Computer vision

Image Filtration

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Filtration

Linear methods

in spatial domain – using convolution

-> direct manipulation of pixels in an image

in frequency domain

-> using Fourier transform (or Cosine tranform)

Non linear methods

Image non-linear filtration

Non linear methods
2-D order-statistic filtering

Order filter

Special cases: minimum filter, maximum filter, median filter

Steps:

1. Define the neighbourhood around the pixel
2. Sort values of this neighbourhood pixels in the numerical order
3. Replaces each element in the output image by the order-th element in the sorted set of neighbours.

Non linear methods
2-D median filter

The median filter is a non-linear digital filtering technique, often used to remove noise from images or other signals.

(special case of the order-statistic filtering)

Steps:

1. Define the neighbourhood around a pixel
2. Sort values of this neighbourhood in the numerical order
3. Pick the median of the sorted set as the new pixel value

Median filter - Example

Median filter 5x5

…reduce noise and preserve (partially) edges

Image linear filtration

Smooth filters

Convolution

Linear operation

The discrete convolution of two functions f(x, y) and h(x, y) of size M X N is denoted by

f(x,y) * h(x,y) and is defined by the expression

$$f(x,y) * h(x,y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) h(x-m, y-n).$$

Example of 2D Convolution

6	5	13 (0,3)	25 (0,5)	24 (0,3)	9
5	6	19 (0,5)	26 (1)	20 (0,5)	5
5	9	24 (0,3)	25 (0,5)	13 (0,3)	4
5	15	25	23	7	4
9	24	26	17	5	5

12,8	24,7	47,1	68	60,3	33,2
17,7	39,4	72,7	92,7		

12,8	24,7	47,1	68,2	60,3	33,2
17,7	39,4	72,7	92,7	72,9	37,3
20,8	50,2	84	89,3	58,4	27,8
29,4	65,7	91,5	80,4	44,8	22,6
46,6	87,8	97,3	69	35,2	22,5

Border Effects

- zero: set all pixels outside the source image to 0
- constant: set all pixels outside the source image to a specified border value
- mirror: reflect pixels across the image edge
- reduced size: size of output image differ from size of input image

Mean (blur) filter
Smooth filter

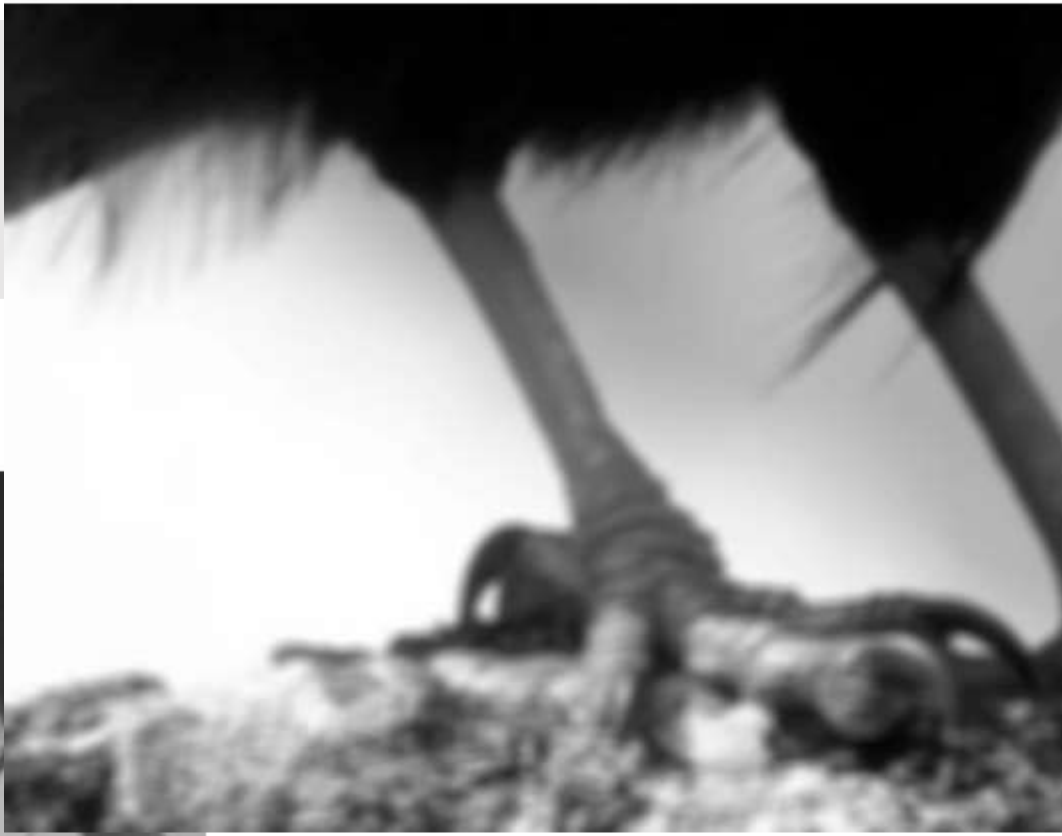
Convolution kernel 3x3:

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Convolution kernel 5x5:

$$\frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

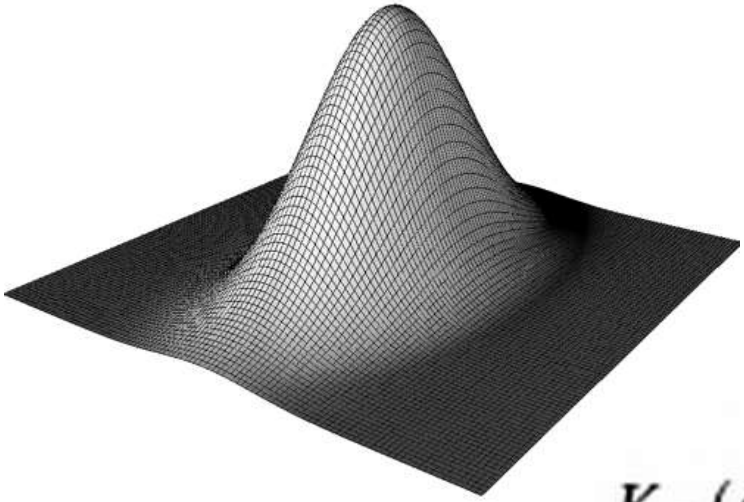
Mean filter



Gauss filter
Smooth filter

Gauss convolution kernel 5x5 (integer): $\frac{1}{273}$

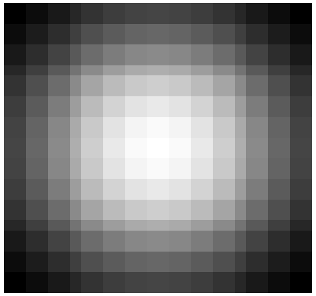
1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1



$$K_\sigma(x,y) := \frac{1}{2\pi\sigma^2} \cdot exp\left(\frac{-x^2 - y^2}{2\sigma^2}\right)$$

Example of 2D Convolution
Gaussian filter kernel

Gaussian filter kernel size:15x15



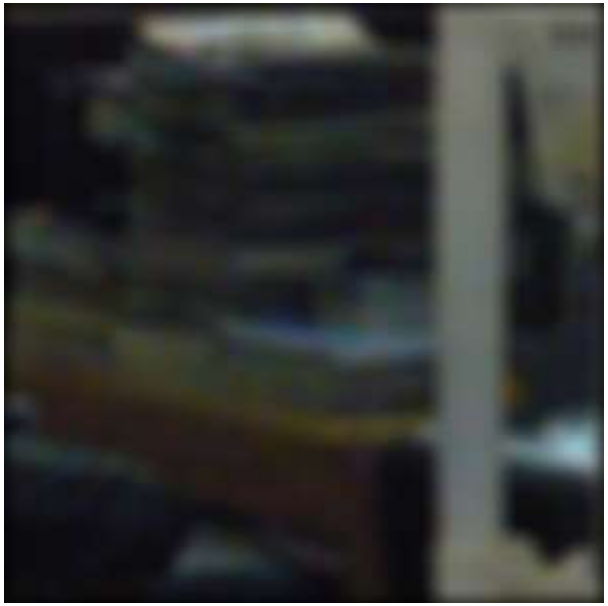


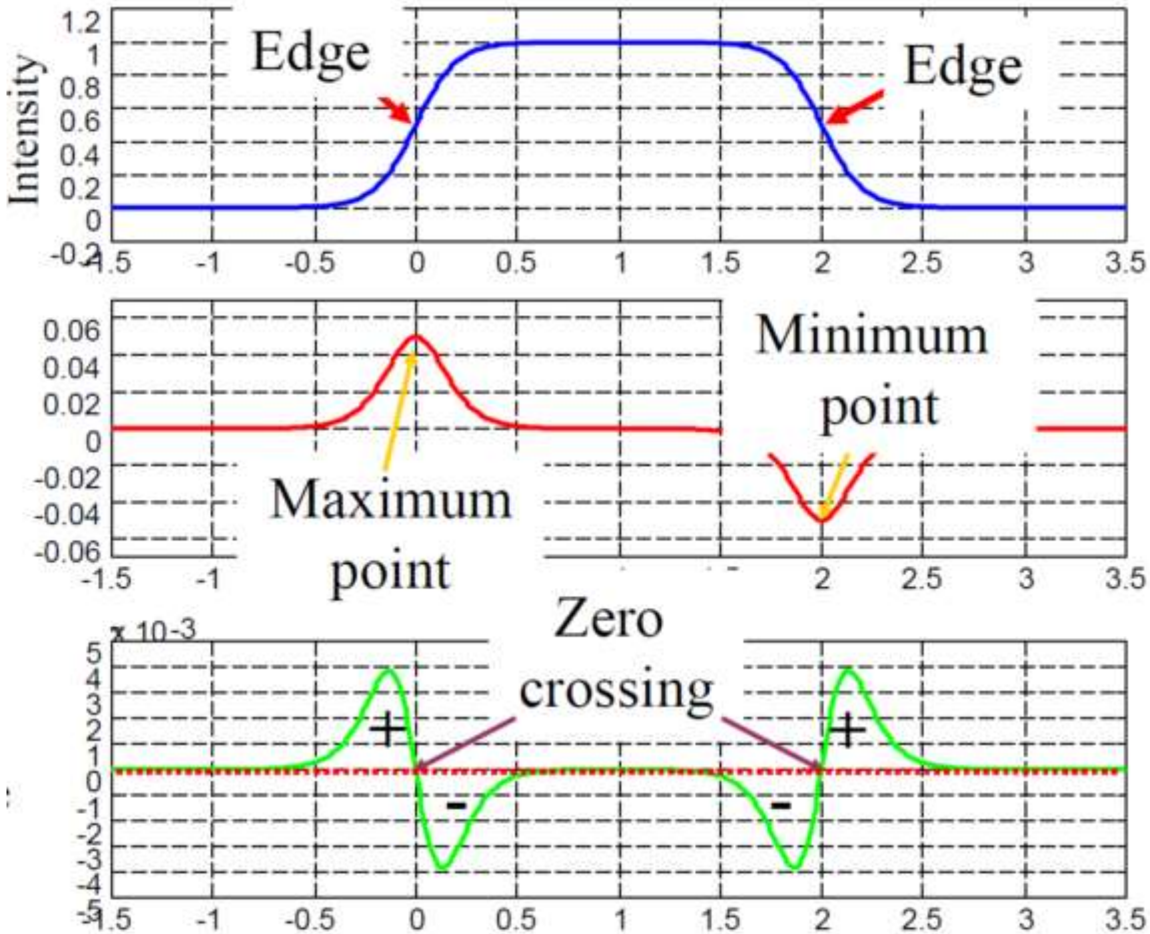
Image linear filtration-
Edge filters

Edge detection

Gray level profile line

The 1-st derivative

The 2-nd derivative



Prewitt filter

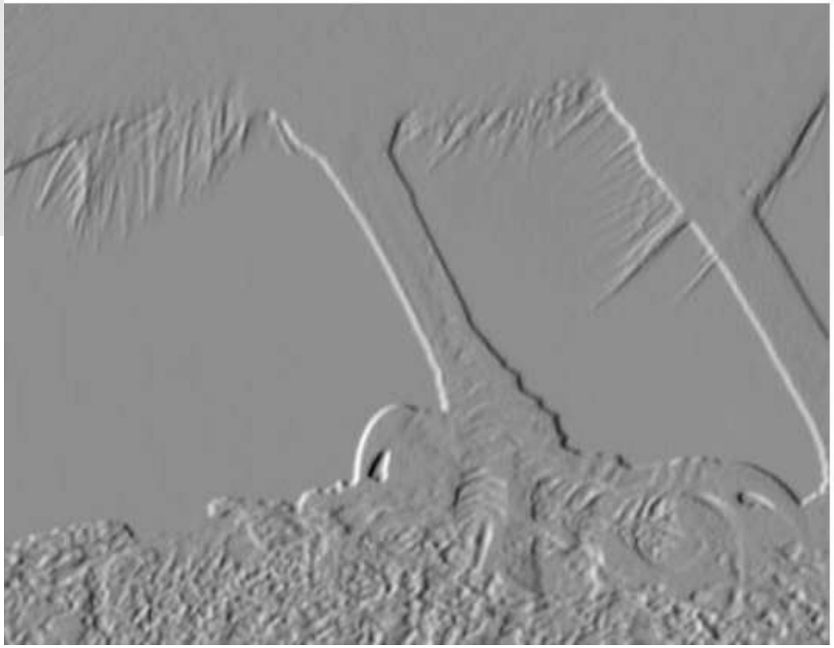
Gradient edge filter (1-st deriv.)

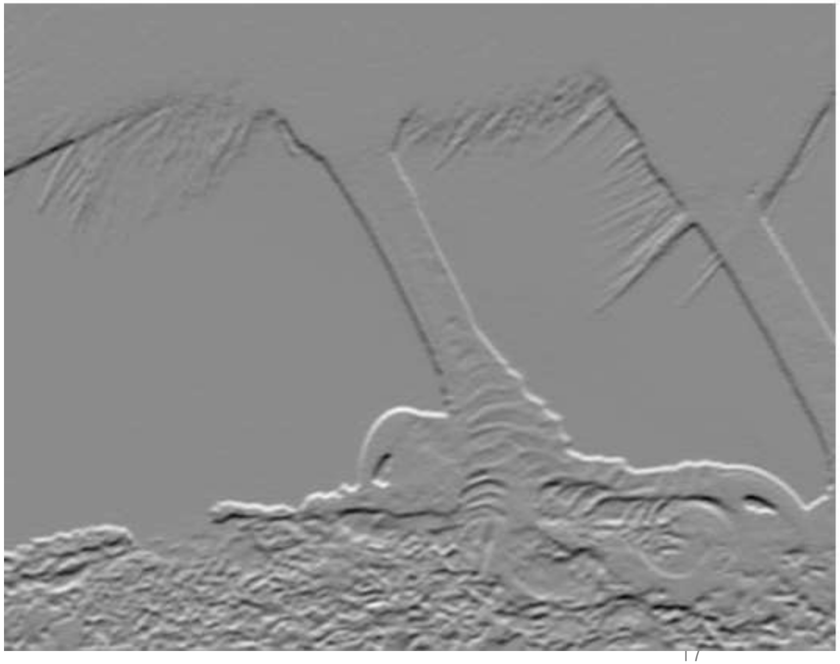
Vertical convolution kernel:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Horizontal convolution kernel:

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$





Sobel filter

Gradient edge filter
(1-st deriv.)

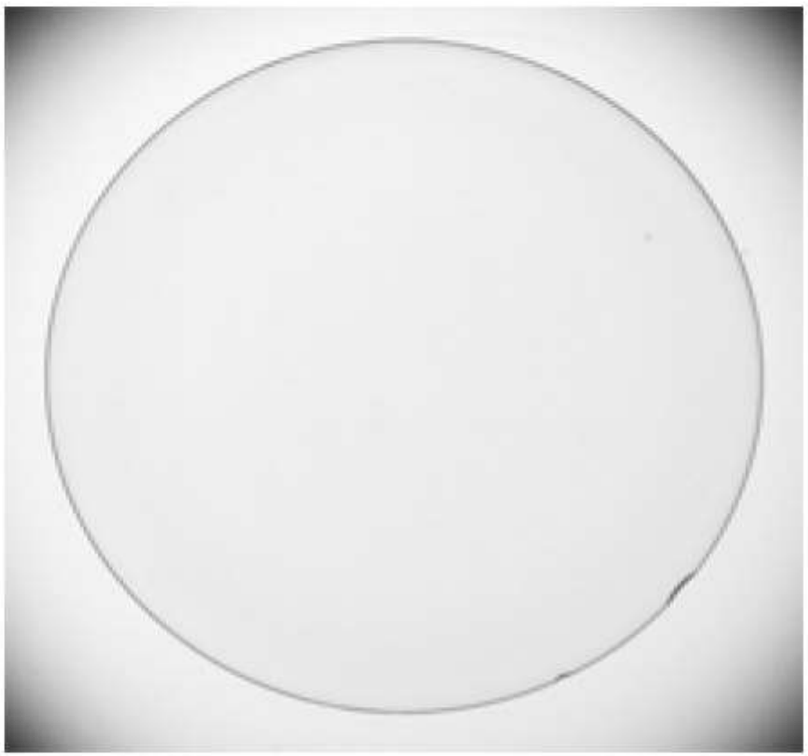
Sobel convolution kernels:

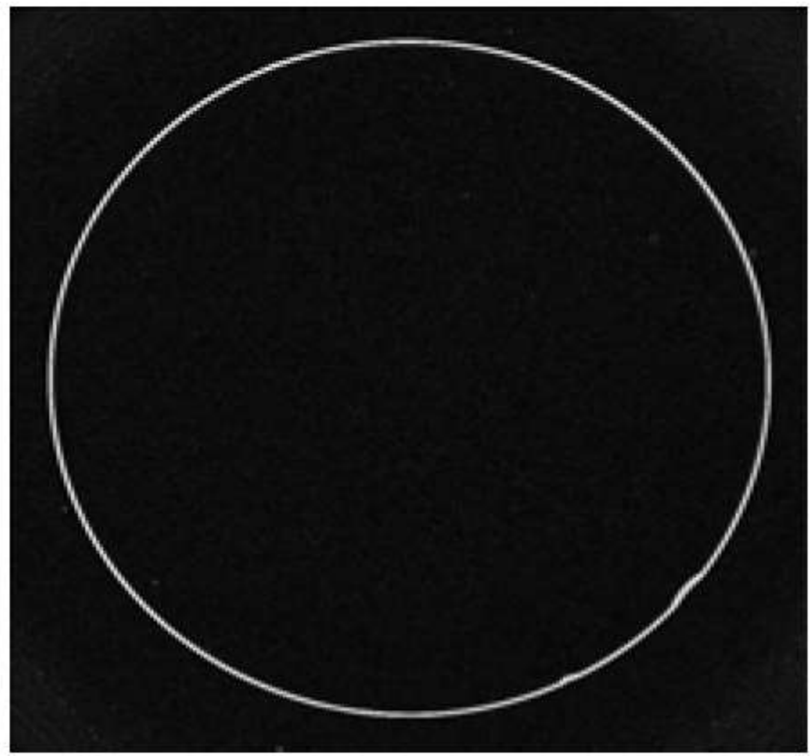
$$\mathbf{G}_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} * \mathbf{A} \quad \text{and} \quad \mathbf{G}_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} * \mathbf{A}$$

Use of First Derivatives for Enhancement - The Sobel Gradient

(a)Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).

(b) Sobel gradient.





Roberts filter

Gradient edge filter (1-st deriv.)

Roberts convolution kernels:

$$h_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad h_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

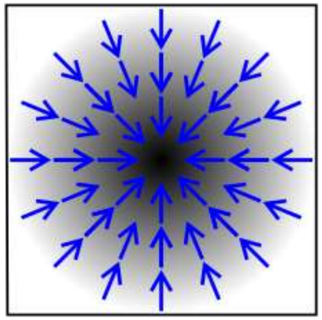
Image gradient

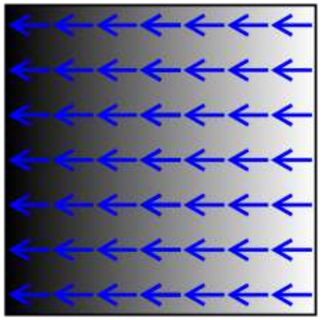
- gradient vector of the image intensity
- vector at each point in the image

The gradient of an image is given by the formula:

$$\nabla I = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

image gradient (example):





edge filtered image (example):

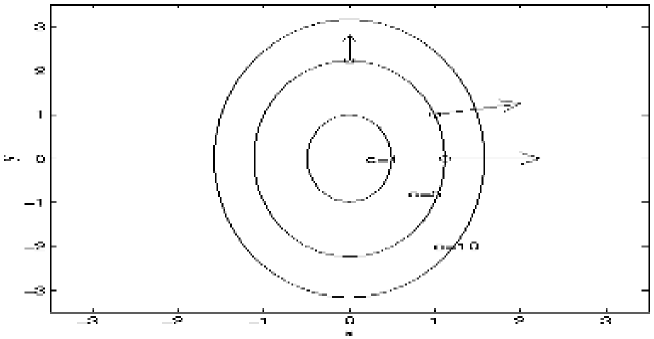
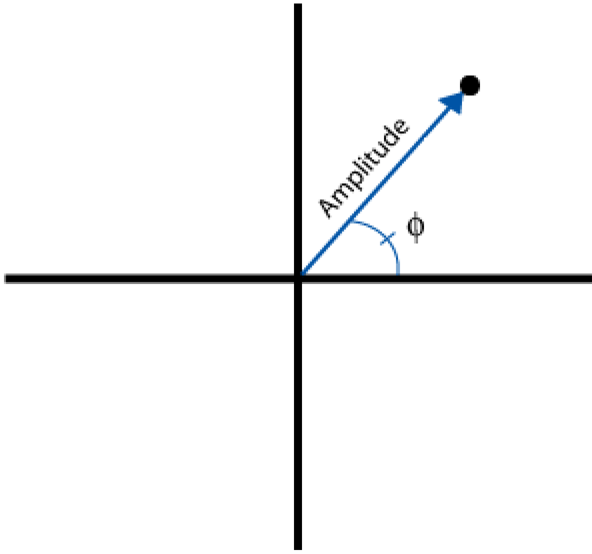


Image gradient

Calculates the gradient vector of the image intensity at each point using Prewitt (Sobel) filer



Magnitude (amplitude)

$$\mathbf{G} = \sqrt{\mathbf{G}_x^{\,2} + \mathbf{G}_y^{\,2}},$$

angle

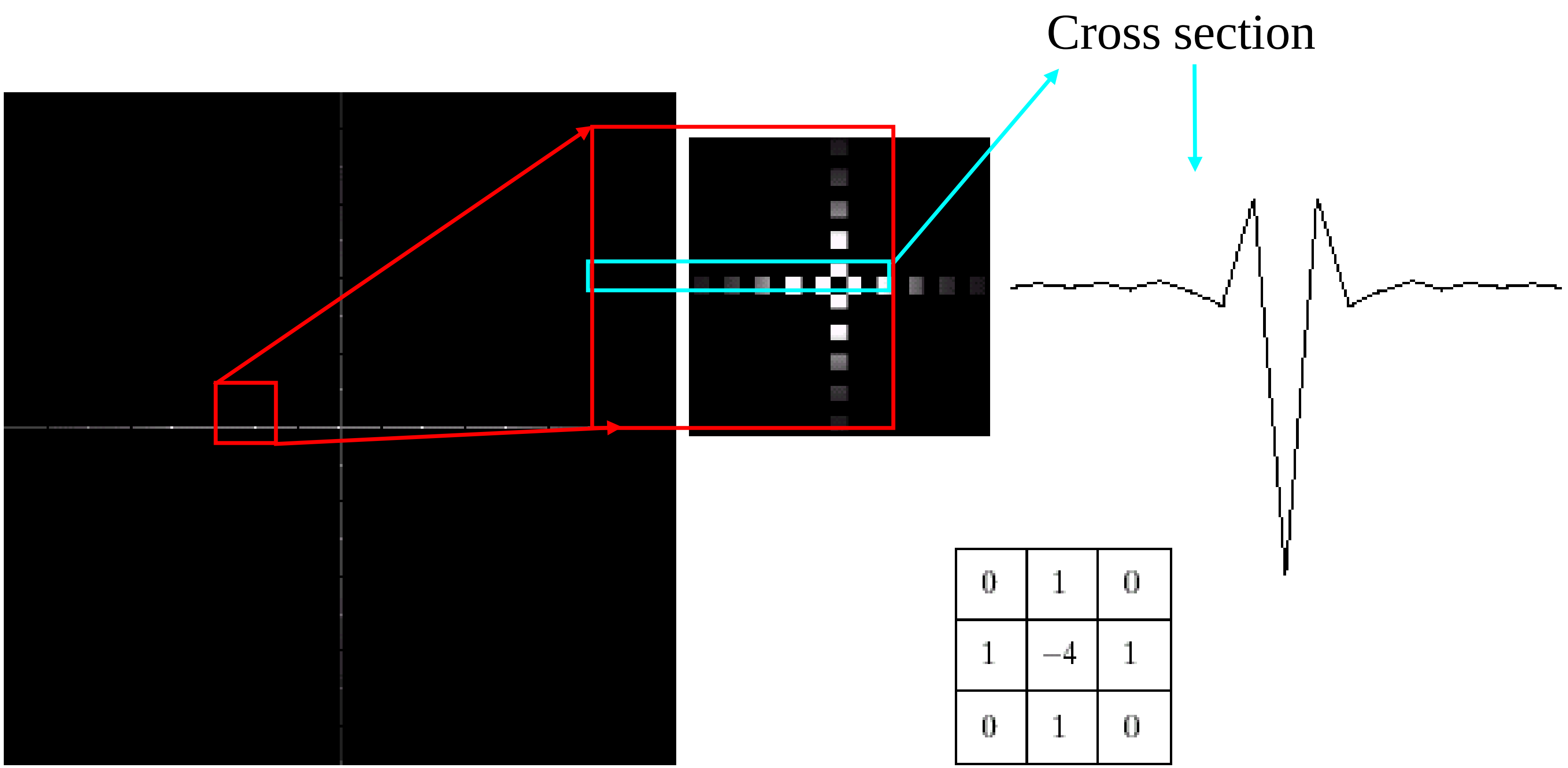
$$\Theta = \arctan\left(\mathbf{G}_y, \mathbf{G}_x\right)$$

Laplacian filter
Edge filter (2-nd derivation)

Laplacian for a function (image) f(x, y) of two variables is defined as:

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Laplacian Filter



0	1	0
1	−4	1
0	1	0

Laplacian mask in Chapter 3

Laplacian convolutional kernel (filter mask)

Mask includes/or not includes also the diagonal neighbors.

convolutional kernels 3x3:

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

convolutional kernel 5x5:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 1 & 2 & -16 & 2 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Image Enhancement

Sharpening

Image restoration

Noise reduction

Image Enhancement

The principal objective of enhancement is to process an image so that the result is more suitable than the original image for a specific application.

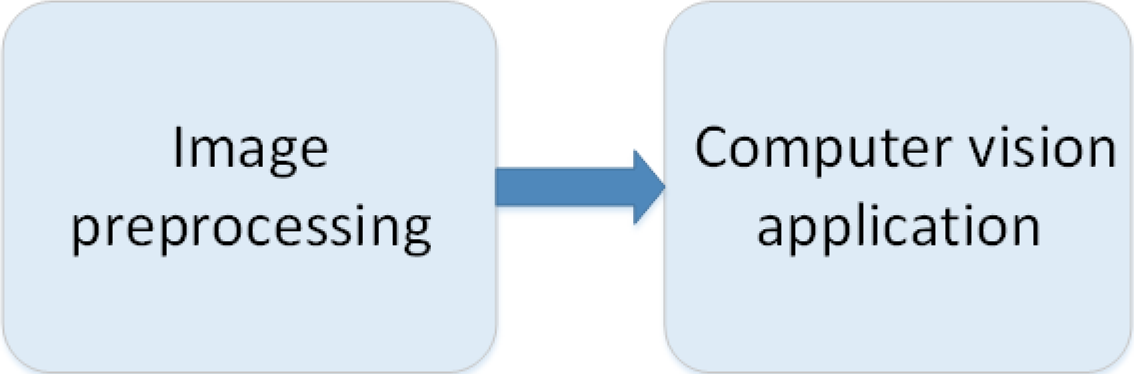


Image Enhancement

Sharpening

Sharpening - example

Sharpening using Spatial Filters

the principal objective of sharpening is

- to highlight fine details (edges) in an image
- or to enhance fine details that have been blurred, either disturbed
- or as a natural effect of a particular method of image acquisition.

Sharpening approaches

- Using blur of the original image:

Subtract the blurred image from the original, creating a mask.
Add the mask to the original image

- Using Laplacian image:

Add/subtract the Laplacian from the original image

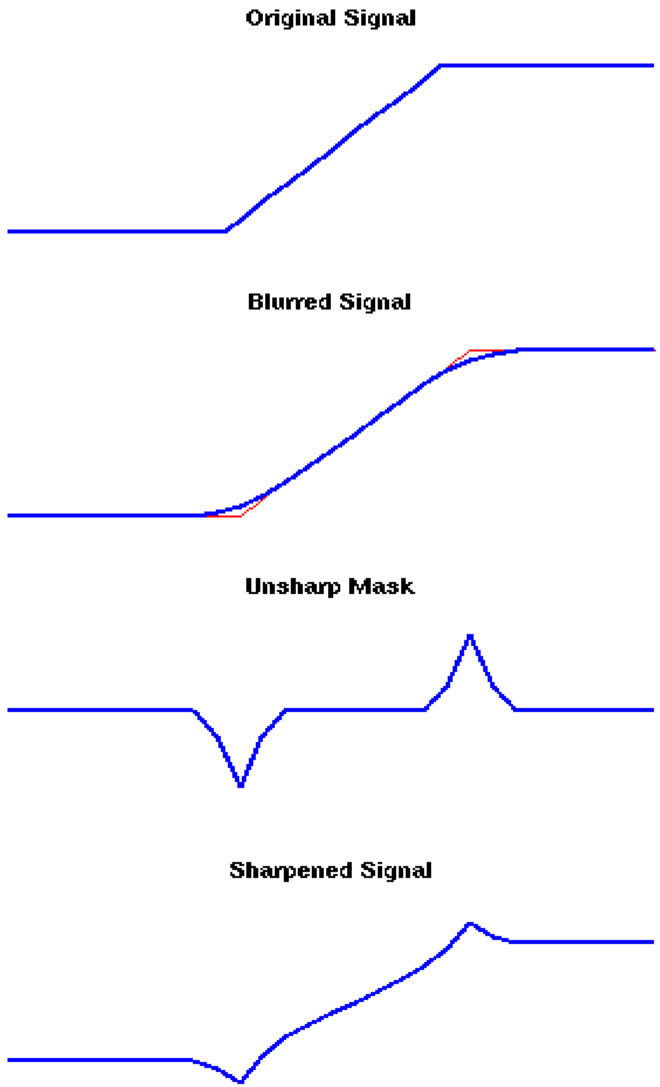
Sharpening 1D example

Original signal So:

Blurred signal Sb

Dif = So-Sb

So + Dif



Sharpening by Laplacian
Add/Subtract the original and Laplacian images

If the definition of Laplacian filter kernel used has a negative center coefficient, then we subtract the image

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

If the definition of Laplacian filter kernel used has a positive center coefficient, then we add the image

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases}$$

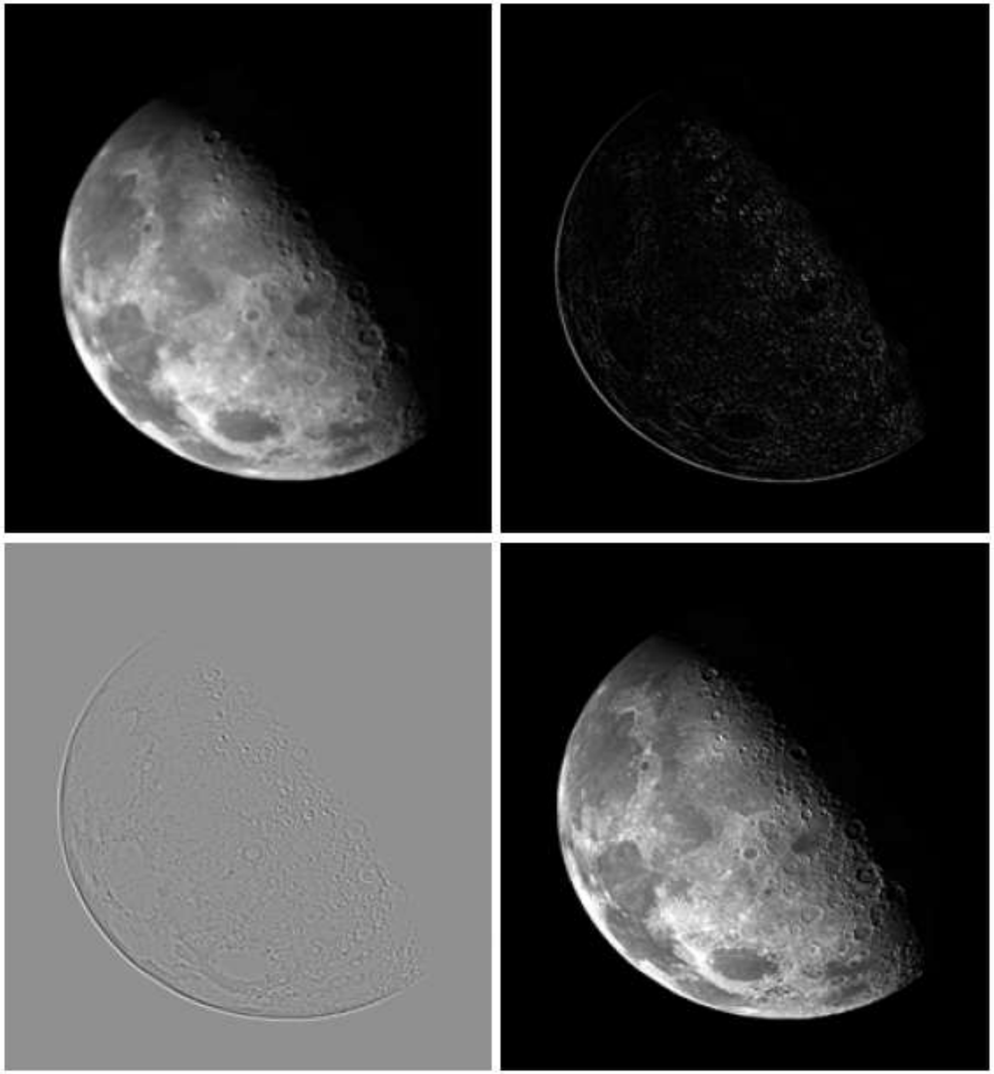
Example of sharpening using Laplacian

(a) Image of the North Pole of the moon.

(b) Laplacian filtered image.

(c) Laplacian image scaled for display purposes.

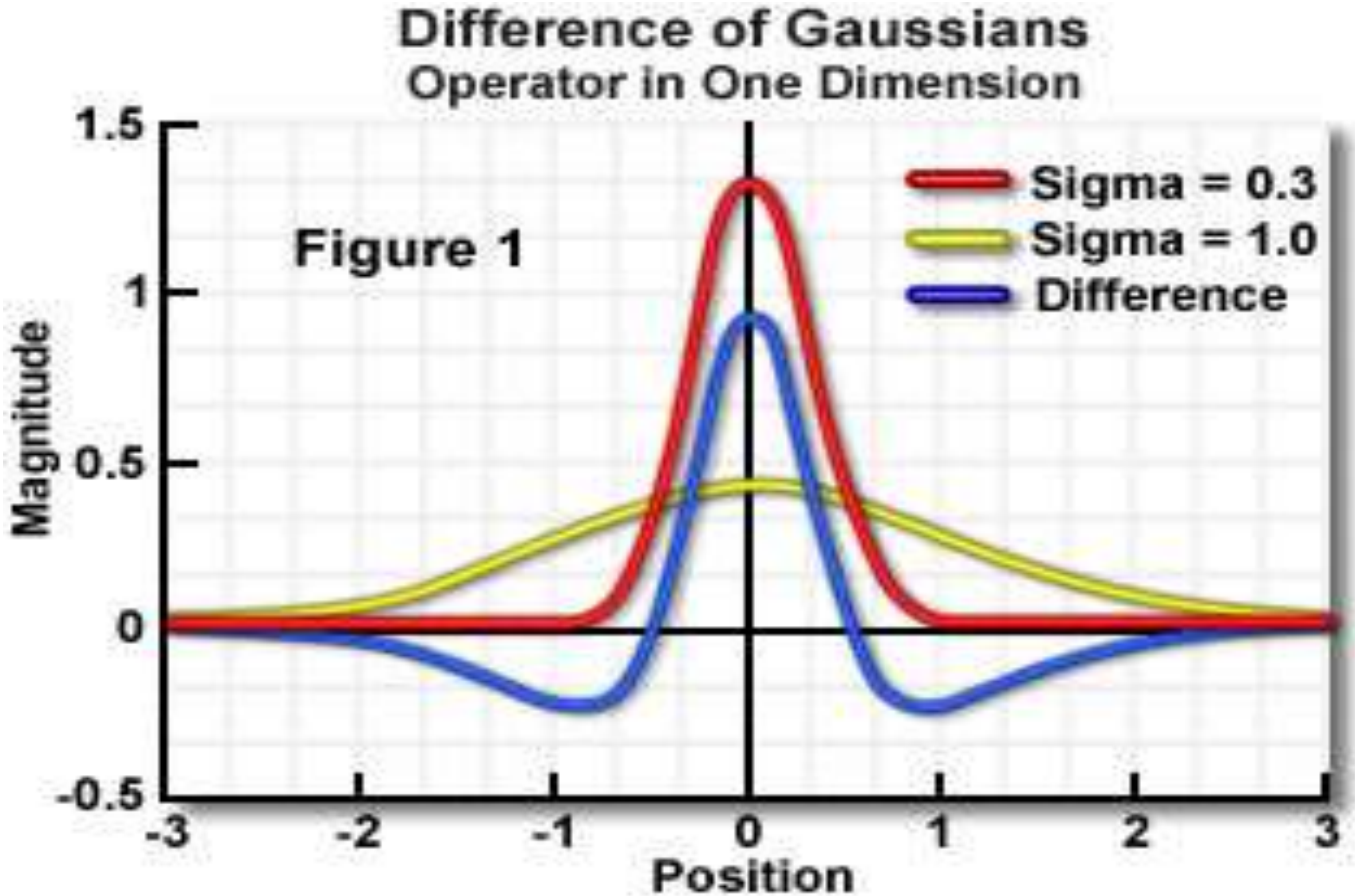
(d) Image enhanced by using sharpening



Edge detection

...more methods

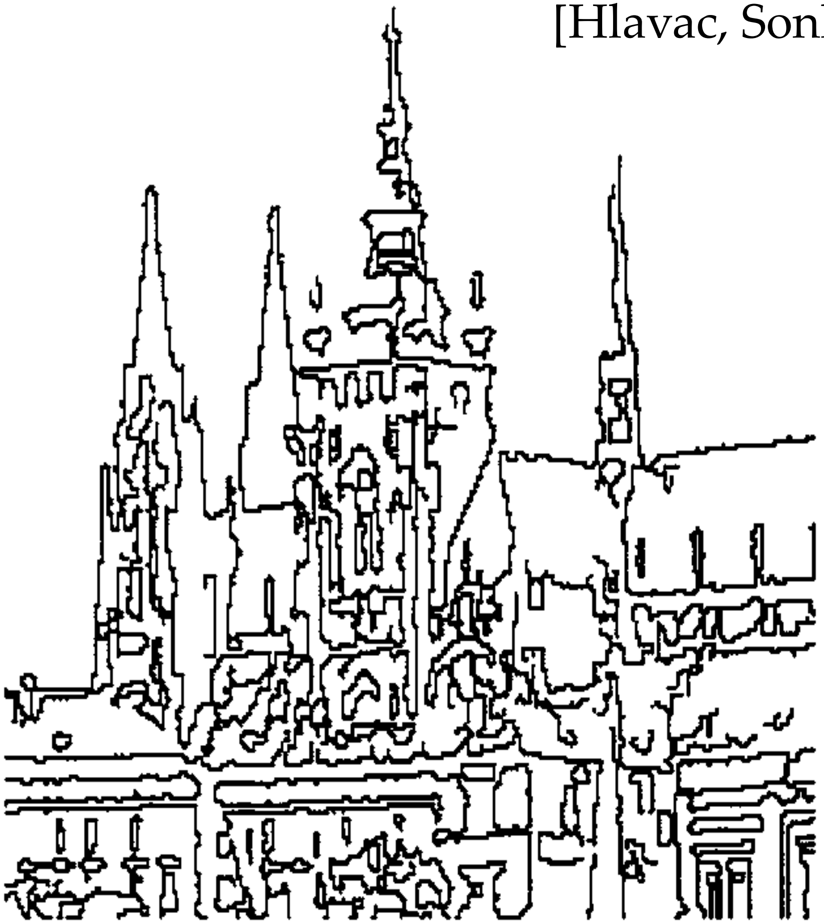
Difference of Gaussians (DoG) Edge detection

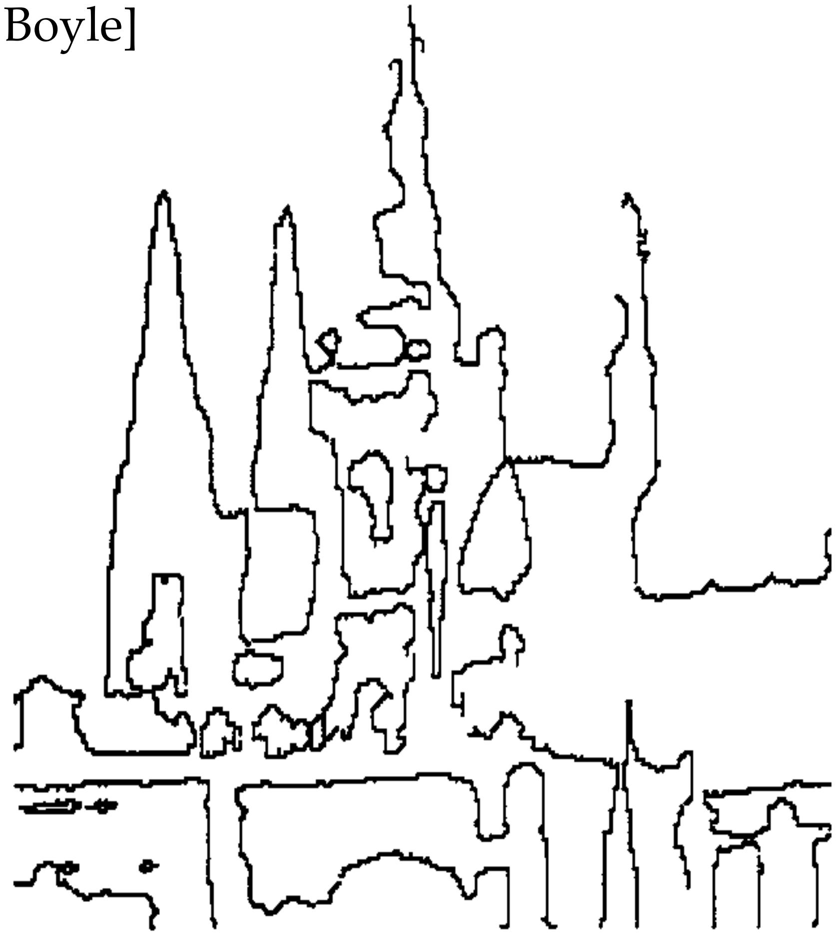


Canny edge detection -> binary contours

Example - Canny edge detection at two different scales

[Hlavac, Sonka, Boyle]





Canny edge detection
Algorithm

1. Noise Reduction – Gaussian blur

Convolve an image f with a Gaussian (std σ)

2. Finding Intensity Gradient of the Image

Estimate local edge normal directions for each pixel in the image.

Smoothened image is then filtered with a Sobel kernel in both horizontal and vertical direction to get first derivative in horizontal direction () and vertical direction (). From these two images, we can find edge gradient and direction for each pixel.

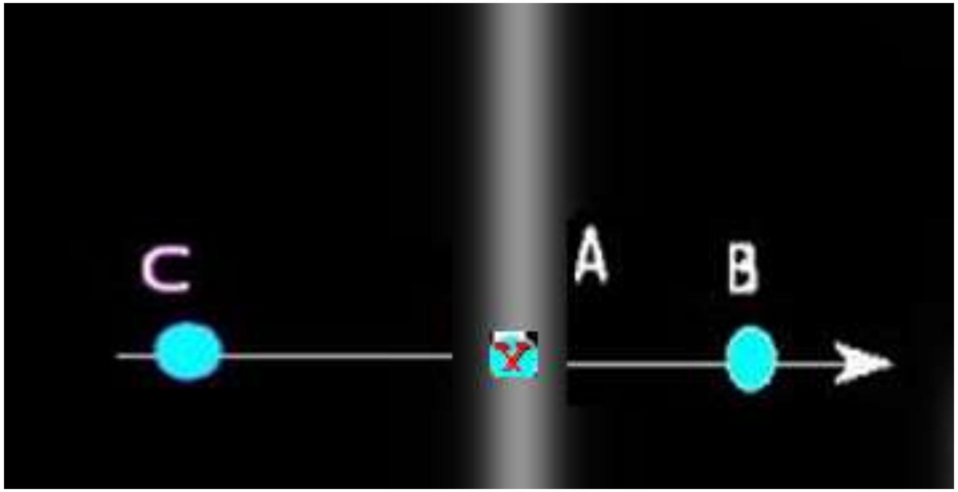
See „image gradient"

Canny edge detection Algorithm – cont.

3. Non-maximum Suppression

Eliminate points that do not lie in important edges - find the location of the edges using non-maximal suppression.

Point A is on the edge (in vertical direction). Gradient direction is normal to the edge. Point B and C are in gradient directions. So point A is checked with point B and C to see if it forms a local maximum. If so, it is considered for next stage, otherwise, it is suppressed (put to zero).



-> Compute the magnitude of the edge

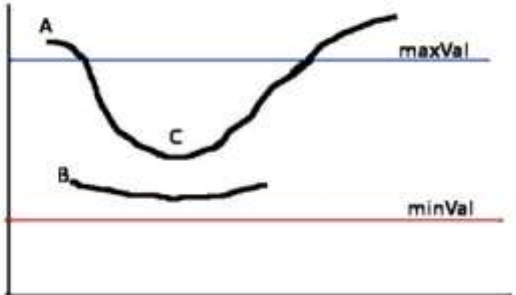
Canny edge detection
Algorithm – cont.

4. Hysteresis Thresholding

Threshold edges in the image with hysteresis to eliminate spurious responses.

Parameters: two threshold values, minVal and maxVal.

Any edges with intensity gradient more than maxVal are sure to be edges and those below minVal are sure to be non-edges, so discarded. Those who lie between these two thresholds are classified edges or non-edges based on their connectivity. If they are connected to "sure-edge" pixels, they are considered to be part of edges. Otherwise, they are also discarded.



5. Repeat steps (1) through (4) for ascending values of the standard deviation σ

6. Aggregate the final information about edges at multiple scale using the 'feature synthesis' approach.

Reduction of Noise

Noise

Uncorrelated noise is defined as the random graylevel variations within an image that have no spatial dependences from image to image.

Classification of noise is based upon:

- the shape of probability density function (analog case of noise)
- the shape of histogram function (discrete case of noise)

Typical image noise models

Typical image noise models are:

- Uniform
- Gaussian (normal)
- Salt-and-Pepper (impulse)
- Gamma noise
- Rayleigh distribution
- ...and more

Uniform noise

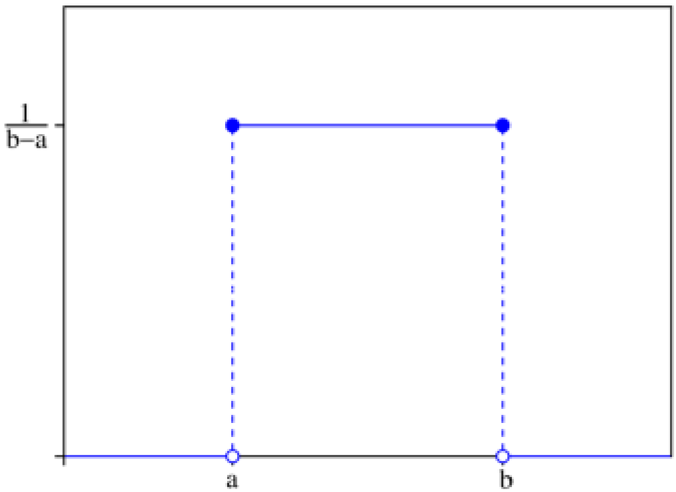
Uniform noise can be analytically described by:

$$HISTOGRAM_{Uniform} = \begin{cases} \dfrac{1}{b-a} & for\ a \le g \le b \\ 0 & elsewhere \end{cases}$$

$$mean = \frac{a+b}{2}$$

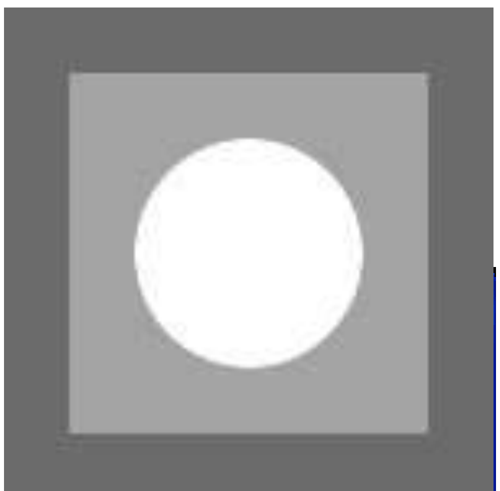
$$variance = \frac{(b-a)^2}{12}$$

Histogram of Uniform noise:



Example of Uniform Noise

Original image
and histogram:



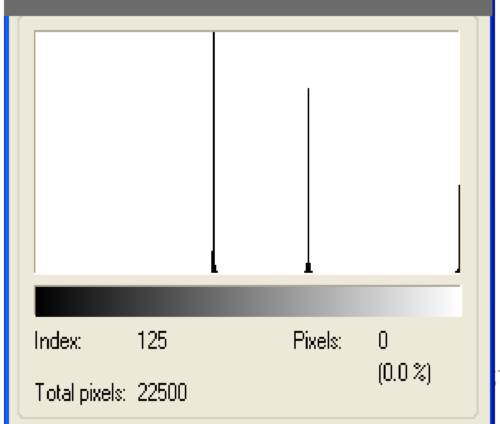
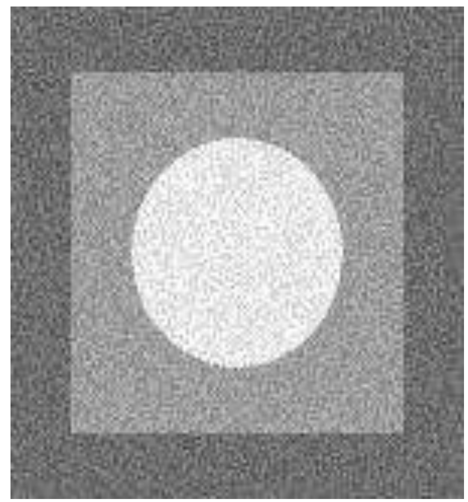
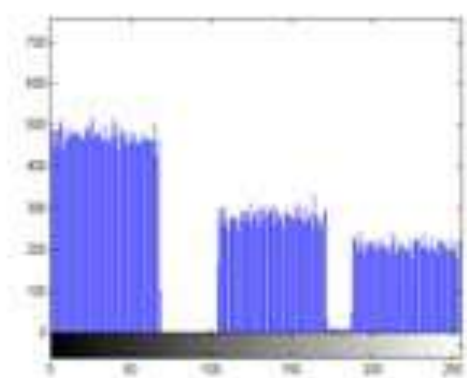


Image disturbed by uniform noise
and histogram:





Uniform noise

Quantization noise has an approximately uniform distribution

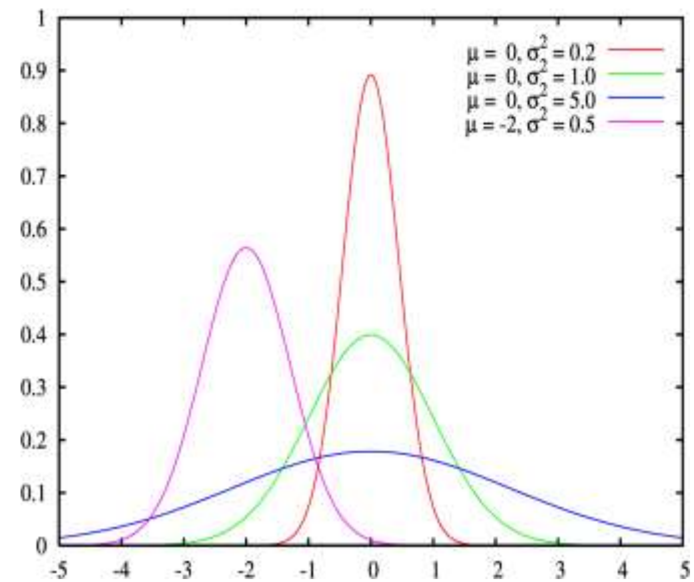
Uniform noise can be used to generate any other type of noise distribution, and is often used to degrade images for the evaluation of image restoration algorithms since it provides the most unbiased or neutral noise model

Gaussian noise (Amplifier noise)

… is statistical noise that has a probability density function (pdf) of the normal distribution (also known as Gaussian distribution).

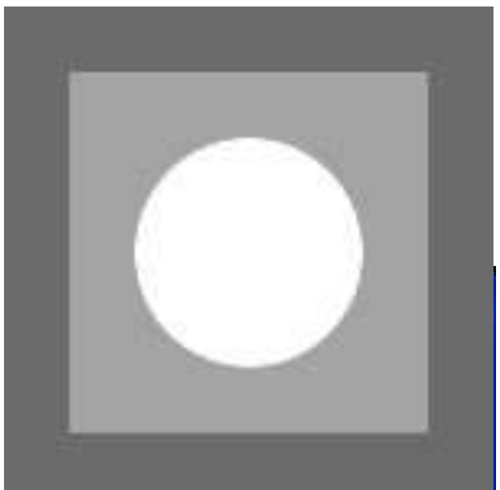
…is a major part of the "read noise" of an image sensor, that is, of the constant noise level in dark areas of the image.

PDF (Propability density function)



Example of Gaussian Noise

Original image
and histogram:



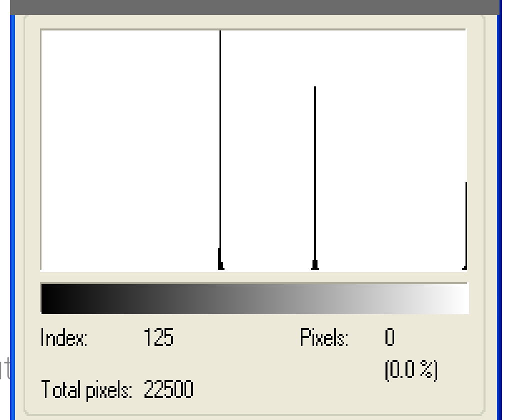
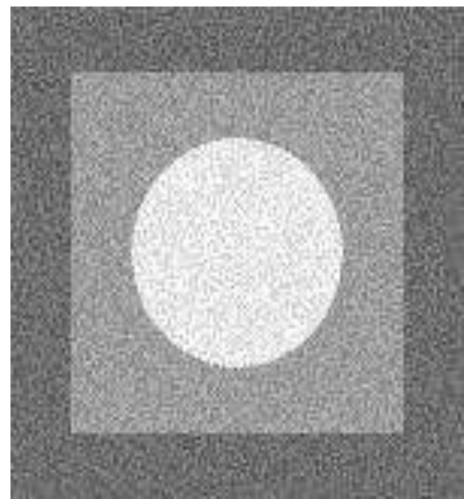
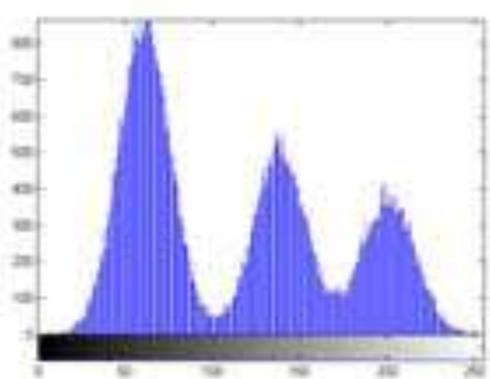


Image disturbed by Gaussian noise
and histogram:





Salt and Pepper noise

Salt and Pepper noise can be analytically described by:

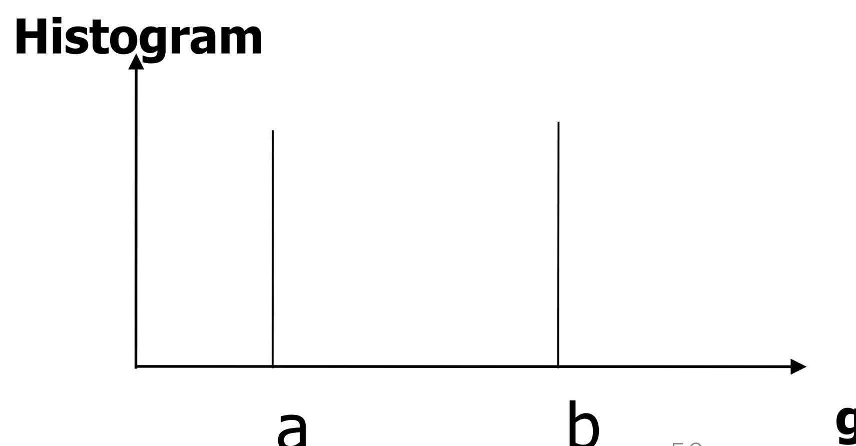
$$HISTOGRAM_{Salt\,\&\,Pepper} = \begin{cases} A & \text{for } g = a \text{ ("pepper")} \\ B & \text{for } g = b \text{ ("salt")} \end{cases}$$

There are only two possible values, a and b.

For an 8-bit image,

the typical value for pepper-noise

is 0, and 255 for salt-noise



Salt and Pepper noise

The salt-and-pepper type noise (also called impulse noise, shot noise or spike noise) is typically caused by

malfunctioning pixel elements in the camera sensors,

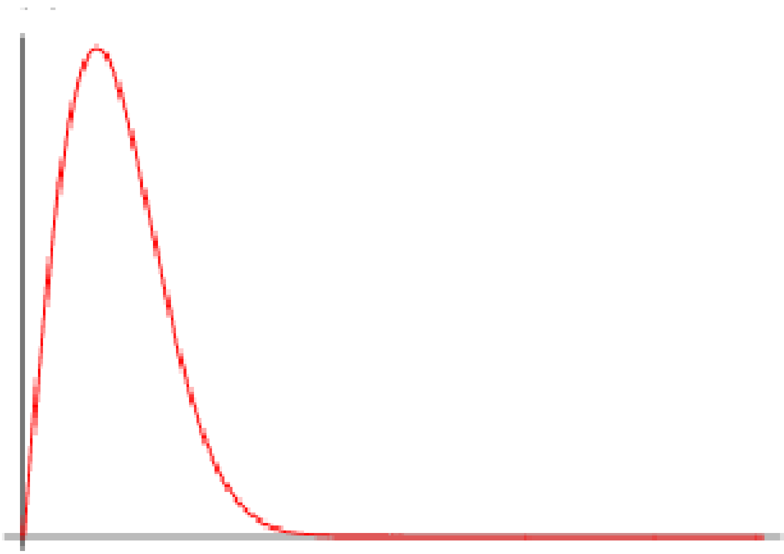
faulty memory locations,

or timing errors in the digitization process

Rayleigh noise

PDF (Propability density function)

Rayleigh distribution is defined as:



$$HISTOGRAM_{Rayleigh} = \frac{2g}{\alpha} e^{-g^2/\alpha}$$

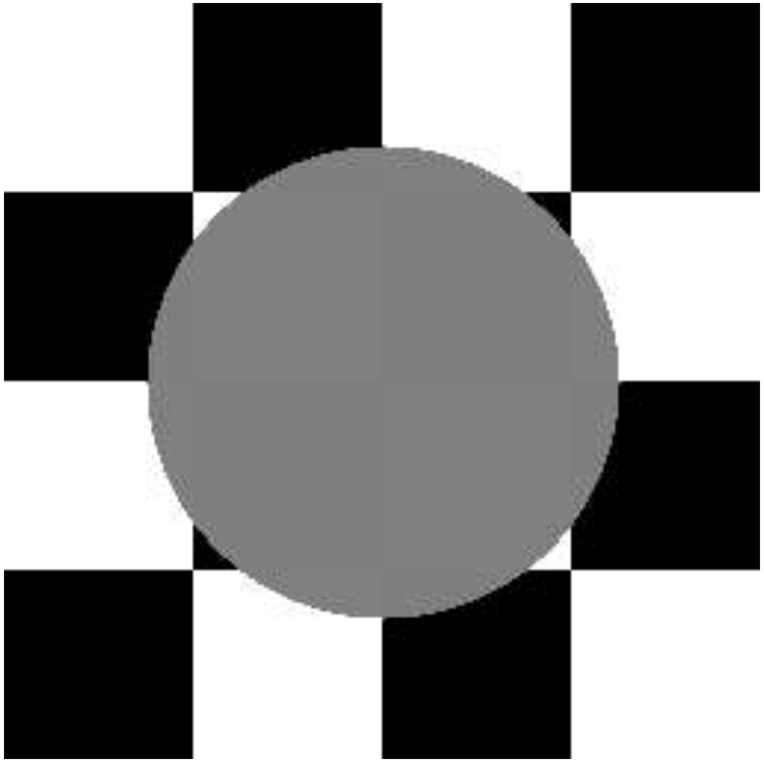
$$where: \quad mean = \sqrt{\frac{\pi\alpha}{4}}$$

$$variance = \frac{\alpha(4-\pi)}{4}$$

Radar range and velocity images typically contain noise that can be modeled by the Rayleigh distribution

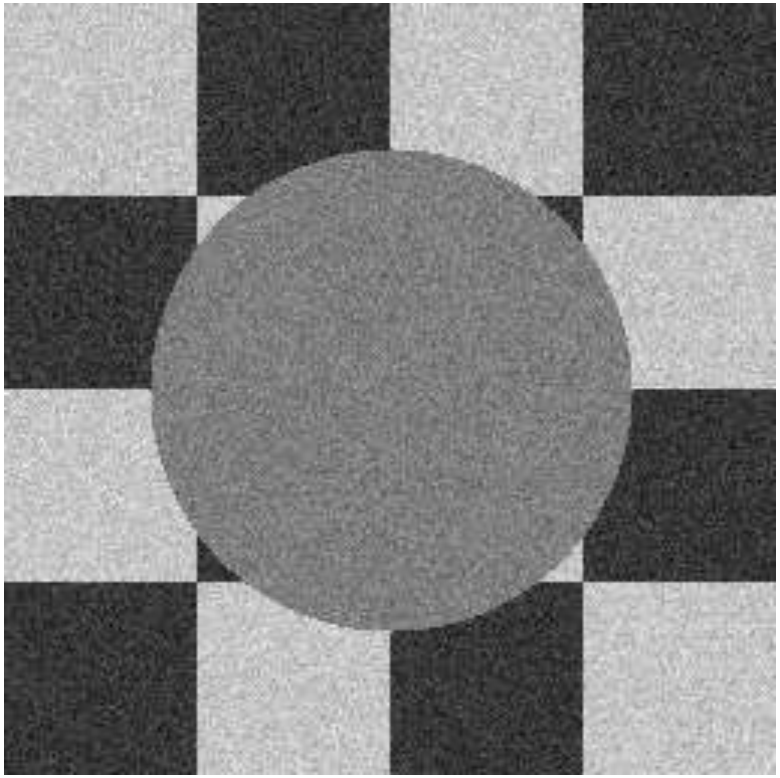
Example of Gaussian Noise

Original image



Original image without noise

Image with added Gaussian noise



Examples of Uniform Noise and Salt-and-pepper Noise

Image with added uniform noise

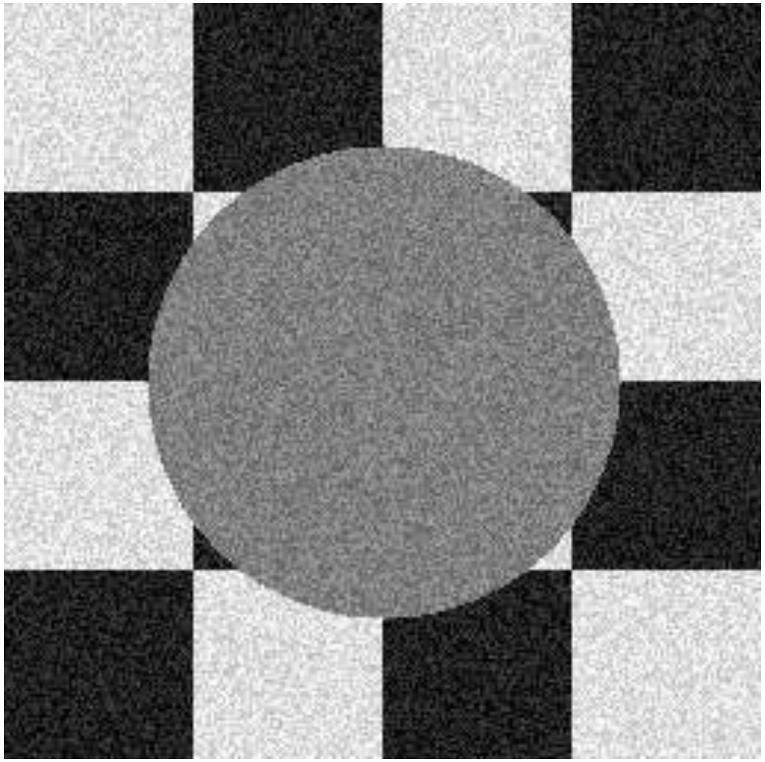
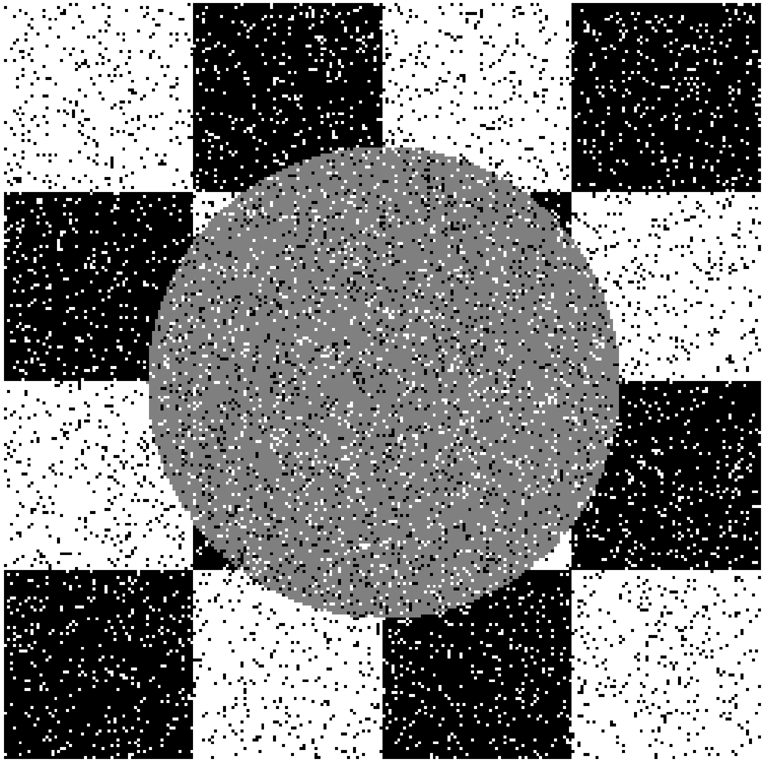
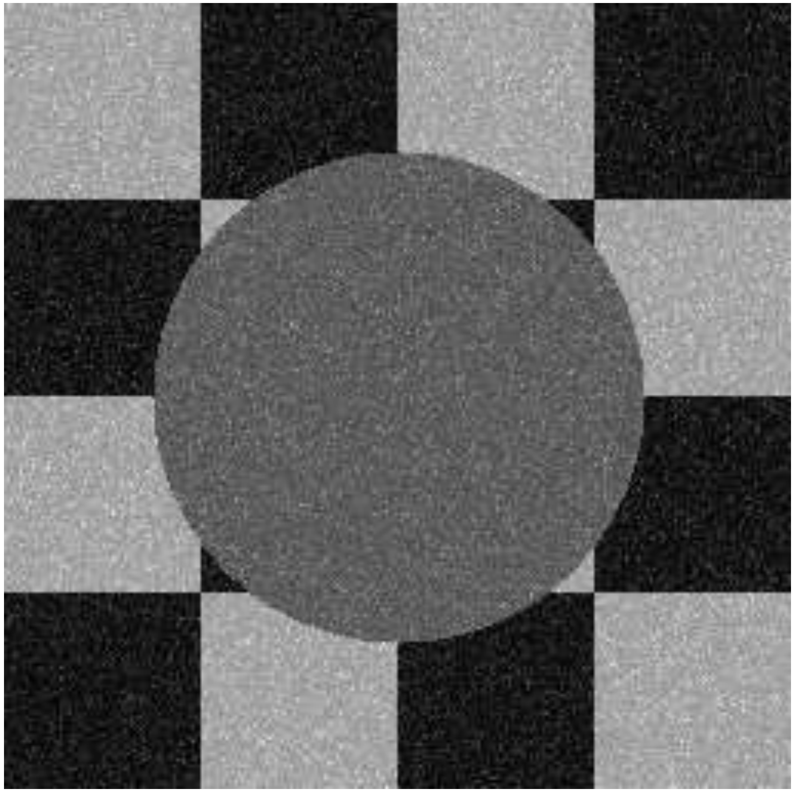


Image with added salt-and-pepper noise



Examples of Rayleigh noise

Image with added Rayleigh noise



Noise – additive component

Model of degraded image by additive noise :

$$d = I + n$$

Where:

d is degraded image

I is original image

n is additive noise function

Methods of noise reduction

Spatial / frequency filter

Specific noise -> tailor-made filter...

The two primary categories of spatial filters for noise removal are

Order filters (Median filter...)

Linear filters (Low-Pass filter...)

Image restoration using median filtration

Filtering of high frequency content

In most noiseless images the spatial frequency energy is concentrated in the low frequencies

In an image with added noise, much of the high frequency content is due to noise

This information is useful in the development of models for noise removal

Periodic Noise

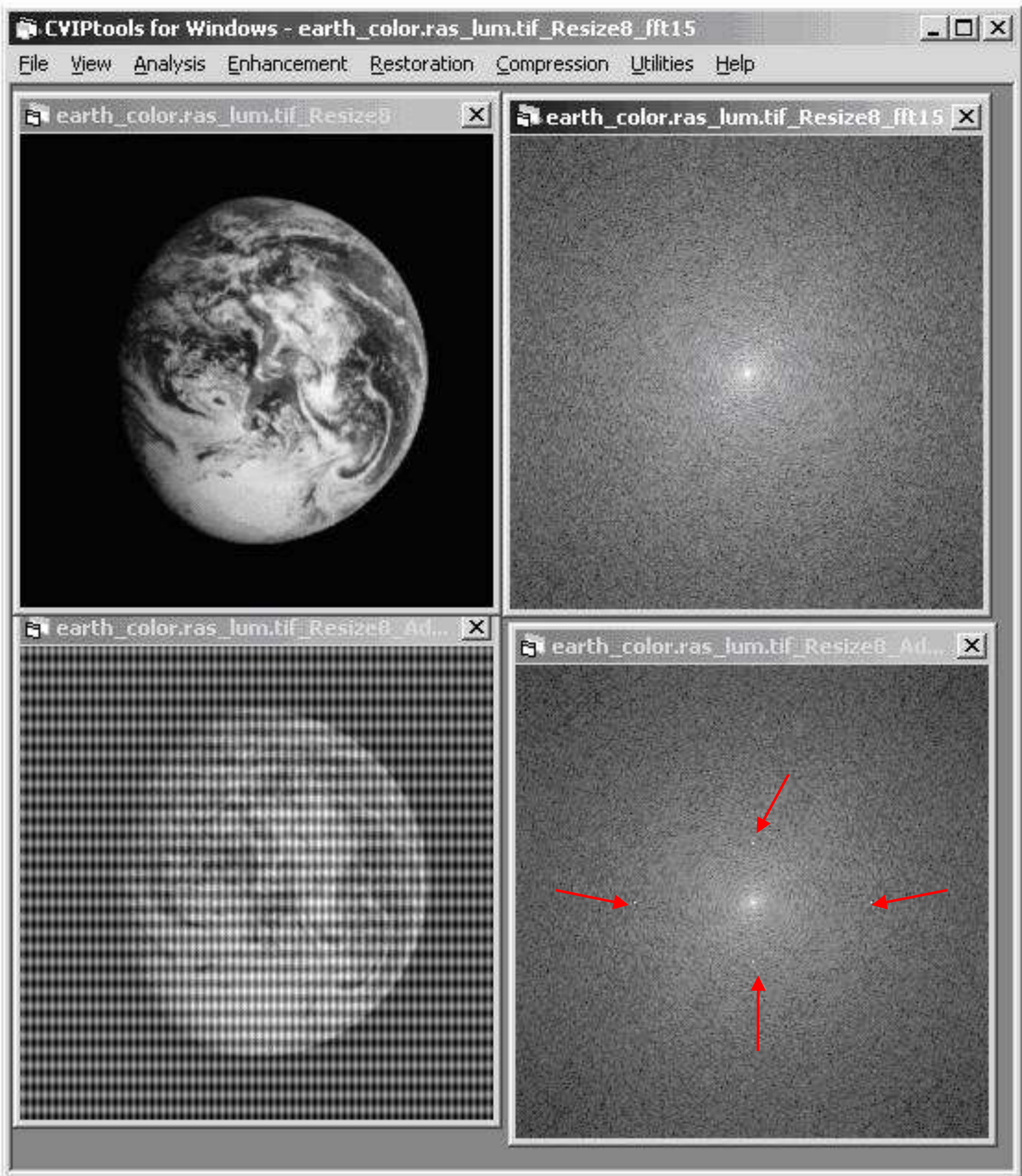
Periodic noise in images is typically caused by electrical and/or mechanical systems, such as mechanical jitter (vibration) or electrical interference in the system during image acquisition

It appears in the frequency domain as impulses corresponding to sinusoidal interference

It can be removed with band reject and notch filters

Image Corrupted by Periodic Noise.

On the top are the original image and its spectrum; under it are the image with additive sinsuiodal noise, and its spectrum. Note the four impulses corresponding to the noise appearing as white dots – two on the vertical axis and two on the horizontal axis



Estimation of Noise

Consists of finding an image (or sub-image) that contains only noise, and then using its histogram for the noise model

Noise only images can be acquired by aiming the imaging device (e.g. camera) at a blank wall

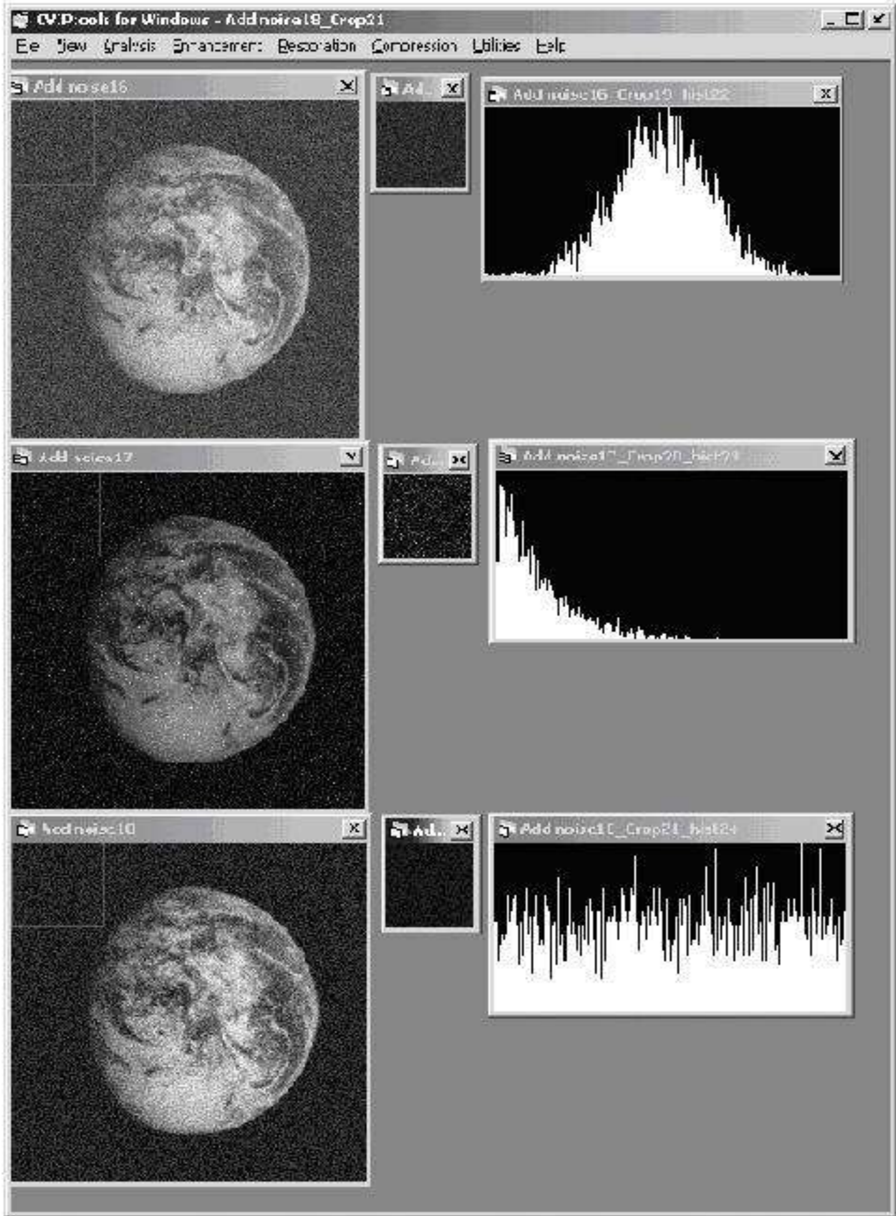
A "noise-only“ portion of the image is selected that has a known histogram, and that knowledge is used to determine the noise characteristics

After a portion of the image is selected, we subtract the known values from the histogram, and what is left is our noise model

To develop a valid model many sub-images need to be evaluated

Estimating the Noise with Crop and Histogram.

On the left are three images with different noise types added. The upper left corner is cropped from the image and is shown in the middle. The histogram for the cropped subimage is shown on the right. Although the noise images look similar, the histograms are quite distinctive – Gaussian, negative exponential and uniform



Compu

Removing of noise by Order filters

Implemented by arranging the neighborhood pixels in order from smallest to largest gray level value, and using this ordering to select the "correct" value

Order filters such as the median can be used to smooth images

Order filters work best with salt-and-pepper, negative exponential, or Rayleigh noise

! Salt-and-pepper noise could be efficiently removed using the median filter

Removing of noise - Linear filters

Low-pass filters can be used to reduce noise effects

The linear filters work properly in the case of Gaussian or uniform noise

The linear filters have the disadvantage of blurring the image edges or details

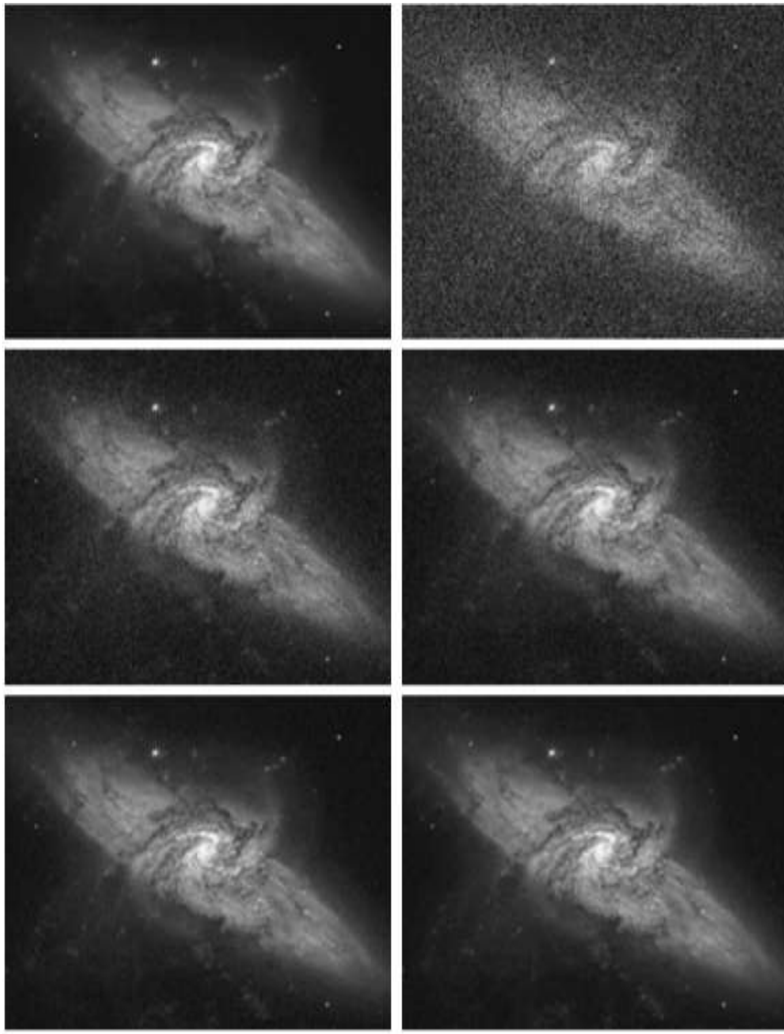
Removing of noise - Linear filters - example

(a)Image of Galaxy Pair NGC 3314.

(b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels.

(c)–(f) Results of averaging the noisy images (mean filter)

Size : K=8, 16, 64, and 128.



Pixels intensity direct

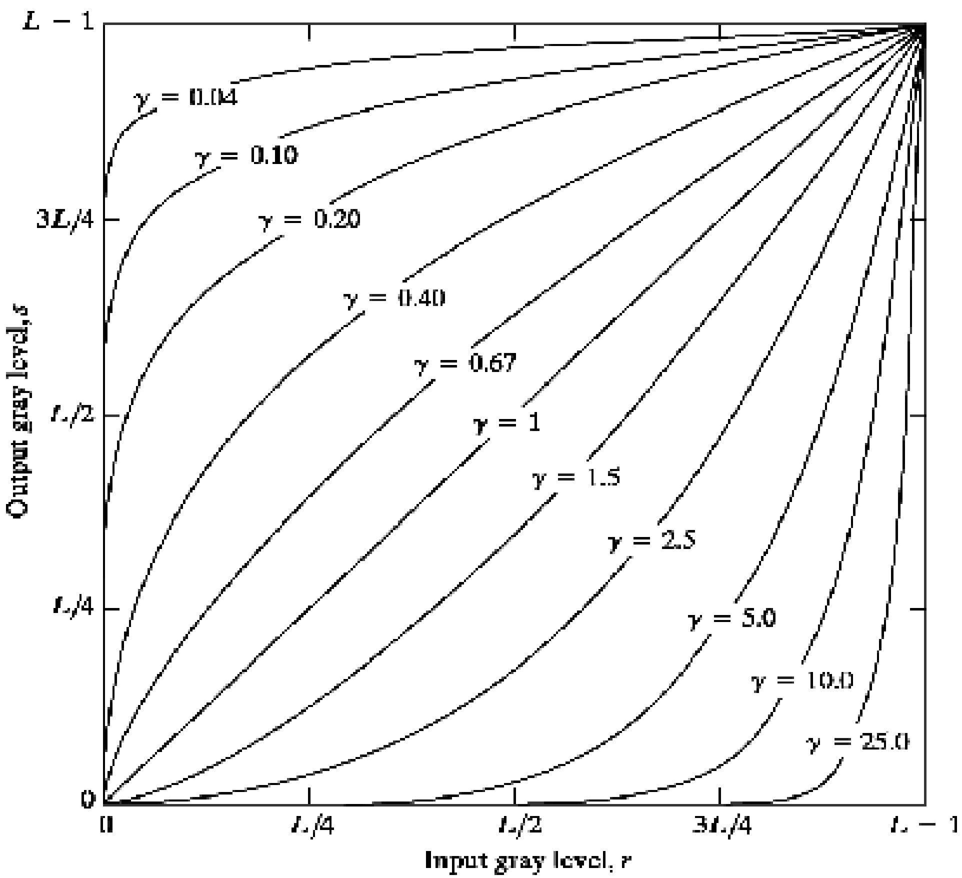
Power-Law Transformations

Power-law transformations have the basic form:

$$s = cr^{\gamma}$$

where c and g are positive constants.

Power-Law Transformations



Plots of the equation

$$s = cr^{\gamma}$$

various values of γ

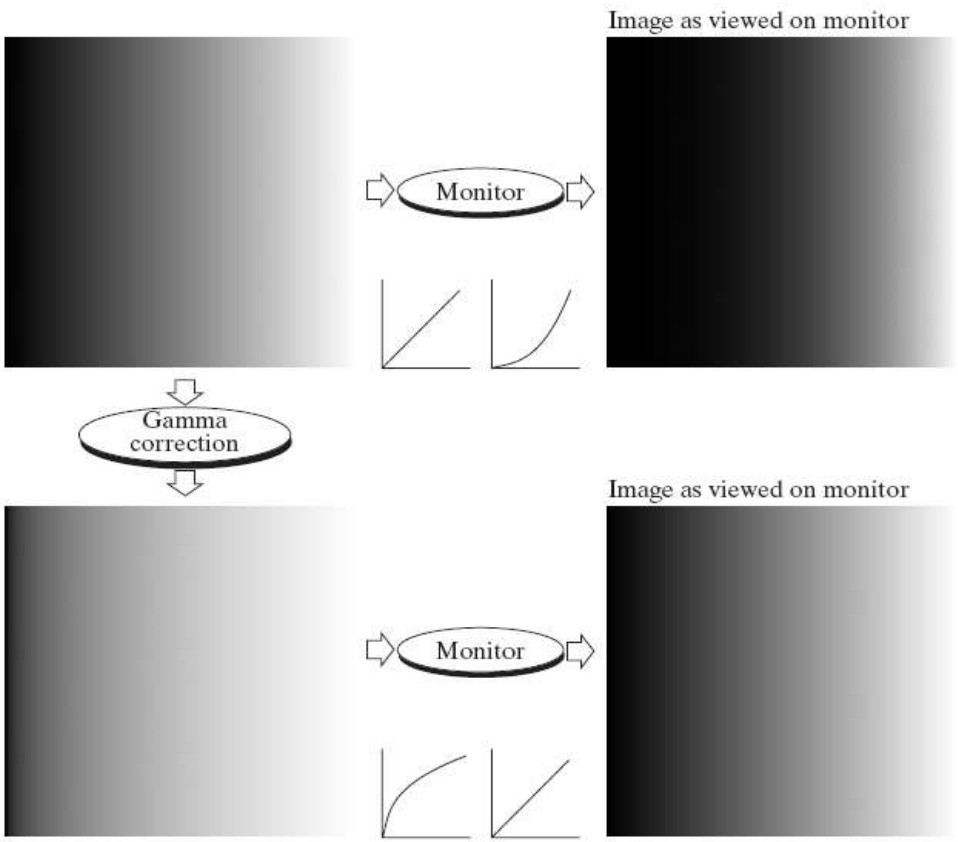
Signal range: 0-1

Power-Law Transformations used for gamma correction

The process of used the Power-Law Transformations is called gamma correction.

For example, cathode ray tube (CRT) devices have an intensity-to-voltageresponse that is a power function, with exponents varying from approximately 1.8 to 2.5

Gamma Correction



(a) Linear scale gray-scale image.

(b) Response of monitor to mean wedge.

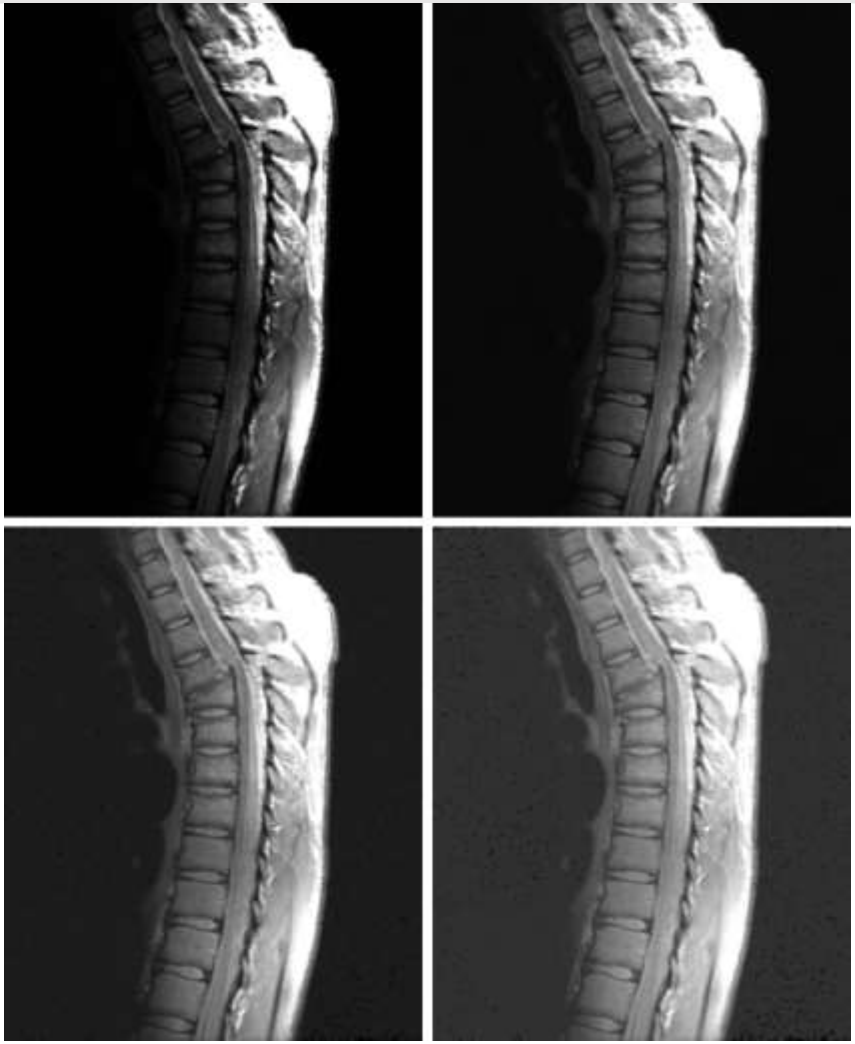
(c) Gammacorrected wedge.

(d) Output of monitor.

Power-Law Transformations - example

(a) Magnetic resonance (MR) image of a fractured human spine.

(b)–(d) Results of applying the power-law transformation with c=1 and g=0.6, 0.4, and 0.3, respectively.



Advanced methods

Adaptive filtration

Adaptive filtration

Adaptive filtration applies a linear filter to an image adaptively, tailoring itself to the local image features (variance...etc)

Example:

Where the variance is large, then performs little smoothing.
Where the variance is small, than performs more smoothing.

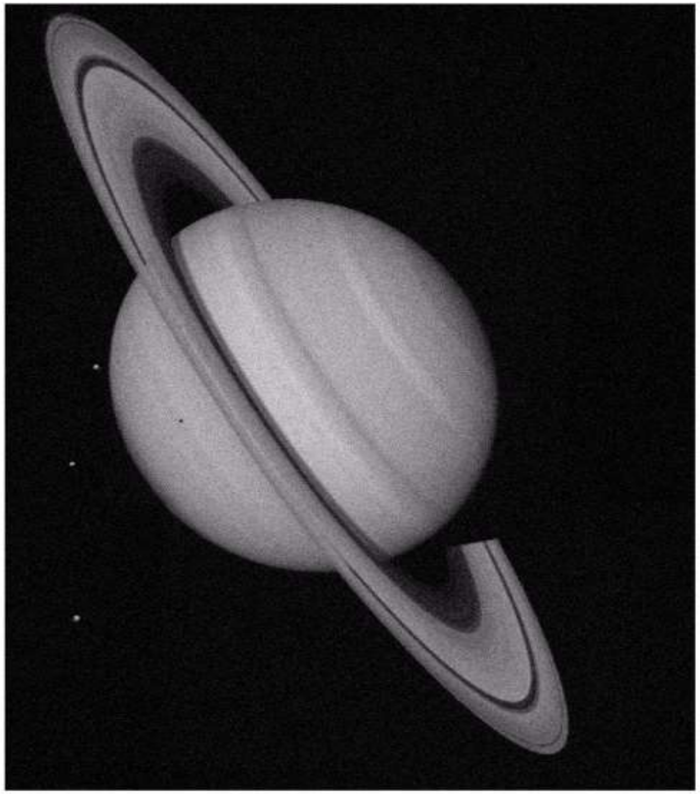
Adaptive filtration

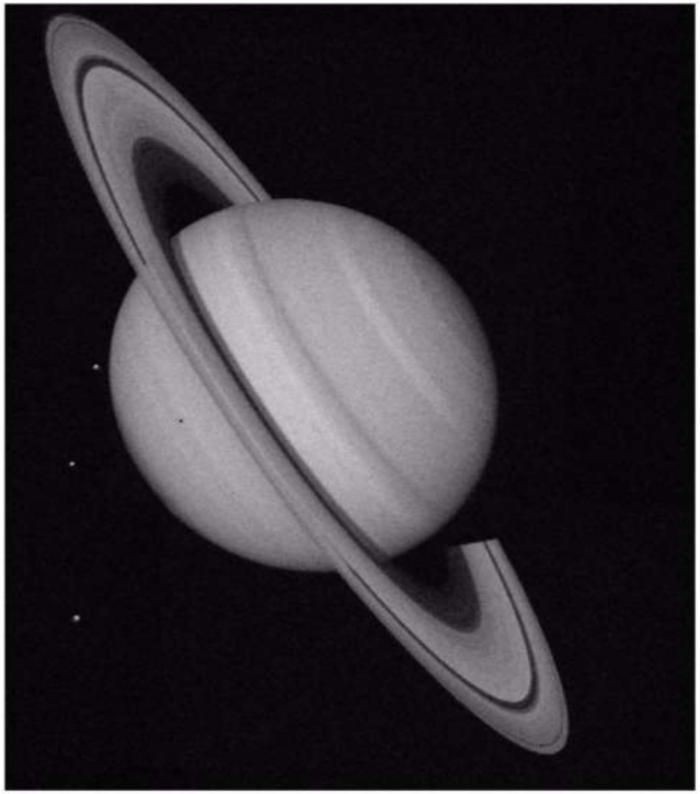
This approach often produces better results than linear filtering.

The adaptive filter is more selective than a comparable linear filter, preserving edges and other high-frequency parts of an image.

Example of adaptive filtration

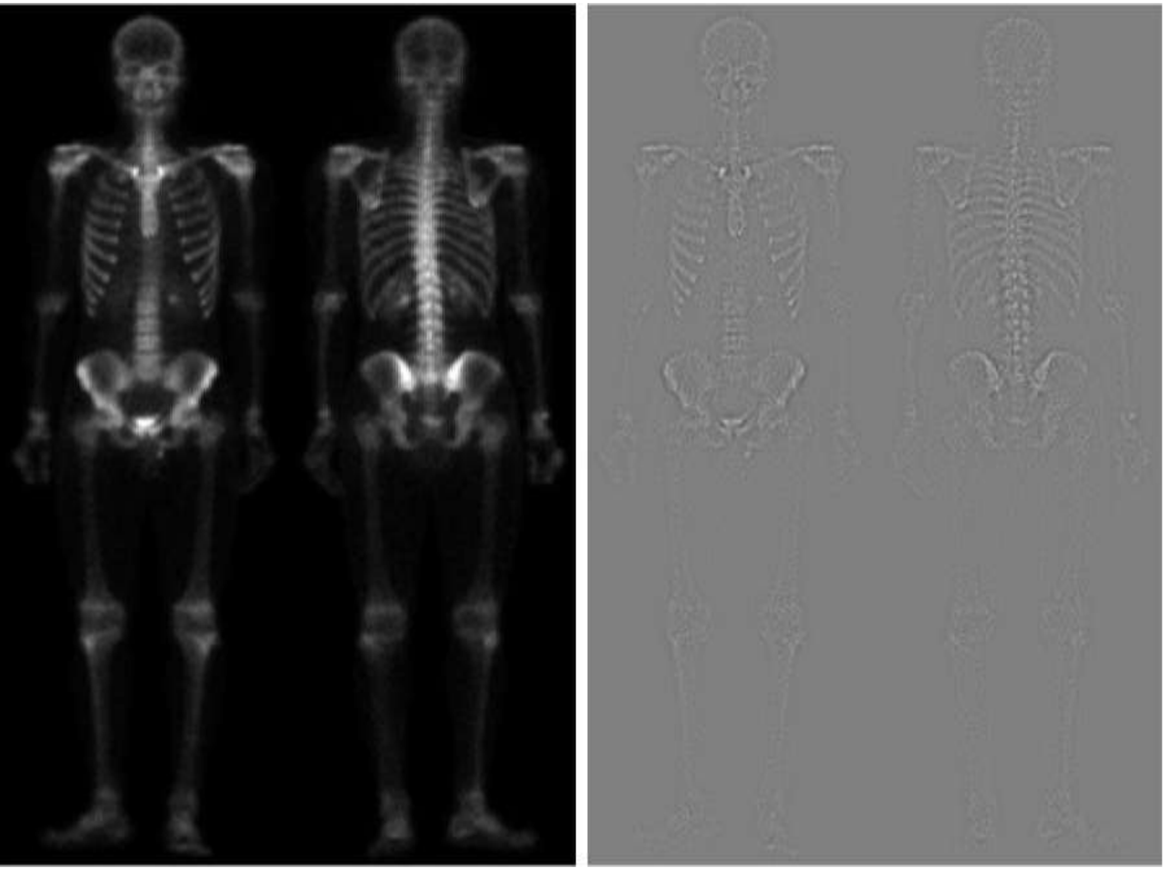
The example below applies adaptive filtration using the Wiener filter to an image of Saturn that has had Gaussian noise added.





Original Image Courtesy of NASA

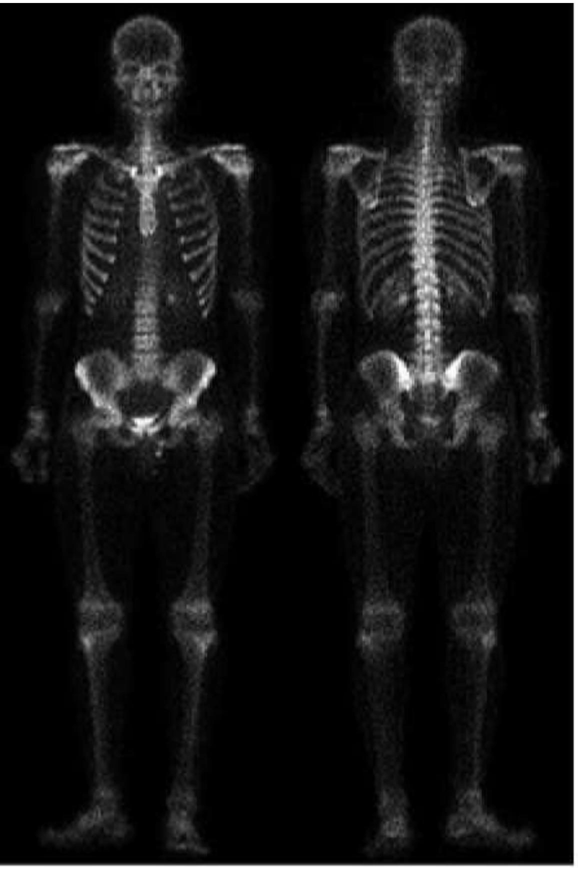
Example of Combining Spatial Enhancement Methods

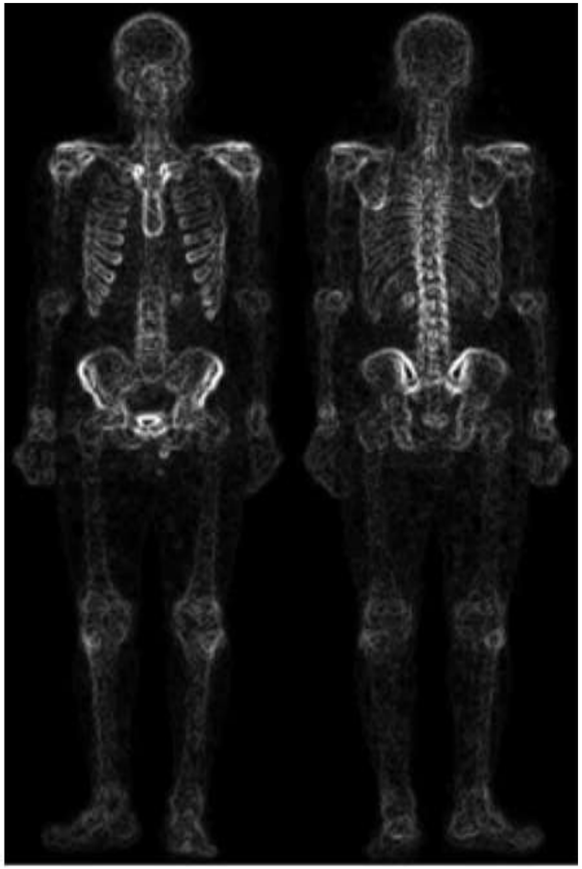


(a) Image of whole body bone scan.

(b) Laplacian of (a)

Example of Combining Spatial Enhancement Methods

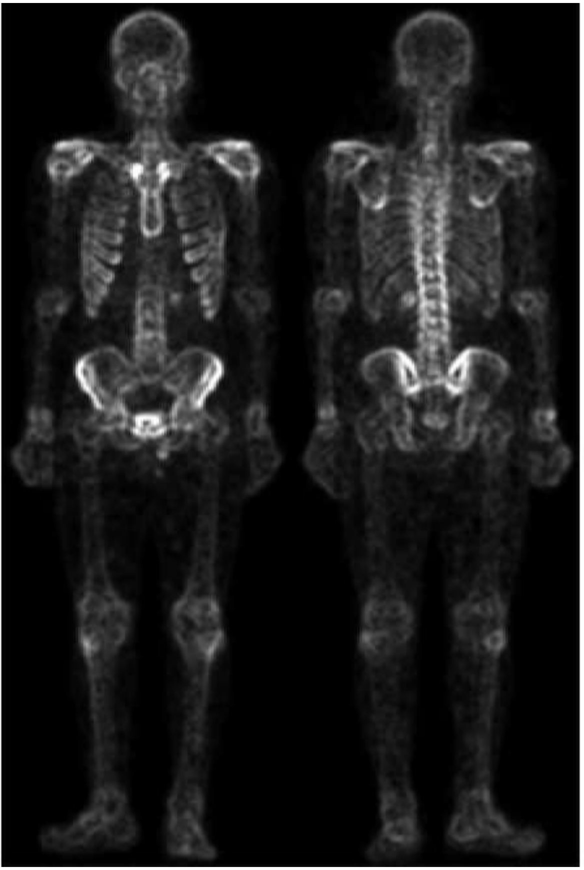


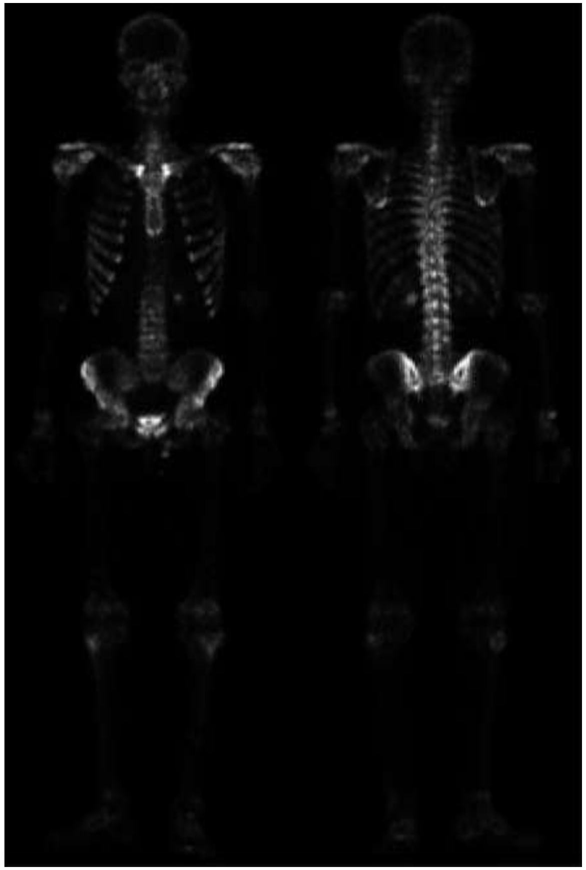


(c) Sharpened image obtained by adding (a) and (b).

(d) Sobel of (a)

Example of Combining Spatial Enhancement Methods

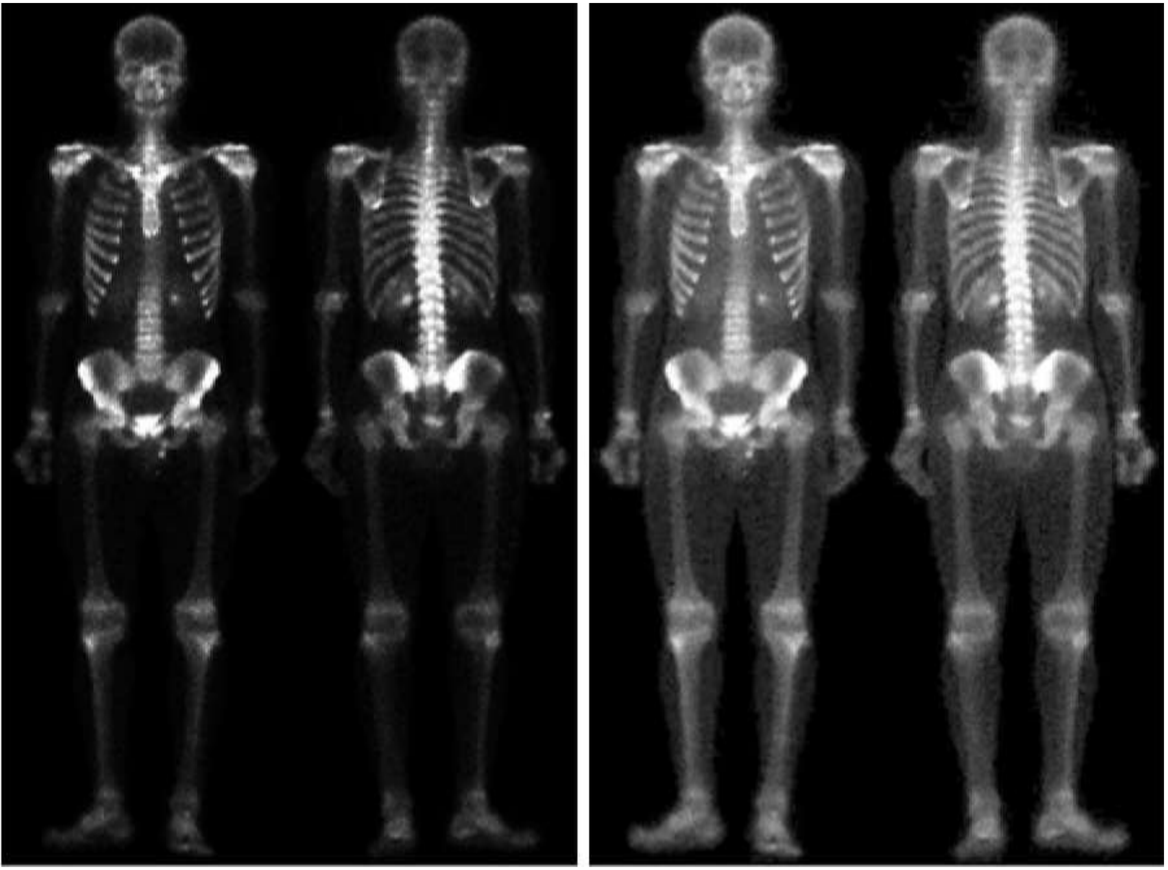




(e) Sobel image smoothed with a 5*5 averaging filter.

(f) Mask image formed by the product of (c) and (e).

Example of Combining Spatial Enhancement Methods



(g) Sharpened image obtained by the sum of (a) and (f).

(h) Final result obtained by applying a power-law transformation to (g).

Image Deblurring

The blurring, or degradation, of an image can be caused by many factors:

Movement during the image capture process, by the camera or, when long exposure times are used, by the subject

Out-of-focus optics, use of a wide-angle lens, atmospheric turbulence, or a short exposure time, which reduces the number of photons captured

Image Deblurring

A blurred or degraded image can be approximately described by this equation

$$g = H*f + n, \text{ where}$$

g The blurred image

H The distortion operator- point spread function (PSF)
The distortion operator, when convolved with the image, creates the distortion.

f The original true image

n Additive noise, introduced during image acquisition, that corrupts the image

Image Deblurring using Deconvolution

Based on this model, the fundamental task of deblurring is to deconvolve the blurred image with the PSF that exactly describes the distortion.

Deconvolution is the process of reversing the effect of convolution.

Note! The quality of the deblurred image is mainly determined by knowledge of the PSF.

Example of Image Deblurring

Original Image

Blurred Image

Restored, True PSF

Pyramid processing