# Computer vision

## Morphological image processing

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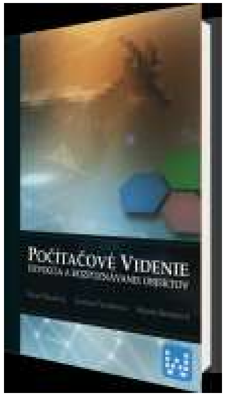
# Scope

Neighbours of pixel, (neighbourhood's pixel)
Connectivity
Distances

Morphological processing

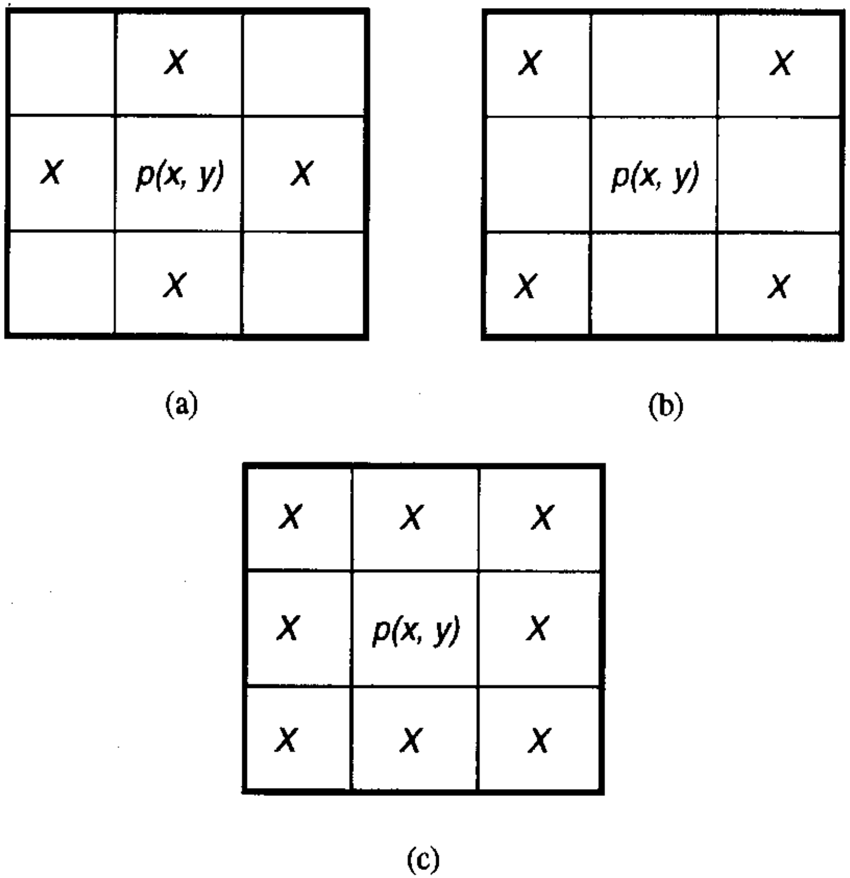
*Kapitola 4 Morfologické operácie od str. 52*



# Basic terms

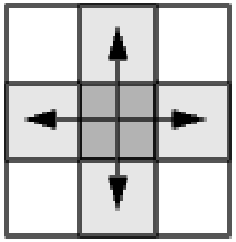
# Neighbors of Pixel
*(Susednosť)*

a) 4-neighbors
b) diagonal neighbors
c) 8-neighbors

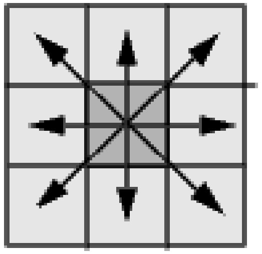


# Connectivity *(Súvislosť)*

4-connected :
Two pixels p and q are
4-connected if they are
4-neighbors and p ϵV and q ϵV

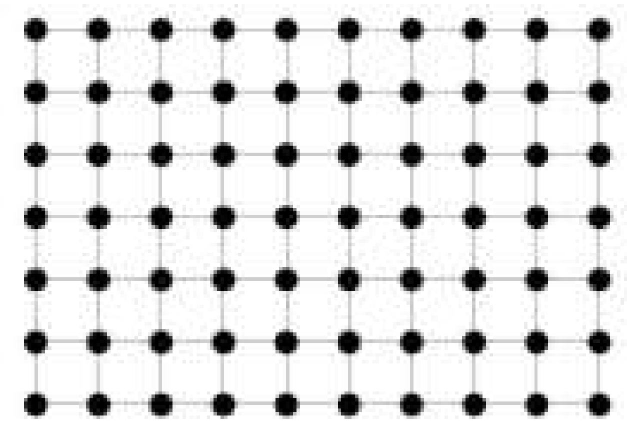


8-connected
Two pixels p and q are
8-connected if they are
8-neighbor and p ϵV and q ϵV



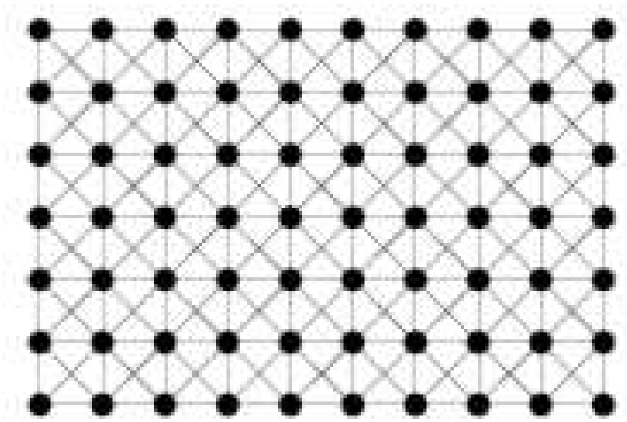
# 4-connectivity, 8-connectivity
# *(4-súvislosť, 8-súvislosť)*

4-connected



8-connected

:



# Metric

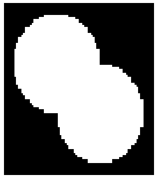
A metric on the points p, q is a function called the distance function.

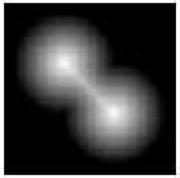
The following conditions are satisfied:

1. $D(p,q) \geq 0$ non−negativity
2. $D(p,q) = 0 \Leftrightarrow p = q$ identity
3. $D(p,q) = D(q,p)$ symmetry
4. $D(p,z) \leq D(p,q) + D(q,z)$ triangle inequality

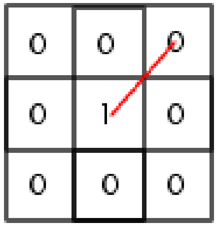
# Distance functions

Distance between pixels p and q can be defined by one of the following:

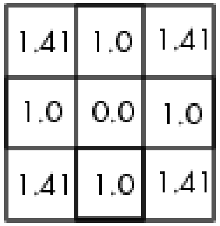




Euclidean distance: $D_E(p,q) = \sqrt{(x1-x2)^2 + (y1-y2)}$




Image

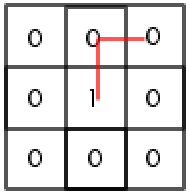



Distance transform

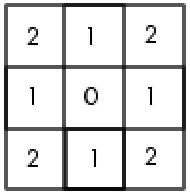
City-block distance : $D_4(p,q) = |x1-x2| + |y1-y2|$

*(Vzdialenosť medzi dvomi bodmi v obraze je daná počtom*

*krokov (jednotkovej veľkosti) po pravouhlej diskrétnej mriežke)*

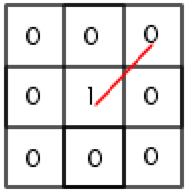



Image

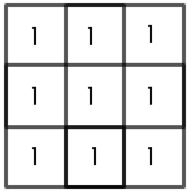



Distance transform

Chess-board distance $D_8(p,q) = max(|x1-x2|, |y1-y2|)$



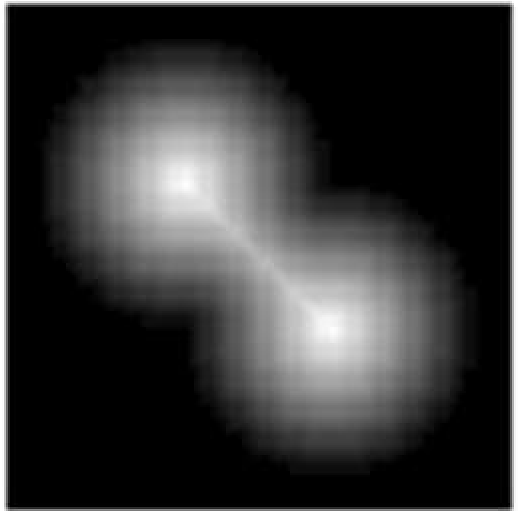

Image




Distance transform

# Distance image

Euclidean distance:



# Grayscale image , binary image

grayscale image example:

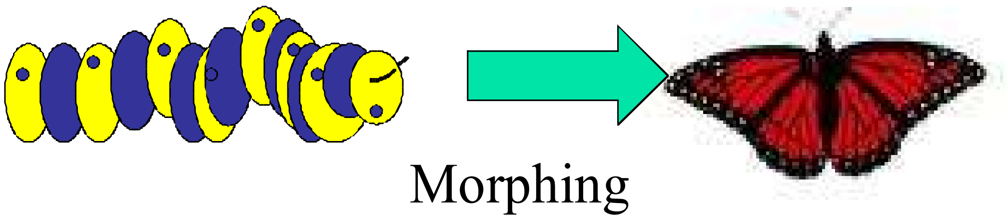
binary image example:

# Morphological image processing

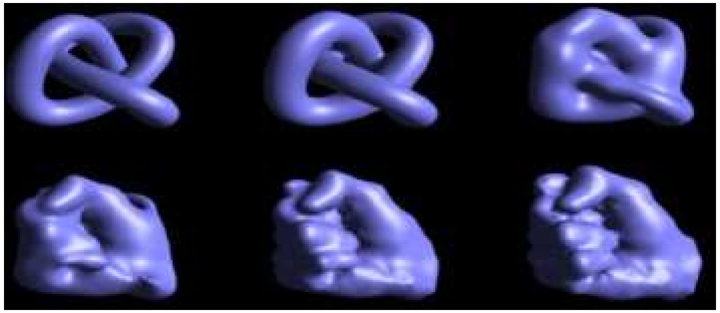
# What are Morphological Operations?

Morphological operations are intended to affect the shape of the object.

“Morphological operations” come from the word “morphing” in biology which means changing a shape.




“Morphing” refers also  to an animation technique in which one image is gradually turned into another



# Morphological image processing

Morphological image processing uses information about the neighbours in the topological surroundings of the processing pixel.

Neighbourhood operator : structuring element

Used inputs:

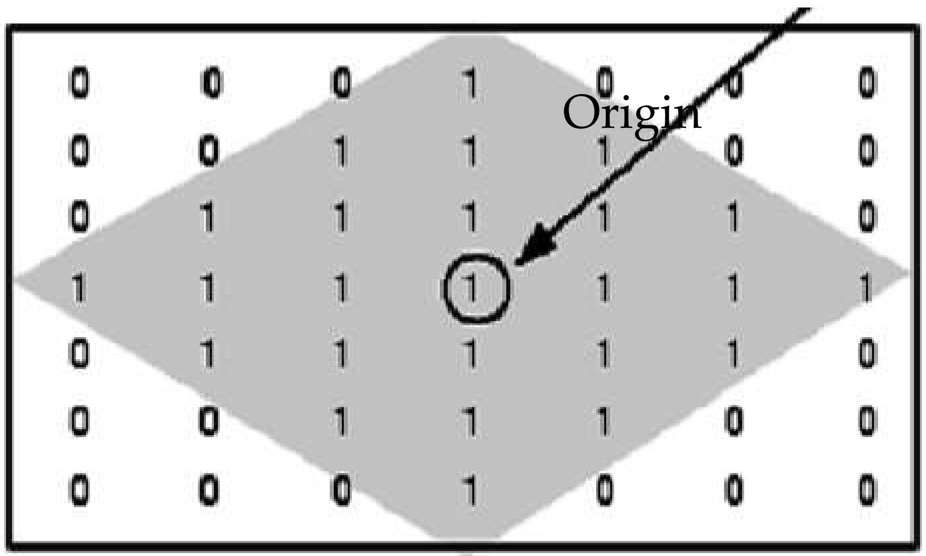
binary images, grayscale images (colour images)

The foundation of morphological processing is in the mathematically rigorous field of *set theory*, however, this level of sophistication is seldom needed.

# Flat Structuring Element

A structuring element is a matrix consisting of only 0's and 1's that can have any arbitrary shape and size.

The pixels with values of 1 define the neighborhood .



# Basic morphological operations: Dilation & Erosion

Dilation adds pixels to the boundaries of an (white) objects in an image, while erosion removes pixels on the object boundaries.

The number of pixels added or removed from the objects in an image depends on the size and shape of the structuring element used to process the image.

Morphological operations are typically applied to process binary images, but they are also utilized on gray-scale images.

# Rules for Dilation and Erosion Operation

## Dilation

The value of the output pixel is the maximum value of all pixels in the input pixel's neighbourhood - selected by values of 1 in the structuring element. $F \oplus SE$

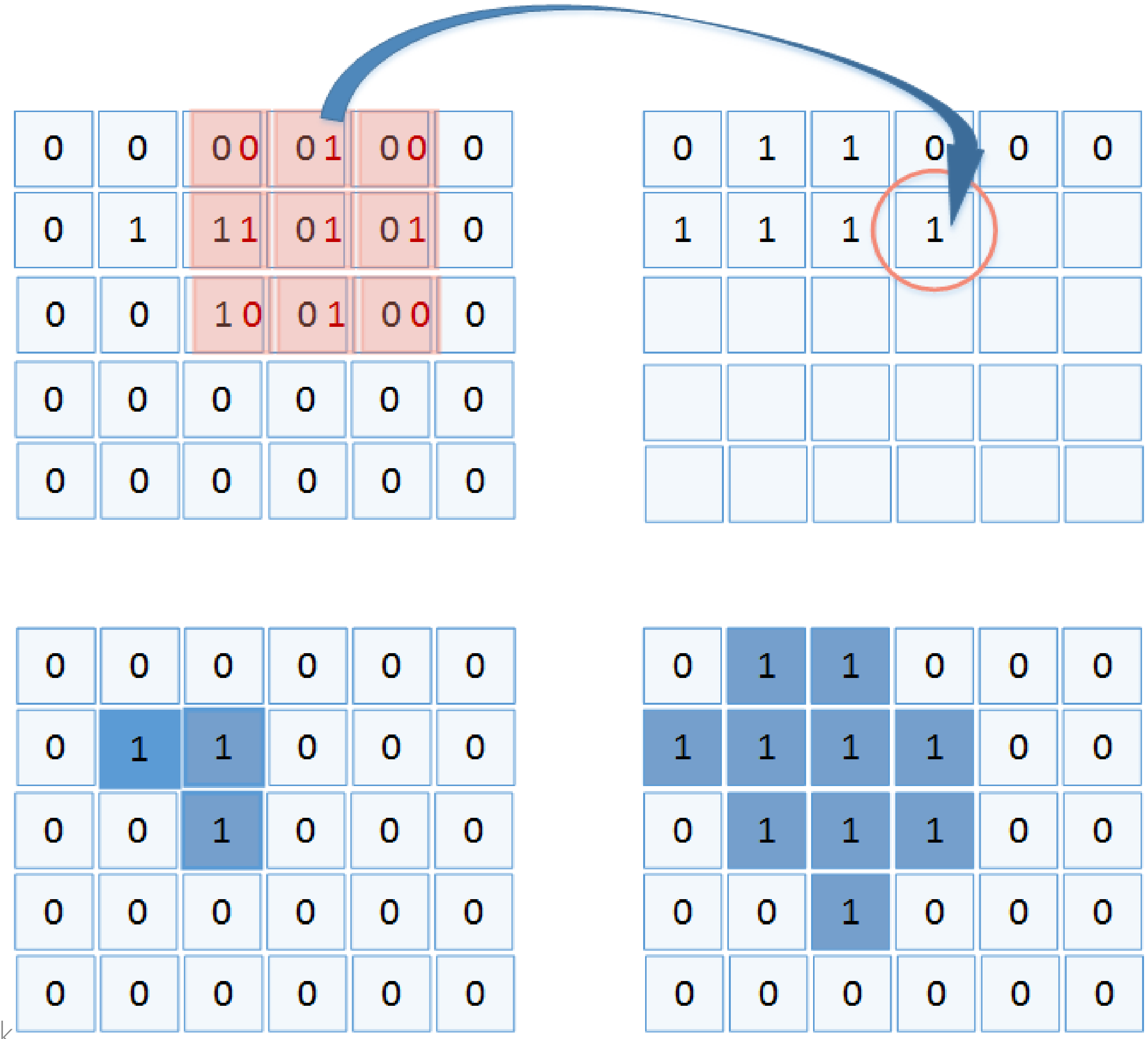
In a binary image, if any of the selected pixels has value 1, the output pixel is set to 1.

## Erosion

The value of the output pixel is the minimum value of all pixels in the input pixel's neighbourhood - selected by values of 1 in the structuring element. $F \ominus SE$

In a binary image, if any of the selected pixels has value 0, the output pixel is set to 0.

# Example of dilation on a binary image



# Binary dilation - example

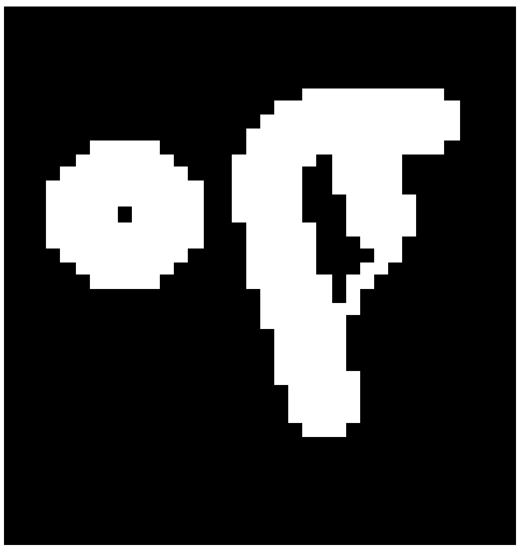
Input binary image

Image after dilation

# Applications of dilation for bridging gaps in an image

Input binary image

Image after dilation





Str. elem.:

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |

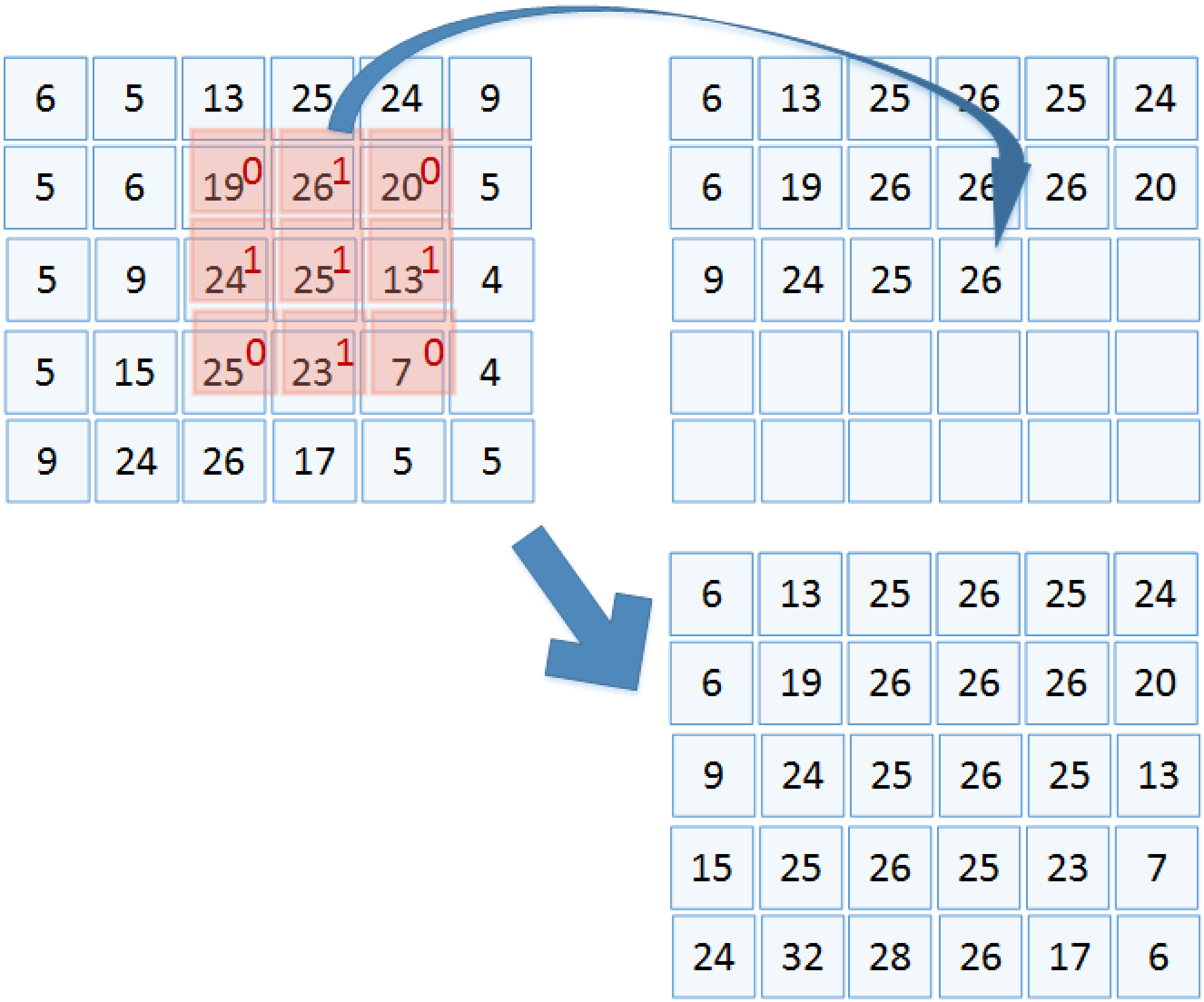
# Dilation - an example

structuring element:

Effect of dilation using the 7×7 structuring element :

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |

# Example of dilation on a gray-scale image



# Example of dilation on a gray-scale image

Input image

Image after dilation

# Applications of dilation

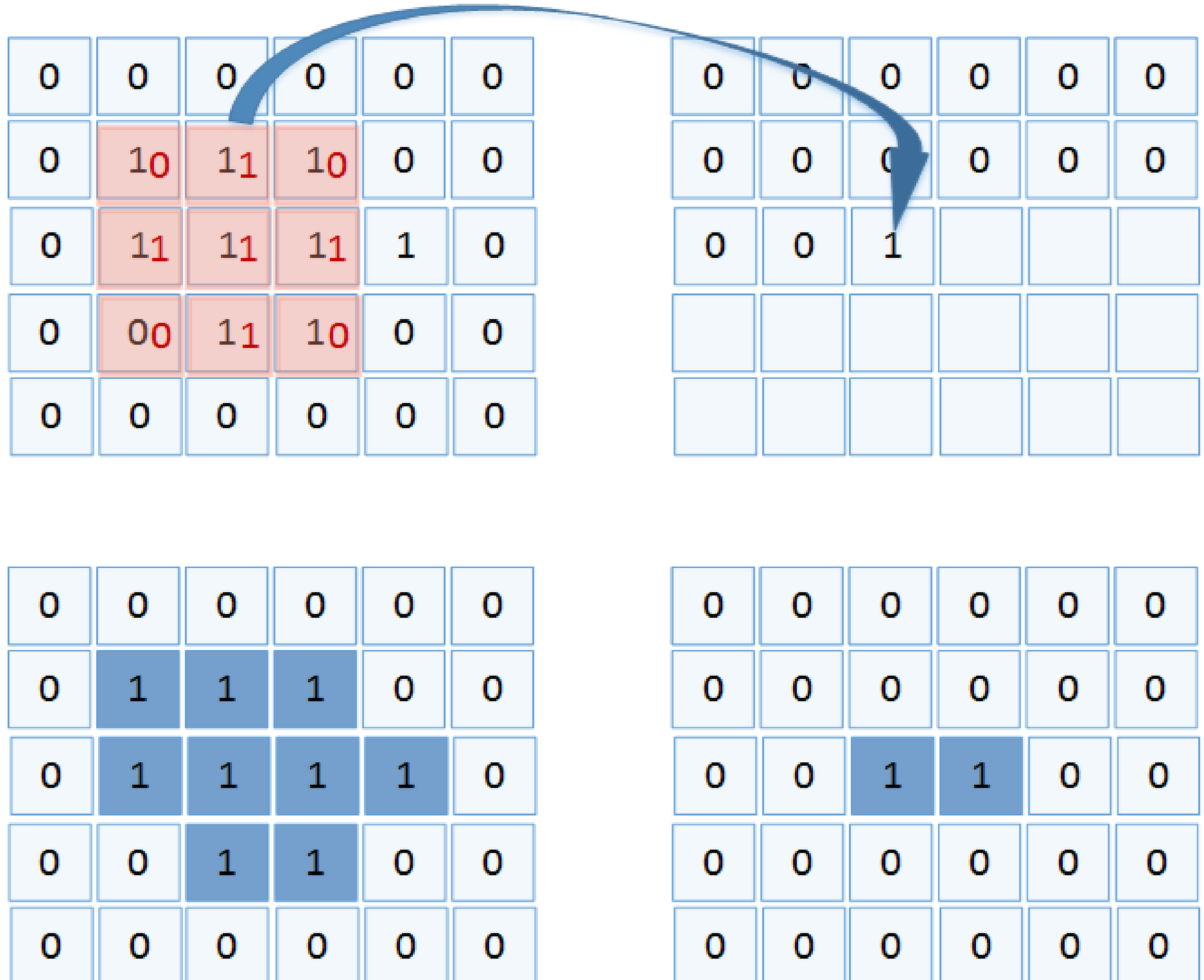
Expand / enlarge white objects in the image

Fill gaps or bays of insufficient width

Fill small holes of sufficiently small size

Connects objects separated by a distance less than the size of the neighbourhood

# Example of erosion on a binary image

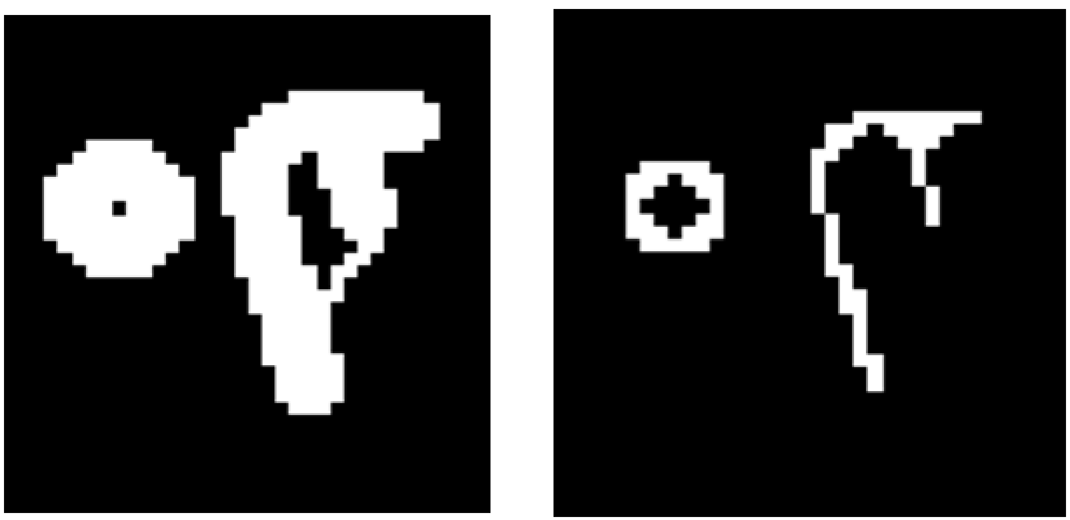


# Applications of erosion

Cut back / decrease white objects in the image

Thus areas of foreground pixels shrink in size, and holes within those areas become larger.

input erode



Str. elem.:

$$\begin{matrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{matrix}$$

# Binary erosion - example

Input binary image

Image after erosion

# Examples of Erosion

original image

eroded image with 3 by 3

eroded image with 5 by 5

eroded image with 7 by 7

# Erosion - example

Applications of erosion : Eliminating unwanted detail

structuring element:

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |

# Example of erosion on a gray-scale image

| | | | | | |
|---|---|---|---|---|---|
| 6 | 5 | 13 | 25 | 24 | 9 |
| 5 | 6 | $19^0$ | $26^1$ | $20^0$ | 5 |
| 5 | 9 | $24^1$ | $25^1$ | $13^1$ | 4 |
| 5 | 15 | $25^0$ | $23^1$ | $7^0$ | 4 |
| 9 | 24 | 26 | 17 | 5 | 5 |

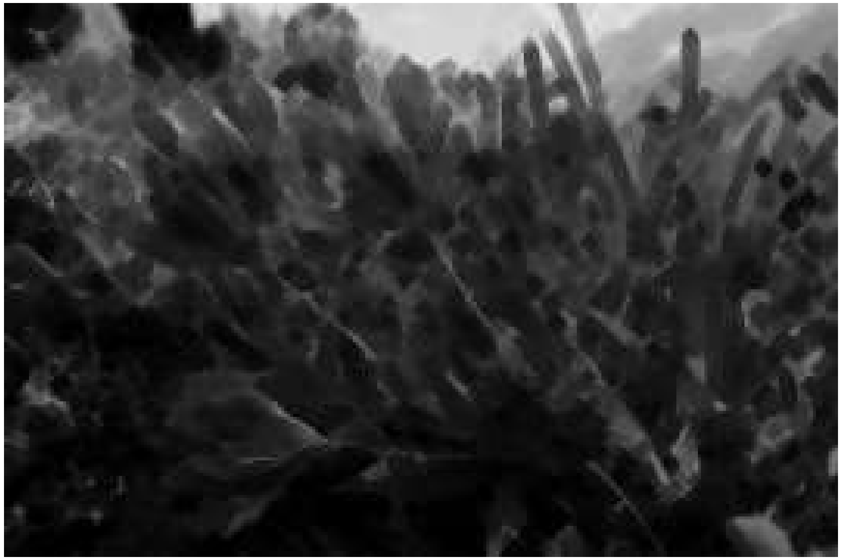
| | | | | | |
|---|---|---|---|---|---|
| 5 | 5 | 5 | 13 | 9 | 5 |
| 5 | 5 | 6 | 19 | 5 | 4 |
| 5 | 5 | 9 | 13 | | |
| | | | | | |
| | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| 5 | 5 | 5 | 13 | 9 | 5 |
| 5 | 5 | 6 | 19 | 5 | 4 |
| 5 | 5 | 9 | 13 | 4 | 4 |
| 5 | 5 | 15 | 7 | 4 | 4 |
| 9 | 18 | 11 | 5 | 5 | 5 |

# Example of erosion on a gray-scale image

Input image

Image after erosion



# Relating Erosion & Dilation
## Erosion & dilation are not exact inverse

Dilation cannot

- recreate protuberances eliminated (removed) by erosion
- recreate small objects eliminated (removed) by erosion

Erosion cannot

- recreate holes filled (removed) by dilation
- recreate gaps or bays filled (removed) by dilation

-> *we can use sequences of morph. operations*

# Opening & Closing

The definition of a morphological opening: $(\mathbf{F} \ominus \mathbf{S}) \oplus \mathbf{S}$

an erosion followed by a dilation using the same structuring element for both operations.

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The related operation, morphological closing of an image, is the reverse: $(\mathbf{F} \oplus \mathbf{S}) \ominus \mathbf{S}$

it consists of dilation followed by an erosion with the same structuring element.

# Example of opening

Input binary image

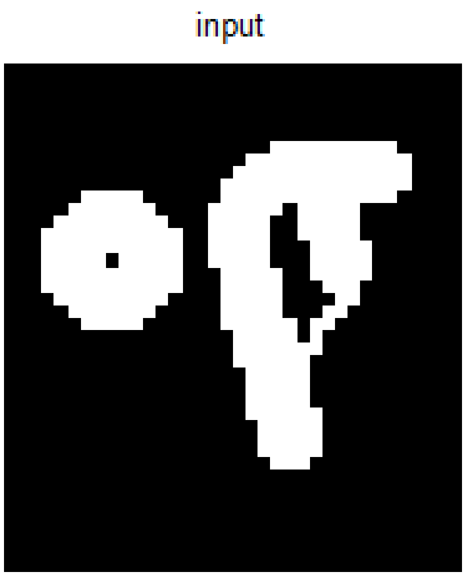
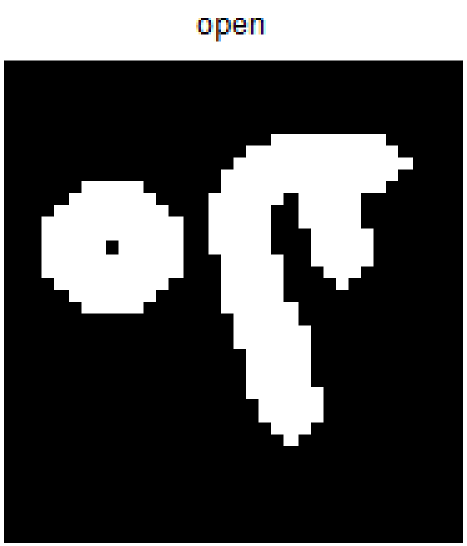



Image after opening

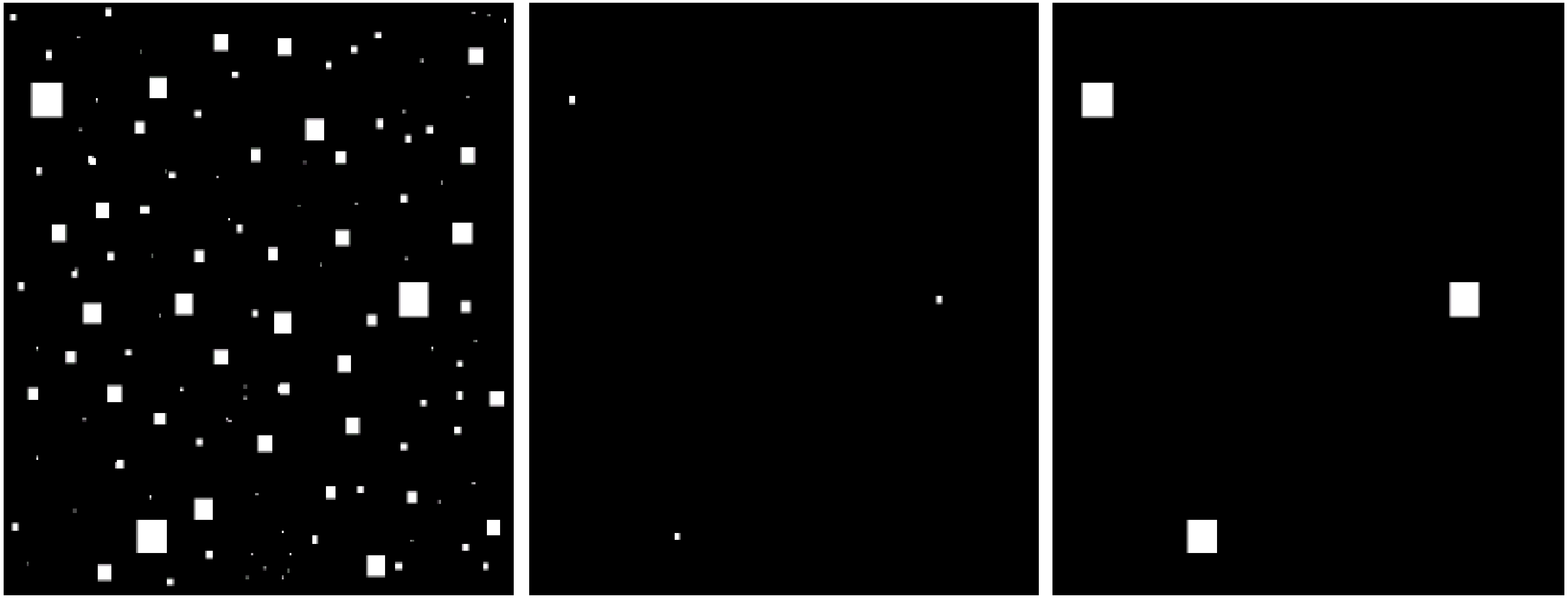



Used structuring element:

$$\begin{matrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{matrix}$$

# Example of Opening

Remove small objects



a b c

**FIGURE 9.7** (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

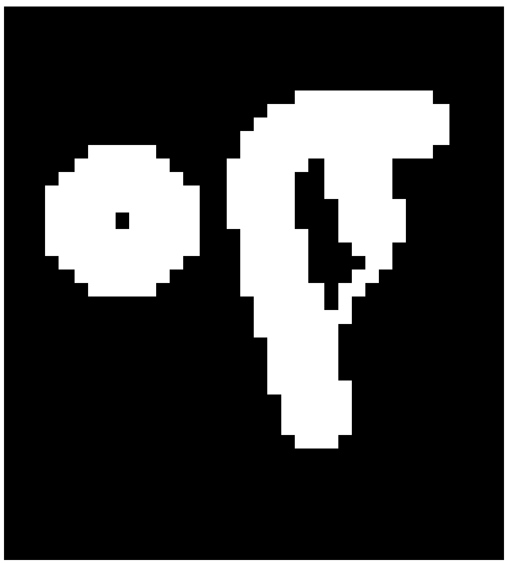


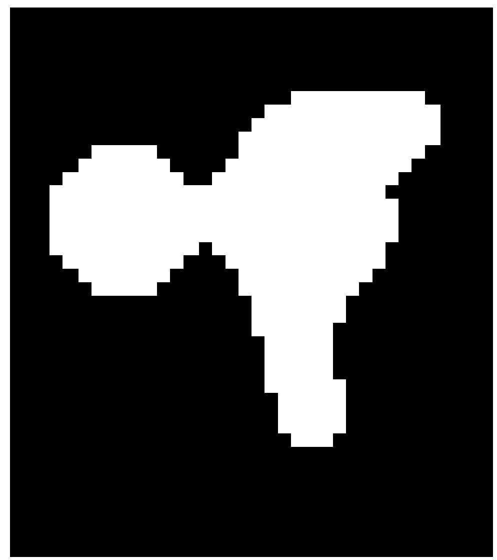
Source: Digital Image Processing
Rafael C. Gonzalez,Richard E. Woods,

# Example of closing

Input binary image

Image after closing




Used structuring element:

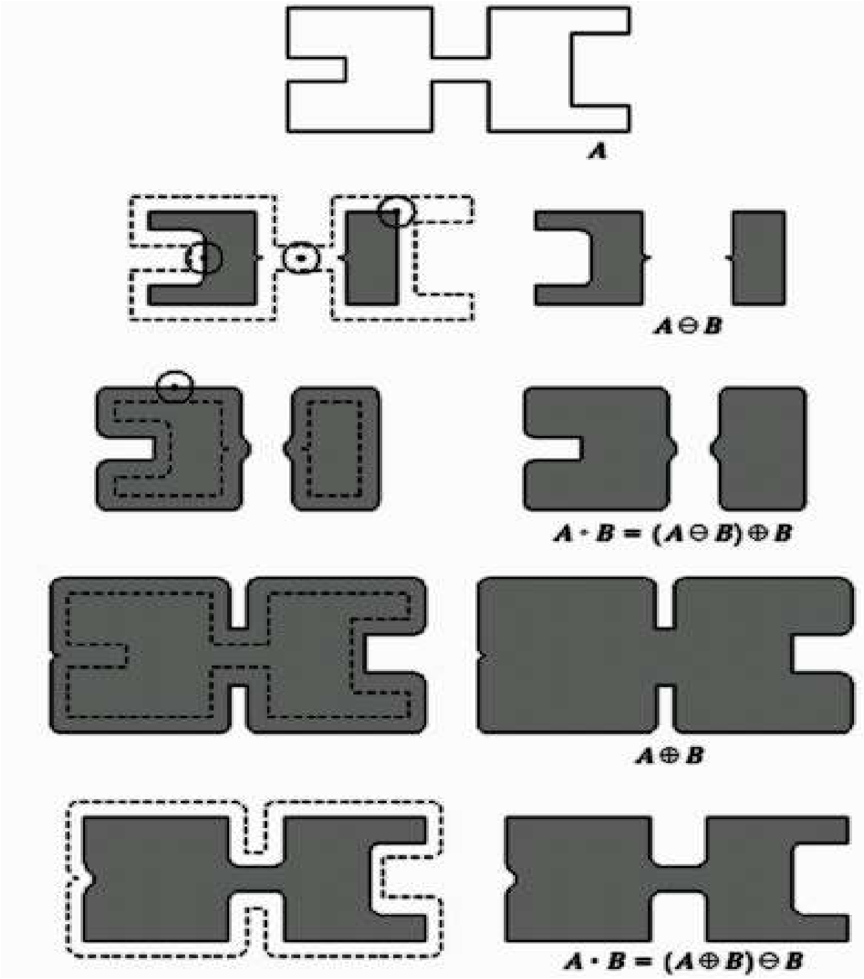
| | | | | |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |

# Examples

Input image

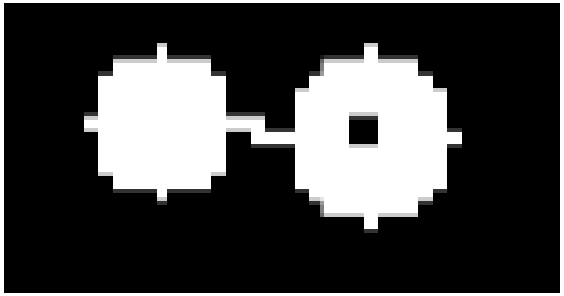
Example of opening

Example of closing



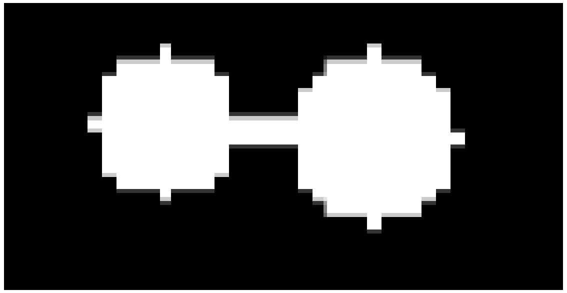

Source: Digital Image Processing
Rafael C. Gonzalez,Richard E. Woods,

# Open & Close



A

opening of A
→removal of small protrusions, thin connections, ...



closing of A
→ removal of holes

# Examples of binary opening, closing

Input binary iamge Open Close

# Distance transform

Every non-zero pixel will be replaced by his distance to the next zero pixel

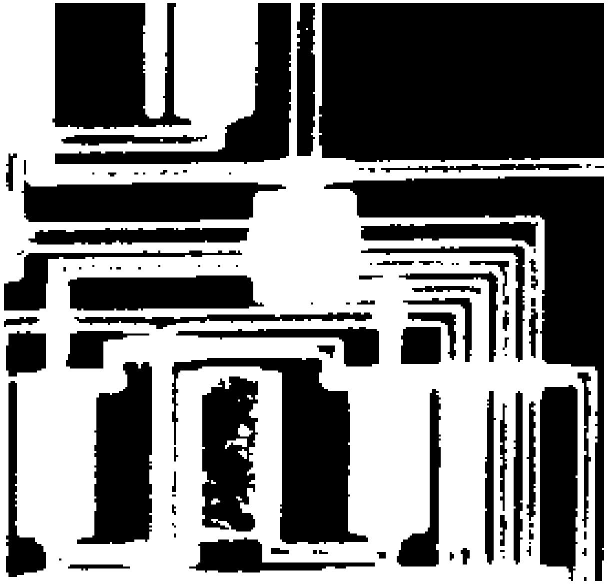
Input image

Distance transformed image

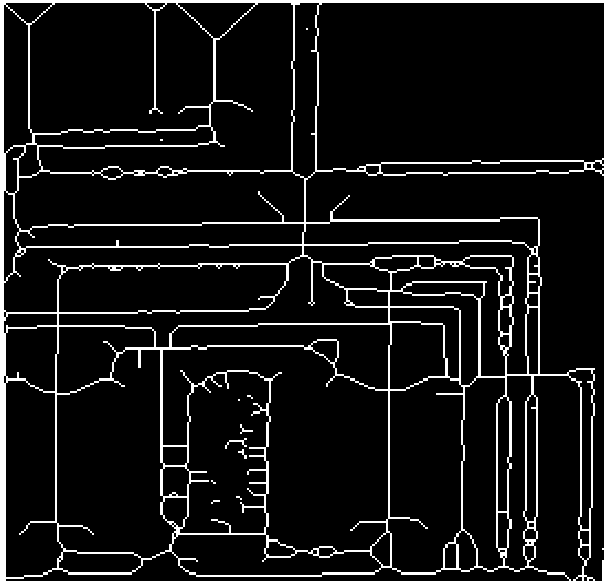
# Skeletonization

To reduce all objects in an image to lines, without changing the essential structure of the image

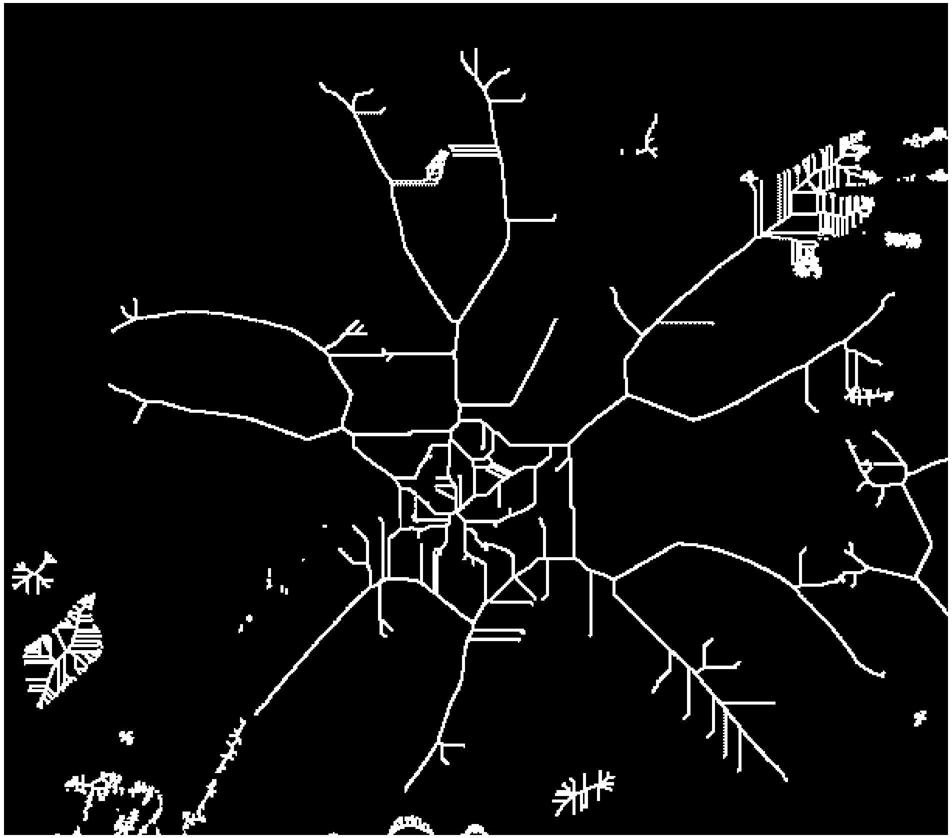
Input Image



Skeleton

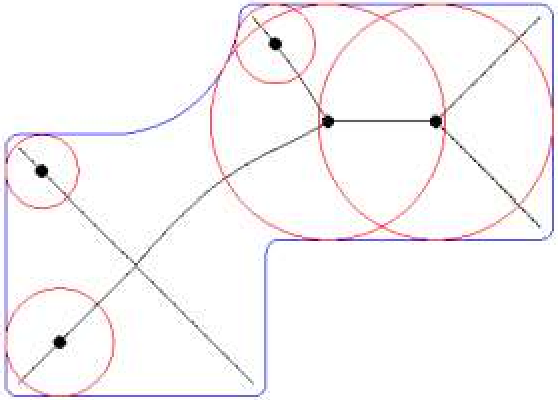


# Skeleton



# Skeleton algorithms

- algorithm using maximal circles
- sequential morphological thinning
- using distance transform



# Morphological "top hat" operation

Morphological "top hat" operation returns the image minus the morphological opening of the image.

$$\mathbf{F} - ((\mathbf{F} \ominus \mathbf{S}) \oplus \mathbf{S})$$

Input image

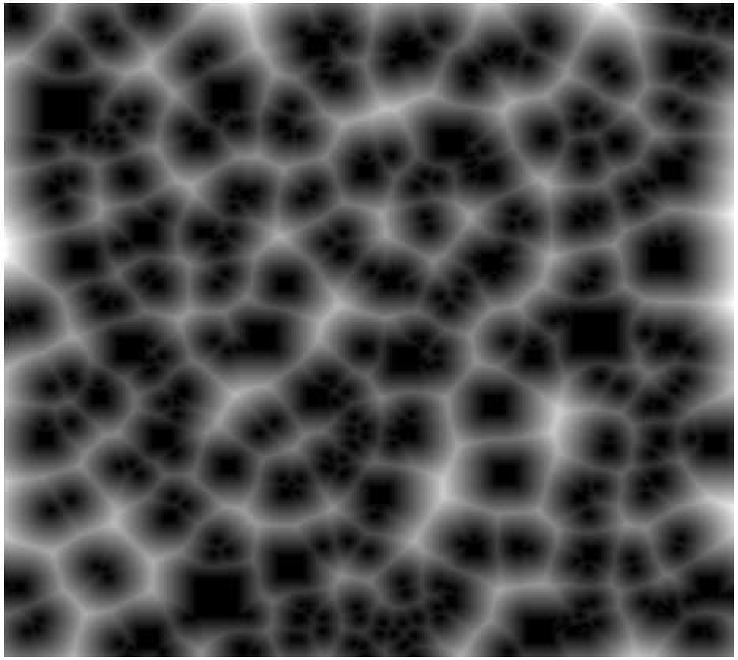
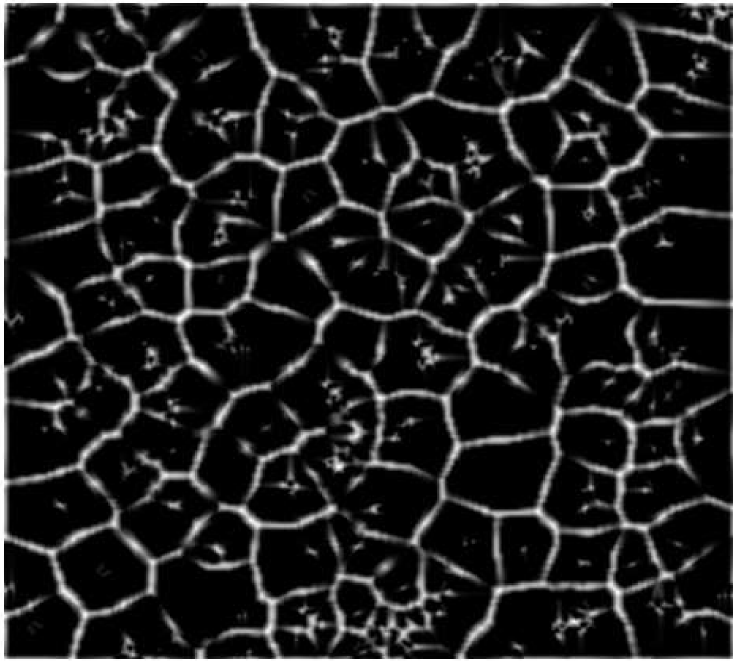


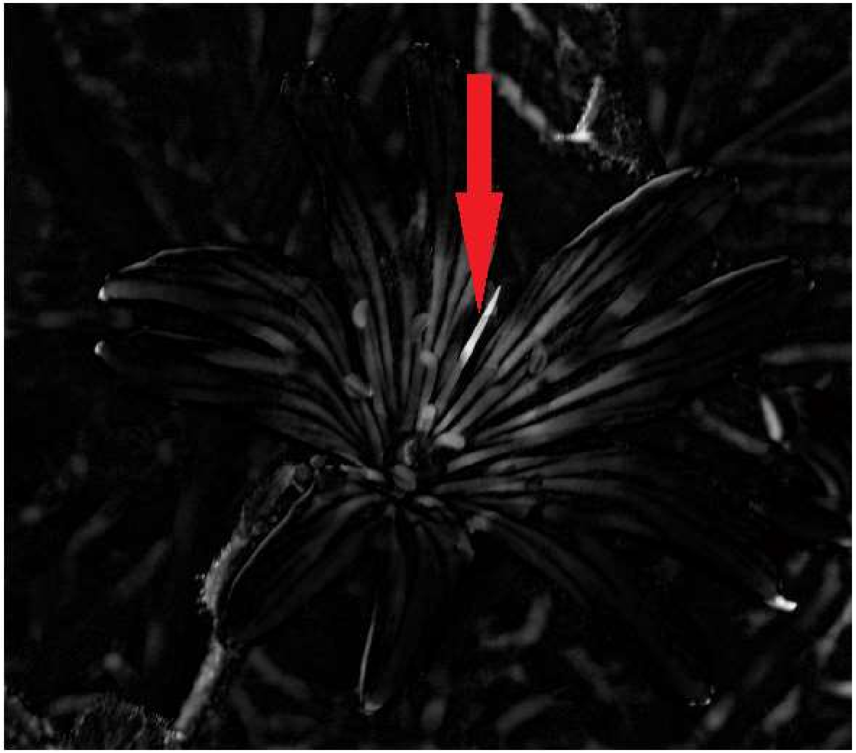
Image after top hat operation



# Example of Morphological "top hat" operation

Input Image

Image after top hat operation



# Morphological "bottom hat" operation

Morphological "bottom hat" operation performs closing (dilation followed by erosion) and than subtracts the original image. $((\mathbf{F} \oplus \mathbf{S}) \ominus \mathbf{S}) - \mathbf{F}$

Input image

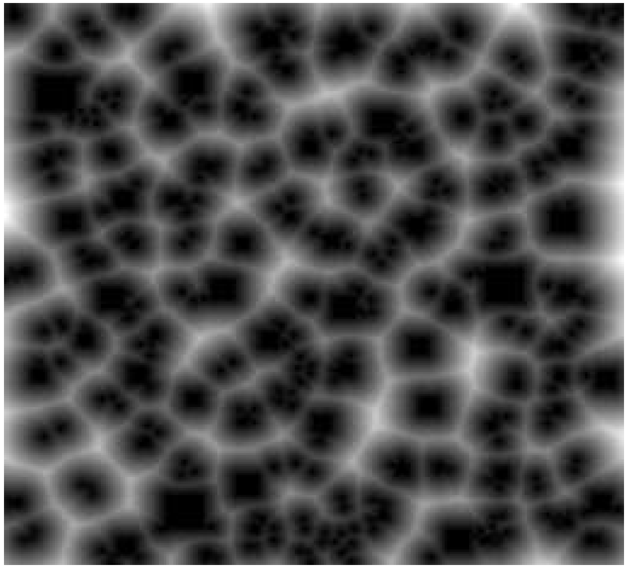
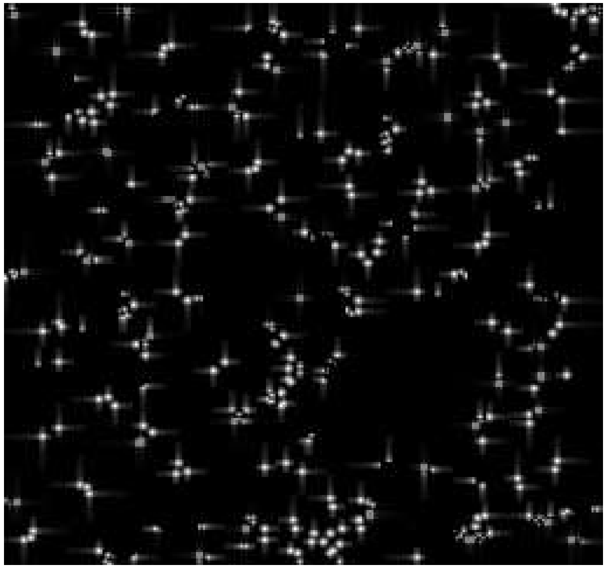


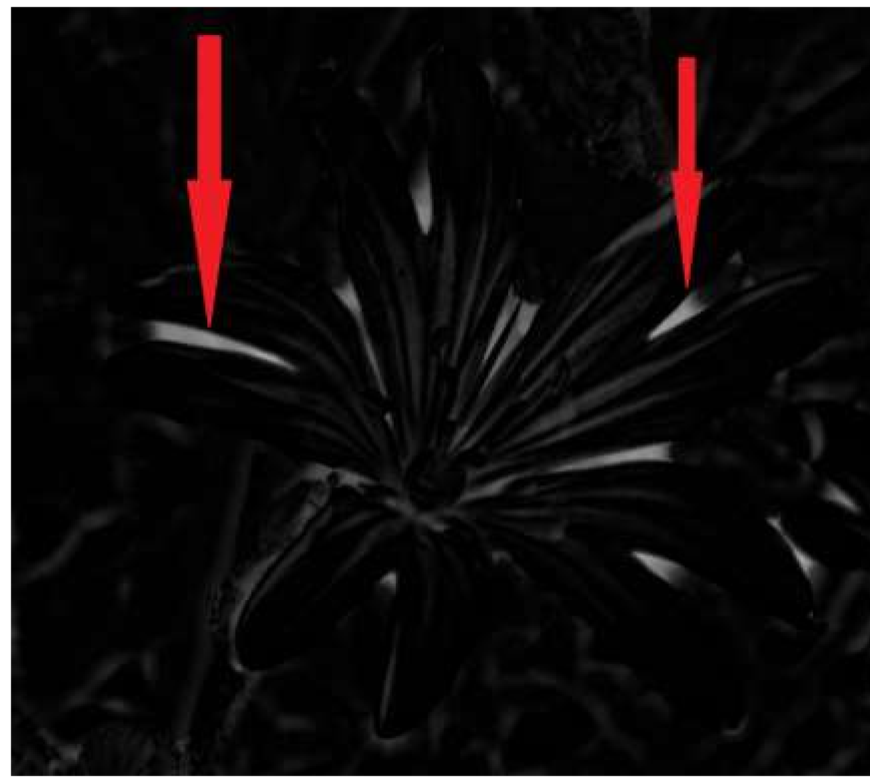
Image after bottom hat operation



# Example of Morphological „bottom hat" operation

Input Image

Image after bottom hat operation



# Morphological Gradient

This morphological operator is a composition of

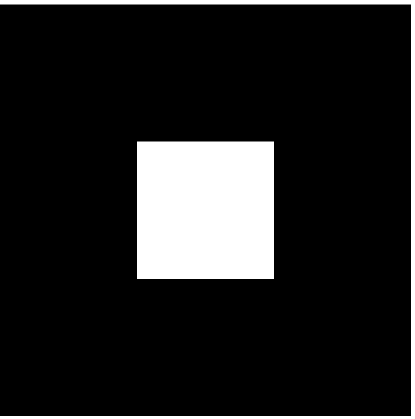
- a dilation an erosion of the input image by the same structuring element  and than the subtraction of these two results.

$$g = (f \oplus b) - (f \ominus b)$$

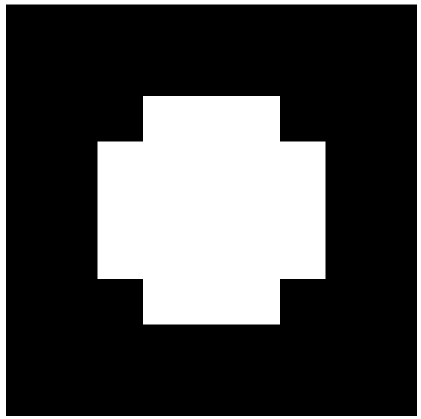
One important application of the morphological gradient on binary images is the boundary extraction (edge detection).

The same operator can be applied to gray level images.

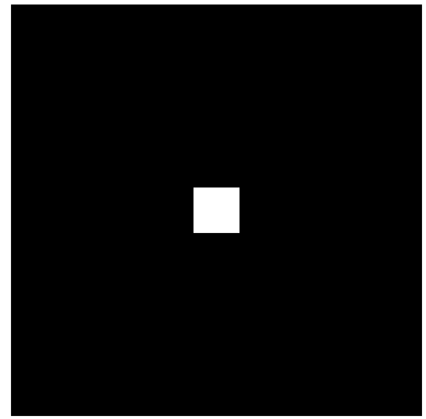
# Morphological Gradient - Example



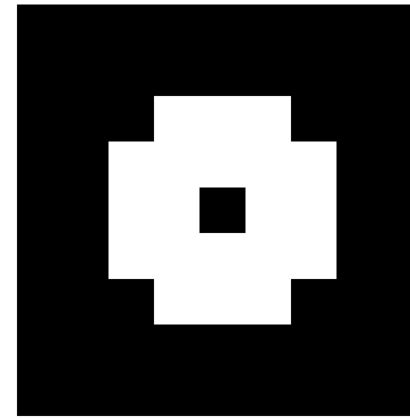
Input image



Dilation



Erosion



Subtraction

$$g = (f \oplus b) - (f \ominus b)$$

Str. element: $\begin{matrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{matrix}$

# Morphological Gradient: An Example of a Gray-scale Image

$$g = (f \oplus b) - (f \ominus b)$$

Input image

Morphological Gradient

# Boundary Extraction – Edge detection

Simple boundary extraction:

The boundary is obtained by eroding (or dilating) then taking the difference of the input image and it's erosion (or dilation).

The thickness of the boundary depends on the size of the structuring element.

# Hit-or-Miss operation for shape detection

This operation requires two structuring elements: SEhm1, SEhm2.

The neighborhoods of those structuring elements SEhm1 and SEhm2 should not have any overlapping elements.

The hit-or-miss operation preserves pixels whose neighborhoods match the shape of structuring element SEhm1 and don't match the shape of SEhm2

hit-or-miss is defined as the intersection of the erosion of A by the first structure element and the erosion of the complement of A by the second structure element:

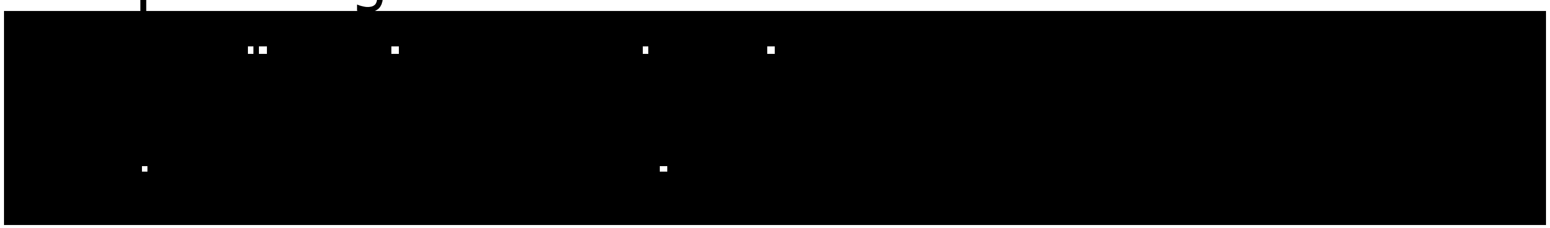
HitOrMiss = Intersect (Erosion (A, SEhm1), Erosion (~A, SEhm2)).

$$HM = (F \ominus SEhm1) \cap (F^c \ominus SEhm2)$$

# Example of the Hit-or-Miss Operation

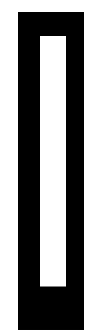
We will develop morphological image processing techniques that use combinations and sequences of operations. These techniques are useful in a wide range of applications.

Input image



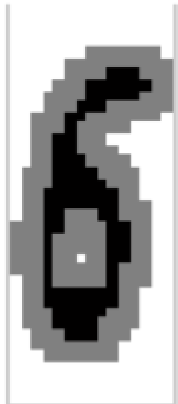
Result image

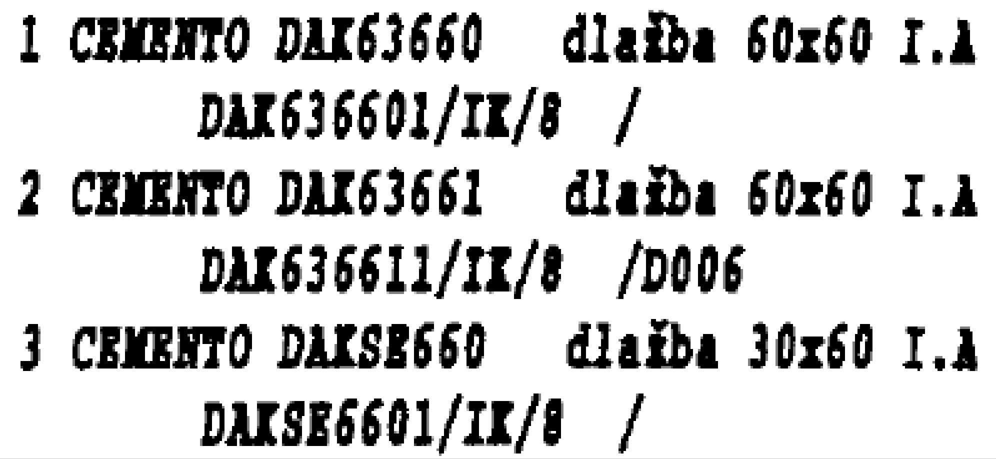
Structuring elements:

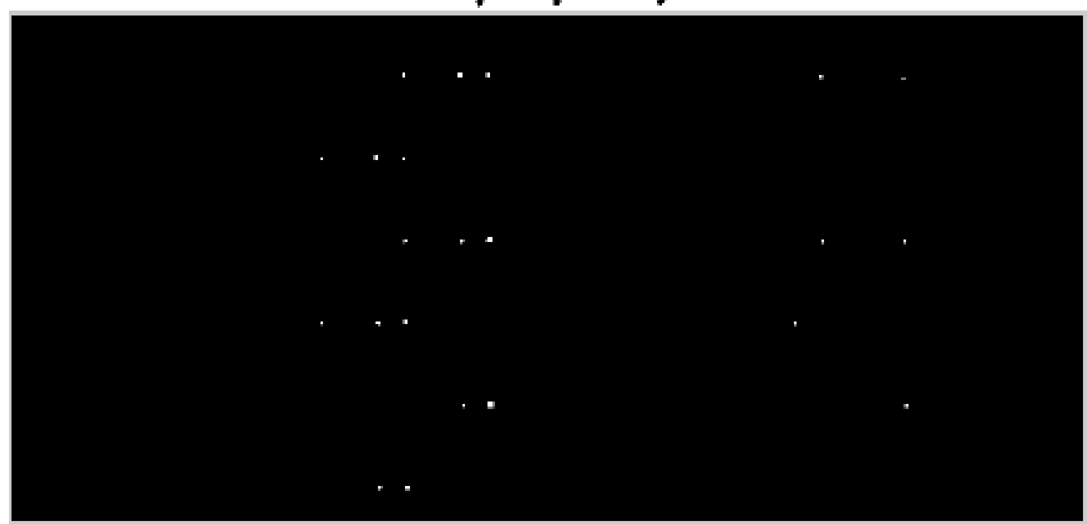


# Example: Hit-Or –Miss

Structuring element:




1 CEMENTO DAK63660 dlažba 60x60 I.A
DAK636601/IK/8 /
2 CEMENTO DAK63661 dlažba 60x60 I.A
DAK636611/IK/8 /D006
3 CEMENTO DAKSE660 dlažba 30x60 I.A
DAKSE6601/IK/8 /



white pixels(value: +1)

are corresponding to the SEhm1,

black pixels(value : -1)

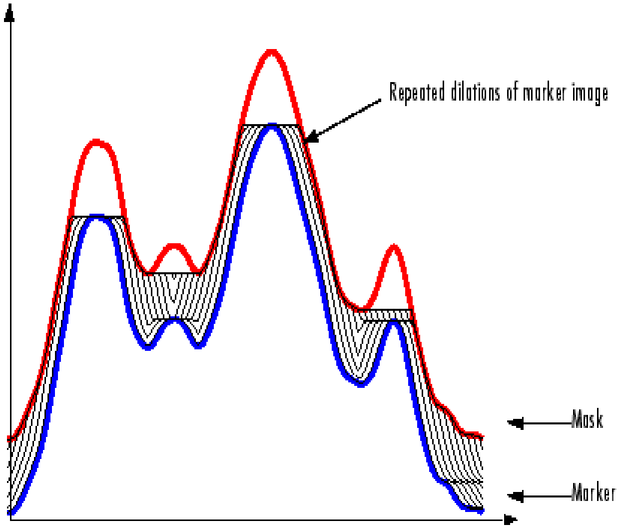
are corresponding to the SEhm2,

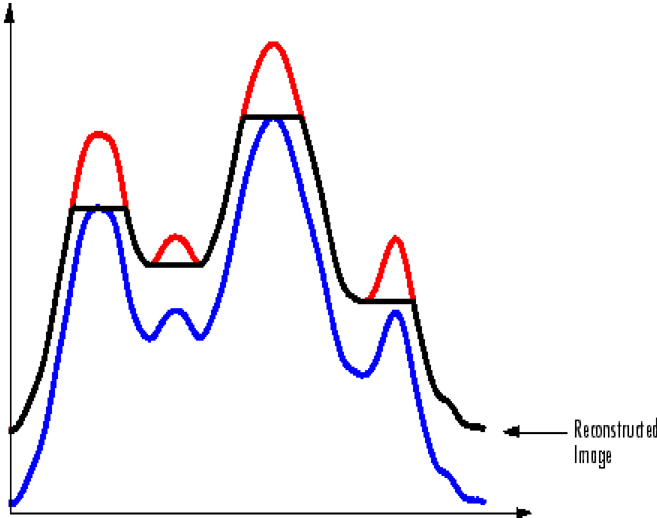
gray pixels (value 0) means ignore.

# Morphological Reconstruction

Morphological reconstruction can be thought of conceptually as repeated dilations of an image, called the marker image, until the contour of the marker image fits under a second image, called the mask image.

In morphological reconstruction, the peaks in the marker image "spread out,"



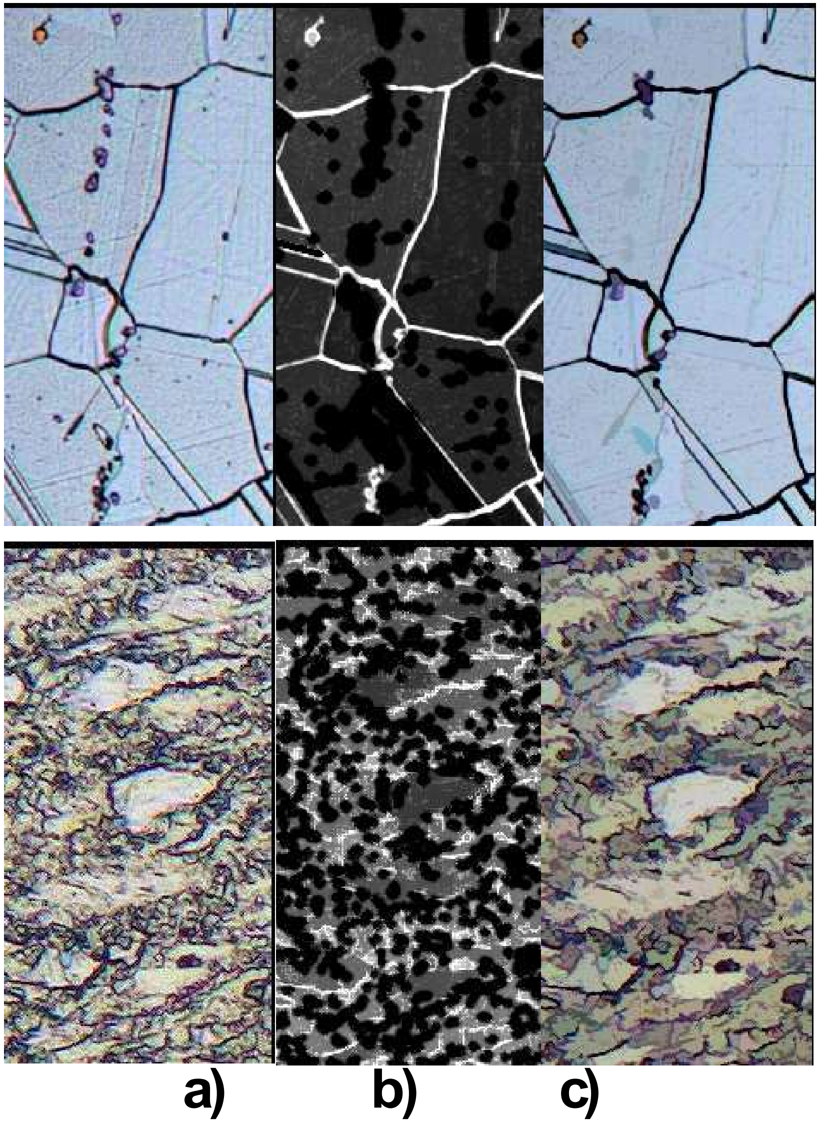
# Morphological Reconstruction

Examples of the image pre- processing and segmentation using mophological reconstruction.

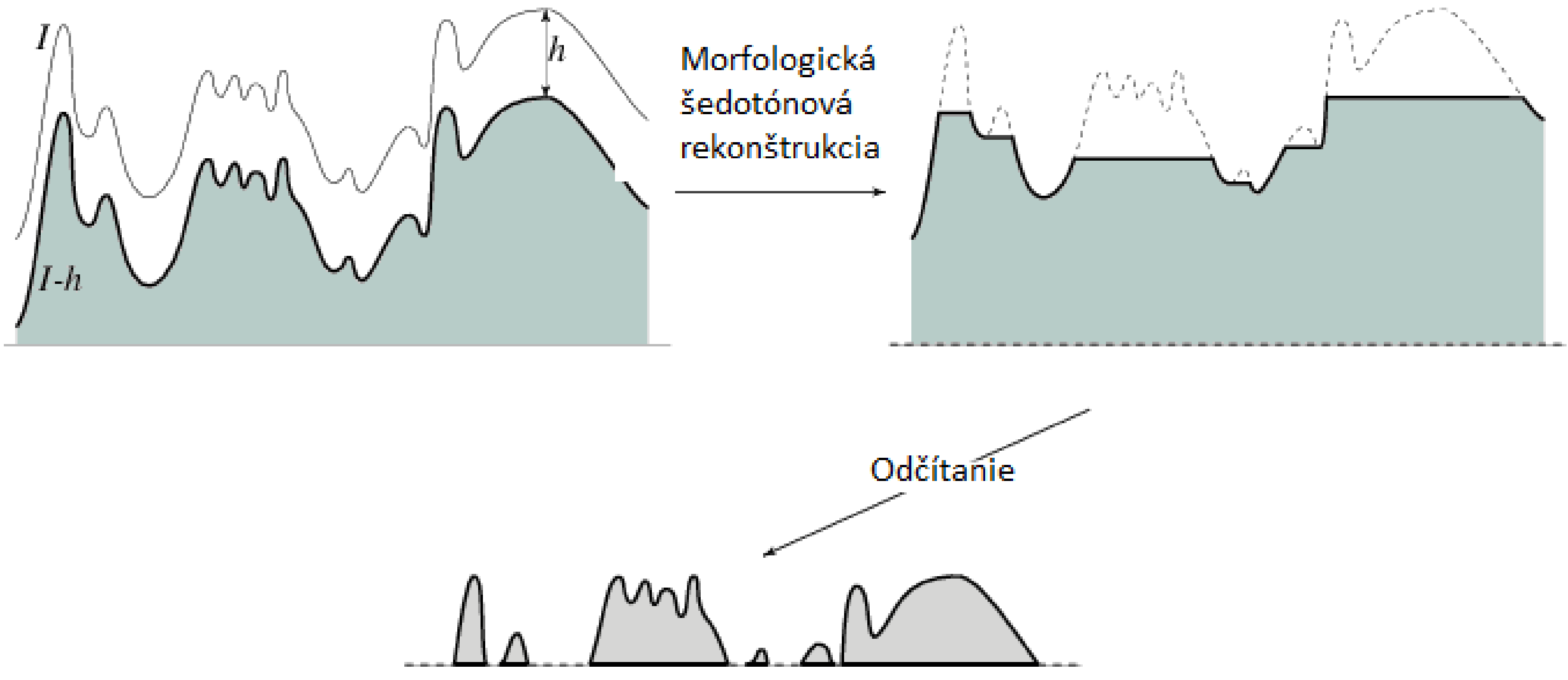
a) original image,

b) marker image

c) reconstructed image



# Morphological Reconstruction
# Extracting of Local Maxima



# Morphological Reconstruction Extracting of Local Maxima - example

Input image

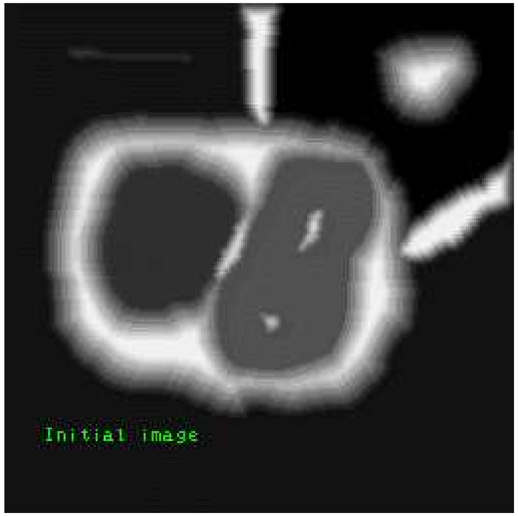
Local Maxima

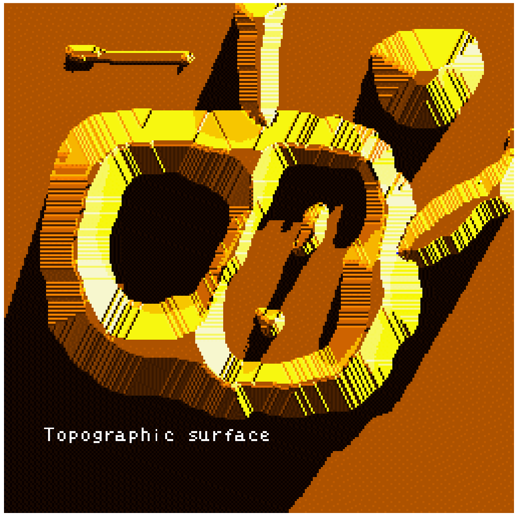
# Watershed Transformation

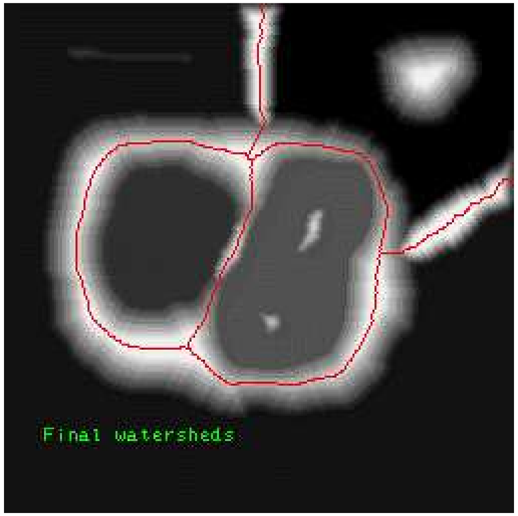
Principle:

Any grey-scale image can be considered as a topographic surface.

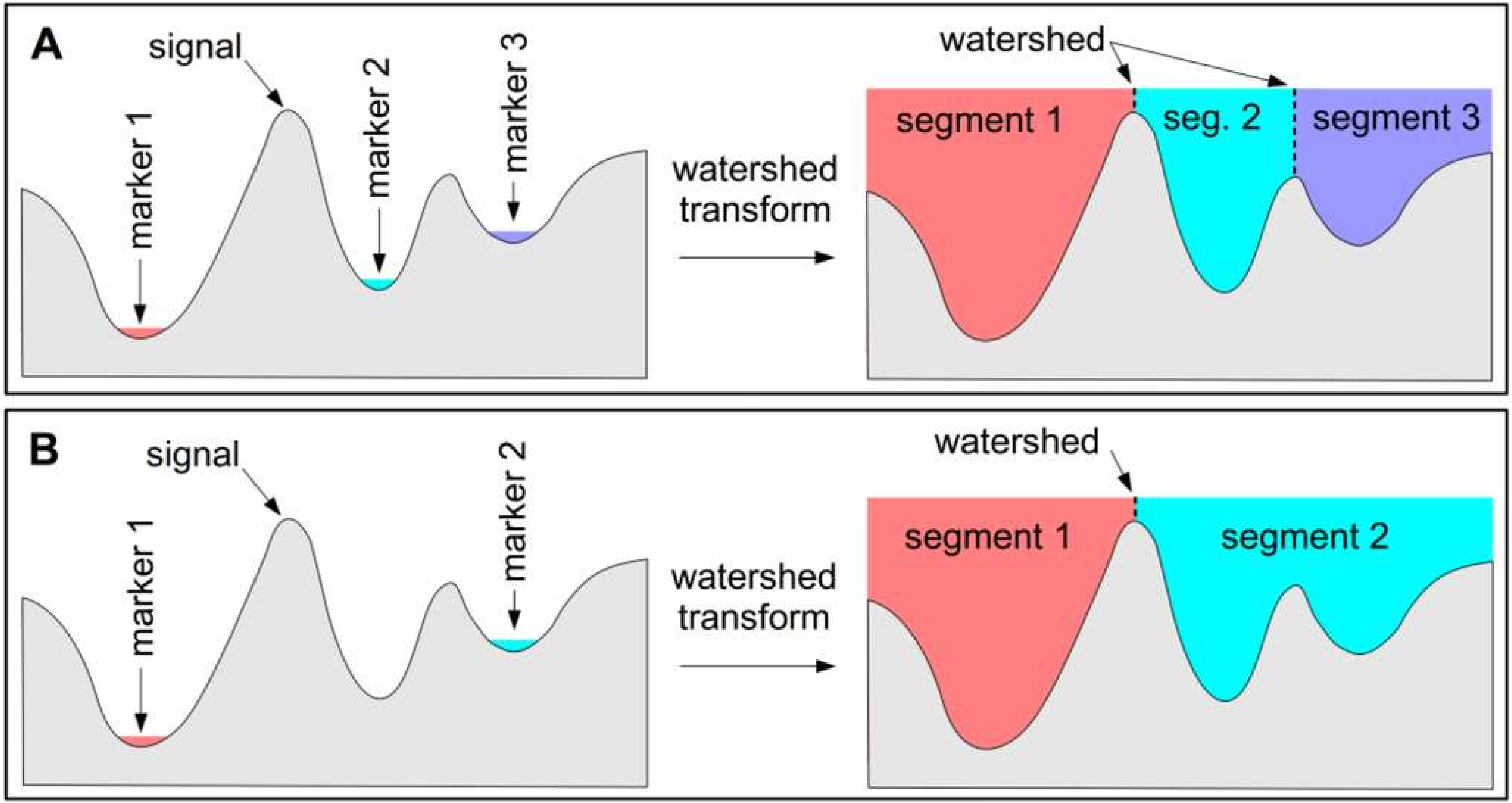
The watershed algorithm is an image processing segmentation algorithm that splits an image into areas, based on the topology of the image.



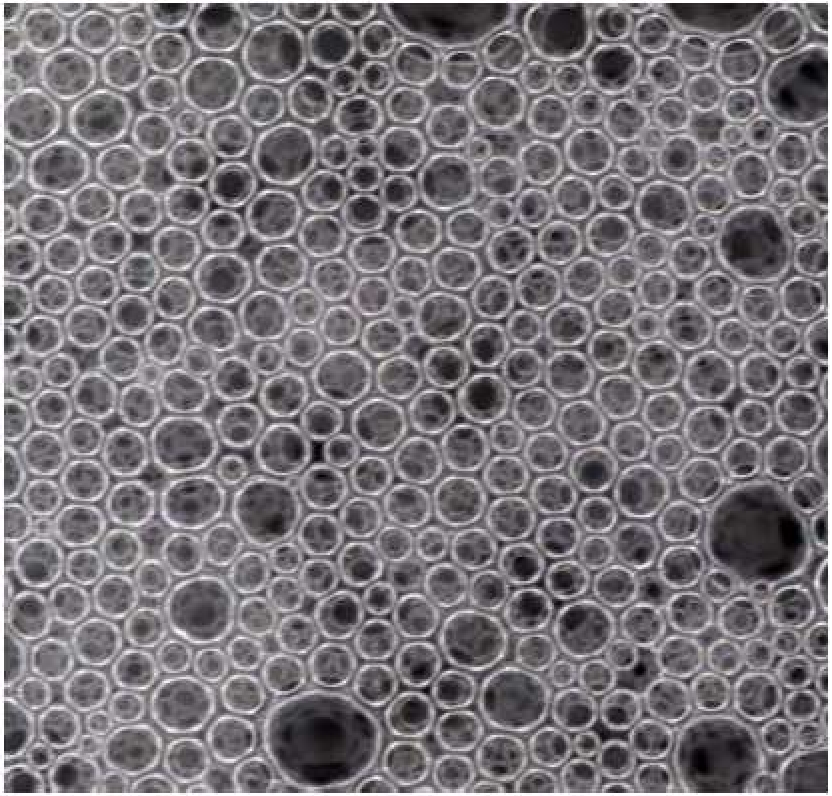



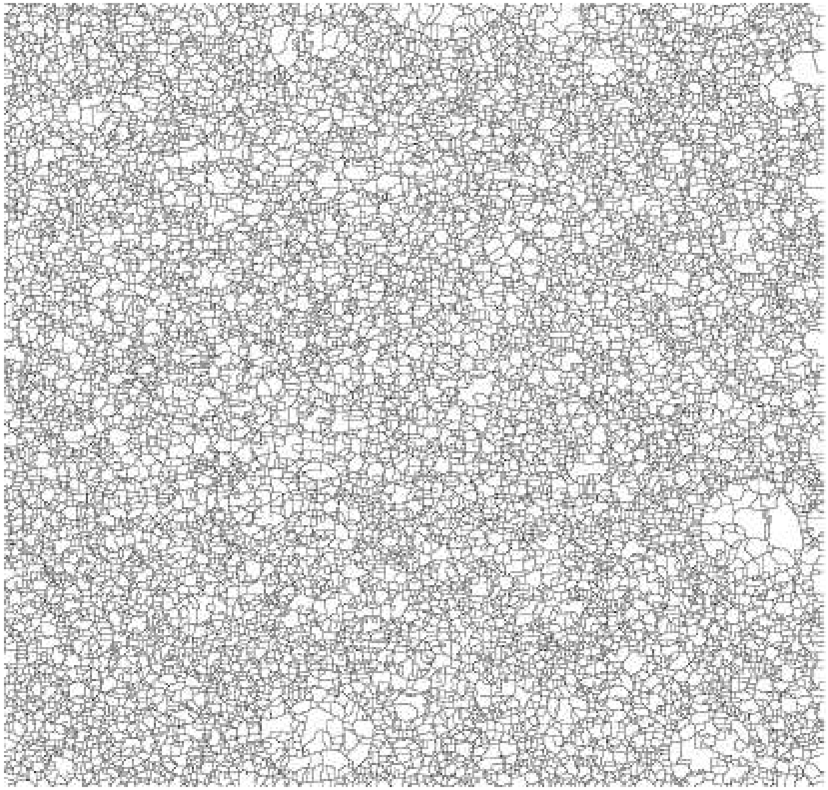



# Watershed Transform

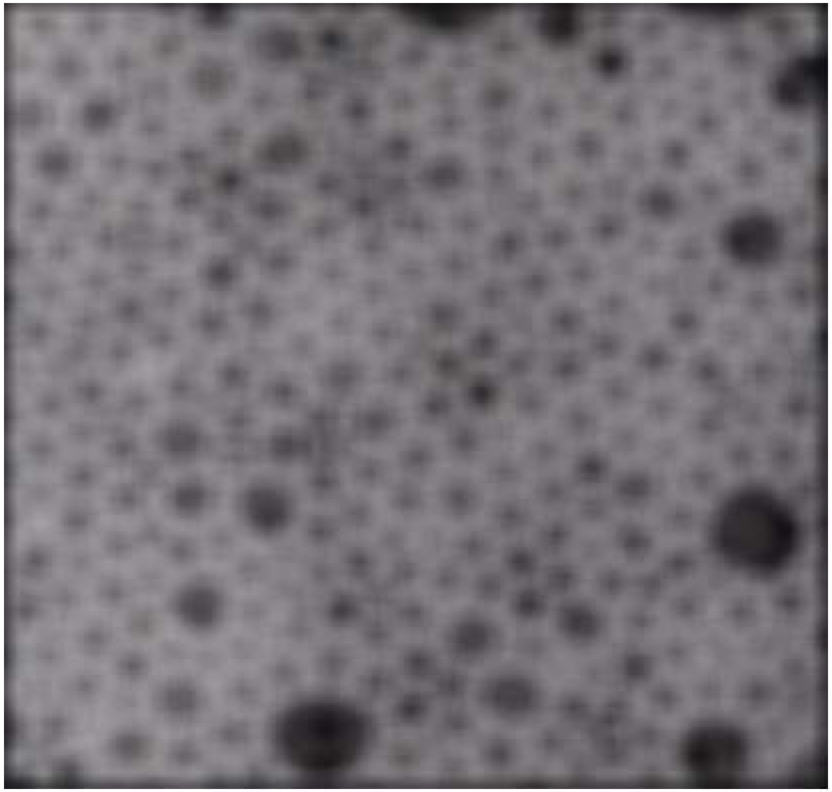


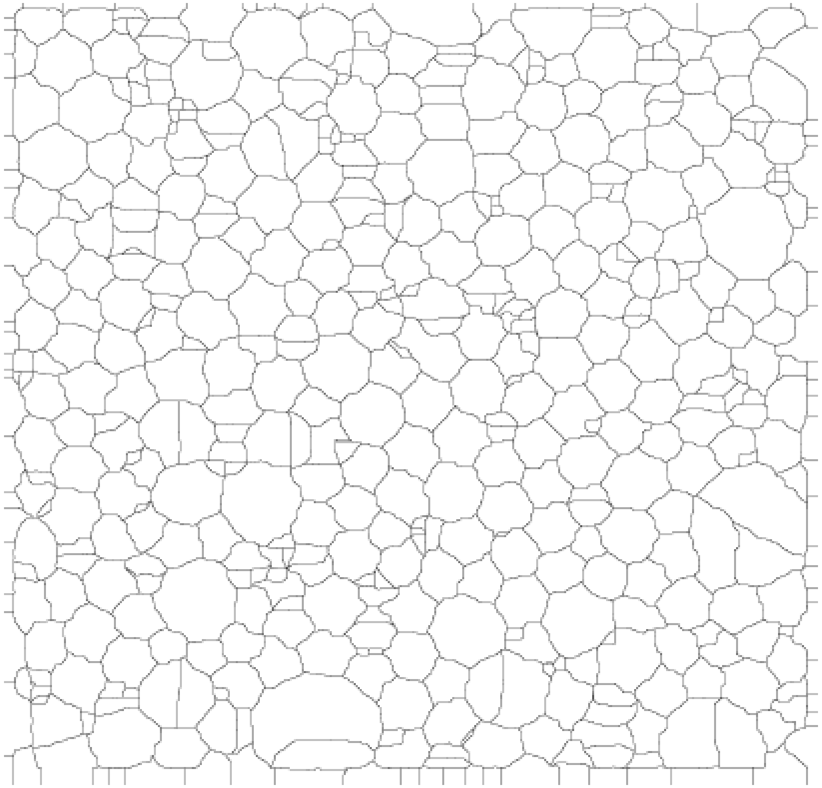
# Watershed – oversegmentation problem





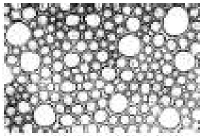
# Watershed after Gauss filtering

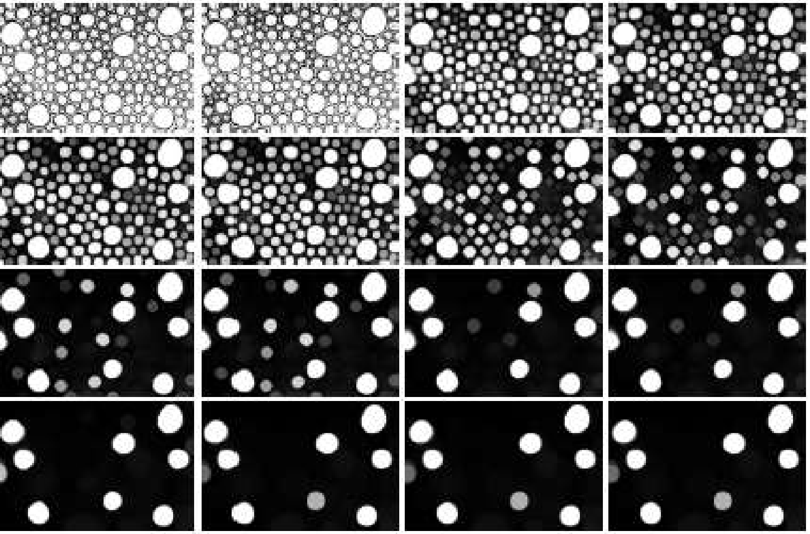


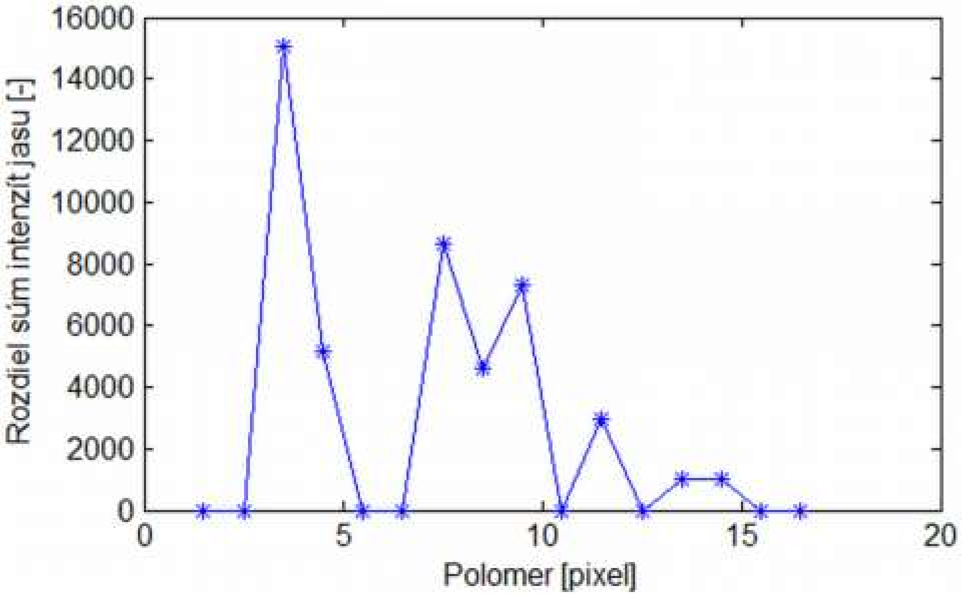


# Granulometry example

Input image








Morphological opening with gradually varying size of structural elements.

Dependence of the difference of brightness before and after the opening operation by using the morphological structural element of circular shape for varying radius.