Online Appendixes

Appendix A. Equilibrium Derivation for Scenario NB

The equilibrium results of scenario NA are shown in Table A.1, with key time thresholds defined as $\underline{T} = Q\sqrt{\frac{k_1}{2C_m}} + Q\sqrt{\frac{k_2}{2C_g}}$ and $\overline{T} = Q\sqrt{\frac{k_1}{2C_m}} + Q\sqrt{\frac{k_2}{2R_g}}$. To ensure that both parties have positive profits, the following conditions must hold: $M = \frac{\sqrt{2k_1C_m}}{2} + \frac{\sqrt{2k_2C_g}}{2} + C_g(\underline{T} - T)$ and $w > \frac{\sqrt{2k_1C_m}}{2}$. The detailed derivation for these equilibrium results is presented below.

Table A.1. The equilibrium decisions and profits of both players in NB scenario

VARIABLES	VALUE
r_p^{NB}	$\sqrt{\frac{2C_m}{k_1}}$
L^{NB}	$Q\sqrt{\frac{k_1}{2C_m}}$
r_a^{NB}	$\begin{cases} \sqrt{\frac{2R_g}{k_2}}, & T \geq \overline{T} \\ \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T - \sqrt{k_1}Q}, & \underline{T} \leq T \leq \overline{T} \\ \sqrt{\frac{2C_g}{k_2}}, & T \leq \underline{T} \end{cases}$
π_{PM}^{NB}	$wQ - Q\sqrt{\frac{k_1C_m}{2}}$
π_{GC}^{NB}	$\begin{cases} (M-w)Q - \frac{\sqrt{2R_gk_2}}{2}Q + R_g(T - \overline{T}), & T \geq \overline{T} \\ (M-w)Q - \frac{1}{2}k_2Q\frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T - \sqrt{k_1}Q}, & \underline{T} \leq T \leq \overline{T} \\ (M-w)Q - \frac{\sqrt{2C_gk_2}}{2}Q - C_g(\underline{T} - T), & T \leq \underline{T} \end{cases}$

Lemma 1. For any given L, π_{PM}^{NB} is continuous in $r_p \in (0, +\infty)$, and concave in $r_p \in (0, \frac{Q}{L}]$ and $r_p \in (\frac{Q}{L}, +\infty)$ respectively. The optimal decision of the prefab manufacturer

regarding production rate can be characterized as: (1) ${r_p}^* = \sqrt{\frac{2R_M}{k_1}}\ if\ L \in L_1$; (2) ${r_p}^* = \frac{Q}{L}\ if\ L \in L_2$; (3) ${r_p}^* = \sqrt{\frac{2C_M}{k_1}}\ if\ L \in L_3$, where we have $L_1 \overset{\Delta}{=} \{L \geq Q\sqrt{\frac{k_1}{2R_M}}\}$, $L_2 \overset{\Delta}{=} \{Q\sqrt{\frac{k_1}{2C_M}} \leq L \leq Q\sqrt{\frac{k_1}{2R_M}}\}$ and $L_3 \overset{\Delta}{=} \{0 \leq L \leq Q\sqrt{\frac{k_1}{2C_M}}\}$.

Proof

By partitioning the decision space $r_p \in (0, +\infty)$, we can simplify (1) as follows:

$$\Pi_{PM} = \begin{cases} \Pi_{PM}^{(H)} = wQ - \dfrac{1}{2}k_1 Q r_p + R_m(L - \frac{Q}{r_p}) & , r_p \geq \frac{Q}{L} \\ \Pi_{PM}^{(L)} = wQ - \dfrac{1}{2}k_1 Q r_p - C_m(\frac{Q}{r_p} - L) & , r_p \leq \frac{Q}{L} \end{cases} \tag{A.1}$$

It is easy to see that, $\Pi_{PM}^{(H)}|_{r_p = \frac{Q}{L}} = \Pi_{PM}^{(L)}|_{r_p = \frac{Q}{L}} = wQ - \dfrac{1}{2}k_1 Q \cdot \dfrac{Q}{L}$. Therefore, Π_{PM} is continuous at $r_p = \dfrac{Q}{L}$. Then we take the first and second order partial derivatives of Π_{PM} with respect to r_p and have:

(1) $\dfrac{\partial \Pi_{PM}^{(H)}}{\partial r_p} = -\dfrac{1}{2}k_1 Q + \dfrac{R_m Q}{r_p^2}, \dfrac{\partial^2 \Pi_{PM}^{(H)}}{\partial r_p^2} = -\dfrac{2R_m Q}{r_p^3} < 0$, thereby $\Pi_{PM}^{(H)}$ is concave on the interval $r_p \in [\dfrac{Q}{L}, +\infty)$;

(2) $\dfrac{\partial \Pi_{PM}^{(L)}}{\partial r_p} = -\dfrac{1}{2}k_1 Q + \dfrac{C_m Q}{r_p^2}, \dfrac{\partial^2 \Pi_{PM}^{(L)}}{\partial r_p^2} = -\dfrac{2C_m Q}{r_p^3} < 0$, thereby $\Pi_{PM}^{(L)}$ is also concave on the interval $r_p \in (0, \dfrac{Q}{L}]$.

However, it is obvious that $\dfrac{\partial \Pi_{PM}}{\partial r_p}$ is not continuous at $r_p = \dfrac{Q}{L}$, and Π_{PM} is continuous bimodal function of r_p, which is concave on the intervals $r_p \in (0, \dfrac{Q}{L}]$ and $r_p \in [\dfrac{Q}{L}, +\infty)$, respectively. Let ${r_p}^{(L)}$ and ${r_p}^{(H)}$ be the optimal solutions of Π_{PM} on the two intervals, we have:

$${r_p}^{(L)} = \min\{\frac{Q}{L}, \sqrt{\frac{2C_m}{k_1}}\}, \quad {r_p}^{(H)} = \max\{\frac{Q}{L}, \sqrt{\frac{2R_m}{k_1}}\} \tag{A.2}$$

In order to obtain the global optimal solution for Π_{PM} with respect to r_p, we need to compare the magnitudes of the two local optimal solutions. Let

$$\Delta = \Pi_{PM}^{(H)} - \Pi_{PM}^{(L)} = \frac{1}{2}k_1 Q(r_p^{(L)} - r_p^{(H)}) + R_m(L - \frac{Q}{r_p^{(H)}}) + C_m(\frac{Q}{r_p^{(L)}} - L) \tag{A.3}$$

The positivity or negativity of L determines the optimal solution for $\prod_{PM}$. Here we assume $C_m > R_m$, meaning the penalty for delay is higher than the reward for early delivery, and divide the values of L into 3 intervals: (1) $L_1 \overset{\Delta}{=} \{L \geq Q\sqrt{\frac{k_1}{2R_m}}\}$; (2) $L_2 \overset{\Delta}{=} \{Q\sqrt{\frac{k_1}{2C_m}} \leq L \leq Q\sqrt{\frac{k_1}{2R_m}}\}$; (3) $L_3 \overset{\Delta}{=} \{0 \leq L \leq \sqrt{\frac{k_1}{2C_m}}\}$. Based on the partition, we have:

(1) If $L \in L_1$, then $r_p^{(L)} = \frac{Q}{L}$, $r_p^{(H)} = \sqrt{\frac{2R_m}{k_1}}$, and $\Delta = \frac{1}{2}k_1 Q(\frac{Q}{L} - \sqrt{\frac{2R_m}{k_1}}) + R_m(L - Q\sqrt{\frac{k_1}{2R_m}}) = \frac{(\sqrt{2R_m}L - \sqrt{k_1}Q)^2}{2L} \geq 0$, which means $\Pi_{PM}^{(H)} \geq \Pi_{PM}^{(L)}$ holds. Therefore, we have $r_p^{*} = r_p^{(H)} = \sqrt{\frac{2R_m}{k_1}}$.

(2) If $L \in L_2$, then $r_p^{(L)} = \frac{Q}{L}$, $r_p^{(H)} = \frac{Q}{L}$, and $\Delta = 0$ which indicates that $\Pi_{PM}^{(H)} = \Pi_{PM}^{(L)}$. Therefore, we have $r_p^{*} = \frac{Q}{L}$.

(3) If $L \in L_3$, then $r_p^{(L)} = \sqrt{\frac{2C_m}{k_1}}$, $r_p^{(H)} = \frac{Q}{L}$, and $\Delta = \frac{1}{2}k_1 Q(\sqrt{\frac{2C_m}{k_1}} - \frac{Q}{L}) + C_m(Q\sqrt{\frac{k_1}{2C_m}} - L) = -\frac{(\sqrt{2C_m}L - \sqrt{k_1}Q)^2}{2L} \leq 0$ which means that $\Pi_{PM}^{(H)} \leq \Pi_{PM}^{(L)}$ holds. Therefore, we have $r_p^{*} = r_p^{(L)} = \sqrt{\frac{2C_m}{k_1}}$.

Lemma 2. The optimal decision of the GC regarding assembly rate and quoted delivery lead time can be characterized as:

$$L^* = Q\sqrt{\frac{k_1}{2C_M}}, \quad r_a^* = \begin{cases} \sqrt{\frac{2R_{GC}}{k_2}}, & T \geq Q\sqrt{\frac{k_1}{2C_M}} + Q\sqrt{\frac{k_2}{2R_{GC}}} \\ \frac{\sqrt{2C_M}Q}{\sqrt{2C_M}T - \sqrt{k_1}Q}, & Q\sqrt{\frac{k_1}{2C_M}} + Q\sqrt{\frac{k_2}{2C_{GC}}} \leq T \leq Q\sqrt{\frac{k_1}{2C_M}} + Q\sqrt{\frac{k_2}{2R_{GC}}} \\ \sqrt{\frac{2C_{GC}}{k_2}}, & T \leq Q\sqrt{\frac{k_1}{2C_M}} + Q\sqrt{\frac{k_2}{2C_{GC}}} \end{cases}$$

Proof

Based on the value of L, we discuss three cases:

(1) When $L \geq Q\sqrt{\frac{k_1}{2R_m}}$, then $r_p = \sqrt{\frac{2R_m}{k_1}}, \frac{Q}{r_p} \leq L$, and we have:

$$\Pi_{GC} = (M-w)\ Q - \frac{1}{2}k_2 r_a Q - c_g(\frac{Q}{r_p} + \frac{Q}{r_a} - T)^+ + R_g(T - \frac{Q}{r_p} - \frac{Q}{r_a})^+ - R_m(L - \frac{Q}{r_p}) \quad \text{(A.4)}$$

(a) If $\frac{Q}{r_p} + \frac{Q}{r_a} - T \leq 0$, then we have $\Pi_{GC} = (M-w)Q - \frac{1}{2}k_2 r_a Q + R_g(T - \frac{Q}{r_p} - \frac{Q}{r_a}) - R_m(L - \frac{Q}{r_p})$. Its Lagrange dual function is:

$$\Psi(L, r_a) = -[(M-w)Q - \frac{1}{2}k_2 r_a Q + R_g(T - \frac{Q}{r_p} - \frac{Q}{r_a}) - R_m(L - \frac{Q}{r_p})] + \alpha_1(\frac{Q}{r_p} + \frac{Q}{r_a} - T) + \alpha_2(\frac{Q}{r_p} - L)$$

, where $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$ are the Lagrange multipliers. Based on the KKT optimality conditions, we have:

$$\alpha_1 \geq 0, \frac{Q}{r_p} + \frac{Q}{r_a} - T \leq 0, \alpha_1(\frac{Q}{r_p} + \frac{Q}{r_a} - T) = 0 \quad \text{(A.5)}$$

$$\alpha_2 \geq 0, \frac{Q}{r_p} - L \leq 0, \alpha_2(\frac{Q}{r_p} - L) = 0 \quad \text{(A.6)}$$

$$\frac{\partial \Psi(L, r_a)}{\partial L} = R_m - \alpha_2 = 0 \quad \text{(A.7)}$$

$$\frac{\partial \Psi(L, r_a)}{\partial r_a} = \frac{1}{2}k_2 Q - \frac{R_g Q}{r_a^2} - \frac{\alpha_1 Q}{r_a^2} = 0 \quad \text{(A.8)}$$

According to Eq. (A.7), we have $\alpha_2 = R_m > 0$. Referencing Eq. (A.6), we have $\frac{Q}{r_p} - L = 0$, then $L = \frac{Q}{r_p} = Q\sqrt{\frac{k_1}{2R_m}}$. According to Eq. (A.8), we have $\alpha_1 = \frac{1}{2}k_2 r_a^2 - R_g \geq 0$, that is $r_a \geq \sqrt{\frac{2R_g}{k_2}}$. According to Eq. (A.5), we have $r_a \geq \frac{Q}{T - \frac{Q}{r_p}} = \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T - \sqrt{k_1}Q}$. To satisfy the KKT optimality conditions, we have $r_a^* = \max\{\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T - \sqrt{k_1}Q}, \sqrt{\frac{2R_g}{k_2}}\}$.

(b) If $\frac{Q}{r_p} + \frac{Q}{r_a} - T \geq 0$, then we have $\Pi_{GC} = (M-w)Q - \frac{1}{2}k_2 r_a Q - C_g(\frac{Q}{r_p} + \frac{Q}{r_a} - T) - R_m(L - \frac{Q}{r_p})$. Its Lagrange dual function is:

$$\Psi(L,r_a)=-[(M-w)Q-\frac{1}{2}k_2 r_a Q-C_g(\frac{Q}{r_p}+\frac{Q}{r_a}-T)-R_m(L-\frac{Q}{r_p})]+\alpha_1(T-\frac{Q}{r_p}-\frac{Q}{r_a})+\alpha_2(\frac{Q}{r_p}-L)$$

Similar to case (a), based on the KKT optimality conditions, we can obtain $r_a^*=\min\{\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q},\sqrt{\frac{2C_g}{k_2}}\}$.

In summary, when $L\geq Q\sqrt{\frac{k_1}{2R_m}}$, we have $L^*=\frac{Q}{r_p}=Q\sqrt{\frac{k_1}{2R_m}}$ and $r_a^*=\begin{cases}\max\{\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q},\sqrt{\frac{2R_g}{k_2}}\}, & r_a\geq\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}\\ \min\{\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q},\sqrt{\frac{2C_g}{k_2}}\}, & r_a\leq\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}\end{cases}$. To obtain the global optimal solution, let $r_a^{(H)}=\max\{\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q},\sqrt{\frac{2R_g}{k_2}}\}$ and $r_a^{(L)}=\min\{\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q},\sqrt{\frac{2C_g}{k_2}}\}$, then we discuss by cases:

(i) If $\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}\leq\sqrt{\frac{2R_g}{k_2}}$, then $r_a^{(H)}=\sqrt{\frac{2R_g}{k_2}}$, $r_a^{(L)}=\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}$, and we have $\frac{Q}{r_p}+\frac{Q}{r_a^{(L)}}=T$, $\Pi_{GC}^{(H)}=(M-w)Q-\frac{1}{2}k_2 r_a^{(H)}Q+R_g(T-\frac{Q}{r_p}-\frac{Q}{r_a^{(H)}})-R_m(L-\frac{Q}{r_p})$, and $\Pi_{GC}^{(L)}=(M-w)Q-\frac{1}{2}k_2 r_a^{(L)}Q-R_m(L-\frac{Q}{r_p})$. Let $\Delta=\Pi_{GC}^{(H)}-\Pi_{GC}^{(L)}$, we have $\Delta=\frac{1}{2}k_2Q(r_a^{(L)}-r_a^{(H)})+R_g(T-\frac{Q}{r_p}-\frac{Q}{r_a^{(H)}})=R_gQ\cdot\frac{(r_a^{(L)}-r_a^{(H)})^2}{r_a^{(L)}(r_a^{(H)})^2}\geq 0$. Therefore, $\Pi_{GC}^{(H)}\geq\Pi_{GC}^{(L)}$, and then we have $r_a^*=r_a^{(H)}=\sqrt{\frac{2R_g}{k_2}}$.

(ii) If $\sqrt{\frac{2R_g}{k_2}}\leq\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}\leq\sqrt{\frac{2C_g}{k_2}}$, then $r_a^{(H)}=r_a^{(L)}=\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}$, obviously we have $r_a^*=\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}$.

(iii) If $\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}\geq\sqrt{\frac{2C_g}{k_2}}$, then $r_a^{(H)}=\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}$, $r_a^{(L)}=\sqrt{\frac{2C_g}{k_2}}$, and we have $\Delta=\Pi_{GC}^{(H)}-\Pi_{GC}^{(L)}=C_gQ\cdot\frac{-(r_a^{(L)}-r_a^{(H)})^2}{r_a^{(H)}(r_a^{(L)})^2}\leq 0$. Therefore, $\Pi_{GC}^{(H)}\leq\Pi_{GC}^{(L)}$ and then $r_a^*=r_a^{(L)}=\sqrt{\frac{2C_g}{k_2}}$.

To sum up, when $L\geq Q\sqrt{\frac{k_1}{2R_m}}$, we have $r_p^*=\sqrt{\frac{2R_m}{k_1}}$, $L^*=\frac{Q}{r_p}=Q\sqrt{\frac{k_1}{2R_m}}$, and

$$r_a^* = \begin{cases} \sqrt{\frac{2R_g}{k_2}}, & \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2R_g}{k_2}} \\ \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}, & \sqrt{\frac{2R_g}{k_2}} \le \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2C_g}{k_2}} \\ \sqrt{\frac{2C_g}{k_2}}, & \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q} \ge \sqrt{\frac{2C_g}{k_2}} \end{cases}.$$

(2) When $Q\sqrt{\frac{k_1}{2C_M}} \le L \le Q\sqrt{\frac{k_1}{2R_M}}$, then $r_p = \frac{Q}{L}$, and we have $\Pi_{GC} = (M-w)Q - \frac{1}{2}k_2 r_a Q - C_g(L + \frac{Q}{r_a} - T)^+ + R_g(T - L - \frac{Q}{r_a})^+$. Similarly, discussing cases based on the relationship between $\frac{Q}{r_p} + \frac{Q}{r_a}$ and T, we can obtain $L^* = Q\sqrt{\frac{k_1}{2C_m}}$, $r_p^* = \frac{Q}{L^*} = \sqrt{\frac{2C_m}{k_1}}$,

and $$r_a^* = \begin{cases} \sqrt{\frac{2R_g}{k_2}}, & \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2R_g}{k_2}} \\ \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q}, & \sqrt{\frac{2R_g}{k_2}} \le \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2C_g}{k_2}} \\ \sqrt{\frac{2C_g}{k_2}}, & \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \ge \sqrt{\frac{2C_g}{k_2}} \end{cases}.$$

(3) When $0 \le L \le Q\sqrt{\frac{k_1}{2C_m}}$, then $r_p = \sqrt{\frac{2C_m}{k_1}}$ and $\frac{Q}{r_p} \ge L$, we have $\Pi_{GC} = (M-w)Q - \frac{1}{2}k_2 r_a Q - C_g(\frac{Q}{r_p} + \frac{Q}{r_a} - T)^+ + R_g(T - \frac{Q}{r_p} - \frac{Q}{r_a})^+ + (C_m - C_{idl})(\frac{Q}{r_p} - L)$. By discussing the cases separately, we can obtain $r_p^* = \sqrt{\frac{2C_m}{k_1}}$, $L^* = \frac{Q}{r_p^*} = Q\sqrt{\frac{k_1}{2C_m}}$, and

$$r_a^* = \begin{cases} \sqrt{\frac{2R_g}{k_2}}, & \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2R_g}{k_2}} \\ \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q}, & \sqrt{\frac{2R_g}{k_2}} \le \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2C_g}{k_2}} \\ \sqrt{\frac{2C_g}{k_2}}, & \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \ge \sqrt{\frac{2C_g}{k_2}} \end{cases}.$$

To compare Π_{GC} in the two cases (i.e., $L \ge Q\sqrt{\frac{k_1}{2R_m}}$ and $L \le Q\sqrt{\frac{k_1}{2R_m}}$), let $\Delta r_a^{(1)} = \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q} - \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q}$ and $\Delta r_a^{(2)} = \sqrt{\frac{2C_g}{k_2}} - \sqrt{\frac{2R_g}{k_2}}$, and then proceed with the discussion in two scenarios: (1) $\Delta r_a^{(1)} \ge \Delta r_a^{(2)}$; (2) $\Delta r_a^{(1)} < \Delta r_a^{(2)}$. In scenario (1), we can further divide the discussion into four subcases. First, when $\frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q} < \sqrt{\frac{2R_g}{k_2}}$, then $r_p^{(1)} = \sqrt{\frac{2R_m}{k_1}}$, $L^{(1)} = Q\sqrt{\frac{k_1}{2R_m}}$,

$r_a^{(1)} = \sqrt{\frac{2R_g}{k_2}}$, $r_p^{(2)} = \sqrt{\frac{2C_m}{k_1}}$, $L^{(2)} = Q\sqrt{\frac{k_1}{2C_m}}$, and $r_a^{(2)} = \sqrt{\frac{2R_g}{k_2}}$. We have $\Delta_{12} = \Pi_{GC}^{(1)} - \Pi_{GC}^{(2)} = R_g(\frac{Q}{r_p^{(2)}} - \frac{Q}{r_p^{(1)}})$. Since $r_p^{(2)} > r_p^{(1)}$, then $\Delta_{12} < 0$, namely $\Pi_{GC}^{(1)} < \Pi_{GC}^{(2)}$. We can obtain $r_p^* = r_p^{(2)} = \sqrt{\frac{2C_m}{k_1}}$, $L^* = L^{(2)} = Q\sqrt{\frac{k_1}{2C_m}}$, $r_a^* = r_a^{(2)} = \sqrt{\frac{2R_g}{k_2}}$.

Similarly, we can deduce that $\Pi_{GC}^{(2)}$ is always larger in the other three cases (i.e., $\frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2R_g}{k_2}} < \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q} < \sqrt{\frac{2C_g}{k_2}}$, $\sqrt{\frac{2R_g}{k_2}} < \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} < \sqrt{\frac{2C_g}{k_2}} < \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}$, and $\sqrt{\frac{2C_g}{k_2}} \le \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} < \frac{\sqrt{2R_m}Q}{\sqrt{2R_m}T-\sqrt{k_1}Q}$). By following a similar line of reasoning, we can derive that $\Pi_{GC}^{(2)}$ is consistently larger in all five cases of scenario (2). Consequently, combining the results from both scenarios, the equilibrium solution we arrive at is $r_p^* = r_p^{(2)} = \sqrt{\frac{2C_m}{k_1}}$, $L^* = L^{(2)} = Q\sqrt{\frac{k_1}{2C_m}}$,

$$r_a^* = r_a^{(2)} = \begin{cases} \sqrt{\frac{2R_g}{k_2}}, & \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2R_g}{k_2}} \\ \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q}, & \sqrt{\frac{2R_g}{k_2}} \le \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \le \sqrt{\frac{2C_g}{k_2}} \\ \sqrt{\frac{2C_g}{k_2}}, & \frac{\sqrt{2C_m}Q}{\sqrt{2C_m}T-\sqrt{k_1}Q} \ge \sqrt{\frac{2C_g}{k_2}} \end{cases}.$$

Appendix B. Equilibrium Derivation for Scenario AB

The equilibrium results are presented in Table B.1. The updated time thresholds, which account for the efficiency gains from blockchain, are defined as: $\overline{T'} = \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}} + \frac{Q}{1+\beta}\sqrt{\frac{k_2}{2R_g}}$ and $\underline{T'} = \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}} + \frac{Q}{1+\beta}\sqrt{\frac{k_2}{2C_g}}$. For blockchain adoption to be profitable for both parties, the efficiency gains (α for the PM and β for the GC) must exceed certain thresholds. Specifically, the following conditions must be met: $\alpha > \frac{\sqrt{2k_1C_m}}{2(w-b)} - 1$ and $\beta > \frac{\sqrt{2k_2C_g}Q}{(M-w)Q - C_g(T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}) - bQ} - 1$. A detailed derivation of these results is presented below.

Lemma 3. The optimal solutions of the PM and the GC in the equilibrium of the PCSC with blockchain adoption are given in Table B.1, wherein $\overline{T}' = \frac{Q}{(1+\alpha)}\sqrt{\frac{k_1}{2C_M}} + \frac{Q}{(1+\beta)}\sqrt{\frac{k_2}{2R_{GC}}}$ and $\underline{T}' = \frac{Q}{(1+\alpha)}\sqrt{\frac{k_1}{2C_M}} + \frac{Q}{(1+\beta)}\sqrt{\frac{k_2}{2C_{GC}}}$.

Table B.1 Equilibrium outcomes in the PCSC with blockchain

VARIABLES	VALUE
r_p^{AB}	$\sqrt{\frac{2C_m}{k_1}}$
L^{AB}	$\frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}$
r_a^{AB}	$\begin{cases} \sqrt{\frac{2R_g}{k_2}}, & T > \overline{T}' \\ \frac{\sqrt{2C_m}(1+\alpha)Q}{\sqrt{2C_m}(1+\alpha)(1+\beta)T-\sqrt{k_1}(1+\beta)Q}, & \underline{T}' \le T \le \overline{T}' \\ \sqrt{\frac{2C_g}{k_2}}, & T < \underline{T}' \end{cases}$
π_{PM}^{AB}	$wQ - \frac{\sqrt{2k_1C_m}}{2(1+\alpha)}Q - bQ$
π_{GC}^{AB}	$\begin{cases} (M-w)Q - \frac{\sqrt{2k_2R_g}}{2(1+\beta)}Q + R_g(T - \frac{Q}{(1+\alpha)}\sqrt{\frac{k_1}{2C_m}} - \frac{Q}{(1+\beta)}\sqrt{\frac{k_2}{2R_g}}) - bQ, & T > \overline{T}' \\ (M-w)Q - \frac{k_2Q}{2(1+\beta)}\frac{\sqrt{2C_m}(1+\alpha)Q}{\sqrt{2C_m}(1+\alpha)(1+\beta)T-\sqrt{k_1}(1+\beta)Q} - bQ, & \underline{T}' \le T \le \overline{T}' \\ (M-w)Q - \frac{\sqrt{2k_2C_g}}{2(1+\beta)}Q - C_g(\frac{Q}{(1+\alpha)}\sqrt{\frac{k_1}{2C_m}} + \frac{Q}{(1+\beta)}\sqrt{\frac{k_2}{2Cg}} - T) - bQ, & T < \underline{T}' \end{cases}$

Proof of Lemma 3

Proof of Lemma 3 is similar to that of Proof of Lemma 2, hence omitted.

Appendix C. Proofs of Propositions

Proof of Proposition 1

According to the results of Lemma 2 and Lemma 3, we can deduce that $\pi_{PM}^{AB} - \pi_{PM}^{NB} = \frac{\sqrt{2k_1C_m}Q}{2} - \frac{\sqrt{2k_1C_m}Q}{2(1+\alpha)} - bQ$. By letting $\pi_{PM}^{AB} - \pi_{PM}^{NB} > 0$, we obtain $b < \frac{2\sqrt{2k_1C_m}}{2(1+\alpha)}$.

Proof of Proposition 2

According to Lemma 2, the value of r_a^{NB} depends on the relationship between T, $\underline{T}$, and $\overline{T}$. Similarly, Lemma 3 indicates that the value of r_a^{AB} is determined by the relationship between T,

$\underline{T}'$, and $\overline{T}'$. To compare π_{GC}^{AB} and π_{GC}^{NB} based on the values of r_a^{AB} and r_a^{NB}, we divide the range of T into five intervals. It is evident that $\underline{T}' < \underline{T}$ and $\overline{T}' < \overline{T}$, but the relative positions of $\underline{T}$ and $\overline{T}'$ are uncertain and can affect the comparison results. Therefore, we discuss the five intervals of T in two cases based on the relationship between $\underline{T}$ and $\overline{T}'$.

(1) When $\underline{T}' < \underline{T} < \overline{T}' < \overline{T}$, the five intervals of T are as follows:

(i) If $T < \underline{T}'$, then $\pi_{GC}^{AB} - \pi_{GC}^{NB} = \frac{\alpha}{1+\alpha} C_g Q\sqrt{\frac{k_1}{2C_m}} + \frac{\beta}{1+\beta}\sqrt{2k_2C_g}Q - bQ$. Let $\pi_{GC}^{AB} - \pi_{GC}^{NB} > 0$, we have $b < \frac{\alpha}{1+\alpha} C_g \sqrt{\frac{k_1}{2C_m}} + \frac{\beta}{1+\beta}\sqrt{2k_2C_g}$. Let $f_1(\alpha,\beta) = \frac{\alpha}{1+\alpha} C_g \sqrt{\frac{k_1}{2C_m}} + \frac{\beta}{1+\beta}\sqrt{2k_2C_g}$, we can calculate the partial derivatives and have $\frac{\partial f_1(\alpha,\beta)}{\partial T} = 0$, $\frac{\partial f_1(\alpha,\beta)}{\partial \alpha} > 0$, and $\frac{\partial f_1(\alpha,\beta)}{\partial \beta} > 0$. Therefore, for any α and β, we have $f_1(\alpha,\beta) > f_1(0,0) = 0$. Consequently, there exists a $\overline{b}_2 = f_1(\alpha,\beta) > 0$ such that when $b < \overline{b}_2$, $\pi_{GC}^{AB} > \pi_{GC}^{NB}$.

(ii) If $T \in (\underline{T}', \underline{T})$, then $\pi_{GC}^{AB} - \pi_{GC}^{NB} = \sqrt{2k_2C_g}Q - \frac{k_2 Q}{2(1+\beta)^2}\frac{Q}{T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}} - bQ + C_g(Q\sqrt{\frac{k_1}{2C_m}} - T)$. Let $\pi_{GC}^{AB} - \pi_{GC}^{NB} > 0$, we have $b < \sqrt{2k_2C_g} - \frac{k_2 Q}{2(1+\beta)^2}\frac{1}{T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}} - bQ + C_g\sqrt{\frac{k_1}{2C_m}} - \frac{C_g T}{Q}$. Let $f_2(\alpha,\beta) = \sqrt{2k_2C_g} - \frac{k_2 Q}{2(1+\beta)^2}\frac{1}{T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}} - bQ + C_g\sqrt{\frac{k_1}{2C_m}} - \frac{C_g T}{Q}$, we calculate the partial derivative of $f_2(\alpha,\beta)$ with respect to T and obtain $\frac{\partial f_2(\alpha,\beta)}{\partial T} = \frac{k_2 Q}{2(1+\beta)^2}\frac{1}{(T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}})^2} - \frac{C_g}{Q}$ and $\frac{\partial^2 f_2(\alpha,\beta)}{\partial T^2} = \frac{k_2 Q}{(1+\beta)^2}\frac{-1}{(T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}})^3} < 0$. Therefore, when $\underline{T}' < T < \underline{T}$, $\frac{\partial f_2(\alpha,\beta)}{\partial T} < \frac{\partial f_2(\alpha,\beta)}{\partial T}|_{T=\underline{T}'} = 0$, indicating that $f_2(\alpha,\beta)$ is strictly decreasing on the interval $(\underline{T}', \underline{T})$. Therefore, it follows that for any α and β, we have $f_2(\alpha,\beta) > f_2(\alpha,\beta)|_{T=\underline{T}} = \frac{\sqrt{2k_2C_g}}{2} - \frac{k_2}{2(1+\beta)^2}\frac{1}{\frac{\alpha}{1+\alpha}\sqrt{\frac{k_1}{2C_m}} + \sqrt{\frac{k_2}{2C_g}}}$. It is evident that $\frac{\partial f_2(\alpha,\beta)|_{T=\underline{T}}}{\partial \alpha} > 0$ and $\frac{\partial f_2(\alpha,\beta)|_{T=\underline{T}}}{\partial \beta} > 0$. As a result, $f_2(\alpha,\beta)|_{T=\underline{T}} > f_2(0,0)|_{T=\underline{T}} = 0$. Hence, for any α and β, we have $f_2(\alpha,\beta) > 0$. Consequently, there exists a

$\overline{b_2}=f_2(\alpha,\beta)>0$ such that when $b<\overline{b_2}$, $\pi_{GC}^{AB}>\pi_{GC}^{NB}$.

(iii) If $T\in(\underline{T},\overline{T}')$, then $\pi_{GC}^{AB}-\pi_{GC}^{NB}=\frac{k_2Q^2}{2}\frac{1}{T-Q\sqrt{\frac{k_1}{2C_m}}}-\frac{k_2Q}{2(1+\beta)^2}\frac{Q}{T-\frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}}-bQ$. Let $\pi_{GC}^{AB}-\pi_{GC}^{NB}>0$, we have $b<\frac{k_2Q}{2}\frac{1}{T-Q\sqrt{\frac{k_1}{2C_m}}}-\frac{k_2Q}{2(1+\beta)^2}\frac{1}{T-\frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}}$. Let $f_3(\alpha,\beta)=\frac{k_2Q}{2}\frac{1}{T-Q\sqrt{\frac{k_1}{2C_m}}}-\frac{k_2Q}{2(1+\beta)^2}\frac{1}{T-\frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}}$, we can readily deduce the result that $\frac{\partial f_3(\alpha,\beta)}{\partial\alpha}>0$ and $\frac{\partial f_3(\alpha,\beta)}{\partial\beta}>0$. As a result, $f_3(\alpha,\beta)>f_3(0,0)=0$. Therefore, there exists a $\overline{b_2}=f_3(\alpha,\beta)>0$ such that when $b<\overline{b_2}$, $\pi_{GC}^{AB}>\pi_{GC}^{NB}$.

(vi) If $T\in(\overline{T}',\overline{T})$, then $\pi_{GC}^{AB}-\pi_{GC}^{NB}=\frac{k_2Q^2}{2}\frac{1}{T-Q\sqrt{\frac{k_1}{2C_m}}}-\frac{Q\sqrt{2k_2R_g}}{1+\beta}+R_gT-\frac{R_gQ}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}-bQ$. Let $\pi_{GC}^{AB}-\pi_{GC}^{NB}>0$, we have $b<\frac{k_2Q}{2}\frac{1}{T-Q\sqrt{\frac{k_1}{2C_m}}}-\frac{\sqrt{2k_2R_g}}{1+\beta}+\frac{R_gT}{Q}-\frac{R_g}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}$. Let $f_4(\alpha,\beta)=\frac{k_2Q}{2}\frac{1}{T-Q\sqrt{\frac{k_1}{2C_m}}}-\frac{\sqrt{2k_2R_g}}{1+\beta}+\frac{R_gT}{Q}-\frac{R_g}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}$, we can obtain that $\frac{\partial f_4(\alpha,\beta)}{\partial T}=\frac{k_2Q}{2}\frac{-1}{(T-Q\sqrt{\frac{k_1}{2C_m}})^2}+\frac{R_g}{Q}$ and $\frac{\partial^2 f_4(\alpha,\beta)}{\partial T^2}=\frac{k_2Q}{(T-Q\sqrt{\frac{k_1}{2C_m}})^3}$. Since $T>Q\sqrt{\frac{k_1}{2C_m}}$, it follows that $\frac{\partial^2 f_4(\alpha,\beta)}{\partial T^2}>0$. As a result, $\frac{\partial f_4(\alpha,\beta)}{\partial T}<\frac{\partial f_4(\alpha,\beta)}{\partial T}|_{T=\overline{T}}=0$, which means that $f_4(\alpha,\beta)$ is strictly decreasing on the interval $(\overline{T}',\overline{T})$. Therefore, it follows that for any α and β, we have $f_4(\alpha,\beta)>f_4(\alpha,\beta)|_{T=\overline{T}}=\frac{\beta\sqrt{2k_2R_g}}{1+\beta}+\frac{\alpha}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}>0$. There exists a $\overline{b_2}=f_4(\alpha,\beta)>0$ such that when $b<\overline{b_2}$, $\pi_{GC}^{AB}>\pi_{GC}^{NB}$.

(v) If $T<\underline{T'}$, then $\pi_{GC}^{AB}-\pi_{GC}^{NB}=\frac{\alpha}{1+\alpha}R_gQ\sqrt{\frac{k_1}{2C_m}}+\frac{\beta}{1+\beta}\sqrt{2k_2C_g}Q-bQ$. Let $\pi_{GC}^{AB}-\pi_{GC}^{NB}>0$, we have $b<\frac{\alpha}{1+\alpha}R_g\sqrt{\frac{k_1}{2C_m}}+\frac{\beta}{1+\beta}\sqrt{2k_2C_g}$. Let $f_5(\alpha,\beta)=\frac{\alpha}{1+\alpha}R_g\sqrt{\frac{k_1}{2C_m}}+\frac{\beta}{1+\beta}\sqrt{2k_2C_g}$. It is straightforward to deduce that $\frac{\partial f_3(\alpha,\beta)}{\partial\alpha}>0$ and $\frac{\partial f_3(\alpha,\beta)}{\partial\beta}>0$. Hence, $f_5(\alpha,\beta)>f_5(0,0)=0$. Therefore, it follows that for any α and β, there exists a $\overline{b_2}=f_5(\alpha,\beta)>0$ such that when $b<\overline{b_2}$, $\pi_{GC}^{AB}>\pi_{GC}^{NB}$.

(2) When $\underline{T}'<\overline{T}'<\underline{T}<\overline{T}$, except for the third case (i.e., $T\in(\overline{T}',\underline{T})$), the calculations for the remaining four intervals are identical to those in case (1). If $T\in(\overline{T}',\underline{T})$, then $\pi_{GC}^{AB}-\pi_{GC}^{NB}=(R_g-C_g)T+\sqrt{2k_2C_g}Q+C_gQ\sqrt{\frac{k_1}{2C_m}}-\frac{R_gQ\sqrt{k_1/2C_m}}{1+\alpha}-\frac{\sqrt{2k_2R_g}Q}{1+\beta}$. Let $\pi_{GC}^{AB}-\pi_{GC}^{NB}>0$, we

have $b < (R_g - C_g)\frac{T}{Q} + \sqrt{2k_2C_g} + C_g\sqrt{\frac{k_1}{2C_m}} - \frac{R_g\sqrt{k_1/2C_m}}{1+\alpha} - \frac{\sqrt{2k_2R_g}}{1+\beta}$. Let $f_3^{'}(\alpha,\beta) = (R_g - C_g)\frac{T}{Q} + \sqrt{2k_2C_g} + C_g\sqrt{\frac{k_1}{2C_m}} - \frac{R_g\sqrt{k_1/2C_m}}{1+\alpha} - \frac{\sqrt{2k_2R_g}}{1+\beta}$, we can deduce that $\frac{\partial f_3^{'}(\alpha,\beta)}{\partial T} = \frac{R_g - C_g}{Q} < 0$, which indicates that $f_3^{'}(\alpha,\beta)$ is strictly decreasing on the interval $(\overline{T}',\underline{T})$. Hence, $f_3^{'}(\alpha,\beta) > f_3^{'}(\alpha,\beta)|_{T=\underline{T}}$. It is easy to find that $\frac{\partial f_3^{'}(\alpha,\beta)|_{T=\underline{T}}}{\partial\alpha} > 0$ and $\frac{\partial f_3^{'}(\alpha,\beta)|_{T=\underline{T}}}{\partial\beta} > 0$, then $f_3^{'}(\alpha,\beta)|_{T=\underline{T}} > f_3^{'}(0,0)|_{T=\underline{T}} > 0$. Therefore, there exists a $\overline{b_2} = f_3^{'}(\alpha,\beta) > 0$ such that when $b < \overline{b_2}$, $\pi_{GC}^{AB} > \pi_{GC}^{NB}$.

In conclusion, regardless of the interval T belongs to, there always exists an upper bound $\overline{b_2} > 0$ such that $\pi_{GC}^{AB} > \pi_{GC}^{NB}$ when $b < \overline{b_2}$.

Proof of Corollary 1

From Propositions 1 and 2, it directly follows that when $b < \min\{\overline{b_1},\overline{b_2}\}$, both $\pi_{PM}^{AB} > \pi_{PM}^{NB}$ and $\pi_{GC}^{AB} > \pi_{GC}^{NB}$ hold simultaneously.

Proof of Proposition 3

According to Proposition 2, $f_2(\alpha,\beta) - f_1(\alpha,\beta) = -\frac{C_g}{Q}\frac{(T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}} - \frac{Q}{1+\beta}\sqrt{\frac{k_2}{2C_g}})^2}{(T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}})} \le 0$, the equality holds when $T = \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}} + \frac{Q}{1+\beta}\sqrt{\frac{k_2}{2C_g}} = \underline{T}'$.

To compare $f_3(\alpha,\beta)$ and $f_2(\alpha,\beta)$, we calculate the partial derivative of $f_3(\alpha,\beta)$ with respect to T and obtain $\frac{\partial f_3(\alpha,\beta)}{\partial T} = \frac{k_2Q}{2}(\frac{1}{(T - Q\sqrt{\frac{k_1}{2C_m}})^2} + \frac{1}{(1+\beta)^2(T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}})^2})$. Let $(1+\beta)(T - \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}) > (T - Q\sqrt{\frac{k_1}{2C_m}})$, we have $T > \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}(1 - \frac{\alpha}{\beta})$. When $\alpha \ge \beta$, T strictly exceeds $\frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}(1 - \frac{\alpha}{\beta})$ throughout the interval $(\underline{T},\overline{T}')$, and consequently, $\frac{\partial f_3(\alpha,\beta)}{\partial T} < 0$. When $\alpha < \beta$, $\frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}(1 - \frac{\alpha}{\beta}) < \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}} \le Q\sqrt{\frac{k_1}{2C_m}} < Q\sqrt{\frac{k_1}{2C_m}} + Q\sqrt{\frac{k_2}{2C_g}} = \underline{T}$. Hence, $T > \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}}(1 - \frac{\alpha}{\beta})$ holds when $T \in (\underline{T},\overline{T}')$. In summary, for any $T \in (\underline{T},\overline{T}')$, we have $\frac{\partial f_3(\alpha,\beta)}{\partial T} < 0$. By comparing $f_3(\alpha,\beta)|_{T=\underline{T}}$ and $f_2(\alpha,\beta)|_{T=\underline{T}}$, we have

$f_3(\alpha,\beta)|_{T=\underline{T}} - f_2(\alpha,\beta)|_{T=\underline{T}} = 0$, then $f_3(\alpha,\beta) \le f_3(\alpha,\beta)|_{T=\underline{T}} = f_2(\alpha,\beta)|_{T=\underline{T}} \le f_2(\alpha,\beta)$. Moreover, we also find that $f_3^{'}(\alpha,\beta)|_{T=\underline{T}} - f_2(\alpha,\beta)|_{T=\underline{T}} = 0$, then we have $f_3^{'}(\alpha,\beta) \le f_3^{'}(\alpha,\beta)|_{T=\underline{T}} = f_2(\alpha,\beta)|_{T=\underline{T}} \le f_2(\alpha,\beta)$.

Similarly, we can deduce that $f_4(\alpha,\beta)|_{T=\overline{T'}} - f_3(\alpha,\beta)|_{T=\overline{T'}} = 0$. Since $\frac{\partial f_4(\alpha,\beta)}{\partial T} < 0$ in $(\overline{T'},\overline{T})$, we have $f_4(\alpha,\beta) \le f_4(\alpha,\beta)|_{T=\overline{T'}} = f_3(\alpha,\beta)|_{T=\overline{T'}} \le f_3(\alpha,\beta)$. In a similar way, we can deduce the result that $f_4(\alpha,\beta) \le f_4(\alpha,\beta)|_{T=\overline{T'}} = f_3^{'}(\alpha,\beta)|_{T=\overline{T'}} \le f_3^{'}(\alpha,\beta)$ Likewise, we observe that $f_5(\alpha,\beta) - f_4(\alpha,\beta)|_{T=\overline{T}} = 0$, so $f_5(\alpha,\beta) = f_4(\alpha,\beta)|_{T=\overline{T}} < f_4(\alpha,\beta)$. In summary, we have $f_5(\alpha,\beta) \le f_4(\alpha,\beta) \le \{f_3(\alpha,\beta), f_3'(\alpha,\beta)\} \le f_2(\alpha,\beta) \le f_1(\alpha,\beta)$. The equalities hold only at the common boundary of the two intervals to which T belongs.

Proof of Proposition 4

By comparing the results of Lemma 2 and Lemma 3, we can readily obtain $r_p^{AB} = r_p^{NB}$, $L^{AB} = \frac{1}{1+\alpha} L^{NB}$, and if $T < \underline{T'}$ or $T > \overline{T'}$, then $r_a^{AB} = r_a^{NB}$; if $\underline{T'} < T < \overline{T'}$, then $r_a^{AB} < r_a^{NB}$.

Proof of Proposition 5

(1) Since we assume that $\alpha \in (0,1)$ and $\beta \in (0,1)$, it is readily to verify that $\underline{T'} = \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}} + \frac{Q}{1+\beta}\sqrt{\frac{k_2}{2C_g}} < Q\sqrt{\frac{k_1}{2C_m}} + Q\sqrt{\frac{k_2}{2C_g}} = \underline{T}$.

(2) We find that $\Delta^{'} = \overline{T'} - \underline{T'} = \frac{Q}{1+\beta}\sqrt{\frac{k_2}{2R_g}} - \frac{Q}{1+\beta}\sqrt{\frac{k_2}{2C_g}}$ and $\Delta = \overline{T} - \underline{T} = Q\sqrt{\frac{k_2}{2R_g}} - Q\sqrt{\frac{k_2}{2C_g}}$, implying that $\Delta^{'} = \frac{1}{1+\beta}\Delta$.

(3) $\overline{T'} = \frac{Q}{1+\alpha}\sqrt{\frac{k_1}{2C_m}} + \frac{Q}{1+\beta}\sqrt{\frac{k_2}{2R_g}} < Q\sqrt{\frac{k_1}{2C_m}} + Q\sqrt{\frac{k_2}{2R_g}} = \overline{T}$