Chapter8

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8.1

introducing state feedback. $u = r - [k_1 \quad k_2]\, x$, we can obtain

$$\dot{x} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} x - \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} r$$

the new:A-matrix has characteristic polynomial

$$\begin{aligned} \Delta_f(s) &= (s - 2 + k_1)(s - 1 + 2k_2) - (-1 - 2k_1)(1 - k_2) \\ &= s^2 + (k_1 + 2k_2 - 3)\, s + (k_1 - 5k_2 + 3) \end{aligned}$$

the desired characteristic polynomial is $(s+1)(s+2) = s^2 + 3s + 2\cdot$, equating $k_1 + 2k_2 - 3 = 3$ and $k_1 - 5k_2 + 3 = 2$, yields $k_1 = 4$ and $k_2 = 1$ so the desired state feedback gain k is $k = \begin{bmatrix} 4 & 1 \end{bmatrix}$

8.2

$$\begin{aligned} &\Delta(s) = (s-2)(s-1) + 1 = s^2 - 3s + 1 \\ &\Delta_f(s) = (s+1)(s+2) = s^2 + 3s + 2 \\ &\bar{k} = [3 - (-3) \quad 2 - 3] = \begin{bmatrix} 6 & -1 \end{bmatrix} \\ &\bar{C}^{-1} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \quad \bar{C} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$C = \begin{bmatrix} b & Ab \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \quad C^{-1} = \begin{bmatrix} -1/7 & 4/7 \\ 2/7 & -1/7 \end{bmatrix}$ C is nonsingular, so (A, b) is can trollable using (8.13)

$$k = \bar{k}\bar{C}C^{-1} = \begin{bmatrix} 6 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1/7 & 4/7 \\ 2/7 & -1/7 \end{bmatrix} = \begin{bmatrix} 4 & 1 \end{bmatrix}$$

8.3

(A, b) is controllable

selecting $F = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ and $\overline{k} = \begin{bmatrix} 1 & 1 \end{bmatrix}\cdot$ then

$AT - TF = b\bar{k} \Rightarrow T = \begin{bmatrix} 0 & \frac{1}{13} \\ 1 & \frac{9}{13} \end{bmatrix} \quad T^{-1} = \begin{bmatrix} -9 & 1 \\ 13 & 0 \end{bmatrix}$

$k = \overline{k}T^{-1} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -9 & 1 \\ 13 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 1 \end{bmatrix}$

8.4

(A, b) is controllable

$\Delta(s) = (s-1)^3 = s^3 - 3s^2 + 3s - 1$

$\Delta_f(s) = (s+2)(s+1+j1)(s+1-j1) = s^3 + 4s^2 + 6s + 4$

$\bar{k} = \begin{bmatrix} 4-(-3) & 6-3 & 4-(-1) \end{bmatrix} = \begin{bmatrix} 7 & 3 & 5 \end{bmatrix}$

$\bar{C}^{-1} = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \quad \bar{C} = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \quad C^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -2 & -3 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

$k = \overline{k}\bar{C}C^{-1} \begin{bmatrix} 7 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ -2 & -3 & 2 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 15 & 47 & -8 \end{bmatrix}$

8.5

$$\hat{g}_f(s) = \frac{(s-1)}{(s+2)(s+3)} = \frac{(s-1)(s+2)}{(s+2)^2(s+3)}$$

We can easily see that it is possible to change $\hat{g}(s)$ to $\hat{g}_f(s)$ by state feedback and the resulting system is asymptotically stable and BIBO stable:

8.6

$$\hat{g}_f(s) = \frac{1}{s+3} = \frac{(s-1)(s+2)}{(s-1)(s+2)(s+3)}$$

It is possible to change $\hat{g}(s)$ to $\hat{g}_f(s)$ by state feedback and the resulting system is BIBO stable,however it is not asymptotically stable,because it has a 1 eigenvalue.

8.7

$$\hat{g}(s) = c(sI-A)b = \begin{bmatrix} \frac{1}{s-1} & \frac{1}{(s-1)^2} & \frac{1}{(s-1)^3} - \frac{1}{(s-1)^2} \\ 0 & \frac{1}{s-1} & \frac{1}{(s-1)^2} \\ 0 & 0 & \frac{1}{s-1} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \frac{2s^2-8s+8}{s^3-3s^2+3s-1}$$

$$\Delta_f(s) = s^3 + 4s^2 + 6s + 4$$

$$P = \frac{\overline{\alpha}_3}{\beta_3} = \frac{4}{8} = 0.5$$

from the problem 8.4,we know:

$$k = \begin{bmatrix} 15 & 47 & -8 \end{bmatrix}$$

8.8

(A, b) is controllable

$$\Delta(z) = z^3 - 3s + 3z - 1$$
$$\Delta_f(z) = z^3$$

$\bar{k} = \begin{bmatrix} 3 & -3 & 1 \end{bmatrix}$

$$k = \bar{k}\bar{C}C^{-1} = \begin{bmatrix} 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ -2 & -3 & 2 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 2 \end{bmatrix}$$

The state feedback equation becomes

$$\dot{\bar{x}}[k+1] = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \end{bmatrix} \bar{x}[k] + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} r[k] = \begin{bmatrix} 0 & -4 & -4 \\ 0 & 1 & 1 \\ -1 & -5 & -1 \end{bmatrix} \bar{x}[k] + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} r[k]$$

$$y[k] = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} x[k]$$

we can present the zero-mput response of the feedback 'system as following $y_{zi}[k] = \overline{C}\bar{A}^k x[0]$

$$\bar{A} = \begin{bmatrix} 0 & -4 & -4 \\ 0 & 1 & 1 \\ -1 & -5 & -1 \end{bmatrix}$$

$$\bar{A}^2 = \begin{bmatrix} 4 & 16 & 0 \\ -1 & -4 & 0 \\ 1 & 4 & 0 \end{bmatrix}$$

$$\bar{A}^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

so when $k \geq 3$,the zero-input response of the feedback system becomes identically zero.

8.9

$$\hat{g}(z) = \frac{2z^2-8s+8}{z^3-3s^2+3z-1} \quad \Delta(z) = z^3 \quad u[k] = pr[k] - kx[k]$$
$$\hat{g}_f(z) = p\frac{2z^2-8z+8}{z^3}$$

(A, b) is controllable all poles of $\hat{g}_f(z)$ can be assigned to lie inside the section in fig 8.3(b) under this condition; if the reference input is a step function with magnitude a, then the output y[k] will approach the constant $\hat{g}_f(1) \cdot a$ when $k \to +\infty$. thus in order for y[k] to track any step reference input we need. $\hat{g}_f(1) = 1$ $\hat{g}_f(1) = 2p = 1 \Rightarrow p = 0.5$

Let $r[k] = a$,its z-transform is $\hat{r}(z) = \frac{az}{z-1}$

$$\hat{y}(z) = \hat{y_f}(z).\hat{r}(z) = \frac{z^2-4z+4}{z^3}\frac{az}{z-1} = \frac{az}{z-1} - a - \frac{4a}{z^2}$$

its inverse z-transform:

$$y[k] = a - a\delta[k] - 4a\delta[k-2]$$

when k=0,$y[0] = a - a\delta[0] = 0$
when k=1,$y[1] = a$
when k=2,$y[2] = a - 4a = -3a$
when $k \geq 3$,$y[k] = a = r[k]$

8.10

With the transformation matrix is

$$P^{-1} = \begin{bmatrix} 0 & 1 & 4 & 0 \\ 1 & 2 & 4 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

the uncontrollable state equation can be transformed into

$$\begin{bmatrix} \dot{\bar{x}}_c \\ \dot{\bar{x}}_{\bar{c}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -4 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_{\bar{c}} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$= \begin{bmatrix} \overline{A_c} & 0 \\ 0 & \overline{A_{\bar{c}}} \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_{\bar{c}} \end{bmatrix} + \begin{bmatrix} \bar{b}_c \\ 0 \end{bmatrix} u$$

$\left(\overline{A}_C, \bar{b}_c\right)$ is controllable and the uncontrollable eigenvalues of the system is negative,so the equation is stabilizable,the eigenvalue of $\overline{A}_{\bar{C}}$ is 1 is not affected by the state feedback.while the eigenvalues of the controllable sub-state equation can be arbitrarily assigned in pairs so by using state feedback , it is possible for the resulting system to have eigenvalues $-2; -2; -1, -1$, also eigenvalues -2,-2,-2,-1 It is impossible to have $-2, -2, -2, -2$, as it eigenvalues because state feedback can not affect a eigenvalue -1

8.11

the eigenvalues of the full-dimensional state estimator must be selected as $-2 \pm j2$, the A, b and c matxices in problem 8.1 are

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad c = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

the full-dimensional state estimator can be written as $\dot{\hat{x}} = (A - lc) = \hat{x} + bu + ly$

let $l = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$ then the characteristic polynomial of $\dot{\hat{x}} = (A - lc)$ is

$$\begin{aligned} \Delta(s) &= (s - 2 + l_1)(s - 1 + l_2) + (1 + l_2)(1 - l_1) \\ &= s^2 + (l_1 + l_2 - 3)s + 3 - 2l_1 - l_2 \\ &= (s + 2)^2 + 4 = s^2 + 4s + 8 \\ \Rightarrow \quad & l_1 = -12 \quad l_2 = 19 \end{aligned}$$

thus a full-dimensional state estimor with eigenvalues $-2 \pm j2$has been designed as following

$$\dot{\hat{x}} = \begin{bmatrix} 14 & 13 \\ -20 & -18 \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u + \begin{bmatrix} -12 \\ 19 \end{bmatrix} y$$

designing a reduced-dimensional state estimator with eigenvalue -3.

(1)$\rightarrow$ select $|x|$ stable matrix $F = -3$

(2)$\rightarrow$ select |x| vector L $= 1,($ F, L). IS controllable

(3)$\rightarrow$ solve T =: TA – FT $= lc$

$$T = \begin{bmatrix} t_1 & t_2 \end{bmatrix}$$
$$\begin{bmatrix} t_1 & t_2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} + 3 \begin{bmatrix} t_1 & t_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$
$$\Rightarrow T = \begin{bmatrix} \frac{3}{21} & \frac{4}{21} \end{bmatrix}$$

(4) $\rightarrow$ THE l-dimensional 'state estimutor with eigenvalue -3 is

$$\dot{z} = -3x + \tfrac{13}{21} u + y$$
$$\hat{x} = \begin{bmatrix} 1 & 1 \\ \frac{5}{21} & \frac{4}{21} \end{bmatrix}^{-1} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -4 & 21 \\ 5 & -21 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix}$$

8.12

three overall transfer function are the same

$$\hat{y}_{r \rightarrow y} = \frac{3s - 4}{(s+1)(s+2)}$$

8.13

select a 4×4 matrix $F = \begin{bmatrix} -4 & -3 & 0 & 0 \\ 3 & -4 & 0 & 0 \\ 0 & 0 & -5 & -4 \\ 0 & 0 & 4 & -5 \end{bmatrix}$ the eigenvalues of F are $-4 \pm 3j$ and $-5 \pm 4j$

(1)if $\overline{k_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ so $(F, \overline{k_1})$ is observable

$$AT_1 - T_1 F = B\overline{K_1} \Rightarrow T_1 = \begin{bmatrix} -0.0013 & -0.0059 & 0.0005 & -0.0042 \\ -0.0126 & 0.0273 & -0.0193 & 0.0191 \\ 0.1322 & -0.0714 & 0.1731 & -0.0185 \\ 0.0006 & -0.0043 & 0.2451 & -0.1982 \end{bmatrix}$$

$$K_1 = \bar{K}_1 T_1^{-1} = \begin{bmatrix} -606.2000 & -168.0000 & -14.2000 & -2.0000 \\ 371.0670 & 119.1758 & 14.8747 & 2.2253 \end{bmatrix}$$

(3)if $\bar{K}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\left(F, \overline{k_2}\right)$ is observable

$$AT_2 - T_2F = B\overline{K_2} \Rightarrow T_2 = \begin{bmatrix} -0.0046 & -0.0071 & 0.0024 & -0.0081 \\ -0.0399 & 0.0147 & -0.0443 & 0.0306 \\ 0.2036 & 0.0607 & 0.3440 & 0.0245 \\ 0.0048 & -0.0037 & 0.2473 & 0.2007 \end{bmatrix}$$

$$K_2 = \bar{K}_2 T_2^{-1} = \begin{bmatrix} -252.9824 & -55.2145 & -0.0893 & -3.2509 \\ 185.5527 & 59.8815 & 7.4593 & 2.5853 \end{bmatrix}$$