Light and Radiometry

Geometrical Radiometry

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By housing one or more radiant energy detectors in watertight assemblies and by appropriately channeling the direction of the photons arriving at the detector, we can measure the flow of radiant energy as a function of direction at any location within a water body. By adding appropriate filters to the instrument, we can also measure the wavelength dependence and state of polarization of the light field. From such measurements we can develop precise descriptions of radiative transfer in natural waters. Thus we are led to the science of *geometrical radiometry*, the union of Euclidean geometry and radiometry.

We first define *radiance*, the fundamental quantity that describes light in radiometric terms. We then define various *irradiances* and other quantities that are derivable from the radiance, and which are often easier to measure and of more relevance to a particular problem.

Radiance

Consider an instrument designed as schematically shown in Fig. go to. A hole at one end of the housing or collecting tube and a system of internal light baffles allows photons that enter the hole at angles of α or less (measured from the axis of the tube) to fall onto a translucent diffusing surface of area ΔA . The diffuser makes the light field homogeneous in the region near the detector, so that it is necessary to sample only a part of the internal light field in order to measure the total energy entering the instrument. Photons of all wavelengths passing through the diffuser are then filtered so that only photons in a wavelength interval $\Delta\lambda$ centered on wavelength λ can reach the radiant energy detector. To the accuracy with which $\cos\alpha = 1$, the solid angle $\Delta\Omega$ of the hole as seen by any point on the diffuser surface is the same. This $\Delta\Omega$ is the solid angle subtended by the instrument. The instrument is pointing in the $-\hat{\xi}$ direction, so as to collect photons traveling in a set of directions of solid angle $\Delta\Omega$ centered on direction $\hat{\xi}$. We assume that the instrument is small compared to the scale of spatial (positional) changes in the light field, so that we can think of the instrument as being located at position $\vec{x} = (x_1, x_2, x_3) = x_1\hat{i}_1 + x_2\hat{i}_2 + x_3\hat{i}_3$ within a water body. Suitable calibration of the current or voltage output of the detector gives the amount of radiant energy ΔQ entering the instrument during a time interval Δt centered on time t . An *operational definition* of the *unpolarized spectral radiance* is then

$$L(\vec{x}, t, \hat{\xi}, \lambda) \equiv \frac{\Delta Q}{\Delta t\, \Delta A\, \Delta\Omega\, \Delta\lambda} \qquad (\mathrm{J\ s^{-1}\ m^{-2}\ sr^{-1}\ nm^{-1}}).$$

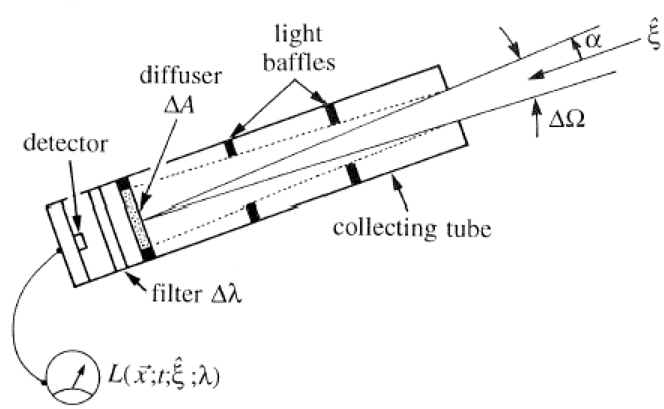

Figure: Schematic design of an instrument for measuring unpolarized spectral radiance.

In practice the intervals Δt , ΔA , $\Delta\Omega$, and $\Delta\lambda$ are taken small enough to get a useful resolution of L over the various parameter domains, but not so small as to encounter diffraction effects or fluctuations from photon shot noise at low light levels. Typical values are $\Delta t \approx 10^{-2}$ to 10^3 s (depending on whether one wishes an "instantaneous" measurement or wishes to average out sea surface wave effects), $\Delta A \approx 10^{-4}$ to 10^{-3} m^2, $\Delta\Omega \approx 0.01$ to 0.1 sr, and $\Delta\lambda \approx$ 1 to 10 nm. We shall always let $\hat{\xi}$ *denote the direction of photon travel*, which is the convention in radiative transfer theory. The *viewing direction*, which is the direction an instrument is pointed to detect $L(\vec{x}, t, \hat{\xi}, \lambda)$, is then $-\hat{\xi} = (\pi - \theta, \phi + 2\pi)$.

In the conceptual limit of infinitesimal parameter intervals, the spectral radiance is given by

$$L(\vec{x},t,\hat{\xi},\lambda) \;=\; \frac{\partial^4 Q}{\partial t\,\partial A\,\partial \Omega\,\partial \lambda}. \qquad \textbf{(1)}$$

Spectral radiance is the fundamental radiometric quantity of interest in hydrologic optics. It specifies the spatial ($\vec{x}$), temporal (t), directional ($\hat{\xi}$), and wavelength (λ) structure of the light field. As we shall see, *all other radiometric quantities can be derived from L.*

Although radiance is an extremely useful concept and is adequate for most needs of optical oceanography, it is an approximation to the exact description of light in terms of electric and magnetic fields. We therefore anticipate that there are situations for which radiance fails to give an adequate description of the light field. When that happens, we must resort to Maxwell's equations and compute the electric and magnetic fields themselves. The limitations of radiance are discussed in Level 2. Moreover, both because of instrumental difficulties and because such detailed directional information often is not needed for specific applications, the most commonly measured radiometric quantities are various irradiances.

Plane Irradiance

If the collecting tube is removed from the instrument of Fig. go to, then photons from an entire hemisphere of directions can reach the detector, as illustrated in Fig. go to. Such an instrument, when pointed "straight up" (in the $-\xi_3$ direction) so as to detect photons headed *downward* (all $\hat{\xi}$ in Ξ_d) measures the *spectral downward plane irradiance* E_d:

$$E_d(\vec{x},t,\lambda) \;\equiv\; \frac{\Delta Q}{\Delta t\,\Delta A\,\Delta\lambda} \qquad (\mathrm{W\ m^{-2}\ nm^{-1}}). \qquad \textbf{(2)}$$

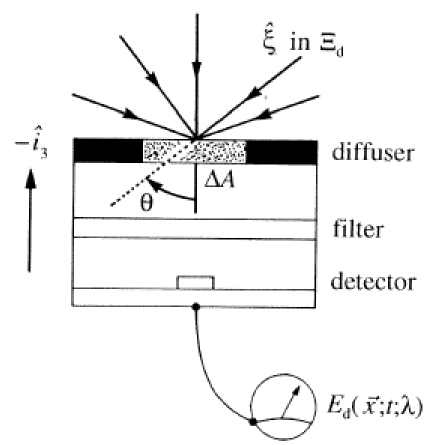

Figure: Schematic design of an instrument for measuring spectral plane irradiance.

Implicit in this definition is the assumption that each point of the collector surface is equally sensitive to photons incident onto the surface from any angle. If this is the case, however, the collector *as a whole* is *not* equally sensitive to all photons headed in downward directions. Imagine a collimated beam of light headed straight downward (e.g. from the sun straight overhead). This beam, assumed to be larger than the collector surface, sees the full area ΔA of the collector surface. However, the same large beam traveling at an angle θ relative to the instrument axis sees a collector surface of effective area $\Delta A|\cos\theta|$ (the area ΔA as projected onto a plane perpendicular to the beam direction). Otherwise identical light beams therefore generate detector responses that are proportional to the cosines of the incident photon directions. Such instruments are called *cosine collectors.*

The *cosine law for irradiance* is simply the statement that a collimated beam of photons intercepting a plane surface produces an irradiance that is proportional to the cosine of the angle between the photon directions and the normal to the collector surface.

Since the instrument of Fig. go to collects photons traveling in all downward directions, but with detector's surface area weighted by the cosine of the photon's incident angle θ, the instrument is in essence integrating $L(\vec{x},t,\hat{\xi},\lambda)|\cos\theta|$ over all downward directions. The spectral downward plane irradiance is therefore related to the spectral radiance by

$$E_d(\vec{x},t,\lambda) \;=\; \int_{\hat{\xi}\in\Xi_d} L(\vec{x},t,\hat{\xi},\lambda)\,|\cos\theta|\,d\Omega(\hat{\xi}) =$$

(3)

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} L(\vec{x}, t, \theta, \phi, \lambda) |\cos\theta| \sin\theta d\theta\, d\phi$$

If the same instrument is oriented *downward*, so as to detect photons heading *upward*, then the quantity being measured is the *spectral upward plane irradiance* E_u :

$$E_u(\vec{x}, t, \lambda) = \int_{\phi=0}^{2\pi} \int_{\theta=\pi/2}^{\pi} L(\vec{x}, t, \theta, \phi, \lambda) |\cos\theta| \sin\theta d\theta\, d\phi \qquad \textbf{(4)}$$

Note that it is necessary to take the absolute value of cosθ in Eq. (go to) because, with our choice of coordinates, $\cos\theta < 0$ for $\hat{\xi}$ in Ξ_u. The absolute value is superfluous in Eq. (go to).

Scalar Irradiance

Now consider an instrument that is designed to be *equally* sensitive to all photons headed in the downward direction. Such an instrument is shown in Fig. go to . The spherical shape of the diffuser insures that the instrument is equally sensitive to photons from any direction. If each point on the diffuser surface behaves like a cosine collector, then the effective area of the collector is $\Delta A = \pi r^2$, where r is the radius of the diffuser. The large shield blocks upward-traveling photons. The upper surface of the shield is assumed to be completely absorbing, so that it cannot reflect downward traveling photons back upward into the diffuser. This instrument, when oriented upward as shown in Fig. go to , measures the *spectral downward scalar irradiance* E_{od}, which is related to the spectral radiance by

$$\begin{aligned} E_{od}(\vec{x}, t, \lambda) &= \int_{\hat{\xi} \in \Xi_d} L(\vec{x}, t, \hat{\xi}, \lambda) d\Omega(\hat{\xi}) \\ &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} L(\vec{x}, t, \theta, \phi,\, \lambda) \sin\theta d\theta\, d\phi \qquad \textbf{(5)} \end{aligned}$$

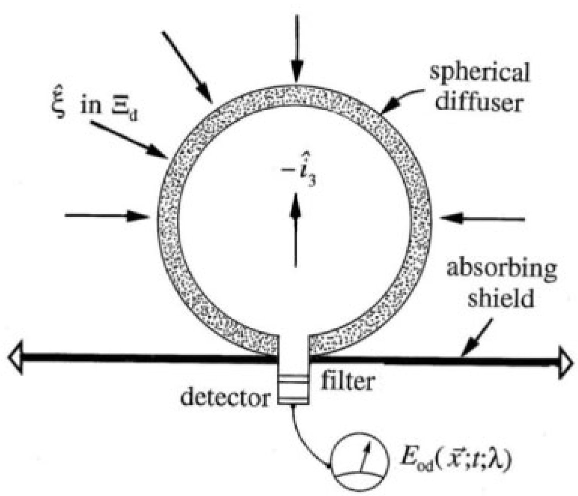

Figure: Schematic design of an instrument for measuring spectral scalar irradiance.

If the instrument of Fig. go to is inverted, so as to collect only upward traveling photons, then it measures the *spectral upward scalar irradiance* E_{ou}. If this shield is removed, photons traveling in all directions are collected. The quantity then measured is the *spectral total scalar irradiance* E_o :

$$\begin{aligned} E_o(\vec{x}, t, \lambda) &= \int_{\hat{\xi} \in \Xi} L(\vec{x}, t, \hat{\xi}, \lambda) d\Omega(\hat{\xi}) \\ &= E_{od}(\vec{x}, t, \lambda) + E_{ou}(\vec{x}, t, \lambda). \qquad \textbf{(6)} \end{aligned}$$

Other possible instrument designs are discussed in Højerslev (1975). However, the three shown in Figs. go to to go to are sufficient for most of the needs of hydrologic optics. Such instruments are commercially available.

Vector Irradiance

The *spectral vector irradiance* $\vec{E}$ is defined as

$$\vec{E}(\vec{x},t,\lambda) \;=\; \int_{\hat{\xi}\in\Xi} L(\vec{x},t,\hat{\xi},\lambda)\,\hat{\xi}\,d\Omega(\hat{\xi}) \qquad \textbf{(7)}$$

Recalling from Eq. (go to) that the vertical component of $\hat{\xi}$ is just $\hat{i}_3 \cdot \hat{\xi} = \cos\theta$, we can write the vertical component of the vector irradiance as

$$\begin{aligned}(\vec{E})_3 &= \hat{i}_3 \cdot \vec{E} \\ &= \int_{\Xi} L(\vec{x},t,\hat{\xi},\lambda)\;\cos\theta\, d\Omega(\hat{\xi}) \\ &= E_d - E_u \end{aligned} \qquad \textbf{(8)}$$

In developing Eq. (go to), we have noted that $\cos\theta > 0$ in Ξ_d and $\cos\theta < 0$ in Ξ_u. The quantity $E_d - E_u$ is called the *net downward irradiance*. This net downward irradiance often is often called the "vector" irradiance, although strictly speaking it is only the vertical component of the vector irradiance. If the radiance distribution is horizontally homogeneous, the horizontal components of the vector irradiance are zero.

EXAMPLE: IRRADIANCES OF AN ISOTROPIC RADIANCE DISTRIBUTION

Consider an *isotropic*, or directionally uniform, radiance distribution: $L(\vec{x},t,\hat{\xi},\lambda) = L_o(\vec{x},t,\lambda)$ for all $\hat{\xi}$ in Ξ. Then by Eq. (go to), the downward plane irradiance is

$$\begin{aligned}E_d(\vec{x},t,\lambda) \;&= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} L_o(\vec{x},t,\lambda)|\cos\theta|\,\sin\theta d\theta\, d\phi \\ &= \pi L_o(\vec{x},t,\lambda)\end{aligned}$$

In the last equation, π carries units of steradian from the integration over solid angle; thus E_d has units of irradiance when L_o has units of radiance. Likewise, $E_u = \pi L_o$, so that the net downward irradiance is zero. The scalar irradiance E_{od} is given by Eq. go to

$$\begin{aligned}E_{od}(\vec{x},t,\lambda) \;&= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} L_o(\vec{x},t,\lambda)\,\sin\theta d\theta\, d\phi \\ &= 2\pi L_o(\vec{x},t,\lambda)\end{aligned}$$

In the same manner we find $E_{ou} = 2\pi L_o$, so that the total scalar irradiance $E_o = 4\pi L_o$.

Photosynthetically Available Radiation

Photosynthesis is a quantum process. That is to say, it is the *number* of available photons rather than their total energy that is relevant to the chemical transformations. This is because a photon of, say, wavelength 400 nm, *if absorbed* by a cholorphyll molecule, induces the same chemical change as does a less energetic photon of wavelength 500 nm. (However, photons of different wavelengths are not equally likely to be absorbed.) Only a part of the photon's energy goes into photosynthesis; the excess appears as heat or is re-radiated. Moreover, chlorophyll is equally able to absorb a photon regardless of the photon's direction of travel.

Now recall that the spectral total scalar irradiance $E_o(\vec{x},\lambda)$ is the total radiant power per square meter at wavelength λ coursing through point $\vec{x}$ owing to photons traveling in all directions. By Eq. (go to), the number of photons generating $E_o(\vec{x},\lambda)$ is $E_o(\vec{x},\lambda)\lambda/hc$. Therefore, in studies of phytoplankton biology, a useful measure of the underwater light field is the *photosynthetically available radiation*, *PAR* or E_{PAR}, defined by

$$PAR(\vec{x}) \equiv \int_{400\,\mathrm{nm}}^{700\,\mathrm{nm}} E_o(\vec{x},\lambda)\,\frac{\lambda}{hc}\,d\lambda \qquad (\mathrm{photons\ s^{-1}\ m^{-2}}). \qquad \textbf{(9)}$$

Note that *PAR* is by definition a broadband quantity; there is no "spectral" *PAR*. Bio-optical literature often states *PAR* values in units of $\mathrm{mol\ photons\ s^{-1}m^{-2}}$ or $\mathrm{einst\ s^{-1}m^{-2}}$. Recall from the table of derived units that one einstein is one mole of photons (6.023×10^{23} photons).

Morel and Smith (1974) found that over a wide variety of water types from very clear to turbid, the conversion factor for energy to photons varies by only $\pm 10\%$ about the value

$$2.5 \times 10^{18} \text{ photons s}^{-1} \text{ W}^{-1} \;=\; 4.2\; \mu\text{einst s}^{-1} \text{ W}^{-1}\,.$$

PAR is usually estimated using only the visible wavelengths, 400-700 nm, although some investigators include near-ultraviolet wavelengths in the integral of Eq. (go to). PAR is also sometimes estimated using E_d rather than E_o. This can lead to underestimation of *PAR* by 30% or more because E_d is always less than E_o.

Instruments for the direct measurement of *PAR*, often called *quanta meters,* can be constructed along the lines of Fig. go to by the incorporation of suitable wavelength filters. Such instruments are commercially available. Engineering details can be found in Jerlov and Nygård (1969) and in Kirk (1994), which also discusses *PAR* and photosynthesis in great detail.

Although *PAR* has a venerable history as a simple parameterization of available light in phytoplankton growth models, it should be noted that *PAR* is an imperfect measure of how light contributes to photosynthesis. This is because different species of phytoplankton, or even the same species under different environmental conditions, have different suites of pigments. Phytoplankton with different pigments absorb light differently as a function of wavelength. Thus phytoplankton with different pigments use the same $E_o(\lambda)$ with different efficiencies, thus giving one an advantage over the other. Recent ecosystem models therefore replace *PAR* by spectral scalar irradiance $E_o(\lambda)$ and also account for different pigments in different functional classes of phytoplankton. Such models can better simulate how light is utilized by different components of the ecosystem (e.g., Bissett et al. (1999); Fujii et al. (2007).

Intensity

Another family of radiometric quantities can be defined from the measurements employed in the operational definition of radiance. The *spectral intensity I* is defined as

$$I(\vec{x},t,\hat{\xi},\lambda) \;=\; \frac{\Delta Q}{\Delta t\,\Delta\Omega\,\Delta\lambda} \qquad (\text{W sr}^{-1}\text{ nm}^{-1})\,,$$

or

$$I(\vec{x},t,\hat{\xi},\lambda) \;=\; \int_{\Delta A} L(\vec{x},t,\hat{\xi},\lambda)\,dA\,.$$

In Eq. (go to), ΔA is the surface of the collector that sees the solid angle $\Delta\Omega$, and dA is an element of area. Just as for irradiance, we can define various plane, scalar and vector intensities by insertion of the appropriate factors into the integrand of Eq. (go to), which as written represents the scalar intensity I_o. The concept of intensity is useful in the radiometry of point light sources. We will use it in Chapter 3 in the definition of the volume scattering function.

Terminology and Notation

The adjective "spectral," as in spectral radiance, as commonly used can mean either "as a function of wavelength" or "per unit wavelength interval." Committees on international standards recommend an argument λ, e.g. $L(\lambda)$, for the first meaning and a subscript λ, e.g. L_λ, for the second meaning (Meyer-Arendt (1968)). However, this subscript convention is seldom used in optical oceanography, perhaps because the symbols already are cluttered with subscripts. Most authors seem to write $L(\lambda)$ and consider it to mean radiance per unit wavelength interval, as a function of wavelength. The adjective spectral and argument λ are often omitted for brevity, although strictly speaking, a term without the adjective "spectral" refers to a quantity integrated or measured over a finite band of wavelengths, as in the computation of *PAR* in Eq. (go to). Most radiative transfer *theory* assumes the energy to be monochromatic, i.e., radiance per unit wavelength interval at a particular wavelength. However, most *measurements* are made over a wavelength band of 5 to 20 nm, which complicates the comparison of theory and observation.

Spectral quantities are sometimes expressed in terms of a unit frequency interval rather than a unit wavelength interval. To establish the conversion, consider the radiant energy $Q(\lambda)d\lambda$ contained in a wavelength interval $d\lambda$. The same amount of energy is contained in a corresponding frequency interval $Q(\nu)d\nu$. Since an increase in wavelength ($d\lambda > 0$) implies a decrease in frequency ($d\nu < 0$), and vice versa, we can write

$$Q(\lambda)d\lambda = -Q(\nu)d\nu.$$

Using $\lambda = c/\nu$ we then get

$$Q(\nu) = -Q(\lambda)\frac{d\lambda}{d\nu} = \frac{c}{\nu^2}Q(\lambda) = \frac{\lambda^2}{c}Q(\lambda),$$

which is the desired connection between $Q(\lambda)$ and $Q(\nu)$, or between any other radiometric quantities. A wavelength interval $d\lambda$ therefore corresponds to a frequency interval $d\nu$ of size

$$d\nu \;=\; \frac{c}{\lambda^2} d\lambda.$$

The *wavenumber* $\kappa = 1/\lambda$ is also sometimes used (the value usually given in inverse centimeters, cm^{-1}). An analysis parallel to that just given yields

$$d\kappa \;=\; \frac{1}{\lambda^2} d\lambda.$$

Since sampling times Δt are generally long compared to the time ($\approx 10^{-6}$ s) required for the light field in a water body to reach steady state after a change in the environment, time-independent radiative transfer theory is usually sufficient for hydrologic optics studies. In this case, the time is implicitly understood and the argument t can be omitted. A notable exception to the use of time-independent radiative transfer theory occurs with the use of pulsed lasers to determine water depth or detect underwater objects. In this application, the laser pulses last only for nanoseconds and time-dependent theory must be used.

Finally, time-averaged (over seconds to minutes) horizontal variations (on a scale of meters to kilometers) in the environment and in the optical properties of natural water bodies illuminated by the sun are usually much less than vertical variations. In that case, underwater light fields depend spatially only on the depth $x_3 = z$ to a good approximation. Thus, for example, we often can refer to just "the radiance $L(z, \theta, \phi, \lambda)$" without generating confusion. An important exception of this situation occurs with artificial light sources, such as an underwater light or a laser being used for bathymetric mapping. In these cases the horizontal variations in the radiance can be large, and the three-dimensional spatial variations of the light field must be considered.

Although this web site uses the terminology and notation commonly seen in the current optical oceanographic literature, much published work uses a different nomenclature. Prior to the late 1970's, different symbols were often employed for the radiometric concepts just defined. This historical notation is seen, for example, in the monumental monograph (Hydrologic Optics by Preisendorfer (1976). There is a Level 2 discussion of historical notation and of further subdivisions of radiometric concepts into *field* (energy falling onto a surface) vs. *surface* (energy emitted by a surface) quantities.

Much of the radiative transfer work in atmospheric, astrophysical, and biomedical optics, and in neutron transport theory, is relevant to hydrologic optics. However, to a considerable extent, these fields have developed independently and each has its own nomenclature. For example, in atmospheric and astrophysical optics, radiance is often called "intensity" or "specific intensity" and is given the symbol I ; this use of "intensity" is not be confused with intensity as defined above. The classic text by van de Hulst (1957) uses "intensity" for irradiance. The letter "I" often is used as the symbol for irradiance. The word "flux" is sometimes used to mean irradiance and is sometimes used to mean power. Those who use "flux" for power generally use "flux density" for irradiance. The ambiguity associated with "flux" can be avoided simply by using "power" or "irradiance," as is appropriate. Matters are further complicated by the occasional misuse of photometric terms such as "brightness" and "luminance" for radiometric quantities; these matters are discussed in the Level 2 section on photometry.