Chapter3

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7.1

$$\hat{g(s)} = \frac{s-1}{(s^2-1)(s+2)} = \frac{s-1}{s^3+2s^2-s-2}$$

so,from the equation (7.9),we can get the three-dimensional controllable realization:

$$\dot{x} = \begin{bmatrix} -2 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} x$$

it is obviously the $\hat{g(s)}$ is not coprime fraction,so the controllable realization is not observable.

7.2

from the quation (7.14), we can easily get the three-dimensional observable realization:

$$\dot{x} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

similarly, it is not controllable.

7.3

from the inverse canonical decomposition,we can add an uncontrollable state to problem 7.1:

$$\dot{x} = \begin{bmatrix} -2 & 1 & 2 & a_1 \\ 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & a_3 \\ 0 & 0 & 0 & a_4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & -1 & c_4 \end{bmatrix} x$$

where a_i and c_i are arbitrary value,this is an uncontrollable and unobservable realization. the transfer function can be reduced to a coprime fraction,which is:

$$\hat{g(s)} = \frac{1}{(s+1)(s+2)} = \frac{1}{s^2+3s+2}$$

A minimal realization can be realized through controllable realization, this realization is two-dimensional

$$\dot{x} = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

7.4

from the equation (7.27),we can get:

$$\begin{aligned} D(s) &= -2 - s + 2s^2 + s^3 \\ N(s) &= -1 + s \\ \overline{D}(s) &= \overline{D}_0 + \overline{D}_1 s + \overline{D}_2 s^2 \\ \overline{N}(s) &= \overline{N}_0 + \overline{N}_1 s + \overline{N}_2 s^2 \end{aligned}$$

the Sylvester resultant:

$$\mathbf{SM} := \begin{bmatrix} -2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -2 & -1 & 0 & 0 \\ 2 & 0 & -1 & 1 & -2 & -1 \\ 1 & 0 & 2 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -\overline{N}_0 \\ \overline{D}_0 \\ -\overline{N}_1 \\ \overline{D}_1 \\ -\overline{N}_2 \\ \overline{D}_2 \end{bmatrix} = 0$$

the rank of **S** is 5, so there are 2 linearly indepedent N-columns,the degree of the transfer function is 2 .we can also calculate a monic null vector

$$z = \begin{bmatrix} -1 & 2 & 0 & 3 & 0 & 1 \end{bmatrix}'$$

7.5

just as the problem (7.4) show,in the same way:

$$D(s) = -1 + 4s^2$$
$$N(s) = -1 + 2s$$
$$\overline{D}(s) = \overline{D}_0 + \overline{D}_1 s$$
$$\overline{N}(s) = \overline{N}_0 + \overline{N}_1 s$$

the Sylvester resultant:

$$\mathbf{SM} := \begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & -1 \\ 4 & 0 & 0 & 2 \\ 0 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} -\overline{N}_0 \\ \overline{D}_0 \\ -\overline{N}_1 \\ \overline{D}_1 \end{bmatrix} = 0$$

the rank of $\mathbf{S}$ is 3,we can calculate a monic null vector

$$z = \begin{bmatrix} -0.5 & 0.5 & 0 & 1 \end{bmatrix}'$$

so the coprime fraction is:

$$\hat{g(s)} = \frac{1}{2s+1}$$

7.6

The Sylvester resultant by arranging the coefficients of $N(s)$ and $D(s)$ in descending powers:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

the second D-columns is linearly depedent of its LHS columns, so it is not true that all D-columns are linearly indepedent of their LHS columns. the degree of $\hat{g(s)}$ is 1,but the linearly indepedent N-columns is 2.

7.7

the realization is a controllable realization,the controllable realization is observable if and only if the transfer function is coprime fraction.$D(s)$ and $N(s)$ are coprime if and only if the Sylvester resultant is nonsingular.

$$\hat{g(s)} = \frac{N(s)}{D(s)} = \frac{\beta_1 s + \beta_2}{s^2 + \alpha_1 s + \alpha_2}$$

the Sylvester resultant:

$$\mathbf{S} := \begin{bmatrix} \alpha_2 & \beta_2 & 0 & 0 \\ \alpha_1 & \beta_1 & \alpha_2 & \beta_2 \\ 1 & 0 & \alpha_1 & \beta_1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The determinant of $\mathbf{S}$ is $-\alpha_2\beta_1^2 + \alpha_1\beta_1\beta_2 - \beta_2^2$ for the controllable realization, its observability matrix:

$$O = \begin{bmatrix} \beta_1 & \beta_2 \\ -\alpha_1\beta_1 + \beta_2 & -\alpha_2\beta_1 \end{bmatrix}$$

The realization is observable if and only if the observability matrix is full column rank,the determinant of O is $-\alpha_2\beta_1^2 + \alpha_1\beta_1\beta_2 - \beta_2^2$ Thus the two condition is same.

7.8

Let us consider a transfer function:

$$\hat{g(s)} = \frac{N(s)}{D(s)} = \frac{\beta_1 s^2 + \beta_2 s + \beta_3}{s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3}$$

its Sylvester resultant:

$$\mathbf{S} := \begin{bmatrix} \alpha_3 & \beta_3 & 0 & 0 & 0 & 0 \\ \alpha_2 & \beta_2 & \alpha_3 & \beta_3 & 0 & 0 \\ \alpha_1 & \beta_1 & \alpha_2 & \beta_2 & \alpha_3 & \beta_3 \\ 1 & 0 & \alpha_1 & \beta_1 & \alpha_2 & \beta_2 \\ 0 & 0 & 1 & 0 & \alpha_1 & \beta_1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The observability matrix:

$$O = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \\ -\alpha_1\beta_1 + \beta_2 & -\alpha_2\beta_1 + \beta_3 & -\alpha_3\beta_1 \\ (\alpha_1^2 - \alpha_2)\beta_1 - \alpha_1\beta_2 + \beta_3 & (\alpha_1\alpha_2 - \alpha_3)\beta_1 - \alpha_2\beta_2 & \alpha_1\alpha_3\beta_1 - \alpha_3\beta_2 \end{bmatrix}$$

7.9

its controllable realization is:

$$\dot{x} = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

its observability matrix is:

$$O = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

its determinant is nonzero, so the realization is observable. The Sylvester resultant of D(s) and N(S) is:

$$\mathbf{S} := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

its determinant is also nonzero,so the Sylvester resultant is nonsingular.

7.10

$$g\hat{(s)} = \frac{1}{(s+1)^2} = s^{-2} - 2s^{-3}$$

the irreducible companion form:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$