Lecture 5

Game Theory

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PS4168: Economic Psychology

Game Theory

https://www.youtube.com/embed/AJ5aIvjNgao?si=RWOztM-oMpazRJZ_

Key Concepts in Todays lecture / Overview

- Strict Dominance
- Nash Equilibrium
- Mixed Strategy Nash Equilibrium
- Predicting others' behaviour
- In-class Activities

Recap on last week!

- Definition of Rationality?
  - Instrumental rationality
    - "our mental states or processes are rational when they help us to achieve our goals" (Over, 2004, p. 3)
- Two approaches to the study of decision making
  - Normative Theories *versus* Behavioural Theories
- Theories of decision making
  - Expected Value / Expected Utility
  - Prospect Theory
  - Social Functionalist Theory

Introduction to Game Theory

- Branch of Applied Mathematics
  - Provides a framework for modelling and predicting behaviour in Social situations of
    - cooperation
    - coordination
    - conflict (Dowling, 2007; Von Neuman & Morgenstern, 1944, 1947)

The Prisoner Dilemma

The Prisoner Dilemma

- Two suspects (e.g., Bob and Susan) are arrested on suspicion of a serious crime
- However:
  - Only sufficient evidence to convict them of a minor crime
- Prisoners are kept in isolation and offered a deal:
  - If neither confess they each get 1 year
  - If one confesses and the other doesn't the one that confessed will be let free and the other is sentenced for 20 years
  - If they both confess they each get 5 years (adapted from Dowling, 2007; originally devised by Merrill Flood and Melvin Dresher in 1950 see Dowling, 2007, p. 107)

The Prisoner Dilemma

https://app.sli.do/event/aDybaWhRyTBynrEGdd5vPX

The Prisoner Dilemma

		Player 2	
		keep quiet	confess
Player 1	keep quiet	-1 , -1	-20 , 0
	confess	0 , -20	-5 , -5

(adapted from Dowling, 2007, p. 108)

Strict Dominance

- The strategy x is a **dominant** strategy if it is a *strict best response* to any feasible strategy that the others might play
- We say that a strategy x strictly dominates strategy y for a player if
  - strategy x provides a greater payoff for that player than strategy y
  - regardless of what the other player(s) do.
- What is the dominant strategy in the Prisoner Dilemma?

Combining the Options

This leaves `<confess><confess>` as the ***Rational*** outcome

		Player 2	
		keep quiet	**confess**
Player 1	keep quiet	-1 , -1	-20 , 0*
	confess	0* , -20	-5* , -5*

(adapted from Dowling, 2007, p. 108)

Some Alternatives to the Prisoner Dilemma

- Import Tax
- Advertising
- International Relations(taken from Spaniel, 2013)
- Any other examples?

Import Tax

- Should countries introduce tax on imports/trading tariffs?
- Placing tariffs (a tax) on imported goods can
  - protect domestic industries
  - though this leads to higher prices overall
- The best outcome for a country is to tax imports while not having the other country tax its exports.
- Free trade is the next best outcome
- Mutual tariffs is the next best outcome
  - ultimately, this leads to higher prices than the free trade outcome
- The worst possible outcome is to levy no taxes while the other country enforces a tariff

Import Tax

		Country 2	
		No Tax	Tax
Country 1	No Tax	3 , 3	1 , 4
	Tax	4 , 1	2 , 2

Advertising

- Consider two rival firms considering whether to advertise their products
- Would the firms ever want the government to pass a law forbidding advertisement?
- If advertising campaigns only persuade a consumer to buy a certain **brand** of product *rather than the product in general*
  - If one side places ads and the other does not, the firm with the advertising campaign cuts into the other's share of the market.
  - If they both advertise, the ads cancel each other out, but they still have to pay for the campaigns.

Advertising

		Company 2	
		No Ads	Ads
Company 1	No Ads	4 , 4	2 , 5
	Ads	5 , 2	3 , 3

Going to War

- Should two states go to war?
- Peace is preferable to war
- BUT
  - Striking first leads to a large advantage
  - Being struck first is very costly
    - Striking at the same time as opponent is preferable to being attacked

Going to war

		Country 2	
		Defend	Attack
Country 1	Defend	3 , 3	-3 , 4
	Attack	4 , -3	-1 , -1

Asymmetric Games

		Player 2	
		Left	Right
Player 1	Up	9 , -2	3 , 0
	Down	8 , 5	-1 , 6

Another Asymmetric Game

- Two Clubs ONE and TWO in a town
- Will run either a Salsa night or a Disco night
- ONE is centrally located but TWO is outside the town
  - If TWO runs the same night as ONE nobody will show
- Three types of customers
  - 60 hardcore Salsa fans - will only go to Salsa
  - 20 hardcore Disco fans - will only go to Disco
  - 20 people prefer going to a disco theme but will attend a salsa night if that is the only option

Another Asymmetric Game

		TWO	
		Salsa	Disco
ONE	Salsa	80 , 0	60 , 40
	Disco	40 , 60	40 , 0

Iterated Elimination of Strictly Dominated Strategies

	Left	Center	Right
Up	13 , 3	1 , 4	7 , 3
Middle	4 , 1	3 , 3	6 , 2
Down	-1 , 9	2 , 8	8 , -1

Different types of Games

Stag Hunt

- Two hunters enter a range filled with hares and a single stag
  - Hares are easy to capture
    - but not worth much meat (1)
  - Catching the stag requires working together
    - worth much more meat (6 - to share between 2)
- If they both hunt hares, they each capture half of the hares in the range
- If one hunts the stag and the other hunts hares
  - the stag hunter goes home empty-handed
  - the hare hunter captures all of the hares
- If both hunt the stag, they share the stag (value of the stag is greater than the value of all of the hares)

Stag Hunt

		Player 2	
		Stag	Hare
Player 1	Stag	3 , 3	0 , 2
	Hare	2 , 0	1 , 1

No Strict Dominace in the Stag Hunt?

		Player 2	
		Stag	Hare
Player 1	Stag	3 , 3	0 , 2
	Hare	2 , 0	1 , 1

No Strict Dominace in the Stag Hunt?

- **Nash equilibrium**
  - a set of strategies, one for each player
  - no player has incentive to change his or her strategy (given what the other players are doing)
- **Nash equilibrium** is the best strategy given the strategy chosen by the other participants(Dowling, 2007)
- Individuals have no incentive to deviate (not group deviations)
  - no regrets at the end of the game
    - once the other player's strategy has been revealed
- Nash equilibria are ***inherently stable***

Nash Equilibria in real life

- Traffic
  - Traffic Lights
  - Driving on the Left/Right
  - Other Examples?

Driving on the Left/Right

A Beautiful Mind

Matching Pennies

Matching Pennies

- 2 players have a penny
- Each put penny down either Heads up or Tails up
- If both show heads or both show tails (they match) Player 1 wins
- If one shows heads and the other shows tails (they do not match) Player 2 wins

- Strictly competitive / *zero sum* game
- Players actively want the opponent to perform poorly

Matching Pennies

		Player 2	
		Heads	Tails
Player 1	Heads	1 , -1	-1 , 1
	Tails	-1 , 1	1 , -1

Matching Pennies and Nash Equilibrium

- Every finite game has a Nash equilibrium in mixed strategies
- **Nash Existence Theorem**: If each player in an *n*-player game has a finite number of pure strategies, then the game has a (not necessarily unique) Nash equilibrium in (possibly) mixed strategies (Gintis, 2009, p. 44)
- If I could read your mind, how would you beat me at Matching Pennies?
- mixed strategy refers to how we are randomizing over multiple strategies (across multiple trials) rather than playing a single "pure" strategy.

Mixed Strategy Algorithm

Mixed Strategy Algorithm

		Player 2	
		Left	Right
Player 1	Up	3 , -3	-2 , 2
	Down	-1 , 1	0 , 0

Mixed Strategy Algorithm

- In matching pennies, flipping against the mind reader was intended to make the Expected Utility of each of the opponents strategies the same
    - Calculate best strategy for Player 1
        - $EU_L = EU_R$
- $EU_L = f(\sigma_u)$
- $EU_R = f(\sigma_u)$
- Express - EU_L and EU_R in terms of σ_u and solve for σ_u

Mixed Strategy Algorithm

Expected Utility of Left

- EU_L is -3 sometimes (when Player 1 plays Up) and 1 the rest of the time
- $EU_L = \sigma_u(-3) + (1 - \sigma_u)(1)$

Mixed Strategy Algorithm

Expected Utility of Right

- EU_R is 2 sometimes (when Player 1 plays Up) and 0 the rest of the time
- $EU_R = \sigma_u(2) + (1 - \sigma_u)(0)$

Mixed Strategy Algorithm

- Let
$$EU_L = EU_R$$
- $$\sigma_u(-3) + (1 - \sigma_u)(1) = \sigma_u(2) + (1 - \sigma_u)(0)$$
- $$-3\sigma_u + 1 - \sigma_u = 2\sigma_u + 0$$
- $$6\sigma_u = 1$$
- $$\sigma_u = \frac{1}{6}$$
- If Player 1 plays Up $\frac{1}{6}$ of time Player 2 is indifferent between Left and Right

Mixed Strategy Algorithm

- Calculate Player 2's best strategy using the same calculation but letting
  - $EU_u = f(\sigma_L)$
  - $EU_d = f(\sigma_L)$
- and solving for σ_L
- Yields

$$\sigma_L = \frac{1}{3}$$

Mixed Strategy Algorithm

Mixed Strategy Nash Equilibrium:

- $\sigma_u = \frac{1}{6}, \sigma_L = \frac{1}{3}$

Number Beauty Contest

- Everyone picks a number between 1 and 100
- Everyone's response is collated to calculate the Mean
- The Mean is multiplied by a constant $\frac{2}{3}$
- Congratulations if you picked the *most beautiful number*

Strategy for the Number Beauty Contest

- Midpoint of 1 and 100 is 50
  - Mean of uniformly drawn sample is 50
  - $\frac{2}{3}$ of 50 is $33.33...$
- Best strategy is to guess around $33.33 \ldots$?
- Everyone guessing 33 brings the Mean closer to 33
  - ***BUT*** $\frac{2}{3}$ of 33 is $22.22...$
- Best strategy to guess around $22. \ldots$?(Dowling, 2007; Keynes, 1936)

Basic Ultimatum Game (Dictator Version 1)

- In pairs identify who is Player 1 and who is Player 2

- Player 1 receives €100
- Player 1 may divide the €100 between you and Player 2 however you like.
- End of the Game
- What is the ***rational*** choice?

Basic Ultimatum Game (Dictator Version 2)

- In pairs identify who is Player 1 and who is Player 2
- Player 1 receives €100
- Player 1 may divide the €100 between you and Player 2 however you like.
- Player 2 can accept or reject the offer
- End of the Game
- What is the ***rational*** choice?
  - for Player 1
  - for Player 2

Ultimatum Game

- In pairs identify who is Player 1 and who is Player 2
- Player 1 receives €100
- Player 1 may divide the €100 between you and Player 2 however you like.
- Player 2 can
  - accept the offer
  - reject the offer
    - If Player 2 rejects, **both** players get **nothing**
- End of the Game

Ultimatum Game

- What is the ***rational*** choice?
  - for Player 1
  - for Player 2

Fairness vs Rationality

https://www.youtube.com/embed/-KSryJXDpZo

Backward Induction

- Sum of money to be divided between Ann and Ben
- Ann starts with 100p and can give some to Ben
- Ben can accept or reject
  - if Ben rejects:
    - Money is reduced to 25p and Ben is in charge of dividing the money
- Ann can accept or reject
  - if Ann rejects, both receive nothing

Backward Induction

(taken from Dowling, 2007, p. 118)

Backward Induction and the Centipede Game

- A dime (10c) is put on the table
- Player 1 has the option to take it
- If Player 1 leaves it, another dime is placed on top and Player 2 has the option to take it
- If Player 2 leaves it another dime is added
    - and so on. . .

Backward Induction and the Centipede Game

- Assume there are 10 rounds, this will leave \$1 on the table at Player 2's decision
  - Final round, Player 2 will take the money
- By backwards induction Player 1 should take at round 9
  - but Player 2 should anticipate this and take the money at round 8 an so on. . .
- Nash Equilibrium is for Player 1 to take the dime on round 1
  - Without prior collusion participants have waited till the last round and split the money

Beyond Individual Games

- Game theory works for investigating behvaiour at the individual level
- Can also be used to test the emergence of ***best*** or ***most robust*** strategies through computer simulations
- The effectiveness of strategies relative to each other can be established

Repeated Trials of the Prisoner Dilemma

- Axelrod (1984) famously simulated the Prisoner Dilemma using a points system

		B	
		cooperate	defect
A	cooperate	3 , 3	0 , 5
	defect	5 , 0	1 , 1

Repeated Trials of the Prisoner Dilemma

- A “tournament” for strategies was devised and the winner was ***Tit for Tat***
  - Initially offer cooperation
  - Respond to defection with defection
  - Observed during WW1
- Nowak, May, & Sigmund (1995) used learning algorithms
  - cooperation eventually prevailed
    - *generous tit for tat* (failed to retaliate occasionally)
    - *Pavlov*

In-Class Activity

In Class Activity

- In groups identify features of games discussed in everyday activities
- Pick an example of everyday activity that illustrates some of the concepts discussed today

References

References

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