Linear Regression using Gradient Descent

In this notebook, I've built a simple linear regression model using gradient descent to fit a line ($y = mx + c$) through input data consisting of (x, y) values. The purpose is to better understand gradient descent by calculating the gradients by hand and implementing it without using out-of-the-box libraries/ methods.

Problem Statement

Given a set of x and y values, the problem statement here is to find a line "$y = mx + b$" that best fits the data. This involes finding the values of m and b that minimizes an *Error/ Loss* function for the given dataset. We will use *Mean Squared Error (denoted as E)* as the error function to be minimized, and it is given by -

$$E = \frac{1}{N} \sum_{i=1}^{N} (y_i - (mx_i + b))^2$$

To find the optimial value of m and b that minimizes the error function using Gradient Descent, the values are randomly initialized first and then updated $number of steps$ times or until the error function value converges. Below equations are used to update the values of m and b at each step -

$$m = m - learning\_rate \times \frac{\partial E}{\partial m}$$

$$b = b - learning\_rate \times \frac{\partial E}{\partial b}$$

Calculating Gradients for Mean Squared Error

The Gradient Descent update formulae given above needs the partial derivatives of error function E with respect to both m and b ($\frac{\partial E}{\partial m}$ and $\frac{\partial E}{\partial b}$ respectively). These partial derivatives are calculated as follows -

$$\begin{aligned} E &= \frac{1}{N} \sum_{i=1}^{N} (y_i - (mx_i + b))^2 \\ &= \frac{1}{N} \sum_{i=1}^{N} y_i^2 - 2y_i(mx_i + b) + (mx_i + b)^2 \\ &= \frac{1}{N} \sum_{i=1}^{N} y_i^2 - 2x_i y_i m - 2y_i b + x_i^2 m^2 + 2x_i mb + b^2 \end{aligned}$$

$$\begin{aligned} \frac{\partial E}{\partial m} &= \frac{1}{N} \sum_{i=1}^{N} 0 - 2x_i y_i - 0 + 2x_i^2 m + 2x_i b + 0 \\ &= \frac{-2}{N} \sum_{i=1}^{N} x_i (y_i - (mx_i + b)) \end{aligned}$$

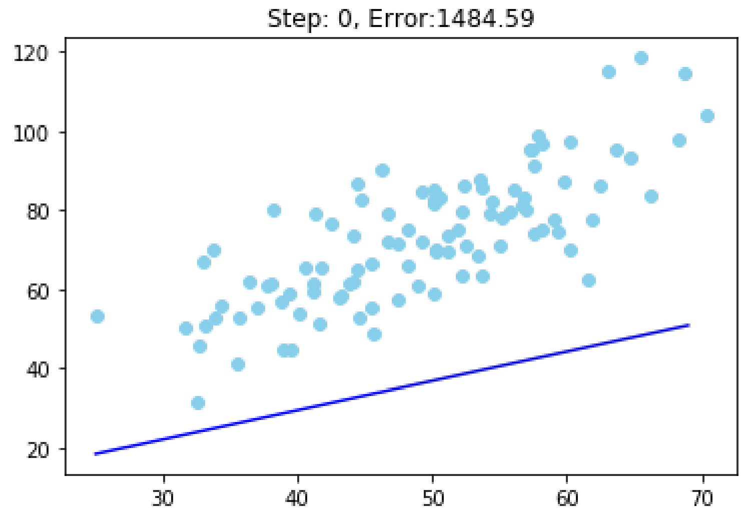
$$
\begin{aligned}
\frac{\partial E}{\partial b} &= \frac{1}{N}\sum_{i=1}^{N} 0 - 0 - 2y_i + 0 + 2x_i m + 2b \\
&= \frac{-2}{N}\sum_{i=1}^{N}(y_i - (mx_i + b))
\end{aligned}
$$

Implementation

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

%matplotlib inline

def plot_fit(points, m, b, step, error):
    '''
    Plots a scatterplot of points, with a line defined by m and b.
    Also shows the current step number in Gradient Descent and
    current error value.
    '''

    x_values = [p[0] for p in points]
    y_values = [p[1] for p in points]

    plt.scatter(x_values, y_values, color='skyblue')
    fit_xvalues = list(range(int(min(x_values)), int(max(x_values))))
    fit_yvalues = [m * fit_x + b for fit_x in fit_xvalues]
    plt.plot(fit_xvalues, fit_yvalues, color='blue')
    plt.title('Step: ' + str(step) + ', Error:' + str(round(error, 2)))
    plt.show()

def mean_squared_error(points, m, b):
    '''
    Calculates Mean Squared Error for the line defined by m & b
    with the given points.
    '''
    n = len(points)
    error = 0

    for i in range(n):
        x_i = points[i, 0]
        y_i = points[i, 1]

        error += (y_i - (m * x_i + b)) **2

    error = float(error) / n

    return error

def gradient_step(points, current_m, current_b, learning_rate):
    '''
    Calculates gradients and new values for m and b based on
    current values of m and b and given learning rate.
    '''
```

```
    gradient_m = 0
    gradient_b = 0

    n = len(points)

    for i in range(n):
        x_i = float(points[i, 0])
        y_i = float(points[i, 1])

        gradient_m += (-2 / n) * (x_i * (y_i - (current_m * x_i + curren
t_b)))
        gradient_b += (-2 / n) * (y_i - (current_m * x_i + current_b))

    new_m = current_m - learning_rate * gradient_m
    new_b = current_b - learning_rate * gradient_b
    new_error = mean_squared_error(points, new_m, new_b)

    return [new_m, new_b, new_error]

def gradient_descent(points, initial_m, initial_b, learning_rate, num_st
eps):
    '''
    Runs Gradient Descent for the given points, starting with the initia
l
    m and b values, and the given learning rate for the given number of
 steps.
    '''
    m = initial_m
    b = initial_b

    error_vec = list()

    # Update m and b for each gradient descent step
    for step in range(num_steps):

        [m, b, error] = gradient_step(points, m, b, learning_rate)

        # Plot the current fit once every 100 steps to track progress
        if not step % 100:
            plot_fit(points, m, b, step, error)

        error_vec.append(error)

    # Plot the error function value for each step
    plt.plot(error_vec, color='red')
    plt.xlabel('Number of Steps')
    plt.ylabel('Mean Squared Error')
    plt.show()

    return [m, b]

def run():
    points = np.genfromtxt('data.csv', delimiter=',')

    initial_m = 0
    initial_b = 0
```

```
    learning_rate = 0.0001
    num_steps = 1000

    m, b = gradient_descent(points, initial_m, initial_b, learning_rate,
 num_steps)

if __name__ == '__main__':
    run()
```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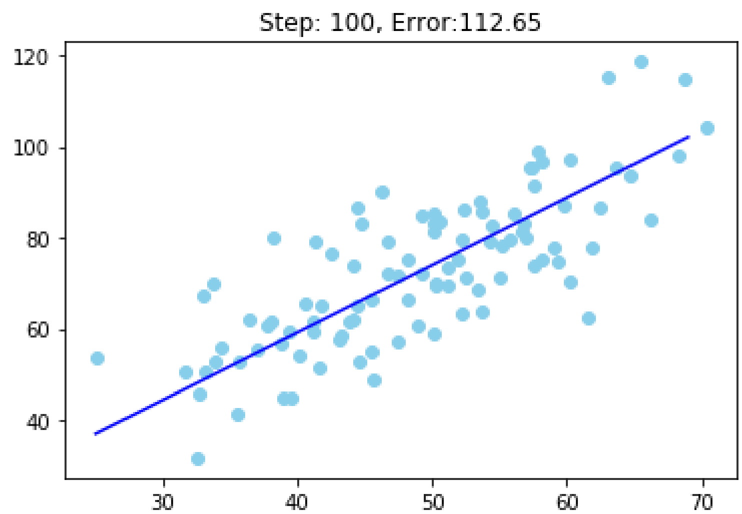
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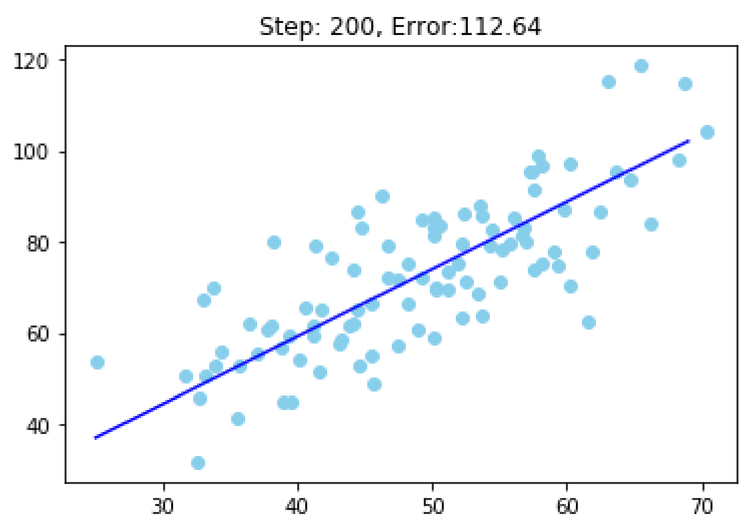
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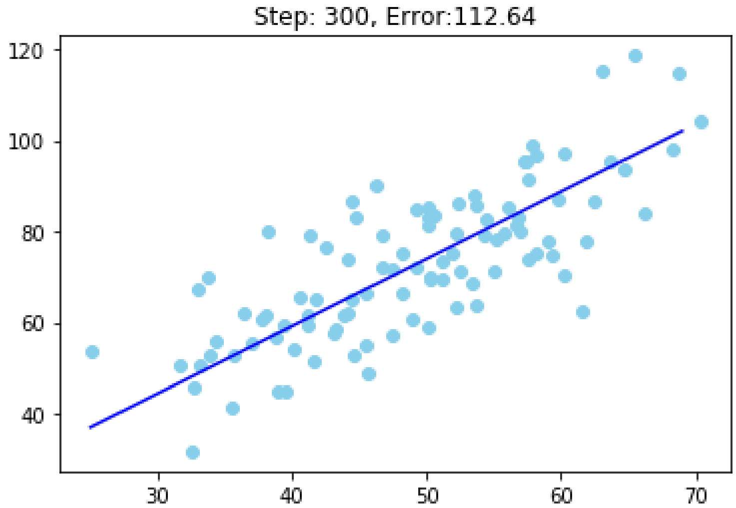
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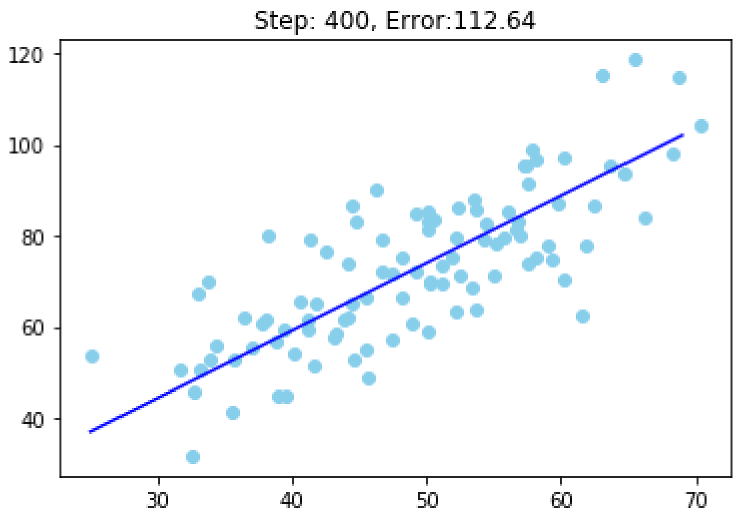
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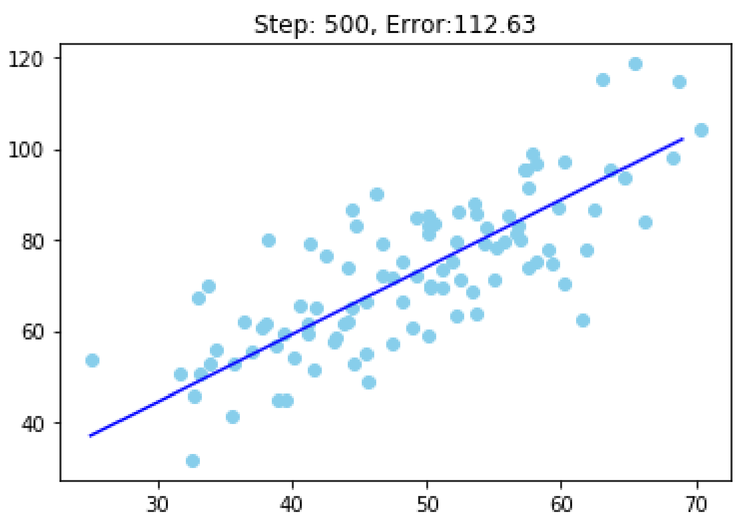
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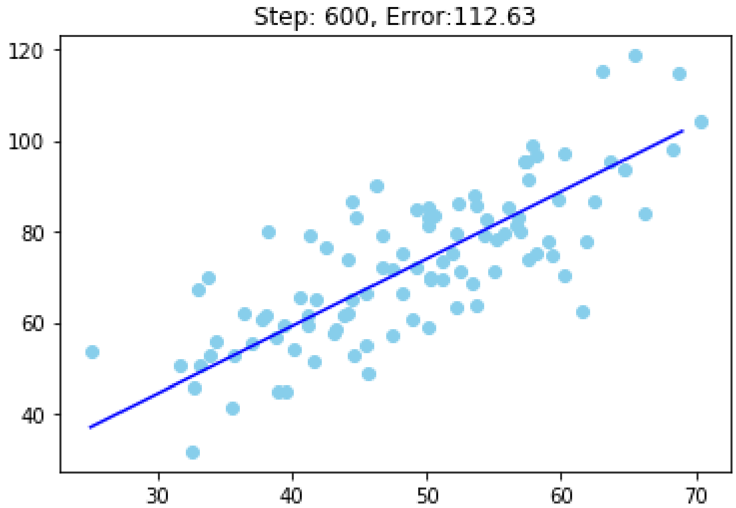
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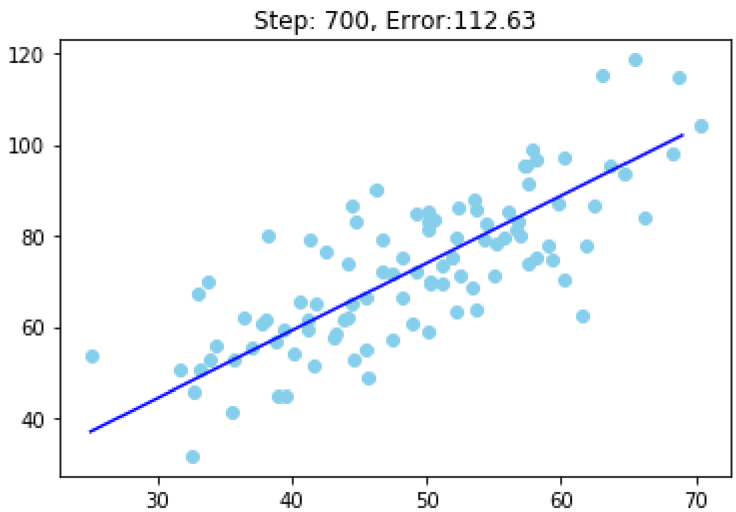
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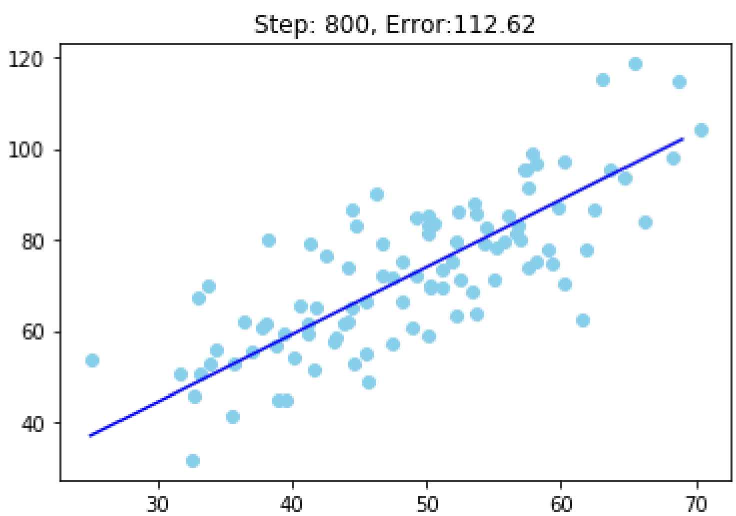
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