## PCTMC methods

## April 28, 2022

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[1]: import sympy as sp
import numpy as np
import time
from scipy.integrate import odeint
import matplotlib.pyplot as plt
import numpy.random as rnd
from math import ceil
[2]: class Symbols:
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[2]: class Symbols:
         def __init__(self):
             # initialize
             self.reference = []
             self.values = []
             # initialize map from string to int and back
             self.reference2id = {}
             self.names2id = {}
             self.id2reference = {}
             self.dimension = 0
         #Adds a new symbol to the symbol array, creating a sympy object
         def add(self, name, value):
             symbol = sp.symbols(name)
             index = len(self.reference)
             self.reference.append(symbol)
             self.values.append(value)
             self.reference2id[symbol] = index
             self.names2id[name] = index
             self.id2reference[index] = symbol
             self.dimension += 1
             return symbol
         # sets the value of a symbol
         def set(self, name, value):
             try:
                 index = self.names2id[name]
                 self.values[index] = value
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except:
    print("Symbol " + name + " is not defined")

def get_value(self,name):
    try:
        index = self.names2id[name]
        return self.values[index]
    except:
        print("Symbol " + name + " is not defined")

#finalizes the symbol array, generating a numpy array for values
def finalize(self):
    self.values = np.array(self.values)

def get_id(self, name):
    return self.names2id[name]
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[3]: class Transition:
         def __init__(self, update, rate, sympy_symbols):
             #this will convert rate into a sympy expression.
             self.update = None
             self.update_dictionary = update
             try:
                 self.rate = sp.sympify(rate, sympy_symbols, evaluate=False)
             except sp.SympifyError as e:
                 print("An error happened while parsing expression", rate,":",e)
         # finalizes transition by turning the update list into a numpy array
         def finalize(self, variables):
             self.update = np.zeros(variables.dimension)
             for var_name in self.update_dictionary:
                 index = variables.get_id(var_name)
                 self.update[index] = self.update_dictionary[var_name]
             self.update = np.reshape(self.update, (1,variables.dimension))
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[5]: class Model:

def __init__(self):
    #init variables and parameters
    self.variables = Symbols()
    self.parameters = Symbols()
    # this contains a map of names to sympy variables, to be used later for

→parsing expressions
    self.names2sym = {}
    # init transition list
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self.transitions = []
    self.transition number = 0
    self.system_size = 0;
    self.system_size_reference = None
    self.system_size_name = ''
    self.variables_names = []
def set_system_size(self, name, value):
    self.add parameter(name, value)
    self.system_size_reference = self.names2sym[name]
    self.system_size_name = name
    self.system_size = value
def add_variable(self, name, value):
    if name in self.names2sym:
        raise ModelError("Name " + name + " already defined!")
    var = self.variables.add(name, value)
    self.variables_names.append(name)
    self.names2sym[name] = var
def add_parameter(self, name, value):
    if name in self.names2sym:
        raise ModelError("Name " + name + " already defined!")
    par = self.parameters.add(name, value)
    self.names2sym[name] = par
# Changes the initial value of a variable
def set_variable(self, name, value):
    self.variables.set(name, value)
# Changes the value of a parameter
def set_parameter(self, name, value):
    if self.system_size_name == name:
        self.parameters.set(name, value)
        self.system_size = value
    else:
        self.parameters.set(name, value)
def get_parameter_value(self, name):
    return self.parameters.get_value(name)
# Adds a transition to the model
def add_transition(self, update, rate):
    t = Transition(update, rate, self.names2sym)
    self.transitions.append(t)
    self.transition_number += 1
```

```
# Finalizes the initialization
  def finalize_initialization(self):
      self.variables.finalize()
      self.parameters.finalize()
      for t in self.transitions:
           t.finalize(self.variables)
      self.__generate_vector_field()
      self.__generate_diffusion()
      self.__generate_jacobian()
      self.__generate_numpy_functions()
   #generates the mean field vector field
  def __generate_vector_field(self):
       self._vector_field_sympy = np.zeros(self.transitions[0].update.shape,_
→dtype=object)
       111
       CODE HERE: fluid equation
       ,,,
       self._vector_field_sympy = sp.simplify(self._vector_field_sympy)
   #generates the diffusion term
  def __generate_diffusion(self):
      self._diffusion_sympy = 0
       111
       CODE HERE: diffusion term
       111
   # computes symbolically the jacobian of the vector field
  def __generate_jacobian(self):
      n = self.variables.dimension
      f = self._vector_field_sympy
      x = self.variables.reference
      J = np.zeros((n, n), dtype=object)
       111
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CODE HERE: javobian (sympy.diff)
             111
             self._jacobian_sympy = J
         # generate numpy expressions and the mean field VF
         def __generate_numpy_functions(self):
             sympy_ref = self.variables.reference + self.parameters.reference
             self.rates = sp.lambdify(sympy_ref, [t.rate for t in self.transitions],_

¬"numpy")
             self.vector_field = sp.lambdify(sympy_ref, self._vector_field_sympy,__

¬"numpy")
             self.diffusion = sp.lambdify(sympy_ref, self._diffusion_sympy, "numpy")
             self.jacobian = sp.lambdify(sympy_ref, self._jacobian_sympy, "numpy")
         #evaluates and returns vector field, diffusion, ...
         def evaluate_all_vector_fields(self, var_values):
             f = self.vector_field(*var_values, *self.parameters.values)
             D = self.diffusion(*var_values, *self.parameters.values)
             J = self.jacobian(*var_values, *self.parameters.values)
             return np.asarray(f), np.asarray(D), np.asarray(J)
         def evaluate_MF_vector_field(self, var_values):
             f = self.vector_field(*var_values, *self.parameters.values)
             return np.asarray(f)
         def evaluate_rates(self, var_values):
             r = self.rates(*var_values, *self.parameters.values)
             return np.asarray(r)
[6]: class Simulator:
         def __init__(self, model):
             self.model = model
             self.t0 = 0
             self.x0 = model.variables.values
         def _unpack(self, z):
             n = self.model.variables.dimension
             phi = z[0:n] # mean field
             c = np.reshape(z[n:], (n, n)) # c term
             return phi, c
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def _pack(self, f, dc):
      z = np.concatenate((f.flatten(), dc.flatten()))
      return z
   # computes the full vector field for the linear noise ODE
  def _linear_noise_ODE(self, z, t):
      x_t, c_t = self._unpack(z)
       111
       CODE HERE: linear noise eq
       111
       dz = self._pack(dx_dt, dc_dt)
      return dz
   # computes the vector field for the classic mean field
  def _mean_field_ODE(self, x, t):
       111
       CODE HERE: mean field eq
       111
      return dx.flatten()
  def _generate_time_stamp(self, final_time, points):
       Generates a time stamp from time self.tO to final_time,
       with points+1 number of points.
       :param final_time: final time of the simulation
       :param points: number of points
       :return: a time stamp numpy array
       step = (final_time - self.t0) / points
      time = np.arange(self.t0, final_time + step, step)
      return time
  def _SSA_single_simulation(self, final_time, time_stamp, model_dimension,_
→trans_number):
      A single SSA simulation run
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:param final_time: final simulation time
       :param time_stamp: time array containing time points to save
       :param model_dimension: dimension of the model
       : param\ trans\_number:\ transitions'\ number
       :return: the variables computed along the trajectory
       # tracks simulation time and state
       time = 0
       state = self.x0
       # tracks index of the time stamp vector, to save the array
       print_index = 1
       x = np.zeros((len(time_stamp), model_dimension))
       # save initial state
       x[0, :] = self.x0
       # main SSA loop
       trans_code = range(trans_number)
       while time < final_time:</pre>
           # compute rates and total rate
           # sanity check, to avoid negative numbers close to zero
           # COMPLETE HERE
           # check if total rate is non zero.
           # store values in the output array
           while print_index < len(time_stamp) and time_stamp[print_index] <=__
→time:
               x[print_index, :] = state
               print_index += 1
       return x
   def SSA_simulation(self, final_time, runs=100, points=1000, update=1):
       11 11 11
       Runs SSA simulation for a given number of runs and returns the average
       :param final_time: final simulation time
       :param runs: number of runs, default is 100
       :param points: number of points to be saved, default is 1000
       :param update: percentage step to update simulation time on screen
       :return: a Trajectory object, containing the average
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time_stamp = self._generate_time_stamp(final_time, points)
      n = self.model.variables.dimension
      m = self.model.transition_number
       average = np.zeros((len(time_stamp), n))
       # LOOP ON RUNS, count from 1
      update_runs = ceil(runs * update / 100.0)
       c = 0
      for i in range(1, runs + 1):
           c = c + 1
           # updates every 1% of simulation time
           if c == update runs:
               print(ceil(i * 100.0 / runs), "% done")
           y = self._SSA_single_simulation(final_time, time_stamp, n, m)
           # WARNING, works with python 3 only.
           # updating average
           average = (i - 1) / i * average + y / i
       time_stamp = np.reshape(time_stamp, (len(time_stamp), 1))
       trajectory = Trajectory(time_stamp, average, "SSA average", self.model.
→variables_names)
      return trajectory
  def MF_simulation(self, final_time, points=1000):
       Numerically integrates standard mean field equations
       :param final_time: final simulation time
       :param points: number of points to be saved
       :return: a trajectory object for model observables
       t = self._generate_time_stamp(final_time, points)
       111
       CODE HERE: mean field solution
       111
       t = np.reshape(t, (len(t), 1))
      trajectory = Trajectory(t, x, "Mean Field", self.model.variables_names)
      return trajectory
  def LN_simulation(self, final_time, points=1000):
       Numerivally integrates the linear noise equations
       :param final time: final simulation time
```

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:param points: number of points to be saved
:return: a trajectory object for corrected model observables
"""

n = self.model.variables.dimension
t = self._generate_time_stamp(final_time, points)

'''

CODE HERE: linear noise solution

'''

return
```

```
[7]: class Trajectory:
         def __init__(self, t, x, desc, labels):
             self.time = t
             self.data = x
             self.labels = labels
             self.description = desc
         def plot(self, var_to_plot=None):
             if var_to_plot is None:
                 var_to_plot = self.labels
             fig = plt.figure()
             ax = fig.add_subplot(111)
             ax.set_prop_cycle(plt.cycler('color', ['r', 'g', 'b', 'c', 'm', 'y', _
      \rightarrow'k']))
             handles = []
             labels = []
             for v in var_to_plot:
                 try:
                     i = self.labels.index(v)
                     h, = ax.plot(self.time, self.data[:, i])
                     handles.append(h)
                     labels.append(v)
                 except:
                     print("Variable", v, "not found")
             fig.legend(handles, labels)
             plt.title(self.description)
             plt.xlabel('Time')
             plt.show()
         def plot_comparing_to(self, trajectory, var_to_plot=None):
             if var_to_plot is None:
                 var_to_plot = self.labels
             fig = plt.figure()
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```
ax = fig.add_subplot(111)
      \hookrightarrow 'c', 'c', 'm', 'm', 'y', 'y', 'k', 'k']))
      handles = []
      labels = []
      for v in var_to_plot:
          try:
             i = self.labels.index(v)
             h, = ax.plot(self.time, self.data[:, i])
             handles.append(h)
             labels.append(self.description + " " + v)
             h, = ax.plot(trajectory.time, trajectory.data[:, i], '--')
             handles.append(h)
             labels.append(trajectory.description + " " + v)
          except Exception as e:
             print("Probably variable", v, "not found")
             print("Exception is", e)
      fig.legend(handles, labels)
      plt.title(self.description + " vs " + trajectory.description)
      plt.xlabel('Time')
      plt.show()
```

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