Lecture Notes

PET504E
Advanced Well Test Analysis

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1 Introduction

The term "Well Testing" as it is used in Petroleum Industry means the measuring of a formation's (or reservoir's) pressure (and/or rate) response to flow from a well. The term "Well Testing" is generally used with the term "Pressure Transient Analysis", interchangeably. It is an indirect measurement technique as opposed to direct methods such as fluid sampling or coring. Well testing provides dynamic information on the reservoir whereas direct measurements only provide static information, which is not sufficient for predicting the behavior of the reservoir.

Simply, the objective of well testing is to deduce quantitative information about the well/reservoir system under consideration from its response to a given input. Input (or input signal) is used for perturbing one or more wells so that the output (signal) exhibiting the response of the reservoir is obtained at the perturbated well and/or adjacent wells. In practice, the input is equivalent to controlling the well behavior created by changing the flow rate or the pressure at the well (Mathematically specifying the well behavior is equivalent to specifying a boundary condition). A common example for creating an input signal is a build up test where we change the rate to zero by shutting-in the well. Reservoir response, also called output signal, to a given input is monitored by measuring the pressure change (or rate change) at the well. This process is illustrated as,

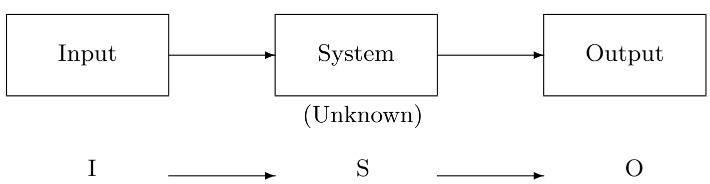


Fig. 1.1. Block diagram ?????

Typical examples for input and output signals as used in petroleum industry are shown in Fig. 1.2.

From reservoir response as monitored by the "output signal", we would like to determine information related to the followings:

– Fluid in place; pore volume, $\phi h A$.

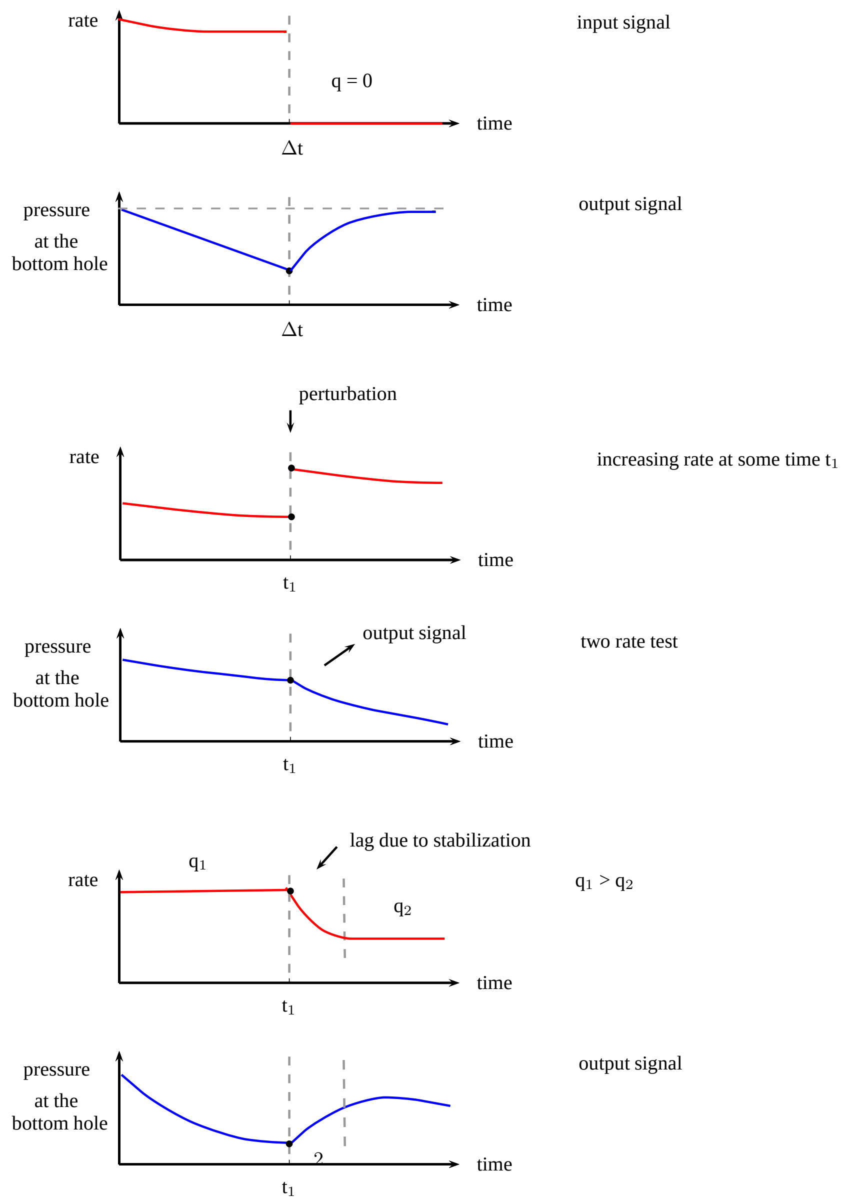


Fig. 1.2. Typical input and output signals - Transient phenomena.

- Ability of reservoir to transfer fluid, kh (or transmissibility, $\frac{kh}{\mu}$).
- Determination of average reservoir pressure, $\overline{P}$, which is the driving force in the reservoir
- Prediction of rate versus time data.
- Initial recovery, is the reservoir worth producing.
- Is there any damage around the wellbore impeding the flow? skin factor, s.
- Reservoir description (type of reservoir, flow boundaries (faults)).
- Distance to fluid interface that is important determining swept zone for secondary and tertiary methods.

Interpretation of well test data consist of basically three steps:

(i) Determination of the one most appropriate reservoir / wellbore (mathematical) model of the actual system. We also call such a model as the interpretation model. Here our intention is to find a representative mathematical model that reproduces, as close as possible, the output of the actual system for a given input. This is known as the inverse problem. We are trying to obtain information about the physical system by using observed measurements. Unfortunately, the solution of inverse problem often yields non-unique results. By non-unique results, we mean that several different interpretation models may generate an output signal (response) to a given input that is similar (or identical) to that of the actual system. The inverse problem can be represented by the following equation.

$$\Sigma = O/I \ \approx S \tag{1.1}$$

where Σ denotes the interpretation model, S denotes the actual system. In inverse problem, as can be seen from Eq.1.1, it may be possible to obtain the same outputs to a given I for different Σ_i's,however, the number of alternative models (solutions) can be reduced as the number and the range of output signal measurements.

(ii) Once the appropriate model is determined, estimate the parameters of the actual system S. These parameters are kh, s, ϕ, C, λ, w etc. This is known as parameter estimation and achieved by adjusting the parameters of the model by different mathematical methods to obtain an output signal, Ω, that is always qualitatively identical (within some tolerance) to that of the actual system, O. The computation of Ω is known as the "forward problem" in mathematics. Contrary to the inverse problem, the solution of the forward problem is always unique for a given system; that is,

$$I \times \Sigma = \Omega \approx 0 \tag{1.2}$$

The adjusted parameters of the interpretation model are assumed to represent the parameters of the real system S.

(iii) Validate the results of the interpretation. This can be achieved by using the parameters determined from part (ii) in the model to generate output signals for the entire range of the test and by comparing these outputs with the physical measurements.

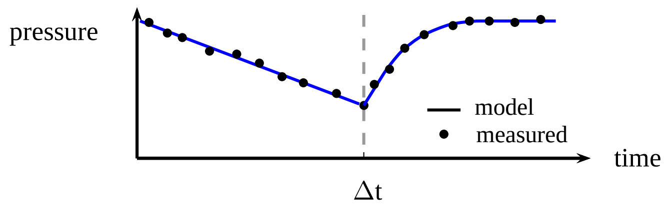


Fig. 1.3. Parameters estimated based on the analysis of buildup data.

Now we consider single phase flow in a cylindrical reservoir produced by a well at the center. Thepartial differential equation (p.d.e.) describing the flow is given by,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{kr}{\mu}\frac{\partial p}{\partial r}\right) = \phi c_t \frac{\partial p}{\partial t} \tag{1.3}$$

or if k, μ are constant,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\phi c_t \mu}{k}\frac{\partial p}{\partial t} = \frac{1}{\eta}\frac{\partial p}{\partial t} \tag{1.4}$$

$\eta = \frac{k}{\phi c_t \mu}$ is the hydraulic diffusivity; a measure of the speed at which a pressure disturbance propagates through the formation. If we specify, k, ϕ, c_t, μ, and the flow rate, then $p(r,t)$ is uniquely determined. This is an example for the forward problem.

Inverse problem, given q and p, helps us to

1. determine the p.d.e. that describes the reservoir best
2. find k, ϕ, etc.

Analytical solutions have been presented in the literature for a variety of different well and reservoir settings for single phase flow. A summary of these responses are given by Bourdet [5].

2 Flow Equations

In this chapter, we will derive the equations which describe the fluid flow in porous media. Such equations are derived from the conservation of mass and the momentum equation as given by Darcy's semi-empirical equation. With the exception of thermal recovery schemes, all well-testing models assume isothermal conditions in the reservoir and thus the energy conservation is not needed.

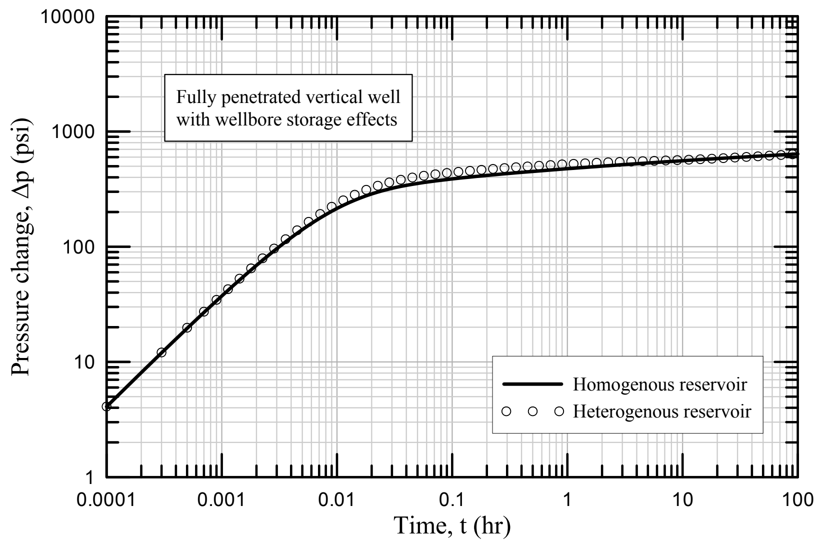


Fig. 1.4. Homogeneous vs heterogeneous reservoir, pressure difference .

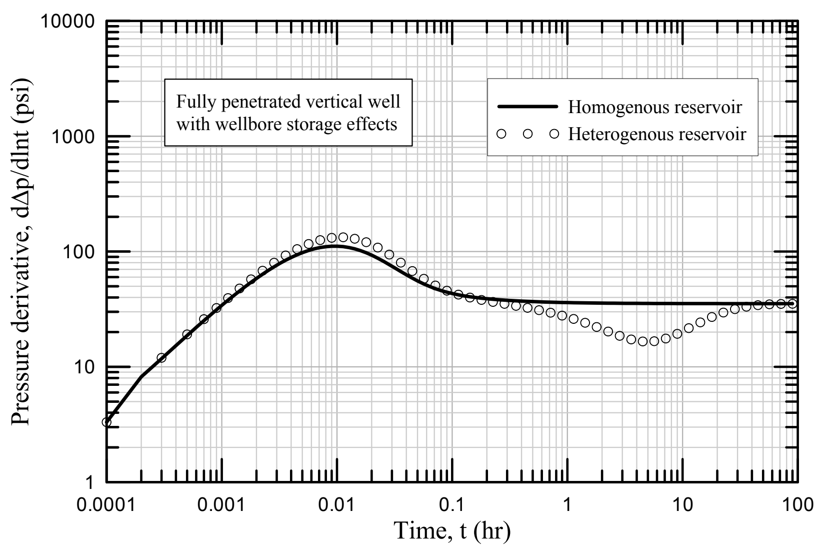


Fig. 1.5. Homogeneous vs heterogeneous reservoir - logarithmic derivative.

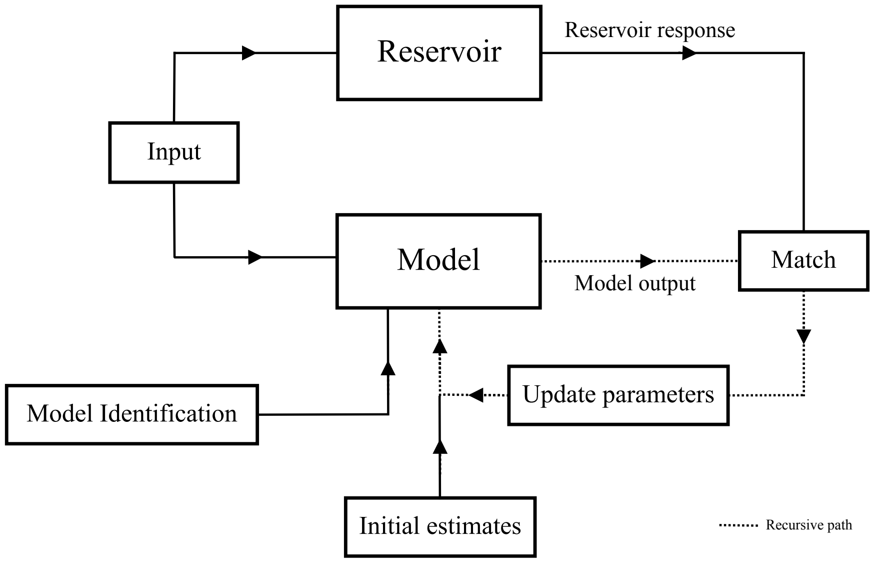


Fig. 1.6. Flow diagram of computer aided parameter estimation.

2.1 Conservation of mass

For a single phase fluid , the mathematical form of the mass balance in porous media is given by

$$-\nabla \cdot (\rho \mathbf{v}) = \frac{\partial (\rho \phi)}{\partial t} \tag{2.1}$$

where ρ is the density of the fluid in $\mathbf{M}/\mathbf{L}^3$ and $\mathbf{v}$ is the fluid velocity vector in $\mathbf{L}/\mathbf{T}$. Note that the units of Eq. 2.1 is $\mathbf{M}/\mathbf{L}^3\mathbf{T}$ It is also important to note that Eq. 2.1 applies for any coordinate system and can be derived either from a mass balance done on a control volume for a coordinate system under consideration or from divergence theorem (or Gauss Theorem see Supplement II).

InCartesian coordinate system,

$$\nabla \cdot (\rho \mathbf{v}) = \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) \tag{2.2}$$

Incylindrical coordinate system,

$$\nabla \cdot (\rho \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) \tag{2.3}$$

2.2 Conservation of momentum in porous media

The principle of momentum conservation is described by the equation of motion. For most hydrocarbon fluids, the shear stress - shear rate behavior is described by the Newton's law of friction, combined with the equation of motion, results in the well known Navier-Stokes equation. Solution of the Navier-Stokes equation with the appropriate boundary conditions yields the velocity distribution of a given problem. Although, it is possible to solve Navier-Stokes in pipe flow, it is almost impossible to solve due to complexity of the pore geometry and its distribution. This hinders the formation of the boundary conditions for flow through a porous medium. Therefore, a different approach is taken. In 1856, Darcy discovered that for a single phase viscous flow in porous media, the velocity is proportional to the pressure gradient with a proportionality constant k. The general form of Darcy's Law including gravity effects is given by Eq.2.4.

$$\mathbf{v} = -\frac{\mathbf{k}}{\mu}\left(\nabla p - \rho g \nabla z'\right) \tag{2.4}$$

where $\mathbf{v}$ is defined as a volumetric flow rate across a unit cross-section area (solid+fluid) averaged over a small region of space. The unit of $\mathbf{v}$ is in $\mathbf{L/T}$. Eq. 2.4 yields a velocity vector that replaces the solution of Navier-Stokes equation. In Eq. 2.4 $\mathbf{k}$ is the permeability tensor. Operationally, $\mathbf{k}$ acts like a matrix in coordinate system, we usually assume

$$\mathbf{k}\,\nabla p = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix} \begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix} = \begin{bmatrix} k_x \frac{\partial p}{\partial x} \\ k_y \frac{\partial p}{\partial y} \\ k_z \frac{\partial p}{\partial z} \end{bmatrix}$$

$$\nabla z' = \begin{bmatrix} \frac{\partial z'}{\partial x} \\ \frac{\partial z'}{\partial y} \\ \frac{\partial z'}{\partial z} \end{bmatrix}$$

where, z' is the direction in which gravity acts, i.e., the direction towards the center of the earth. In Cartesian coordinate system, for each velocity component,

$$v_\xi = -\frac{k_\xi}{\mu}\left(\frac{\partial p}{\partial \xi} - \rho g \frac{\partial z'}{\partial \xi}\right), \quad \xi = x, y, z \tag{2.5}$$

We generally denote γ as the specific weight of fluid and define as,

$$\gamma = \rho g$$

It follows from Eq. 2.5, 2.2, and 2.1 that the p.d.e. describing conservation of mass in Cartesian coordinate system is

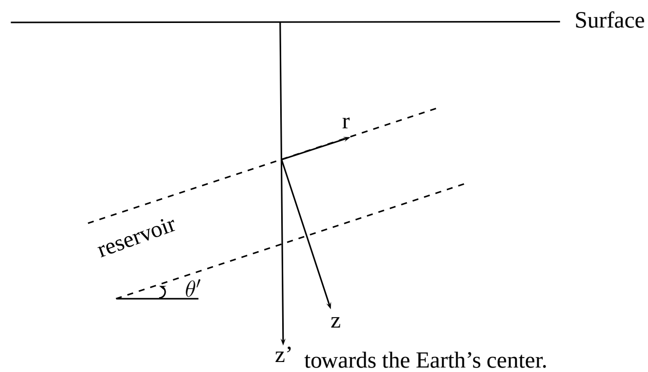
$$\begin{aligned} &\frac{\partial}{\partial x}\left[\rho \frac{k_x}{\mu}\left(\frac{\partial p}{\partial x} - \gamma \frac{\partial z'}{\partial x}\right)\right] \\ &+ \frac{\partial}{\partial y}\left[\rho \frac{k_y}{\mu}\left(\frac{\partial p}{\partial y} - \gamma \frac{\partial z'}{\partial y}\right)\right] \\ &+ \frac{\partial}{\partial z}\left[\rho \frac{k_z}{\mu}\left(\frac{\partial p}{\partial z} - \gamma \frac{\partial z'}{\partial z}\right)\right] = \frac{\partial}{\partial t}\left(\rho \phi\right) \end{aligned} \tag{2.6}$$

In Cylindrical coordinates (neglecting flow in θ direction)

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k_r}{\mu}\rho\left(\frac{\partial p}{\partial r}-\gamma\frac{\partial z'}{\partial r}\right)\right]+\frac{\partial}{\partial z}\left[\frac{k_z}{\mu}\rho\left(\frac{\partial p}{\partial z}-\gamma\frac{\partial z'}{\partial z}\right)\right]=\frac{\partial}{\partial t}(\rho\phi) \tag{2.7}$$

Remark on gravity term:

In Cylindrical coordinates,assume z and z' are in the same direction



then,

$$\frac{\partial z'}{\partial r}=\frac{\partial z'}{\partial \theta}=0 \quad and \quad \frac{\partial z'}{\partial z}=1$$

if the reservoir is dipping with an angle of θ',

$$\frac{\partial z'}{\partial r}=\sin\theta' \quad \frac{\partial z'}{\partial \theta}=\cos\theta' \quad \frac{\partial z'}{\partial z}=\cos\theta'$$

Now consider a single well at the center of a cylindrical reservoir, then Eq. 2.7 correctly describes the fluid flow for both partially penetrating and fully penetrating cases, see Fig 2.1.

Now define formation volume factor B as,

$$B=\frac{V_{reservoir}}{V_{SC}}=\frac{m/\rho}{m/\rho_{\ SC}}=\frac{\rho_{SC}}{\rho} \quad ; \quad \rho_{SC}\,\text{is constant} \tag{2.8}$$

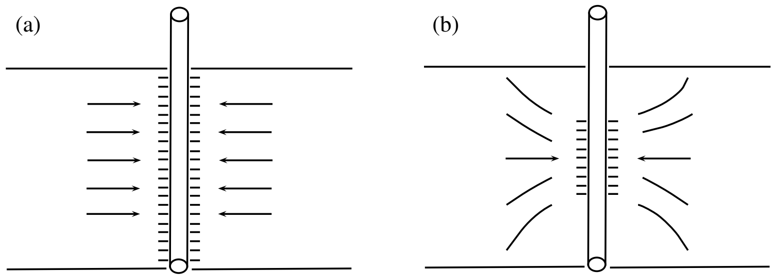


Fig. 2.1. (a) Fully penetrated vertical well. (b) Partially penetrated vertical well.

Recall general continuity equation, Eq. 2.1 and insert Eq. 2.8, then we have,

$$-\nabla \cdot \left(\frac{\mathbf{v}}{B}\right) = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) \tag{2.9}$$

$B = B(p)$, $\rho = \rho(p)$, and $\phi = \phi(p) \rightarrow$ single valued functions of p Expanding right hand side (RHS) of Eq. 2.9,

$$\begin{aligned} \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) &= \phi\frac{\partial}{\partial t}\left(\frac{1}{B}\right) + \frac{1}{B}\frac{\partial \phi}{\partial t} \\ &= \phi\left(-\frac{1}{B^2}\frac{dB}{dp}\right)\frac{\partial p}{\partial t} + \frac{1}{B}\frac{d\phi}{dt}\frac{\partial p}{\partial t} \\ &= \frac{\phi}{B}\left(-\frac{1}{B}\frac{dB}{dp} + \frac{1}{\phi}\frac{d\phi}{dt}\right)\frac{\partial p}{\partial t} \end{aligned} \tag{2.10}$$

Fluid and rock compressibilities are defined, respectively, by,

$$c_{fluid} = -\frac{1}{V}\frac{dV}{dp} = -\frac{1}{B}\frac{dB}{d\rho} = \frac{1}{\rho}\frac{d\rho}{dp} \tag{2.11}$$

$$c_r = c_f = \frac{1}{\phi}\frac{d\phi}{dp} \tag{2.12}$$

(here p is the fluid pressure in the pore, therefore, $\frac{d\phi}{dp} > 0$.) Using Eqs. 2.11 and 2.12 in Eq. 2.10 and the resulting equation in Eq. 2.9 gives,

$$-\nabla \cdot \left(\frac{\mathbf{v}}{B}\right) = \frac{\phi c_t}{B}\frac{\partial p}{\partial t} \tag{2.13}$$

Note that here the total compressibility c_t is defined as $c_t = c_{fluid} + c_r$. Under the assumptions of Darcy's Law we have,

$$\nabla \cdot \left(\frac{\mathbf{k}}{\mu B} \left(\nabla p - \gamma \nabla z' \right) \right) = \frac{\phi c_t}{B} \frac{\partial p}{\partial t} \tag{2.14}$$

Slightly compressible fluid of constant compressibility Now assuming negligible gravity effects and k/μ is constant then,

$$\frac{k}{\mu} \nabla \cdot (\rho \nabla p) = \phi c_t \rho \frac{\partial p}{\partial t} \tag{2.15}$$

Assuming $c(\nabla p)^2$ is small here $c = \frac{1}{\rho}\frac{\partial \rho}{\partial p}$, Eq. 2.15 is well approximated by

$$\frac{k}{\mu} \nabla^2 p = \phi c_t \frac{\partial p}{\partial t} \tag{2.16}$$

Remark on coordinate systems:

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \quad \text{in Cartesian coordinates}$$

$$\nabla^2 p = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} \quad \text{in cylindrical coordinates}$$

$$\begin{aligned} \nabla^2 p = & \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial p}{\partial r} \right) + \frac{1}{r^2 sin\theta} \frac{\partial}{\partial \theta} \left(sin\theta \frac{\partial p}{\partial \theta} \right) \\ & + \frac{1}{r^2 sin^2\theta} \frac{\partial^2 p}{\partial \phi^2} \end{aligned} \quad \text{in spherical coordinates}$$

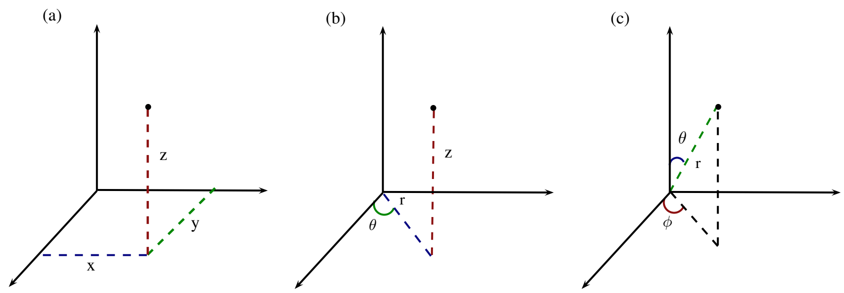


Fig. 2.2. (a) Cartesian coordinates. (b) Cylindrical coordinates. (c) Spherical coordinates.

Further assuming,

$$c = \frac{1}{\rho}\frac{\partial \rho}{\partial p} = \text{constant} \tag{2.17}$$

$$\int_{p_i}^{p} c dp = \int_{\rho_i}^{\rho} \frac{1}{\rho} d\rho$$

$$c\left(p - p_i\right) = \ln\left(\frac{\rho}{\rho_i}\right) \quad ; \quad p_i \text{ initial pressure}$$

or

$$\rho = \rho_i \exp\left[-c\left(p_i - p\right)\right] \tag{2.18}$$

Using Taylor series representation of $\exp\left[-c\left(p_i - p\right)\right]$ gives,

$$\begin{aligned}\rho &= \rho_i \left[1 - c\left(p_i - p\right) + \frac{c^2}{2}\left(p_i - p\right)^2 + \cdots\right] \\ &= \rho_i \left[1 - c\left(p_i - p\right) + \frac{c^2}{2}\left(p_i - \widetilde{p}\right)^2\right] \quad p < \widetilde{p} < p_i\end{aligned}$$

$$\rho = \rho_i \left[1 - c\left(p_i - p\right)\right] \tag{2.19}$$

Note that c is very small for oil (or liquids); $c \approx 10^{-5} \sim 10^{-6}$. Using Eq. 2.19 in Eq. 2.16.

$$\frac{k}{\mu}\nabla^2 p = \phi c_t \frac{\partial p}{\partial t} \tag{2.20}$$

2.3 Multiphase flow

Three distinct phases, gas, oil, and water occur i a petroleum reservoir. Varying pressure conditions (isothermal system assumed) cause a mass exchange between two hydrocarbon phases - oil-gas (water-oil and gas-water systems is assumed immiscible). The mass transfer between oil and gas is described by solution gas-oil ration, R_s, which gives the amount of gas dissolved in oil as a function of pressure, i.e. $[V_{dissolved\,gas}/V_o]_{STC}$.

The fluid flow equations (based on β-model) with the introduction of phase saturations for oil-water-gas system is written as;

$$-\nabla \cdot \left(\frac{\mathbf{v}_o}{B_o}\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) \quad for\,oil \tag{2.21}$$

$$-\nabla \cdot \left(\frac{\mathbf{v}_w}{B_w}\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) \quad for\,water \tag{2.22}$$

$$-\nabla \cdot \left(\frac{R_s\mathbf{v}_o}{B_o} + \frac{\mathbf{v}_g}{B_g}\right) = \frac{\partial}{\partial t}\left[\phi\left(\frac{R_s}{B_o}S_o + \frac{S_g}{B_g}\right)\right] \quad for\,gas \tag{2.23}$$

With introducing relative permeability , the velocity vector for each phase is given by,

$$\mathbf{v}_\varphi = -\frac{\mathbf{k}\,k_{r\varphi}}{\mu_\varphi}\left(\nabla p_\varphi - \gamma_\varphi \nabla z'\right) \tag{2.24}$$

where $\varphi = o, w, or g$.

Using Eq. 2.24 in Eqs. 2.21, 2.22, and 2.23 for corresponding phase,

$$\nabla \cdot \left(\frac{\mathbf{k}\,k_{ro}}{B_o\mu_o}\left(\nabla p_o - \gamma_o \nabla z'\right)\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) \tag{2.25}$$

$$\nabla \cdot \left(\frac{\mathbf{k}\,k_{rw}}{B_w\mu_w}\left(\nabla p_w - \gamma_w \nabla z'\right)\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) \tag{2.26}$$

$$\begin{aligned}\nabla \cdot \left(\frac{R_s\mathbf{k}\,k_{ro}}{B_o\mu_o}\left(\nabla p_o - \gamma_o \nabla z'\right) + \frac{\mathbf{k}\,k_{rg}}{B_g\mu_g}\left(\nabla p_g - \gamma_g \nabla z'\right)\right) \\ = \frac{\partial}{\partial t}\left[\phi\left(\frac{R_s}{B_o}S_o + \frac{S_g}{B_g}\right)\right]\end{aligned} \tag{2.27}$$

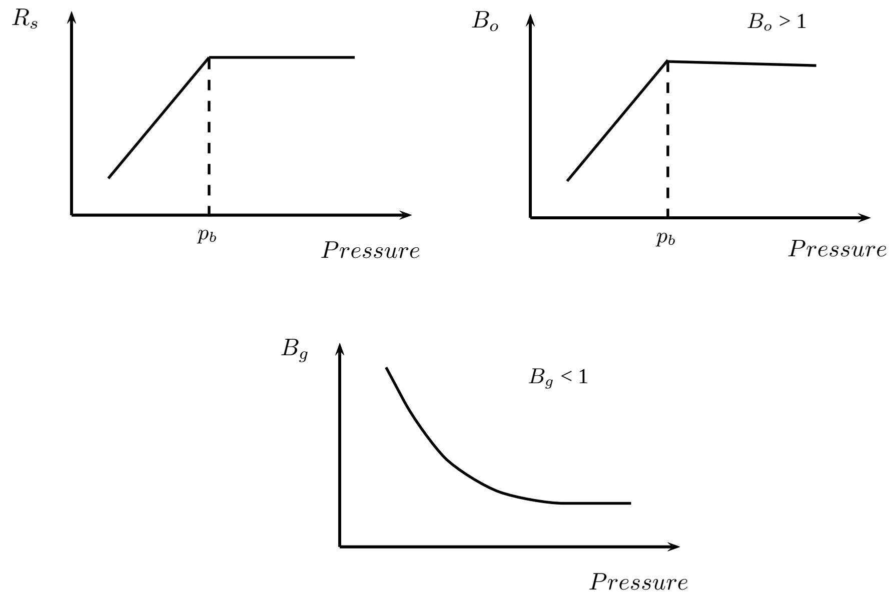


Fig. 2.3. R_s, B_o, B_g behavior.

It is important to note that B defined by Eq. 2.8, considering oil, is valid if oil is single component or "black-oil" with no dissolved gas ($R_s = 0$). On the other hand, Eq. 2.14 is valid for black-oil problems with $R_s \neq 0$ provided that the pressure is above bubble-point. Eq. 2.25 and Eq. 2.27 becomes identical if the pressure is above bubble point.

In some cases, the flow equation is written in terms of pseudo potential.

$$\psi = \int_{p_0}^{p} \frac{1}{\gamma} dp - (z' - {z_0}') \tag{2.28}$$

where z' is measured positive in the direction of gravity and ${z_0}'$ is the datum where p_0 is measured. Also $\gamma = \mathbf{M}/\mathbf{L}^2\mathbf{T}^2)$ and $\gamma = \gamma(p)$.

$$\begin{aligned}\nabla\psi &= \nabla\int_{p_0}^{p}\frac{1}{\gamma}dp - \nabla\left(z' - {z_0}'\right)\\ &= \frac{d}{dp}\left\{\int_{p_0}^{p}\frac{1}{\gamma}dp\right\}\nabla p - \nabla z'\\ &= \frac{1}{\gamma}\nabla p - \nabla z'\end{aligned}$$

Then,

$$\gamma\,\nabla\psi = \nabla p - \gamma\,\nabla z' \tag{2.29}$$

and

$$\frac{\partial\psi}{\partial t} = \frac{1}{\gamma}\frac{\partial p}{\partial t} \tag{2.30}$$

Using Eqs. 2.29 and 2.30 in Eq. 2.14 we obtain,

$$\nabla\cdot\left(\frac{\mathbf{k}\gamma}{B\mu}\nabla\psi\right) = \frac{\phi c_t\gamma}{B}\frac{\partial\psi}{\partial t} \tag{2.31}$$

If we consider that we have stress dependent reservoir, that is permeability decreases as the fluid pressure in the pores decreases, then

$$\mathbf{k} = k\left(p\right)\widetilde{\mathbf{k}}$$

where the entries of $\widetilde{\mathbf{k}}$ are independent of pressure. It has been observed that tight (and geothermal) reservoirs are examples of stress dependent reservoirs. Then Eq. 2.31 is expressed as

$$\nabla\cdot\left(\frac{k\left(p\right)\widetilde{\mathbf{k}}\gamma}{B\mu}\nabla\psi\right) = \frac{\phi c_t\gamma}{B}\frac{\partial\psi}{\partial t} \tag{2.32}$$

A pseudo pressure function is defined to partially linearize Eq. 2.32 (or Eq. 2.14) under the assumption that the gravity effects are not important.

$$m\left(p\right) = \int_{p_0}^{p}\frac{k\left(p\right)}{\mu\left(p\right)B\left(p\right)}dp \tag{2.33}$$

$$\nabla m\left(p\right) = \frac{d}{dp}m\left(p\right)\nabla p = \frac{k\left(p\right)}{\mu\left(p\right)B\left(p\right)}\nabla p \tag{2.34}$$

$$\frac{\partial m(p)}{\partial t} = \frac{k(p)}{\mu(p) B(p)} \frac{\partial p}{\partial t} \tag{2.35}$$

If gravity effects are ignored, Eq. 2.14 reduces to

$$\nabla \cdot \left(\frac{k(p)\,\widetilde{\mathbf{k}}}{B(p)\,\mu(p)} \nabla p \right) = \frac{\phi c_t}{B} \frac{\partial p}{\partial t} \tag{2.36}$$

Using Eqs. 2.33, 2.34, and 2.35 in 2.36 gives

$$\nabla \cdot \left(\widetilde{\mathbf{k}} \nabla m(p) \right) = \frac{\phi(p)\, c_t(p)\, \mu(p)}{k(p)} \frac{\partial m(p)}{\partial t} \tag{2.37}$$

Note Eq. 2.37 is still non-linear. Eq. 2.32 is the expression of basic flow equations in terms of the potential ψ. Therefore, to partially linearize Eq. 2.32, a "pseudo pressure" is defined as

$$m(\psi) = \int_{\psi_0}^{\psi} \frac{k(\psi)\,\gamma(\psi)}{\mu(\psi)\, B(\psi)} d\psi \tag{2.38}$$

2.4 Diffusivity equation for single phase gas flow - real gas flow

For gases μ, ρ are strong functions of pressure. Permeability typically is independent of pressure, however, at low pressures Klinkenberg effect may cause some pressure dependence in permeability and/or tight reservoirs are considered as discussed earlier. To account for the dependence of $k/\mu B_g$ on pressure, Eq.2.33 or 2.38 is used. Note that Eq. 2.37 is also valid for flow of real gases in porous media.

With some modifications, the above procedure is the current approach used to derive the p.d.e. for gas flow. The method was first introduced in the literature by Al-Hussainy, Ramey and Crawford [2]. Below the p.d.e. is derived using Al-Hussainy et. al. [2] approach. Note that Eq. 2.36 holds for real gases. Assuming $k(p)\mathbf{k}$ is independent of pressure and same in all directions, then Eq. 2.36 becomes

$$\nabla \cdot \left(\frac{1}{B(p)\,\mu(p)} \nabla p \right) = \frac{\phi c_t}{kB} \frac{\partial p}{\partial t} \tag{2.39}$$

Since $B = \frac{(\rho_g)_{SC}}{\rho_g}$ and $(\rho_g)_{SC}$ is constant Eq. 2.39 is equivalent to

$$\nabla \cdot \left(\frac{\rho}{\mu} \nabla p \right) = \frac{\phi c_t \rho}{k} \frac{\partial p}{\partial t} \tag{2.40}$$

Recall that ρ for real gases is given by the following equation of state (EOS),

$$\rho = \frac{pM}{zRT} \tag{2.41}$$

Using Eq. 2.41 in 2.40 gives

$$\nabla \cdot \left(\frac{pM}{zRT\mu} \nabla p \right) = \frac{pM}{zRT} \frac{\phi c_t}{k} \frac{\partial p}{\partial t} \tag{2.42}$$

Since M/RT is constant, then Eq. 2.42 reduces to

$$\nabla \cdot \left(\frac{p}{z\mu} \nabla p \right) = \frac{\phi c_t p}{zk} \frac{\partial p}{\partial t} \tag{2.43}$$

Al-Hussainy et. al. [2] defined the integral transform $m'(p)$ to be

$$m'(p) = 2 \int_{p_0}^{p} \frac{p'}{\mu z} dp' \tag{2.44}$$

$$\nabla m'(p) = \frac{2p}{\mu z} \nabla p \tag{2.45}$$

$$\frac{\partial m'}{\partial t} = 2 \frac{p}{\mu z} \frac{\partial p}{\partial t} \tag{2.46}$$

Using Eqs. 2.44, 2.45, and 2.46 in 2.43 gives,

$$\nabla \cdot \left[\nabla m'(p) \right] = \frac{\phi c_t \mu(p)}{k} \frac{\partial m'(p)}{\partial t} \tag{2.47}$$

2.5 1-D Radial flow equation

Consider a completely penetrating well in an infinite porous medium of uniform thickness filled with a single phase fluid. Further assume that flow is axisymmetric, i.e., no variation in θ-direction or in a plane z'=constant equipotential curves are circles - see Figure 2.4,

If reservoir is not horizontal, Eq. 2.14, applies with $v_\theta = 0$ and so Eq. 2.14 becomes in $r - z$ coordinates.

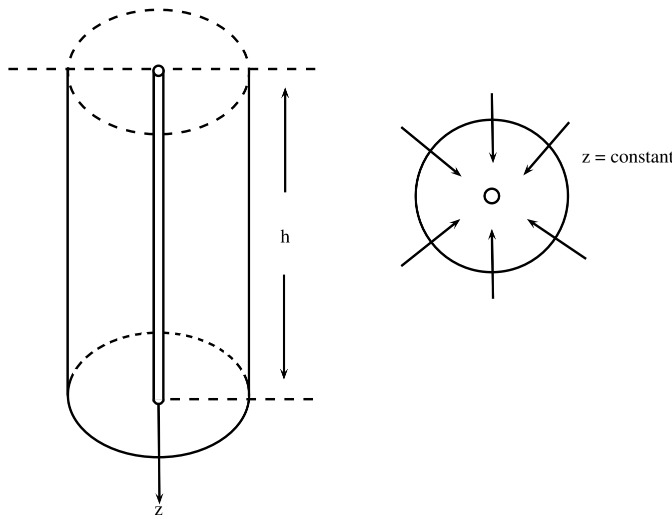


Fig. 2.4. Radial flow geometry.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{rk_r}{\mu B}\left(\frac{\partial p}{\partial r}-\gamma\frac{\partial z'}{\partial r}\right)\right] + \frac{\partial}{\partial z}\left[\frac{k_z}{\mu B}\left(\frac{\partial p}{\partial z}-\gamma\frac{\partial z'}{\partial z}\right)\right] = \frac{\phi c_t}{B}\frac{\partial p}{\partial t} \tag{2.48}$$

Now assume $z = z'$ and $\theta = 0$, then $\frac{\partial z'}{\partial r} = 0$

For a completely penetrating well, it is physically reasonable to assume $v_z \approx 0$ ($k_z \ll k_r$), i.e.,

$$v_z = -\frac{k_z}{\mu}\left(\frac{\partial p}{\partial z}-\gamma\frac{\partial z'}{\partial z}\right) = 0 \quad ; \quad \frac{\partial z'}{\partial z} = 1$$
$$\frac{\partial p}{\partial z} - \gamma = 0 \quad ; \quad p\left(z_2\right) = p\left(z_1\right) + \gamma\left(z_2 - z_1\right) \quad z_z > z_1 \tag{2.49}$$

Then the general radial flow problem becomes,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{k_r}{\mu B}\frac{\partial p}{\partial r}\right) = \frac{\phi c_t}{B}\frac{\partial p}{\partial t} \tag{2.50}$$

where ϕ, c_t, B, k_r, and μ are functions of pressure.

Recall pseudo-pressure function defined as,

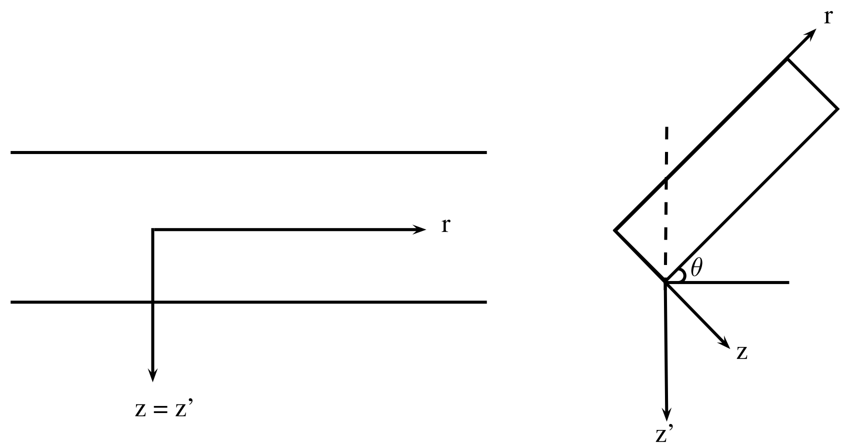


Fig. 2.5. r-z coordinates.

$$m(p) = \int_{p_b}^{p} \frac{k_r(p)}{\mu(p) B(p)} dp \tag{2.51}$$

Using Eq. 2.51, Eq. 2.50 is written as,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m(p)}{\partial r}\right) = \frac{\phi c_t \mu}{k_r}\frac{\partial m(p)}{\partial t} \tag{2.52}$$

Dimensionless Variables In well testing dimensionless variables are used for two main reasons;

(i) minimize number of variables (by grouping parameters)
(ii) provide general solutions

Dimensionless time is defined as,

$$t_D = \frac{k_i t}{(\phi c_t \mu)_i r_w^2} \tag{2.53}$$

where subscript "i" refers to initial conditions, i.e.,

$$k_i = k(p_i) \;\; ; \;\; \mu_i = \mu(p_i) \;\; etc.$$

here p_i is the initial reservoir pressure (at some datum) and we assume p_i is independent of r, then

$$\frac{\partial m}{\partial t} = \frac{\partial m}{\partial t_D}\frac{\partial t_D}{\partial t} = \frac{\partial m}{\partial t_D}\left(\frac{k_i}{(\phi c_t \mu)_i r_w^2}\right) \tag{2.54}$$

Using Eq. 2.54 in Eq. 2.52, and simplifying gives,

$$r_w^2\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial m\,(p)}{\partial r}\right)\right] = \left(\frac{\phi c_t \mu}{k}\right)\left[\frac{k_i}{(\phi c_t \mu)_i}\right]\frac{\partial m}{\partial t_D} \tag{2.55}$$

Now define,

$$r_D = \frac{r}{r_w} \tag{2.56}$$

and dimensionless diffusivity,

$$\eta_D = \frac{k/(\phi c_t \mu)}{k_i/(\phi c_t \mu)_i} \tag{2.57}$$

then Eq. 2.55 becomes

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m\,(p)}{\partial r_D}\right) = \frac{1}{\eta_D}\frac{\partial m\,(p)}{\partial t_D} \tag{2.58}$$

If $\frac{1}{\eta_D} = 0$, then Eq. 2.58 is a linear p.d.e. .
Consider production at a specified rate q; i.e.,

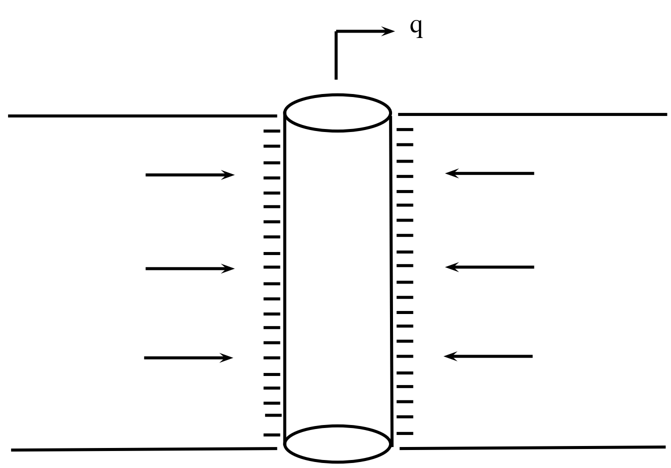


Fig. 2.6. Production from a vertical full penetrating well. Volumetric flux, q, into the wellbore (flow out of the reservoir the boundary represented by wellbore).

Flow rate out $= qB = \int\limits_S \mathbf{v}\,\mathbf{n}\,dS$ and $\mathbf{n}$ is the unit outward normal to S and is equal to

$$\begin{aligned} \mathbf{n} &= -i_r + 0i_\theta + 0i_z = (1,0,0) \\ \mathbf{v} &= (v_r + v_\theta + v_z) \end{aligned}$$

$$\begin{aligned} qB &= \int\limits_S -v_r\big|_{r=r_w} dS \quad ; \quad ds = r_w d\theta dz \\ qB &= \int_0^{2\pi}\int_0^h (-v_r)_{r_w} r_w d\theta dz \\ q &= \int_0^{2\pi}\int_0^h \frac{k}{\mu B}\left(r\frac{\partial p}{\partial r}\right)_{r=r_w} d\theta dz \\ q &= 2\pi\int_0^h \left(r\frac{\partial m}{\partial r}\right)_{r_w} dz \end{aligned}$$

Now we assume that variation of $r\frac{\partial m(p)}{\partial r}$ in z-direction is insignificant or

$$\int\limits_0^h \left(r\frac{\partial m(p)}{\partial r}\right)_{r_w} dz = \left(r\frac{\partial m(p)}{\partial r}\right)_{r_w,\widehat{z}} h$$

where $\widehat{z}$ is a mean value between $0 \le \widehat{z} \le h$. Then the boundary condition is

$$q = 2\pi h\left(r\frac{\partial m(p)}{\partial r}\right)_{r_w} \tag{2.59}$$

Define,

$$\begin{aligned} m_D &= \frac{2\pi h\left[m(p_i) - m(p)\right]}{q} \\ &= \frac{2\pi h}{q}\left[m(p_i) - m(p)\right] \\ &= \frac{2\pi h}{q}\int\limits_p^{p_i} \frac{k(p)}{\mu(p)\,B(p)} dp \end{aligned} \tag{2.60}$$

Then we can show that Eqs. 2.58 and 2.59 is written as

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m_D}{\partial r_D}\right)=\frac{1}{\eta_D}\frac{\partial m_D}{\partial t_D} \tag{2.61}$$

$$\left(r_D\frac{\partial m_D}{\partial r_D}\right)_{r_D=1}=-1 \tag{2.62}$$

Note that $r_D=1$ corresponds to $r=r_w$. Initial condition, $p=p_i$ at values of r at $\widehat{z}$. $m_D=0$ at $t_D=0$ then,

$$p\left(r,t\right)|_{t=0}=p_i \tag{2.63}$$

Infinite acting reservoir is considered implying,

$$\lim_{r\to\omega}p\left(r,t\right)=p_i$$

which corresponds to

$$\lim_{r_D\to\infty}m_D\left(r_D,t_D\right)=0 \tag{2.64}$$

In summary, the following initial boundary value problem (IBVP) is achieved with the appropriate boundary conditions.

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial m_D}{\partial r_D}\right)=\frac{1}{\eta_D}\frac{\partial m_D}{\partial t_D} \tag{2.65}$$

$$\left(r_D\frac{\partial m_D}{\partial r_D}\right)_{r_D=1}=-1 \tag{2.66}$$

$$\lim_{r_D\to\infty}m_D\left(r_D,t_D\right)=0 \tag{2.67}$$

$$m_D\left(r_D,t_D=0\right)=0 \tag{2.68}$$

Eqs. 2.65-2.68 lead to give a complete mathematical description of the physical problem. Because of η_D term, it is a non-linear IBVP. It can also be solved analytically (see Kale and Mattar[8] or Peres et.al.[12])

For simplicity, assume that variations in k, ϕ, c_t, and B are small ("negligible") for the pressure change considered. Then,

$$\eta_D=\frac{(k/\phi c_t\mu\ )}{(k/\phi c_t\mu\ )_{p_i}}\approx 1 \tag{2.69}$$

and

$$\eta_D = \frac{(k/\phi c_t \mu\ )}{(k/\phi c_t \mu\ )_{p_i}} \approx 1 \tag{2.70}$$

$$m(p_i) - m(p) = \int_p^{p_i} \frac{k(p)}{\mu(p)\, B(p)} dp \approx \frac{k_i}{\mu_i B_i}(p_i - p) \tag{2.71}$$

and then it follows from Eq. 2.60 that

$$m_D = \frac{2\pi k_i h (p_i - p)}{q B_i \mu_i} = p_D = \frac{2\pi k h (p_i - p)}{q B \mu} \tag{2.72}$$

that is the definition of dimensionless pressure p_D in well testing. Considering $\frac{1}{\eta_D} \approx 1$ in Eq. 2.65, we have

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D \frac{\partial m_D}{\partial r_D}\right) = \frac{\partial m_D}{\partial t_D} \tag{2.73}$$

$$\left(r_D \frac{\partial m_D}{\partial r_D}\right)_{r_D=1} = -1 \tag{2.74}$$

$$\lim_{r_D \to \infty} m_D(r_D, t_D) = 0 \tag{2.75}$$

$$m_D(r_D, t_D = 0) = 0 \tag{2.76}$$

Note that Eq. 2.73 is a linear p.d.e. . We seek a solution to the IBVP given by Eqs. 2.73 - 2.76. To find a solution we assume that

$$m_D = m_D(\varepsilon_D)$$

where

$$\varepsilon_D = \frac{r_D^2}{4t_D} = \varepsilon_D(r_D, t_D)$$

is called dimensionless Boltzman transform. To use the Boltzman transform, there must be no characteristic length in the system (such as $r_D = 1$, r_w). Since the inner boundary condition, Eq. 2.74, is for a finite wellbore problem, it involves a characteristic length. Thus, to be able to use Boltzman transform, Eq. 2.74 with one that does not involve characteristic length, that is "line source well" inner boundary condition.

$$\lim_{r_D \to 0}\left(r_D \frac{\partial m_D}{\partial r_D}\right) = -1 \tag{2.77}$$

Under the preceding assumptions, approximate problem becomes: Find $m_D = m_D(\varepsilon_D)$ such that m_D satisfies

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D \frac{\partial m_D}{\partial r_D}\right) = \frac{\partial m_D}{\partial t_D} \tag{2.78}$$

$$\lim_{r_D \to 0}\left(r_D \frac{\partial m_D}{\partial r_D}\right) = -1 \tag{2.79}$$

$$\lim_{r_D \to \infty} m_D\left(r_D, t_D\right) = 0 \tag{2.80}$$

$$m_D\left(r_D, t_D = 0\right) = 0 \tag{2.81}$$

where,

$$m_D = \frac{2\pi h \int_p^{p_i} \frac{k(p)}{\mu(p)B(p)} dp}{q} \tag{2.82}$$

Use of Boltzman transformation, $\varepsilon_D = \frac{r_D^2}{4t_D}$, changes the p.d.e. to second order ordinary differential equation o.d.e., further collapses three auxiliary conditions into two conditions,

$$\frac{\partial m_D}{\partial r_D} = \frac{\partial m_D}{\partial \varepsilon_D}\frac{\partial \varepsilon_D}{\partial r_D} = \frac{\partial m_D}{\partial \varepsilon_D}\frac{2r_D}{4t_D} \tag{2.83}$$

and

$$r_D \frac{\partial m_D}{\partial r_D} = 2\left(\frac{r_D^2}{4t_D}\right)\frac{dm_D}{d\varepsilon_D} = 2\varepsilon_D \frac{dm_D}{d\varepsilon_D} \tag{2.84}$$

Using Eq. 2.84 and the chain rule,

$$\begin{aligned}\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D \frac{\partial m_D}{\partial r_D}\right) &= \frac{1}{r_D}\frac{d}{d\varepsilon_D}\left(2\varepsilon_D \frac{dm_D}{d\varepsilon_D}\right)\frac{d\varepsilon_D}{dr_D} \\ &= \frac{1}{t_D}\frac{d}{d\varepsilon_D}\left(2\varepsilon_D \frac{dm_D}{d\varepsilon_D}\right)\end{aligned} \tag{2.85}$$

Similarly,

$$\frac{\partial m_D}{\partial t_D} = \frac{dm_D}{d\varepsilon_D}\frac{\partial \varepsilon_D}{\partial t_D} = \frac{dm_D}{d\varepsilon_D}\left(-\frac{r_D^2}{4t_D^2}\right) = -\frac{1}{t_D}\varepsilon_D\frac{dm_D}{d\varepsilon_D} \tag{2.86}$$

Using Eqs. 2.85 and 2.86 in Eq. 2.78 gives,

$$\varepsilon_D\frac{dm_D}{d\varepsilon_D} + \frac{d}{d\varepsilon_D}\left(\varepsilon_D\frac{dm_D}{d\varepsilon_D}\right) = 0 \tag{2.87}$$

Using Eq. 2.84 the inner boundary condition (see Eq. 2.79) is written as

$$\lim_{\varepsilon_D \to 0} 2\varepsilon_D\frac{dm_D}{d\varepsilon_D} = -1 \tag{2.88}$$

Initial condition (I.C.) Eq. 2.81,

$$t_D \to 0 \quad \varepsilon_D \to \infty \quad \Rightarrow \quad \lim_{\varepsilon_D \to \infty} m_D\left(\varepsilon_D\right) = 0$$

Outer boundary condition (O.B.C.) Eq. 2.80,

$$r_D \to \infty \quad \varepsilon_D \to \infty \quad \Rightarrow \quad \lim_{\varepsilon_D \to \infty} m_D\left(\varepsilon_D\right) = 0$$

Thus both I.C. and O.B.C. is represented by,

$$\lim_{\varepsilon_D \to \infty} m_D\left(\varepsilon_D\right) = 0 \tag{2.89}$$

We need to solve boundary value problem (B.V.P.) given by Eqs. 2.87 - 2.89. Let,

$$w_D = \frac{dm_D}{d\varepsilon_D} \tag{2.90}$$

Substituting Eq. 2.90 into Eq. 2.87 gives,

$$\frac{d}{d\varepsilon_D}\left(\varepsilon_D w_D\right) + \varepsilon_D w_D = 0$$

$$\varepsilon_D\frac{dw_D}{d\varepsilon_D} + w_D + \varepsilon_D w_D = 0$$

$$\varepsilon_D\frac{dw_D}{d\varepsilon_D} + \left(1+\varepsilon_D\right) w_D = 0 \tag{2.91}$$

Eq. 2.91 is separable ordinary differential equation, thus separating variables,

$$\frac{dw_D}{w_D} = -\frac{1+\varepsilon_D}{\varepsilon_D} \tag{2.92}$$

Interpreting both sides yields,

$$\ln w_D = -\varepsilon_D - \ln \varepsilon_D + c_1 \quad ; \quad c_1 \text{ is an integrating constant}$$

or

$$\begin{aligned} \ln w_D &= -\varepsilon_D - \ln \varepsilon_D + c_1 \quad ; \quad c_1 \text{ is an integrating constant} \\ w_D &= \exp\left[-\varepsilon_D - \ln \varepsilon_D + c_1\right] \\ &= e^{c_1} \frac{1}{\varepsilon_D} e^{-\varepsilon_D} = \frac{c}{\varepsilon_D} e^{-\varepsilon_D} \quad ; \quad c = e^{c_1} \end{aligned}$$

At this point, we have

$$w_D = \frac{dm_D}{d\varepsilon_D} = \frac{c}{\varepsilon_D} \exp\left[-\varepsilon_D\right]$$

thus

$$2\varepsilon_D \frac{dm_D}{d\varepsilon_D} = 2c \exp\left[-\varepsilon_D\right] \tag{2.93}$$

It follows from Eq. 2.88 and 2.93 that

$$c = -1/2 \tag{2.94}$$

Thus Eq. 2.93 becomes

$$\frac{dm_D}{d\varepsilon_D} = -\frac{1}{2\varepsilon_D} \exp\left[-\varepsilon_D\right] \tag{2.95}$$

Integrating Eq. 2.95 from ε_D to ∞ gives,

$$\int_{\varepsilon_D}^{\infty} \frac{dm_D}{d\varepsilon_D}\, d\varepsilon_D = -\frac{1}{2} \int_{t_D}^{\infty} \frac{\exp\left[-u\right]}{u}\, du$$

in limit

$$\lim_{\varepsilon_D \to \infty} m_D(\varepsilon_D) - m_D(\varepsilon_D) = -\frac{1}{2}\int_{t_D}^{\infty} \frac{\exp[-u]}{u}\, du \tag{2.96}$$

It follows from Eq. 2.89 and 2.96 that

$$m_D(\varepsilon_D) = \frac{1}{2}\int_{\frac{t_D^2}{4t_D}}^{\infty} \frac{\mathrm{e}^{-u}}{u}\, du \tag{2.97}$$

which is known as the Theis solution [14]. Eq. 2.97 is also referred to as the line source or the exponential integral or E_i solution.

Exponential integral is defined by

$$-E_i[-x] = \int_x^{\infty} \frac{\mathrm{e}^{-u}}{u}\, du \quad ; \quad E_i \text{ function}$$

Exponential integral function is also referred to as $E_[1](x)$,

$$-E_i[-x] = E_1(x) \tag{2.98}$$

with this notation Eq. 2.97 becomes

$$m_D(\varepsilon_D) = -E_i\left[-\frac{r_D^2}{4t_D}\right] \tag{2.99}$$

The series expansion of E_i function [1]

$$E_i[-x] = \widetilde{\gamma} + \ln x + \sum_{n=1}^{\infty} \frac{x^n}{nn!} \tag{2.100}$$

Let $x \to 0$ in Eq. 2.99, then we obtain

$$\lim_{x \to 0} E_i[-x] = \widetilde{\gamma} + \ln x \tag{2.101}$$

where $\widetilde{\gamma}$ is Euler's constant and defined by,

$$\widetilde{\gamma} = \lim_{n \to \infty} \left\{ \sum_{k=1}^{n} \frac{1}{k} - \ln(n) \right\} = 0.5772156649... \tag{2.102}$$

If we consider that $\frac{r_D^2}{4t_D}$ is sufficiently small so that $E_i\left[-\frac{r_D^2}{4t_D}\right]$ is approximated by Eq. 2.101. Then, Eq. 2.99 is approximated as

$$m_D(\varepsilon_D) = -\frac{1}{2}\left[\ln\left(\frac{r_D^2}{4t_D}\right) + \widetilde{\gamma}\right] \tag{2.103}$$

or

$$m_D = \frac{1}{2}\ln\left(\frac{4t_D}{r_D^2 e^{\widetilde{\gamma}}}\right) \tag{2.104}$$

which is referred to as the semilog approximation to Eq. 2.99.

Although Eq. 2.104 is derived as a limiting form of Eq. 2.99 as $\frac{r_D^2}{4t_D} \to 0$, Eq. 2.99 is well approximated by Eq. 2.104 if

$$\frac{r_D^2}{4t_D} \leq \frac{1}{100}$$

which is equivalent to;

$$\frac{t_D}{r_D^2} \geq 25 \tag{2.105}$$

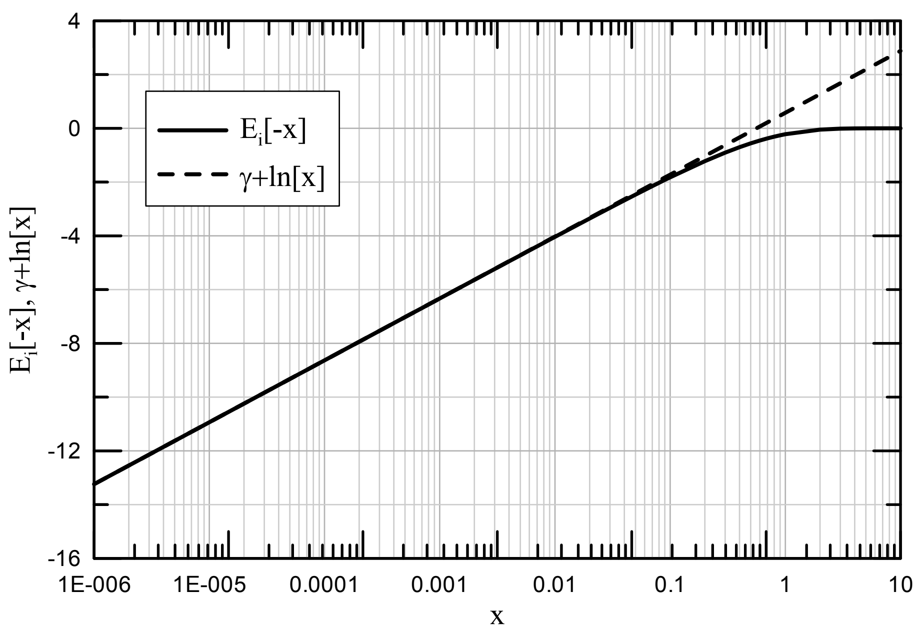


Fig. 2.7. Comparison of exponential integral with logarithmic approximation given by Eq. 2.101.

Remark on exponential integral approximation:

At $r = r_w$, $r_D = r/r_w = 1$ and Eq. 2.104 becomes,

$$\frac{2.637 \times 10^{-3} k_i t}{(\phi \mu c_t)_i r_w^2} \geq 25$$

where t is in hrs. Now suppose, $r_w = 0.35$ ft, $\mu_i = 0.8$ cp, $\phi_i = 0.1$, $k_i = 10$ md, and $c_t = 1.1 \times 10^5$. then it follows that

$$t_D \approx 2.5 \times 10^5 t$$

In order for Eq. 2.101 to hold, we need

$$2.5 \times 10^5 t \geq 25 \quad \Rightarrow \quad t \geq 1.0 \times 10^{-4} hrs (= 0.36 seconds)$$

This example illustrates that at the wellbore ($r_D = 1$), the semilog approximation of the line source solution becomes valid within several seconds to a very few minutes for reasonable values of physical parameters; (e.g. k_i, c_t...). If the semilog approximation of the line source solution is used to analyze interference data with an observation well say at $r = 1000 r_w$, ($r_D = 1000$) away from a flowing well, then for the values of physical parameters considered, Eq. 2.105 requires,

$$t \geq 10^{-4} r_D^2 = 100 hrs$$

In short, the semilog approximation of the line source solution is used, i.e., Eq. 2.101 to analyze well test pressure data measured either at the active (producing) well or at the observation (shut-in) well provided that test time satisfies Eq. 2.105.

Line source solution given by Eq. 2.78 we used the line source well boundary condition Eq. 2.77 that is

$$\lim_{r_D \to 0} \left(r_D \frac{\partial m_D}{\partial r_D} \right) = -1$$

Differentiating Eq. 2.78 with respect to r_D gives,

$$\frac{\partial m_D}{\partial r_D} = \frac{1}{2}\frac{\partial}{\partial r_D}\left[\int_{\frac{t_D^2}{4t_D}}^{\infty} \frac{\mathrm{e}^{-u}}{u}\,du\right] = -\frac{1}{2}\frac{\partial}{\partial r_D}\left[\int_{\infty}^{\frac{t_D^2}{4t_D}} \frac{\mathrm{e}^{-u}}{u}\,du\right] \tag{2.106}$$

Recall,

$$\frac{d}{dx}\left[\int_a^{b(x)} f(z)\,dz\right] = f\left[b(x)\right]\frac{db(x)}{dx} \tag{2.107}$$

where a is aconstant. Using the formula given by Eq. 2.107 in Eq. 2.106 and simplifying the resulting equation;

$$\frac{\partial m_D}{\partial r_D} = -\frac{1}{r_D}\exp\left[-\frac{r_D^2}{4t_D}\right] \tag{2.108}$$

Multiplying Eq. 2.108 by r_D

$$r_D\frac{\partial m_D}{\partial r_D} = -\exp\left[-\frac{r_D^2}{4t_D}\right] \tag{2.109}$$

Thus the line source solution satisfies line source wellbore B.C., i.e.,

$$\lim_{r_D\to 0}\left(r_D\frac{\partial m_D}{\partial r_D}\right) = -1$$

It is important to note that Eq. 2.78 assumes a well with vanishingly small radius,however, the more rigorous solution is the one considering a well with finite wellbore. To obtain a solution for a well with finite wellbore producing at a constant rate, an inner boundary condition such that

$$\lim_{r_D\to 0}\left(r_D\frac{\partial m_D}{\partial r_D}\right) = -1 \tag{2.110}$$

Therefore, the finite wellbore radius I.B.V.P. differs from the line source I.B.V.P. in the boundary condition given by Eq. 2.110Line source solution is closed with Eq. 2.77.

Note that $r_D = 1$ (see Eq. 2.106), the line source solution satisfies

$$\left(r_D\frac{\partial m_D}{\partial r_D}\right)_{r_D=1} = -\exp\left[-\frac{1}{4t_D}\right] \tag{2.111}$$

As $t_D \to \infty$, the RHS of Eq. 2.111 approaches -1, i.e.,

$$\lim_{t_D \to \infty} \left(r_D \frac{\partial m_D}{\partial r_D} \right)_{r_D=1} = -1 \tag{2.112}$$

from which intuitively we expect that at sufficiently large times, the line source solution for $r_D = 1$ will be very close to the finite wellbore radius solution. In fact for $t_D > 25$, the RHS of Eq. 2.111 is within 1% of -1.

Mueller and Witherspoon [9] were the ones first to investigate the validity of line source solution. They compared the finite wellbore radius solution and the line source solution. Such comparison is shown in Fig. 2.8 - log-log plot of $p_D(= m_D)$ versus t_D/r_D^2.

The dashed curve in the figure represents the exponential integral solution(or the line source solution) given by Eq. 2.78. The top solid curve corresponds to the finite wellbore radius solution evaluated at the wellbore (i.e. $r_D = 1$). All other solid curves represent finite wellbore radius solution at various locations (different r_D values) in the reservoir.

Figure indicates that when $t_D/r_D^2 \geq 25$, the difference between the finite wellbore radius solution and the line source solution is negligible. Thus at $r_D = 1$ (at the wellbore) $t_D \geq 25$ is required for two solutions to be essentially the same. Also note that if $r_D \geq 20$, then the two solutions are essentially equal for all values of the dimensionless time t_D of practical interest.

For the interference testing purposes, the difference between the line source solution and the finite wellbore radius solution is negligible. To use Eq. 2.78 for the analysis of pressure data measured at a flowing well, we evaluate Eq. 2.78 at $r_D = 1$ and use the notation

$$p\left(r_D = 1, t_D\right) = p\left(r = r_w, t\right) = p_{wf} = p_{wf}\left(t\right) \tag{2.113}$$

2.6 Semilog analysis

In the previous section, we showed that if $t_D/r_D^2 \geq 25$, the semilog approximation of the line source solution is given by Eq. 2.104 i.e.;

$$m_D = \frac{1}{2} \ln \left(\frac{4t_D}{r_D^2 e^{\tilde{\gamma}}} \right)$$

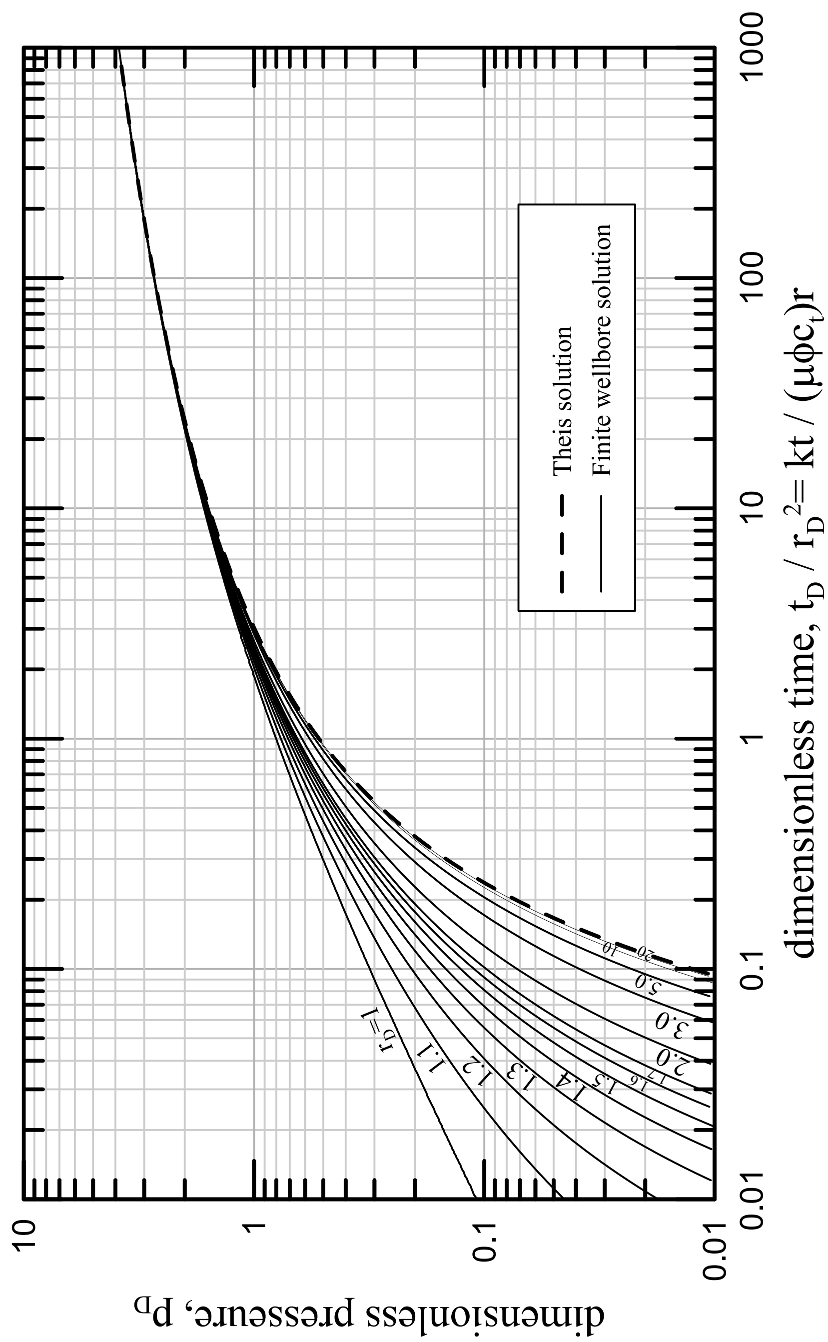


Fig. 2.8. Comparison of line source solution to finite radius wellbore solution[9].

At $r_D = 1$,

$$m_D(r_D = 1) = \frac{2\pi h}{q} \int_{p_{wf}}^{p_i} \frac{k(p)}{\mu(p) B(p)} dp = \frac{1}{2}\left[\frac{4t_D}{e^{\tilde{\gamma}}}\right] = \frac{1}{2}(\ln t_D + 0.80907) \tag{2.114}$$

or

$$m_D(r_D = 1) = \underbrace{\frac{1}{2\log(e)}}_{1.15129} \left\{\log(t) + \log\left[\frac{k_i}{(\phi c_t \mu)_i r_w^2}\right] + \underbrace{\log(e)\ln\left(\frac{4}{e^{\tilde{\gamma}}}\right)}_{0.351378}\right\} \tag{2.115}$$

Note that

$$m_D(r_D = 1) = \frac{2\pi h}{q} \int_{p_{wf}}^{p_i} \frac{k(p)}{\mu(p) B(p)} dp = \frac{2\pi h}{q}\left[m(p_i) - m(p_{wf})\right] \tag{2.116}$$

Then it follows from Eq. 2.115 and 2.116 that

$$m(p_{wf}) = m(p_i) - 1.15129 \frac{q}{2\pi h} \left\{\log(t) + \log\left[\frac{k_i}{(\phi c_t \mu)_i r_w^2}\right] + 0.351378\right\} \tag{2.117}$$

which indicates that a semilog plot of $m(p_{wf})$ versus t that yields a straight line with a slope

$$\text{slope} = \widetilde{m} = -1.15129 \frac{q}{2\pi h} \tag{2.118}$$

h is computed from the slope of semilog straight line. Now differentiate Eq. 2.115 w.r.t. *logt*.

$$\frac{2\pi h}{q}\frac{d}{d\log}\int_{p_{wf}}^{p_i}\frac{k(p)}{\mu(p)B(p)}dp = 1.15129 \tag{2.119}$$

or

$$\frac{2\pi h}{q}\frac{d}{dp_{wf}}\left\{\int_{p_{wf}}^{p_i}\frac{k(p)}{\mu(p)B(p)}dp\right\}\frac{dp_{wf}}{d\log t} = 1.15129$$

or

$$\frac{2\pi h}{q}\left[-\frac{k(p_{wf})}{\mu(p_{wf})B(p_{wf})}\right]\frac{dp_{wf}}{d\log t} = 1.15129$$

or

$$\frac{k(p_{wf})}{\mu(p_{wf})B(p_{wf})} = -1.15129\frac{q}{2\pi h\left(dp_{wf}/d\log t\right)} \tag{2.120}$$

Aside:

For solution gas drive reservoirs, Bøe et.al.[4] showed that pressure and oil saturation, S_o are unique function of the Boltzman variable $r_D^2/4t_D$, or r^2/t provided that $t_D/r_D^2 \geq 25$.

Serra et. al. [13] used this observation to show how to compute effective permeability from wellbore B.C. for constant oil rate.

$$q_o = 2\pi\left(\frac{kk_{ro}}{\mu_o}rh\frac{\partial p}{\partial r}\right)_{r=r_w} \tag{2.121}$$

Suppose p is a function of the Boltzman variable $z = r^2/t$ then,

$$\frac{\partial p}{\partial r} = \frac{dp}{dz}\frac{dz}{dr} = \frac{dp}{dz}\frac{2r}{t} \tag{2.122}$$

and

$$\frac{\partial p}{\partial t} = \frac{dp}{dz}\frac{dz}{dt} = \frac{dp}{dz}\left(-\frac{r^2}{t^2}\right) \tag{2.123}$$

It follows that

$$r\frac{\partial p}{\partial r} = 2\frac{r^2}{t}\frac{dp}{dz} = (-2t)\,\frac{-r^2}{t^2} = -2t\frac{dp}{dt} = -2\frac{dp}{d\ln t}$$

or at $r = r_w$

$$\left(r\frac{\partial p}{\partial r}\right)_{r_w} = -2t\frac{dp_{wf}}{dt} = -2\frac{dp_{wf}}{d\ln t} \tag{2.124}$$

Using Eq. 2.121 in Eq. 2.124 gives,

$$q_o = 2\pi h\left(\frac{kk_{ro}}{\mu_o B_o}\right)_{r_w}\left(-2\frac{dp_{wf}}{d\ln t}\right) \tag{2.125}$$

Rearranging Eq. 2.125, we obtain,

$$\left(\frac{kk_{ro}}{\mu_o B_o}\right)_{r_w} = -\frac{q_o}{4\pi h\left(\frac{dp_{wf}}{d\ln t}\right)} \tag{2.126}$$

that provides a way of computing $(kk_{ro})_{p_{wf}}$ as a function of p_{wf} from

$$\begin{aligned}(kk_{ro})_{p_{wf}} &= -\frac{q_o(\mu_o B_o)_{p_{wf}}}{4\pi h\left(\frac{dp_{wf}}{d\ln t}\right)} \\ &= -\frac{2.303 q_o(\mu_o B_o)_{p_{wf}}}{4\pi h\left(\frac{dp_{wf}}{d\log t}\right)} \quad ; \quad \ln t = 2.303\log t\end{aligned} \tag{2.127}$$

Eq. 2.127 implies that k_{ro} is a function of pressure. Therefore, oil saturation is considered as a function of pressure, i.e. $S_o = S_o(p)$.

Now we consider the case when the variations in $k(p)$, $\mu(p)$,and $B(p)$ are negligible,

$$m_D = p_D = \frac{2\pi kh\left[p_i - p\left(r,t\right)\right]}{qB\mu} \tag{2.128}$$

$$t_D = \frac{kt}{\phi c_t \mu r_w^2} \tag{2.129}$$

Suppose that pressure at any point $r \neq r_w$ in the reservoir is measured. If the semilog approximation to the line source holds, it follows that

$$p_D = \frac{1}{2}\ln\left[\frac{4t_D}{e^{\widetilde{\gamma}}r_D^2}\right] \tag{2.130}$$

or using the dimensional variables

$$\begin{gathered} p\left(r,t\right) = p_i - 1.15129\frac{qB\mu}{2\pi kh} \\ \left\{\log\left(t\right) + \log\left[\frac{kh}{\phi c_t h\mu r^2}\right] + 0.351378\right\} \end{gathered} \tag{2.131}$$

which also indicates that a semilog plot of p versus t gives a straight line with a slope equal to

$$\widetilde{m} = -1.15129\frac{qB\mu}{2\pi kh} \Rightarrow kh = -1.15129\frac{qB\mu}{2\pi\widetilde{m}} \tag{2.132}$$

Note that the pressure drop could also be plotted as a function of t, i.e., $\Delta p\left(r,t\right) = p_i - p\left(r,t\right)$. In this case, a semilog plot of Δp versus t yields a semilog slope of

$$\widetilde{m} = 1.15129\frac{qB\mu}{2\pi kh} \quad ; \quad \widetilde{m} \geq 0 \tag{2.133}$$

If we extrapolate the semilog straight line to zero pressure drop in a plot of Δp versus $logt$ and denote time time at which $\Delta p = 0$ as t_0 then it follows from Eq. 2.131 that

$$\begin{gathered} \Delta p(r,t)_{t=t_0} = 1.15129\frac{qB\mu}{2\pi kh} \\ \left\{\log\left(t_0\right) + \log\left[\frac{kh}{\phi c_t h\mu r^2}\right] + 0.351378\right\} = 0 \end{gathered} \tag{2.134}$$

then,

$$\log\left(t_0\right) = -\log\left[\frac{kh}{\phi c_t h\mu r^2}\right] - 0.351378$$

or

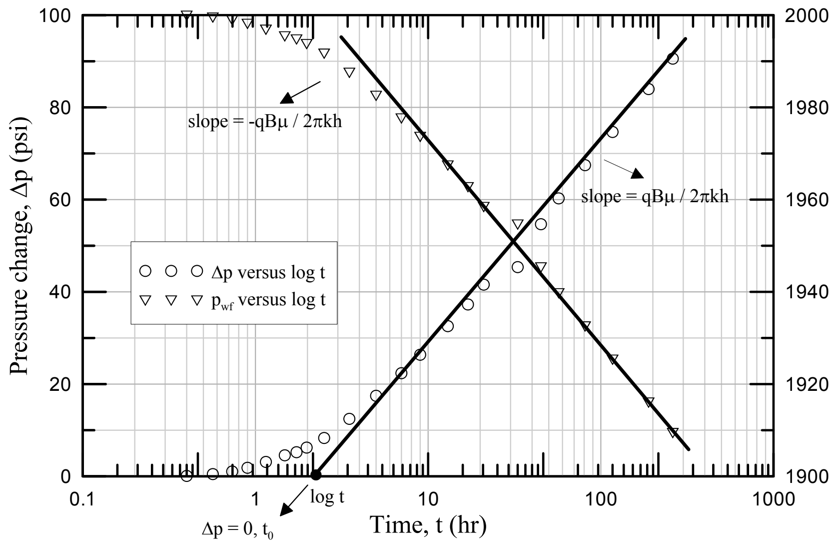


Fig. 2.9. Semilog analysis.

$$\log(t_0) = -\log\left[\frac{kh}{\phi c_t h \mu r^2} 10^{0.351378}\right] \tag{2.135}$$

Taking anti-log of both sides then,

$$t_0 = \frac{\mu r^2 \phi c_t h}{kh} \frac{1}{10^{0.351378}}$$

$$\phi c_t h = \frac{t_0 kh}{\mu r^2} 10^{0.351378} \tag{2.136}$$

where kh is obtained from the slope of the semilog straight line.

Semilog plot of $p(r)$ versus t could also be used by extrapolating the semilog straight line to $p = p_i$ to determine $\phi c_t h$, then again Eq. 2.136 is used.

2.7 Pressure derivative

Pressure derivative (the rate of change of pressure with respect to time or with respect to lnt) has improves our ability to analyze well test data. Especially in identifying various flow regimes exhibited by the measured pressure data and helps us to identify the proper semilog straight line on a pressure (pressure change) versus lnt plot.

Derivative analysis in well testing goes back to Tiab and Crichlow [15]. The study presented a type curve matching technique, based on dP_D/t_D, for interpreting the pressure transient behavior of multiple-sealing-fault systems and closed rectangular reservoirs. Later, Bourdet et. al. [6] presented derivative type curves for the wellbore storage and skin problem. They suggested using $dp_D/dt_D = (dt_D dp_D/t_D)$ because of its convenience.

Now recall line source solution; i.e.;

$$p_D = -\frac{1}{2}E_i\left[-\frac{r_D^2}{4t_D}\right] = -\frac{1}{2}\int_{\infty}^{\frac{r_D^2}{4t_D}} \frac{\mathrm{e}^{-u}}{u}\,du \tag{2.137}$$

Taking the derivative of Eq. 2.137 w.r.t. t_D gives

$$\begin{aligned}
\frac{dp_D}{dt_D} &= -\frac{1}{2}\frac{d}{dt_D}\left\{\int_{\infty}^{\frac{r_D^2}{4t_D}} \frac{\mathrm{e}^{-u}}{u}\,du\right\} \\
&= -\frac{1}{2}\left.\frac{\mathrm{e}^{-u}}{u}\right|_{u=\frac{r_D^2}{4t_D}} \frac{d}{dt_D}\left[\frac{r_D^2}{4t_D}\right] \\
&= \frac{1}{2t_D}\frac{\exp\left[-\frac{r_D^2}{4t_D}\right]}{\frac{r_D^2}{4t_D}}\left[\frac{r_D^2}{4t_D}\right]
\end{aligned}$$

$$\frac{dp_D}{dt_D} = \frac{1}{2t_D}\exp\left[-\frac{r_D^2}{4t_D}\right] \tag{2.138}$$

Multiplying Eq. 2.138 by t_D and recall,

$$t_D\frac{dp_D}{dt_D} = \frac{dp_D}{d\ln t_D} \tag{2.139}$$

we obtain

$$t_D\frac{dp_D}{dt_D} = \frac{1}{2}\exp\left[-\frac{r_D^2}{4t_D}\right] \tag{2.140}$$

Using the definition of dimensionless variables, Eq. 2.140 is written in terms of dimensional variables as,

$$\frac{\partial\left(p_i - p\left(r,t\right)\right)}{\partial \ln t} = \frac{qB\mu}{4\pi kh}\exp\left[-\frac{r^2\mu\phi c_t h}{4tkh}\right] \tag{2.141}$$

When $t_D/r_D^2 > 25$, then $\exp\left[-\frac{r_D^2}{4t_D}\right] \approx 1$, which corresponds to the semilog approximation of the line source solution, then Eq. 2.141 reduces to

$$\begin{aligned} \frac{\partial\left(p_i - p\left(r,t\right)\right)}{\partial \ln t} &= \frac{qB\mu}{4\pi kh} \\ \frac{\partial p\left(r,t\right)}{\partial \ln t} &= -\frac{qB\mu}{4\pi kh} \end{aligned} \tag{2.142}$$

This suggest two possibilities: First using our measured values of $p(r,t)$ (if $r = r_w$, $p(r = r_w, t) = p_{wf}(t)$), we can numerically calculate the values of $\frac{dp(r,t)}{d\ln t}$ whenever numerical values of $\frac{dp}{d\ln t}$ become constant, kh can be computed from Eq. 2.142 as follows,

$$kh = -\frac{qB\mu}{4\pi\left[\frac{\partial p}{\partial \ln t}\right]} \tag{2.143}$$

To calculate $\frac{dp}{d\ln t}$ numerically, we may use the following forward difference approximation given by

$$\frac{dp\left(r,t_i\right)}{d\ln t_i} \approx \frac{p\left(t_{i+1}\right) - p\left(t_i\right)}{\ln\left(t_{i+1}\right) - \ln\left(t_i\right)} \quad ; \quad i = 1,2,3,\ldots,N-1 \tag{2.144}$$

where N is the number of data points.

One way of numerically differentiating the pressure data is given by Bourdet[5] (see Eq. 2.145). Bourdet suggests to use three points one before and one after the point of interest. Forward and backward slopes of the point of interest are estimated and combined by weighing with respect to point i.

$$\frac{dp}{d\ln t_i} \approx \frac{\left(\frac{\Delta p}{\Delta \ln t}\right)_1 \Delta \ln t_2 + \left(\frac{\Delta p}{\Delta \ln t}\right)_2 \Delta \ln t_1}{\Delta \ln t_1 + \Delta \ln t_2} \tag{2.145}$$

Numerical differentiation usually amplifies the error that are inherent in nature of gauge measurements. Smoothing and/or post processing is required in many cases. Probably the most convenient way of smoothing is applied by choosing an appropriate step size ($ln\Delta t_1$ and $ln\Delta t_2$) for differentiation. Step size is increased until the response is smooth enough. The procedure is applied carefully as over smoothing may some hinder some features. Usually the step size is around 0.2.

2.8 Type curve analysis (or type curve matching)

The idea behind type curve analysis (TCA) becomes evident when $log(p_D)$ and $log(t_D/r_D^2)$ is consider

$$\begin{aligned}\log p_D &= \log\left[\frac{2\pi kh}{qB\mu}\Delta p\right]\\ &= \log\left[p_i - p\left(r,t\right)\right] + \log\left[\frac{2\pi kh}{qB\mu}\right]\end{aligned} \tag{2.146}$$

$$\begin{aligned}\log\left(\frac{t_D}{t_D^2}\right) &= \log\left[\frac{kt}{\phi c_t \mu r^2}\right]\\ &= \log\left[t\right] + \log\left[\frac{k}{\phi c_t \mu r^2}\right]\end{aligned} \tag{2.147}$$

Eqs. 2.146 and 2.147 show us that log-log plot of Δp versus t should have the same shape as a log-log graph of p_D versus t_D/r_D^2. The translation between the two graphs involves the second terms on the RHS of Eqs. 2.146 and 2.147.

Once p_D or (m_D) versus t_D/r_D^2 curve with the graph of Δp versus t is matched, we can find a match pointa match point is recorded. The match point consists of values for $((t)_m$,$(\Delta p)_m)$ from Δp versus t graph and $[(t_D/r_D^2)_m,\ (p_D)_m]$, a corresponding point on the p_D versus t_D/r_D^2 graph,

$$\left(\frac{t_D}{r_D^2}\right)_m = \frac{kt_m}{\phi c_t \mu r^2} \tag{2.148}$$

and rearranging Eq. 2.148 we obtain an estimate of ϕc_t from,

$$\phi c_t = \frac{k}{\mu r^2}\frac{(t)_m}{(t_D/r_D^2\ )_m} \tag{2.149}$$

In addition permeability is estimated by using the following relation

$$(p_D)_m = \frac{2\pi kh}{qB\mu}(\Delta p)_m \tag{2.150}$$

$$k = \frac{qB\mu}{2\pi h}\frac{(p_D)_m}{(\Delta p)_m} \tag{2.151}$$

The results from TCA with the line source solution, within some tolerance, should be the same as the results that are calculated from the semilog straight line. TCA, however, can also be used when log approximation does not apply. This suggests a general approach to the analysis of pressure tests. Apply type curve analysis prior and check if proper semilog straight line exists. That is, if there is data available when $t_D/r_D^2 > 25$. If the semilog straight line exists, then semilog analysis is applied.

Interference testing An interference test involves measurements of pressure at a given point due to a rate change at another point in the reservoir. The two points involve the same well or multiple wells.

Single well interference tests: The purpose of this kind of tests is to determine if there is vertical communication and quantify vertical permeability, k_z.

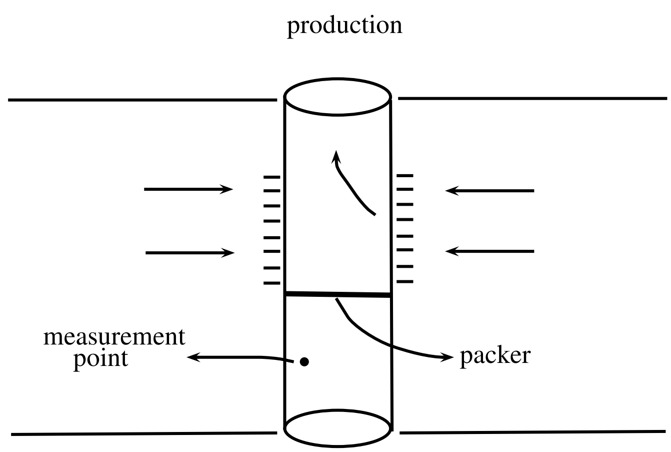


Fig. 2.10. A schematic of a single well interference test.

Multiple interference tests are run basically for the following two purposes,

(1) to determine if there is hydraulic communication between wells.
(2) quantify the inter-well formation properties, k and ϕc_t

The classical interference test procedure involves

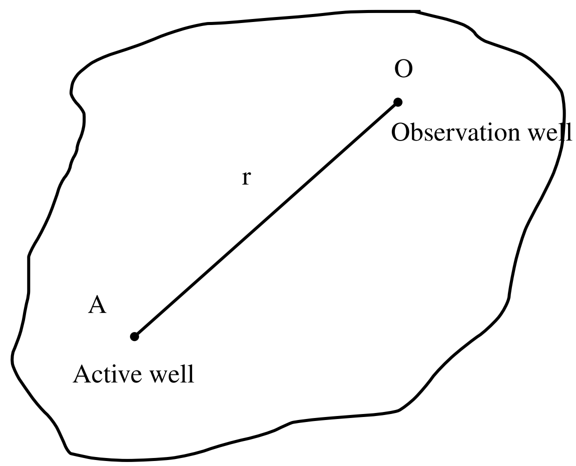


Fig. 2.11. Well A is the active well and is where the production occurs. Well O is the observation well where the pressure is measured.

(i) shut in both wells and allow them to reach the average reservoir pressure, p_i or $\overline{p}$
(ii) produce well A at a constant rate, q.
(iii) measure the pressure response in well O.

If there is a measured response, then our first objective is achieved. Inter-well values of k and ϕc_t, are obtained by semilog and TCA analysis techniques described earlier.

TCA procedure

(i) Overlay tracing paper (or transparency) on the type curve and plot the grid onto the tracing paper.
(ii) Plot all the data points.
(iii) Keeping the axes parallel move the Δp versus t graph with vertically and horizontally until the best match of Δp versus t with the type curve; i.e., p_D versus t_D/rD^2 graph is obtained.
(iv) Pick the match point; $[(t)_m,(\Delta p)_m]$ and $[(t_D/r_D^2)_m,\ (p_D)_m]$. It is convenient to use an intersection point of the grid on Δp versus t graph.
(v) Calculate k and ϕc_t from Eqs. 2.149 and 2.151 using the match point values.

(vi) Determine from $t_D/r_D^2 > 25$ criterion whether there are any data which will lie on a straight line on a plot of Δp or (p) versus lnt plot. If you determine that you will have such data, then proceed with the semilog analysis to determine k and ϕc_t.

Use of derivative in TCA Type curve analysis, before 1980, is based on the plotting Δp versus t and matching this data with the appropriate type curve (is a graphical representation of a theoretical model or interpretation model). After 1980, due primarily to Bourdet et al.'s work [6], type curves incorporating the pressure derivatives (i.e.; $dp_D/dlnt_D$) become popular. Recall that for constant rate production,

$$p_D = \frac{2\pi kh\Delta p}{qB\mu}$$

and

$$\frac{t_D}{r_D^2} = \frac{kt}{\phi c_t \mu r^2}$$

then

$$\frac{t_D}{r_D^2}\frac{\partial p_D}{\partial t_D/r_D^2} = \frac{\partial p_D}{\partial \ln t_D/r_D^2} = \frac{\partial p_D}{\partial \ln t_D} = \frac{2\pi kh}{qB\mu}t\frac{\partial \Delta p}{\partial t}$$

$$\frac{t_D}{r_D^2}\frac{\partial p_D}{\partial t_D/r_D^2} = \frac{2\pi kh}{qB\mu}\frac{\partial \Delta p}{\partial \ln t} \tag{2.152}$$

Eq. 2.152 when compared to Eq. 2.128 reveals that $\frac{\partial p_D}{\partial \ln t_D}$ is nondimensionalized in the same way as Δp. Thus, p_D and $\frac{\partial p_D}{\partial t_D/r_D^2}$ could be plotted on the same graph. For the line source solution, one can construct a type curve that involves both p_D versus t_D/r_D^2 and $\frac{\partial p_D}{\partial t_D/r_D^2}$ versus t_D/r_D^2 ,$p'_D = \partial p_D/\partial \ln t_D$. Such a type curve is shown below for the line source solution - Fig. 2.12.

It is argued that using derivative type curves for type curve analysis is more accurate as it requires to match both both Δp and $\Delta p'$ versus t simultaneously with the appropriate p_D and p'_D type curves. Since $\partial p_D/\partial \ln t_D$ is the instantaneous slope of p_D plotted on a semilog graph, after $t_D/r_D^2 > 25$ $\frac{\partial p_D}{\partial t_D/r_D^2}$ should become constant and have a value of 1/2. Similarly if the test data reach the time

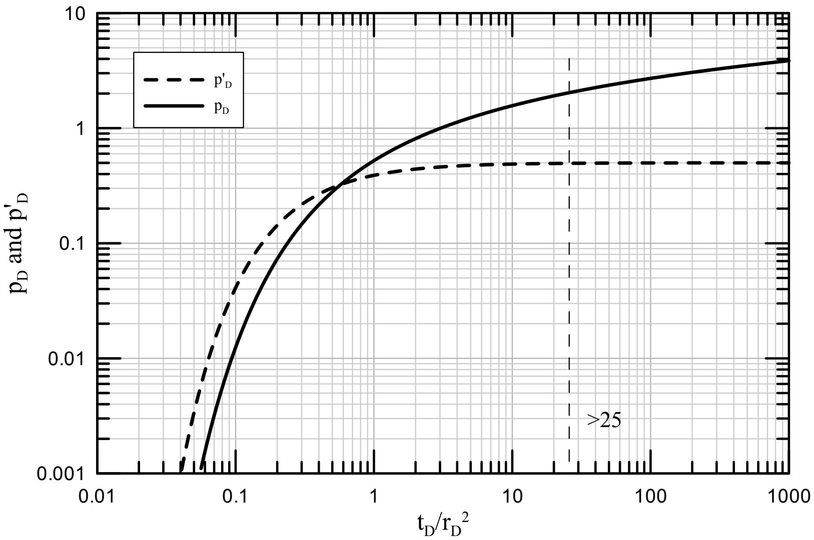


Fig. 2.12. Line source solution.

when the semilog approximation is applicable, $\partial \Delta p / \partial \ln t$ becomes constant. In terms of type curve matching, this means that vertical match is fixed since the constant on flat portion of $\partial \Delta p / \partial \ln t$ should lie on the top of the constant portion of $\frac{\partial p_D}{\partial t_D / r_D^2}$. To complete the match, the test data only have to be moved in the horizontal direction until the rest of the Δp and $\partial \Delta p / \partial \ln t$ data lie on the type curves. Another approach alternative to derivative type curve matching has been proposed by Onur and Reynolds in 1988 [11] simplifying type curve matching procedure and reducing non-uniqueness the problem (i.e., the data may seem to match the type curve equally well for different translation between the two sets of axes.). Onur and Reynolds proposed a derivative group based on a ratio of dimensionless pressure, p_D to the pressure derivative, $p_D' = dp_D / d_l n t_D$; i.e.,

$$Rpp' = \frac{p_D}{2 p_D{}'} = \frac{p_D}{2 \left[\frac{dp_D}{d \ln \left(t_D / r_D^2 \right)} \right]} \tag{2.153}$$

to construct pressure derivative type curves. Since,

$$p_D = \frac{2\pi k h \Delta p}{q B \mu}$$

and

$$p_D = \frac{dp_D}{d \ln t_D/r_D^2} = \frac{2\pi kh}{qB\mu}\Delta p'$$

then

$$\frac{p_D}{2p_D{}'} = \frac{\Delta p}{2\Delta p'} \tag{2.154}$$

which indicates that the vertical scale of a plot $p_D/(2p_D{}')$ vs t_D/r_D^2 is identical to the vertical scale of a field data plot of $\frac{\Delta p}{2\Delta p'}$ versus t. Such a type curve is shown in Fig. 2.13 for the line source solution. Note that for the line source solution,

$$\frac{p_D}{2p_D{}'} = \frac{\Delta p}{2\Delta p'} = -\frac{1}{2}Ei\left[-\frac{r_D^2}{4t_D}\right]/\exp\left[-\frac{r_D^2}{4t_D}\right] \tag{2.155}$$

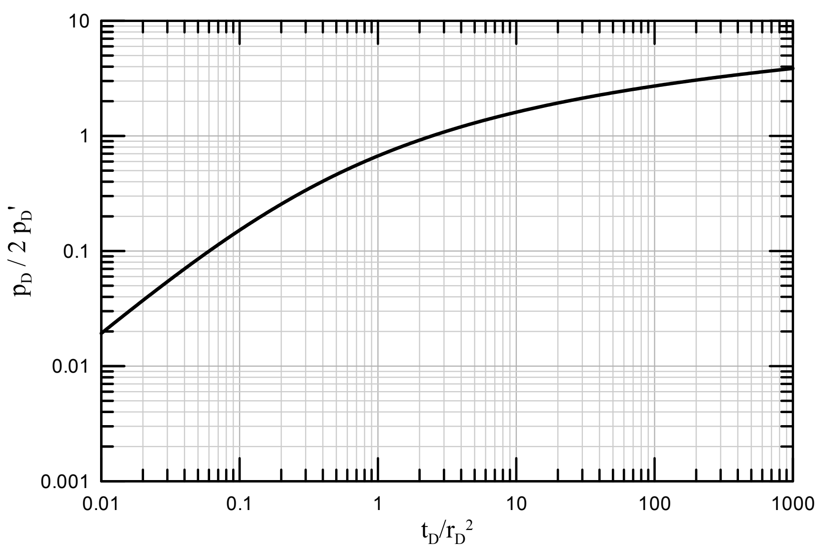


Fig. 2.13. Line source solution.

Use of Rpp' type curve

(i) A match of $\Delta p/2\Delta p'$ versus t data with the $p_D/(2p_D{}')$ type curve is obtained only by moving the field data plot in the horizontal direction. Once match is obtained, time match point valuesare determined, $[t_m, (t_D/r_D^2)_m]$.

(ii) Fix the correspondence between time match point values, match the Δp versus t graph with p_D versus t_D/r_D^2 graph only by the vertical movement. Once the match is obtained, the pressure match point values are determined, $[(\Delta p)_m, (p_D)_m]$.
(iii) Since all match point values are determined kh and ϕc_t are computed from Eqs. 2.149 and 2.151.

If the semilog straight line is reached during the test, then

$$\Delta p = p_i - p\,(r,t) = 1.15129 \frac{qB\mu}{2\pi kh}$$
$$\left\{ \log\,(t) + \log\left[\frac{kh}{\phi c_t \mu h r_w^2}\right] + 0.351378 \right\}$$

and

$$\Delta p' = \frac{d\Delta p}{d\ln t} = \underbrace{\frac{1.15129}{2\pi\ln[10]}}_{0.0795773} \frac{qB\mu}{kh} = \text{constant} \tag{2.156}$$

Data in a time range where $\Delta p' =$ constant determines whether a semilog straight line is reached or not. Alternatively, if the semilog straight line is reached during the test then,

$$\frac{\Delta p}{2\Delta p'} = \frac{p_D}{2p_D{}'} = 1.15129 \left\{ \begin{array}{l} \log\,(t) + \\ \log\left[\dfrac{kh}{\phi c_t \mu h r_w^2}\right] + 0.351378 \end{array} \right\} \tag{2.157}$$

which indicates that a semilog plot of $\Delta p/2\Delta p'$ versus t should yield a straight with a slope of 1.15129 regardless of the values of the reservoir parameters, k, h, etc. Therefore, a semilog plot of $\Delta p/2\Delta p'$ versus $t$allow one to identify the time period (or periods) when the semilog straight line exists. Once such a plot identifies a semilog straight line, semilog analysis is employed with confidence (see Fig 2.14).

Although pressure derivatives are useful in data analysis, it requires the computation of $\Delta p'$ versus t by numerical differentiation. Resulting derivative data is often noisy to analyze properly with the derivative type curves. To eliminate this problem, use of pressure integral functions proposed. Unlike the derivative, these functions

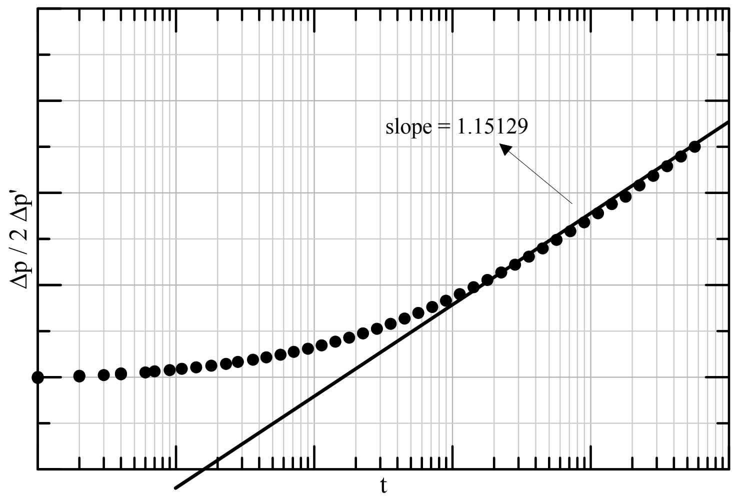


Fig. 2.14. Rpp' semilog analysis.

help us to generate data essentially free of noise while preserving the basic advantage of the pressure derivative (see Onur [10] and Blasingame [3]).

2.9 Formation Damage

Formation damage is a decrease in absolute permeability in the near wellbore region. Anything anything that causes a decrease in the permeability of the formations to hydrocarbons in the vicinity of wellbore will be referred to as true formation damage (invasion of drilling fluids, deposition of asphaltenes, or paraffins etc.). This will be associated with the skin factor (which is generally called true or mechanical skin factor).

Skin factor arising from other than a change in permeability will be referred to as pseudo-skin factor. The following may give rise to pseudo-skin factor ,

(i) insufficient perforations
(ii) non-Darcy flow effect
(iii) restricted entry wells
(iv) multi-phase effects

The formation damage is corrected by stimulation such as,

(i) injecting acids to dissolve reservoir rock to increase ϕ and k.
(ii) injecting solvent to dissolve scale.
(iii) injecting steam to dissolve contaminants and/or increase mobility.
(iv) creating propped hydraulic fractures.

"Skin effect" concept was introduced to the petroleum industry by van Everdingen [16] and Hurst [7]. Now, we investigate the skin effect by following the steps below.

(i) Analyze a pressure test using basic semilog equation and calculate a value for permeability knowing $\phi c)th$ and μ.
(ii) Using the values of k and $\phi c_t h \mu$, compute the theoretical pressures from the line source solution at $r = r_w$.
(iii) Plot the test data and the theoretical pressures on a semilog graph

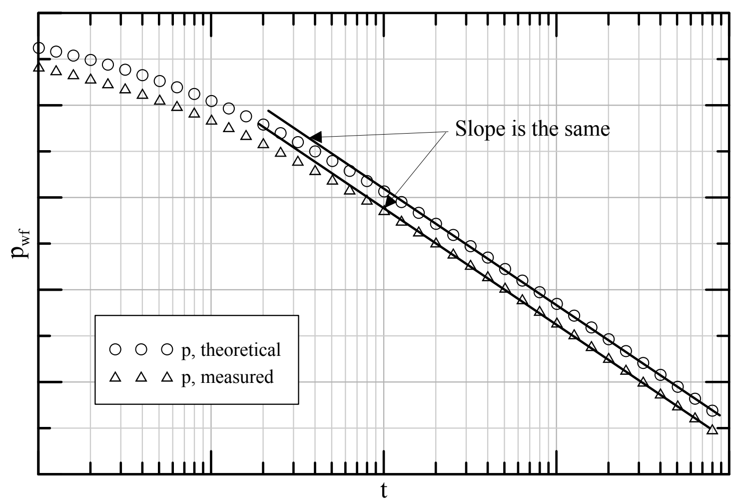


Fig. 2.15. Skin effect.

As Fig. 2.15 indicates, the theoretical curve is shifted with a constant value. This is the same observation both van Everdingen and Hurst made measured values of p_{wf} were displaced below the theoretical straight line by a constant, i.e.;

$$\Delta p_{measured} = \Delta p_{theoretical} + \Delta p_{skin} \tag{2.158}$$

when Δp_{skin} is constant, van Everdingen and Hurst attributed the extra pressure drop to a low permeability region around the well. They called this region of altered permeability as the skin zone and the additional pressure drop due to the skin zone Δp_{skin}. They believed this low permeability region did not extend very far into the reservoir.

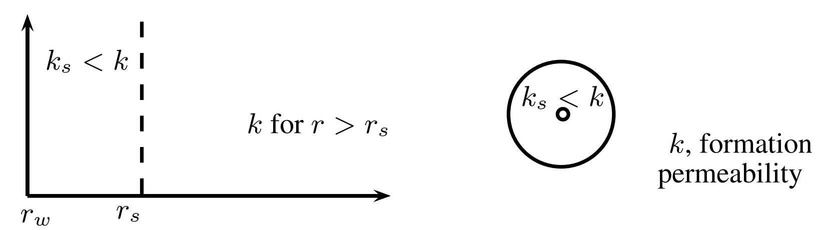


Fig. 2.16. Skin zone.

Now investigate the pressure profile due to an altered permeability for a constant rate production.

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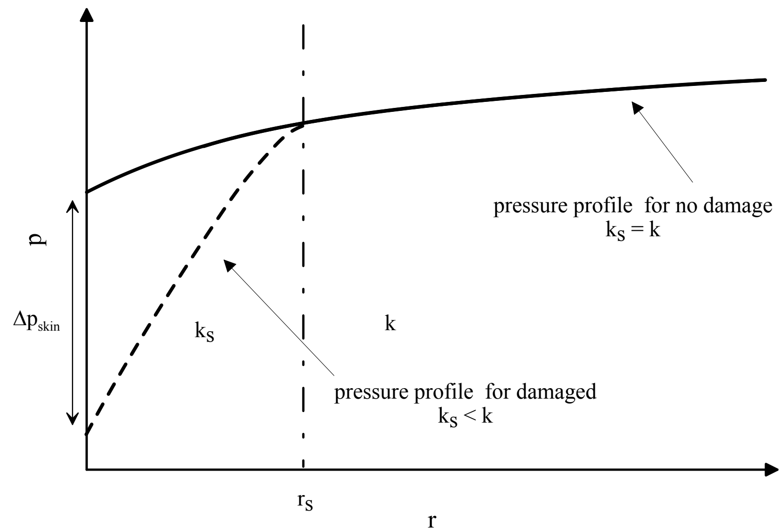

Fig. 2.17. Effect of skin on pressure profiles.

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