$$B = 2[\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2) - \mathrm{Var}(\hat{\theta}_2)] \quad (2)$$

$$C = \mathrm{Var}(\hat{\theta}_2)$$

Minimizing with respect to c.

$$\frac{d}{dc}\mathrm{Var}(\hat{\theta}_c) = \frac{d}{dc}(Ac^2 + Bc + C) = 0$$

$$= 2Ac + B = 0$$

$$\Leftrightarrow 2Ac = -B$$

$$c^* = -\frac{B}{2A}$$

Thus

$$\cancel{c^* = \frac{2[\mathrm{Cov}(\hat{\theta}_1, \theta_2) - \mathrm{Var}(\hat{\theta}_2)]}{2[\mathrm{Var}(\hat{\theta}_1), \hat{\theta}_2) - \mathrm{Var}[\hat{\theta}_2]}}$$

$$\cancel{c^* = \frac{\mathrm{Var}(\hat{\theta}_2) - \mathrm{Cov}(\hat{\theta}_1, \theta_2)}{\mathrm{Var}(\hat{\theta}_1, \hat{\theta}_2) - 2\mathrm{Var}(\hat{\theta}_2)}}$$

$$c^* = -\frac{\cancel{2}[\mathrm{Cov}(\hat{\theta}_1, \theta_2) - \mathrm{Var}(\hat{\theta}_2)]}{\cancel{2}[\mathrm{Var}(\hat{\theta}_1) + \mathrm{Var}(\hat{\theta}_2) - 2\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2)}$$

$$\boxed{c^* = \frac{\mathrm{Var}[\hat{\theta}_2] - \mathrm{Cov}(\hat{\theta}_1, \theta_2)}{\mathrm{Var}(\hat{\theta}_1) + \mathrm{Var}(\hat{\theta}_2) - 2\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2)}}$$

For special case where $\hat{\theta}_1$ and $\hat{\theta}_2$ are antithetic and identically distributed $\sigma_1^2 = \sigma_2^2$ Then

$$\cancel{c^* = \frac{\sigma_2^2 - \sigma_{12}}{2\sigma_1^2 + 2\sigma_2^2} \; 1/2} \qquad c^* = \frac{\sigma_2^2 - \sigma_{12}}{2\sigma_2^2 - 2\sigma_{12}} = 1/2$$

$(\hat{\theta}_2)]$ (2)

$c + c) = 0$

$= 0$

$(\hat{\theta}_2)]$

$(\hat{\theta}_2)$

$- \mathrm{Var}(\hat{\theta}_2)]$

$\mathrm{Var}(\hat{\theta}_2) - 2\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2)$

$\boxed{(\hat{\theta}_1, \hat{\theta}_2) \quad - 2\mathrm{Cov}(\hat{\theta}_1, \hat{\theta}_2)}$

are antithetic and identically

$$\frac{\sigma_2^2 - \sigma_{12}}{2\sigma_2^2 - 2\sigma_{12}} = 1/2$$