Chapter9

31202008881 Bao Ze an

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9.1

for $D(s) = s^2 - 1$,$N(s) = s - 2$,because $D(s)$ and $N(s)$ are coprime.The solution is exist in

$$A(s)D(s) + B(s)N(s) = s^2 + 2s + 2$$

from the example 9.1,we have for any polynomial $Q(s)$

$$A(s) = \frac{1}{3}(s^2 + 2s + 2) + Q(s)(-s + 2)$$
$$B(s) = \frac{-1}{3}(s + 2)(s^2 + 2s + 2) + Q(s)(s^2 - 1)$$

is a solution we can't find a solution with degB(s) $\leq$ degA(s) in the equation.
for any $Q(s) = q_0$ of degree 0,we have degB(s) $>$ degA(s).
for any $Q(s) = q_0 + q_1 s$ of degree 1,we have degB(s) $>$ degA(s).
for any $Q(s) = q_0 + q_1 s + q_2 s^2$ of degree 3,we also have degB(s) $>$ degA(s).
proceeding forward,we can conclude that there exist no solution with degB(s) $\leq$ degA(s) in the equation.

9.2

for $F(s) = (s+2)(s+1+j1)(s+1-j1) = s^3 + 4s^2 + 6s + 4$,$N(s) = s - 1$,$D(s) = s^2 - 4$ $N(s)$ and $D(s)$ are coprime.

$$D_0 = -4, D_1 = 0, D_2 = 1$$
$$N_0 = -1, N_1 = 1, N_2 = 0$$

Suppose

$$A(s) = A_0 + A_1 s$$
$$B(s) = B_0 + B_1 s$$

$$\begin{bmatrix} A_0 & B_0 & A_1 & B_1 \end{bmatrix} \begin{bmatrix} -4 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -4 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 4 & 1 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} A_0 & B_0 & A_1 & B_1 \end{bmatrix} = \begin{bmatrix} -6 & 20 & 1 & 10 \end{bmatrix}$
The compensator equation is $C(s) = \frac{B(s)}{A(s)} = \frac{10s+20}{s-6}$
To track any step reference input,$\hat{g}_0(0) = 1$,so $p = \frac{F_0}{B_0 N_0} = \frac{4}{20\times -1} = -0.2$.

9.3

for the compensator computed in the problem 9.2,if the transfer function changes to $\overline{\hat{g}}(s) = \frac{s-0.9}{s^-4.1}$

$$\begin{aligned} \hat{g}_0(s) =& \frac{pB(s)N(s)}{A(s)D(s) + B(s)N(s)} \\ =& \frac{-0.2(20+10s)(s-0.9)}{(s-6)(s^2-4.1) + (10s+20)(s-0.9)} \end{aligned}$$

$$\hat{g}_0(0) = \frac{3.6}{6.6} = 0.55$$

so the overall system can't track asymptotically any step reference input.

1. The compensator of degree 3
   First,we introduce a internal model $\frac{1}{\Phi(s)} = \frac{1}{s}$,Then $B(s)/A(s)$ can be solved form:

   $$A(s)D(s)\Phi(s) + B(s)N(s) = F(s)$$

   Because $\overline{D}(s) = D(s)\Phi(s)$ has degree 3,we may select $A(s)$ and $B(s)$ to have degree 2.Then $F(s)$ has degree 5.
   $F(s) = (s+2)(s+1+j1)(s+1-j1)(s+3)(s+3) = s^5 + 10s^4 + 39s^3 + 76s^2 + 78s + 36$

   $$\begin{aligned} &\overline{D}(s) = D(s)\Phi(s) = (s^2-4.1)s = 0 - 4.1s + 0\dot{s}^2 + 1\dot{s}^3 \\ &N(s) = s - 0.9 = -0.9 + 1\dot{s} + 0\dot{s}^2 + 0\dot{s}^3 \end{aligned}$$

   $$\begin{bmatrix} A_0 & B_0 & A_1 & B_1 & A_2 & B_2 \end{bmatrix} \begin{bmatrix} 0 & -4.1 & 0 & 1 & 0 & 0 \\ -0.9 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.1 & 0 & 1 & 0 \\ 0 & -0.9 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4.1 & 0 & 1 \\ 0 & 0 & -0.9 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 36 & 78 & 76 & 39 & 10 & 1 \end{bmatrix}$$

   $\Rightarrow \begin{bmatrix} A_0 & B_0 & A_1 & B_1 & A_2 & B_2 \end{bmatrix} = \begin{bmatrix} -78.5 & -40 & 10 & 226.4 & 1 & 121.6 \end{bmatrix}$ The compensator is :

   $$C(s) = \frac{B(s)}{A(s)\Phi(s)} = \frac{121.6s^2 + 226.4s - 40}{(s^2+10s-78.5)s}$$

2. The compensator of degree 2

   through using the free parameter, we may able to include an internal model in the compensator selecting the $F(s)$ have degree 4.

   $F(s) = (s^3 + 4s^2 + 6s + 4)(s + 3) = s^4 + 7s^3 + 18s^2 + 22s + 12$

$$\overline{D}(s) = D(s)\Phi(s) = (s^2 - 4.1) = -4.1 + 0\dot{s} + 1\dot{s}^2$$
$$N(s) = s - 0.9 = -0.9 + 1\dot{s} + 0\dot{s}^2$$

$$\begin{bmatrix} A_0 & B_0 & A_1 & B_1 & A_2 & B_2 \end{bmatrix} \begin{bmatrix} -4.1 & 0 & 1 & 0 & 0 \\ -0.9 & 1 & 0 & 0 & 0 \\ 0 & -4.1 & 0 & 1 & 0 \\ 0 & -0.9 & 1 & 0 & 0 \\ 0 & 0 & -4.1 & 0 & 1 \\ 0 & 0 & -0.9 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 22 & 18 & 7 & 1 \end{bmatrix}$$

   in order for proper compensator, $C(s) = \frac{B_0+B_1s+B_2s^2}{A_0+A_1s+A_2s^2}$, to have $1/s$ as a factor, we require $A_0 = 0$. $\Rightarrow \begin{bmatrix} A_0 & B_0 & A_1 & B_1 & A_2 & B_2 \end{bmatrix} = \begin{bmatrix} 0 & -13.3 & -18.5 & 45.1 & 1 & 25.5 \end{bmatrix}$ thus the compensator is:

$$C(s) = \frac{B(s)}{A(s)} = \frac{25.5s^2 + 45.1s - 13.3}{s^2 - 18.5s}$$

9.4

$$\hat{g}(s) = \frac{(s-1)}{s(s-2)}$$

There is no need to introduce a feedforward gain to achieve tracking of any step reference input. Because $D(s) = s(s-2)$, for $r(s) = \frac{a}{s} = \frac{N_r(s)}{D_r(s)}$ the unstable roots of $D_r(s) = s$ is canceled by $D(s) = s(s-2)$, so it has achieved robustly tracking any step reference input.

9.5

yes, the design is robust.

9.6

for $\hat{g}(s) = \frac{1}{s-1}$, $w(t) = asin(2t + \theta)$

in order to achieve the design, the polynomial A(s) must contain the disturbancce model $(s^2 + 4)$ and the step input s.

consider $A(s)D(s) + B(s)N(s) = F(s)$, for this equation, we have deg $D(s) = n = 1$, thus if $m = n - 1 = 0$, then the solution is unique and we have no freedom in assigning $A(s)$. if m=2, then we have two free parameters that can be used to assign A(s)

Let

$$A(s) = \tilde{A}_0 s(s^2+4)$$
$$B(s) = B_0 + B_1 s + B_2 s^2 + B_3 s^3$$

Define $\overline{D}(s) = D(s)(s^2+4)s = (s^2-s)(s^2+4) = 0 - 4s + 4s^2 - s^3 + s^4$

for $\tilde{A}_0\overline{D}(s) + B(s)N(s) = F(s)$

$$\begin{bmatrix} \tilde{A}_0 & B_0 & B_1 & B_2 & B_3 \end{bmatrix} \begin{bmatrix} \overline{D}_0 & \overline{D}_1 & \overline{D}_2 & \overline{D}_3 & \overline{D}_4 \\ N_0 & N_1 & 0 & 0 & 0 \\ 0 & N_0 & N_1 & 0 & 0 \\ 0 & 0 & N_0 & N_1 & 0 \\ 0 & 0 & 0 & N_0 & N_1 \end{bmatrix} = \begin{bmatrix} F_0 & F_1 & F_2 & F_3 & F_4 \end{bmatrix}$$

$F(s) = (s+1+j2)(s+1-j2)(s+2+j1)(s+2-j1) = (s^2+2s+5)(s^2+4s+5) = s^4+6s^3+18s^2+30s+25$

$$\begin{bmatrix} \tilde{A}_0 & B_0 & B_1 & B_2 & B_3 \end{bmatrix} \begin{bmatrix} 0 & -4 & 4 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 25 & 30 & 18 & 6 & 1 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} \tilde{A}_0 & B_0 & B_1 & B_2 & B_3 \end{bmatrix} = \begin{bmatrix} 1 & 25 & 34 & 14 & 7 \end{bmatrix}$ thus the compensator is:

$$C(s) = \frac{B(s)}{A(s)} = \frac{7s^3+14s^2+34s+25}{s(s^2+4)}$$

9.7

the output can track robustly any step reference input. Because

$$\hat{e}(s) = \frac{A(s)D(s)\Phi(s)}{F(s)}\frac{N_r(s)}{D_r(s)}$$

because the unstable roots of $D_r(s) = s$ are canceled by $D(s) = s(s-2)$,so we conclude $e(t) \to 0$ as t $\to \infty$ But ,the system can't reject any step disturbancce.

$$\hat{y}_w(s) = \frac{N(s)A(s)\Phi(s)}{F(s)}\frac{N_w(s)}{D_w(s)}$$

the $D_w(s) = s$ can't be canceled.

9.8

$$\hat{g}_{yr}(s) = \frac{1/s}{1+1/s} = \frac{1}{s+1}$$

so the transfer function from r to y is BIBO stable,but the system is not totally stable. the transfer function from n_1 to n_2 is:

$$\hat{g}_{n_2 n_1}(s) = \frac{1/(s-2)}{1+1/s} = \frac{s}{(s+1)(s-2)}$$

is not BIBO stable.

9.9

1. the every possible closed-loop transfer function

$$\begin{aligned}
g_{n_2 r} &= C(s)(1+C(s)\hat{g}(s))^{-1} \\
g_{n_3 r} &= C(s)\hat{g}(s)(1+C(s)\hat{g}(s))^{-1} \\
g_{n_3 n_2} &= \hat{g}(s)(1+C(s)\hat{g}(s))^{-1} \\
g_{r n_2} &= -\hat{g}(s)(1+C(s)\hat{g}(s))^{-1} \\
g_{r n_3} &= -(1+C(s)\hat{g}(s))^{-1}
\end{aligned}$$

   so the closed-loop transfer function of every possible input-output pair contains the factor $(1+C(s)\hat{g}(s))^{-1}$.

2. $(1+C(s)\hat{g}(s))^{-1}$ is proper $\Leftrightarrow |(1+C(\infty)\hat{g}(\infty))^{-1}| < \infty \Leftrightarrow (1+C(\infty)\hat{g}(\infty)) \neq 0$

3. if $C(\infty)\hat{g}(\infty) \neq -1$,then every possible input-output pair transfer function is a product of two proper fucntion.Then every closed-loop transfer function is proper and well posed.

9.10

from the corollary 9.4,we can find
$\frac{s-1}{(s+1)^2}$ and $\frac{(s-1)(b_0 s+b_1)}{(s+2)^2(s^2+2s+2)}$ is implementable,the others are not implementable.

9.11

For $\hat{g}(s) = \frac{(s-1)}{s(s-2)}$ and $\hat{g}_0(s) = \frac{-2(s-1)}{s^2+2s+2}$
open loop configurations:

$$C(s) = \frac{\hat{g}_0(s)}{\hat{g}(s)} = \frac{\frac{-2(s-1)}{s^2+2s+2}}{\frac{(s-1)}{s(s-2)}} = \frac{-2s(s-2)}{s^2+2s+2}$$

because it involves unstable pole-zero cancellations of $(s-1)$,so it is not totally stable,the implementations can't be used in practice.

unity-feedback configurations: The general form of a unity-feedback configurations is:$\hat{g}_0(s) = \frac{c(s)g(s)}{1+c(s)g(s)}$
we can compute the compensator as:

$$C(s) = \frac{\hat{g}_0(s)}{\hat{g}(s)[1-\hat{g}_0(s)]} = \frac{-2(s-2)}{s+4}$$

it also involves unstable pole-zero cancellations of $(s-1)$,so it is not totally stable,it can't be used in practice.

9.12

First we compute:

$$\frac{\hat{g}_0(s)}{N(s)} = \frac{-2(s-1)}{(s^2+2s+2)(s-1)} = \frac{-2}{(s^2+2s+2)} = \frac{\overline{E}(s)}{\overline{F}(s)}$$

Because the degree of $\overline{F}(s)$ is 2,we introduce $F(s) = s+3$,so that the degree of $\overline{F}(s)\hat{F}(s) = (s^2+2s+2)(s+3)$ is 3=2n-1
thus we have

$$\hat{g}_0(s) = \frac{\overline{E}(s)N(s)\hat{F}(s)}{\overline{F}(s)\hat{F}(s)} = \frac{L(s)N(s)}{A(s)D(s)+M(s)N(s)}$$

$$L(s) = \overline{E}(s)\hat{F}(s) = -2$$

and $A(s)$ and $M(s)$ can be solved from

$$A(s)D(s)+M(s)N(s) = \overline{F}(s)\hat{F}(s) = (s^2+2s+2)(s+3) = s^3+5s^2+8s+6$$

$$D(s) = s(s-2) = s^2-2s = 0-2s+1.s^2$$

$$N(s) = s-1 = -1+1.s+0.s^2$$

$$\begin{bmatrix} A_0 & M_0 & A_1 & M_1 \end{bmatrix} \begin{bmatrix} 0 & -2 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 8 & 5 & 1 \end{bmatrix}$$

$\Rightarrow \begin{bmatrix} A_0 & M_0 & A_1 & M_1 \end{bmatrix} = \begin{bmatrix} -21 & -6 & 1 & 28 \end{bmatrix}$
thus we have $A(s) = s-21$ and $M(s) = 28s-6$ then the compensators are:

$$C_1(s) = \frac{L(s)}{A(s)} = \frac{-2(s+3)}{s-21}$$

$$C_2(s) = \frac{M(s)}{A(s)} = \frac{28s-6}{s-21}$$

we can see the $A(s)$ is not a Hurwitz polynomial we can't implement the two compensators as show in Fig 9.4(a),because the $A(s)$ is not a Hurwitz polynomial,the output of $C_1(s)$ will grow without bound and the overall system is not totally stable.

Implementing the two compensators in Fig(9.4)d:

$$\hat{u}(s) = A^{-1}(s)\left[\begin{array}{cc} L(s) & -M(s) \end{array}\right]\left[\begin{array}{c} r(s) \\ y(s) \end{array}\right] = (\left[\begin{array}{cc} -2 & 28 \end{array}\right] + \frac{1}{s-21}\left[\begin{array}{cc} -48 & 582 \end{array}\right])(\left[\begin{array}{c} \hat{r}(s) \\ \hat{y}(s) \end{array}\right])$$

Its state-space realization is

$$\dot{x} = -21x + \left[\begin{array}{cc} -48 & 582 \end{array}\right]\left[\begin{array}{c} r \\ y \end{array}\right]$$

$$y = x + \left[\begin{array}{cc} -2 & 28 \end{array}\right]\left[\begin{array}{c} r \\ y \end{array}\right]$$

9.13

According to the final value theorem, $r(t) = at \Rightarrow \hat{r}(s) = \frac{a}{s^2}$

$$\hat{y}(s) = \hat{g}_0(s)\hat{r}(s) = \hat{g}_0(s)\frac{a}{s^2} = \frac{k_1}{s} + \frac{k_2}{s^2} + (fraction\ of\ the\ poles\ of\ \hat{g}_0(s))$$

solve the coefficients,according to the residue method:

$$k_2 = \hat{g}_0(s).\frac{a}{s^2}.s^2\bigg|_{s=0} = \hat{g}_0(0).a$$

$$k_1 = \frac{d}{ds}[\hat{g}_0(s).a]\bigg|_{s=0} = g_0^{'}(0).a$$

if $\hat{g}_0(s)$ is BIBO stable, then every pole lies inside the left half plane will approaches 0 as $t \to \infty$ thus we have

$$y_{ss}(t) = \lim_{t\to\infty} y(t) = \mathcal{L}^{-1}[\frac{k_1}{s} + \frac{k_2}{s^2}] = g_0(0)'.a + \hat{g}_0(0).at$$

Thus if the output is to track asymptotically the ramp reference input we require $\hat{g}_0(0) = 1$ and $\hat{g}_0^{'}(0) = 0$

9.14

necessity:

$$\hat{g}_0(0) = 1 \Rightarrow \hat{g}_0(0) = \frac{b_0}{a_0} = 1 \Rightarrow b_0 = a_0$$

$\hat{g}^{'}(0) = 0$

$$\hat{g}_0^{'}(s) = \frac{(b_1 + 2b_2 s + \cdots + mb_m s^{m-1})(a_0 + a_1 s + \cdots + a_n s^n) - (b_0 + b_1 s + \cdots + b_m s^m)(a_1 + 2a_2 s + \cdots + na_n s^{n-1})}{(a_0 + a_1 s + \cdots + a_n s^n)^2}$$

$$\hat{g}_0^{'}(0) = \frac{b_1 a_0 - b_0 a_1}{a_0^2} = 0 \Rightarrow$$

$$b_1 a_0 = b_0 a_1$$

because $a_0 = b_0$,$a_1 = b_1$

sufficiency: if $a_0 = b_0$ and $a_1 = b_1$,$\hat{g}_0(0) = \frac{b_0}{a_0} = 1$

$$\hat{g}_0^{'}(0) = \frac{b_1 a_0 - b_0 a_1}{a_0^2} = 0$$

9.15

(1) according to the corollary 9.4 all roots of $s^2 + 2s + a$ have negative real parts,so we need:

$$\begin{cases} s_1 + s_2 = -2 \le 0 \\ s_1 s_2 = a > 0 \end{cases}$$

all zeros of $N(s) = (s+3)(s-2)$ with zero or positive real parts are retained in $(b_1 s + b_0)$

$$2b_1 + b_0 = 0 \Rightarrow b_0 = -2b_1$$

(2)

$$\hat{g}_0(0) = \frac{-2b_0}{2 \times 2} = 1$$

so $b_0 = -2$

from problem 9.13,we can get:

$$\hat{g}_0^{'}(s) = \frac{[b_1 s + b_0 + (s-2)b_1][(s+2)(s^2+2s+2)] - [(s-2)(b_1 s + b_0)][(s^2+2s+2) + (s+2)(2s+2)]}{[(s+2)(s^2+2s+2)]^2}$$

$$\hat{g}_0^{'}(0) = \frac{16b_0 - 8b_1}{4} = 0$$

$2b_0 = b_1$ so $b_1 = -4$

9.16

$$\hat{G}(s) = \begin{bmatrix} \frac{s+1}{s(s+1)} \\ \frac{1}{s^2-1} \end{bmatrix} = \begin{bmatrix} (s+1)^2 \\ s \end{bmatrix} \left[s(s^2-1)^{-1} \right]$$

$$D(s) = s(s^2-1) = s^3 - s = 0 - 1.s + 0.s^2 + 1.s^3$$

$$N(s) = \begin{bmatrix} s^2 + 2s + 1 \\ s \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} s + \begin{bmatrix} 1 \\ 0 \end{bmatrix} s^2 + \begin{bmatrix} 0 \\ 0 \end{bmatrix} s^3$$

$$S_m = \begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

here we need linearly indepedent rows from top to bottom of S_m,there we will apply QR decomposition to the transpose of S_m. we see that there are two linearly indepedent N_1-rows and one linearly indepedent N_2 row.The degree of $G(s)$ is 3 and we have found three linearly indepedent N-rows.Therefore there is no need to search further and we have $v_1 = 2$ and $v_2 = 1$.Thus the row index is $v = 2$.we select $m_1 = m_2 = m = v - 1 = 1$ thus for any column-reduced F(s) of column degrees $m + \mu = 4$,we can find a proper compensator

$$F(s) = (s+2)(s+1+j1)(s+1-j1)(s+3) = s^4 + 7s^3 + 18s^2 + 22s + 12$$

$$\begin{bmatrix} A_0 & B_0 & A_1 & B_1 \end{bmatrix} \tilde{S}_1 = \begin{bmatrix} A_0 & B_0 & A_1 & B_1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 22 & 18 & 7 & 1 \end{bmatrix}$$

we knew $\tilde{S}_1$ is full column rank,when seraching the row index,we knew that the last of $\tilde{S}_1$ is a linearly depedent row. we delete the row, and assign the second column of B_1 as 0 using matlab, we have

$$\begin{bmatrix} A_0 & B_0 & A_1 & B_1 \end{bmatrix} = \begin{bmatrix} 3.5 & 12 & -2 & 1 & 3.5 & 0 \end{bmatrix}$$

so we can get $A(s) = s + 3.5$ and $B(s) = \begin{bmatrix} 3.5s + 12 & -2 \end{bmatrix}$

$$\hat{G}(s) = N(s)F^{-1}(s)B(s) = \begin{bmatrix} s^2 + 2s + 1 \\ s \end{bmatrix} (s^4 + 7s^3 + 18s^2 + 22s + 12)^{-1} \begin{bmatrix} 3.5s + 12 & -2 \end{bmatrix}$$

$$\hat{G}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{12} \begin{bmatrix} 12 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{6} \\ 0 & 0 \end{bmatrix}$$

so the system can't track asymptotically any step reference input.

9.17

This problem can be seen as s dual problem as problem 9.16

$$\hat{G}(s) = [s(s^2-1)]^{-1} \begin{bmatrix} (s+1)^2 & s \end{bmatrix}$$

$$\overline{D}(s) = s(s^2-1) = s^3 - s = 0 - 1.s + 0.s^2 + 1.s^3$$

$$\overline{N}(s) = \begin{bmatrix} (s+1)^2 & s \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} s + \begin{bmatrix} 1 & 0 \end{bmatrix} s^2 + \begin{bmatrix} 0 & 0 \end{bmatrix} s^3$$