On the Use of DELEqC-III in Bilevel Problems with Linear Equality Constraints

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1 Test problems

The problems used in the computational experiments are detailed here. These problems are those used in [1].

Problem P.1'

$$\begin{aligned}
&\max_{\mathbf{x},\mathbf{y}\geq 0} F(\mathbf{x},\mathbf{y}) = 8x_1 + 4x_2 - 4y_1 + 40y_2 + 4y_3 \\
&\text{s.t.} \quad \max f(\mathbf{x},\mathbf{y}) = -x_1 - 2x_2 - y_1 - y_2 - 2y_3 \\
&\qquad\quad \text{s.t.} \quad E_{\mathbf{x}}\mathbf{x} + E_{\mathbf{y}}\mathbf{y} = \mathbf{c}
\end{aligned}$$

Problem P.4

$$\begin{aligned}
&\min_{\mathbf{x},\mathbf{y}\geq 0} F(\mathbf{x},\mathbf{y}) = -8x_1 - 4x_2 + 4y_1 - 40y_2 - 4y_3 \\
&\text{s.t.} \quad \min f(\mathbf{x},\mathbf{y}) = \frac{1 + x_1 + x_2 + 2y_1 - y_2 + y_3}{6 + 2x_1 + y_1 + y_2 - 3y_3} \\
&\qquad\quad \text{s.t.} \quad E_{\mathbf{x}}\mathbf{x} + E_{\mathbf{y}}\mathbf{y} = \mathbf{c}
\end{aligned}$$

Problem P.5

$$\begin{aligned}
&\min_{\mathbf{x},\mathbf{y}\geq 0} F(\mathbf{x},\mathbf{y}) = \mid -8x_1 - 4x_2 + 4y_1 - 40y_2 - 4y_3 + 29.2 \mid \\
&\text{s.t.} \quad \min f(\mathbf{x},\mathbf{y}) = \frac{1 + x_1 + x_2 + 2y_1 - y_2 + y_3}{6 + 2x_1 + y_1 + y_2 - 3y_3} \\
&\qquad\quad \text{s.t.} \quad E_{\mathbf{x}}\mathbf{x} + E_{\mathbf{y}}\mathbf{y} = \mathbf{c}
\end{aligned}$$

where

$$E_{\mathbf{x}} = \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 2 & 0 \end{bmatrix}, E_{\mathbf{y}} = \begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ -1 & 2 & -0.5 & 0 & 1 & 0 \\ 2 & -1 & -0.5 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

From [1–4], the best solution of the original formulation of problem P.1 (which has linear inequality constraints) is (0, 0.9, 0, 0.6, 0.4) leading to the values $F^* = 29.2$ and $f^* = -3.2$. Also, as pointed out in [1, 5, 6], the best solution of P.4 is (0.00, 0.90, 0.00, 0.60, 0.40) that leads to the optimal values $F^* = -29.2$ and $f^* = 0.31$. Finally, the best solution of P.5 is (0.00, 0.90, 0.00, 0.60, 0.40) with $F^* = 0$ and $f^* = 0.31$ [6, 7].

Problem P.2'

$$\begin{aligned}
\max_{\mathbf{x},\mathbf{y}\geq 0} \ & F(\mathbf{x},\mathbf{y}) = 2x_1 - x_2 - 0.5y_1 \\
\text{s.t.} \quad & \max f(\mathbf{x},\mathbf{y}) = -x_1 - x_2 + 4y_1 - y_2 \\
& \quad \text{s.t.} \quad E_{\mathbf{x}}\mathbf{x} + E_{\mathbf{y}}\mathbf{y} = \mathbf{c}
\end{aligned}$$

where

$$E_{\mathbf{x}} = \begin{bmatrix} -2 & 0 \\ 1 & -3 \\ 1 & 1 \end{bmatrix}, E_{\mathbf{y}} = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} -2.5 \\ 2 \\ 2 \end{bmatrix}$$

Problem P.3'

$$\begin{aligned}
\min_{\mathbf{x},\mathbf{y}\geq 0} \ & F(\mathbf{x},\mathbf{y}) = (x_1 - 1)^2 + 2y_1^2 - 2x_1 \\
\text{s.t.} \quad & \min f(\mathbf{x},\mathbf{y}) = (2y_1 - 4)^2 + (2y_2 - 1)^2 + x_1 y_1 \\
& \quad \text{s.t.} \quad E_{\mathbf{x}}\mathbf{x} + E_{\mathbf{y}}\mathbf{y} = \mathbf{c}
\end{aligned}$$

where

$$E_{\mathbf{x}} = \begin{bmatrix} 4 \\ -4 \\ 4 \\ -4 \end{bmatrix}, E_{\mathbf{y}} = \begin{bmatrix} 5 & 4 & 1 & 0 & 0 & 0 \\ -5 & 4 & 0 & 1 & 0 & 0 \\ -4 & 5 & 0 & 0 & 1 & 0 \\ 4 & 5 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} 12 \\ 4 \\ 4 \\ 4 \end{bmatrix}$$

The original version problem P.2 (which has linear inequality constraints) was solved in [8], where the best solution is presented as (2, 0, 1.5, 0) leading to the values $F^* = 3.25$ and $f^* = 4$. Finally, the original formulation of problem P.3 (which has linear inequality constraints) has (17/9, 8/9, 0) as the best solution, that leads to the values $F^* = -1.40741$ and $f^* = 7.617284$ [1, 4, 9].

2 Performance Profiles

The parameters `CR`, `F`, and the Differential Evolution (DE) variant of the proposed BL-DELEqC-III were analyzed. Here, we present the Performance Profiles of the median results obtained for the best parameters concerning each DE variant. One can observe that BL-DELEqC-III with `CR`=0.7, `F`=0.7 and DE/target-to-best/1/bin obtained the best results. This parameter setting (i) reached the best median results in most of the problems (larger $\rho(1)$, (ii) achieved the best overall performance (area under the PPs curves), and (iii) is the most reliable approach (smallest τ such that $\rho(\tau) = 1$).

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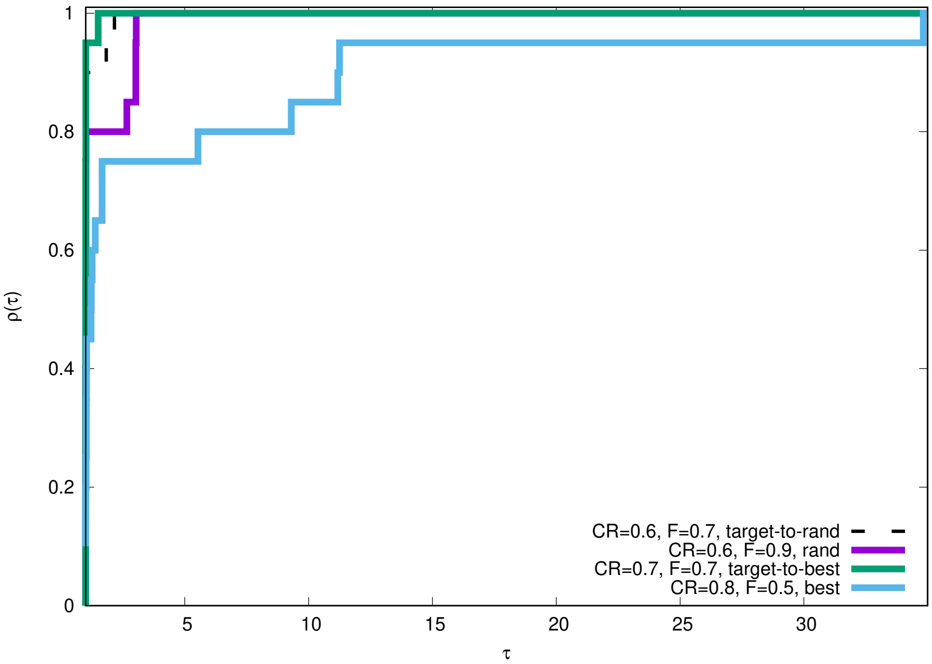

Figure 1. Performance profiles.

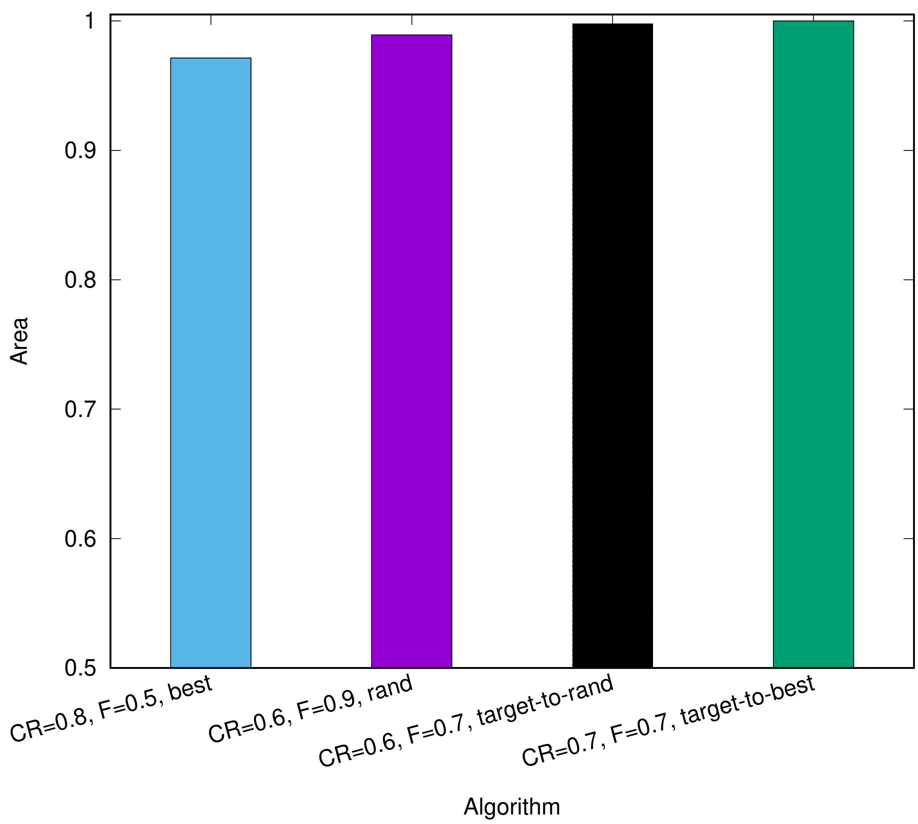

Figure 2. Areas under the performance profiles curves.