

## RAPID RESPONSE PAPER

### Primary production of the ocean water column as a function of surface light intensity: algorithms for remote sensing

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**Abstract**—Empirically, the phytoplankton production per unit area of sea surface, normalised to total pigment biomass in the water column, is a linear function of the surface insolation. The slope  $\psi$  varies by only a factor of two in samples from a variety of environments. A theory is developed that describes the empirical results. A simplifying assumption of the theory, that photosynthesis is a linear function of available light, leads to a bias (overestimate) that can be calculated as a function of a measureable dimensionless parameter. With the bias corrected,  $\psi$  is shown to be equal, to within a dimensionless constant, to the initial slope  $\alpha$  of the photosynthesis–light curve. The theory can be applied to the calculation of primary production from satellite surveys of ocean colour or from survey data collected by ship.

#### INTRODUCTION

ONE OF the major tasks facing biological oceanography today is the quantification of the mean and variance of primary production over the world's oceans. The information is required not just for its traditional application in marine food chain studies (MOISEEV, 1971), but also for more contemporary calculations concerning the biogeochemical cycling of elements such as carbon and oxygen (NATIONAL ACADEMY OF SCIENCES, 1984; MALONE and ROEDERER, 1985). It has become clear that reconciliation of conflicting reports (SHULENBERGER and REID, 1981; JENKINS, 1982; PLATT, 1984; JENKINS and GOLDMAN, 1985; REID and SHULENBERGER, 1986) about the magnitude of the biogenic fluxes of carbon and oxygen in the ocean depends on the understanding of the significance of their variances (PLATT and HARRISON, 1985, 1986). All of these considerations bear on the question of the response of the autotrophic ocean biota to increasing concentrations of CO<sub>2</sub> in the atmosphere and hence on the prediction of future trends in the climate of the earth.

Even the most rapid of conventional methods for measuring primary production (LEWIS and SMITH, 1983) is not suitable for mapping on the regional or global scale. Supplementary data, collected in synoptic survey, will have to be supplied. To fill the gap, one looks, naturally to data remotely sensed from satellites (REVELLE, 1985). It is

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well accepted (PLATT *et al.*, 1981; FASHAM, 1984) that ecological problems require information on fluxes as well as biomass for their resolution. On the other hand, although remote sensing has achieved moderate success (GORDON *et al.*, 1980; HOVIS *et al.*, 1980; SMITH, 1981; SMITH and BAKER, 1982; BROWN *et al.*, 1985) in the mapping of phytoplankton pigments (biomass), no method has emerged for calculation of phytoplankton production from the electromagnetic signals detectable by satellite (but see TOPLISS and PLATT, in press). The procedures used so far (SMITH *et al.*, 1982; PLATT and HERMAN, 1983) therefore have been based on empirical correlation, and have ignored, for the most part, the accumulated knowledge on phytoplankton physiology (but see SMITH and BAKER, 1978).

In this paper I present a new, analytic relationship between the primary production of the oceanic water column and the sunlight available at the sea surface. From data collected by members of this laboratory, and from data collected by other workers, it appears that the relationship is remarkably stable. Furthermore, the magnitude of the slope of the relationship may be rationalised in terms of known phytoplankton physiology. The independent variable (surface light) is assumed known, or measurable from satellite data (GAUTIER *et al.*, 1980; GAUTIER, 1982; GAUTIER and KATSAROS, 1984). The slope is controlled by properties potentially accessible to remote sensing. The approach should be useful in estimating phytoplankton production over large tracts of the ocean from remotely sensed data.

#### BACKGROUND

For at least 15 years (LARRANCE, 1971), workers have noticed (or it is implicit in their data) that fluctuations between days in surface light intensity account for a high proportion of the variance in water column primary production normalised to water column pigment concentration (MALONE, 1976; FALKOWSKI, 1981; HARRISON *et al.*, 1982; JORDAN and JOINT, 1984). Because of the diverse systems of units and presentation of data employed by different authors, any quantitative similarity between these independent reports has largely escaped notice (but see JORDAN and JOINT, 1984). During analysis of data on primary production in the Canadian high arctic (PLATT, unpublished data) it was remarked that these various studies were, in fact, remarkably coherent. This observation led to the following analysis.

#### ANALYSIS

It may be noticed first that the empirical evidence refers to quantities expressed per unit area of sea surface. Let  $I_0$  be the flux of photosynthetically active radiation (PAR) available at the surface. Following PLATT *et al.* (1984), let  $S$  be the rate of absorption of energy by photosynthetic pigments per unit area per unit time:

$$S = \int_0^{\infty} Q(z) dz, \quad (1)$$

where  $Q(z)$  is the rate of absorption by pigments per unit volume at depth  $z$ . The limits of integration are 0 and  $\infty$ ; in fact the non-trivial contributions to the integral, by definition, come only from the euphotic zone.

The magnitude of  $Q$  depends on  $I(z)$ , the light available at  $z$ , and on the total

absorption cross-section of pigments in the unit volume around  $z$ :

$$Q(z) = kB(z) I(z), \quad (2)$$

where  $k$  is the pigment-specific attenuation coefficient and  $B(z)$  is the pigment biomass. In turn,  $I(z)$  depends in part on the product  $kB(z)$ :

$$I(z) = I_0 e^{-\xi z - \chi z - k \int_0^z B(z') dz'}, \quad (3)$$

where  $\xi$  is the attenuation coefficient of the water itself and  $\chi$  is that due to suspensoids other than pigmented phytoplankton cells (PLATT *et al.*, 1977). In equation (3) the implicit assumption is that  $\chi$  and  $k$  do not vary with depth in the interval  $(0, z)$ .

We can now evaluate the integral in equation (1). It is convenient to make the simplifying assumption that the pigment biomass is distributed uniformly with depth,  $B(z) = B$ , constant. This leads to the result

$$S = \left( \frac{kB}{\xi + \chi + kB} \right) I_0. \quad (4)$$

We assume also that the realised water column production will be a linear function of the light absorbed:

$$\int P dz = \eta S, \quad (5)$$

where  $\eta$  is an efficiency factor to be determined. These assumptions, particularly the latter, will be discussed at length below.

We now define the quantity  $\Lambda$

$$\Lambda \equiv \int P dz / \int B dz, \quad (6)$$

where the pigments are integrated through the photic zone. Under the assumption of uniform pigment distribution we have,

$$\int_0^{Z_p} B dz = B Z_p, \quad (7)$$

where  $Z_p$  is the thickness of the photic zone, leading to the simple result

$$\Lambda = \frac{\eta S}{B Z_p}. \quad (8)$$

Writing out equation (8) explicitly for  $\Lambda$  we find

$$\Lambda = \frac{k I_0 \eta}{Z_p (\xi + \chi + kB)}. \quad (9)$$

Thus we conclude that a plot of  $\Lambda$  vs  $I_0$  should be a straight line with slope  $\psi$  (notation of FALKOWSKI, 1981), where

$$\psi = \frac{k \eta}{Z_p (\xi + \chi + kB)}. \quad (10)$$

ESTIMATING THE MAGNITUDE OF  $\psi$ 

Consider the case that  $I_0$  is given as a photon flux in units of Einsteins (E)  $\text{m}^{-2} \text{h}^{-1}$ . The efficiency factor  $\eta$  depends on maximum quantum yield of photosynthesis ( $\phi$ ) and some thermodynamic efficiency factor  $\theta$

$$\eta = \theta\phi. \quad (11)$$

We take the quantum requirement of photosynthesis to be 10 ( $\phi = 0.1 \text{ mol C (E)}^{-1}$ ) and  $\theta$  to be 10%. Then  $\eta \approx 10^{-2} \text{ mol C (E)}^{-1}$  or  $1.2 \times 10^{-1} \text{ g C (E)}^{-1}$ .

For the rest of the quantities in equation (10) we refer to Table 1 in JORDAN and JOINT (1984). There we estimate, roughly,  $Z_p \approx 40 \text{ m}$ ;  $B \approx 1 \text{ mg Chl } a \text{ m}^{-3}$ ; the quantity  $(\xi + \chi) \approx 0.1 \text{ m}^{-1}$ . We take  $k = 0.016 \text{ m}^2 (\text{mg Chl } a)^{-1}$ , following BANNISTER (1974). Then  $\psi \approx 4 \times 10^{-3} \eta \text{ mol C m}^2 (\text{mg Chl } a)^{-1} \text{ E}^{-1}$ . Converting to gC and substituting for  $\eta$  we find  $\psi \approx 4 \times 10^{-3} \times 12 \times 10^3 \times 10^{-2}$  or  $\psi \approx 0.5 \text{ g C (g Chl } a)^{-1} \text{ m}^2 (\text{E})^{-1}$ . This is entirely consistent with the empirical slope of  $0.48 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ E}^{-1}$  found by JORDAN and JOINT (1984) for the primary production experiments corresponding to the background data in their Table 1.

It is also instructive to estimate  $\Lambda$  from the typical magnitude of  $\alpha^B$ , the initial slope of the photosynthesis–light curve for phytoplankton. The pertinent quantity here is the efficiency  $\eta$ . A characteristic value of  $\alpha^B$  is  $\approx 0.05 \text{ mg C (mg Chl } a)^{-1} \text{ h}^{-1} (\text{W m}^{-2})^{-1}$  (e.g. PLATT *et al.*, 1980, 1982). It is important to note that  $\alpha^B$  is based on the light available, not the light absorbed,

$$\alpha^B = \frac{1}{B} \cdot \frac{\partial P}{\partial I} \bigg|_{I \rightarrow 0}. \quad (12)$$

An alternative definition for  $\eta$  is then similar to that for  $\alpha^B$ , but based on the light *absorbed*

$$\eta = \frac{1}{kB} \cdot \frac{\partial P}{\partial I} \bigg|_{I \rightarrow 0} = \frac{\alpha^B}{k}. \quad (13)$$

Noting that  $1 \text{ W m}^{-2} \approx 1.5 \times 10^{-2} \text{ E m}^{-2} \text{ h}^{-1}$ , we can then estimate  $\eta$  as

$$\eta \approx \frac{0.05}{1.5} \times 10^2 \times \frac{1}{0.016} \times 10^{-3} \text{ g C (E)}^{-1} \approx 2.0 \times 10^{-1} \text{ g C (E)}^{-1},$$

or roughly the same order of magnitude as before, leading to a similar estimate for  $\psi$ . We can thus proceed with the reassurance that the analysis and the empirical results relating water column production to surface light are securely consistent with what has been learned from independent experiments on phytoplankton physiology.

STABILITY OF  $\psi$ 

It may now be considered whether  $\psi$  can be expected to remain constant to any useful degree. The question is best approached through examination of equation (10). Only a relatively small number of quantities is involved. The value of  $Z_p$  itself will depend on the magnitude of the bracket  $(\xi + \chi + kB)$ . In particular, if all contributions to optical

attenuation are uniform with depth, and if  $Z_p$  is defined (conventionally) as the depth at which light is reduced to 1% of its surface value, we have simply

$$Z_p = \frac{4.6}{\xi + \chi + k_B} . \quad (14)$$

In this case equation (10) reduces to

$$\psi = \frac{k\eta}{4.6} . \quad (15)$$

An even simpler result emerges if we substitute for  $\eta$  according to equation (13)

$$\psi = \frac{\alpha^B}{4.6} . \quad (16)$$

Hence  $\psi$  can be considered constant to the extent that  $\alpha^B$  is uniform with depth and that  $\alpha^B$  is constant with respect to time and station.

#### THE EMPIRICAL EVIDENCE

The principal problem encountered in assembling the empirical data is knowing the averaging time to which the primary production measurements refer. Taking equation (16) literally, the time scale does not occur, since  $\psi$  is more in the nature of an efficiency than a rate; but one is not always sure that the irradiance data are calculated for the same time period over which the average primary production rates are quoted. The duration of the incubation affects the interpretation of the  $^{14}\text{C}$  results (MARRA *et al.*, 1981; DRING and JEWSON, 1982; SMITH, 1982; SMITH and PLATT, 1984).

Another problem is the interconversion between different sets of irradiance units, which cannot be done exactly if the spectral composition of the light is not known. The integrals over depth for  $P$  and  $B$  may not be precise if only a few depths are sampled (a problem that becomes less serious the more uniform the distribution of properties with depth). The integral over depth for  $P$  may be inaccurate if a simulated *in situ* procedure is used and the light levels have been improperly matched. In other words one expects a certain amount of inconsistency between data sets collected by different workers just from the differences in methodology and sampling.

#### LARRANCE (1971)

Primary production data collected during the period 1966–1968 in the mid-subarctic Pacific (south of the Aleutians) were published by LARRANCE (1971). Samples were collected at five relative light depths and incubated on deck in a simulated *in situ* format. Sampling depths were calculated from Secchi disc readings. Incubation lasted for one half the daylight period, but results were quoted as daily rates. Some 56 data points were reported. Incoming radiation was quoted as a daily rate, not corrected for PAR.

Some one-tenth of Larrance's data came from south of 46°N and therefore in the Transitional Domain between the subarctic and subtropical water masses. If these stations were excluded, an excellent regression of  $\Lambda$  on  $I_0$  was obtained by Larrance. The

intercept is not significantly different from zero. In the units used in this paper, the slope  $\psi = 0.66 \text{ g C (Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$ , where a factor of 0.5 has been used to correct for PAR. The data points from the transitional water appeared to have a steeper regression slope (LARRANCE, 1971, Fig. 3).

*MALONE (1976)*

Seasonal data from the New York Bight area were reported by MALONE (1976). Samples were collected at five relative light depths (as determined by Secchi disc) and incubated on deck by the simulated *in situ* method for 24 h. The results were quoted as daily rates. Incoming radiation was quoted as a daily rate, corrected for PAR. Some 30 data points were collected. For these data, Malone found an excellent regression with slope, converted to the units used in this paper,  $\psi = 2.5 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$ . Some 80% of the variance in  $\Lambda$  could be ascribed to variations in  $I_0$ . The slope is roughly five times the average of the slopes found in the other studies reported here, including one for the New York Bight itself (FALKOWSKI, 1981), and one may suspect a systematic error in the data, perhaps in the solar radiation figures, which seem low (MALONE, 1976, Fig. 11).

*MOREL (1978)*

Some 32 stations in the subtropical North Atlantic and the eastern tropical Pacific were reported by MOREL (1978). A variety of trophic regimes was sampled, from the less productive Sargasso Sea to the Mauritanian upwelling. Although the water column biomass varied by a factor of  $10^2$  between stations, the photosynthetically stored energy was a much less variable proportion of the available PAR, roughly  $7 \pm 3\%$ . In the units used here this is equivalent to a slope  $\psi = 0.44 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$ .

*FALKOWSKI (1981)*

A further seasonal study of the New York Bight area was reported by FALKOWSKI (1981). Some 37 data points were collected. For these data FALKOWSKI (1981, Fig. 6) could explain 86% of the variance in  $\Lambda$  by a regression on  $I_0$  for which the slope was  $\psi = 0.43 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$  (units incorrectly written in original paper). Comparing the abscissa in FALKOWSKI (1981, Fig. 6) with that in MALONE (1976, Fig. 11) for the same region, one sees after converting the units, that the irradiance scale is low by a factor of about 5 in Malone's data compared to Falkowski's. Correcting for this discrepancy would bring Malone's data into line with those of the rest of the studies discussed here.

*HARRISON et al. (1982)*

Data from 14 stations made during the summer of 1978 off the coast of northeast Baffin Island and in the open waters of Baffin Bay were published by HARRISON *et al.* (1982). Sampling was at five relative light depths chosen with the aid of the Secchi disc. The samples were incubated for 24 h on deck by the simulated *in situ* method. Incident radiation was measured directly on the ship and adjusted for PAR. Note that the conversion factor between  $\text{W m}^{-2}$  and  $\text{Langley h}^{-1}$  was incorrectly printed in the original paper.

These data could be described very well (HARRISON *et al.*, 1982, Fig. 9) by a regression (59% variance explained) of slope  $\psi = 0.33 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$  in the units used here and after correcting the error referred to in the previous paragraph. It is worth

noting that these stations (September, latitude  $\approx 70^\circ\text{N}$ ) represent conditions of almost continuous daylight.

*JORDAN and JOINT (1984)*

Primary production measurements made at various stations in the western English Channel during 1980 and 1981 were published by JORDAN and JOINT (1984). The results of some 13 *in situ* experiments were tabulated. Samples were collected at six fixed depths and incubated apparently for from 4 to 6 h (text not clear). Results were quoted as hourly rates. Insolation was recorded continuously during the incubations. The depth-averaged diffuse attenuation coefficient was calculated from submarine photometer readings. A regression slope of  $0.43 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$  was obtained by Jordan and Joint for these data (units incorrectly stated in original paper).

The extent to which the assumption of uniform distribution of properties is justified can be judged from Fig. 5 of JORDAN and JOINT (1984) which shows profiles for chlorophyll and primary production on two different sampling dates. The assumption would be a good one for the 5 May data but less so for the 5 August data.

*PLATT (unpublished data)*

*In situ* primary production data were collected at seven stations during August and September 1983 in Jones Sound (at  $\approx 76^\circ\text{N}$ ), eastern Canadian Arctic. Samples were taken at five fixed depths and incubated for 6 or 24 h nominal incubation times. Results are reported as daily rates. Incident radiation was measured directly on the ship and corrected for PAR. Conditions were open pack ice and almost 24 h continuous sunlight. Variations between days in insolation were considerable due to the sporadic incidence of thick fog.

These data could be described by a regression of slope  $\psi = 0.36 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$ .

*PLATT (unpublished data)*

*In situ* primary production measurements were made on 22 occasions during the spring bloom of 1985 in Bedford Basin, Nova Scotia. Both 6 and 24 h incubations were done at 1, 5 and 10 m. Incident radiation was measured directly at the Bedford Institute and converted to PAR. The 6 h incubations could be described by a regression slope  $\psi = 0.34 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$  with 70% of the variance explained. The 24 h data could be described by a regression slope  $\psi = 0.31 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$  with 59% of the variance explained.

## DISCUSSION

The slope of the regression of  $\Lambda$  on  $I_0$  varies remarkably little in the studies collected here (range  $0.31\text{--}0.66 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$ ). The theoretical analysis shows that, to the extent that properties can be assumed uniformly distributed throughout the photic zone, the slope  $\psi$  in the equation

$$\Lambda = \psi I_0 \quad (17)$$

is given to within a dimensionless constant by the value of the initial slope of the

photosynthesis–light curve ( $\alpha^B$ ). This is a particularly simple result, and we now examine the underlying assumptions.

*Assumption that photosynthesis is linear in absorbed light*

The assumption, stated in equation (5), that photosynthesis is linear in the absorbed light deserves considerable discussion. It will lead to bias to the extent that part of the water column lies outside the range of light-limited photosynthesis. The bias is likely to be negligible in turbid waters, but may be significant at transparent ocean stations. In general also, the agreement should be better in higher latitudes than low, and for the same reason.

The appropriate index for the light-limited region is the adaptation parameter  $I_k \equiv P_m^B/\alpha^B$ , where  $P_m^B$  is the assimilation number (PLATT and JASSBY, 1976). Data on the typical magnitude and variability of  $I_k$  on a range of time and space scales may be found in MACCAULL and PLATT (1977); HARRISON and PLATT (1980, 1981, 1986); CÔTÉ and PLATT (1983); SUBBA RAO and PLATT (1984). The light-limited range of photosynthesis may be described very roughly as the interval  $0 < I(z) \leq I_k$ .

The dimensionless quantity  $I_* \equiv I_0/I_k$  is then the relevant scale for judging the potential bias introduced by the linear approximation. The practical problem in applying it, however, is that  $I_0$  changes throughout the day. For further discussion, it is therefore essential to distinguish between the following quantities: (a)  $I_0''$ , the maximum instantaneous value of  $I_0$  as encountered at local noon; (b)  $I_0(t)$ , the instantaneous value of  $I_0$  measured at any time  $t$ ; and (c)  $I_0$ , as used in equations (3), (4), (9) and (17), with the implication of averaging or integrating over some period of time  $\leq 1$  day. Of course,  $I_0$  and  $I_0(t)$  become indistinguishable if the integration time is sufficiently short, and  $I_0$  may be used instead of  $I_0(t)$  where it is not essential to retain the explicit time dependence. Similar distinctions can be made for other quantities used in this paper, such as  $\psi$ ,  $I_*$ ,  $\Lambda$  and  $S$ .

We consider first the bias introduced for the instantaneous integral  $\int P(z)dz$ . Let us call this integral  $P_Z$ . It will be a function of time,  $P_Z(t)$ . For the *linear* model,

$$P_Z = \alpha I_0 \int_0^\infty e^{-\kappa z} dz, \quad (18)$$

where  $\kappa \equiv (\xi + \chi + kB)$  is the total attenuation coefficient and the lack of superscript on  $\alpha$  indicates no normalisation to pigment. Equation (18) reduces

$$P_Z = \frac{\alpha I_0}{\kappa}. \quad (19)$$

Substituting for  $\alpha$ , we find

$$P_Z = \frac{P_m}{\kappa} \cdot \frac{I_0}{I_k} = I_* \frac{P_m}{\kappa}, \quad (20)$$

where the lack of superscript in  $P_m$  again implies no normalisation to pigment.

We now calculate  $P_Z$  for a *non-linear* representation of the photosynthesis–light curve. Alternative representations have been discussed by JASSBY and PLATT (1976) and by PLATT *et al.* (1977). As a standard (LEDERMAN and TETT, 1981), we may take Smith's equation (SMITH, 1936).



We now calculate  $P_Z$  when  $P(z)$  is given by Smith's equation:

$$P(z) = \frac{P_m I_k^{-1} I(z)}{[1 + (I_k^{-1} I(z))^2]^{1/2}}. \quad (21)$$

In this case

$$P_Z = \frac{P_m}{K} \log_e (I_* + \sqrt{1 + I_*^2}). \quad (22)$$

To find the relative error  $\varepsilon$  incurred by the linear approximation we subtract equation (22) from (20) and divide by (22)

$$\varepsilon = \frac{I_* - \log_e (I_* + \sqrt{1 + I_*^2})}{\log_e (I_* + \sqrt{1 + I_*^2})}. \quad (23)$$

The result is plotted in Fig. 1. We see that the relative error of the linear approximation is 13% at  $I_* = 1$ , rising to 65% at  $I_* = 3$ . These are the magnitude of errors that might be expected for instantaneous evaluations of water column integrals using the linear approximation.

The next step is to take into account the time variation in  $I_0$ . A standard representation (IKUSHIMA, 1967) is

$$I_0(t) = I_0^n \sin^3 \left( \frac{\pi t}{D} \right), \quad (24)$$

where  $D$  is the daylength. The average light intensity throughout the day,  $\langle I_0 \rangle$ , can then be calculated as

$$\langle I_0 \rangle = \frac{1}{D} \int_0^D I_0^n \sin^3 \left( \frac{\pi t}{D} \right) dt \quad (25)$$

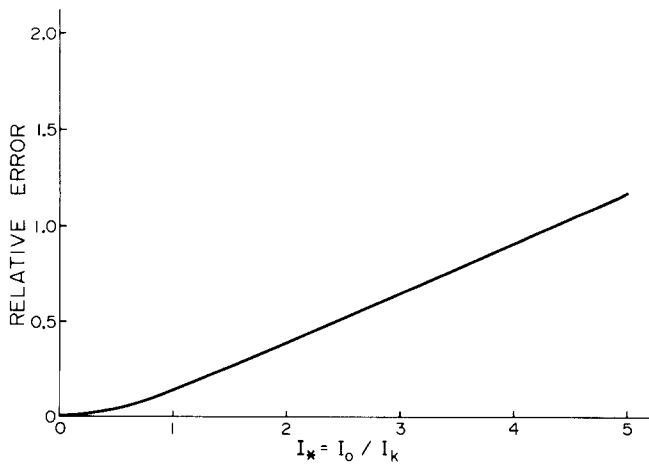


Fig. 1. Relative error  $\varepsilon$  (overestimate), as function of dimensionless parameter  $I_*$ , of linear approximation compared to Smith's equation for computing instantaneous water column integrals of primary production.

or

$$\langle I_0 \rangle = 0.42 I_0^m. \quad (26)$$

The factor 0.42 gives a crude indication of the reduction in the relative error on  $P_Z$  from working with the average surface light rather than the maximum (for which the error is maximal). For example, if the instantaneous relative error is  $\varepsilon^m$  at  $I_*^m$ , the average error  $\langle \varepsilon \rangle$  for the day would be roughly  $0.42 \varepsilon^m$ , or the error when  $I_* = \langle I_* \rangle$ .

Because the underlying relationships are nonlinear, evaluation of the average error for the day requires, more strictly, a complete double integration over depth and time. This is the problem we address next. We calculate  $P_{Z,T}$ , where

$$P_{Z,T} = \int_0^D \int_0^\infty P(z,t) dz, dt \quad (27)$$

with  $P(z,t)$  and  $I_0(t)$  specified as before.

In general, integrals of the form equation (27), where  $P(z)$  is nonlinear will be closer to their linear approximations than the instantaneous integrals evaluated at  $I_*(t) = I_*^m$  because  $I_*(t) < I_*^m$  for all times except noon. When  $P_Z$  is given by the simple *linear* approximation (equation 18), it is easy to show from equation (27) that

$$P_{Z,T} = 0.42 I_*^m D \frac{P_m}{\kappa}. \quad (28)$$

Equation (27) may be evaluated by numerical integration when  $P(I)$  is *nonlinear*. We can then calculate the relative error,  $\varepsilon$ , as before. As an example, we have computed equation (27) with  $P(z)$  specified by the standard Smith's equation and then the relative error between it and equation (28). This function  $\varepsilon(I_*^m)$  is plotted in Fig. 2. There we see that a linear approximation is in error by only 16% at  $I_*^m = 2$  and by 36% at  $I_*^m = 4$ .

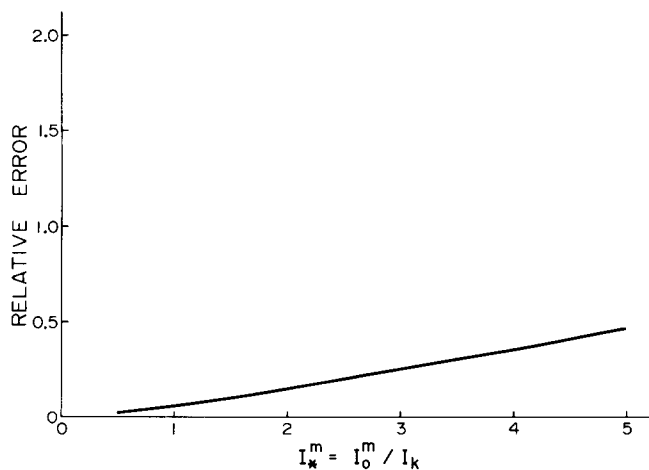


Fig. 2. Relative error  $\varepsilon$  (overestimate), as function of dimensionless parameter  $I_*^m$ , of linear approximation compared to Smith's equation for computing daily water column integrals of primary production.

We can summarise the arguments in this section as follows. The instantaneous water column integrals for primary production may be evaluated using various functional forms for the photosynthesis–light curve. The relative error  $\varepsilon(I_*)$  between any of these integrals and that for the approximation of linear (non-saturating) photosynthesis can be computed as a function of the dimensionless parameter  $I_* = I_0/I_k$ . Taking the Smith equation as standard, it is found that the linear approximation of the instantaneous column integral is within 50% of the correct answer for  $I_* \leq 2.5$ . When the time dependence of  $I_0$  is taken into account, a similar calculation shows that the linear approximation of the daily column integral is within 20% of the correct answer for  $I_*^m \leq 2.5$ . Furthermore the extent of the bias (overestimate) is given as  $I_*^m$  changes.

#### *The dependence of $\psi$ on $\alpha^B$*

The data from Bedford Basin afford an opportunity to test equation (16) connecting  $\psi$  and  $\alpha^B$ , since independent measures of the photosynthesis–light curves were made using shipboard incubators. The average value of  $\alpha^B$  for samples taken at both 1 and 10 m on 22 occasions (different days) during the sampling period was  $0.053 \text{ mg C (mg Chl } a)^{-1} \text{ h}^{-1} (\text{W m}^{-2})^{-1}$ . This is equivalent, from equation (16) to  $\psi = 0.77 \text{ g C (g Chl } a)^{-1} \text{ m}^2 (\text{E})^{-1}$ . The empirical result for the *in situ* incubations was  $\psi \approx 0.33 \text{ g C (g Chl } a)^{-1} \text{ m}^2 (\text{E})^{-1}$ .

We know from equation (23) that the linear approximation embodied in equation (16) should overestimate  $\psi$ , and by an amount depending on the ratio of  $I_0$  to  $I_k$ . The mean value of the derived photosynthetic parameter  $I_k$  for the Bedford Basin experiments was  $62 \text{ W m}^{-2}$ . During the same period, the average value of  $I_0^m$  was  $300 \text{ W m}^{-2}$ . Thus, a representative value of  $I_*^m$  for the sampling period is  $I_*^m \approx 5$ . From Fig. 2, we therefore expect that  $\psi$  estimated from equation (16) will be 47% too high, just from the bias introduced by the linear assumption. The bias may be removed through multiplication by the factor  $1/1.47$ , giving  $\psi = 0.52 \text{ g C (g Chl } a)^{-1} \text{ m}^2 (\text{E})^{-1}$ , to compare with  $\psi \approx 0.33 \text{ g C (g Chl } a)^{-1} \text{ m}^2 (\text{E})^{-1}$  calculated from the *in situ* incubations.

The test of consistency in this one case is most satisfactory. In practice we know that things will not be so simple because the assumption of uniform distribution of properties with depth will not always be respected. In those cases where the depth of the mixed layer exceeds the photic depth, however, equation (16) should be more robust.

Another complication is that when vertical mixing is relatively weak, vertical differences in  $\alpha^B$  can be expected to develop through photoadaptation (LEWIS *et al.*, 1984). The differences will be more pronounced when the photic depth exceeds the depth of the mixed layer (PLATT *et al.*, 1982; GALLEGOS and PLATT, 1982, 1985; GALLEGOS *et al.*, 1983). Note that it is the response of the water column as a whole that is in question, not the response at discrete depths. Thus, in the study reported by MOREL (1978), the fraction of available PAR that was stored in photosynthesis varied by a factor of 10 between surface and base of euphotic zone but the utilisation of PAR per unit area of surface varied by only a factor of three between stations.

One should also be aware of the effect of wavelength on  $\alpha^B$ . It has recently become possible to measure the action spectrum for phytoplankton at sea (LEWIS *et al.*, 1985a,b). It is found that the wavelength dependence is strong. As more field data of this kind are collected, one should be able to construct appropriate weighting functions involving products of the form  $\alpha^B(\lambda) \cdot I_0(\lambda)/k(\lambda)$ , where  $\lambda$  is the wavelength.

The general variability of  $\alpha^B$  on a variety of time and space scales has been discussed in a number of papers from this laboratory (HARRISON and PLATT, 1980, 1981, 1986;

MACCAULL and PLATT, 1977; CÔTÉ and PLATT, 1983, 1984; THERRIAULT and PLATT, 1978; THERRIAULT *et al.*, 1978). Taken together, they provide a good background for judging the utility of equation (16), and also (10) in those cases where explicit data on  $\alpha^B$  are lacking.

With regard to trophic status, the papers of MOREL (1978, 1982) give some idea of the variability in  $\psi$  to be expected between regions. Thus, for oligotrophic and mesotrophic stations  $\psi = 0.47 \pm 0.18 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$  (MOREL, 1978, Table 1a) whereas for the more eutrophic stations  $\psi = 0.35 \pm 0.14 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$  (MOREL, 1978, Table 1b). A somewhat different regime exists at the Guinea Dome ( $\approx 13^\circ\text{N}$ ). Here there is a well-developed, deep chlorophyll maximum at  $\approx 50 \text{ m}$ , with little chlorophyll elsewhere. In other words, the vertical distribution of pigment is very far from uniform. In this case, MOREL (1982) found  $\psi = 0.58 \pm 0.28 \text{ g C (g Chl } a)^{-1} \text{ m}^2 \text{ (E)}^{-1}$ .

One further implication of the analysis is that annual water column productivity should be acutely sensitive to variations between years in insolation. It is known that these can be significant (RYTHER and MENZEL, 1961; PLATT, 1971).

#### CONCLUSIONS

Recently a method has been proposed (TOPLISS and PLATT, in press) for estimation of  $\alpha^B$  from the fluorescence yield of the solar-stimulated or passive fluorescence response of chlorophyll pigments. This signal is detectable by optical profiling from a ship (TOPLISS, 1985) as well as by remote sensing from aircraft (NEVILLE and GOWER, 1977). Its deployment in satellites is foreseen (GOWER and BORSTAD, 1981). The possibility then arises that  $\Lambda$  could be indexed routinely and synoptically in standard surveys. The water column productivity  $\int P(z)dz$  follows immediately from equation (6) if an estimate of biomass is available, for example, from Coastal Zone Colour Scanner Imagery (HOVIS *et al.*, 1980; GORDON *et al.*, 1980). A surface layer estimate would be adequate here since the theory assumes  $B(z)$  constant throughout the photic zone and since it has been shown that surface values are good predictors of the water column integrals (PLATT and HERMAN, 1983). For ship surveys, vertical profiles of pigment biomass can be developed in minutes on station (HERMAN *et al.*, 1984) or continuously under way (HERMAN and DENMAN, 1977).

Remotely sensed information refers to the upper layers of the ocean where photosynthesis is less likely to be light-limited. The fact that  $\psi$  is shown to depend on  $\alpha^B$ , a characteristic of the light-limited part of the photosynthesis–light curve, does not, however, invalidate the exploitation of their interrelationship in remote sensing. The explanation lies partly in the points made in the preceding paragraph and partly in that, for the primary production of the water column as a whole,  $\alpha^B$  is the characteristic photosynthesis parameter. It is the production per unit of sea surface for the photic zone that is in question, not the production of any smaller depth interval, whether it be light-limited or light-saturated.

The formalism developed in this paper therefore appears to offer a secure framework for organisation of major programs of mapping the primary production of the ocean over large regions of space and with high frequency in time (NATIONAL ACADEMY OF SCIENCES, 1984; MALONE and ROEDERER, 1985). It has the advantage of being an approach that takes into account what is known about the physiological ecology of natural assemblages of marine phytoplankton.

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## REFERENCES

- BANNISTER T. T. (1974) Production equations in terms of chlorophyll concentration, quantum yield and upper limit to production. *Limnology and Oceanography*, **19**, 1–12.
- BROWN O. B., R. H. EVANS, J. W. BROWN, H. R. GORDON, R. C. SMITH and K. S. BAKER (1985) Phytoplankton blooming off the U.S. east coast: a satellite description. *Science*, **229**, 163–167.
- CÔTÉ B. and T. PLATT (1983) Day-to-day variations in the spring–summer photosynthetic parameters of coastal marine phytoplankton. *Limnology and Oceanography*, **28**, 320–344.
- CÔTÉ B. and T. PLATT (1984) Utility of light–saturation curve as an operational model for quantifying the effects of environmental conditions on phytoplankton photosynthesis. *Marine Ecology Progress Series*, **18**, 57–66.
- DRING M. J. and B. H. JEWSON (1982) What does  $^{14}\text{C}$  uptake by phytoplankton really measure? A theoretical modelling approach. *Proceedings of the Royal Society of London*, **B214**, 354–368.
- FALKOWSKI P. G. (1981) Light–shade adaptation and assimilation numbers. *Journal of Plankton Research*, **3**, 203–216.
- FASHAM M. J. R., editor (1984) *Flows of energy and materials in marine ecosystems: theory and practice*. Plenum Publishing Corporation, London, 773 pp.
- GALLEGOS C. L. and T. PLATT (1982) Phytoplankton production and turbulence in surface mixed layers. *Deep-Sea Research*, **28**, 65–76.
- GALLEGOS C. L. and T. PLATT (1985) Vertical advection of phytoplankton and productivity estimates: a dimensional analysis. *Marine Ecology Progress Series*, **26**, 125–134.
- GALLEGOS C. L., T. PLATT, W. G. HARRISON and B. IRWIN (1983) Photosynthetic parameters of arctic marine phytoplankton: vertical variations and time scales of adaptation. *Limnology and Oceanography*, **28**, 698–708.
- GAUTIER C. (1982) Mesoscale insolation variability derived from satellite data. *Journal of Applied Meteorology*, **21**, 51–58.
- GAUTIER C. and K. B. KATSAROS (1984) Insolation during STREX: comparisons between surface measurements and satellite estimates. *Journal of Geophysical Research*, **89**, 11,779–11,788.
- GAUTIER C., G. DIAK and S. MASSE (1980) A simple physical model to estimate incident solar radiation at the surface from GOES satellite data. *Journal of Applied Meteorology*, **19**, 1005–1012.
- GORDON H. R., D. K. CLARK, J. L. MUELLER and W. A. HOVIS (1980) Phytoplankton pigments from the Nimbus-7 coastal zone colour scanner: comparisons with surface measurements. *Science*, **210**, 63–66.
- GOWER J. F. R. and G. A. BORSTAD (1981) Use of the *in vivo* fluorescence line at 685 nm for remote sensing surveys of surface chlorophyll *a*. In: *Oceanography from space*. J. F. R. GOWER, editor, Plenum Press, New York, pp. 329–338.
- HARRISON W. G. and T. PLATT (1980) Variations in assimilation number of coastal marine phytoplankton: effects of environmental co-variables. *Journal of Plankton Research*, **2**, 249–260.
- HARRISON W. G. and T. PLATT (1981) Primary production and nutrient fluxes off the northern coast of Peru: a summary. In: *Investigacion Cooperativa De La Anchoveta Y Su Ecosistema (Icane) Entre Peru Y Canada*. Instituto Del Mar Del Peru Boletín. Volumen Extraordinario, pp. 15–21.
- HARRISON W. G. and T. PLATT (1986) Photosynthetic characteristics of phytoplankton in the Eastern Canadian Arctic. *Polar Biology*, in press.
- HARRISON W. G., B. IRWIN and T. PLATT (1982) Primary production and nutrient assimilation by natural phytoplankton populations of the eastern Canadian Arctic. *Canadian Journal of Fisheries and Aquatic Sciences*, **39**, 335–345.
- HERMAN A. W. and K. L. DENMAN (1977) Rapid underway profiling of chlorophyll with an *in situ* fluorometer mounted on a Batfish vehicle. *Deep-Sea Research*, **24**, 385–397.
- HERMAN A. W., M. R. MITCHELL and S. R. YOUNG (1984) A continuous pump sampler for profiling copepods and chlorophyll in the upper oceanic layers. *Deep-Sea Research*, **31**, 439–450.
- HOVIS W. A., D. K. CLARK, F. ANDERSON, R. W. AUSTIN, W. H. WILSON, E. T. BAKER, D. BALL, H. R. GORDON, J. L. MUELLER, S. Z. EL-SAYED, B. STURM, R. C. WRIGLEY and C. S. YENTSCH (1980) Nimbus-7 Coastal Zone Colour Scanner: system description and initial imagery. *Science*, **210**, 60–63.
- IKUSHIMA I. (1967) Ecological studies on the productivity of aquatic plant communities. III. Effect of depth on daily photosynthesis in submerged macrophytes. *Botanical Magazine, Tokyo*, **80**, 57–67.
- JASSBY A. D. and T. PLATT (1976) Mathematical formulation of the relationship between photosynthesis and light for phytoplankton. *Limnology and Oceanography*, **21**, 540–547.
- JENKINS W. J. (1982) Oxygen utilization rates in the North Atlantic subtropical gyre and primary production in oligotrophic systems. *Nature*, **300**, 246–248.

- JENKINS W. J. and J. C. GOLDMAN (1985) Seasonal oxygen cycling and primary production in the Sargasso Sea. *Journal of Marine Research*, **43**, 465–491.
- JORDON M. B. and I. R. JOINT (1984) Studies on phytoplankton distribution and primary production in the western English Channel in 1980 and 1981. *Continental Shelf Research*, **3**, 25–34.
- LARRANCE J. D. (1971) Primary production in the Mid-Subarctic Pacific Region, 1966–68. *Fisheries Bulletin*, **69**, 595–613.
- LEDERMAN T. C. and P. TETT (1981) Problems in modelling the photosynthesis–light relationship for phytoplankton. *Botanica Marina*, **24**, 125–134.
- LEWIS M. and J. SMITH (1983) A small volume, short-incubation-time method for measurement of photosynthesis as a function of incident irradiance. *Marine Ecology*, **13**, 99–102.
- LEWIS M. R., E. P. W. HORNE, J. J. CULLEN, N. S. O'KEY and T. PLATT (1984) Turbulent motions may control phytoplankton photosynthesis in the upper ocean. *Nature*, **311**, 49–50.
- LEWIS M. R., R. E. WARNOCK, B. IRWIN and T. PLATT (1985a) Measuring photosynthetic action spectra of natural phytoplankton populations. *Journal of Phycology*, **21**, 310–315.
- LEWIS M. R., R. E. WARNOCK and T. PLATT (1985b) Absorption and photosynthesis action spectra for natural phytoplankton populations: implications for production in oligotrophic ocean. *Limnology and Oceanography*, **30**, 794–806.
- MALONE T. C. (1976) Phytoplankton productivity in the apex of the New York Bight: environmental regulation of productivity/chlorophyll *a*. In: *The Middle Atlantic Continental Shelf and New York Bight*, M. G. GROSS, editor, *Limnology and Oceanography Special Symposium*, **2**, 260–272.
- MALONE T. F. and J. G. ROEDERER (1985) *Global change*. Cambridge University Press, Cambridge, 512 pp.
- MARRA J., G. LANDRIAU and H. W. DUCKLOW (1981) Tracer kinetics and plankton rate processes in oligotrophic oceans. *Marine Biology Letters*, **2**, 215–223.
- MACCAULL W. A. and T. PLATT (1977) Diel variations in the photosynthetic parameters of coastal marine phytoplankton. *Limnology and Oceanography*, **22**, 723–731.
- MOISEEV P. A. (1971) The living resources of the World oceans. Transcript from Russian. In: *Israel Program for Scientific Translations, Jerusalem*, B. GOLEK, editor. Published for the National Marine Fisheries Service, National Oceanic and Atmospheric Administration, U.S. Department of Commerce and The National Science Foundation, Washington, D.C., 334 pp.
- MOREL A. (1978) Available, usable and stored radiant energy in relation to marine photosynthesis. *Deep-Sea Research*, **25**, 673–688.
- MOREL A. (1982) Optical properties and radiant energy in the waters of the Guinea Dome and the Mauritanian upwelling area in relation to primary production. *Rapports et Proces-Verbaux des Reunions, Conseil International pour l'Exploration de la Mer*, **180**, 94–107.
- NATIONAL ACADEMY OF SCIENCES (1984) Global Ocean Flux Study: Proceedings of a workshop, September 10–14, 1984, National Academy of Sciences, Woods Hole Study Center, Woods Hole, Massachusetts. Academy Press, 360 pp.
- NEVILLE R. A. and J. F. R. GOWER (1977) Passive remote sensing of phytoplankton via chlorophyll *a* fluorescence. *Journal of Geophysical Research*, **82**, 3487–3493.
- PLATT T. (1971) The annual production by phytoplankton in St. Margaret's Bay, Nova Scotia. *Journal de Conseil, Conseil International pour l'Exploration de la Mer*, **33**, 324–334.
- PLATT T. (1984) Primary productivity in the central North Pacific: comparison of oxygen and carbon fluxes. *Deep-Sea Research*, **31**, 1311–1319.
- PLATT T. and A. D. JASSBY (1976) The relationship between photosynthesis and light for natural assemblages of coastal marine phytoplankton. *Journal of Phycology*, **12**, 421–430.
- PLATT T. and A. W. HERMAN (1983) Remote sensing of phytoplankton in the sea: surface layer chlorophyll as an estimate of water column chlorophyll and primary production. *International Journal of Remote Sensing*, **4**, 343–351.
- PLATT T., M. LEWIS and R. GEIDER (1984) Thermodynamics of the pelagic ecosystem: elementary closure conditions for biological production in the open ocean. In: *Flows of energy and materials in marine ecosystem: theory and practice*, M. J. R. FASHAM, editor, Plenum Publishing Corporation, London, pp. 49–84.
- PLATT T. and W. G. HARRISON (1985) Biogenic fluxes of carbon and oxygen in the ocean. *Nature*, **318**, 55–58.
- PLATT T. and W. G. HARRISON (1986) Letter to the Editor. Reconciliation of carbon and oxygen fluxes in the upper ocean. *Deep-Sea Research*, **318**, 55–58.
- PLATT T., K. L. DENMAN and A. D. JASSBY (1977) Modelling the productivity of phytoplankton. In: *The sea: ideas and observations on progress in the study of the sea*, E. D. GOLDBERG, editor, John Wiley, New York, pp. 807–856.
- PLATT T., C. L. GALLEGOS and W. G. HARRISON (1980) Photoinhibition of photosynthesis in natural assemblages in marine phytoplankton. *Journal of Marine Research*, **38**, 687–701.
- PLATT T., K. H. MANN and R. E. ULANOWICZ, editors (1981) *Mathematical models in biological oceanography*. UNESCO Monographs on Oceanographic Methodology, The UNESCO Press, Paris, 156 pp.

- PLATT T., W. G. HARRISON, B. IRWIN, E. P. HORNE and C. L. GALLEGOS (1982) Photosynthesis and photoadaptation of marine phytoplankton in the Arctic. *Deep-Sea Research*, **29**, 1159–1170.
- REID J. L. and E. SHULENBERGER (1986) Letter to the Editor. Oxygen saturation and carbon uptake near 28°N, 155°W. *Deep-Sea Research*, **33**, 267–271.
- REVELLE R. (1985) Oceanography from Space. *Science*, **228**, 133.
- RYTHER J. H. and D. W. MENZEL (1961) Primary production in the southwest Sargasso Sea, January–February, 1960. *Bulletin of Marine Science of the Gulf and Caribbean*, **11**, 381–388.
- SHULENBERGER E. and J. L. REID (1981) The Pacific shallow oxygen maximum, deep chlorophyll maximum, and primary productivity, reconsidered. *Deep-Sea Research*, **28**, 901–919.
- SMITH E. L. (1936) Photosynthesis in relation to light and carbon dioxide. *Proceedings of the National Academy of Sciences*, **22**, 504–511.
- SMITH R. C. (1981) Remote sensing and depth distribution of ocean chlorophyll. *Marine Ecology Progress Series*, **5**, 359–361.
- SMITH R. E. H. (1982) The estimation of phytoplankton production and excretion by <sup>14</sup>C. *Marine Biology Letters*, **3**, 325–334.
- SMITH R. C. and K. S. BAKER (1978) The bio-optical state of ocean waters and remote sensing. *Limnology and Oceanography*, **23**, 247–259.
- SMITH R. C. and K. S. BAKER (1982) Oceanic chlorophyll concentrations as determined by satellite (Nimbus-7 Coastal Zone Colour Scanner). *Marine Biology*, **66**, 1–11.
- SMITH R. E. H. and T. PLATT (1984) Carbon exchange and <sup>14</sup>C tracer methods in a nitrogen-limited diatom, *Thalassiosira pseudonana*. *Marine Ecology Progress Series*, **16**, 75–87.
- SMITH R. C., R. W. EPPLEY and K. S. BAKER (1982) Correlation of primary production as measured aboard ship in southern California coastal waters and as estimated from satellite chlorophyll images. *Marine Biology*, **66**, 1–8.
- SUBBA RAO D. V. and T. PLATT (1984) Primary production in the Arctic Ocean. *Polar Biology*, **3**, 191–201.
- THERRIAULT J. C. and T. PLATT (1978) Spatial heterogeneity of phytoplankton biomass and related factors in the near-surface waters of an exposed coastal embayment. *Limnology and Oceanography*, **23**, 888–899.
- THERRIAULT J. C., D. J. LAWRENCE and T. PLATT (1978) Spatial variability of phytoplankton turnover in relation to physical processes in a coastal embayment. *Limnology and Oceanography*, **23**, 900–911.
- TOPLISS B. J. (1985) Optical measurements in the Sargasso Sea: solar stimulated chlorophyll fluorescence. *Oceanological Acta*, **8**, 263–270.
- TOPLISS B. J., T. PLATT and B. IRWIN Passive fluorescence and photosynthesis in the ocean. *Deep Sea Research*, in press.