

Differential Evolution for linear equality constraint satisfaction via unconstrained search in the null space

Supplementary Material

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This document contains the supplementary material of the paper “Differential Evolution for linear equality constraint satisfaction via unconstrained search in the null space”.

Tables 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 present the statistical results obtained by the proposed approach, DELEqC-III, and those found by DELEqC-II.

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Table 1: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

TP/n/m	method	best	median	mean	std	worst
1/10/5	DELEqC-II*	32.14	32.15	32.15	8.36e-03	32.19
1/10/5	DELEqC-III-trand*	32.16	32.66	32.78	6.20e-01	35.75
1/10/5	DELEqC-III-rand	32.14	32.14	32.14	1.97e-03	32.15
1/20/5	DELEqC-II*	32.14	32.14	32.14	2.03e-06	32.14
1/20/5	DELEqC-III-trand*	32.14	32.14	32.15	1.74e-01	33.89
1/20/5	DELEqC-III-rand	32.14	32.14	32.14	0.00e+00	32.14
1/20/10	DELEqC-II	64.27	64.27	64.27	9.95e-08	64.27
1/20/10	DELEqC-III-trand*	64.27	64.27	64.40	7.82e-01	71.78
1/20/10	DELEqC-III-rand	64.27	64.27	64.27	1.42e-14	64.27
1/30/5	DELEqC-II*	32.15	43.65	51.99	2.35e+01	144.19
1/30/5	DELEqC-III-trand*	32.14	32.14	32.52	3.83e+00	70.61
1/30/5	DELEqC-III-rand	32.14	32.14	32.14	0.00e+00	32.14
1/30/10	DELEqC-II*	64.27	64.27	64.34	3.50e-01	67.25
1/30/10	DELEqC-III-trand*	64.27	64.27	64.34	5.38e-01	69.66
1/30/10	DELEqC-III-rand	64.27	64.27	64.27	1.42e-14	64.27
1/30/15	DELEqC-II	96.41	96.41	96.41	2.84e-14	96.41
1/30/15	DELEqC-III-trand*	96.41	96.41	96.76	1.39e+00	109.28
1/30/15	DELEqC-III-rand	96.41	96.41	96.41	2.84e-14	96.41
1/40/5	DELEqC-II*	94.54	363.76	419.24	2.05e+02	1182.53
1/40/5	DELEqC-III-trand*	32.14	32.14	32.26	1.08e+00	42.86
1/40/5	DELEqC-III-rand	32.14	32.14	32.14	0.00e+00	32.14
1/40/10	DELEqC-II*	77.68	181.70	211.94	1.02e+02	554.51
1/40/10	DELEqC-III-trand*	64.27	64.27	64.65	1.97e+00	80.29
1/40/10	DELEqC-III-rand	64.27	64.27	64.27	1.42e-14	64.27
1/40/15	DELEqC-II*	96.43	107.64	117.08	2.23e+01	206.45
1/40/15	DELEqC-III-trand*	96.41	96.42	96.99	1.61e+00	104.59
1/40/15	DELEqC-III-rand	96.41	96.41	96.41	2.84e-14	96.41
1/40/20	DELEqC-II*	128.55	128.55	128.61	3.52e-01	131.46
1/40/20	DELEqC-III-trand*	128.55	128.68	129.89	2.74e+00	143.81
1/40/20	DELEqC-III-rand	128.55	128.55	128.55	5.68e-14	128.55
1/50/5	DELEqC-II*	384.05	1054.30	1066.19	3.62e+02	2271.65
1/50/5	DELEqC-III-trand*	32.14	32.14	32.38	1.68e+00	44.54
1/50/5	DELEqC-III-rand	32.14	32.14	32.14	0.00e+00	32.14
1/50/10	DELEqC-II*	260.53	690.84	728.50	2.43e+02	1379.39
1/50/10	DELEqC-III-trand*	64.27	64.27	64.51	1.19e+00	75.43
1/50/10	DELEqC-III-rand	64.27	64.27	64.27	1.42e-14	64.27
1/50/15	DELEqC-II*	177.02	446.75	484.13	1.87e+02	988.51
1/50/15	DELEqC-III-trand*	96.41	96.42	98.59	9.06e+00	182.27
1/50/15	DELEqC-III-rand	96.41	96.41	96.41	2.84e-14	96.41
1/50/20	DELEqC-II*	132.54	290.59	292.53	1.10e+02	646.29
1/50/20	DELEqC-III-trand*	128.55	128.63	131.13	8.25e+00	197.57
1/50/20	DELEqC-III-rand	128.55	128.55	128.55	5.68e-14	128.55
1/50/25	DELEqC-II*	160.69	171.59	179.80	2.69e+01	307.70
1/50/25	DELEqC-III-trand*	160.68	162.13	164.67	5.86e+00	188.16
1/50/25	DELEqC-III-rand	160.68	160.68	160.68	5.68e-14	160.68

Table 2: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

TP/n/m	method	best	median	mean	std	worst
1/100/5	DELEqC-II*	2889.46	4914.76	4938.24	8.13e+02	6945.10
1/100/5	DELEqC-III-trand*	32.14	32.39	34.79	1.09e+01	137.22
1/100/5	DELEqC-III-rand	32.14	32.14	32.14	0.00e+00	32.14
1/100/10	DELEqC-II*	2745.22	4500.80	4496.47	8.90e+02	6881.03
1/100/10	DELEqC-III-trand*	64.27	66.53	73.89	2.07e+01	229.68
1/100/10	DELEqC-III-rand	64.27	64.27	64.27	1.42e-14	64.27
1/100/15	DELEqC-II*	2416.66	4044.79	4128.93	8.05e+02	6229.37
1/100/15	DELEqC-III-trand*	96.42	101.19	110.47	2.63e+01	278.89
1/100/15	DELEqC-III-rand	96.41	96.41	96.41	2.84e-14	96.41
1/100/20	DELEqC-II*	2024.06	3594.11	3579.69	6.71e+02	5184.28
1/100/20	DELEqC-III-trand*	128.69	137.81	151.00	3.28e+01	304.23
1/100/20	DELEqC-III-rand	128.55	128.55	128.55	5.68e-14	128.55
1/100/25	DELEqC-II*	2135.03	3302.55	3409.00	7.02e+02	4989.38
1/100/25	DELEqC-III-trand*	161.02	173.59	185.09	3.05e+01	319.51
1/100/25	DELEqC-III-rand	160.68	160.68	160.68	5.68e-14	160.68
1/100/30	DELEqC-II*	2065.21	3018.93	3105.38	6.24e+02	4823.09
1/100/30	DELEqC-III-trand*	193.18	217.03	226.38	3.59e+01	373.68
1/100/30	DELEqC-III-rand	192.82	192.82	192.82	5.68e-14	192.82
1/100/35	DELEqC-II*	1562.84	2649.04	2662.98	6.05e+02	4135.14
1/100/35	DELEqC-III-trand*	225.89	253.12	280.14	6.31e+01	487.32
1/100/35	DELEqC-III-rand	224.96	224.96	224.96	0.00e+00	224.96
1/100/40	DELEqC-II*	1393.18	2361.56	2360.78	5.26e+02	3887.97
1/100/40	DELEqC-III-trand*	257.86	309.19	330.60	8.83e+01	990.22
1/100/40	DELEqC-III-rand	257.10	257.10	257.10	1.14e-13	257.10
1/100/45	DELEqC-II*	1291.02	1987.70	2081.03	4.69e+02	3669.80
1/100/45	DELEqC-III-trand*	292.72	347.86	369.93	7.64e+01	785.36
1/100/45	DELEqC-III-rand	289.23	289.23	289.23	5.68e-14	289.23
1/100/50	DELEqC-II*	918.76	1613.53	1689.56	4.45e+02	3541.94
1/100/50	DELEqC-III-trand*	332.90	387.88	407.12	8.28e+01	848.34
1/100/50	DELEqC-III-rand	321.37	321.37	321.37	5.68e-14	321.37

Table 3: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

TP/n/m	method	best	median	mean	std	worst
2/10/5	DELEqC-II	35.38	35.38	35.38	7.11e-15	35.38
2/10/5	DELEqC-III-trand*	35.38	35.39	35.41	1.01e-01	36.30
2/10/5	DELEqC-III-rand	35.38	35.38	35.38	7.11e-15	35.38
2/20/5	DELEqC-II	35.23	35.23	35.23	7.11e-15	35.23
2/20/5	DELEqC-III-trand*	35.23	35.23	35.23	5.87e-03	35.29
2/20/5	DELEqC-III-rand	35.23	35.23	35.27	2.29e-01	36.87
2/20/10	DELEqC-II	115.69	115.69	115.71	4.63e-02	115.83
2/20/10	DELEqC-III-trand*	115.70	115.95	116.60	2.69e+00	136.00
2/20/10	DELEqC-III-rand*	115.69	115.71	115.88	1.18e+00	127.59
2/30/5	DELEqC-II*	37.96	87.40	101.34	5.14e+01	310.91
2/30/5	DELEqC-III-trand	35.20	35.20	35.20	2.74e-06	35.20
2/30/5	DELEqC-III-rand	35.20	35.20	35.26	3.26e-01	36.86
2/30/10	DELEqC-II*	115.64	116.04	120.37	6.55e+00	142.52
2/30/10	DELEqC-III-trand*	115.65	115.77	115.80	1.50e-01	116.60
2/30/10	DELEqC-III-rand	115.64	115.65	116.30	2.57e+00	128.61
2/30/15	DELEqC-II	242.15	242.15	242.42	1.43e+00	256.23
2/30/15	DELEqC-III-trand*	242.43	244.35	245.15	5.07e+00	287.28
2/30/15	DELEqC-III-rand*	242.15	242.16	242.54	5.86e-01	244.51
2/40/5	DELEqC-II*	230.91	792.14	813.87	2.86e+02	1705.64
2/40/5	DELEqC-III-trand	35.18	35.18	35.18	9.95e-08	35.18
2/40/5	DELEqC-III-rand*	35.18	35.18	35.61	1.80e+00	47.67
2/40/10	DELEqC-II*	231.75	482.90	539.13	2.22e+02	1584.27
2/40/10	DELEqC-III-trand*	115.62	115.67	115.69	7.02e-02	115.96
2/40/10	DELEqC-III-rand	115.62	115.62	117.18	4.07e+00	128.61
2/40/15	DELEqC-II*	248.33	305.58	317.27	5.48e+01	529.92
2/40/15	DELEqC-III-trand*	242.04	242.65	243.96	5.69e+00	279.57
2/40/15	DELEqC-III-rand	241.94	241.94	242.08	4.36e-01	244.31
2/40/20	DELEqC-II*	400.60	402.01	405.23	6.68e+00	428.97
2/40/20	DELEqC-III-trand*	401.11	402.97	406.86	9.32e+00	458.70
2/40/20	DELEqC-III-rand	400.60	401.25	402.64	2.80e+00	412.16
2/50/5	DELEqC-II*	917.49	1889.41	1920.50	5.45e+02	3603.68
2/50/5	DELEqC-III-trand	35.17	35.17	35.48	2.46e+00	59.74
2/50/5	DELEqC-III-rand	35.17	35.17	35.99	2.62e+00	50.57
2/50/10	DELEqC-II*	795.82	1454.06	1529.16	4.20e+02	2920.15
2/50/10	DELEqC-III-trand	115.61	115.63	115.86	1.54e+00	129.21
2/50/10	DELEqC-III-rand	115.61	115.61	118.95	6.23e+00	142.52
2/50/15	DELEqC-II*	440.50	1042.90	1114.27	3.16e+02	2005.85
2/50/15	DELEqC-III-trand*	241.85	242.15	242.72	3.63e+00	278.38
2/50/15	DELEqC-III-rand	241.84	241.84	242.40	5.01e+00	292.18
2/50/20	DELEqC-II*	469.07	757.45	788.38	1.83e+02	1419.47
2/50/20	DELEqC-III-trand*	400.21	401.95	403.05	3.75e+00	423.57
2/50/20	DELEqC-III-rand	400.03	400.03	401.17	2.22e+00	408.52
2/50/25	DELEqC-II*	593.55	646.93	657.49	4.65e+01	813.00
2/50/25	DELEqC-III-trand*	590.81	600.10	605.89	1.72e+01	695.19
2/50/25	DELEqC-III-rand	589.30	596.93	599.04	9.51e+00	638.73

Table 4: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

2/100/5	DELEqC-II*	9864.88	13093.95	13198.65	1.99e+03	19352.34
2/100/5	DELEqC-III-trand	35.15	35.15	38.83	7.27e+00	67.76
2/100/5	DELEqC-III-rand	35.15	36.85	41.57	7.40e+00	55.52
2/100/10	DELEqC-II*	8736.06	12136.65	12161.09	1.66e+03	16555.50
2/100/10	DELEqC-III-trand	115.59	115.86	120.74	7.32e+00	158.09
2/100/10	DELEqC-III-rand*	115.59	128.61	131.33	9.65e+00	160.11
2/100/15	DELEqC-II*	7551.76	11235.13	11356.86	1.60e+03	16907.32
2/100/15	DELEqC-III-trand*	241.67	241.78	243.87	9.63e+00	310.09
2/100/15	DELEqC-III-rand	241.67	241.67	246.95	9.44e+00	272.79
2/100/20	DELEqC-II*	6922.78	10015.67	10142.04	1.53e+03	15568.00
2/100/20	DELEqC-III-trand	399.23	401.90	406.21	1.47e+01	480.90
2/100/20	DELEqC-III-rand*	399.17	404.89	406.28	8.89e+00	438.69
2/100/25	DELEqC-II*	6421.49	9271.77	9298.71	1.34e+03	12327.87
2/100/25	DELEqC-III-trand*	586.66	588.39	593.89	1.28e+01	659.71
2/100/25	DELEqC-III-rand	586.43	586.43	591.31	1.09e+01	626.71
2/100/30	DELEqC-II*	5887.15	8616.59	8732.41	1.35e+03	13152.53
2/100/30	DELEqC-III-trand	820.19	830.78	839.13	3.32e+01	1029.71
2/100/30	DELEqC-III-rand	819.79	828.24	836.50	2.03e+01	920.62
2/100/35	DELEqC-II*	5563.13	7669.10	7823.66	1.23e+03	11584.01
2/100/35	DELEqC-III-trand	1077.16	1109.66	1117.17	4.55e+01	1431.77
2/100/35	DELEqC-III-rand	1074.96	1116.32	1126.66	4.77e+01	1255.78
2/100/40	DELEqC-II*	4669.74	6915.00	7001.13	1.01e+03	10443.57
2/100/40	DELEqC-III-trand	1394.84	1435.04	1450.11	5.72e+01	1771.36
2/100/40	DELEqC-III-rand*	1387.07	1494.33	1490.80	6.32e+01	1657.81
2/100/45	DELEqC-II*	4337.18	6509.64	6463.83	9.43e+02	10007.56
2/100/45	DELEqC-III-trand	1746.26	1814.88	1827.69	6.37e+01	2166.91
2/100/45	DELEqC-III-rand*	1719.56	1901.80	1907.64	8.11e+01	2102.47
2/100/50	DELEqC-II*	4072.78	5883.04	5865.06	8.01e+02	9006.94
2/100/50	DELEqC-III-trand	2164.64	2239.74	2270.07	9.95e+01	2777.55
2/100/50	DELEqC-III-rand*	2157.51	2441.79	2428.55	1.02e+02	2812.51

Table 5: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

TP/n/m	method	best	median	mean	std	worst
3/10/5	DELEqC-II*	37.27	42.94	43.11	3.59e+00	55.05
3/10/5	DELEqC-III-trand*	37.01	37.94	38.39	2.21e+00	56.89
3/10/5	DELEqC-III-rand	36.98	36.98	37.30	8.03e-01	41.86
3/20/5	DELEqC-II*	74.26	107.14	106.16	1.02e+01	130.94
3/20/5	DELEqC-III-trand*	36.98	37.03	37.23	6.79e-01	42.95
3/20/5	DELEqC-III-rand	36.98	36.98	37.55	9.03e-01	42.95
3/20/10	DELEqC-II*	75.09	123.81	122.32	1.37e+01	150.74
3/20/10	DELEqC-III-trand*	73.96	74.84	75.71	3.66e+00	108.82
3/20/10	DELEqC-III-rand	73.95	73.95	74.49	9.99e-01	78.93
3/30/5	DELEqC-II*	146.25	182.06	181.37	1.09e+01	209.99
3/30/5	DELEqC-III-trand	36.98	36.99	37.50	3.57e+00	72.79
3/30/5	DELEqC-III-rand*	36.98	37.97	40.20	6.53e+00	72.95
3/30/10	DELEqC-II*	113.74	204.30	201.55	1.82e+01	229.47
3/30/10	DELEqC-III-trand	73.97	74.30	74.88	1.26e+00	80.07
3/30/10	DELEqC-III-rand*	73.95	74.45	76.00	6.13e+00	124.42
3/30/15	DELEqC-II*	111.34	227.22	223.92	2.06e+01	252.94
3/30/15	DELEqC-III-trand*	111.37	113.63	114.47	4.31e+00	149.81
3/30/15	DELEqC-III-rand	110.93	111.92	112.92	3.51e+00	133.73
3/40/5	DELEqC-II*	205.23	253.05	253.41	1.45e+01	279.11
3/40/5	DELEqC-III-trand	36.98	36.98	37.57	1.18e+00	43.94
3/40/5	DELEqC-III-rand*	36.98	40.96	55.67	2.19e+01	118.96
3/40/10	DELEqC-II*	128.61	280.59	277.70	2.02e+01	301.93
3/40/10	DELEqC-III-trand	73.95	74.95	76.09	8.20e+00	155.54
3/40/10	DELEqC-III-rand*	73.95	75.94	78.61	1.03e+01	138.17
3/40/15	DELEqC-II*	122.37	302.81	288.21	4.88e+01	329.10
3/40/15	DELEqC-III-trand*	110.97	112.85	115.50	1.14e+01	185.55
3/40/15	DELEqC-III-rand	110.93	112.42	113.86	6.30e+00	168.91
3/40/20	DELEqC-II*	151.99	321.19	305.00	4.99e+01	358.31
3/40/20	DELEqC-III-trand*	147.96	152.13	156.08	1.10e+01	199.64
3/40/20	DELEqC-III-rand	147.90	148.90	150.30	5.02e+00	184.67
3/50/5	DELEqC-II*	267.53	333.09	329.96	1.58e+01	355.99
3/50/5	DELEqC-III-trand	36.98	37.97	38.39	2.34e+00	54.88
3/50/5	DELEqC-III-rand*	36.98	113.38	108.77	3.27e+01	174.67
3/50/10	DELEqC-II*	157.19	357.67	349.67	3.30e+01	385.12
3/50/10	DELEqC-III-trand	73.95	74.95	76.04	3.42e+00	96.32
3/50/10	DELEqC-III-rand*	74.95	80.92	101.44	3.35e+01	197.99
3/50/15	DELEqC-II*	175.27	382.72	354.23	6.71e+01	421.00
3/50/15	DELEqC-III-trand	110.93	112.93	117.09	1.38e+01	200.47
3/50/15	DELEqC-III-rand	110.93	113.91	119.79	1.81e+01	213.87
3/50/20	DELEqC-II*	186.70	404.81	379.93	6.61e+01	440.76
3/50/20	DELEqC-III-trand*	147.92	153.01	156.43	1.58e+01	259.47
3/50/20	DELEqC-III-rand	147.90	149.89	154.53	1.41e+01	241.69
3/50/25	DELEqC-II*	220.48	415.64	373.82	8.15e+01	465.84
3/50/25	DELEqC-III-trand*	185.10	193.64	202.32	2.28e+01	324.39
3/50/25	DELEqC-III-rand	184.88	187.36	190.84	1.35e+01	267.58

Table 6: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

3/100/5	DELEqC-II*	326.45	736.12	685.15	1.24e+02	776.68
3/100/5	DELEqC-III-trand	53.78	81.03	84.74	2.75e+01	286.75
3/100/5	DELEqC-III-rand*	478.11	547.25	547.39	2.98e+01	606.56
3/100/10	DELEqC-II*	479.95	775.31	752.42	7.58e+01	808.62
3/100/10	DELEqC-III-trand	78.02	119.53	127.54	4.68e+01	493.93
3/100/10	DELEqC-III-rand*	475.24	564.23	564.18	3.38e+01	634.95
3/100/15	DELEqC-II*	441.48	797.10	766.96	8.50e+01	840.42
3/100/15	DELEqC-III-trand	122.85	161.16	175.18	4.76e+01	385.29
3/100/15	DELEqC-III-rand*	444.48	587.06	579.55	4.55e+01	685.23
3/100/20	DELEqC-II*	512.34	822.26	770.05	1.12e+02	869.48
3/100/20	DELEqC-III-trand	160.03	199.41	222.73	6.83e+01	531.71
3/100/20	DELEqC-III-rand*	436.21	611.60	603.81	5.97e+01	702.68
3/100/25	DELEqC-II*	526.09	836.00	754.53	1.31e+02	899.45
3/100/25	DELEqC-III-trand	197.72	263.38	272.33	5.43e+01	511.75
3/100/25	DELEqC-III-rand*	417.34	625.58	619.41	7.12e+01	761.69
3/100/30	DELEqC-II*	528.20	808.54	753.85	1.37e+02	937.68
3/100/30	DELEqC-III-trand	246.74	313.69	334.94	7.71e+01	689.19
3/100/30	DELEqC-III-rand*	383.26	675.33	655.57	7.70e+01	778.75
3/100/35	DELEqC-II*	588.31	677.62	752.71	1.32e+02	953.00
3/100/35	DELEqC-III-trand	281.13	368.16	392.27	8.67e+01	706.99
3/100/35	DELEqC-III-rand*	379.45	718.35	696.55	8.64e+01	826.56
3/100/40	DELEqC-II*	620.35	700.80	742.70	1.10e+02	979.64
3/100/40	DELEqC-III-trand	328.06	431.56	455.79	1.03e+02	1084.15
3/100/40	DELEqC-III-rand*	361.19	751.71	729.78	1.08e+02	872.75
3/100/45	DELEqC-II*	617.62	717.01	747.70	9.88e+01	1002.80
3/100/45	DELEqC-III-trand	383.27	500.43	519.49	1.05e+02	932.20
3/100/45	DELEqC-III-rand*	398.82	796.98	779.09	9.58e+01	923.60
3/100/50	DELEqC-II*	645.45	742.76	786.19	1.10e+02	1041.15
3/100/50	DELEqC-III-trand	400.23	532.20	557.77	1.02e+02	975.98
3/100/50	DELEqC-III-rand*	374.77	849.49	836.54	8.26e+01	953.27

Table 7: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

TP/n/m	method	best	median	mean	std	worst
4/10/5	DELEqC-II	21485.30	21485.31	21485.30	2.88e-05	21485.31
4/10/5	DELEqC-III-trand*	21485.31	21485.31	21485.33	4.21e-02	21485.56
4/10/5	DELEqC-III-rand*	21485.31	21485.31	21485.31	0.00e+00	21485.31
4/20/5	DELEqC-II	21495.27	21495.27	21499.37	1.50e+01	21561.12
4/20/5	DELEqC-III-trand*	21495.28	21551.15	21575.88	2.61e+02	24165.91
4/20/5	DELEqC-III-rand*	21495.27	21495.27	21504.27	1.98e+01	21553.86
4/20/10	DELEqC-II	43035.03	43035.03	43035.03	1.03e-04	43035.03
4/20/10	DELEqC-III-trand*	43035.03	43035.03	43317.85	1.64e+03	57161.55
4/20/10	DELEqC-III-rand*	43035.03	43035.03	43035.03	7.28e-12	43035.03
4/30/5	DELEqC-II*	21531.89	23084.39	31395.42	2.18e+04	148436.29
4/30/5	DELEqC-III-trand*	21505.17	21561.16	21664.85	7.21e+02	28431.36
4/30/5	DELEqC-III-rand	21505.17	21505.17	21523.23	2.21e+01	21551.39
4/30/10	DELEqC-II*	43044.97	43051.78	43153.54	6.20e+02	49151.41
4/30/10	DELEqC-III-trand*	43044.97	43101.38	43153.78	3.21e+02	46158.28
4/30/10	DELEqC-III-rand	43044.97	43044.97	43046.03	7.40e+00	43098.65
4/30/15	DELEqC-II	64584.83	64584.83	64584.83	1.73e-06	64584.83
4/30/15	DELEqC-III-trand*	64584.83	64598.43	68373.56	1.84e+04	209336.32
4/30/15	DELEqC-III-rand*	64584.83	64584.83	64584.83	7.28e-12	64584.83
4/40/5	DELEqC-II*	58636.35	684030.04	997940.83	1.10e+06	8196801.86
4/40/5	DELEqC-III-trand*	21515.60	21611.01	21614.76	9.41e+01	22454.12
4/40/5	DELEqC-III-rand	21515.07	21515.07	21531.45	1.78e+01	21566.19
4/40/10	DELEqC-II*	45046.92	166267.17	275545.74	3.26e+05	2507052.96
4/40/10	DELEqC-III-trand*	43055.07	43120.16	44206.25	5.43e+03	94738.85
4/40/10	DELEqC-III-rand	43054.87	43054.87	43075.65	2.25e+01	43100.06
4/40/15	DELEqC-II*	64667.15	71299.38	84165.96	4.48e+04	473773.49
4/40/15	DELEqC-III-trand*	64594.77	64706.06	72935.21	4.46e+04	508482.70
4/40/15	DELEqC-III-rand	64594.77	64594.77	64594.77	1.46e-11	64594.77
4/40/20	DELEqC-II*	86134.63	86136.95	86319.18	6.24e+02	91227.58
4/40/20	DELEqC-III-trand*	86134.63	86941.24	98952.01	4.17e+04	467801.96
4/40/20	DELEqC-III-rand	86134.63	86134.63	86134.63	1.46e-11	86134.63
4/50/5	DELEqC-II*	1038892.96	3881430.34	4239970.88	2.56e+06	13032764.87
4/50/5	DELEqC-III-trand*	21525.36	21648.13	22044.35	1.87e+03	37061.10
4/50/5	DELEqC-III-rand	21524.97	21524.97	21535.35	1.25e+01	21550.30
4/50/10	DELEqC-II*	195381.22	1966953.58	2534779.02	1.92e+06	10205378.76
4/50/10	DELEqC-III-trand*	43071.38	43172.71	44754.99	5.83e+03	82737.91
4/50/10	DELEqC-III-rand	43064.77	43064.77	43078.18	1.71e+01	43100.06
4/50/15	DELEqC-II*	145144.99	669957.15	946629.68	8.23e+05	5713910.43
4/50/15	DELEqC-III-trand*	64615.27	64985.77	73263.01	4.46e+04	505352.71
4/50/15	DELEqC-III-rand	64604.67	64604.67	64621.39	2.18e+01	64649.85
4/50/20	DELEqC-II*	92097.14	217628.99	317390.51	3.12e+05	2252585.84
4/50/20	DELEqC-III-trand*	86144.57	87127.57	102847.54	5.27e+04	522479.20
4/50/20	DELEqC-III-rand	86144.56	86144.56	86144.56	1.46e-11	86144.56
4/50/25	DELEqC-II*	107992.21	121596.41	137165.26	4.18e+04	367327.85
4/50/25	DELEqC-III-trand*	107684.43	113947.92	149255.80	1.06e+05	867204.44
4/50/25	DELEqC-III-rand	107684.42	107684.42	107684.42	1.46e-11	107684.42

Table 8: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

TP/n/m	method	best	median	mean	std	worst
4/100/5	DELEqC-II*	19767594.53	45755571.14	51480364.73	2.47e+07	172329099.75
4/100/5	DELEqC-III-trand*	22160.46	31306.02	61101.67	1.03e+05	839071.98
4/100/5	DELEqC-III-rand	21573.72	21574.46	21575.44	1.96e+00	21582.88
4/100/10	DELEqC-II*	16426064.34	42588363.11	44919667.44	1.68e+07	93608430.46
4/100/10	DELEqC-III-trand*	43724.39	65844.83	116274.18	2.07e+05	1773695.43
4/100/10	DELEqC-III-rand	43114.26	43114.26	43117.40	4.21e+00	43127.73
4/100/15	DELEqC-II*	9401930.70	33919111.74	36911259.88	1.46e+07	94310090.54
4/100/15	DELEqC-III-trand*	65059.30	100981.50	233764.62	4.06e+05	2823400.29
4/100/15	DELEqC-III-rand	64654.16	64654.16	64660.38	6.91e+00	64675.46
4/100/20	DELEqC-II*	12099705.35	29730973.41	32021657.73	1.10e+07	67279513.73
4/100/20	DELEqC-III-trand*	87843.97	143043.48	261654.61	3.15e+05	2431064.58
4/100/20	DELEqC-III-rand	86194.06	86194.06	86202.35	9.80e+00	86223.40
4/100/25	DELEqC-II*	8017193.66	24999392.80	27184816.37	1.05e+07	65267913.65
4/100/25	DELEqC-III-trand*	110504.39	196676.65	295267.27	2.94e+05	2280732.19
4/100/25	DELEqC-III-rand	107733.96	107733.96	107741.47	1.08e+01	107761.55
4/100/30	DELEqC-II*	4707507.74	18184434.11	21530038.45	1.08e+07	55873818.42
4/100/30	DELEqC-III-trand*	130535.05	257536.14	473352.84	5.33e+05	3593624.29
4/100/30	DELEqC-III-rand	129273.86	129273.86	129284.54	1.47e+01	129311.22
4/100/35	DELEqC-II*	5991178.31	15475545.28	18224078.14	9.25e+06	44681001.73
4/100/35	DELEqC-III-trand*	157860.34	340656.06	561548.46	5.57e+05	3198086.42
4/100/35	DELEqC-III-rand	150813.76	150813.76	150830.55	1.87e+01	150859.27
4/100/40	DELEqC-II*	3992040.17	13248214.36	14292605.12	6.94e+06	36817944.74
4/100/40	DELEqC-III-trand*	186428.09	448024.77	739187.97	8.54e+05	6955800.16
4/100/40	DELEqC-III-rand	172353.65	172353.65	172372.55	2.27e+01	172408.52
4/100/45	DELEqC-II*	2715332.73	9461396.22	10658519.16	5.28e+06	27715015.98
4/100/45	DELEqC-III-trand*	212977.90	548684.99	874606.39	1.02e+06	8386464.21
4/100/45	DELEqC-III-rand	193893.55	193893.55	193910.22	2.56e+01	193962.19
4/100/50	DELEqC-II*	1875178.26	6054605.35	7252473.21	4.33e+06	31057093.16
4/100/50	DELEqC-III-trand*	231040.56	752186.89	1258778.84	1.77e+06	11682742.53
4/100/50	DELEqC-III-rand	215433.41	215433.41	215433.42	4.99e-02	215433.86

Table 9: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

TP/n/m	method	best	median	mean	std	worst
5/10/5	DELEqC-II*	0.21	0.35	0.36	7.21e-02	0.51
5/10/5	DELEqC-III-trand*	0.16	0.28	0.28	6.40e-02	0.42
5/10/5	DELEqC-III-rand	0.15	0.22	0.23	4.98e-02	0.39
5/20/5	DELEqC-II*	0.20	0.48	0.54	1.59e-01	0.82
5/20/5	DELEqC-III-trand	0.15	0.22	0.22	5.06e-02	0.36
5/20/5	DELEqC-III-rand*	0.15	0.24	0.30	1.60e-01	0.62
5/20/10	DELEqC-II*	0.80	0.91	0.89	4.28e-02	0.96
5/20/10	DELEqC-III-trand*	0.70	0.91	0.89	3.93e-02	1.00
5/20/10	DELEqC-III-rand	0.40	0.88	0.84	1.12e-01	0.91
5/30/5	DELEqC-II*	0.63	0.99	0.97	1.19e-01	1.31
5/30/5	DELEqC-III-trand	0.15	0.21	0.23	7.63e-02	0.62
5/30/5	DELEqC-III-rand*	0.20	0.24	0.40	2.00e-01	0.82
5/30/10	DELEqC-II*	0.84	0.93	0.94	2.82e-02	1.00
5/30/10	DELEqC-III-trand*	0.80	0.91	0.89	3.02e-02	0.96
5/30/10	DELEqC-III-rand	0.80	0.91	0.89	3.83e-02	0.96
5/30/15	DELEqC-II*	0.94	1.00	0.99	7.89e-03	1.00
5/30/15	DELEqC-III-trand*	0.97	0.97	0.98	9.94e-03	1.02
5/30/15	DELEqC-III-rand	0.93	0.97	0.98	1.21e-02	1.00
5/40/5	DELEqC-II*	1.13	1.44	1.48	2.12e-01	2.14
5/40/5	DELEqC-III-trand	0.15	0.21	0.29	1.54e-01	0.62
5/40/5	DELEqC-III-rand*	0.20	0.62	0.49	1.90e-01	0.82
5/40/10	DELEqC-II*	1.01	1.17	1.20	1.29e-01	1.96
5/40/10	DELEqC-III-trand	0.80	0.91	0.90	3.35e-02	1.01
5/40/10	DELEqC-III-rand	0.80	0.91	0.91	3.06e-02	0.96
5/40/15	DELEqC-II*	0.98	1.03	1.04	2.69e-02	1.11
5/40/15	DELEqC-III-trand*	0.94	0.97	0.98	1.31e-02	1.02
5/40/15	DELEqC-III-rand	0.94	0.97	0.98	1.00e-02	1.00
5/40/20	DELEqC-II*	1.00	1.02	1.02	6.58e-03	1.04
5/40/20	DELEqC-III-trand*	1.01	1.02	1.02	9.78e-03	1.04
5/40/20	DELEqC-III-rand	1.01	1.01	1.01	5.62e-03	1.02
5/50/5	DELEqC-II*	1.36	2.06	2.12	3.94e-01	3.79
5/50/5	DELEqC-III-trand	0.15	0.28	0.34	1.81e-01	0.96
5/50/5	DELEqC-III-rand*	0.20	0.62	0.57	1.55e-01	0.82
5/50/10	DELEqC-II*	1.23	1.76	1.81	3.09e-01	2.81
5/50/10	DELEqC-III-trand	0.80	0.91	0.90	3.20e-02	1.02
5/50/10	DELEqC-III-rand*	0.80	0.91	0.91	2.86e-02	0.96
5/50/15	DELEqC-II*	1.11	1.47	1.51	2.27e-01	2.23
5/50/15	DELEqC-III-trand*	0.94	0.97	0.98	1.63e-02	1.04
5/50/15	DELEqC-III-rand	0.97	0.97	0.98	1.17e-02	1.02
5/50/20	DELEqC-II*	1.05	1.18	1.21	1.09e-01	1.55
5/50/20	DELEqC-III-trand	1.01	1.02	1.02	9.78e-03	1.05
5/50/20	DELEqC-III-rand*	1.01	1.02	1.01	6.00e-03	1.03
5/50/25	DELEqC-II*	1.03	1.05	1.07	4.94e-02	1.37
5/50/25	DELEqC-III-trand*	1.03	1.04	1.04	5.61e-03	1.05
5/50/25	DELEqC-III-rand	1.03	1.03	1.03	2.32e-03	1.04

Table 10: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

5/100/5	DELEqC-II*	3.86	5.58	5.61	1.00e+00	9.66
5/100/5	DELEqC-III-trand*	0.15	0.62	0.64	1.54e-01	0.98
5/100/5	DELEqC-III-rand	0.20	0.62	0.65	1.13e-01	0.82
5/100/10	DELEqC-II*	3.45	4.96	5.09	9.60e-01	7.78
5/100/10	DELEqC-III-trand*	0.81	0.96	0.96	4.76e-02	1.03
5/100/10	DELEqC-III-rand	0.87	0.91	0.93	2.90e-02	1.01
5/100/15	DELEqC-II*	2.97	4.68	4.75	7.69e-01	6.77
5/100/15	DELEqC-III-trand*	0.97	1.01	1.01	2.16e-02	1.07
5/100/15	DELEqC-III-rand	0.97	1.00	0.99	1.13e-02	1.02
5/100/20	DELEqC-II*	3.03	4.30	4.46	9.01e-01	7.33
5/100/20	DELEqC-III-trand*	1.01	1.03	1.03	1.25e-02	1.08
5/100/20	DELEqC-III-rand	1.01	1.02	1.02	4.24e-03	1.03
5/100/25	DELEqC-II*	2.50	4.00	4.05	7.29e-01	5.63
5/100/25	DELEqC-III-trand*	1.03	1.04	1.04	9.18e-03	1.08
5/100/25	DELEqC-III-rand	1.03	1.03	1.03	2.01e-03	1.04
5/100/30	DELEqC-II*	2.43	3.72	3.81	8.28e-01	6.50
5/100/30	DELEqC-III-trand*	1.04	1.05	1.06	9.60e-03	1.09
5/100/30	DELEqC-III-rand	1.04	1.04	1.04	2.39e-03	1.05
5/100/35	DELEqC-II*	2.26	3.29	3.38	6.56e-01	5.40
5/100/35	DELEqC-III-trand*	1.05	1.06	1.07	1.57e-02	1.12
5/100/35	DELEqC-III-rand	1.05	1.05	1.05	1.51e-03	1.06
5/100/40	DELEqC-II*	2.10	3.00	3.02	5.26e-01	4.56
5/100/40	DELEqC-III-trand*	1.06	1.08	1.08	2.19e-02	1.25
5/100/40	DELEqC-III-rand	1.06	1.06	1.06	6.57e-04	1.06
5/100/45	DELEqC-II*	1.85	2.70	2.75	4.60e-01	4.22
5/100/45	DELEqC-III-trand*	1.07	1.09	1.09	1.92e-02	1.20
5/100/45	DELEqC-III-rand	1.07	1.07	1.07	1.52e-04	1.07
5/100/50	DELEqC-II*	1.47	2.29	2.32	4.35e-01	3.75
5/100/50	DELEqC-III-trand*	1.08	1.10	1.10	2.06e-02	1.21
5/100/50	DELEqC-III-rand	1.08	1.08	1.08	2.22e-16	1.08

Table 11: Statistical results of DELEqC-II-CR0.7-F0.9-trand-NP100 (DELEqC-II), DELEqC-III-CR0.1-F0.5-trand-NP30 (DELEqC-III-trand) and DELEqC-III-CR0.3-F0.6-rand-NP20 (DELEqC-III-rand). A * indicate that the results are statistical different (Mann-Whitney non-parametric test with p -value < 0.05) to those obtained by the best variant (with respect to the median).

TP/n/m	method	best	median	mean	std	worst
13/16/8	DELEqC-II	216.04	301.99	298.33	3.25e+01	379.85
13/16/8	DELEqC-III-trand*	3210.47	30646.47	38151.91	2.47e+04	143238.68
13/16/8	DELEqC-III-rand*	3210.47	43810.68	47425.86	3.15e+04	154392.78
14/10/2	DELEqC-II*	1.54	1.54	1.54	3.49e-04	1.54
14/10/2	DELEqC-III-trand*	1.80	9.79	13.21	9.63e+00	55.45
14/10/2	DELEqC-III-rand	0.10	0.77	0.72	2.03e-01	1.27
15/10/8	DELEqC-II*	26.27	26.27	26.27	3.55e-15	26.27
15/10/8	DELEqC-III-trand	0.00	0.00	0.00	0.00e+00	0.00
15/10/8	DELEqC-III-rand	0.00	0.00	0.00	0.00e+00	0.00
16/30/6	DELEqC-II	426.18	478.01	480.28	2.65e+01	562.97
16/30/6	DELEqC-III-trand*	6533.57	10797.39	11239.12	2.32e+03	16946.10
16/30/6	DELEqC-III-rand*	21332.77	35573.01	36533.21	7.34e+03	55192.38
17/30/24	DELEqC-II*	1451.30	1451.30	1451.30	8.02e-06	1451.30
17/30/24	DELEqC-III-trand*	2.09	11.01	14.48	1.11e+01	58.78
17/30/24	DELEqC-III-rand	0.00	0.01	0.02	2.73e-02	0.13
18/60/15	DELEqC-II	6933.36	7866.78	7917.74	5.41e+02	9681.18
18/60/15	DELEqC-III-trand*	114906.27	176702.84	175895.69	2.08e+04	223595.86
18/60/15	DELEqC-III-rand*	239047.96	527550.79	524232.91	8.45e+04	708527.08
19/60/45	DELEqC-II	19015.34	19018.06	19018.18	1.36e+00	19022.02
19/60/45	DELEqC-III-trand*	75293.84	177205.62	179278.52	5.12e+04	335935.88
19/60/45	DELEqC-III-rand*	211511.90	451569.04	471227.20	1.43e+05	907088.14
20/100/25	DELEqC-II	70100.07	73816.77	73876.36	1.61e+03	78493.09
20/100/25	DELEqC-III-trand*	186979.09	297208.54	293522.01	3.57e+04	374173.76
20/100/25	DELEqC-III-rand*	1294476.80	1644242.80	1669694.52	1.71e+05	2075921.20
28/1000/200	DELEqC-II	406863111.43	466560138.15	463747866.07	1.26e+07	469111545.73
28/1000/200	DELEqC-III-trand*	1602585656.08	1669397057.72	1667666627.82	2.82e+07	1737506060.41
28/1000/200	DELEqC-III-rand*	2275681550.51	2755201276.53	2747243522.68	1.60e+08	3118391639.24
29/1000/800	DELEqC-II*	1709348333.19	1709426361.05	1709433166.40	4.46e+04	1709564395.61
29/1000/800	DELEqC-III-trand	529365475.04	624368456.34	620885013.72	3.66e+07	708768591.73
29/1000/800	DELEqC-III-rand*	1288827526.15	2092913396.09	2063026235.91	2.82e+08	2620277191.84

Description of the test-problems

Three sets of test-problems were used in the experimental analyses. Their descriptions are presented in the following.

Set 1. The first five test-problems 1 to 5 were taken from [1]. Those problems are subject to the same set of linear constraints. In [2] a scheme for scaling those problems was proposed by varying the number of variables (n) and the number of constraints (m). The description of the problems and the scaling procedure are given in [2].

Set 2. Test-problem 6 has $n = 16$ variables and $m = 8$ linear equality constraints [3, 4]. The description and the optimal solution of the problem are as follows

Objective function:

$$f(x) = \sum_{i=1}^{16} \sum_{j=1}^{16} a_{ij}(x_i^2 + x_i + 1)(x_j^2 + x_j + 1)$$

where a_{ij} is presented in Table 12.

Optimal solution:

$$x^* = [.03984735, .7919832, .2028703, .8483579, 1.126991, .9347987, \\ 1.681962, .1553009, 1.567870, 0, 0, 0, .6602041, 0, .6742559, 0]$$

Constraints:

$$\sum_{j=1}^{16} b_{ij}x_j - c_j = 0, \quad i = 1, \dots, 8 \\ 0 \leq x_i \leq 5, \quad i = 1, \dots, 16$$

where b_{ij} and c_j are given in Table 13.

Set 3. Test-problem 7 is composed of highly non-convex problems as described in [5]. For those problems, the objective function and the set of linear equality constraints are given by

Objective function:

$$f(x) = rn \sum_{i=1}^n (x_i - x_i^*)^2 + (\sin(g_1 P_1(x)))^2 + (\sin(g_2 P_2(x)))^2,$$

where

$$P_1(x) = \sum_{i=1}^n (x_i - x_i^*)^2 + \sum_{i=1}^n (x_i - x_i^*)^2, \\ P_2(x) = \sum_{i=1}^n (x_i - x_i^*),$$

Table 12: Matrix A for test-problem 6

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
a_{1j}	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	1
a_{2j}	0	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
a_{3j}	0	0	1	0	0	0	1	0	1	1	0	0	0	1	0	0
a_{4j}	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0
a_{5j}	0	0	0	0	1	1	0	0	0	1	0	1	0	0	0	1
a_{6j}	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0
a_{7j}	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0
a_{8j}	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0
a_{9j}	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1
a_{10j}	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
a_{11j}	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
a_{12j}	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
a_{13j}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
a_{14j}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
a_{15j}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
a_{16j}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 13: Matrix B and vector c for test-problem 6

j	b_{1j}	b_{2j}	b_{3j}	b_{4j}	b_{5j}	b_{6j}	b_{7j}	b_{8j}	c_j
1	0.22	-1.46	1.29	-1.10	0	0	1.12	0	2.5
2	0.20	0	-0.89	-1.06	0	-1.72	0	0.45	1.1
3	0.19	-1.3	0	0.95	0	-0.33	0	0.26	-3.1
4	0.25	1.82	0	-0.54	-1.43	0	0.31	-1.10	-3.5
5	0.15	-1.15	-1.16	0	1.51	1.62	0	0.58	1.3
6	0.11	0	-0.96	-1.78	0.59	1.24	0	0	2.1
7	0.12	0.8	0	-0.41	-0.33	0.21	1.12	-1.03	2.3
8	0.13	0	-0.49	0	-0.43	-0.26	0	0.10	-1.5
9	1	0	0	0	0	0	-0.36	0	
10	0	1	0	0	0	0	0	0	
11	0	0	1	0	0	0	0	0	
12	0	0	0	1	0	0	0	0	
13	0	0	0	0	1	0	0	0	
14	0	0	0	0	0	1	0	0	
15	0	0	0	0	0	0	1	0	
16	0	0	0	0	0	0	0	1	

$r = 0.025$, $g_1 = 1$ and $g_2 = 1$. These problems have simple bound constraints on all variables ($x \in [-100, 100]^n$).

Optimal solution: The optimal solution is generated in Algorithm 1, lines 2-3. The optimal values are integer numbers starting in $-n/2$ and going up to $n/2$, with an increment of 1, except for zero. For instance, for a problem with $n = 10$, the optimal solution is $x^* = [-5, -4, -3, -2, -1, 1, 2, 3, 4, 5]$.

Constraints: The linear equality constraints are randomly generated using Algorithm 1.

Algorithm 1: Generate linear equality constraints

input : n (number of variables) and m (number of linear equality constraints)

- 1 i) Generate the solution x^* by using the following procedure:
- 2 **for** $i \leftarrow 1$ **to** n **do**
- 3 **if** $(i - (n/2) < 0)$ **then**
- 4 $x^*[i] = (i - (n/2))$;
- 5 **else**
- 6 $x^*[i] = (i - (n/2) + 1)$;
- 7 ii) Randomly generate the elements of the matrix E from the uniform distribution in $[-10, 10]$.
Denote the rows of E by $e_k, k = 1, \dots, m$;
- 8 iii) Let $c_k = e_k x^* + y$, where y is a random number drawn from a uniform distribution in $[1, 10]$
and $k = 1, \dots, m$;

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