

Lecture Notes

PET504E
Advanced Well Test
Analysis

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1 Introduction

The term "Well Testing" ^{c1}as it is used in Petroleum Industry means the measuring of a formation's (or reservoir's) pressure (and/or rate) response to flow from a well. The term "Well Testing" is generally used with the term "Pressure Transient Analysis", interchangeably. It is an indirect measurement technique as opposed to direct methods such as fluid sampling or coring. Well testing provides dynamic information on the reservoir whereas direct measurements only provide static information, which is not sufficient for predicting the behavior of the reservoir.

^{c1} Murat Çınar: Text added.

Simply, the objective of well testing is to deduce quantitative information about the well/reservoir system under consideration from its response to a given input. Input (or input signal) is used for perturbing one or more wells so that the output (signal) exhibiting the response of the reservoir is obtained at the perturbed well and/or adjacent wells. In practice, the input is equivalent to controlling the well behavior ^{c2} created by changing the flow rate or the pressure at the well (Mathematically specifying the well behavior is equivalent to specifying a boundary condition). A common example for creating an input signal is ^{c3}a build up test ^{c4}where we change the rate to zero by shutting-in the well. Reservoir response, ^{c5} also called output signal, to a given input is monitored by measuring the pressure change (or rate change) at the ^{c6} well. This process is illustrated as,

^{c2} Murat Çınar: and

^{c3} Murat Çınar: Text added.

^{c4} Murat Çınar: in which

^{c5} Murat Çınar: which is

^{c6} Murat Çınar: same

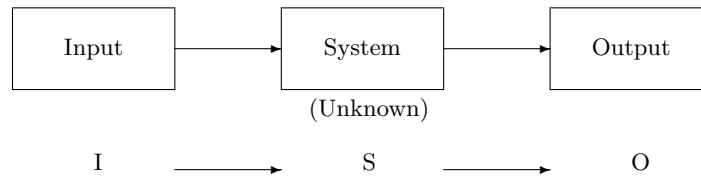


Fig. 1.1. Block diagram ?????

Typical examples for input and output signals as used in petroleum industry are shown in Fig. 1.2.

From reservoir response as monitored by the "output signal", we would like to determine information related to the followings:

- Fluid in place; pore volume, ϕhA .

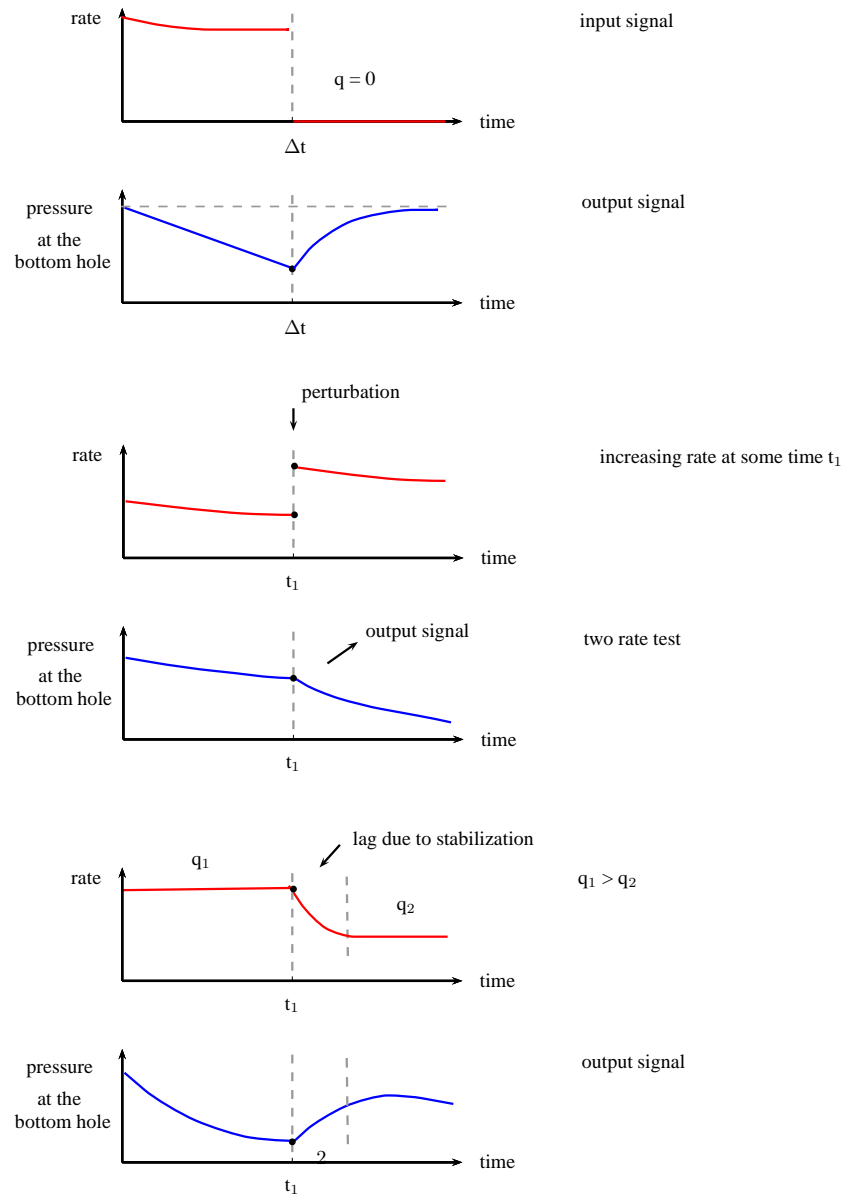


Fig. 1.2. Typical input and output signals - Transient phenomena.

- Ability of reservoir to transfer fluid, kh (or transmissibility, $\frac{kh}{\mu}$).
- Determination of average reservoir ^{c1}pressure, \bar{P} , which is the driving force in the reservoir ^{c2}
- Prediction of rate versus time data.
- Initial recovery, is the reservoir worth producing.
- Is there any damage around the wellbore impeding the flow? skin factor, s .
- Reservoir description (type of reservoir, flow boundaries (faults)).
- Distance to fluid interface ^{c3}that is important determining swept zone for secondary and tertiary methods.

Interpretation of well test data consist of basically three steps:

- (i) Determination of the one most appropriate reservoir / wellbore (mathematical) model ^{c4}of the actual system. We also call such a model as the interpretation model. ^{c5}Here our intention is to find a representative mathematical model that reproduces, as close as possible, the output of the actual system for a given input. This is known as the inverse problem. ^{c6}We are trying to obtain information about the physical system by using observed measurements. Unfortunately, the solution of inverse problem often yields non-unique results. ^{c7}By non-unique results, we mean that several different interpretation models ^{c8}may generate an output signal (response) to a given input ^{c9}that is similar (or identical) to that of the actual system. The inverse problem can be represented by the following equation.

$$\Sigma = O/I \approx S \quad (1.1)$$

where Σ denotes the interpretation model, S denotes the actual system. In inverse problem, as can be seen from Eq.1.1, it may be possible to obtain the same outputs to a given I for different Σ_i 's ^{c10},however, the number of alternative models (solutions) can be reduced as the number and the range of output signal measurements.

- (ii) Once the appropriate model is determined, estimate the parameters of the actual system S . ^{c11}These parameters are kh , s , ϕ , C , λ , w etc. This is known as ^{c12}parameter estimation ^{c13c14}and achieved by adjusting the parameters of the model by different ^{c15}mathematical methods to obtain an output signal, Ω , that is always qualitatively identical (within some tolerance) to that of ^{c16}the actual system, O . The computation of Ω is known as the "^{c17}forward problem" in mathematics. Contrary to the inverse problem, the solution of the ^{c18}forward problem is always unique for a given system; that is,

$$I \times \Sigma = \Omega \approx 0 \quad (1.2)$$

The adjusted parameters of the interpretation model are assumed to represent the parameters of the real system S . ^{c19}

- (iii) Validate the results of the interpretation. This can be achieved by using the parameters determined from part (ii) in the model to generate output

^{c1} Murat Çınar: average

^{c2} Murat Çınar: Based upon the explanation you gave about the pressure decline recently, I am not sure if this statement is correct.

^{c3} Murat Çınar: which

^{c4} Murat Çınar: to

^{c5} Murat Çınar: Here our hope is that the model chosen will produce an output signal to a given input which is as close as possible to that of the actual system.

^{c6} Murat Çınar: Text added.

^{c7} Murat Çınar: With

^{c8} Murat Çınar: can

^{c9} Murat Çınar: which

^{c10} Murat Çınar: However

^{c11} Murat Çınar: Such parameters can be

^{c12} Murat Çınar: the

^{c13} Murat Çınar: problem

^{c14} Murat Çınar: It is

^{c15} Murat Çınar: Text added.

^{c16} Murat Çınar: Text added.

^{c17} Murat Çınar: direct

^{c18} Murat Çınar: direct

^{c19} Murat Çınar: I think we should discuss this phrase. Does the real system have any parameters? I do not think so... This is something conceptual we need to think about

signals for the entire range of ^{c20}the test and by comparing these outputs with the ^{c21}physical measurements.

^{c20} Murat Çınar: Text added.

^{c21} Murat Çınar: measured ones

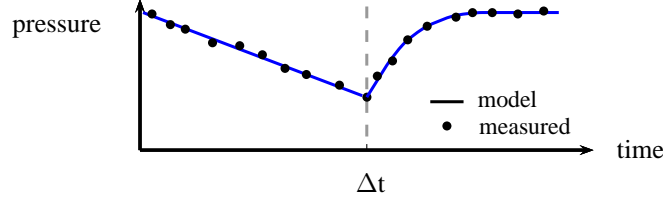


Fig. 1.3. Parameters estimated based on the analysis of buildup data.

^{c1}Now we consider single phase flow in a cylindrical reservoir produced by a well at the center. The ^{c2}partial differential equation (PDE) describing the flow is given by,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{kr}{\mu} \frac{\partial p}{\partial r} \right) = \phi c_t \frac{\partial p}{\partial t} \quad (1.3)$$

or if k, μ are constant,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) = \frac{\phi c_t \mu}{k} \frac{\partial p}{\partial t} = \frac{1}{\eta} \frac{\partial p}{\partial t} \quad (1.4)$$

^{c3} $\eta = \frac{k}{\phi c_t \mu}$ is the hydraulic diffusivity^{c4}; a measure of the ^{c5}speed at which a pressure disturbance ^{c6}propagates through the formation. If we specify, k, ϕ, c_t, μ , and the flow rate, then $p(r, t)$ is uniquely determined. This is an example for the ^{c7}forward problem.

Inverse problem, given q and p , ^{c8}helps us to

1. determine the PDE that describes the reservoir best
2. find k, ϕ , etc.

^{c9}

^{c10}Analytical solutions have been presented in the literature for a variety of different well and reservoir settings for single phase flow. A summary of these responses are given by Bourdet [1].

2 Flow Equations

In this chapter, we will derive the equations which describe the fluid flow in porous media. Such equations are derived from the conservation of mass and the momentum equation as given by Darcy's semi-empirical equation. With the exception of thermal recovery schemes, all well-testing models assume isothermal conditions in the reservoir and thus the energy conservation is not needed.

^{c1} Murat Çınar: To illustrate an example of direct and inverse problems, let's consider single phase flow in a closed cylindrical reservoir produced by a single well at the center.

^{c2} Murat Çınar: p.d.E

^{c3} Murat Çınar: $\eta = \frac{\phi c_t \mu}{k}$

^{c4} Murat Çınar: which is

^{c5} Murat Çınar: rapidity with

^{c6} Murat Çınar: Text added.

^{c7} Murat Çınar: direct

^{c8} Murat Çınar: Text added.

^{c9} Murat Çınar: During the past ten years, a lot of work has been done to develop models to a wide variety of reservoir / well configurations such as fractures, layered reservoirs, multiple porosities (composite zones), fractured wells, slanted and horizontal wells, etc. Well testing literature is almost complete for single phase problems, but some work needs to be done for multi-phase problems and heterogeneous reservoir systems.

Recently, people are much focused on the model recognition problem and the computer-aided parameter estimation by using non-linear regression techniques. Pressure derivatives and integrals are proven to be very useful in identifying the appropriate

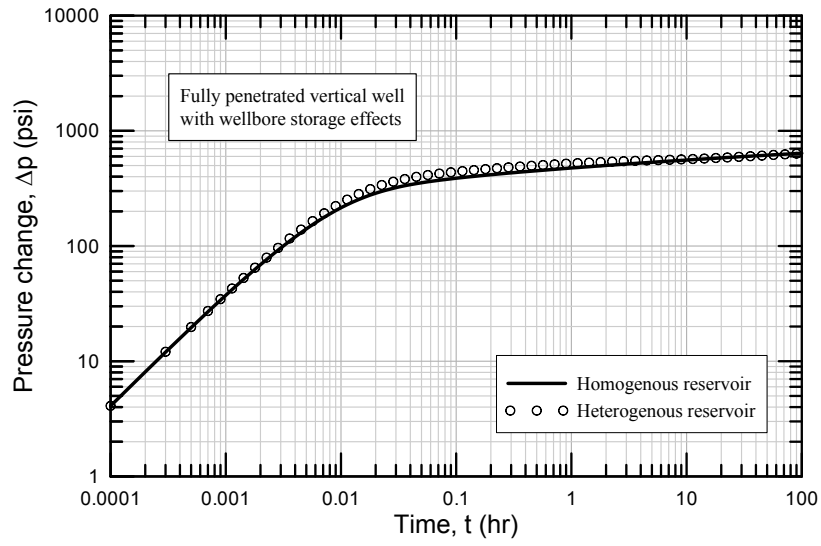


Fig. 1.4. Homogeneous vs heterogeneous reservoir, pressure difference .

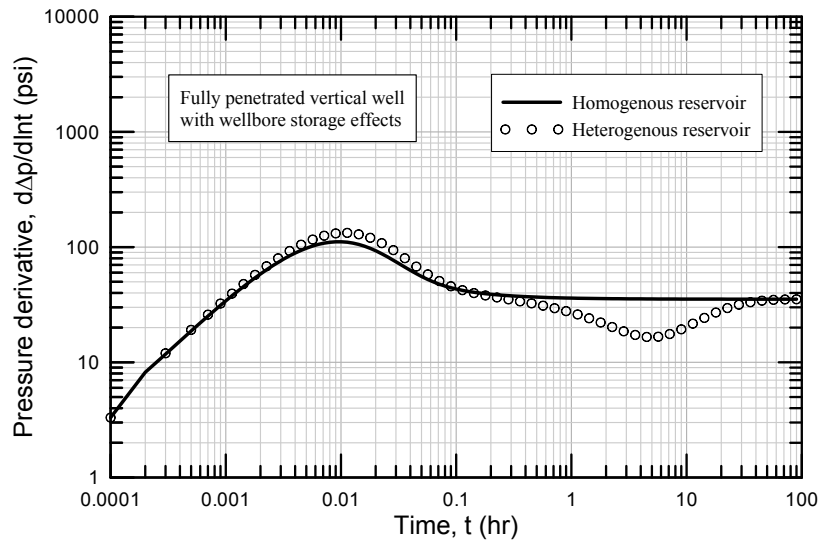


Fig. 1.5. Homogeneous vs heterogeneous reservoir - logarithmic derivative.

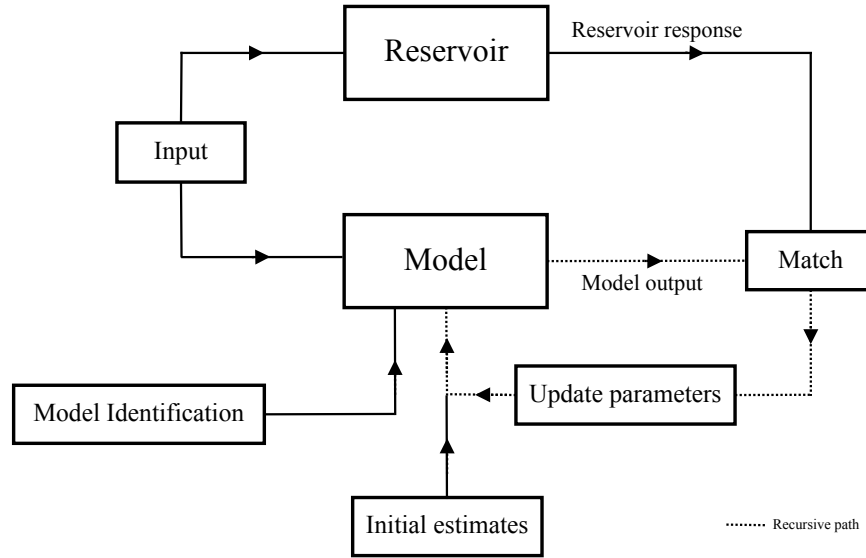


Fig. 1.6. Flow diagram of computer aided parameter estimation.

2.1 Conservation of mass

For a single phase fluid ^{c11}, the mathematical form of the mass balance in porous media is given by

$$-\nabla \cdot (\rho \mathbf{v}) = \frac{\partial (\rho \phi)}{\partial t} \quad (2.1)$$

^{c1} where ρ is the density of the fluid in ^{c2} M/L^3 and \mathbf{v} is the fluid velocity vector in ^{c3} L/T . Note that the units of Eq. 2.1 is ^{c4} $\text{M/L}^3\text{T}$. It is also important to note that Eq. 2.1 applies for any coordinate system and can be derived either from a mass balance done on a control volume for a coordinate system under consideration or from ^{c5} divergence theorem (or Gauss Theorem see Supplement II).

In ^{c6}Cartesian coordinate system,

$$\nabla \cdot (\rho \mathbf{v}) = \frac{\partial}{\partial x} (\rho v_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho v_z) \quad (2.2)$$

In ^{c7}cylindrical coordinate system,

$$\nabla \cdot (\rho \mathbf{v}) = \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) \quad (2.3)$$

^{c8}

^{c11} Murat Çınar: (or a fluid composed of one hydrocarbon component)

^{c1} Murat Çınar: I removed the constant -5.615 from the equation

^{c2} Murat Çınar: lbm/ft^3

^{c3} Murat Çınar: $\text{RB/ft}^2\text{day}$

^{c4} Murat Çınar: $\text{lbm/ft}^3\text{day}$

^{c5} Murat Çınar: a

^{c6} Murat Çınar: x-y-x

^{c7} Murat Çınar: r-θ-z

^{c8} Murat Çınar: typo in the equation corrected

2.2 Conservation of momentum in porous media

The principle of momentum conservation is described by the equation of motion. For most hydrocarbon fluids, the shear stress - shear rate behavior ^{c9}is described by the Newton's law of friction, combined with the equation of motion, results in the well known Navier-Stokes equation. Solution of the Navier-Stokes equation with the appropriate boundary conditions yields the velocity distribution of a given problem. Although, it is possible to solve Navier-Stokes in pipe flow, it is almost impossible to solve due to complexity of the pore geometry and its distribution. This hinders the formation of the boundary conditions for flow through a porous medium. Therefore, a different approach ^{c10}is taken. In 1856, ^{c11}Darcy discovered that ^{c12}for a single phase viscous flow in porous media, the velocity is proportional to the pressure gradient with a proportionality constant k . The general form of Darcy's Law including gravity effects is given by Eq.2.4.

$$\mathbf{v} = -\frac{\mathbf{k}}{\mu} (\nabla p - \rho g \nabla z') \quad (2.4)$$

^{c1} where \mathbf{v} is defined as a volumetric flow rate across a unit cross-section area (solid+fluid) averaged over a small region of space. The unit of \mathbf{v} is in ^{c2} \mathbf{L}/\mathbf{T} . Eq. 2.4 yield ^{c3} \mathbf{s} a velocity vector ^{c4}that replaces the solution of Navier-Stokes equation. In Eq. 2.4 \mathbf{k} is ^{c5}the permeability tensor. Operationally, \mathbf{k} acts like a matrix in ^{c6}coordinate system, we usually ^{c7}assume

$$\mathbf{k} \nabla p = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix} \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix} = \begin{bmatrix} k_x \frac{\partial p}{\partial x} \\ k_y \frac{\partial p}{\partial y} \\ k_z \frac{\partial p}{\partial z} \end{bmatrix}$$

$$\nabla z' = \begin{bmatrix} \frac{\partial z'}{\partial x} \\ \frac{\partial z'}{\partial y} \\ \frac{\partial z'}{\partial z} \end{bmatrix}$$

where, z' is the direction in which gravity acts, i.e., the direction towards the center of the earth. In ^{c8}Cartesian coordinate system, ^{c9} for each velocity component,

$$v_\xi = -\frac{k_\xi}{\mu} \left(\frac{\partial p}{\partial \xi} - \rho g \frac{\partial z'}{\partial \xi} \right), \quad \xi = x, y, z \quad (2.5)$$

We generally denote γ as the specific weight of fluid and define as,

$$\gamma = \rho g$$

It follows from Eq. 2.5, 2.2, and 2.1 that the PDE describing conservation of mass in ^{c10}Cartesian coordinate system is

$$\begin{aligned}
 & \frac{\partial}{\partial x} \left[\rho \frac{k_x}{\mu} \left(\frac{\partial p}{\partial x} - \gamma \frac{\partial z'}{\partial x} \right) \right] \\
 & + \frac{\partial}{\partial y} \left[\rho \frac{k_y}{\mu} \left(\frac{\partial p}{\partial y} - \gamma \frac{\partial z'}{\partial y} \right) \right] \\
 & + \frac{\partial}{\partial z} \left[\rho \frac{k_z}{\mu} \left(\frac{\partial p}{\partial z} - \gamma \frac{\partial z'}{\partial z} \right) \right] = \frac{\partial}{\partial t} (\rho \phi)
 \end{aligned} \tag{2.6}$$

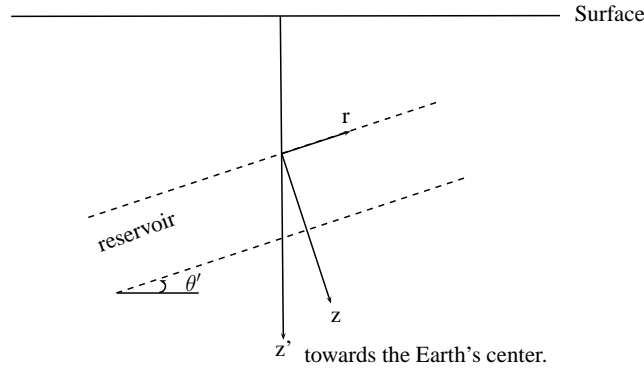
In ^{c1}Cylindrical coordinates (neglecting flow in θ direction)

^{c1} Murat Çınar: ~~r-z~~

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{k_r}{\mu} \rho \left(\frac{\partial p}{\partial r} - \gamma \frac{\partial z'}{\partial r} \right) \right] + \frac{\partial}{\partial z} \left[\frac{k_z}{\mu} \rho \left(\frac{\partial p}{\partial z} - \gamma \frac{\partial z'}{\partial z} \right) \right] = \frac{\partial}{\partial t} (\rho \phi) \tag{2.7}$$

Remark on gravity term:

In ^{c2}Cylindrical coordinates, ^{c3}assume z and z' are in the same direc-



tion then,

$$\frac{\partial z'}{\partial r} = \frac{\partial z'}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial z'}{\partial z} = 1$$

^{c4}if the reservoir is dipping with an angle of θ' ,

$$\frac{\partial z'}{\partial r} = \sin \theta' \quad \frac{\partial z'}{\partial \theta} = \cos \theta' \quad \frac{\partial z'}{\partial z} = \cos \theta'$$

^{c2} Murat Çınar: ~~r- θ -z~~

^{c3} Murat Çınar: if reservoir is horizontal; i.e. ~~$\theta' = 0$~~

^{c4} Murat Çınar: If we had a dip angle ~~θ'~~

^{c5}Now consider a single well ^{c6}at the center of a cylindrical reser-

^{c5} Murat Çınar: If we

^{c6} Murat Çınar: in

voir, then Eq. 2.7 correctly describes the fluid flow for both partially penetrating and fully penetrating cases, see Fig 2.1.

^{c7} ^{c8} Now define formation volume factor B as,

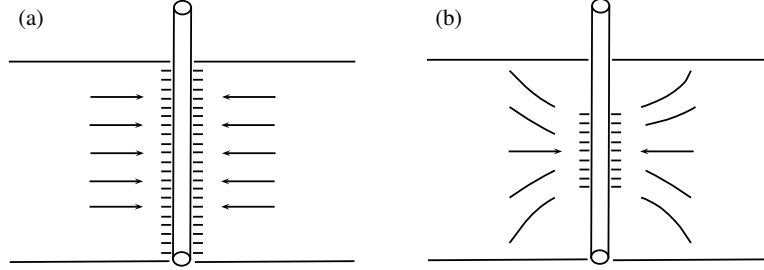


Fig. 2.1. (a) Fully penetrated vertical well. (b) Partially penetrated vertical well.

^{c7} Murat Çınar: Recalling our general continuity equation for the single phase flow gives,

^{c8} Murat Çınar: Text added.

$$B = \frac{V_{\text{reservoir}}}{V_{SC}} = \frac{m/\rho}{m/\rho_{SC}} = \frac{\rho_{SC}}{\rho} \quad ; \quad \rho_{SC} \text{ is constant} \quad (2.8)$$

^{c1} Murat Çınar: which we call it as Formation volume factor, then we can write Equation (2.8) as

^{c2} Murat Çınar: Text added.

^{c1} ^{c2} Recall general continuity equation, Eq. 2.1 and insert Eq. 2.8, then we have,

$$-\nabla \cdot \left(\frac{\mathbf{v}}{B} \right) = \frac{\partial}{\partial t} \left(\frac{\phi}{B} \right) \quad (2.9)$$

$B = B(p)$, $\rho = \rho(p)$, and $\phi = \phi(p) \rightarrow$ single valued functions of p

^{c3} Expanding right hand side (RHS) of Eq. 2.9,

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{\phi}{B} \right) &= \phi \frac{\partial}{\partial t} \left(\frac{1}{B} \right) + \frac{1}{B} \frac{\partial \phi}{\partial t} \\ &= \phi \left(-\frac{1}{B^2} \frac{dB}{dp} \right) \frac{\partial p}{\partial t} + \frac{1}{B} \frac{d\phi}{dt} \frac{\partial p}{\partial t} \\ &= \frac{\phi}{B} \left(-\frac{1}{B} \frac{dB}{dp} + \frac{1}{\phi} \frac{d\phi}{dt} \right) \frac{\partial p}{\partial t} \end{aligned} \quad (2.10)$$

^{c4} Murat Çınar: Typo in Eq corrected.

^{c4} ^{c5} Fluid and rock compressibilities are defined, respectively, by,

^{c5} Murat Çınar: but

$$c_{\text{fluid}} = -\frac{1}{V} \frac{dV}{dp} = -\frac{1}{B} \frac{dB}{dp} = \frac{1}{\rho} \frac{d\rho}{dp} \quad (2.11)$$

$$c_r = c_f = \frac{1}{\phi} \frac{d\phi}{dp} \quad (2.12)$$

(here p is the fluid pressure in the pore, therefore, $\frac{d\phi}{dp} > 0$.)
 Using Eqs. 2.11 and 2.12 in Eq. 2.10 and the resulting equation in Eq. 2.9 gives,

$$-\nabla \cdot \left(\frac{\mathbf{v}}{B} \right) = \frac{\phi c_t}{B} \frac{\partial p}{\partial t} \quad (2.13)$$

^{c1}Note that here the total compressibility c_t is defined as $c_t = c_{fluid} + c_{rock}$. Murat Gımar: Text added.
 Under the assumptions of Darcy's Law we have,

$$\nabla \cdot \left(\frac{\mathbf{k}}{\mu B} (\nabla p - \gamma \nabla z') \right) = \frac{\phi c_t}{B} \frac{\partial p}{\partial t} \quad (2.14)$$

References

1. D. Bourdet. *Well test analysis: the use of advanced interpretation models*. Handbook of petroleum exploration and production. Elsevier, 2002.