum
- Python:
  - Link to intro in Moodle
  - We work in Google CoLab (also in Moodle)



About me

- Instructor: Liron Cohen
- Research topics: Logic, Type systems, proof assistants, formal verification, category theory, Homotopy type theory

- Office hours: Zoom (e-mail me in advance)/TAs
- E-mail: cliron@cs.bgu.ac.il

The CS Problem:

Computing with large/small numbers

Computing with Numbers

1000,2100,1500,300,1700,500

Similar magnitude

1,100000000,0.000000000001

$10^{-10}, 10^{10}$

Not too large/small

$10^{-1000}, 10^{1000}$

Computing with Numbers

Sometimes the large numbers are implicit

$$10000! = \Pi_{i=1}^{10000} i$$

$$= 10000 \cdot 9999 \cdot 9998 \cdot \ldots$$

$$a_i = \exp(i^2) \quad \text{for} \quad i \in \{1,\ldots,10000\}$$

$$a_5 = \exp(25) = 72004899337.4$$

What can we do?

We can compute on logarithmic scale!

and exponentiate the final result if needed

The math technique:

Logarithm & exponent

Logarithm — Definition

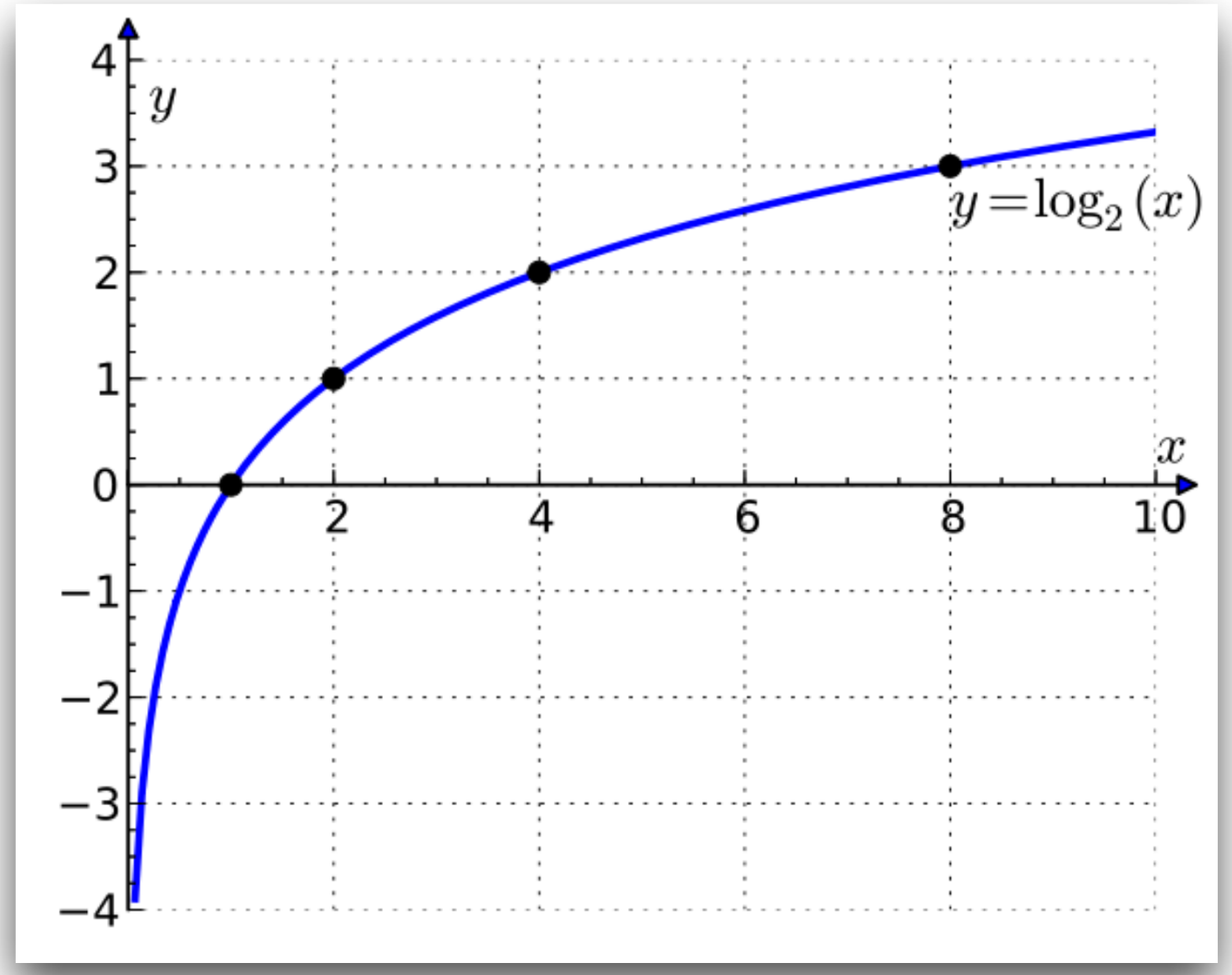
Logarithm is a function $\mathbb{R}^+ \to \mathbb{R}$

$$\log_b x = y \iff b^y = x$$

base

power

Logarithm — Shape



- Defined on $\mathbb{R}^+$
- maps to $\mathbb{R}$
- increasing and concave

Logarithm — Properties

$$\log 1 = 0$$

$$\log (x \cdot y) = \log x + \log y$$

$$\log \left(\frac{x}{y}\right) = \log x - \log y$$

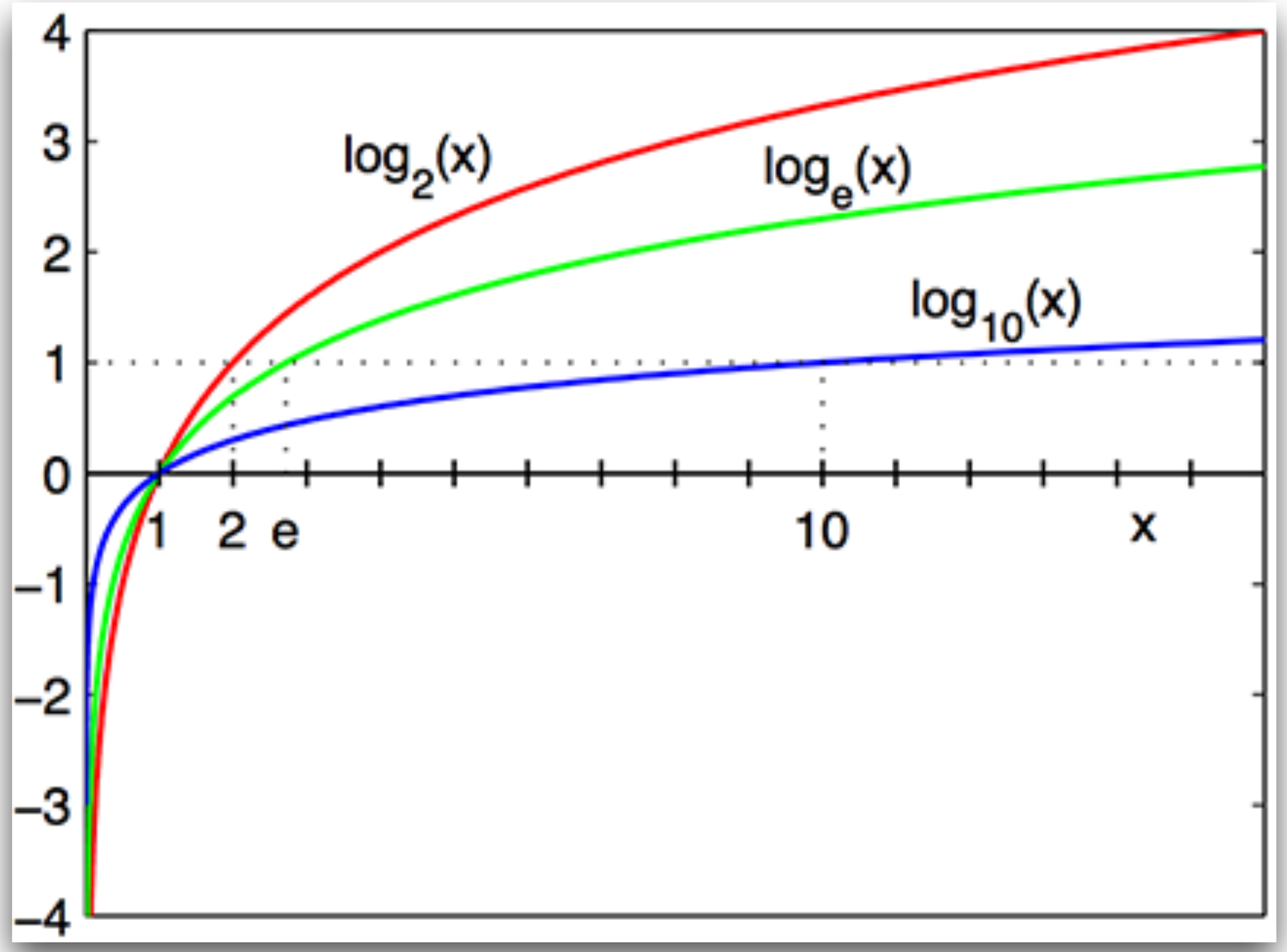
$$\log (x^a) = a \cdot \log x$$

$$b^{\log_b x} = x$$

Logarithm — Bases

Base	Notation	Used in
2 (binary)	$\mathrm{lb}\ x$	CS, information theory
e (natural)	$\ln x$	Statistics, engineering
10 (decimal)	$\lg x$	Physics, engineering

Logarithm — Bases



Let's try...