(1)

6.11 Setup

Unbiased estimators $\hat{\theta}_1$ and $\hat{\theta}_2$

Combined estimator $\hat{\theta}_c = c\,\hat{\theta}_1 + (1-c)\,\hat{\theta}_2$

Rearranging the estimator $\hat{\theta}_c$

$$\hat{\theta}_c = c\hat{\theta}_1 + \hat{\theta}_2 - c\hat{\theta}_2$$
$$\hat{\theta}_c = \hat{\theta}_2 + c\,(\hat{\theta}_1 - \hat{\theta}_2)$$

Now $\hat{\theta}_c$ is convenient for variance expansion.

The equation 6.11 is

$$\mathrm{Var}(\hat{\theta}_c) = \mathrm{Var}(\hat{\theta}_2) + c^2\,\mathrm{Var}(\hat{\theta}_1 - \hat{\theta}_2) + 2c\,\mathrm{Cov}(\hat{\theta}_2, \hat{\theta}_1 - \hat{\theta}_2) \quad (1)$$

The variance of the difference b/w the estimators:

$$\mathrm{Var}(\hat{\theta}_2 - \hat{\theta}_1) = \mathrm{Var}(\hat{\theta}_1) + \mathrm{Var}(\hat{\theta}_2) - 2\,\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2) \quad (2)$$

Covariance term

$$\mathrm{Cov}(\hat{\theta}_2, \hat{\theta}_1 - \hat{\theta}_2) = \mathrm{Cov}(\hat{\theta}_2, \hat{\theta}_1) - \mathrm{Var}(\hat{\theta}_2) \quad (3)$$

replacing equation (2) and (3) in (1)

$$\mathrm{Var}[\hat{\theta}_c] = \mathrm{Var}(\hat{\theta}_2) + c^2\left[\mathrm{var}(\hat{\theta}_1 + \mathrm{Var}(\hat{\theta}_2) - 2\,\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2)\right] + 2c\left[\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2) - \mathrm{var}(\hat{\theta}_2)\right]. \quad (4)$$

The quadratic equation function of c is ~~obtained~~:

$$\mathrm{Var}(\hat{\theta}_c) = Ac^2 + Bc + C$$