

Homework #1

Due date: October 3, 2022 at 10am

1. [Sedgewick & Wayne] 1.1.6, 1.1.11 – 1.1.16, 1.1.18 – 1.1.20, 1.1.31, 1.1.33, 1.2.1, 1.2.6, 1.2.7.
2. Algorithm A takes $1ms = 1 \times 10^{-3} \text{seconds}$ to solve problem P for an input size of 1000. How long will it take for an input size of 10000 if the running time was given by the following running time functions?
 - (a) $T(n) = n$
 - (b) $T(n) = n \lg n$ (base 2)
 - (c) $T(n) = n^2$
 - (d) $T(n) = n^3$
3. Algorithm A takes $1ms = 1 \times 10^{-3} \text{seconds}$ to solve problem P for an input size of 1000. How large a problem can be solved in $1min$ if the running time was given by the following running time functions?
 - (a) $T(n) = n$
 - (b) $T(n) = n \lg n$ (base 2)
 - (c) $T(n) = n^2$
 - (d) $T(n) = n^3$
4. Write an algorithm that given an $N \times N$ matrix (a 2-dimensional array) A of integers and an integer X , determines whether X is in the matrix. Integers in each row are arranged in ascending order from left to right and integers in each column are arranged in ascending order from top to bottom. Note that the naive (brute-force search) is not efficient.
5. Given the following algorithm.

```
MYSTERY( $A[0 : n - 1], p, r$ )
1  if  $r - p + 1 == 1$  // Only one element
2      return  $A[p]$ 
3  if  $r - p + 1 == 2$  // Only two elements
4      return MAXIMUM( $A[p], A[r]$ )
5   $mid = p + (r - p) / 2$ 
6   $x_1 = \text{MYSTERY}(A, p, mid)$ 
7   $x_2 = \text{MYSTERY}(A, mid + 1, r)$ 
8  return MAXIMUM( $x_1, x_2$ )
```

Given $A = [3, 5, -1, 4, 6, 2, 1, 5, 5, 7, 4, 0, 3, 2, -1, 3]$, draw the recursion tree of the calls when the algorithm is executed with the call $\text{MYSTERY}(A, 0, 15)$.

6. Write a RAM program to read $n = 5$ positive integers and print them in nondecreasing order. What is the uniform cost time and space complexity of your RAM program in terms on n ?