Chapter3

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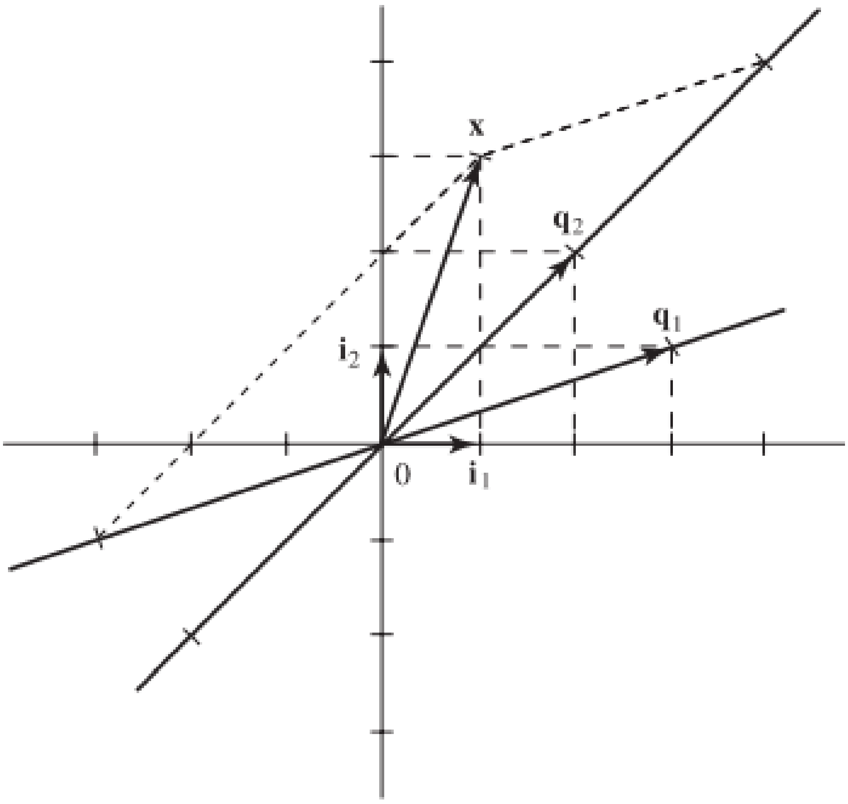
3.1



From the above figure,The three vectors $\boldsymbol{q}_1 = [3 \quad 1]'$,$\boldsymbol{i}_2 = [0 \quad 1]'$ and $\boldsymbol{q}_2 = [2 \quad 2]'$
The representation of $\boldsymbol{x}$ with respect to $\{\boldsymbol{q}_1, \boldsymbol{i}_2\}$ is $[\frac{1}{3} \quad \frac{8}{3}]'$
The representation of $\boldsymbol{q}_1$ with respect to $\{\boldsymbol{i}_2, \boldsymbol{q}_2\}$ is $[-2 \quad \frac{3}{2}]'$
These can be verified like this:

$$x = \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{q}_1 & \boldsymbol{i}_2 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{8}{3} \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{8}{3} \end{bmatrix}$$

3.2

i:The norm of $\boldsymbol{x}_1$

1-norm:$\|\boldsymbol{x}_1\|_1$=$\sum_{i=1}^{3}|x_i|$=|2|+|-3|+|1|=6
2-norm:$\|\boldsymbol{x}_1\|_2$=$(\sum_{i=1}^{3}|x_i|^2)^{\frac{1}{2}} = \sqrt{2^2+|-3|^2+1^2} = \sqrt{14}$
infinite-norm:$\|\boldsymbol{x}_1\|_\infty$=$max_i|x_i|$=3

ii:The norm of $\boldsymbol{x}_2$

1-norm:$\|\boldsymbol{x}_2\|_1$=$\sum_{i=1}^{3}|x_i|$=|1|+|1|+|1|=3
2-norm:$\|\boldsymbol{x}_2\|_2$=$(\sum_{i=1}^{3}|x_i|^2)^{\frac{1}{2}} = \sqrt{1^2+1^2+1^2} = \sqrt{3}$
infinite-norm:$\|\boldsymbol{x}_2\|_\infty$=$max_i|x_i|$=1

3.3

This is just the orthonormalization procedure.

$$\begin{cases} \boldsymbol{u}_1 = \boldsymbol{\alpha}_1 & \boldsymbol{q}_1 = \boldsymbol{u}_1/\left\|\boldsymbol{u}_1\right\| \\ \boldsymbol{u}_2 = \boldsymbol{\alpha}_2 - (\boldsymbol{q}_1^{'}\boldsymbol{\alpha}_2)q_1 & \boldsymbol{q}_2 = \boldsymbol{u}_2/\left\|\boldsymbol{u}_2\right\| \end{cases}$$

This is the ordinary method,what we find is the two vector are orthogonal. so,we just need to make the length of vector is 1.

$$q_1 = \frac{u_1}{\|u_1\|} = [\frac{2}{\sqrt{14}} \quad -\frac{3}{\sqrt{14}} \quad \frac{1}{\sqrt{14}}]^{'}$$
$$q_1 = \frac{u_2}{\|u_2\|} = [\frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{3}} \quad \frac{1}{\sqrt{3}}]^{'}$$

3.4

if n>m,$AA^{'}$