Chapter2

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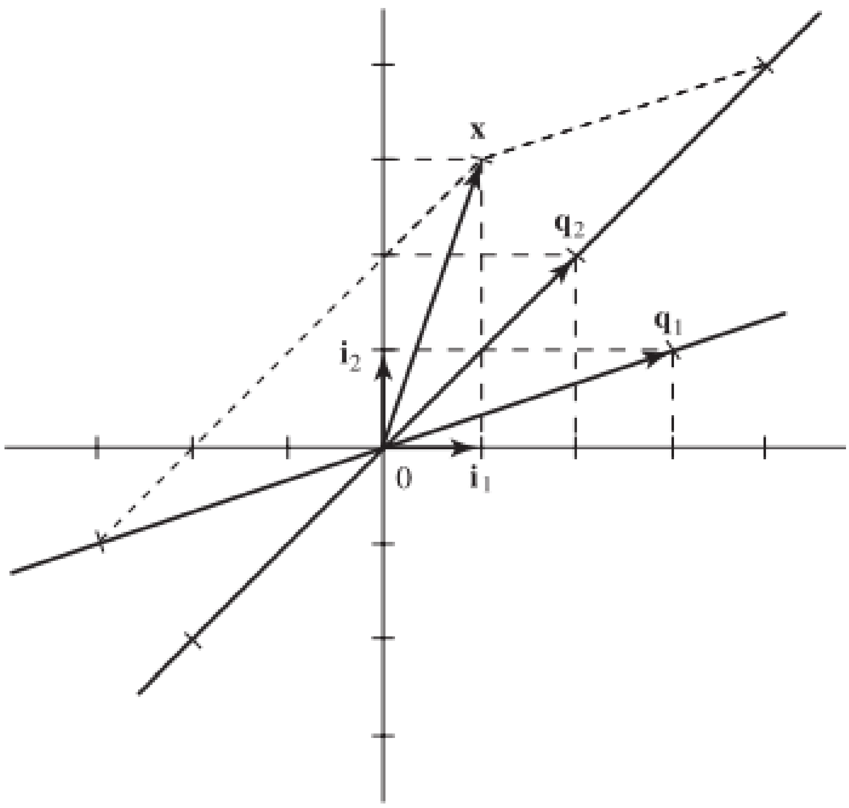
3.1



From the above figure, The three vectors $\boldsymbol{q}_1 = \begin{bmatrix} 3 & 1 \end{bmatrix}', \boldsymbol{i}_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}'$ and $\boldsymbol{q}_2 = \begin{bmatrix} 2 & 2 \end{bmatrix}'$
The representation of x with respect to $\{\boldsymbol{q}_1, \boldsymbol{i}_2\}$ is $\begin{bmatrix} \frac{1}{3} & \frac{8}{3} \end{bmatrix}'$
The representation of $\boldsymbol{q}_1$ with respect to $\{\boldsymbol{i}_2, \boldsymbol{q}_2\}$ is $\begin{bmatrix} -2 & \frac{3}{2} \end{bmatrix}'$
These can be verified like this:

$$x = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{q}_1 & \boldsymbol{i}_2 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{8}{3} \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{8}{3} \end{bmatrix}$$

3.2

i:The norm of x_1

1-norm: $\|x_1\|_1=\sum_{i=1}^{3}|x_i|=|2|+|-3|+|1|=6$
2 -norm: $\|\boldsymbol{x}_1\|_2=\left(\sum_{i=1}^{3}|x_i|^2\right)^{\frac{1}{2}}=\sqrt{2^2+|-3|^2+1^2}=\sqrt{14}$
infinite-norm: $\|\boldsymbol{x}_1\|_\infty=\max_i|x_i|=3$

ii:The norm of x_2

1 -norm: $\|\boldsymbol{x}_2\|_1=\sum_{i=1}^{3}|x_i|=|1|+|1|+|1|=3$
2-norm: $\|\boldsymbol{x}_2\|_2=\left(\sum_{i=1}^{3}|x_i|^2\right)^{\frac{1}{2}}=\sqrt{1^2+1^2+1^2}=\sqrt{3}$
infinite-norm: $\|\boldsymbol{x}_2\|_\infty=\max_i|x_i|=1$

3.3

This is just the orthonormalization procedure.

$$\left\{\begin{array}{ll} u_1=\alpha_1 & q_1=u_1/\|u_1\| \\ u_2=\alpha_2-(q_1'\alpha_2)\,q_1 & q_2=u_2/\|u_2\| \end{array}\right.$$

This is the ordinary method,what we find is the two vector are orthogonal. so,we just need to make the length of vector is 1 .

$$q_1=\frac{u_1}{\|u_1\|}=\begin{bmatrix}\frac{2}{\sqrt{14}} & -\frac{3}{\sqrt{14}} & \frac{1}{\sqrt{14}}\end{bmatrix}'$$
$$q_1=\frac{u_2}{\|u_2\|}=\begin{bmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{bmatrix}'$$

3.4

a

if n>m,$\boldsymbol{A}\boldsymbol{A}^{'}$ is a ordinary vector,which has the rank m

b

if m=n,so $\boldsymbol{A}$ is a nonsingular square matrix,we already have $\boldsymbol{A}^{'}\boldsymbol{A}=\boldsymbol{I}_m$,so $\boldsymbol{A}'=\boldsymbol{A}^{-1}$. $\boldsymbol{A}\boldsymbol{A}^{'}=\boldsymbol{A}\boldsymbol{A}^{-1}=\boldsymbol{I}_n$

3.5

Accoridng to the principle:

$$Nillity(\boldsymbol{A}=numberofcolumnsof\boldsymbol{A}-rank((A)))$$