Chapter3

31202008881 Bao Ze an

Tuesday 16th February, 2021

7.1

$$\hat{g(s)} = \frac{s-1}{(s^2-1)(s+2)} = \frac{s-1}{s^3+2s^2-s-2}$$

so,from the equation (7.9),we can get the three-dimensional controllable realization:

$$\dot{x} = \begin{bmatrix} -2 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} x$$

it is obviously the $\hat{g(s)}$ is not coprime fraction,so the controllable realization is not observable.

7.2

from the quation (7.14), we can easily get the three-dimensional observable realization:

$$\dot{x} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

similarly, it is not controllable.

7.3

from the inverse canonical decomposition,we can add an uncontrollable state to problem 7.1:

$$\dot{x} = \begin{bmatrix} -2 & 1 & 2 & a_1 \\ 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & a_3 \\ 0 & 0 & 0 & a_4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & -1 & c_4 \end{bmatrix} x$$

the transfer function can be reduced to a coprime fraction,which is:

$$\hat{g(s)} = \frac{1}{(s+1)(s+2)} = \frac{1}{s^2+3s+2}$$

A minimal realization can be realized through controllable realization, this realization is two-dimensional

$$\dot{x} = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

7.4

from the equation (7.27),we can get:

$$\begin{aligned} D(s) &= -2 - s + 2s^2 + s^3 \\ N(s) &= -1 + s \\ \overline{D}(s) &= \overline{D}_0 + \overline{D}_1 s + \overline{D}_2 s^2 \\ \overline{N}(s) &= \overline{N}_0 + \overline{N}_1 s + \overline{N}_2 s^2 \end{aligned}$$

the Sylvester resultant is:

$$\mathbf{SM} := \begin{bmatrix} -2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -2 & -1 & 0 & 0 \\ 2 & 0 & -1 & 1 & -2 & -1 \\ 1 & 0 & 2 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -\overline{N}_0 \\ \overline{D}_0 \\ -\overline{N}_1 \\ \overline{D}_1 \\ -\overline{N}_2 \\ \overline{D}_2 \end{bmatrix} = 0$$

the rank of **S** is 5, so there are 2 linearly indepedent N-columns,the degree of the transfer function is 2 .we can also calculate a monic null vector

$$z = \begin{bmatrix} -1 & 2 & 0 & 3 & 0 & 1 \end{bmatrix}'$$

7.5

just as the