

# HW\_DTMC\_Simulation

April 1, 2022

```
[1]: import numpy as np
      # add here the libraries you need
```

## 1 Discrete Time Markov Chains - Part 2

This is an exercise notebook on DTMCs.

Remember to revise of the lecture on DTMC simulation before attempting to solve it! In order to complete this notebook, you need the models implemented in Part 1 notebook on DTMC.

### 1.0.1 1. Simulation of DTMC

Write a method that simulates a DTMC for  $n$  steps, where  $n$  is a parameter of the method, and returns the whole trajectory as output.

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### 1.0.2 2. Statistical analysis

Write methods for:

- 2.1. computing the average of a function  $f$  of the state space, at time step  $n$ .
- 2.2. computing the probability of reaching a target region  $A$  of the state space by time step  $n$ .

Both methods should use simulation, and return an estimate and a confidence interval at a specified confidence level  $\alpha$  (0.95% by default).

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### 1.0.3 3. Branching chain

Consider a population, in which each individual at each generation independently gives birth to  $k$  individuals with probability  $p_k$ . These will be the members of the next generation. Assume  $k \in \{-1, 0, 1, 2\}$ . The population is initial composed of two individuals Adam and Eve.

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Assume now that  $p_0 = p_1 = p_2 = (1 - p_{-1})/3$ . Estimate the average and the confidence interval of the probability of the population to become extinct for increasing values of  $p_{-1}$ .

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