Lecture Notes

PET504E
Advanced Well Test Analysis

January 2012

---

by
Prof. Dr. Mustafa Onur

Dr. Murat Çınar

1 Introduction

The term "Well Testing" [c1]as it is used in Petroleum Industry means the measuring of a formation's (or reservoir's) pressure (and/or rate) response to flow from a well. The term "Well Testing" is generally used with the term "Pressure Transient Analysis", interchangeably. It is an indirect measurement technique as opposed to direct methods such as fluid sampling or coring. Well testing provides dynamic information on the reservoir whereas direct measurements only provide static information, which is not sufficient for predicting the behavior of the reservoir.

[c1] *Murat Çınar: Text added.*

Simply, the objective of well testing is to deduce quantitative information about the well/reservoir system under consideration from its response to a given input. Input (or input signal) is used for perturbing one or more wells so that the output (signal) exhibiting the response of the reservoir is obtained at the perturbated well and/or adjacent wells. In practice, the input is equivalent to controlling the well behavior [c2] created by changing the flow rate or the pressure at the well (Mathematically specifying the well behavior is equivalent to specifying a boundary condition). A common example for creating an input signal is [c3]a build up test [c4]where we change the rate to zero by shutting-in the well. Reservoir response, [c5] also called output signal, to a given input is monitored by measuring the pressure change (or rate change) at the [c6] well. This process is illustrated as,

[c2] *Murat Çınar:* ~~and~~

[c3] *Murat Çınar: Text added.*

[c4] *Murat Çınar:* ~~in which~~

[c5] *Murat Çınar:* ~~which is~~

[c6] *Murat Çınar:* ~~same~~

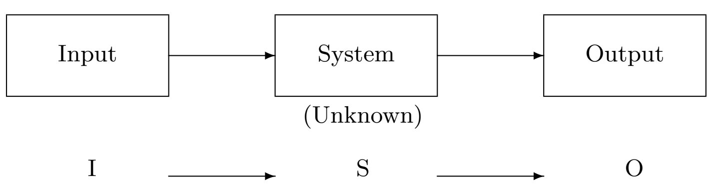


Fig. 1.1. Block diagram ?????

Typical examples for input and output signals as used in petroleum industry are shown in Fig. 6.

From reservoir response as monitored by the "output signal", we would like to determine information related to the followings:

– Fluid in place; pore volume, $\phi h A$.

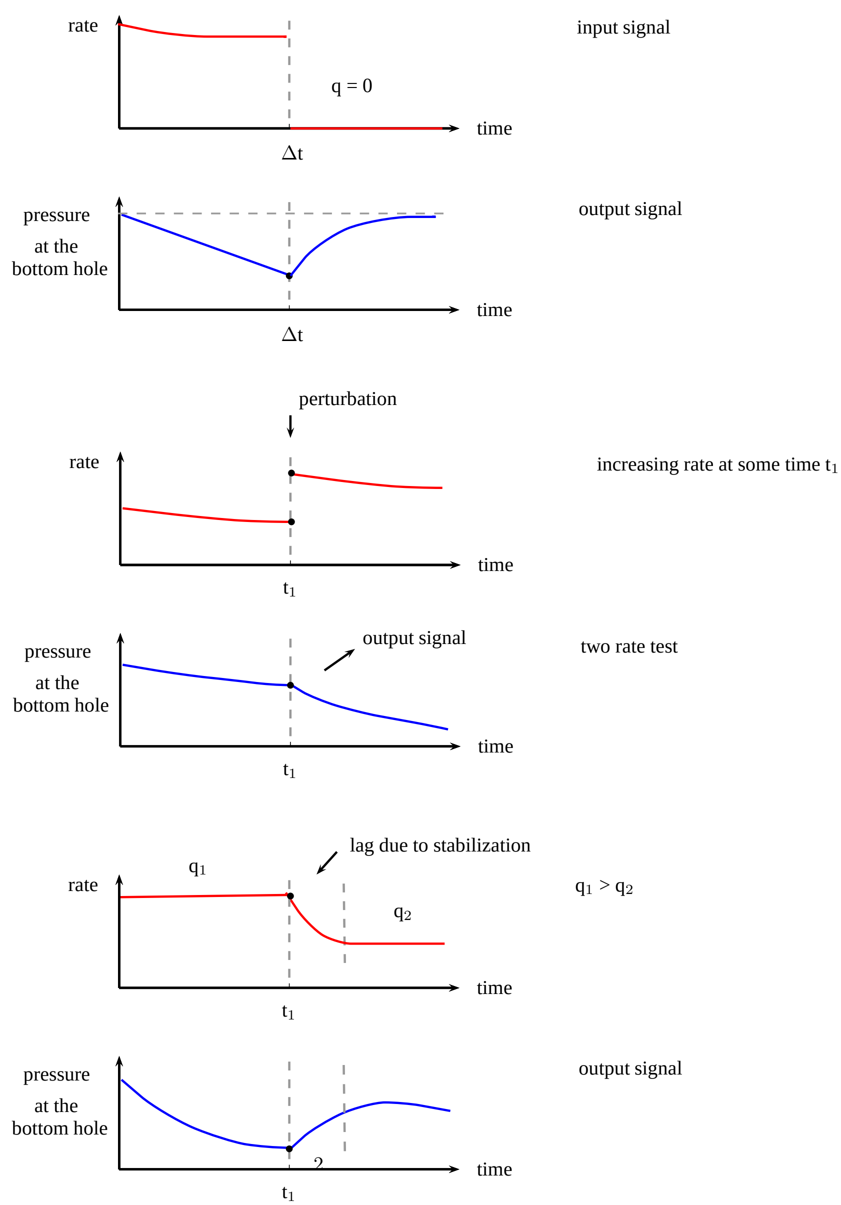


Fig. 1.2. Typical input and output signals - Transient phenomena.

- Ability of reservoir to transfer fluid, kh (or transmissibility, $\frac{kh}{\mu}$).
- Determination of average reservoir [c1]pressure, $\overline{P}$, which is the driving force in the reservoir [c2]
- Prediction of rate versus time data.
- Initial recovery, is the reservoir worth producing.
- Is there any damage around the wellbore impeding the flow? skin factor, s.
- Reservoir description (type of reservoir, flow boundaries (faults)).
- Distance to fluid interface [c3]that is important determining swept zone for secondary and tertiary methods.

Interpretation of well test data consist of basically three steps:

(i) Determination of the one most appropriate reservoir / wellbore (mathematical) model [c4]of the actual system. We also call such a model as the interpretation model. [c5]Here our intention is to find a representative mathematical model that reproduces, as close as possible, the output of the actual system for a given input. This is known as the inverse problem. [c6]We are trying to obtain information about the physical system by using observed measurements. Unfortunately, the solution of inverse problem often yields non-unique results. [c7]By non-unique results, we mean that several different interpretation models [c8]may generate an output signal (response) to a given input [c9]that is similar (or identical) to that of the actual system. The inverse problem can be represented by the following equation.

$$\Sigma = O/I \ \approx S \tag{1.1}$$

where Σ denotes the interpretation model, S denotes the actual system. In inverse problem, as can be seen from Eq.1.1, it may be possible to obtain the same outputs to a given I for different Σ_i's[c10],however, the number of alternative models (solutions) can be reduced as the number and the range of output signal measurements.

(ii) Once the appropriate model is determined, estimate the parameters of the actual system S. [c11]These parameters are kh, s, ϕ, C, λ, w etc. This is known as [c12]parameter estimation[c13][c14] and achieved by adjusting the parameters of the model by different [c15]mathematical methods to obtain an output signal, Ω, that is always qualitatively identical (within some tolerance) to that of [c16]the actual system, O. The computation of Ω is known as the "[c17]forward problem" in mathematics. Contrary to the inverse problem, the solution of the [c18]forward problem is always unique for a given system; that is,

$$I \times \Sigma = \Omega \approx 0 \tag{1.2}$$

The adjusted parameters of the interpretation model are assumed to represent the parameters of the real system S. [c19]

(iii) Validate the results of the interpretation. This can be achieved by using the parameters determined from part (ii) in the model to generate output

[c1] *Murat Çınar*: ~~average~~

[c2] *Murat Çınar*: Based upon the explanation you gave about the pressure decline recently, I am not sure if this statement is correct.

[c3] *Murat Çınar*: ~~which~~

[c4] *Murat Çınar*: ~~to~~

[c5] *Murat Çınar*: ~~Here our hope is that the model chosen will produce an output signal to a given input which is as close as possible to that of the actual system.~~

[c6] *Murat Çınar*: *Text added.*

[c7] *Murat Çınar*: ~~With~~

[c8] *Murat Çınar*: ~~can~~

[c9] *Murat Çınar*: ~~which~~

[c10] *Murat Çınar*: ~~. However~~

[c11] *Murat Çınar*: ~~Such parameters can be~~

[c12] *Murat Çınar*: ~~the~~

[c13] *Murat Çınar*: ~~problem~~

[c14] *Murat Çınar*: ~~. It is~~

[c15] *Murat Çınar*: *Text added.*

[c16] *Murat Çınar*: *Text added.*

[c17] *Murat Çınar*: ~~direct~~

[c18] *Murat Çınar*: ~~direct~~

[c19] *Murat Çınar*: I think we should discuss this phrase. Does the real system have any parameters? I do not think so... This is something conceptual we need to think about

signals for the entire range of [c20]the test and by comparing these outputs with the [c21]physical measurements.

[c20] *Murat Çınar*: *Text added.*
[c21] *Murat Çınar*: ~~measured ones~~

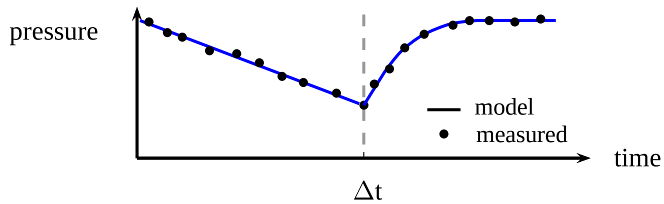


Fig. 1.3. Parameters estimated based on the analysis of buildup data.

[c1]Now we consider single phase flow in a cylindrical reservoir produced by a well at the center. The[c2]partial differential equation (p.d.e.) describing the flow is given by,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{kr}{\mu}\frac{\partial p}{\partial r}\right) = \phi c_t \frac{\partial p}{\partial t} \tag{1.3}$$

or if k, μ are constant,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\phi c_t \mu}{k}\frac{\partial p}{\partial t} = \frac{1}{\eta}\frac{\partial p}{\partial t} \tag{1.4}$$

[c1] *Murat Çınar*: ~~To illustrate an example of direct and inverse problems, let's consider single phase flow in a closed cylindrical reservoir produced by a single well at the center.~~
[c2] *Murat Çınar*: ~~pd.E~~

[c3]$\eta = \frac{k}{\phi c_t \mu}$ is the hydraulic diffusivity[c4]; a measure of the [c5]speed at which a pressure disturbance [c6]propagates through the formation. If we specify, k, ϕ, c_t, μ, and the flow rate, then $p(r,t)$ is uniquely determined. This is an example for the [c7]forward problem.

[c3] *Murat Çınar*: ~~$\eta = \frac{\phi c_t \mu}{k}$~~
[c4] *Murat Çınar*: ~~which is~~
[c5] *Murat Çınar*: ~~rapidity with~~
[c6] *Murat Çınar*: *Text added.*
[c7] *Murat Çınar*: ~~direct~~

Inverse problem, given q and p, [c8]helps us to

1. determine the p.d.e. that describes the reservoir best
2. find k, ϕ, etc.

[c8] *Murat Çınar*: *Text added.*

[c9]

[c10]Analytical solutions have been presented in the literature for a variety of different well and reservoir settings for single phase flow. A summary of these responses are given by Bourdet [4].

[c9] *Murat Çınar*: ~~During the past ten years, a lot of work has been done to develop models to a wide variety of reservoir / well configurations such as fractures, layered reservoirs, multiple porosities (composite zones), fractured wells, slanted and horizontal wells, etc. Well testing literature is almost complete for single phase problems, but some work needs to be done for multi-phase problems and heterogeneous reservoir systems.~~

~~Recently, people are much focused on the model recognition problem and the computer-aided parameter estimation by using non-linear regression techniques. Pressure derivatives and integrals are proven to be very useful in identifying~~

2 Flow Equations

In this chapter, we will derive the equations which describe the fluid flow in porous media. Such equations are derived from the conservation of mass and the momentum equation as given by Darcy's semi-empirical equation. With the exception of thermal recovery schemes, all well-testing models assume isothermal conditions in the reservoir and thus the energy conservation is not needed.

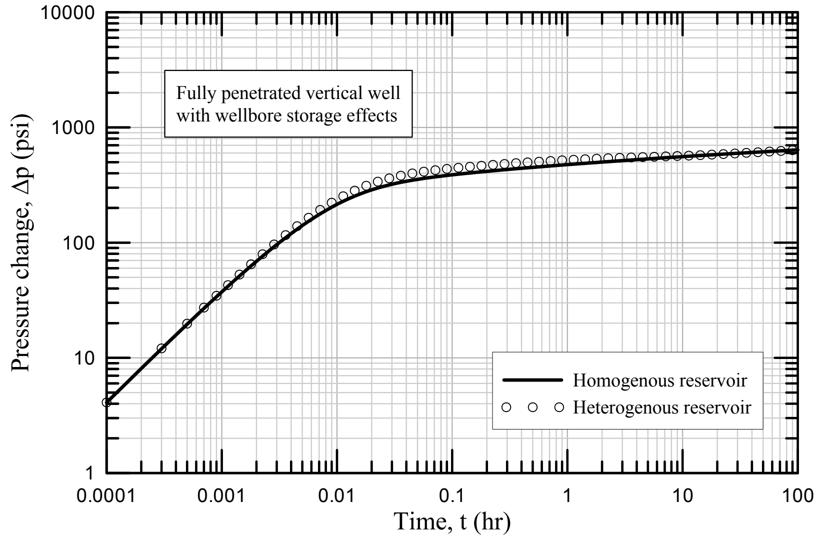


Fig. 1.4. Homogeneous vs heterogeneous reservoir, pressure difference .

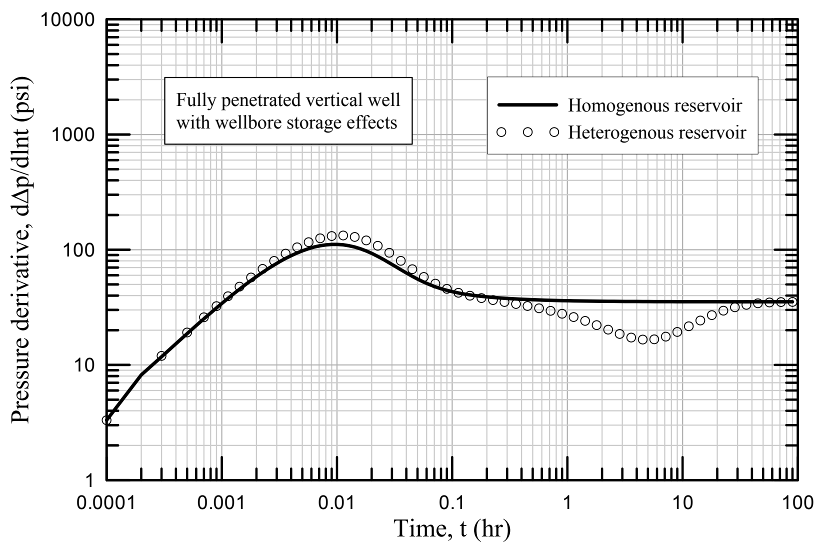


Fig. 1.5. Homogeneous vs heterogeneous reservoir - logarithmic derivative.

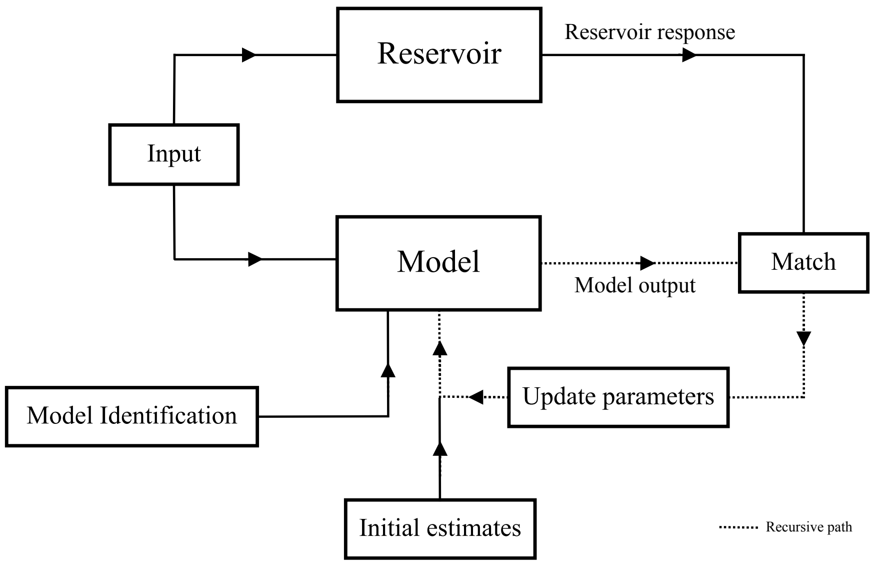


Fig. 1.6. Flow diagram of computer aided parameter estimation.

2.1 Conservation of mass

For a single phase fluid [c11], the mathematical form of the mass balance in porous media is given by

[c11] *Murat Çınar*: ~~(or a fluid composed of one hydrocarbon component)~~

$$-\nabla \cdot (\rho \mathbf{v}) = \frac{\partial (\rho \phi)}{\partial t} \tag{2.1}$$

[c1] where ρ is the density of the fluid in [c2]$\mathbf{M/L^3}$ and $\mathbf{v}$ is the fluid velocity vector in [c3]$\mathbf{L/T}$. Note that the units of Eq. 2.1 is [c4]$\mathbf{M/L^3T}$ It is also important to note that Eq. 2.1 applies for any coordinate system and can be derived either from a mass balance done on a control volume for a coordinate system under consideration or from [c5] divergence theorem (or Gauss Theorem see Supplement II).

[c1] *Murat Çınar*: I removed the constant -5.615 from the equation

[c2] *Murat Çınar*: $\cancel{lbm/ft^3}$

[c3] *Murat Çınar*: $\cancel{RB/ft^2 day}$

[c4] *Murat Çınar*: $\cancel{lbm/ft^3 day}$

[c5] *Murat Çınar*: ~~a~~

In[c6] Cartesian coordinate system,

[c6] *Murat Çınar*: ~~x-y-x~~

$$\nabla \cdot (\rho \mathbf{v}) = \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) \tag{2.2}$$

In[c7] cylindrical coordinate system,

[c7] *Murat Çınar*: ~~r-θ-z~~

$$\nabla \cdot (\rho \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) \tag{2.3}$$

[c8]

[c8] *Murat Çınar*: typo in the equation corrected

2.2 Conservation of momentum in porous media

[c9] *Murat Çınar*: ~~can be~~

The principle of momentum conservation is described by the equation of motion. For most hydrocarbon fluids, the shear stress - shear rate behavior [c9]is described by the Newton's law of friction, combined with the equation of motion, results in the well known Navier-Stokes equation. Solution of the Navier-Stokes equation with the appropriate boundary conditions yields the velocity distribution of a given problem. Although, it is possible to solve Navier-Stokes in pipe flow, it is almost impossible to solve due to complexity of the pore geometry and its distribution. This hinders the formation of the boundary conditions for flow through a porous medium. Therefore, a different approach [c10]is taken. In 1856, [c11]Darcy discovered that [c12]for a single phase viscous flow in porous media, the velocity is proportional to the pressure gradient with a proportionality constant k. The general form of Darcy's Law including gravity effects is given by Eq.2.4.

[c10] *Murat Çınar*: ~~must be~~

[c11] *Murat Çınar*: ~~a French hydraulic engineer Henry~~

[c12] *Murat Çınar*: ~~the velocity vector and the pressure gradient for a single phase viscous flow~~

$$\mathbf{v} = -\frac{\mathbf{k}}{\mu}\left(\nabla p - \rho g \nabla z'\right) \tag{2.4}$$

[c1] where $\mathbf{v}$ is defined as a volumetric flow rate across a unit cross-section area (solid+fluid) averaged over a small region of space. The unit of $\mathbf{v}$ is in [c2]**L/T**. Eq. 2.4 yield[c3]s a velocity vector [c4]that replaces the solution of Navier-Stokes equation. In Eq. 2.4 $\mathbf{k}$ is [c5]the permeability tensor. Operationally, $\mathbf{k}$ acts like a matrix in [c6]coordinate system, we usually [c7]assume

[c1] *Murat Çınar*: I removed the filed unit constants

[c2] *Murat Çınar*: ~~$RB/ft^2 day$~~

[c3] *Murat Çınar*: *Text added.*

[c4] *Murat Çınar*: ~~which~~

[c5] *Murat Çınar*: ~~a~~

[c6] *Murat Çınar*: ~~x-y-z~~

[c7] *Murat Çınar*: ~~use~~

$$\mathbf{k}\,\nabla p = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix} \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix} = \begin{bmatrix} k_x \frac{\partial p}{\partial x} \\ k_y \frac{\partial p}{\partial y} \\ k_z \frac{\partial p}{\partial z} \end{bmatrix}$$

$$\nabla z' = \begin{bmatrix} \frac{\partial z'}{\partial x} \\ \frac{\partial z'}{\partial y} \\ \frac{\partial z'}{\partial z} \end{bmatrix}$$

where, z' is the direction in which gravity acts, i.e., the direction towards the center of the earth. In [c8]Cartesian coordinate system, [c9] for each velocity component,

[c8] *Murat Çınar*: ~~x-y-z~~

[c9] *Murat Çınar*: ~~Eq. 2.5~~

$$v_\xi = -\frac{k_\xi}{\mu}\left(\frac{\partial p}{\partial \xi} - \rho g \frac{\partial z'}{\partial \xi}\right), \quad \xi = x, y, z \tag{2.5}$$

We generally denote γ as the specific weight of fluid and define as,

$$\gamma = \rho g$$

It follows from Eq. 2.5, 2.2, and 2.1 that the p.d.e. describing conservation of mass in [c10]Cartesian coordinate system is

[c10] *Murat Çınar*: ~~x-y-z~~

$$
\begin{aligned}
&\frac{\partial}{\partial x}\left[\rho\frac{k_x}{\mu}\left(\frac{\partial p}{\partial x}-\gamma\frac{\partial z'}{\partial x}\right)\right] \\
&+\frac{\partial}{\partial y}\left[\rho\frac{k_y}{\mu}\left(\frac{\partial p}{\partial y}-\gamma\frac{\partial z'}{\partial y}\right)\right] \\
&+\frac{\partial}{\partial z}\left[\rho\frac{k_z}{\mu}\left(\frac{\partial p}{\partial z}-\gamma\frac{\partial z'}{\partial z}\right)\right]=\frac{\partial}{\partial t}\left(\rho\phi\right)
\end{aligned}
\tag{2.6}
$$

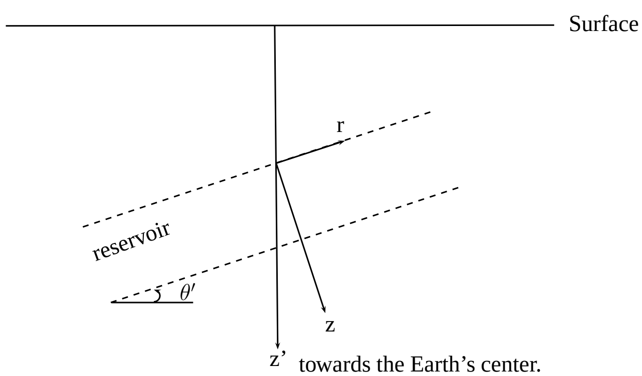
In [c1]Cylindrical coordinates (neglecting flow in θ direction)

[c1] *Murat Çınar*: ~~r-z~~

$$
\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k_r}{\mu}\rho\left(\frac{\partial p}{\partial r}-\gamma\frac{\partial z'}{\partial r}\right)\right]+\frac{\partial}{\partial z}\left[\frac{k_z}{\mu}\rho\left(\frac{\partial p}{\partial z}-\gamma\frac{\partial z'}{\partial z}\right)\right]=\frac{\partial}{\partial t}\left(\rho\phi\right)
\tag{2.7}
$$

Remark on gravity term:

In [c2]Cylindrical coordinates,[c3]assume z and z' are in the same direc-



tion then,

[c2] *Murat Çınar*: ~~r-θ-z~~

[c3] *Murat Çınar*: ~~if reservoir is horizontal; i.e. $\theta' = 0$~~

$$
\frac{\partial z'}{\partial r}=\frac{\partial z'}{\partial \theta}=0 \quad \mathit{and} \quad \frac{\partial z'}{\partial z}=1
$$

[c4]if the reservoir is dipping with an angle of θ',

[c4] *Murat Çınar*: ~~If we had a dip angle θ'~~

$$
\frac{\partial z'}{\partial r}=\sin\theta' \quad \frac{\partial z'}{\partial \theta}=\cos\theta' \quad \frac{\partial z'}{\partial z}=\cos\theta'
$$

[c5]Now consider a single well [c6]at the center of a cylindrical reser-

[c5] *Murat Çınar*: ~~If we~~

[c6] *Murat Çınar*: ~~in~~

voir, then Eq. 2.7 correctly describes the fluid flow for both partially penetrating and fully penetrating cases, see Fig 6.
c7 c8 Now define formation volume factor B as,

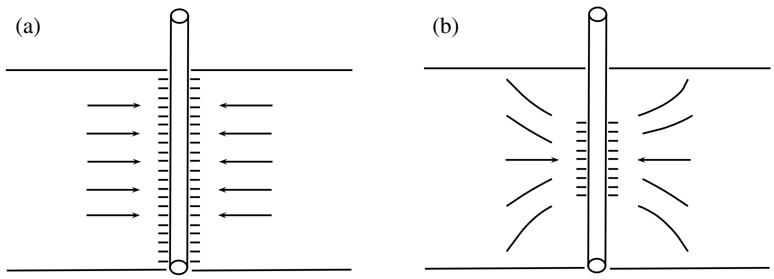


Fig. 2.1. (a) Fully penetrated vertical well. (b) Partially penetrated vertical well.

c7 *Murat Çınar*: ~~Recalling our general continuity equation for the single phase flow gives,~~

c8 *Murat Çınar*: *Text added.*

$$B = \frac{V_{reservoir}}{V_{SC}} = \frac{m/\rho}{m/\rho_{\ SC}} = \frac{\rho_{SC}}{\rho} \quad ; \quad \rho_{SC}\,\text{is constant} \tag{2.8}$$

c1 *Murat Çınar*: ~~which we call it as Formation volume factor, then we can write Equation (2.8) as~~

c2 *Murat Çınar*: *Text added.*

c1 c2 Recall general continuity equation, Eq. 2.1 and insert Eq. 2.8, then we have,

$$-\nabla \cdot \left(\frac{\mathbf{v}}{B}\right) = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) \tag{2.9}$$

$B = B(p)$, $\rho = \rho(p)$, and $\phi = \phi(p) \to$ single valued functions of p

c3 *Murat Çınar*: *Text added.*

c3 Expanding right hand side (RHS) of Eq. 2.9,

$$\begin{aligned} \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) &= \phi\frac{\partial}{\partial t}\left(\frac{1}{B}\right) + \frac{1}{B}\frac{\partial \phi}{\partial t} \\ &= \phi\left(-\frac{1}{B^2}\frac{dB}{dp}\right)\frac{\partial p}{\partial t} + \frac{1}{B}\frac{d\phi}{dt}\frac{\partial p}{\partial t} \\ &= \frac{\phi}{B}\left(-\frac{1}{B}\frac{dB}{dp} + \frac{1}{\phi}\frac{d\phi}{dt}\right)\frac{\partial p}{\partial t} \end{aligned} \tag{2.10}$$

c4 *Murat Çınar*: Typo in Eq corrected.

c5 *Murat Çınar*: ~~but~~

c4 c5 Fluid and rock compressibilities are defined, respectively, by,

$$c_{fluid} = -\frac{1}{V}\frac{dV}{dp} = -\frac{1}{B}\frac{dB}{d\rho} = \frac{1}{\rho}\frac{d\rho}{dp} \tag{2.11}$$

$$c_r = c_f = \frac{1}{\phi}\frac{d\phi}{dp} \tag{2.12}$$

(here p is the fluid pressure in the pore, therefore, $\frac{d\phi}{dp} > 0$.)
Using Eqs. 2.11 and 2.12 in Eq. 2.10 and the resulting equation in Eq. 2.9 gives,

$$-\nabla \cdot \left(\frac{\mathbf{v}}{B}\right) = \frac{\phi c_t}{B}\frac{\partial p}{\partial t} \tag{2.13}$$

[c1]Note that here the total compressibility c_t is defined as $c_t = c_{fluid} + c_f$. Under the assumptions of Darcy's Law we have,

[c1] *Murat Çınar: Text added.*

$$\nabla \cdot \left(\frac{\mathbf{k}}{\mu B}\left(\nabla p - \gamma \nabla z'\right)\right) = \frac{\phi c_t}{B}\frac{\partial p}{\partial t} \tag{2.14}$$

Slightly compressible fluid of constant compressibility Now assuming negligible gravity effects and k/μ is constant then,

$$\frac{k}{\mu}\nabla \cdot (\rho \nabla p) = \phi c_t \rho \frac{\partial p}{\partial t} \tag{2.15}$$

[c2]Assuming $c(\nabla p)^2$ is small here $c = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$, Eq. 2.15 is well approximated by

[c2] *Murat Çınar:* ~~If we assume~~

$$\frac{k}{\mu}\nabla^2 p = \phi c_t \frac{\partial p}{\partial t} \tag{2.16}$$

Remark on coordinate systems:

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \quad \text{in Cartesian coordinates}$$

$$\nabla^2 p = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} \quad \text{in cylindrical coordinates}$$

$$\nabla^2 p = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial p}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 p}{\partial \phi^2} \quad \text{in spherical coordinates}$$

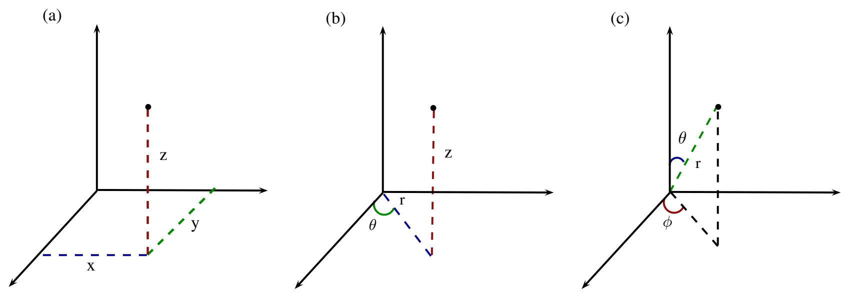


Fig. 2.2. (a) Cartesian coordinates. (b) Cylindrical coordinates. (c) Spherical coordinates.

[c1] *Murat Çınar*: ~~if we assume~~

Further [c1]assuming,

$$c = \frac{1}{\rho}\frac{\partial \rho}{\partial p} = \text{constant} \tag{2.17}$$

$$\int_{p_i}^{p} c dp = \int_{\rho_i}^{\rho} \frac{1}{\rho} d\rho$$

$$c\left(p - p_i\right) = \ln\left(\frac{\rho}{\rho_i}\right) \quad ; \quad p_i \,\text{initial pressure}$$

or

$$\rho = \rho_i \exp\left[-c\left(p_i - p\right)\right] \tag{2.18}$$

Using Taylor series representation of $\exp\left[-c\left(p_i - p\right)\right]$ gives,

$$\begin{aligned} \rho &= \rho_i \left[1 - c\left(p_i - p\right) + \frac{c^2}{2}\left(p_i - p\right)^2 + \cdots\right] \\ &= \rho_i \left[1 - c\left(p_i - p\right) + \frac{c^2}{2}\left(p_i - \widetilde{p}\right)^2\right] \quad p < \widetilde{p} < p_i \end{aligned}$$

$$\rho = \rho_i \left[1 - c\left(p_i - p\right)\right] \tag{2.19}$$

[c2] *Murat Çınar*: *Text added.*

[c2]Note that c is very small for oil (or liquids); $c \approx 10^{-5} \sim 10^{-6}$. Using Eq. 2.19 in Eq. 2.16.

$$\frac{k}{\mu}\nabla^2 p = \phi c_t \frac{\partial p}{\partial t} \tag{2.20}$$

2.3 Multiphase flow

Three distinct phases, gas, oil, and water occur i a petroleum reservoir. Varying pressure conditions (isothermal system assumed) cause a mass exchange between [c3]two hydrocarbon phases [c4]- oil-gas (water-oil and gas-water systems [c5]is assumed immiscible). The [c6]mass transfer between oil and gas is described by solution gas-oil ration, R_s, which gives the amount of gas dissolved in oil as a function of pressure, i.e. $[V_{dissolved\,gas}/V_o]_{STC}$.

[c3] *Murat Çınar*: ~~the~~
[c4] *Murat Çınar*: *Text added.*
[c5] *Murat Çınar*: ~~can be~~
[c6] *Murat Çınar*: ~~material~~

The fluid flow equations (based on β-model) with the introduction of phase saturations for oil-water-gas system [c1]is written as;

[c1] *Murat Çınar*: ~~can be~~

$$-\nabla \cdot \left(\frac{\mathbf{v}_o}{B_o} \right) = \frac{\partial}{\partial t} \left(\frac{\phi S_o}{B_o} \right) \quad for\, oil \tag{2.21}$$

$$-\nabla \cdot \left(\frac{\mathbf{v}_w}{B_w} \right) = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) \quad for\, water \tag{2.22}$$

$$-\nabla \cdot \left(\frac{R_s \mathbf{v}_o}{B_o} + \frac{\mathbf{v}_g}{B_g} \right) = \frac{\partial}{\partial t} \left[\phi \left(\frac{R_s}{B_o} S_o + \frac{S_g}{B_g} \right) \right] \quad for\, gas \tag{2.23}$$

With introducing relative permeability [c2], the velocity vector for each phase is given by,

[c2] *Murat Çınar*: ~~concept~~

$$\mathbf{v}_\varphi = -\frac{\mathbf{k}\, k_{r\varphi}}{\mu_\varphi} \left(\nabla p_\varphi - \gamma_\varphi \nabla z' \right) \tag{2.24}$$

where $\varphi = o, w, or g$.

Using Eq. 2.24 in Eqs. 2.21, 2.22, and 2.23 for corresponding [c3]phase,

[c3] *Murat Çınar*: ~~Hydro Carbon component~~

$$\nabla \cdot \left(\frac{\mathbf{k}\, k_{ro}}{B_o \mu_o} \left(\nabla p_o - \gamma_o \nabla z' \right) \right) = \frac{\partial}{\partial t} \left(\frac{\phi S_o}{B_o} \right) \tag{2.25}$$

$$\nabla \cdot \left(\frac{\mathbf{k}\, k_{rw}}{B_w \mu_w} \left(\nabla p_w - \gamma_w \nabla z' \right) \right) = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) \tag{2.26}$$

$$\begin{aligned} \nabla \cdot \left(\frac{R_s \mathbf{k}\, k_{ro}}{B_o \mu_o} \left(\nabla p_o - \gamma_o \nabla z' \right) + \frac{\mathbf{k}\, k_{rg}}{B_g \mu_g} \left(\nabla p_g - \gamma_g \nabla z' \right) \right) \\ = \frac{\partial}{\partial t} \left[\phi \left(\frac{R_s}{B_o} S_o + \frac{S_g}{B_g} \right) \right] \end{aligned} \tag{2.27}$$

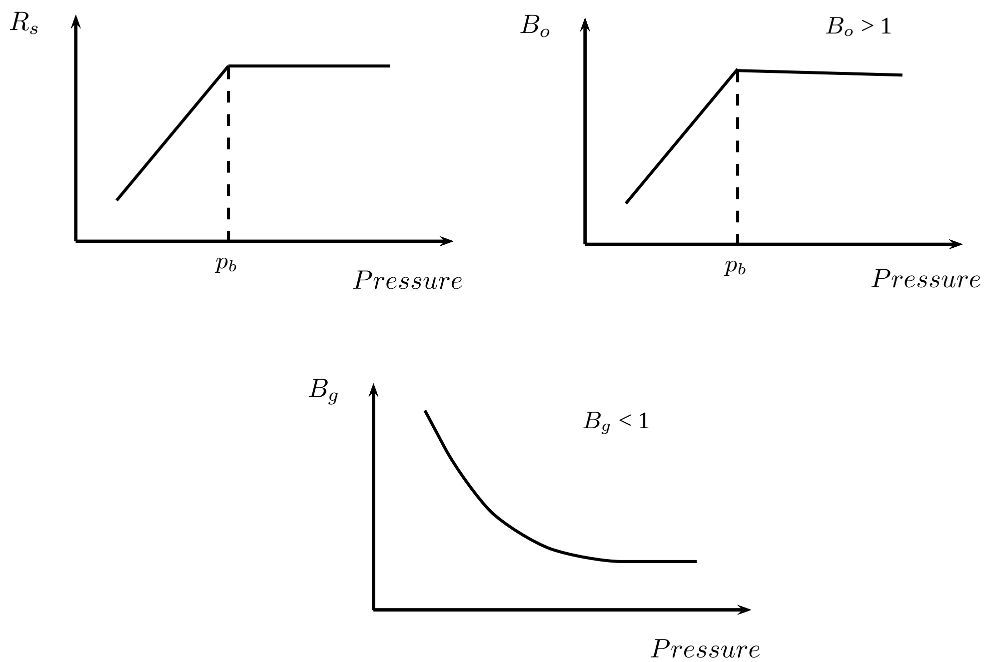


Fig. 2.3. R_s, B_o, B_g behavior.

It is important to note that B defined by Eq. 2.8[c4], considering oil, [c5] is [c6]valid if [c7]oil is single component or [c8] "black-oil" with no dissolved gas ($R_s = 0$). [c9]On the other hand, Eq. 2.14 is [c10]valid for black-oil problems with $R_s \neq 0$ provided that [c11]the pressure is above bubble-point. [c12]Eq. 2.25 and Eq. 2.27 [c13]becomes identical[c14] if the pressure is above bubble point.

[c1]In some cases, the flow equation is written in terms of pseudo potential.

$$\psi = \int_{p_0}^{p} \frac{1}{\gamma} dp - \left(z' - {z_0}'\right) \tag{2.28}$$

where z' is measured positive in the direction of gravity and ${z_0}'$ is the datum where [c2]p_0 [c3]is measured. Also $\gamma = \mathbf{M}/\mathbf{L}^2\mathbf{T}^2)$ and $\gamma = \gamma(p)$.

$$\begin{aligned}
\nabla\psi &= \nabla \int_{p_0}^{p} \frac{1}{\gamma} dp - \nabla\left(z' - {z_0}'\right) \\
&= \frac{d}{dp}\left\{\int_{p_0}^{p} \frac{1}{\gamma} dp\right\} \nabla p - \nabla z' \\
&= \frac{1}{\gamma}\nabla p - \nabla z'
\end{aligned}$$

Then,

$$\gamma\,\nabla\psi = \nabla p - \gamma\,\nabla z' \tag{2.29}$$

and

$$\frac{\partial\psi}{\partial t} = \frac{1}{\gamma}\frac{\partial p}{\partial t} \tag{2.30}$$

Using Eqs. 2.29 and 2.30 in Eq. 2.14 we obtain,

$$\nabla\cdot\left(\frac{\mathbf{k}\gamma}{B\mu}\nabla\psi\right) = \frac{\phi c_t \gamma}{B}\frac{\partial\psi}{\partial t} \tag{2.31}$$

If we consider that we have stress dependent reservoir, that is permeability decreases as the fluid pressure in the pores decreases, then

$$\mathbf{k} = k\left(p\right)\widetilde{\mathbf{k}}$$

where the entries of $\widetilde{\mathbf{k}}$ are independent of pressure. It has been observed that tight (and geothermal) reservoirs are [c4] examples of stress

[c4] *Murat Çınar*: *Text added.*
[c5] *Murat Çınar*: ~~this~~
[c9] *Murat Çınar*: ~~However~~
[c6] *Murat Çınar*: ~~correct~~
[c10] *Murat Çınar*: ~~correct~~
[c7] *Murat Çınar*: ~~we have a~~
[c11] *Murat Çınar*: ~~we are~~
[c8] *Murat Çınar*: ~~if we are talking about~~
[c12] *Murat Çınar*: ~~If we are above bubble point,~~
[c13] *Murat Çınar*: ~~will be~~
[c14] *Murat Çınar*: *Text added.*
[c1] *Murat Çınar*: ~~Sometimes we write the flow equation~~
[c2] *Murat Çınar*: ~~we take as reference~~
[c3] *Murat Çınar*: *Text added.*
[c4] *Murat Çınar*: ~~good~~

dependent reservoirs. Then Eq. 2.31 [c5]is expressed as

[c5] *Murat Çınar*: ~~can be~~

$$\nabla \cdot \left(\frac{k\left(p\right) \widetilde{\mathbf{k}} \gamma}{B\mu} \nabla \psi \right) = \frac{\phi c_t \gamma}{B} \frac{\partial \psi}{\partial t} \tag{2.32}$$

[c1] *Murat Çınar*: ~~It is important to note that Eq. 2.38 is non-linear because ϕ, c_t, γ, μ, B, and k are all pressure dependent (or potential). To partially linearize the Eq. 2.38 (or Eq. 2.14), we normally define a pseudo pressure function if gravity effects are not important.~~

[c2] *Murat Çınar*: Following paragraph is added.

[c1] [c2]A pseudo pressure function is defined to partially linearize Eq. 2.32 (or Eq. 2.14) under the assumption that the gravity effects are not important.

$$m\left(p\right) = \int_{p_0}^{p} \frac{k\left(p\right)}{\mu\left(p\right) B\left(p\right)} dp \tag{2.33}$$

$$\nabla m\left(p\right) = \frac{d}{dp} m\left(p\right) \nabla p = \frac{k\left(p\right)}{\mu\left(p\right) B\left(p\right)} \nabla p \tag{2.34}$$

$$\frac{\partial m\left(p\right)}{\partial t} = \frac{k\left(p\right)}{\mu\left(p\right) B\left(p\right)} \frac{\partial p}{\partial t} \tag{2.35}$$

[c3] *Murat Çınar*: ~~we don't have gravity effects note that~~

If [c3]gravity effects are ignored, Eq. 2.14 reduces to

$$\nabla \cdot \left(\frac{k\left(p\right) \widetilde{\mathbf{k}}}{B\left(p\right) \mu\left(p\right)} \nabla p \right) = \frac{\phi c_t}{B} \frac{\partial p}{\partial t} \tag{2.36}$$

Using Eqs. 2.33, 2.34, and 2.35 in 2.36 gives

$$\nabla \cdot \left(\widetilde{\mathbf{k}} \nabla m\left(p\right) \right) = \frac{\phi\left(p\right) c_t\left(p\right) \mu\left(p\right)}{k\left(p\right)} \frac{\partial m\left(p\right)}{\partial t} \tag{2.37}$$

[c4] *Murat Çınar*: ~~In~~

[c5] *Murat Çınar*: ~~we have written our basic flow equations~~

[c6] *Murat Çınar*: ~~we can define a "pseudo pressure" by~~

Note Eq. 2.37 is still non-linear. [c4] Eq. 2.32 [c5]is the expression of basic flow equations in terms of the potential ψ. Therefore, to partially linearize Eq. 2.32, [c6]a "pseudo pressure" is defined as

$$m\left(\psi\right) = \int_{\psi_0}^{\psi} \frac{k\left(\psi\right) \gamma\left(\psi\right)}{\mu\left(\psi\right) B\left(\psi\right)} d\psi \tag{2.38}$$

2.4 Diffusivity equation for single phase gas flow - real gas flow

[c7] *Murat Çınar*: ~~gas~~

[c8] *Murat Çınar*: ~~although~~

[c9] *Murat Çınar*: ~~if we have tight reservoirs~~

[c10] *Murat Çınar*: ~~we can use~~

[c11] *Murat Çınar*: *Text added.*

[c12] *Murat Çınar*: ~~real flow of~~

For [c7]gases μ, ρ are strong functions of pressure. Permeability typically is independent of pressure[c8], however, at low pressures Klinkenberg effect may cause some pressure dependence in permeability and/or [c9]tight reservoirs are considered as discussed earlier. To account for the dependence of $k/\mu B_g$ on pressure,[c10] Eq.2.33 or 2.38 [c11]is used. Note that Eq. 2.37 is also valid for [c12]flow of real gases in porous media.

With some modifications, the above procedure is the current approach used to derive the p.d.e. for gas flow. The method was first introduced in the literature by Al-Hussainy, Ramey and Crawford [2]. Below [c13]the p.d.e. [c14]is derived using Al-Hussainy et. al. [2] approach. [c15c16] Note that Eq. 2.36 holds for real gases. [c17]Assuming $k(p)\mathbf{k}$ is independent of pressure and same in all directions, then Eq. 2.36 [c18]becomes

$$\nabla \cdot \left(\frac{1}{B\left(p\right)\mu\left(p\right)} \nabla p \right) = \frac{\phi c_t}{kB} \frac{\partial p}{\partial t} \tag{2.39}$$

Since $B = \frac{(\rho_g)_{SC}}{\rho_g}$ and $(\rho_g)_{SC}$ is constant Eq. 2.39 is equivalent to

$$\nabla \cdot \left(\frac{\rho}{\mu} \nabla p \right) = \frac{\phi c_t \rho}{k} \frac{\partial p}{\partial t} \tag{2.40}$$

Recall that ρ for real gases is given by the following equation of state (EOS),

$$\rho = \frac{pM}{zRT} \tag{2.41}$$

Using Eq. 2.41 in 2.40 gives

$$\nabla \cdot \left(\frac{pM}{zRT\mu} \nabla p \right) = \frac{pM}{zRT} \frac{\phi c_t}{k} \frac{\partial p}{\partial t} \tag{2.42}$$

Since M/RT is constant, then Eq. 2.42 reduces to

$$\nabla \cdot \left(\frac{p}{z\mu} \nabla p \right) = \frac{\phi c_t p}{zk} \frac{\partial p}{\partial t} \tag{2.43}$$

Al-Hussainy et. al. [2] defined the integral transform $m'(p)$ to be

$$m'\left(p\right) = 2\int_{p_0}^{p} \frac{p'}{\mu z} dp' \tag{2.44}$$

$$\nabla m'\left(p\right) = \frac{2p}{\mu z} \nabla p \tag{2.45}$$

$$\frac{\partial m'}{\partial t} = 2\frac{p}{\mu z} \frac{\partial p}{\partial t} \tag{2.46}$$

Using Eqs. 2.44, 2.45, and 2.46 in 2.43 gives,

$$\nabla \cdot \left[\nabla m'\left(p\right) \right] = \frac{\phi c_t \mu\left(p\right)}{k} \frac{\partial m'\left(p\right)}{\partial t} \tag{2.47}$$

[c13] *Murat Çınar*: ~~we will derive~~

[c14] *Murat Çınar*: *Text added.*

[c15] *Murat Çınar*: ~~Since Eq. 2.32 is also valid for real gases, we have~~

[c16] *Murat Çınar*: The following sentence is added.

[c17] *Murat Çınar*: ~~Further, assuming~~

[c18] *Murat Çınar*: ~~can be written as~~

2.5 1-D Radial flow equation

[c1] *Murat Çınar*: ~~we have axisymmetric flow~~

Consider a completely penetrating well in an infinite porous medium of uniform thickness filled with a single phase fluid. Further assume that [c1]flow is axisymmetric, i.e., no variation in θ-direction or in a plane z'=constant equipotential curves are circles - see Figure 2.48,

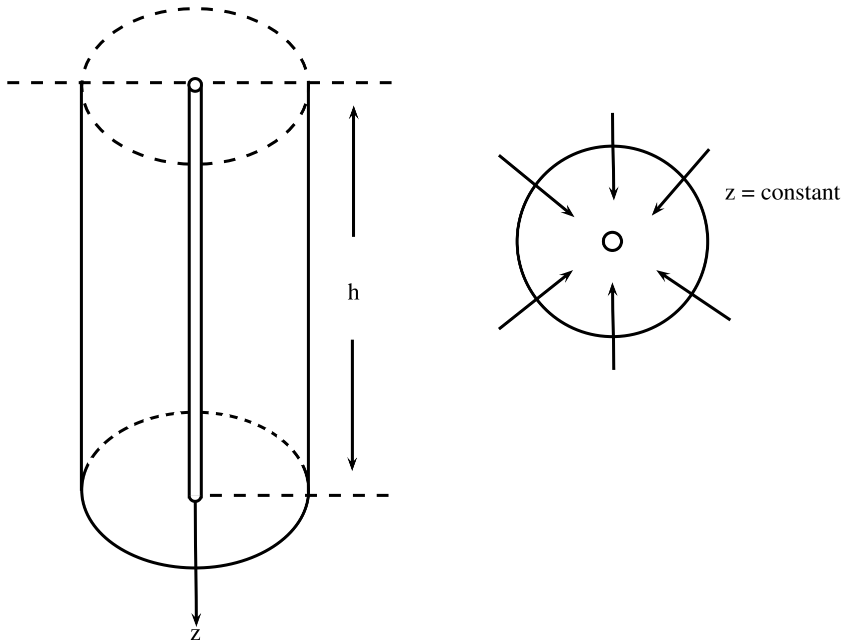


Fig. 2.4. Radial flow geometry.

[c1] *Murat Çınar*: ~~our general equation;~~

If reservoir is not horizontal, [c1] Eq. 2.14, applies with $v_\theta = 0$ and so Eq. 2.14 becomes in $r - z$ coordinates.

$$\begin{aligned}&\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{rk_r}{\mu B}\left(\frac{\partial p}{\partial r} - \gamma\frac{\partial z'}{\partial r}\right)\right]\\&\quad+\frac{\partial}{\partial z}\left[\frac{k_z}{\mu B}\left(\frac{\partial p}{\partial z} - \gamma\frac{\partial z'}{\partial z}\right)\right] = \frac{\phi c_t}{B}\frac{\partial p}{\partial t}\end{aligned} \tag{2.48}$$

[c2] *Murat Çınar*: ~~If we assume~~

[c2]Now assume $z = z'$ and $\theta = 0$, then $\frac{\partial z'}{\partial r} = 0$

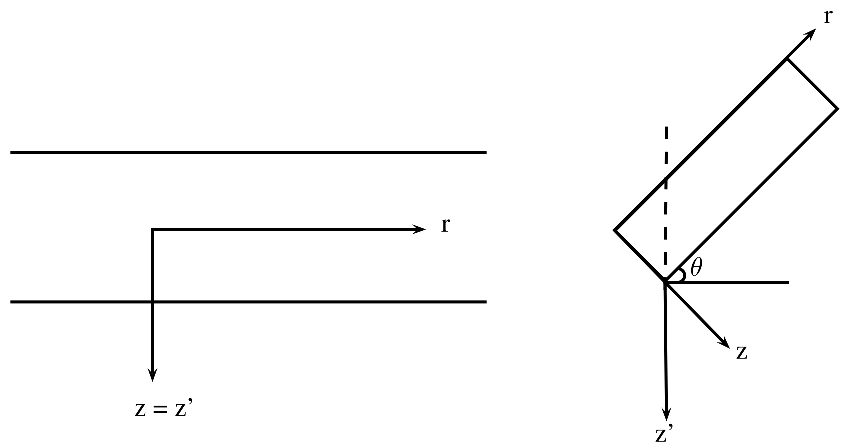


Fig. 2.5. r-z coordinates.

For a completely penetrating well, it is physically reasonable to assume $v_z \approx 0$ ($k_z \ll k_r$), i.e.,

$$\begin{aligned} & v_z = -\frac{k_z}{\mu}\left(\frac{\partial p}{\partial z} - \gamma\frac{\partial z'}{\partial z}\right) = 0 \quad ; \quad \frac{\partial z'}{\partial z} = 1 \\ & \frac{\partial p}{\partial z} - \gamma = 0 \quad ; \quad p\left(z_2\right) = p\left(z_1\right) + \gamma\left(z_2 - z_1\right) \quad z_z > z_1 \end{aligned} \tag{2.49}$$

Then [c1]the general radial flow problem [c2]becomes,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{k_r}{\mu B}\frac{\partial p}{\partial r}\right) = \frac{\phi c_t}{B}\frac{\partial p}{\partial t} \tag{2.50}$$

where ϕ, c_t, B, k_r, and μ [c3]are functions of pressure. [c4]Recall pseudo-pressure function defined as,

$$m\left(p\right) = \int_{p_b}^{p}\frac{k_r\left(p\right)}{\mu\left(p\right)B\left(p\right)}dp \tag{2.51}$$

Using Eq. 2.51, [c5] Eq. 2.50 [c6]is written as,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m\left(p\right)}{\partial r}\right) = \frac{\phi c_t \mu}{k_r}\frac{\partial m\left(p\right)}{\partial t} \tag{2.52}$$

Dimensionless Variables In well testing [c7]dimensionless variables are used for two main reasons;

[c1] *Murat Çınar*: ~~we arrive at~~

[c2] *Murat Çınar*: *Text added.*

[c3] *Murat Çınar*: ~~can be~~

[c4] *Murat Çınar*: ~~Recalling our~~

[c5] *Murat Çınar*: ~~we write~~

[c6] *Murat Çınar*: *Text added.*

[c7] *Murat Çınar*: ~~for two reasons, we are using dimensionless variables to prevent our results,~~

(i) minimize number of variables [c8](by grouping parameters)
(ii) provide general solutions

[c8] *Murat Çınar*: ~~(find group parameters)~~

[c1] *Murat Çınar*: ~~If we define a dimensionless time~~

[c1]Dimensionless time is defined as,

$$t_D = \frac{k_i t}{(\phi c_t \mu)_i r_w^2} \tag{2.53}$$

where subscript "i" refers to initial conditions, i.e.,

$$k_i = k(p_i) \;\; ; \;\; \mu_i = \mu(p_i) \;\; etc.$$

here p_i is the initial reservoir pressure (at some datum) and we assume p_i is independent of r, then

$$\frac{\partial m}{\partial t} = \frac{\partial m}{\partial t_D}\frac{\partial t_D}{\partial t} = \frac{\partial m}{\partial t_D}\left(\frac{k_i}{(\phi c_t \mu)_i r_w^2}\right) \tag{2.54}$$

[c2] *Murat Çınar*: Typo corrected in the following equation

Using Eq. 2.54 in Eq. 2.52, and simplifying gives,[c2]

$$r_w^2\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m\,(p)}{\partial r}\right)\right] = \left(\frac{\phi c_t \mu}{k}\right)\left[\frac{k_i}{(\phi c_t \mu)_i}\right]\frac{\partial m}{\partial t_D} \tag{2.55}$$

[c3] *Murat Çınar*: ~~If we~~

[c3]Now define,

$$r_D = \frac{r}{r_w} \tag{2.56}$$

[c4] *Murat Çınar*: *Text added.*

and [c4]dimensionless diffusivity,

$$\eta_D = \frac{k/(\phi c_t \mu)}{k_i/(\phi c_t \mu)_i} \tag{2.57}$$

[c5] *Murat Çınar*: ~~we can write~~

[c6] *Murat Çınar*: ~~as~~

then[c5] Eq. 2.55 [c6]becomes

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m\,(p)}{\partial r_D}\right) = \frac{1}{\eta_D}\frac{\partial m\,(p)}{\partial t_D} \tag{2.58}$$

If $\frac{1}{\eta_D} = 0$, then Eq. 2.58 is a linear p.d.e. .

[c7] *Murat Çınar*: ~~If we assume~~

[c7]Consider production at a specified rate q; i.e.,
Flow rate out $= qB = \int_S \mathbf{v}\,\mathbf{n}\,dS$ and $\mathbf{n}$ is the unit outward normal to S and is equal to

$$\mathbf{n} = -i_r + 0i_\theta + 0i_z = (1,0,0)$$
$$\mathbf{v} = (v_r + v_\theta + v_z)$$

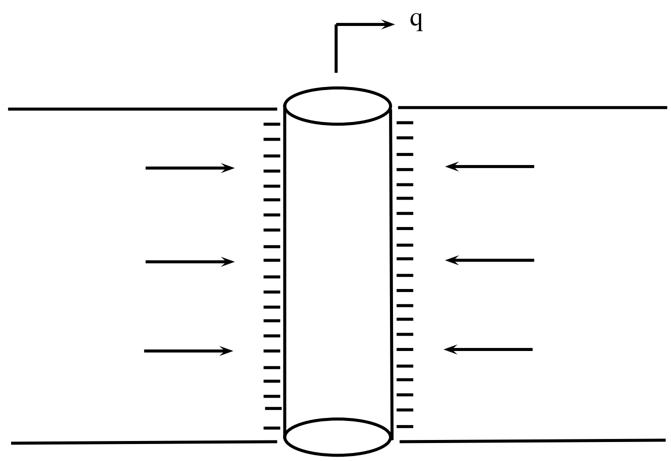


Fig. 2.6. Production from a vertical full penetrating well. Volumetric flux, q, into the wellbore (flow out of the reservoir the boundary represented by wellbore).

$$qB = \int\limits_S -v_r\big|_{r=r_w} dS \quad ; \quad ds = r_w d\theta dz$$

$$qB = \int_0^{2\pi}\int_0^h (-v_r)_{r_w} r_w d\theta dz$$

$$q = \int_0^{2\pi}\int_0^h \frac{k}{\mu B}\left(r\frac{\partial p}{\partial r}\right)_{r=r_w} d\theta dz$$

$$q = 2\pi\int_0^h \left(r\frac{\partial m}{\partial r}\right)_{r_w} dz$$

[c1]Now we assume [c2]that variation of $r\frac{\partial m(p)}{\partial r}$ in z-direction is insignificant or

[c1] *Murat Çınar*: ~~If~~
[c2] *Murat Çınar*: *Text added.*

$$\int\limits_0^h \left(r\frac{\partial m\left(p\right)}{\partial r}\right)_{r_w} dz = \left(r\frac{\partial m\left(p\right)}{\partial r}\right)_{r_w,\widehat{z}} h$$

where $\widehat{z}$ is a mean value between $0 \leq \widehat{z} \leq h$. Then the boundary condition is

$$q = 2\pi h\left(r\frac{\partial m\left(p\right)}{\partial r}\right)_{r_w} \tag{2.59}$$

[c3] *Murat Çınar*: ~~If we define~~

[c3]Define,

$$m_D = \frac{2\pi h\left[m\left(p_i\right) - m\left(p\right)\right]}{q} = \frac{2\pi h}{q}\left[m\left(p_i\right) - m\left(p\right)\right] = \frac{2\pi h}{q}\int\limits_{p}^{p_i}\frac{k\left(p\right)}{\mu\left(p\right)B\left(p\right)}dp \tag{2.60}$$

[c1] *Murat Çınar*: ~~one~~

[c2] *Murat Çınar*: ~~, respectively, can be~~

Then [c1]we can show that Eqs. 2.58 and 2.59[c2] is written as

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m_D}{\partial r_D}\right) = \frac{1}{\eta_D}\frac{\partial m_D}{\partial t_D} \tag{2.61}$$

$$\left(r_D\frac{\partial m_D}{\partial r_D}\right)_{r_D=1} = -1 \tag{2.62}$$

Note that $r_D = 1$ corresponds to $r = r_w$. Initial condition, $p = p_i$ at values of r at $\widehat{z}$. $m_D = 0$ at $t_D = 0$ then,

$$p\left(r,t\right)\big|_{t=0} = p_i \tag{2.63}$$

[c3] *Murat Çınar*: ~~Since we consider an infinite reservoir, then~~

[c3]Infinite acting reservoir is considered implying,

$$\lim_{r\to\omega} p\left(r,t\right) = p_i$$

which corresponds to

$$\lim_{r_D\to\infty} m_D\left(r_D,t_D\right) = 0 \tag{2.64}$$

[c4] *Murat Çınar*: ~~we have the following IBVP,~~

In summary, [c4]the following initial boundary value problem (IBVP) is achieved with the appropriate boundary conditions.

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m_D}{\partial r_D}\right) = \frac{1}{\eta_D}\frac{\partial m_D}{\partial t_D} \tag{2.65}$$

$$\left(r_D\frac{\partial m_D}{\partial r_D}\right)_{r_D=1} = -1 \tag{2.66}$$

$$\lim_{r_D\to\infty} m_D\left(r_D,t_D\right) = 0 \tag{2.67}$$

$$m_D\left(r_D,t_D=0\right) = 0 \tag{2.68}$$

[c5] *Murat Çınar*: ~~our~~

Eqs. 2.65-2.68 lead to give a complete mathematical description of [c5]the physical problem. Because of η_D term, it is a non-linear IBVP. It can also be solved analytically (see Kale and Mattar[5] or Peres et.al.[7])

[c6]For simplicity, assume that variations in k, ϕ, c_t, and B are small ("negligible") for the pressure change considered. [c7]Then,

[c6] *Murat Çınar*: ~~Let's now, for~~

[c7] *Murat Çınar*: ~~wit this assumption~~

$$\eta_D = \frac{(k/\phi c_t \mu\ )}{(k/\phi c_t \mu\ )_{p_i}} \approx 1 \tag{2.69}$$

and

$$\eta_D = \frac{(k/\phi c_t \mu\ )}{(k/\phi c_t \mu\ )_{p_i}} \approx 1 \tag{2.70}$$

$$m\left(p_i\right) - m\left(p\right) = \int_p^{p_i} \frac{k\left(p\right)}{\mu\left(p\right) B\left(p\right)} dp \approx \frac{k_i}{\mu_i B_i}\left(p_i - p\right) \tag{2.71}$$

and then it follows from Eq. 2.60 that

$$m_D = \frac{k_i 2\pi h\left(p_i - p\right)}{qB_i\mu_i} = p_D = \frac{k 2\pi h\left(p_i - p\right)}{qB\mu} \tag{2.72}$$

[c1]that is the definition of dimensionless pressure p_D in well testing. Considering $\frac{1}{\eta_D} \approx 1$ in Eq. 2.65, we have

[c1] *Murat Çınar*: ~~which is the normal~~

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m_D}{\partial r_D}\right) = \frac{\partial m_D}{\partial t_D} \tag{2.73}$$

$$\left(r_D\frac{\partial m_D}{\partial r_D}\right)_{r_D=1} = -1 \tag{2.74}$$

$$\lim_{r_D\to\infty} m_D\left(r_D, t_D\right) = 0 \tag{2.75}$$

$$m_D\left(r_D, t_D = 0\right) = 0 \tag{2.76}$$

Note that Eq. 2.73 is a linear p.d.e. . We seek a solution to the IBVP given by Eqs. 2.73 - 2.76. To find a solution we assume that

$$m_D = m_D\left(\varepsilon_D\right)$$

where

$$\varepsilon_D = \frac{r_D^2}{4t_D} = \varepsilon_D\left(r_D, t_D\right)$$

[c2]is called dimensionless Boltzman transform. To use the Boltzman transform, there must be no characteristic length in the system (such as $r_D = 1$, r_w). Since the inner boundary condition, Eq. 2.74, is for a finite wellbore problem, it involves a characteristic length. Thus, to be able to use Boltzman transform, [c3] Eq. 2.74 [c4] with one that does not involve characteristic length, [c5]that is "line source well" inner boundary condition.

[c2] *Murat Çınar*: ~~and can be~~

[c3] *Murat Çınar*: ~~we replace~~

[c4] *Murat Çınar*: ~~is replaced~~

[c5] *Murat Çınar*: ~~which~~

$$\lim_{r_D \to 0} \left(r_D \frac{\partial m_D}{\partial r_D} \right) = -1 \tag{2.77}$$

Under the preceding assumptions,[c1] approximate problem becomes: Find $m_D = m_D(\varepsilon_D)$ such that m_D satisfies

[c1] *Murat Çınar*: ~~our~~

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial m_D}{\partial r_D} \right) = \frac{\partial m_D}{\partial t_D} \tag{2.78}$$

$$\lim_{r_D \to 0} \left(r_D \frac{\partial m_D}{\partial r_D} \right) = -1 \tag{2.79}$$

$$\lim_{r_D \to \infty} m_D \left(r_D, t_D \right) = 0 \tag{2.80}$$

$$m_D \left(r_D, t_D = 0 \right) = 0 \tag{2.81}$$

where,

$$m_D = \frac{2\pi h \int_p^{p_i} \frac{k(p)}{\mu(p) B(p)} dp}{q} \tag{2.82}$$

Use of Boltzman transformation, $\varepsilon_D = \frac{r_D^2}{4t_D}$, [c2]changes the p.d.e. to second order ordinary differential equation o.d.e., further [c3]collapses three auxiliary conditions into two conditions,

[c2] *Murat Çınar*: ~~will change our~~

[c3] *Murat Çınar*: ~~it will collapse our~~

$$\frac{\partial m_D}{\partial r_D} = \frac{\partial m_D}{\partial \varepsilon_D} \frac{\partial \varepsilon_D}{\partial r_D} = \frac{\partial m_D}{\partial \varepsilon_D} \frac{2r_D}{4t_D} \tag{2.83}$$

[c4]and

[c4] *Murat Çınar*: ~~or~~

$$r_D \frac{\partial m_D}{\partial r_D} = 2 \left(\frac{r_D^2}{4t_D} \right) \frac{dm_D}{d\varepsilon_D} = 2\varepsilon_D \frac{dm_D}{d\varepsilon_D} \tag{2.84}$$

Using Eq. 2.84 and the chain rule,

$$\begin{aligned}\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m_D}{\partial r_D}\right) &= \frac{1}{r_D}\frac{d}{d\varepsilon_D}\left(2\varepsilon_D\frac{dm_D}{d\varepsilon_D}\right)\frac{d\varepsilon_D}{dr_D} \\ &= \frac{1}{t_D}\frac{d}{d\varepsilon_D}\left(2\varepsilon_D\frac{dm_D}{d\varepsilon_D}\right)\end{aligned} \tag{2.85}$$

Similarly,

$$\frac{\partial m_D}{\partial t_D} = \frac{dm_D}{d\varepsilon_D}\frac{\partial \varepsilon_D}{\partial t_D} = \frac{dm_D}{d\varepsilon_D}\left(-\frac{r_D^2}{4t_D^2}\right) = -\frac{1}{t_D}\varepsilon_D\frac{dm_D}{d\varepsilon_D} \tag{2.86}$$

Using Eqs. and in Eq. 2.78 gives,

$$\varepsilon_D\frac{dm_D}{d\varepsilon_D} + \frac{d}{d\varepsilon_D}\left(\varepsilon_D\frac{dm_D}{d\varepsilon_D}\right) = 0 \tag{2.87}$$

Using Eq. 2.84 [c1] the inner boundary condition (see Eq. 2.79) [c2]is written as

[c1] *Murat Çınar*: ~~we can write~~

[c2] *Murat Çınar*: *Text added.*

$$\lim_{\varepsilon_D\to 0} 2\varepsilon_D\frac{dm_D}{d\varepsilon_D} = -1 \tag{2.88}$$

Initial condition (I.C.) Eq. 2.81,

$$t_D \to 0 \quad \varepsilon_D \to \infty \quad \Rightarrow \quad \lim_{\varepsilon_D\to\infty} m_D\left(\varepsilon_D\right) = 0$$

Outer boundary condition (O.B.C.) Eq. 2.80,

$$r_D \to \infty \quad \varepsilon_D \to \infty \quad \Rightarrow \quad \lim_{\varepsilon_D\to\infty} m_D\left(\varepsilon_D\right) = 0$$

Thus both I.C. and O.B.C. [c3]is represented by,

[c3] *Murat Çınar*: ~~can be~~

$$\lim_{\varepsilon_D\to\infty} m_D\left(\varepsilon_D\right) = 0 \tag{2.89}$$

We need to solve boundary value problem (B.V.P.) given by Eqs. 2.87 - 2.89. Let,

$$w_D = \frac{dm_D}{d\varepsilon_D} \tag{2.90}$$

Substituting Eq. 2.90 into Eq. 2.87 gives,

$$\frac{d}{d\varepsilon_D}\left(\varepsilon_D w_D\right) + \varepsilon_D w_D = 0$$

$$\varepsilon_D \frac{dw_D}{d\varepsilon_D} + w_D + \varepsilon_D w_D = 0$$

$$\varepsilon_D \frac{dw_D}{d\varepsilon_D} + (1 + \varepsilon_D)\, w_D = 0 \tag{2.91}$$

[c1] *Murat Çınar*: ~~we have~~

Eq. 2.91 is separable ordinary differential equation, thus separating variables, [c1]

$$\frac{dw_D}{w_D} = -\frac{1 + \varepsilon_D}{\varepsilon_D} \tag{2.92}$$

[c2] *Murat Çınar*: *Text added.*

Interpreting both sides [c2]yields,

$$\ln w_D = -\varepsilon_D - \ln \varepsilon_D + c_1 \quad ; \quad c_1 \text{ is an integrating constant}$$

or

$$\begin{aligned} \ln w_D &= -\varepsilon_D - \ln \varepsilon_D + c_1 \quad ; \quad c_1 \text{ is an integrating constant} \\ w_D &= \exp\left[-\varepsilon_D - \ln \varepsilon_D + c_1\right] \\ &= e^{c_1} \frac{1}{\varepsilon_D} e^{-\varepsilon_D} = \frac{c}{\varepsilon_D} e^{-\varepsilon_D} \quad ; \quad c = e^{c_1} \end{aligned}$$

At this point, we have

$$w_D = \frac{dm_D}{d\varepsilon_D} = \frac{c}{\varepsilon_D} \exp\left[-\varepsilon_D\right]$$

thus

$$2\varepsilon_D \frac{dm_D}{d\varepsilon_D} = 2c \exp\left[-\varepsilon_D\right] \tag{2.93}$$

It follows from Eq. 2.88 and 2.93 that

$$c = -1/2 \tag{2.94}$$

Thus Eq. 2.93 becomes

$$\frac{dm_D}{d\varepsilon_D} = -\frac{1}{2\varepsilon_D} \exp\left[-\varepsilon_D\right] \tag{2.95}$$

Integrating Eq. 2.95 from ε_D to ∞ gives,

$$\int_{\varepsilon_D}^{\infty} \frac{dm_D}{d\varepsilon_D}\, d\varepsilon_D = -\frac{1}{2} \int_{t_D}^{\infty} \frac{\exp\left[-u\right]}{u}\, du$$

in limit

$$\lim_{\varepsilon_D \to \infty} m_D(\varepsilon_D) - m_D(\varepsilon_D) = -\frac{1}{2}\int_{t_D}^{\infty} \frac{\exp[-u]}{u}\, du \tag{2.96}$$

It follows from Eq. 2.89 and 2.96 that

$$m_D(\varepsilon_D) = \frac{1}{2}\int_{\frac{t_D^2}{4t_D}}^{\infty} \frac{\mathrm{e}^{-u}}{u}\, du \tag{2.97}$$

which is known as the Theis solution [9]. Eq. 2.97 is also referred to as the line source or the exponential integral or E_i solution.

[c1] Exponential integral is defined by

[c1] *Murat Çınar*: ~~In the literature, we use the definition given by,~~

$$-E_i[-x] = \int_x^{\infty} \frac{\mathrm{e}^{-u}}{u}\, du \quad ; \quad E_i \text{ function}$$

Exponential integral function is also referred to as $E_[1](x)$,

$$-E_i[-x] = E_1(x) \tag{2.98}$$

with this notation [c2] Eq. 2.97 [c3] becomes

[c2] *Murat Çınar*: ~~we can write~~

[c3] *Murat Çınar*: ~~as~~

$$m_D(\varepsilon_D) = -E_i\left[-\frac{r_D^2}{4t_D}\right] \tag{2.99}$$

The series expansion of E_i function [1]

$$E_i[-x] = \widetilde{\gamma} + \ln x + \sum_{n=1}^{\infty} \frac{x^n}{nn!} \tag{2.100}$$

[c4] Let $x \to 0$ in Eq. 2.99, then we obtain

[c4] *Murat Çınar*: ~~If we let~~

$$\lim_{x \to 0} E_i[-x] = \widetilde{\gamma} + \ln x \tag{2.101}$$

where $\widetilde{\gamma}$ is [c5] Euler's constant and defined by,

[c5] *Murat Çınar*: ~~referred to as~~

$$\widetilde{\gamma} = \lim_{n \to \infty}\left\{\sum_{k=1}^{n} \frac{1}{k} - \ln(n)\right\} = 0.5772156649... \tag{2.102}$$

If we consider that $\frac{r_D^2}{4t_D}$ is sufficiently small so that [c6] $E_i\left[-\frac{r_D^2}{4t_D}\right]$ [c7] is approximated by [c8] Eq. 2.101. Then, Eq. 2.99 [c9] is approximated as

[c6] *Murat Çınar*: ~~we can approximate~~

[c7] *Murat Çınar*: *Text added.*

[c8] *Murat Çınar*: ~~the right hand side of~~

[c9] *Murat Çınar*: ~~can be~~

$$m_D\left(\varepsilon_D\right) = -\frac{1}{2}\left[\ln\left(\frac{r_D^2}{4t_D}\right) + \widetilde{\gamma}\right] \tag{2.103}$$

or

$$m_D = \frac{1}{2}\ln\left(\frac{4t_D}{r_D^2 e^{\widetilde{\gamma}}}\right) \tag{2.104}$$

which is referred to as the semilog approximation to Eq. 2.99. Although [c1]Eq. 2.105 [c2]is derived as a limiting form of Eq. 2.99 as $\frac{r_D^2}{4t_D} \to 0$, Eq. 2.99 [c3]is well approximated by Eq. 2.105 if

[c1] *Murat Çınar*: ~~we derived~~
[c2] *Murat Çınar*: *Text added.*
[c3] *Murat Çınar*: ~~can also be~~

$$\frac{r_D^2}{4t_D} \leq \frac{1}{100}$$

which is equivalent to;

$$\frac{t_D}{r_D^2} \geq 25 \tag{2.105}$$

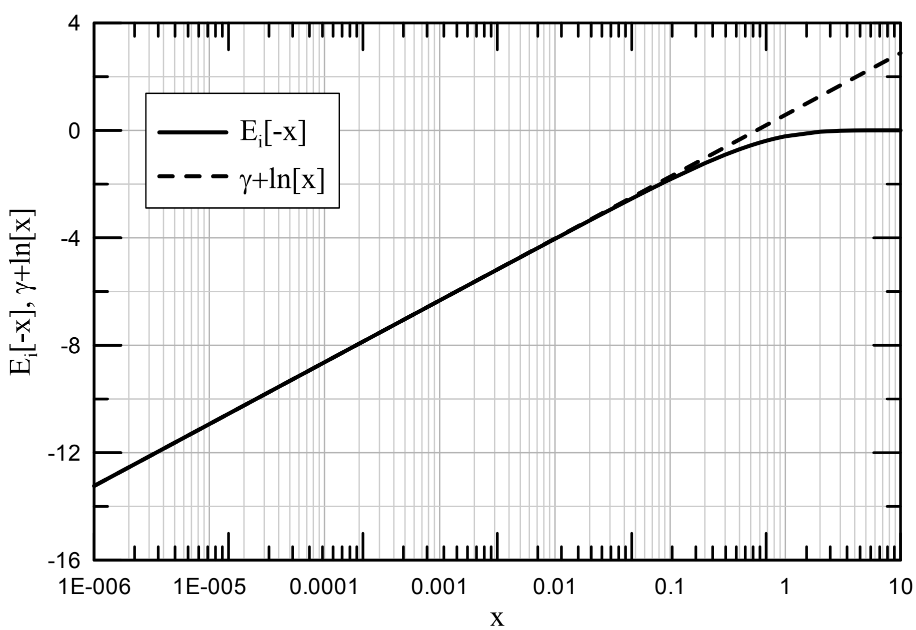


Fig. 2.7. Comparison of exponential integral with logarithmic approximation given by Eq. 2.101.

Remark on exponential integral approximation:

At $r = r_w$, $r_D = r/r_w = 1$ and Eq. 2.105 becomes,

$$\frac{2.637 \times 10^{-3} k_i t}{(\phi \mu c_t)_i r_w^2} \geq 25$$

where t is in hrs. [c1]Now suppose, $r_w = 0.35$ ft, $\mu_i = 0.8$ cp, $\phi_i = 0.1$, $k_i = 10$ md, and $c_t = 1.1 \times 10^5$. then it follows that

[c1] *Murat Çınar*: ~~Let's~~

$$t_D \approx 2.5 \times 10^5 t$$

In order for Eq. 2.101 to hold, we need

$$2.5 \times 10^5 t \geq 25 \quad \Rightarrow \quad t \geq 1.0 \times 10^{-4} hrs (= 0.36 seconds)$$

This [c2]example illustrates that at the wellbore ($r_D = 1$), the semilog approximation of the line source solution becomes valid within several seconds to a very few minutes for reasonable values of[c3] physical parameters; (e.g. k_i, c_t...). If [c4] the semilog approximation of the line source [c5]solution is used to analyze [c6] interference data with [c7]an observation well say at $r = 1000 r_w$, ($r_D = 1000$) [c8]away from [c9]a flowing well, then for the values of physical parameters considered, Eq. 2.105 requires,

[c2] *Murat Çınar*: *Text added.*

[c3] *Murat Çınar*: ~~the~~

[c4] *Murat Çınar*: ~~we are using~~

[c5] *Murat Çınar*: *Text added.*

[c6] *Murat Çınar*: ~~the~~

[c7] *Murat Çınar*: ~~the~~

[c8] *Murat Çınar*: *Text added.*

[c9] *Murat Çınar*: ~~the~~

$$t \geq 10^{-4} r_D^2 = 100 hrs$$

In short, [c10]the semilog approximation of the line source solution [c11]is used, i.e., Eq. 2.101 to analyze well test pressure data measured either at the active (producing) well or at the observation (shut-in) well provided that test time satisfies Eq. 2.105.

[c10] *Murat Çınar*: ~~we can use~~

[c11] *Murat Çınar*: *Text added.*

[c12]Line source solution given by Eq. 2.78 [c13]we used the line source well boundary condition Eq. 2.77 that is

[c12] *Murat Çınar*: ~~To obtain the line source solution which is given by~~

[c13] *Murat Çınar*: ~~is obtained by using the line source boundary condition~~

$$\lim_{r_D \to 0} \left(r_D \frac{\partial m_D}{\partial r_D} \right) = -1$$

Differentiating Eq. 2.78 with respect to r_D gives,

$$\frac{\partial m_D}{\partial r_D} = \frac{1}{2} \frac{\partial}{\partial r_D} \left[\int_{\frac{t_D^2}{4t_D}}^{\infty} \frac{\mathrm{e}^{-u}}{u} \, du \right] = -\frac{1}{2} \frac{\partial}{\partial r_D} \left[\int_{\infty}^{\frac{t_D^2}{4t_D}} \frac{\mathrm{e}^{-u}}{u} \, du \right] \quad (2.106)$$

[c14] Recall,

$$\frac{d}{dx}\left[\int_a^{b(x)} f(z)\,dz\right] = f[b(x)]\,\frac{db(x)}{dx} \tag{2.107}$$

where a is [c1] a constant. Using the formula given by Eq. 2.107 in Eq. 2.106 and simplifying the resulting equation; [c2]

$$\frac{\partial m_D}{\partial r_D} = -\frac{1}{r_D}\exp\left[-\frac{r_D^2}{4t_D}\right] \tag{2.108}$$

Multiplying Eq. 2.108 by r_D

$$r_D\frac{\partial m_D}{\partial r_D} = -\exp\left[-\frac{r_D^2}{4t_D}\right] \tag{2.109}$$

[c3] Thus the line source solution satisfies [c4] line source wellbore B.C., i.e.,

$$\lim_{r_D\to 0}\left(r_D\frac{\partial m_D}{\partial r_D}\right) = -1$$

It is important to note that Eq. 2.78 assumes [c5] a well with vanishingly small radius[c6], however, the more rigorous solution [c7] is the one considering [c8] a well with finite wellbore. To obtain a solution for a well with finite wellbore producing at a constant rate,[c9] an inner boundary condition [c10] such that

$$\lim_{r_D\to 0}\left(r_D\frac{\partial m_D}{\partial r_D}\right) = -1 \tag{2.110}$$

Therefore, the finite wellbore radius I.B.V.P. differs from the line source I.B.V.P. in [c11] the boundary condition [c12] given by Eq. 2.110[c13] Line source solution is closed with Eq. 2.77.

Note that $r_D = 1$ (see Eq. 2.106), the line source solution satisfies

$$\left(r_D\frac{\partial m_D}{\partial r_D}\right)_{r_D=1} = -\exp\left[-\frac{1}{4t_D}\right] \tag{2.111}$$

As $t_D\to\infty$, the RHS of Eq. 2.111 approaches -1, i.e.,

$$\lim_{t_D\to\infty}\left(r_D\frac{\partial m_D}{\partial r_D}\right)_{r_D=1} = -1 \tag{2.112}$$

from which intuitively we expect that at sufficiently large[c14] times,

[c14] *Murat Çınar*: ~~From calculus we know that~~

[c1] *Murat Çınar*: *Text added.*

[c2] *Murat Çınar*: ~~we obtain~~

[c3] *Murat Çınar*: ~~so as expected our~~

[c4] *Murat Çınar*: ~~our~~

[c5] *Murat Çınar*: ~~that we have~~

[c6] *Murat Çınar*: ~~. However the more~~

[c7] *Murat Çınar*: ~~will be~~

[c8] *Murat Çınar*: ~~the~~

[c9] *Murat Çınar*: ~~we need to use~~

[c10] *Murat Çınar*: ~~is used~~

[c11] *Murat Çınar*: ~~that our~~

[c12] *Murat Çınar*: ~~is~~

[c13] *Murat Çınar*: ~~instead of~~

[c14] *Murat Çınar*: ~~values of~~

[c15] *Murat Çınar*: ~~(for $r_D \geq 1$)~~

the line source solution [c15]for $r_D = 1$ will be very close to the finite wellbore radius solution. In fact for $t_D > 25$, the RHS of Eq. 2.111 is within 1% of -1.

Mueller and Witherspoon [6] were the ones first to investigate the validity of line source solution. They compared the finite wellbore radius solution and the line source solution. Such comparison is shown in Fig. 1 [c1]- log-log plot of $p_D (= m_D)$ versus t_D/r_D^2.

[c1] *Murat Çınar*: ~~which is taken from Earlougher's manograph, SPE Monograph 5. It is a~~

The dashed curve in the figure represents the exponential integral solution(or the line source solution) given by Eq. 2.78. The top solid curve corresponds to the finite wellbore radius solution evaluated at the wellbore (i.e. $r_D = 1$). All other solid curves represent finite wellbore radius solution at various locations (different r_D values) in the reservoir.

[c2]Figure indicates that when $t_D/r_D^2 \geq 25$, the difference between the finite wellbore radius solution and the line source solution is negligible. [c3]Thus at $r_D = 1$ (at the wellbore) [c4] $t_D \geq 25$ [c5]is required for [c6] two solutions to be essentially the same. Also note that if $r_D \geq 20$, then the two solutions are essentially equal for all values of the dimensionless time t_D of practical interest.

[c2] *Murat Çınar*: ~~From the figure we see that if~~

[c3] *Murat Çınar*: ~~This means that~~

[c4] *Murat Çınar*: ~~we need~~

[c5] *Murat Çınar*: *Text added.*

[c6] *Murat Çınar*: ~~the~~

For the interference testing purposes, the difference between the line source solution and the finite wellbore radius solution is negligible. To use Eq. 2.78 for the analysis of pressure data measured at a flowing well, we evaluate Eq. 2.78 at $r_D = 1$ and use the notation

$$p\left(r_D = 1, t_D\right) = p\left(r = r_w, t\right) = p_{wf} = p_{wf}\left(t\right) \tag{2.113}$$

2.6 Semilog analysis

[c7]In the previous section, we showed that if $t_D/r_D^2 \geq 25$, the semilog approximation of the line source solution is given by Eq. 2.105 i.e.;

[c7] *Murat Çınar*: ~~From our previous derivations~~

$$m_D = \frac{1}{2}\ln\left(\frac{4t_D}{r_D^2 e^{\tilde{\gamma}}}\right)$$

At $r_D = 1$, [c8]

[c8] *Murat Çınar*: ~~that gives,~~

$$\begin{aligned} m_D\left(r_D = 1\right) &= \frac{2\pi h}{q}\int_{p_{wf}}^{p_i} \frac{k\left(p\right)}{\mu\left(p\right) B\left(p\right)}\, dp \\ &= \frac{1}{2}\left[\frac{4t_D}{e^{\tilde{\gamma}}}\right] \\ &= \frac{1}{2}\left(\ln t_D + 0.80907\right) \end{aligned} \tag{2.114}$$

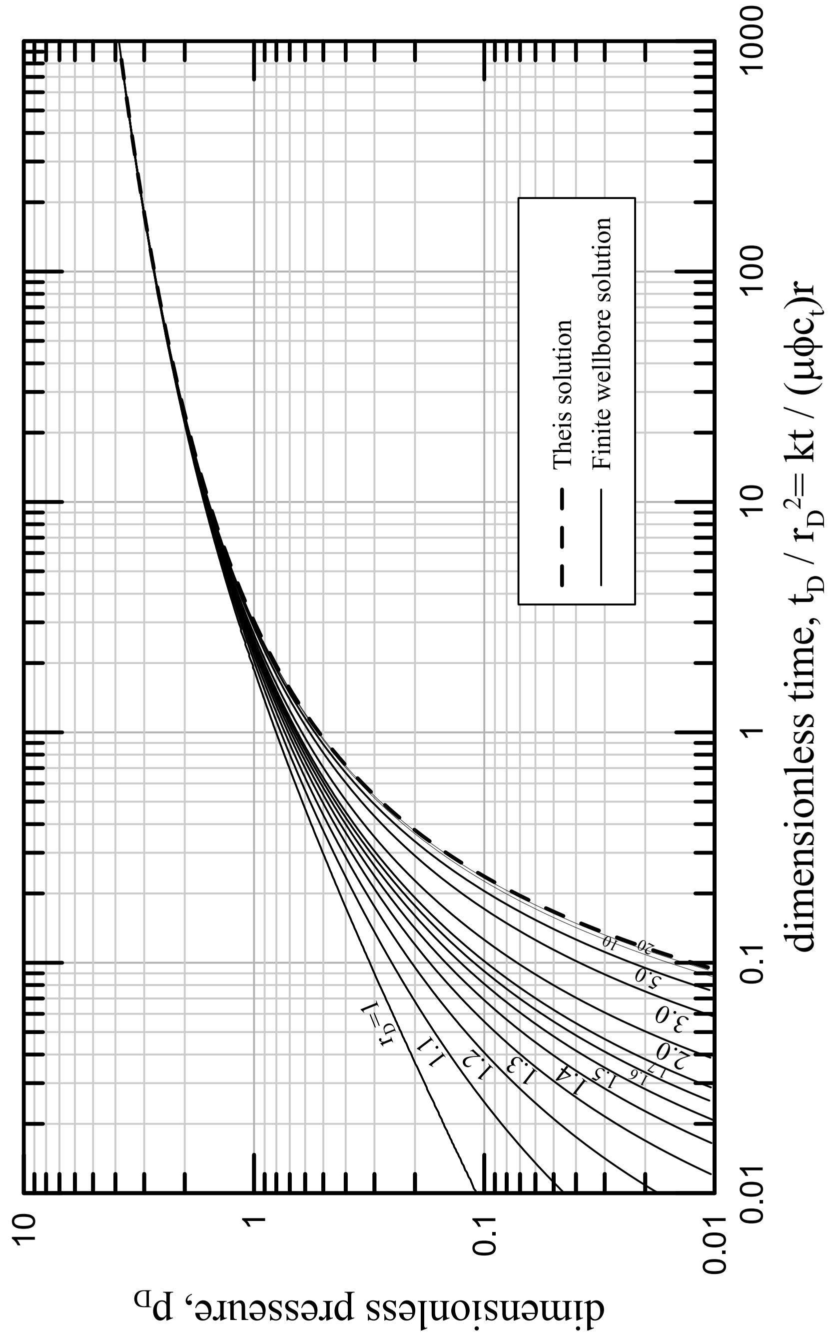


Fig. 2.8. Comparison of line source solution to finite radius wellbore solution[6].

or

$$m_D\left(r_D=1\right)=\underbrace{\frac{1}{2\log\left(e\right)}}_{1.15129}\left\{\log\left(t\right)+\log\left[\frac{k_i}{\left(\phi c_t\mu\right)_i r_w^2}\right]+\underbrace{\log\left(e\right)\ln\left(\frac{4}{e^{\tilde{\gamma}}}\right)}_{0.351378}\right\} \tag{2.115}$$

Note that

$$\begin{aligned} m_D\left(r_D=1\right) &= \frac{2\pi h}{q}\int_{p_{wf}}^{p_i}\frac{k\left(p\right)}{\mu\left(p\right)B\left(p\right)}dp \\ &= \frac{2\pi h}{q}\left[m\left(p_i\right)-m\left(p_{wf}\right)\right] \end{aligned} \tag{2.116}$$

Then it follows from Eq. 2.115 and 2.116 that

$$m\left(p_{wf}\right)=m\left(p_i\right)-1.15129\frac{q}{2\pi h}\left\{\log\left(t\right)+\log\left[\frac{k_i}{\left(\phi c_t\mu\right)_i r_w^2}\right]+0.351378\right\} \tag{2.117}$$

which indicates that a semilog plot of $m(p_{wf})$ versus t [c1]that yields a straight line [c2]with a slope

[c1] *Murat Çınar*: ~~which~~
[c2] *Murat Çınar*: ~~of~~

$$\text{slope}=\widetilde{m}=-1.15129\frac{q}{2\pi h} \tag{2.118}$$

[c3]h is computed from the slope of semilog straight line. [c4]Now differentiate Eq. 2.115 w.r.t. $logt$.

[c3] *Murat Çınar*: ~~from which we can compute h~~
[c4] *Murat Çınar*: ~~Suppose we~~

$$\frac{2\pi h}{q}\frac{d}{d\log}\int_{p_{wf}}^{p_i}\frac{k\left(p\right)}{\mu\left(p\right)B\left(p\right)}dp=1.15129 \tag{2.119}$$

or

$$\frac{2\pi h}{q}\frac{d}{dp_{wf}}\left\{\int_{p_{wf}}^{p_i}\frac{k\left(p\right)}{\mu\left(p\right)B\left(p\right)}dp\right\}\frac{dp_{wf}}{d\log t}=1.15129$$

or

$$\frac{2\pi h}{q}\left[-\frac{k\left(p_{wf}\right)}{\mu\left(p_{wf}\right)B\left(p_{wf}\right)}\right]\frac{dp_{wf}}{d\log t}=1.15129$$

or

$$\frac{k\left(p_{wf}\right)}{\mu\left(p_{wf}\right)B\left(p_{wf}\right)}=-1.15129\frac{q}{2\pi h\left(dp_{wf}/d\log t\right)} \tag{2.120}$$

Aside:

For solution gas drive reservoirs, Bøe et.al.[3] showed that pressure and oil saturation, S_o are unique function of the Boltzman variable $r_D^2/4t_D$, or r^2/t provided that $t_D/r_D^2 \geq 25$.

Serra et. al. [8] used this observation to show how to compute effective permeability from wellbore B.C. for constant oil rate.

$$q_o=2\pi\left(\frac{kk_{ro}}{\mu_o}rh\frac{\partial p}{\partial r}\right)_{r=r_w} \tag{2.121}$$

Suppose p is a function of the Boltzman variable $z=r^2/t$ then,

$$\frac{\partial p}{\partial r}=\frac{dp}{dz}\frac{dz}{dr}=\frac{dp}{dz}\frac{2r}{t} \tag{2.122}$$

and

$$\frac{\partial p}{\partial t}=\frac{dp}{dz}\frac{dz}{dt}=\frac{dp}{dz}\left(-\frac{r^2}{t^2}\right) \tag{2.123}$$

It follows that

$$r\frac{\partial p}{\partial r}=2\frac{r^2}{t}\frac{dp}{dz}=\left(-2t\right)\frac{-r^2}{t^2}=-2t\frac{dp}{dt}=-2\frac{dp}{d\ln t}$$

or at $r=r_w$

$$\left(r\frac{\partial p}{\partial r}\right)_{r_w}=-2t\frac{dp_{wf}}{dt}=-2\frac{dp_{wf}}{d\ln t} \tag{2.124}$$

Using Eq. 2.121 in Eq. 2.124 gives,

$$q_o=2\pi h\left(\frac{kk_{ro}}{\mu_oB_o}\right)_{r_w}\left(-2\frac{dp_{wf}}{d\ln t}\right) \tag{2.125}$$

Rearranging Eq. 2.125, we obtain,

$$\left(\frac{kk_{ro}}{\mu_o B_o}\right)_{r_w} = -\frac{q_o}{4\pi h \left(\frac{dp_{wf}}{d\ln t}\right)} \tag{2.126}$$

[c1]that provides a way of computing $(kk_{ro})_{p_{wf}}$ as a function of p_{wf} from

[c1] *Murat Çınar*: ~~which gives us a way for~~

$$\begin{aligned}(kk_{ro})_{p_{wf}} &= -\frac{q_o(\mu_o B_o)_{p_{wf}}}{4\pi h \left(\frac{dp_{wf}}{d\ln t}\right)} \\ &= -\frac{2.303 q_o(\mu_o B_o)_{p_{wf}}}{4\pi h \left(\frac{dp_{wf}}{d\log t}\right)} \quad ; \quad \ln t = 2.303\log t\end{aligned} \tag{2.127}$$

Eq. 2.127 implies that k_{ro} is a function of pressure. [c1]Therefore, oil saturation is considered as a function of pressure, i.e. $S_o = S_o(p)$. [c2]

[c1] *Murat Çınar*: ~~This means, we must be able to consider $S_o = S_o(p)$ i.e. oil saturation is a function of pressure.~~

[c2] *Murat Çınar*: ~~Aanonsen (PhD, 1985) derived a relation to obtain S_o as afunction of pressure. Then we can construct $k_{ro} versus S_o$.~~

References

1. M. Abramowitz and IA. Stegun. *Handbook of Mathematical Functions with Formulas*. US Government printing office, 1972.
2. R. Al-Hussainy, H.J. Ramey Jr., and P.B. Crawford. The flow of real gases through porous media. *SPE Journal*, 18(5):624 – 636, 1966.
3. A. Bøe, Svein M. Skjaeveland, and Curtis H. Whitson. Two-phase pressure test analysis. *SPE Formation Evaluation*, 17(4):604–610, 1989.
4. D. Bourdet. *Well test analysis: the use of advanced interpretation models*. Handbook of petroleum exploration and production. Elsevier, 2002.
5. D. Kale and L. Mattar. Solution of a non-linear gas flow equation by the perturbation technique. *Journal of Canadian Petroleum Technology*, 19(4), 1980.
6. Thomas D. Mueller and Paul A. Witherspoon. Pressure interference effects within reservoirs and aquifers. *Journal of Petroleum Technology*, 4(4):471–474, 1965.
7. Alvaro M.M. Peres, Kelson V. Serra, and Albert C. Reynolds. Toward a unified theory of well testing for nonlinear-radial-flow problems with application to interference tests. *SPE Formation Evaluation*, 5(2):151–160, 1990.
8. Kelsen V. Serra, Alvaro M.M. Peres, , and Albert C. Reynolds. Well-test analysis for solution-gas-drive reservoirs: Part 1 - determination of relative and absolute permeabilities. *SPE Formation Evaluation*, 5(2):124–132, 1990.
9. C. Theis. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground water storage. *Trans AGU*, pages 519–524, 1935.