Simple Solar Spectral Model for Direct and Diffuse Irradiance on Horizontal and Tilted Planes at the Earth's Surface for Cloudless Atmospheres

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ABSTRACT

In a previous work, we described a simple model for calculating direct normal and diffuse horizontal spectral solar irradiance for cloudless sky conditions. In this paper, we present a new simple model (SPCTRAL2) that incorporates improvements to the simple model approach and an algorithm for calculating spectral irradiance on tilted surfaces. The model was developed using comparisons with rigorous radiative transfer codes and limited outdoor measurements. SPCTRAL2 produces terrestrial spectra between 0.3 and 4.0 μm with a resolution of approximately 10 nm. Inputs to the model include the solar zenith angle, the collector tilt angle, atmospheric turbidity, the amount of precipitable water vapor and ozone, surface pressure, and ground albedo. A major goal of this work is to provide researchers with the capability to calculate spectral irradiance for different atmospheric conditions and different solar collector geometries using microcomputers.

1. Introduction

In previous work (Bird, 1984), we presented a simple model to calculate direct normal and diffuse horizontal spectral irradiance at the earth's surface for cloudless sky conditions. In this paper, we present a new simple model (SPCTRAL2) that incorporates improvements in methodology as well as an algorithm to produce spectra for tilted surfaces. The goal of this work is to give researchers the capability to produce accurate terrestrial spectra using only a microcomputer.

The first model (Bird, 1984) was based on models developed by Leckner (1978) and Brine and Iqbal (1983). Since that work was completed, Justus and Paris (1985) have made improvements to the simple model approach. In the model presented here, we refined the Justus and Paris model and extended it to calculate spectra for tilted surfaces using methods developed by Hay and Davies (1978). Refinements to the Justus and Paris model were based on comparisons with results of rigorous radiative transfer codes and with measured spectra.

The equations used in SPCTRAL2 and examples of the output are presented in the sections that follow. Sections 2 and 3 describe methods for calculating direct normal and diffuse spectral irradiance, respectively. Section 4 gives comparisons of SPCTRAL2 results with rigorous model results and measurements. Examples of the application of the new model are given in section 5.

2. Direct normal irradiance

Minor modifications have been made to the methods we reported in Bird (1984) for calculating direct normal irradiance. The changes include the addition of an earth–sun distance factor, the use of Leckner's water vapor transmittance expression (1978) with some modification of Leckner's absorption coefficients, and the use of Robinson's ozone mass expression as given by Iqbal (1983). These changes and other minor adjustments are described in this section.

The direct irradiance on a surface normal to the direction of the sun at ground level for wavelength λ is given by

$$I_{d\lambda} = H_{o\lambda} D T_{r\lambda} T_{a\lambda} T_{w\lambda} T_{o\lambda} T_{u\lambda}. \quad (1)$$

The parameter $H_{o\lambda}$ is the extraterrestrial irradiance at the mean earth–sun distance for wavelength λ; D is the correction factor for the earth–sun distance; and $T_{r\lambda}$, $T_{a\lambda}$, $T_{w\lambda}$, $T_{o\lambda}$, and $T_{u\lambda}$ are the transmittance functions of the atmosphere at wavelength λ for molecular (Rayleigh) scattering, aerosol attenuation, water vapor absorption, ozone absorption, and uniformly mixed gas absorption, respectively. The direct irradiance on a horizontal surface is obtained by multiplying Eq. (1) by cos$Z$, where Z is the solar zenith angle.

The extraterrestrial spectral irradiance used here was obtained from Fröhlich and Wehrli (1981) of the World Radiation Center. A major segment of this spectrum that is of interest here was taken from the revised Neckel and Labs (1981) spectrum. A 10-nm resolution version of this spectrum is shown in Table 1 for the 122 wavelengths used in this model. The results of integrating the modeled spectra may be more accurate if the variable resolution of the wavelength intervals in the model is used for the extraterrestrial spectrum instead of a fixed 10 nm resolution. However, analyses were performed to select the wavelengths and the res-

TABLE 1. The Neckel and Labs revised extraterrestrial spectrum and atmospheric absorption coefficients at 122 wavelengths.

Wavelength (μm)	Extraterrestrial spectrum (W m^{-2} μm^{-1})	$a_{w\lambda}$	$a_{o\lambda}$	$a_{u\lambda}$	Wavelength (μm)	Extraterrestrial spectrum (W m^{-2} μm^{-1})	$a_{w\lambda}$	$a_{o\lambda}$	$a_{u\lambda}$
0.300	535.9	0.0	10.0	0.0	0.980	767.0	1.48	0.0	0.0
0.305	558.3	0.0	4.80	0.0	0.9935	757.6	0.1	0.0	0.0
0.310	622.0	0.0	2.70	0.0	1.04	688.1	0.00001	0.0	0.0
0.315	692.7	0.0	1.35	0.0	1.07	640.7	0.001	0.0	0.0
0.320	715.1	0.0	0.800	0.0	1.10	606.2	3.2	0.0	0.0
0.325	832.9	0.0	0.380	0.0	1.12	585.9	115.0	0.0	0.0
0.330	961.9	0.0	0.160	0.0	1.13	570.2	70.0	0.0	0.0
0.335	931.9	0.0	0.075	0.0	1.145	564.1	75.0	0.0	0.0
0.340	900.6	0.0	0.040	0.0	1.161	544.2	10.0	0.0	0.0
0.345	911.3	0.0	0.019	0.0	1.17	533.4	5.0	0.0	0.0
0.350	975.5	0.0	0.007	0.0	1.20	501.6	2.0	0.0	0.0
0.360	975.9	0.0	0.0	0.0	1.24	477.5	0.002	0.0	0.05
0.370	1119.9	0.0	0.0	0.0	1.27	442.7	0.002	0.0	0.30
0.380	1103.8	0.0	0.0	0.0	1.29	440.0	0.1	0.0	0.02
0.390	1033.8	0.0	0.0	0.0	1.32	416.8	4.0	0.0	0.0002
0.400	1479.1	0.0	0.0	0.0	1.35	391.4	200.0	0.0	0.00011
0.410	1701.3	0.0	0.0	0.0	1.395	358.9	1000.0	0.0	0.00001
0.420	1740.4	0.0	0.0	0.0	1.4425	327.5	185.0	0.0	0.05
0.430	1587.2	0.0	0.0	0.0	1.4625	317.5	80.0	0.0	0.011
0.440	1837.0	0.0	0.0	0.0	1.477	307.3	80.0	0.0	0.005
0.450	2005.0	0.0	0.003	0.0	1.497	300.4	12.0	0.0	0.0006
0.460	2043.0	0.0	0.006	0.0	1.520	292.8	0.16	0.0	0.0
0.470	1987.0	0.0	0.009	0.0	1.539	275.5	0.002	0.0	0.005
0.480	2027.0	0.0	0.014	0.0	1.558	272.1	0.0005	0.0	0.13
0.490	1896.0	0.0	0.021	0.0	1.578	259.3	0.0001	0.0	0.04
0.500	1909.0	0.0	0.030	0.0	1.592	246.9	0.00001	0.0	0.06
0.510	1927.0	0.0	0.040	0.0	1.610	244.0	0.0001	0.0	0.13
0.520	1831.0	0.0	0.048	0.0	1.630	243.5	0.001	0.0	0.001
0.530	1891.0	0.0	0.063	0.0	1.646	234.8	0.01	0.0	0.0014
0.540	1898.0	0.0	0.075	0.0	1.678	220.5	0.036	0.0	0.0001
0.550	1892.0	0.0	0.085	0.0	1.740	190.8	1.1	0.0	0.00001
0.570	1840.0	0.0	0.120	0.0	1.80	171.1	130.0	0.0	0.00001
0.593	1768.0	0.075	0.119	0.0	1.860	144.5	1000.0	0.0	0.0001
0.610	1728.0	0.0	0.120	0.0	1.920	135.7	500.0	0.0	0.001
0.630	1658.0	0.0	0.090	0.0	1.960	123.0	100.0	0.0	4.3
0.656	1524.0	0.0	0.065	0.0	1.985	123.8	4.0	0.0	0.20
0.6676	1531.0	0.0	0.051	0.0	2.005	113.0	2.9	0.0	21.0
0.690	1420.0	0.016	0.028	0.15	2.035	108.5	1.0	0.0	0.13
0.710	1399.0	0.0125	0.018	0.0	2.065	97.5	0.4	0.0	1.0
0.718	1374.0	1.80	0.015	0.0	2.10	92.4	0.22	0.0	0.08
0.7244	1373.0	2.5	0.012	0.0	2.148	82.4	0.25	0.0	0.001
0.740	1298.0	0.061	0.010	0.0	2.198	74.6	0.33	0.0	0.00038
0.7525	1269.0	0.0008	0.008	0.0	2.270	68.3	0.50	0.0	0.001
0.7575	1245.0	0.0001	0.007	0.0	2.360	63.8	4.0	0.0	0.0005
0.7625	1223.0	0.00001	0.006	4.0	2.450	49.5	80.0	0.0	0.00015
0.7675	1205.0	0.00001	0.005	0.35	2.5	48.5	310.0	0.0	0.00014
0.780	1183.0	0.0006	0.0	0.0	2.6	38.6	15000.0	0.0	0.00066
0.800	1148.0	0.0360	0.0	0.0	2.7	36.6	22000.0	0.0	100.0
0.816	1091.0	1.60	0.0	0.0	2.8	32.0	8000.0	0.0	150.0
0.8237	1062.0	2.5	0.0	0.0	2.9	28.1	650.0	0.0	0.13
0.8315	1038.0	0.500	0.0	0.0	3.0	24.8	240.0	0.0	0.0095
0.840	1022.0	0.155	0.0	0.0	3.1	22.1	230.0	0.0	0.001
0.860	998.7	0.00001	0.0	0.0	3.2	19.6	100.0	0.0	0.8
0.880	947.2	0.0026	0.0	0.0	3.3	17.5	120.0	0.0	1.9
0.905	893.2	7.0	0.0	0.0	3.4	15.7	19.5	0.0	1.3
0.915	868.2	5.0	0.0	0.0	3.5	14.1	3.6	0.0	0.075
0.925	829.7	5.0	0.0	0.0	3.6	12.7	3.1	0.0	0.01
0.930	830.3	27.0	0.0	0.0.	3.7	11.5	2.5	0.0	0.00195
0.937	814.0	55.0	0.0	0.0	3.8	10.4	1.4	0.0	0.004
0.948	786.9	45.0	0.0	0.0	3.9	9.5	0.17	0.0	0.29
0.965	768.3	4.0	0.0	0.0	4.0	8.6	0.0045	0.0	0.025

olution of the extraterrestrial spectrum so that the structure of the modeled and measured (10-nm resolution) spectra would match as closely as possible using a limited number of wavelengths.

The earth–sun distance factor as given by Spencer (1971) is

$$D = 1.00011 + 0.034221 \cos\psi + 0.00128 \sin\psi + 0.000719 \cos 2\psi + 0.000077 \sin 2\psi. \quad (2)$$

The day angle ψ in radians is represented by

$$\psi = 2\pi(d - 1)/365, \quad (3)$$

where d is the day number of a year (1–365).

a. Rayleigh scattering

The expression that we use for the atmospheric transmittance after Rayleigh scattering was adapted from Kneizys *et al.* (1980) and is

$$T_{r\lambda} = \exp\{-M'/[\lambda^4(115.6406 - 1.335/\lambda^2)]\}, \quad (4)$$

where M' is the pressure-corrected air mass. To be completely consistent with the Kneizys *et al.* expression, the coefficient 1.335 would be 1.3366. The relative air mass as given by Kasten (1966) is

$$M = [\cos Z + 0.15(93.885 - Z)^{-1.253}]^{-1}, \quad (5)$$

where Z is the apparent solar zenith angle. The pressure-corrected air mass is $M' = MP/P_0$, where $P_0 = 1013$ mb and P is measured surface pressure in mb.

b. Aerosol scattering and absorption

In our previous work (Bird, 1984), we used an aerosol transmittance expression of the form

$$T_{a\lambda} = \exp(-\beta_n \lambda^{-\alpha_n} M). \quad (6)$$

Values for β and α were derived using a rural aerosol model (Shettle and Fenn, 1975). Two α values were used for this aerosol model: $\alpha_1 = 1.0274$ for wavelengths < 0.5 μm, and $\alpha_2 = 1.2060$ for wavelengths $\geqslant 0.5$ μm. The value of β_n was chosen appropriately for each wavelength interval to produce accurate turbidity values (aerosol optical depth in a vertical path) at 0.5 μm wavelength. For two intervals, this required two β_n values in order to make the turbidity from the two intervals match at 0.5 μm wavelength. The turbidity in Eq. (6) is represented by the Angstrom formula (1961), namely,

$$\tau_{a\lambda} = \beta_n \lambda^{-\alpha_n}. \quad (7)$$

For some types of aerosols, it may be important to separate the aerosol extinction into two or more segments, as we have done here for the rural aerosol model. The form of Eq. (6) allows the turbidity versus the wavelengths on a log–log plot to be nonlinear, which often occurs in the real atmosphere, as shown by King and Herman (1979). However, for the rural aerosol model, this does not appear to significantly improve the accuracy of the modeled results since the function is approximately linear. Also, the approximate nature of this simple model approach sometimes masks the effect of refinements such as this. When a single value of α is used to represent the rural aerosol model, the value should be $\alpha = 1.140$.

c. Water vapor absorption

We adopted the water vapor transmittance expression of Leckner (1978), which has the form

$$T_{w\lambda} = \exp[-0.2385 a_{w\lambda} WM/(1 + 20.07 a_{w\lambda} WM)^{0.45}], \quad (8)$$

where W is the precipitable water vapor (cm) in a vertical path and $a_{w\lambda}$ is the water vapor absorption coefficient as a function of wavelength. The water vapor amount W is not temperature- or pressure-corrected because this has been accounted for in the form of Eq. (8). We modified some of Leckner's values of $a_{w\lambda}$ and added several values to achieve closer agreement with experimental data. The resulting coefficients are given in Table 1. In our previous model, we used a misprinted version of Leckner's expression, which necessitated modifications to the expression and to the absorption coefficients to obtain reasonable agreement with rigorous model results. The correct form, shown in Eq. (8), gives better results.

d. Ozone and uniformly mixed gas absorption

Leckner's ozone transmittance equation (1978) was used, which is

$$T_{o\lambda} = \exp(-a_{o\lambda} O_3 M_o), \quad (9)$$

where $a_{o\lambda}$ is the ozone absorption coefficient, O_3 the ozone amount (atm-cm), and M_o the ozone mass. We used Leckner's ozone absorption coefficients shown in Table 1. The ozone mass expression of Robinson as given by Iqbal (1983) has been adopted. The ozone mass is given by

$$M_o = (1 + h_o/6370)/[\cos^2(Z) + 2h_o/6370]^{0.5}. \quad (10)$$

The parameter h_o is the height of maximum ozone concentration, which is approximately 22 km. The ozone height varies with latitude and time of year. If one does not have ozone measurements available, the ozone amount can be estimated using the expression of Van Heuklon (1979). Since the total ozone amount is an approximation, using $O_3 M_o$ rather than $O_3 M$ may not be an improvement.

Leckner's expression for uniformly mixed gas transmittance is used, and it is expressed as

$$T_{u\lambda} = \exp[-1.41 a_{u\lambda} M'/(1 + 118.93 a_{u\lambda} M')^{0.45}], \quad (11)$$

where $a_{u\lambda}$ is the combination of an absorption coefficient and gaseous amount. We used Leckner's values of $a_{u\lambda}$ shown in Table 1 with a few additions and modifications. The coefficient 118.93 in Eq. (11) was used here, but was later found to be 118.3 in Leckner's paper. This does not have a large effect on the results, but Leckner's value should be used in the future. Final adjustments were made in the gaseous absorption coefficients by comparing the modeled data with measured data, as described in section 4.

3. Diffuse irradiance

The diffuse irradiance is difficult to determine accurately with the simple parameterization methods that were used to calculate direct normal irradiance in the previous section. We used tabulated correction factors in our previous research (Bird, 1984) to make the simple formulation for the diffuse irradiance of Brine and Iqbal (1983) match the results from a rigorous radiative transfer code. Justus and Paris (1985) modified the diffuse formulation and obtained reasonable agreement with rigorous code results without using tabulated correction factors. We examined this new formulation and made some minor adjustments to improve its accuracy. The correction table approach is still valid and may be the most accurate approach; however, this new formulation is more flexible and easier to implement.

In addition, we examined different simple formulations for producing spectra on inclined surfaces. The results of this effort are also reported here.

a. *Diffuse irradiance on a horizontal surface*

The diffuse irradiance on a horizontal surface is divided into three components: 1) the Rayleigh scattering component $I_{r\lambda}$, 2) the aerosol scattering component $I_{a\lambda}$, and 3) the component that accounts for multiple reflection of irradiance between the ground and the air $I_{g\lambda}$. The total scattered irradiance $I_{s\lambda}$ is then given by the sum

$$I_{s\lambda} = I_{r\lambda} + I_{a\lambda} + I_{g\lambda}. \quad (12)$$

If we consider the Rayleigh and aerosol scattering to be independent of each other, the following expressions would be approximately correct:

$$I_{r\lambda} = H_{o\lambda} D \cos(Z) T_{o\lambda} T_{u\lambda} T_{w\lambda} T_{a\lambda} (1 - T_{r\lambda}) 0.5, \quad (13)$$

$$I_{a\lambda} = H_{o\lambda} D \cos(Z) T_{o\lambda} T_{u\lambda} T_{w\lambda} T_{r\lambda} T_{aa\lambda} (1 - T_{as\lambda}) F_s. \quad (14)$$

In these formulas, we have assumed that half of the Rayleigh scatter is downward regardless of the zenith angle of the sun, and that a fraction F_s of the aerosol scatter is downward and can be a function of the solar zenith angle. The transmittance terms $T_{aa\lambda}$ and $T_{as\lambda}$ are for aerosol absorption and aerosol scattering, respectively. In our previous model, we used the assumption of independent scattering, and Eq. (14) has the following form:

$$I_{a\lambda} = H_{o\lambda} D \cos(Z) T_{o\lambda} T_{u\lambda} T_{w\lambda} T_{r\lambda} (1 - T_{a\lambda}) \omega_0 F_a, \quad (15)$$

where ω_0 is the aerosol single scattering albedo at one wavelength and F_a the aerosol forward scattering fraction, which is independent of the sun position. We found that this formula significantly underestimated the scattered irradiance for $Z > 60°$.

In the new simple spectral model reported here (SPCTRAL2), we used modifications of the Justus and Paris expressions (1985) for diffuse irradiance. Comparisons with diffuse irradiance calculated using a rigorous radiative transfer code (BRITE) (Bird, 1983) indicated a tendency for the Justus and Paris model to overestimate the energy in the UV and visible portions of the spectrum. This overestimation increased as the turbidity and air mass increased. By slightly modifying the expression, we were able to obtain closer agreement with BRITE results. We attempted to modify the transmittance expression for aerosols and also tried other modifications, but the changes made to the Rayleigh transmittance terms produced the best results. Table 2 gives examples of the results of these comparisons. The modified expressions are

$$I_{r\lambda} = H_{o\lambda} D \cos(Z) T_{o\lambda} T_{u\lambda} T_{w\lambda} T_{aa\lambda} (1 - T_{r\lambda}^{0.95}) 0.5, \quad (16)$$

$$I_{a\lambda} = H_{o\lambda} D \cos(Z) T_{o\lambda} T_{u\lambda} T_{w\lambda} T_{aa\lambda} T_{r\lambda}^{1.5} (1 - T_{as\lambda}) F_s, \quad (17)$$

$$I_{g\lambda} = (I_{d\lambda} \cos(Z) + I_{r\lambda} + I_{a\lambda}) r_{s\lambda} r_{g\lambda} / (1 - r_{s\lambda} r_{g\lambda}), \quad (18)$$

TABLE 2. Diffuse irradiance (W m^{-2} μm^{-1}) at selected wavelengths calculated using the BRITE code, the Justus and Paris code, and the modified Justus and Paris code. (Model parameters for (a) and (b) are $\alpha = 1.14$, ground albedo = 0.2, O_3 = 0.344 atm-cm, and H_2O = 1.42 cm.)

λ	BRITE	Justus/ Paris	Modified Justus/ Paris	Ratio[a]	Ratio[b]
		(a) $\tau = 0.27$, $Z = 60°$			
0.31	11.9	25.5	17.7	2.14	1.49
0.35	172.6	243.3	174.5	1.41	1.01
0.40	307.2	343.5	268.5	1.12	0.87
0.45	382.7	419.8	368.0	1.10	0.96
0.50	306.6	351.8	317.0	1.15	1.03
0.55	260.6	301.3	278.1	1.16	1.07
0.71	173.6	169.7	163.9	0.98	0.94
0.78	141.9	130.0	126.7	0.92	0.89
0.9935	51.9	60.4	59.8	1.16	1.15
2.1	1.85	2.3	2.3	1.24	1.24
		(b) $\tau = 0.51$, $Z = 80°$			
0.31	0.28	0.34	0.26	1.21	0.93
0.35	37.9	71.2	56.8	1.88	1.50
0.40	80.3	114.8	92.8	1.43	1.16
0.45	125.2	160.8	133.6	1.28	1.07
0.50	126.0	149.3	122.6	1.18	0.97
0.55	115.0	135.0	113.3	1.17	0.99
0.78	81.0	89.5	83.9	1.10	1.04
0.9935	44.6	48.2	47.0	1.08	1.05

[a] Ratio of Justus and Paris data to BRITE data.
[b] Ratio of Modified Justus and Paris data to BRITE data.

$$r_{s\lambda} = T'_{o\lambda}T'_{w\lambda}T'_{aa\lambda}[0.5(1 - T'_{r\lambda}) + (1 - F'_s)T'_{r\lambda}(1 - T'_{as\lambda})], \quad (19)$$

$$T_{as\lambda} = \exp(-\omega_\lambda \tau_{a\lambda} M), \quad (20)$$

$$T_{aa\lambda} = \exp[-(1 - \omega_\lambda)\tau_{a\lambda} M], \quad (21)$$

$$F_s = 1 - 0.5 \exp[(\mathrm{AFS} + \mathrm{BFS} \cos Z) \cos Z], \quad (22)$$

$$\mathrm{AFS} = \mathrm{ALG}[1.459 + \mathrm{ALG}(0.1595 + \mathrm{ALG}\ 0.4129)], \quad (23)$$

$$\mathrm{BFS} = \mathrm{ALG}[0.0783 + \mathrm{ALG} \times (-0.3824 - \mathrm{ALG}\ 0.5874)], \quad (24)$$

$$\mathrm{ALG} = \ln(1 - \langle\cos\theta\rangle), \quad (25)$$

$$F'_s = 1 - 0.5 \exp[(\mathrm{AFS} + \mathrm{BFS}/1.8)/1.8], \quad (26)$$

$$\omega_\lambda = \omega_{0.4} \exp\{-\omega'[\ln(\lambda/0.4)]^2\}. \quad (27)$$

The total scattered irradiance $I_{s\lambda}$ (Eq. 12), is multiplied by C_s where

$$C_s = \begin{cases} (\lambda + 0.55)^{1.8} & \text{for} \quad \lambda \leq 0.45\ \mu\text{m} \\ 1.0 & \text{for} \quad \lambda > 0.45\ \mu\text{m}. \end{cases} \quad (28)$$

The parameter $r_{g\lambda}$ is the ground albedo as a function of wavelength, $r_{s\lambda}$ is the sky reflectivity, and the primed transmittance terms are the regular atmospheric transmittance terms evaluated at $M = 1.8$. The term ω_λ is the aerosol single scattering albedo as a function of wavelength, $\omega_{0.4}$ is the single scattering albedo at 0.4 μm wavelength, ω' is the wavelength variation factor, and $\langle\cos\theta\rangle$ is the aerosol asymmetry factor. For the rural aerosol model, $\omega_{0.4} = 0.945$, $\omega' = 0.095$, and $\langle\cos\theta\rangle = 0.65$. The equations for $T_{as\lambda}$ and $T_{aa\lambda}$ ensure that $T_{a\lambda}$ is equal to $T_{as\lambda}T_{aa\lambda}$ and that the wavelength-dependent single scattering albedo is correctly defined by $\omega_\lambda = s_{a\lambda}/(s_{a\lambda} + k_{a\lambda})$. The parameters $s_{a\lambda}$ and $k_{a\lambda}$ are the aerosol scattering and absorption coefficients, respectively. In a homogeneous medium, the optical depth is related to these coefficients by $\tau_{a\lambda} = (s_{a\lambda} + k_{a\lambda})L$, where L is the path length in the medium.

The only adjustments that we made to the Justus and Paris model (1985) were to take $T_{r\lambda}$ to the 0.95 power instead of to the 1.0 power in (16), to take $T_{r\lambda}$ to the 1.5 power instead of to the 1.0 power in (17), and to multiply $I_{s\lambda}$ (12) by C_s. As mentioned previously, we also changed several absorption coefficients.

It is important to note that the parameters used in SPCTRAL2 for the comparisons in Table 2 were selected to match the atmospheric conditions used in the BRITE code. This includes the use of the rural aerosol model and parameters that represent it. Since the rural aerosol model was used in a rigorous fashion in the BRITE code, our modifications to the Justus and Paris model are based on realistic aerosol data as well as other realistic atmospheric conditions. Some model comparisons could be misleading if sufficient attention is not given to the details of the parameters used. This could be the case for comparisons with the Dave aerosol models (1979). A constant complex index of refraction for all wavelengths is used in these models, which is not representative of real aerosols; it has an effect on the single scattering albedo, the turbidity, and the asymmetry factor as a function of wavelength.

It should also be noted that ω_λ is difficult to determine and is quite variable in the real world. Justus (personal communication, 1984) has derived an expression for ω_λ for the urban aerosol model as a function of relative humidity. This parameter affects only the diffuse component, so the global radiation at the ground should not be overly sensitive to the values used. This is not the case when the upwelling radiation at the top of the atmosphere is calculated as Justus and Paris did.

b. *Diffuse irradiance on inclined surfaces*

Several algorithms have been produced (Hay and Davies, 1978; Liu and Jordan, 1961; Becker and Boyd, 1957; Temps and Coulson, 1977; Klucher, 1979) that convert the broadband global horizontal irradiance to the broadband global irradiance on a tilted surface. Most of these conversion algorithms require the direct normal and the diffuse irradiance on a horizontal surface as input. Several algorithms have been evaluated with measured data (Ma and Iqbal, 1983; Smietana *et al.*, 1984; Perez and Stewart, 1984) in recent years. Some of them appear to be quite accurate for broadband applications for east-, west-, and south-facing surfaces. Perez and Stewart (1984) found that the algorithms were somewhat inadequate for north-facing surfaces. This is partially because there is less irradiance on north-facing slopes.

We used three of these simple conversion algorithms (Hay and Davies, Temps and Coulson, and Klucher) to produce spectra irradiance on tilted surfaces by using the spectral direct and diffuse irradiance calculations of the previous section as inputs to the conversion algorithm. We obtained the best agreement with rigorous modeled data for cloudless-sky conditions using the Hay and Davies algorithm. This was somewhat surprising, because the way in which the algorithms were formulated would favor the Temps and Coulson (1977) algorithms over the Hay and Davies and Klucher algorithms for cloudless-sky applications. The Hay and Davies algorithm is presented in this section and the results of comparisons with rigorous code results and measured data are presented in section 4.

The spectral global irradiance on an inclined surface is represented by

$$I_{T\lambda}(t) = I_{d\lambda}\cos(\theta) + I_{s\lambda}\langle\{I_{d\lambda}\cos(\theta)/[H_{o\lambda}D\cos(Z)]\} + 0.5[1 + \cos(t)][1 - I_{d\lambda}/(H_{o\lambda}D)]\rangle + 0.5 I_{T\lambda} r_{g\lambda}[1 - \cos(t)], \quad (29)$$

where θ is the angle of incidence of the direct beam on the tilted surface and t the tilt angle of the inclined

TABLE 3. Direct normal spectral irradiance comparison of Dave results with SPCTRAL2 results for a Rayleigh atmosphere with molecular absorption (O_3 = 0.31 atm-cm, H_2O = 2.93 cm).

	$Z = 0°$		$Z = 60°$		$Z = 80°$	
λ (μm)	Dave	SPCTRAL2	Dave	SPCTRAL2	Dave	SPCTRAL2
0.31	105.4	103.0	16.1	15.8	0.01	0.0
0.36	607.7	607.1	345.8	346.3	41.5	45.6
0.415	1299.0	1298.0	951.0	951.7	294.5	310.2
0.515	1591.0	1590.0	1381.0	1380.0	811.5	832.0
0.615	1469.0	1469.0	1334.0	1334.0	926.7	953.7
0.7035	1292.0	1291.0	1231.0	1230.0	1032.0	1042.0
0.725	1125.0	1119.0	1033.0	1028.0	793.4	803.1
0.9935	743.4	731.8	730.5	714.5	689.8	667.6
2.1	83.4	70.7	80.0	75.2	72.2	63.4

surface. The tilt angle is zero for a horizontal surface and 90° for a vertical surface. The following relationship holds for the spectral global irradiance on a horizontal surface:

$$I_{T\lambda} = I_{d\lambda} \cos(Z) + I_{s\lambda}. \quad (30)$$

The first term in (29) is the direct component on the inclined surface. The second and third terms account for the circumsolar or aureole component and the diffuse skylight component. The last term in (29) represents the isotropically reflected radiation from the ground. A component that is missing from this model is the horizon-brightening radiation. There are arguments that could be made as to why this algorithm should not be accurate, but the fact that it is reasonably accurate for the cases that we have checked cannot be ignored. It is somewhat surprising that a broadband model can be used for spectral data.

4. Comparisons of the new, simple model with rigorous models and measurements

Comparisons of the output of SPCTRAL2 with results of rigorous radiative transfer codes and with measured data are given in this section. These initial comparisons give the reader some measure of the accuracy of the simple model. Additional high quality spectral measurements, including an error analysis of the measured data, are required for a quantitative analysis of the performance of the model. These measured data are necessary to obtain statistics on the accuracy of the model for different regions of the spectrum and for a range of atmospheric conditions and collector orientations/modes.

a. *Comparison with Dave Rayleigh scattering data*

Dave (1979) produced several data sets using the Spherical Harmonics method of solving the radiative transfer equation. One of the data sets was for a Rayleigh atmosphere (no aerosols) with molecular absorption. The atmospheric model that was used (the Midlatitude Summer model) contained 2.93 cm of precipitable water and 0.31 atm-cm of ozone. A comparison of the results of Dave with the results of SPCTRAL2 is shown in Tables 3 and 4 at a few wavelengths throughout the spectrum. The same extraterrestrial values that were used by Dave were used in SPCTRAL2 for this comparison. Table 3 compares the direct normal irradiance for three solar zenith angles, and Table 4 compares the diffuse irradiance for the same solar zenith angles. The direct normal irradiance was produced using (1) with $T_{a\lambda} = 1.0$ and $D = 1.0$. The diffuse horizontal irradiance was found using (16) with $D = 1.0$, and $T_{aa\lambda} = 1.0$.

b. *Comparisons with BRITE code results*

Examples of comparisons between BRITE (Bird, 1983) code results and results of the SPCTRAL2 code are presented in Figs. 1–4 for global irradiance. Figure 1 is for a horizontal surface with zenith angle (Z) = 0.0°, ozone = 0.344 atm-cm, water vapor = 1.42

TABLE 4. Diffuse horizontal irradiance comparison of Dave results with SPCTRAL2 results for a Rayleigh atmosphere with molecular absorption (O_3 = 0.31 atm-cm, H_2O = 2.93 cm)

	$Z = 0°$		$Z = 60°$		$Z = 80°$	
λ (μm)	Dave	SPCTRAL2	Dave	SPCTRAL2	Dave	SPCTRAL2
0.31	75.5	94.9	21.5	28.5	1.1	0.8
0.36	222.2	221.2	167.7	175.4	72.9	88.1
0.415	234.5	227.7	199.8	198.0	118.0	124.6
0.515	108.1	103.8	99.9	96.3	75.6	73.1
0.615	46.6	44.9	43.7	42.1	34.6	33.0
0.7035	23.4	22.6	22.7	21.8	20.6	19.3
0.725	16.7	17.3	15.6	16.1	13.9	13.0
0.9935	3.3	3.1	3.2	3.1	3.2	2.8

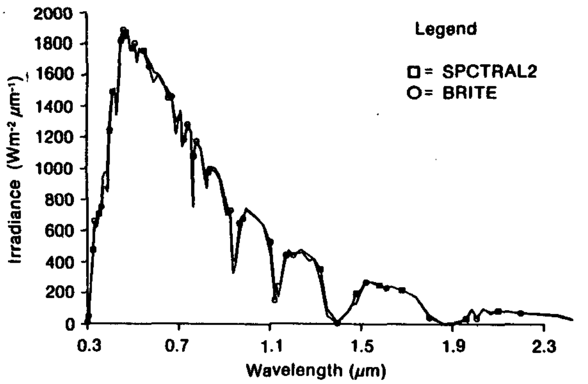

FIG. 1. Comparison of global irradiance calculated using the SPCTRAL2 and BRITE codes for τ_a (0.5 μm) = 0.10, $Z = 0°$, and tilt = 0°.

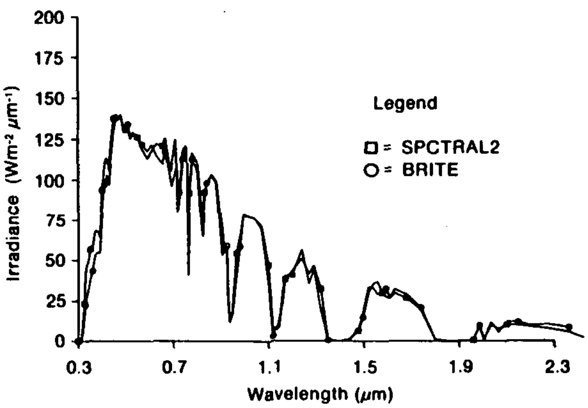

FIG. 2. As in Fig. 1 for $\tau_a = 0.51$ and $Z = 80°$.

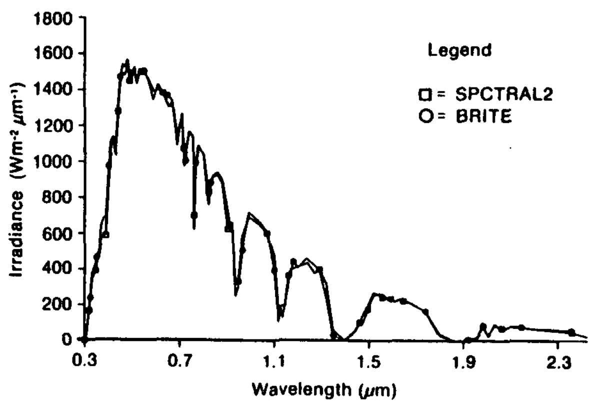

FIG. 3. As in Fig. 1 for $\tau_a = 0.27$, $Z = 48.19°$, and tilt = 37°.

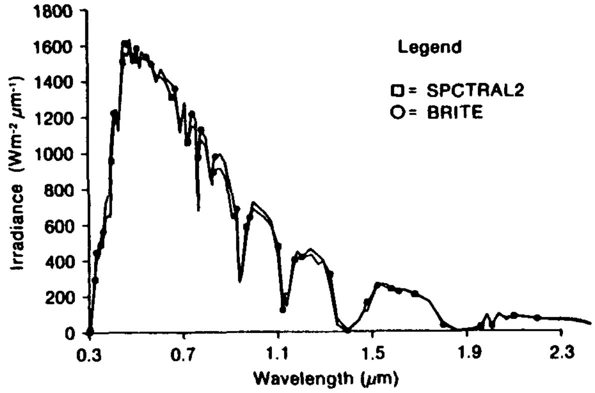

FIG. 4. As in Fig. 1 for $\tau_a = 0.27$, $Z = 37°$, and tilt = 60°.

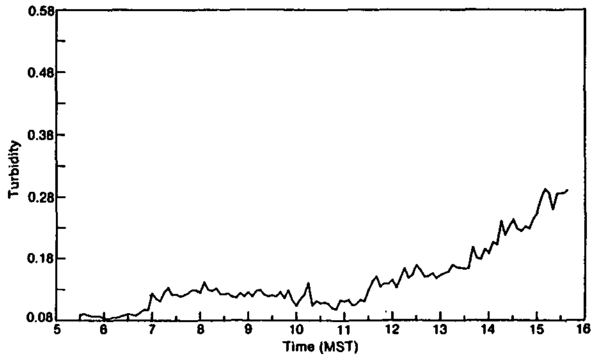

FIG. 5. Turbidity at 0.5 μm vs time of day on 5 August 1981, Golden, CO.

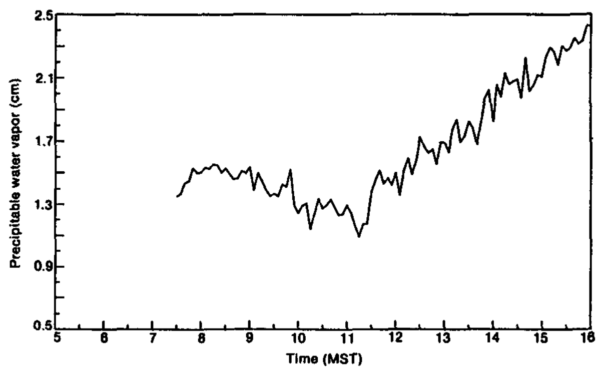

FIG. 6. Precipitable water vapor vs time of day on 5 August 1981, Golden, CO.

TABLE 5. Meteorological and geometrical parameters on 19 August 1981 at Golden, CO.

Spectrum	Zenith angle (°)	Turbidity (0.368 μm)	Turbidity (0.500 μm)	Turbidity (0.862 μm)	H_2O (cm)
Global	31.9	0.180	0.149	0.042	1.36
Direct	30.8	0.176	0.148	0.042	1.35
Tilt	34.66	0.269	0.200	0.065	1.35

cm, ground albedo = 0.2, surface pressure = 1013 mb, and a turbidity at 0.5 μm of 0.1. The only differences for the horizontal spectra shown in Fig. 2 are that $Z = 80.0°$, and a turbidity of 0.51 was used.

Figure 3 is a comparison of BRITE and SPCTRAL2 results for a surface tilted 30° from the horizontal for $Z = 48.19°$ and turbidity at 0.5 μm of 0.27. The spectra compared in Fig. 4 were produced with the same parameters as those of Fig. 3, except that $Z = 37°$ and the surface is tilted 60°.

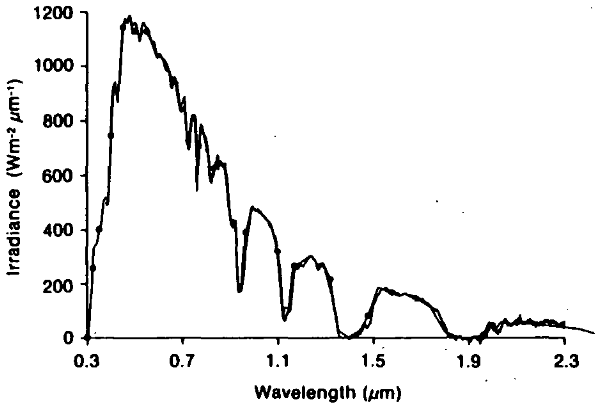

FIG. 7. Comparison between global horizontal irradiance measured on 5 August 1981, Golden, CO, and modeled data using SPCTRAL2.

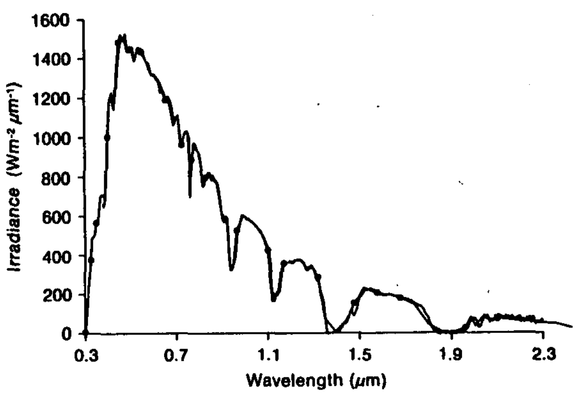

FIG. 8. As in Fig. 7 for 19 August 1981.

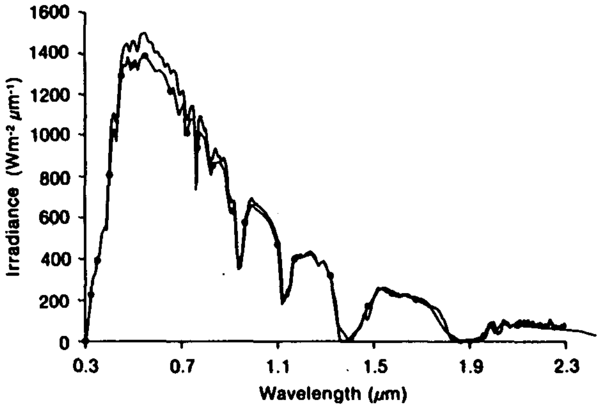

FIG. 9. Comparison between direct normal irradiance measured on 19 August 1981, Golden, CO, and modeled data using SPCTRAL2.

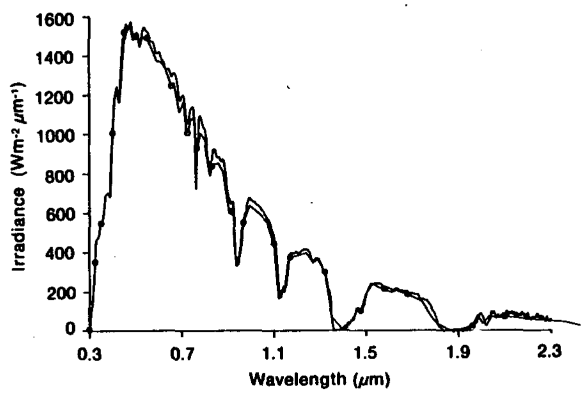

FIG. 10. Comparison between global radiation on a south-facing surface tilted 40°, measured on 19 August 1981, Golden, CO, and modeled data using SPCTRAL2 (circles).

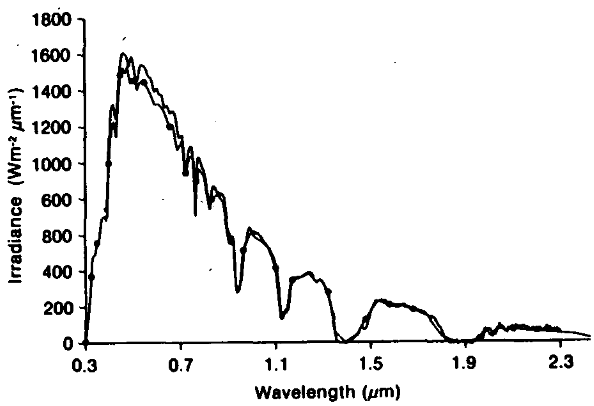

FIG. 11. Comparison between global horizontal irradiance measured on 18 August 1981, Golden, CO, and modeled data using SPCTRAL2.

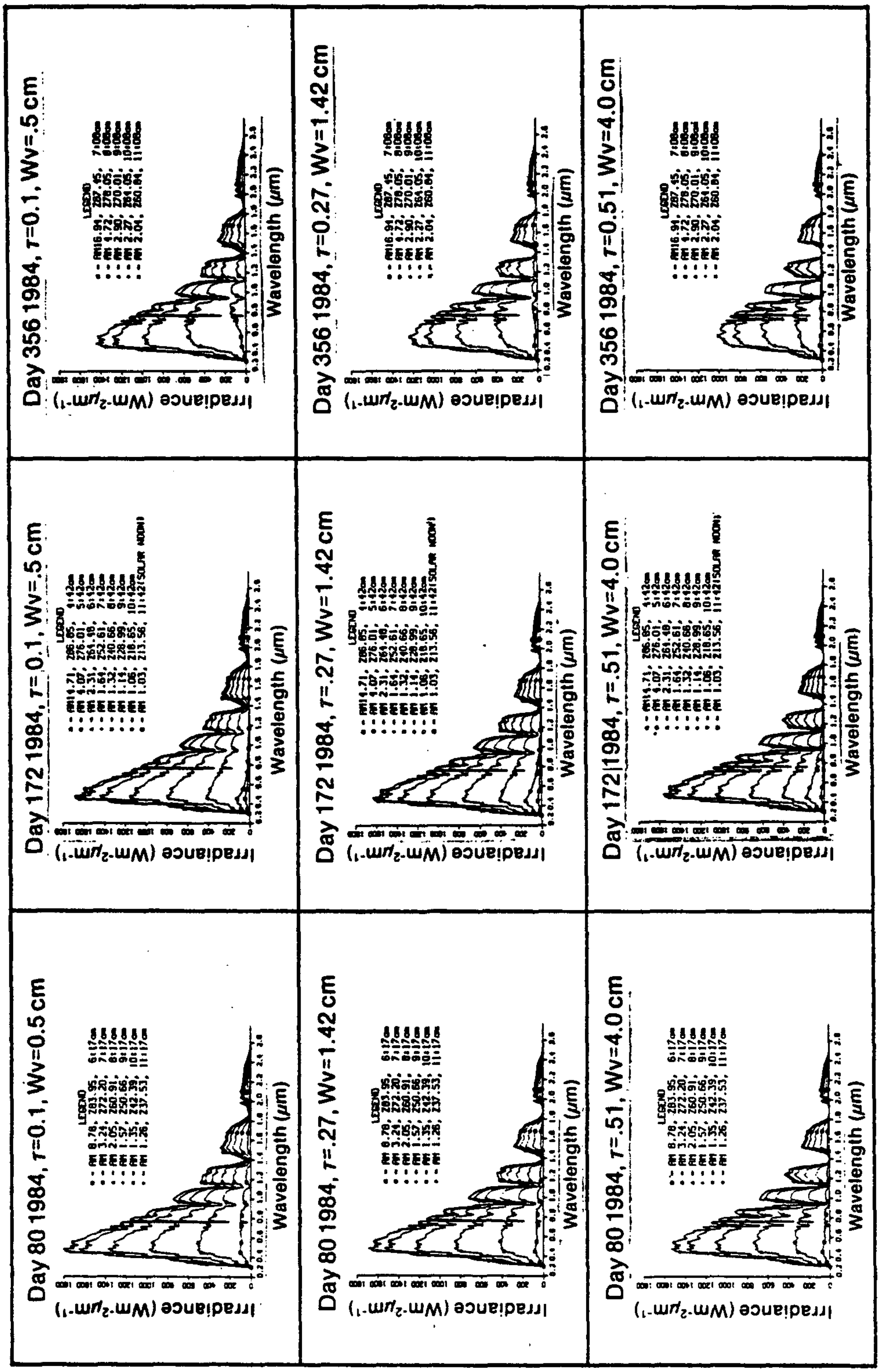

FIG. 12. Global irradiance on a south-facing surface tilted 37° for the equinoxes (days 80 and 266) and for the summer and winter solstices (days 172 and 356); AM is air mass and Z zenith angle.

c. *Comparisons with measured data*

The measured data used to make final adjustments to gaseous absorption coefficients in the model were taken with a unique spectroradiometer (Kliman and Eldering, 1981; Bird *et al.*, 1982) and an automatic sun photometer (Bird and Hulstrom, 1982, 1983). Several comparisons between the spectra produced with the new simple spectral model and measured data are presented here to indicate the extent of agreement.

The first comparison was made with a global horizontal spectrum taken on 5 August 1981 in Golden, Colorado. This site is at 39.75°N lat and 105.156°W long. The spectral measurement was made at 1509 MST, and the sun photometer measurements were made at five-minute intervals throughout the day, as illustrated in Figs. 5 and 6. The following parameters were determined: solar zenith angle = 44.8°, turbidity at 0.368 μm = 0.39, turbidity at 0.500 μm = 0.28, turbidity at 0.862 μm = 0.13, precipitable water = 2.25 cm, and surface pressure = 829.6 mb. The ozone amount was assumed to be 0.31 atm-cm.

The rural aerosol model used in the BRITE code for a turbidity of 0.27 at 0.500 μm wavelength produced turbidities of 0.37 at 0.368 μm wavelength and 0.14 at 0.862 μm wavelength. From this we can infer that the aerosol present during the 5 August measurement was nearly identical to that in the rural aerosol model, which adds validity to this particular comparison.

The results of this comparison are shown in Fig. 7. There is a slight wavelength calibration error evident in the measured data at the infrared end of the spectrum. This calibration error is due to the linear wavelength calibration procedure, which requires that a slope and intercept be determined. The spectroradiometer system determines both slope and intercept in real time in the visible wavelengths. In the infrared wavelengths, only the intercept is calibrated in real time; the slope appears to have changed slightly between laboratory calibration and the time the measurement was taken.

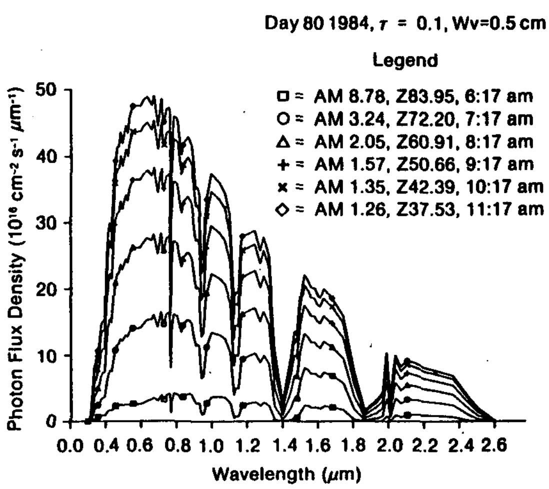

FIG. 13. Photon flux density per wavelength interval corresponding to global irradiance shown in Fig. 12 (day 80).

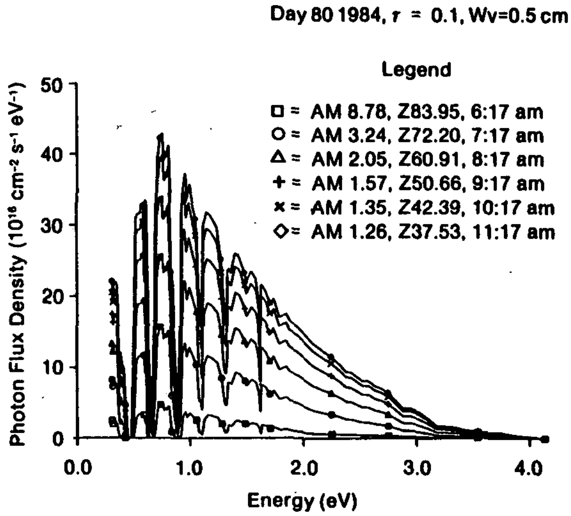

FIG. 14. Photon flux density per photon energy interval corresponding to global irradiance shown in Fig. 12 (day 80).

Several comparisons were also made on 19 August 1981. A global horizontal spectrum was measured at 1044 MST, a direct normal spectrum was measured at 1056 MST, and a global spectrum on a 40° south tilt was measured at 1342 MST. The atmospheric pressure was 832 mb for these measurements. The meteorological and geometrical parameters are shown in Table 5.

Results of these comparisons are shown in Figs. 8 through 10. One has to keep in mind that the circumsolar scattered radiation within a 6° field-of-view (FOV) is included in the direct normal measurements. This could add 1–5% to the irradiance in the 0.5 μm region and could explain why the measured direct normal irradiance is larger. Differences similar in magnitude but in the opposite direction have been observed in measured and modeled diffuse radiation. The circumsolar radiation is missing in the diffuse measurement, which causes the opposite effect.

Another comparison (Fig. 11) was made on 18 August 1981 at 1322 MST for the global horizontal mode. The parameters for this measurement are as follows: solar zenith angle = 30.11°, turbidity at 0.368 μm = 0.320, turbidity at 0.500 μm = 0.225, turbidity at 0.862 μm = 0.069, precipitable water = 1.97 cm, and surface pressure = 830 mb.

The agreement between modeled and measured data is not as good for this set of data. The reason for the disagreement is unknown, but possibly indicates the accuracy limitations of the modeled and the measured results. Justus and Paris (personal communication, 1984) have shown that the use of urban rather than rural aerosol parameters can account for differences of the magnitude and type shown in Fig. 11. It is not known whether or not urban aerosols from nearby

Denver, Colorado, were present during these measurements. Additional measured data will be gathered in the future to assess the impacts of urban aerosols and to make further comparisons of measured versus modeled data.

5. Examples of the application of the new, simple spectral model

The primary goal of this work on simple spectral models is to give researchers the capability to calculate spectral irradiance using microcomputers. The spectra can then be used in models to evaluate solar device performance. For example, scientists can produce spectra by varying input parameters such as air mass, atmospheric turbidity and water vapor, and day of year, and use the spectra to examine the performance of spectrally selective photovoltaic devices under different conditions.

Examples of spectra generated using the simple model for cloudless days at the equinoxes and solstices and three different turbidity and water vapor combinations are shown in Fig. 12. Spectra were calculated at 60-min intervals that are symmetrical about solar noon from sunrise to sunset. Only the morning spectra are plotted since the afternoon spectra are theoretically identical. These spectra were produced for a south-facing surface tilted 37° from the horizontal at sea level for latitude 37° and longitude 100°. The spectra may not be representative of a particular site, but serve as examples of differences in spectral irradiance under different conditions. Of note in these spectra are the effects of high turbidity and air mass on the visible portion of the spectrum, and the differences in spectra content at different times of the year due to differences in air mass values.

These spectra can easily be converted to photon flux per wavelength interval or to photon flux per energy interval (eV) if this format is more useful for particular applications. Examples of conversion results are shown in Figs. 13 and 14.

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