Evaluation of Market-Based Probabilities

SUMMARY: This paper discusses the exploration and utility of market-based probabilities in forecasting financial outcomes, focusing on their predictive power regarding returns, and volatilities. It critically examines trading strategies derived from these probabilities, assessing their effectiveness and exploring enhancements through additional methodologies like machine learning techniques.

1. INTRODUCTION

The exploration of market-based probabilities offers a nuanced perspective on predicting future financial outcomes. These probabilities, extracted from financial markets, reflect t he c ollective a ssessment of future events' likelihood and their potential impacts.

Market-based probabilities offer s ignificant insights for both investors and policymakers. For investors, these probabilities can guide the allocation of resources by highlighting potential future market movements, thereby informing risk management and investment strategies. For policymakers, understanding market expectations through these probabilities can assist in crafting policies that aim to stabilize financial m arkets, adjust to anticipated economic shifts, and address systemic risks.

While offering v aluable i nsights, i t h as l ong been plagued by some issues. Critics often argue that policymakers should use the "true" probability of future events to weigh their decisions. They contend that market-based probability forecasts, derived from financial asset prices, do not match the predictive accuracy of those based on statistical models. This debate highlights the limitations of relying solely on market sentiment for forecasting future events.

These issues are our focus in the paper. In this paper, we delve into the utility of market-based probabilities for forecasting future market outcomes, focusing on their ability to predict returns, volatilities, and reversals. This examination forms the core of our methodology section. Subsequently, we explore trading strategies derived from these estimates, questioning their efficacy an d in vestigating wh ether integrating additional data or methodologies can amplify their effectiveness.

2. Relevant Research Review

2.1. Further Evidence on Option Momentum

Investigates the momentum effect in options trading strategies (specifically straddles, strangles, and butterflies) among large-cap and blue-chip stocks in the S&P 100 Index from 2000 to 2019. The study finds strong momentum for straddles and strangles over one to three years (excluding the most recent month), with no such effect for butterflies. The observed momentum cannot be explained by traditional risk factors, indicating unique market dynamics in option momentum.

2.2. Application of Machine Learning in Option Trading Strategy Decision Making

Investigates the momentum effect in options trading strategies (specifically straddles, strangles, and butterflies) among large-cap and blue-chip stocks in the S&P 100 Index from 2000 to 2019. The study finds strong momentum for straddles and strangles over one to three years (excluding the most recent month), with no such effect for butterflies. The observed momentum cannot be explained by traditional risk factors, indicating unique market dynamics in option momentum.

2.3. Building Equity Market Neutral Strategy Portfolios Using LSTM

Introduces a model for constructing Equity Market Neutral (EMN) strategy portfolios by predicting and selecting S&P 500 consumer stocks using Long Short-Term Memory (LSTM) neural networks. The study highlights the efficacy of LSTM in predicting stock returns and demonstrates that the constructed EMN portfolios outperform benchmarks during non-pandemic periods.

2.4. Convex Optimization under Risk-Neutral Probabilities

Introduces a method for minimizing convex or quasi-convex functions over the set of risk-neutral probabilities, used for computing bounds on various financial metrics such as cumulative distributions, Value at Risk (VaR), Conditional Value at Risk (CVaR), and pricing new derivatives. The method showcases the efficient resolution of such problems using modern convex optimization techniques, offering a viable method for financial analysis and decision-making.

2.5. The Paradox of Risk-Neutral Valuation in Derivative Markets

Discusses the dynamics of derivative markets, specifically how risk-neutral investors can generate endogenous bubbles. A theorem is presented showing that derivative markets may experience extreme price fluctuations, such as spikes and crashes, more frequently than typically expected.

2.6. Momentum and Mean Reversion under Risk-Neutral

Explores the dynamics of financial markets through a risk-neutral lens, specifically the phenomena of momentum and mean reversion. The study proposes a complex model based on Markov martingales and variance gamma processes for analyzing and predicting market price movements.

3. Data

In this study, we analyze option trading data derived from Bloomberg's Option Monitor (OMON) platform, focusing on transactions recorded within a discrete five-day window. These transactions are predicated on actual trades, providing a robust foundation for our analysis. To ensure the integrity and relevance of our dataset, we implement several critical filters. Firstly, we exclude any data entries where the trading volume for an option on any given day is zero. This criterion is pivotal as Bloomberg occasionally reports end-of-day settlement prices for markets experiencing no trading activity, which could skew the analysis. Additionally, we discard observations where the price of a call option does not facilitate the calculation of volatility, effectively removing outliers that could distort our findings.

The dataset encompasses transactions across 53 distinct strike prices, ranging from 3400 to 6800, culminating in a total of 111 observations. It is noteworthy that options at the same strike price may be traded multiple times within the specified period. Given the substantial volume of trades, we opt to utilize the closing prices from each trading day. For analytical purposes, we consolidate the five-day trading window into a singular trading day, selecting the "last price" for each strike price accordingly.

A critical aspect of our methodology involves the association of a risk-free rate with each option. This is accomplished through an interpolation of the U.S. Treasury yield curve, specifically focusing on on-the-run issues as of the price date. This approach ensures that our financial modeling reflects the prevailing economic conditions accurately.

Furthermore, in addressing the valuation of put options, we employ the put-call parity principle, tailored to the asset class under consideration. This conversion of put options into their call option equivalents is a strategic methodological choice, facilitating a more streamlined and unified analysis of the options market. This comprehensive approach allows us to delve into the intricacies of option trading dynamics, offering insights into the underlying patterns and trends that characterize this complex financial landscape.

4. Methodology & Usefulness of risk neutral probabilities

In the analysis of European call and put options, the extraction of risk-neutral densities through a non-parametric approach, grounded solely in the options' payoff structure, marks a pivotal advancement. The Breeden Litzenberger framework delineates how the second derivative's relationship with the strike price directly reveals the risk-neutral distribution, transitioning the challenge from an overdetermined state, constrained by sparse parameters against numerous option prices, to a scenario where, given certain conditions, a singular, definitive solution emerges.

This methodology critically hinges on the assumption of precise option prices across all strikes, an ideal yet often unattainable in practice due to the limited range of available strike prices. To navigate this reality, the adoption of an interpolation strategy becomes essential, bridging the gaps between traded strikes and employing a quadratic smoothing of the volatil-

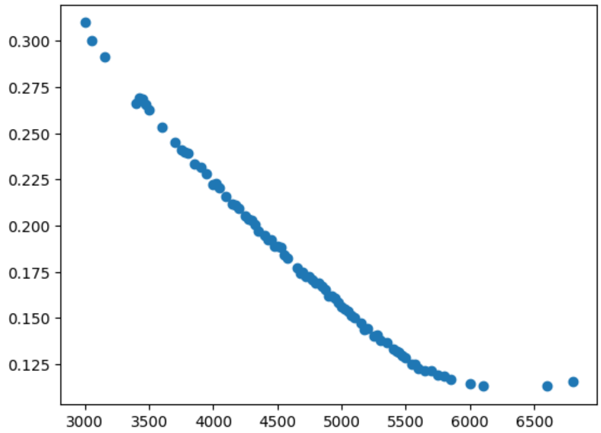

Fig. 1: Data from Bloomberg in Feb. 2024

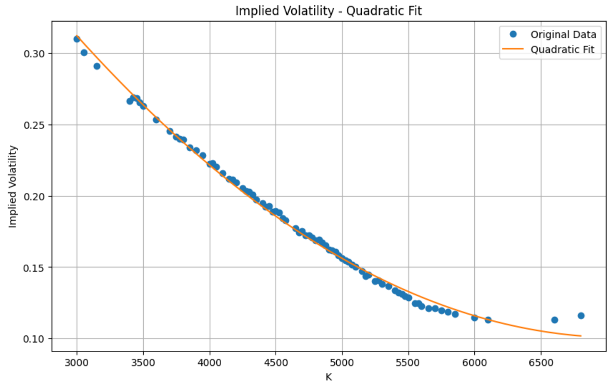

Fig. 2: Implied Vol. and the Fitting

ity curve, thus ensuring a coherent and continuous representation of the market's implied volatility landscape.

In this study, we employ a methodology to estimate Market Probability Density Functions (MPDs) for various asset classes, extending the Shimko (1993) approach. Initially, we gather and analyze option trade data, focusing on actual transactions to accurately model implied volatilities across a comprehensive strike price range. This data collection aims to minimize reliance on extrapolated information, ensuring our analysis is grounded in empirical evidence.

For European call option:

$$c = e^{-\int_0^T r_u du} \int_K^{+\infty} (S_T - K)\phi(S_T) dS_T$$

$$\frac{\partial c}{\partial K} = e^{-\int_0^T r_u du} \int_K^{+\infty} -\phi(S_T) dS_T$$

$$\frac{\partial^2 c}{\partial K^2} = e^{-\int_0^T r_u du} \phi(K)$$

Fig. 1 illustrates the relationship between the implied volatility and strike price from the raw data. Figs. 2, 3, and 4 showcase the functions derived after fitting volatility and subsequent derivative interpolation. Figs. 5 and 6 depict the statistically derived risk-neutral probabilities in the strike price and log return spaces, characterized by left-skewness, thin tails, and high kurtosis. Figs. 7 and 8 represent the changes in risk-neutral probabilities across two different spaces from January to February 2024, indicating larger deviations in means, reduced kurtosis, consistent with statistical descriptions.

Fig. 9 illustrates the relationship between the disparity in probabilities of a 20% increase (prInc) and a 20% decrease (prDec) in SPX options for 6-year and

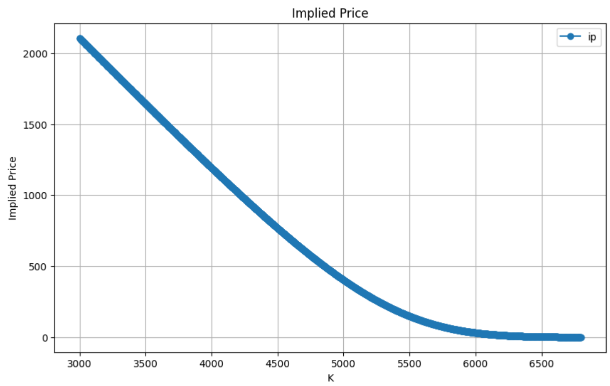

Fig. 3: Implied Price

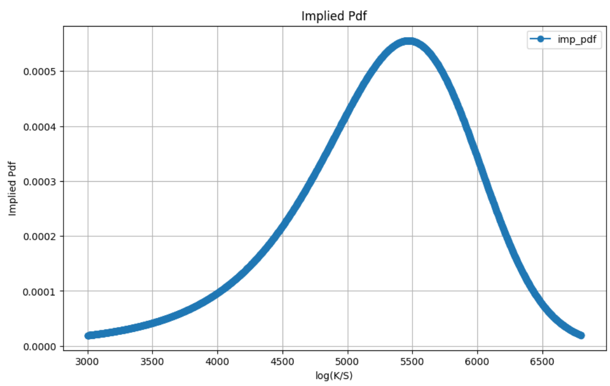

Fig. 4: Implied Pdf in Feb. 2024

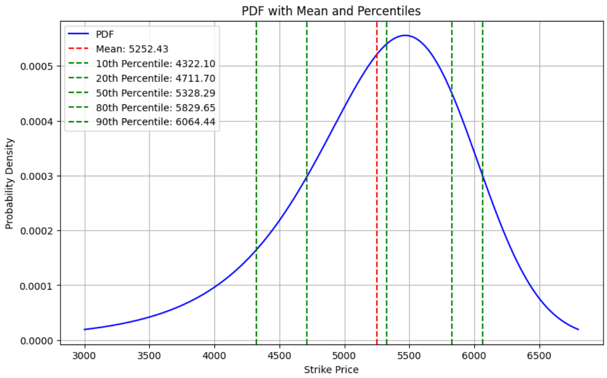

Fig. 5: Statistics of Pdf

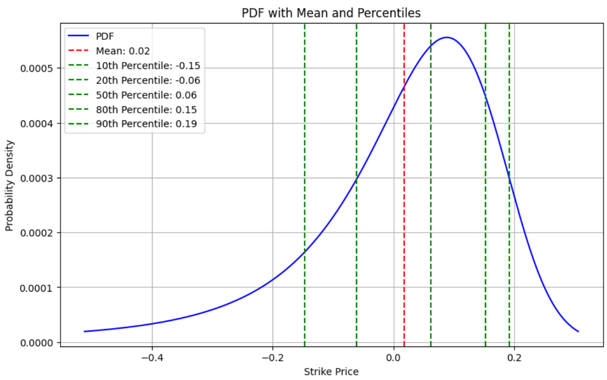

Fig. 6: Statistics of Pdf in log return space

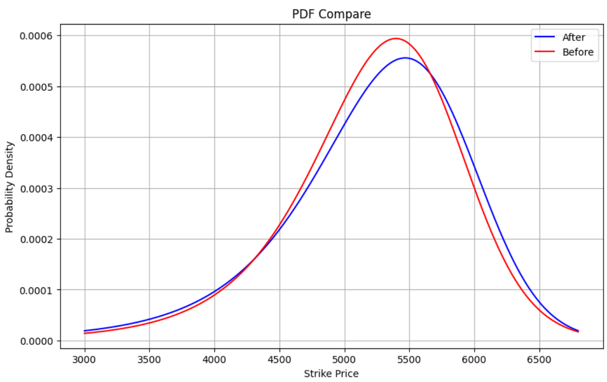

Fig. 7: Pdf Change between Jan. 2024 and Feb. 2024

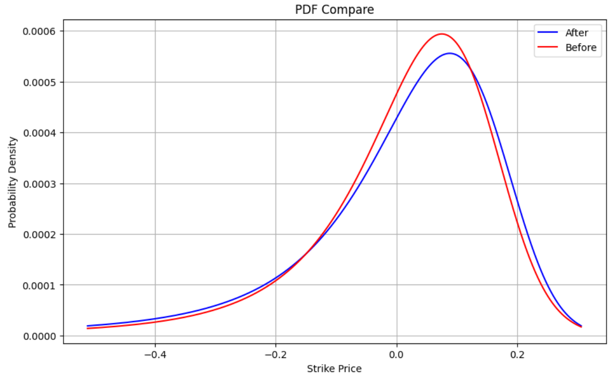

Fig. 8: Pdf Change between Jan. 2024 and Feb. 2024 in log space

12-year maturities, derived from risk-neutral probabilities, and the corresponding log returns of SPX. It is evident that for the 6-year SPX options, a positive gap or one close to zero significantly indicates the potential for SPX to achieve excess returns over the respective period.

Subsequently, we engage in a detailed process involving the calculation of time to expiration, adjustment for risk-free rates, and conversion of put options to call options, utilizing the Black-Scholes model to derive implied volatilities. This step is crucial for accurately capturing the market's expectations and the intrinsic value of the underlying assets.

Finally, by fitting a cubic B-spline to the implied volatility data and calculating the continuous call option prices, we derive the MPD for the underlying asset. This MPD is then transformed into log return space, offering insightful interpretations of market sentiment and future asset valuation. Our methodology, rigorous in its application of financial models and statistical analysis, provides a robust framework for understanding market dynamics and asset price movements.

We examined the viability of using risk-neutral probabilities to forecast market trends. These probabilities, which represent the market's expectation of future volatility, were extracted from the Minneapolis Federal Reserve's dataset for the S&P500 index. Specifically, we focused on the risk-neutral probabilities of large market increases and decreases. The analysis was conducted by regressing these probabilities against the logarithmic differences of the S&P500 index prices, which serve as a proxy for market returns.

4.1. Regression Analysis

Building on the rigorous foundation laid by the exploration of European call and put options and the subsequent derivation of Market Probability Density

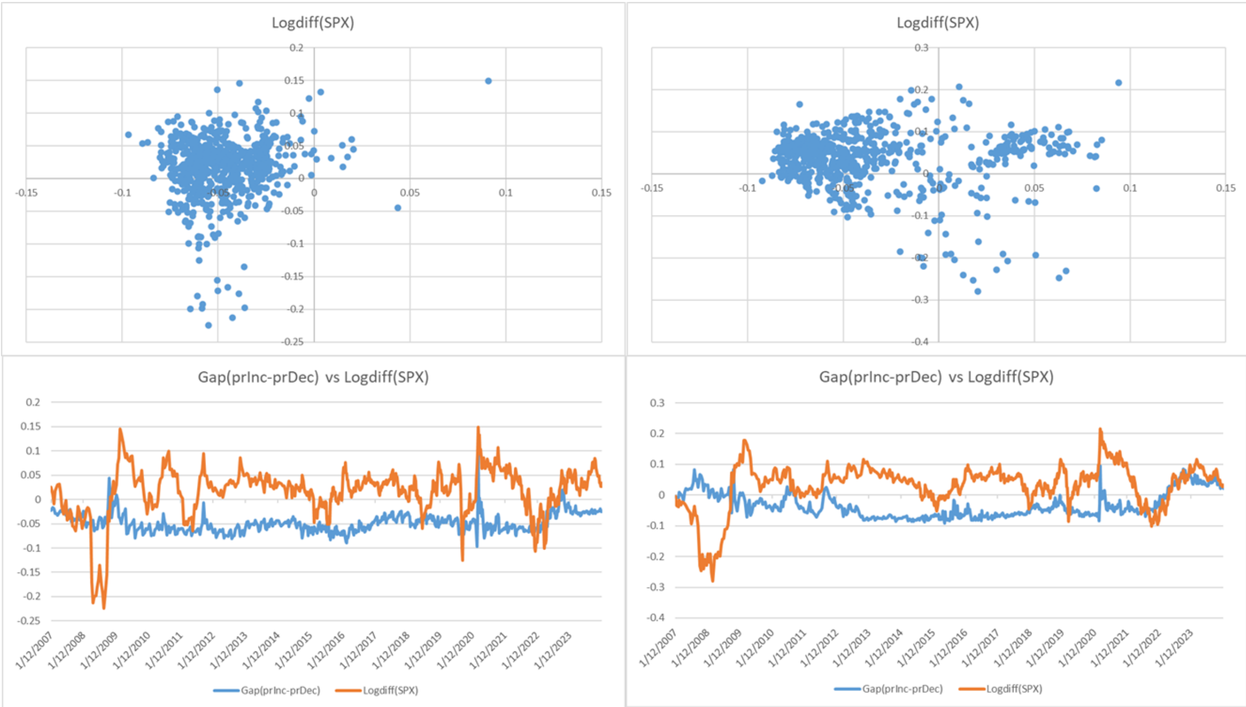

Fig. 9: PDF probabilities and returns since 2007.

```
                            GLSAR Regression Results
=====================================================================================
Dep. Variable:            Logdiff(SPX)   R-squared (uncentered):              0.250
Model:                           GLSAR   Adj. R-squared (uncentered):         0.248
Method:                  Least Squares   F-statistic:                         113.7
Date:                 Thu, 29 Feb 2024   Prob (F-statistic):               2.41e-43
Time:                         19:00:38   Log-Likelihood:                     905.94
No. Observations:                  684   AIC:                                -1808.
Df Residuals:                      682   BIC:                                -1799.
Df Model:                            2
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
prDec          0.3398      0.047      7.306      0.000       0.248       0.431
prInc         -0.1099      0.055     -1.991      0.047      -0.218      -0.002
==============================================================================
Omnibus:                      239.441   Durbin-Watson:                   0.066
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              885.057
Skew:                          -1.625   Prob(JB):                    6.49e-193
Kurtosis:                       7.526   Cond. No.                         5.38
==============================================================================
```

Fig. 10: GLSAR Regression Results

Functions (MPDs), our study extends into the empirical realm by incorporating risk-neutral probabilities for large market movements into our analytical framework. Specifically, we apply Generalized Least Squares with AR covariance structure (GLSAR) to explore the relationship between market probability densities and subsequent market movements. Utilizing risk-neutral probabilities for significant increases (prInc) and decreases (prDec) as independent variables, our model aims to capture the predictive influence these probabilities exert on the logarithmic differences of index prices (Log_Diff). The GLSAR approach, preferred for its capacity to manage the autocorrelation often present in time-series data, aligns

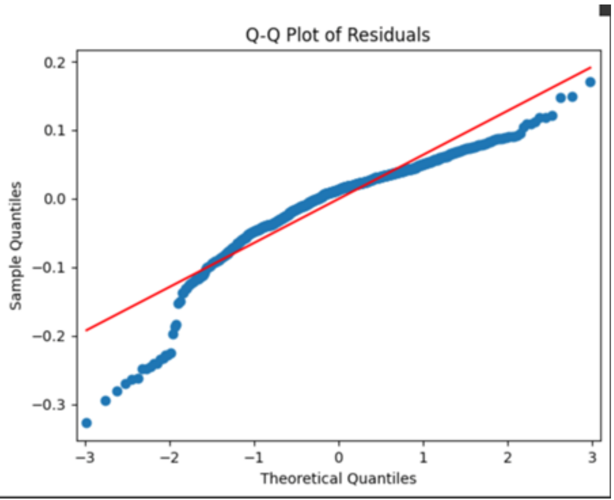

Fig. 11: QQ Plot

with our commitment to methodological rigor. This technique enhances the precision of our coefficient estimates, ensuring robustness in our empirical analysis.

4.2. Findings

The GLSAR model indicates that both prInc and prDec have statistically significant coefficients. The directionality and magnitude of these coefficients suggest a quantifiable influence of risk-neutral probabilities on market movements. A positive coefficient for prInc is associated with an uptrend in market returns, whereas a significant positive coefficient for prDec aligns with a downturn in market performance. These results, derived from a model that accounts for potential autocorrelation, affirm the predictive capacity of risk-neutral probabilities.

4.3. Hypothesis Testing

Hypothesis testing within the GLSAR framework follows a structured approach where the null hypothesis (H0) posits that the coefficients of prIncand prDec are non-influential, set to zero. Conversely, the alternative hypothesis (H1) asserts that these coefficients significantly differ from zero, hence possessing predictive attributes. The t-test outcomes (shown in Fig. 10) from the GLSAR model provide the statistical backing needed to verify or refute the efficacy of risk-neutral probabilities in forecasting market trends.

4.4. Potential Problems

The GLSAR model, while addressing autocorrelation, reveals potential issues that warrant careful consideration. Firstly, the adjusted R-squared value of 0.248 indicates that a considerable proportion of variability in the log differences of index prices remains unexplained by the model. This suggests that other factors not included in the model may be influencing market movements.

The Jarque-Bera test provides a statistic of 885.057 with a p-value effectively at zero, which points to a departure from the normality of residuals. This non-normality, indicated by a high kurtosis value (as seen in Fig. 11), suggests that extreme values are more prevalent than would be expected in a normal distribution and that the tails of the distribution are heavy. The negative skewness also suggests an asymmetry in the distribution of residuals, with a tail that is skewed to the left.

Moreover, the Durbin-Watson statistic is at 0.066, indicating a potential for positive serial correlation in the residuals, albeit this issue should have been mitigated by the GLSAR modeling process. However, a value this close to zero may still suggest some level of correlation that the model has not fully accounted for.

These diagnostics imply that the GLSAR approach does not fully resolve all the issues inherent in the data. Investors and researchers should thus be cautious in interpreting the results, and it may be beneficial to explore additional model refinements or to consider alternative models that could better capture the complexity of the data.

4.5. Implications for Investors

The adoption of Generalized Least Squares with AR covariance structure(GLSAR) for our regression analysis has led to noteworthy findings that bear significant implications for investors. The model's ability to statistically validate the predictive significance of risk-neutral probabilities for both market increases (prInc) and decreases (prDec) provides a strategic advantage. Investors can leverage this information to gauge potential future market movements and make informed asset allocation decisions.

However, the potential problems identified through diagnostic tests need to be carefully considered. The adjusted R-squared value leaves a substantial amount of variability in market returns unexplained. This underscores the complexity of market dynamics and the need for investors to look beyond the model at other factors that could be influencing market outcomes.

The departure from normality as indicated by the Jarque-Bera test, coupled with high kurtosis, suggests the presence of outliers or events that could lead to substantial gains or losses. The skewed residuals further indicate that investors should be cautious about potential downside risks.

Additionally, the Durbin-Watson statistic, despite the application of GLSAR, hints at some remaining positive serial correlation, suggesting that the model may not have fully captured all temporal dependencies. This could affect the predictability of the model under certain market conditions.

For investors, these diagnostics imply that while the model provides a more nuanced approach to understanding risk-neutral probabilities, it should not be used in isolation. Investment strategies should incorporate a mix of models and indicators to better understand risk and to construct a diversified portfolio. The insights from the GLSAR model could be used in conjunction with other analytical tools, economic indicators, and risk management strategies to enhance the robustness of investment decisions.

5. Trading Strategies

Merging trading strategies with risk-neutral probability distributions provides a sophisticated framework for navigating financial markets. This approach utilizes market-derived expectations reflected in derivative prices to inform trading decisions, moving beyond historical data analysis. It facilitates strategies like delta-neutral trading and volatility arbitrage for optimized risk management and precision in exploiting market inefficiencies. One relevant strategy is the Market Neutral Pairs Trading Strategy, grounded in statistical arbitrage and mean reversion. This approach identifies two correlated securities with synchronized price movements, focusing on their relative performance rather than individual behavior. Within the risk-neutral probabilities framework, the strategy assumes the price differential between the paired securities tends to revert to its historical average, creating a profit opportunity through convergence.

5.1. Market Neutral Pairs Trading Strategy

Pairs trading is a market-neutral trading strategy that matches a long position with a short position in a pair of highly correlated instruments such as two stocks, exchange-traded funds (ETFs), currencies, commodities or options. Pairs traders wait for weakness in the correlation, and then go long on the under-performer while simultaneously going short on the over-performer, closing the positions as the relationship returns to its statistical norm. The strategy's profit is derived from the difference in price change between the two instruments, rather than from the direction in which each moves. Therefore, a pairs trade (also known as a relative value trade) is market neutral.

In the context of pairs trading, risk neutrality implies that the trader is indifferent to the individual risks of the two instruments in the pair, as long as the pair as a whole is expected to return to its statistical norm.

Risk neutral densities (RNDs) can be used in pairs trading to estimate the future prices of the two instruments in the pair. By using RNDs, a pairs trader can estimate the probability that the price relationship between the two instruments will return to its statistical norm within a certain time frame, and make trading decisions based on this probability.

5.2. Steps for Implementing Pairs Trading

5.2.1. *Identify Potential Pairs*

The first step in pairs trading is to identify potential pairs of securities which we did using K-means clustering to group securities based on their historical price movements. Securities in the same cluster are likely to be highly correlated and should be suitable candidates for pairs trading.

We used a list of 50 stock tickers and conducted K-Means clustering for the time period of 2012 to 2015 so that we can back-test it eventually and truly understand if the Market Neutral Pairs Trading would have worked in a time period where there was essentially no market uncertainty like Covid-19.

We see that several stocks that shouldn't be related still are in the same cluster (coincidences). We pick V (Visa) and MA (Mastercard) as our pairs since they operate in the same industry as global payment processing companies and are considered rivals in the electronic payments space. (Refer to Fig. 12)

5.2.2. *Risk Neutral Densities*

For our pair, we require the estimated RNDs. Since, it is hard to come across real RNDs of Visa and Mastercard, we could use the Black-Scholes to estimate the risk neutral probabilities.

5.2.3. *Identify Trading Opportunities*

Using the RNDs to identify trading opportunities. We see that the return on both was very miniscule over 2012-2015 but Visa had a slightly higher return. We could implement strategies to long Visa and short Mastercard to account for any potential changes and closely monitor any large change. The correlation between Visa and Mastercard is very high and would be a prime example for Market Neutral Pairs Trading Strategy.

5.2.4. *Check it with 2016-2020*

We see in the graph that Visa and Mastercard have moved so closely together for a large part of the back-tested data. Proving our hypothesis that they are an optimal pair for pairs trading. While we would not have made a big return, we would definitely identify a risk neutral trading strategy. (Refer to Fig. 13)

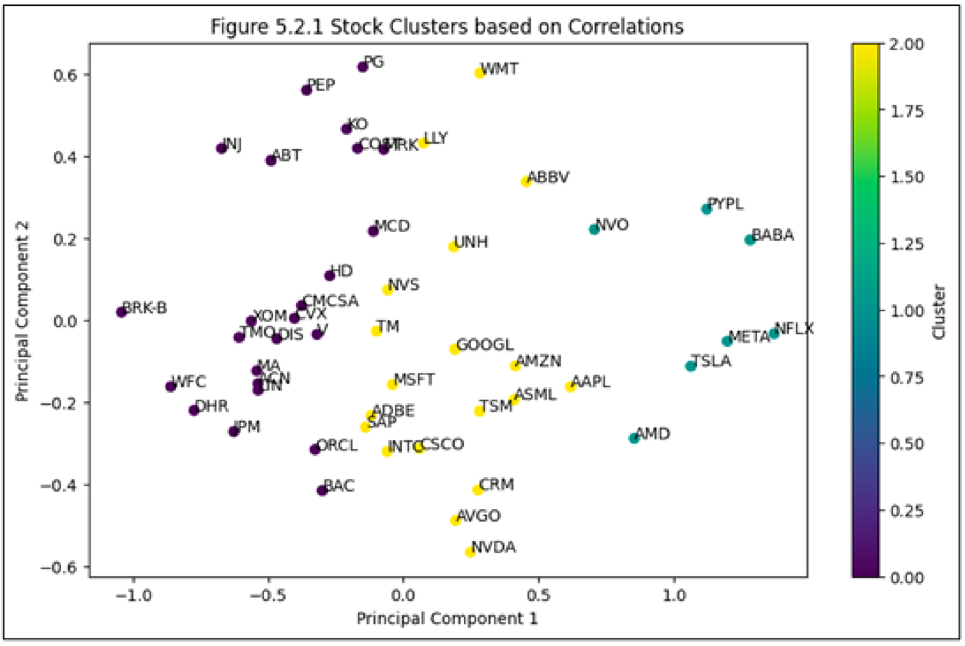

Fig. 12: Stocks Clusters

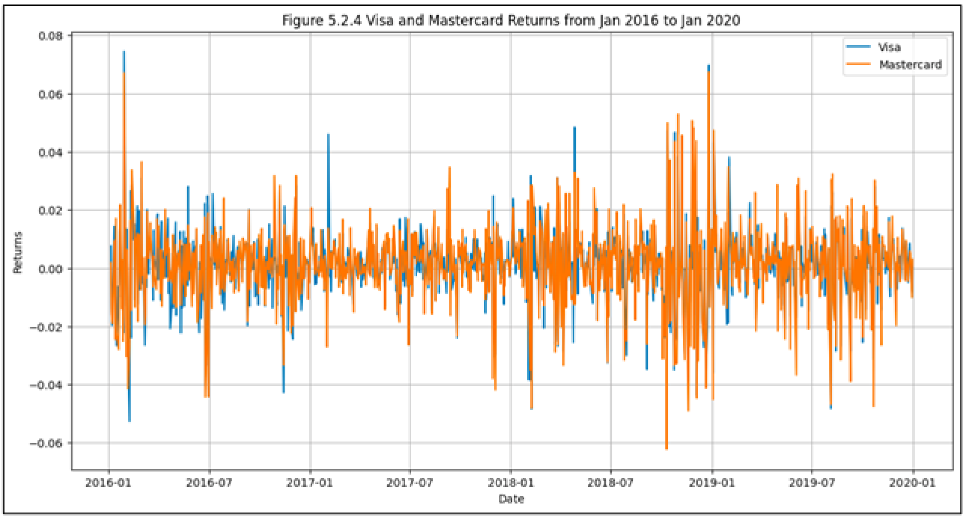

Fig. 13: Returns Compare

5.3. Delta Neutral Options Trading

The delta-neutral portfolio strategy is employed in options trading, utilizing multiple positions with balancing positive and negative deltas to achieve an overall delta of zero. This strategy is particularly effective for managing risk in the face of market movements. Delta-neutral portfolios aim to neutralize the impact of market fluctuations within a certain range, thus maintaining a net change of zero. Options traders leverage delta-neutral strategies to capitalize on either implied volatility or the time decay of options. In the context of risk-neutral probabilities, these strategies hinge on computing expected asset values under the assumption of a hypothetical fair, single probability for an outcome. The PDF can inform hedging strategies where the hedge ratios are adjusted based on the changing probabilities of various price levels.

Fig. 14: Backtesting results for 2023

5.4. Implementation of Delta Neutral Options Trading

The Delta Neutral Options Trading Strategy, especially when integrated with the concept of risk-neutral probabilities distribution, represents a sophisticated approach to options trading that capitalizes on market volatility and time decay, rather than directional price movements of the underlying asset. This strategy is particularly effective in environments where the trader can leverage the predictive power of the risk-neutral probabilities distribution to manage and adjust their positions dynamically.

In the context of options pricing, risk-neutral probabilities represent a theoretical distribution of future prices of the underlying asset under the assumption that investors are indifferent to risk. This framework simplifies the complex task of options pricing by assuming that all assets earn the risk-free rate of return, allowing for the use of a single, unified probability measure to price derivatives.

Combining the delta neutral strategy with risk-neutral probabilities distribution involves adjusting the options portfolio to be insensitive to small movements in the underlying asset's price, while also taking into account the expected movements of the asset as implied by the options market itself. This dual approach allows traders to hedge against immediate price movements while positioning themselves to profit from the actual volatility of the asset as it deviates from the expected path.

5.4.1. Establishing a Delta Neutral Position

The strategy starts by creating a delta neutral position through the purchase or sale of options and possibly the underlying asset. The goal is to achieve a net delta of zero, making the position initially insensitive to small price movements in the underlying asset.

5.4.2. Leveraging Risk-Neutral Distribution

The risk-neutral probabilities distribution is used to inform the selection of strike prices and expirations for the options in the strategy. By analyzing the implied volatility surface and the risk-neutral distribution, traders can identify options that are priced in a way that reflects an expected level of movement in the underlying asset that differs from the historical or realized volatility.

5.4.3. Dynamic Position Adjustment

The delta of the options portfolio will change as the underlying asset's price moves and as time passes. Traders must dynamically adjust their positions to maintain delta neutrality. The adjustments are guided not just by the delta but also by insights gained from the risk-neutral distribution, such as potential shifts in implied volatility or the probability of reaching certain price levels.

5.4.4. *Profit from Volatility and Time Decay*

The strategy aims to profit from the volatility (as captured by the Vega of the options) and time decay (Theta), rather than from directional price movements. By understanding the risk-neutral probabilities, traders can more effectively manage these positions to exploit discrepancies between implied and realized volatility.

5.4.5. *Risk Management*

Incorporating risk-neutral probabilities into the strategy also aids in risk management, providing a more nuanced view of potential price movements and the likelihood of extreme events. This allows for better-informed hedging strategies and more precise control over the risk profile of the portfolio.

By integrating the delta neutral options trading strategy with the concept of risk-neutral probabilities distribution, traders can achieve a more refined approach to options trading. This strategy leverages the market's collective expectations about future price movements, as embedded in options prices, to make informed trading decisions that balance the pursuit of profit with the management of risk.

To assess the effectiveness of our strategy, we conducted a backtest for the year 2023. In the chart below, we can observe that the strategy yielded a return of 4.72% over the course of the year 2023, with relatively low volatility. This demonstrates the strategy's effectiveness in risk management.

6. Optimization

Incorporating machine learning techniques into Delta Neutral Options Trading Strategy presents a promising avenue for optimization. By employing a supervised classification framework and evaluating different models, one could highlight the effectiveness of using gradient tree boosting algorithms. This approach not only outperforms the simplistic trade strategy but also showcases statistically significant advantages. The systematic hyperparameter search further refines the model's performance, setting a strong foundation for applying supervised classification methods to a broader range of derivative trading strategies. This methodology opens up new possibilities for dynamically adjusting trading strategies based on evolving market conditions, thereby improving the timing of adjustments and the selection of options. Through the integration of advanced machine learning techniques, we can anticipate a future where trading strategies are more profitable.

7. Conclusion

Risk-neutral probability estimates are indeed useful to a certain extent in predicting future returns, and volatilities. They provide investors with a framework to understand market expectations embedded in derivative prices, detached from personal risk preferences. Strategies such as the Market Neutral Pairs Trading Strategy and Delta Neutral Options Trading have proven effective by leveraging these risk-neutral probabilities, focusing on exploiting discrepancies and hedging market risks respectively. Furthermore, integrating these strategies with machine learning (ML) techniques can significantly enhance their effectiveness. Machine learning can process vast datasets to uncover patterns and insights that are not immediately apparent, enabling a more nuanced approach to strategy optimization. This combination allows for dynamic adjustments based on evolving market conditions, leading to more robust and adaptable trading strategies that can capitalize on market inefficiencies more efficiently.

8. References

[1] Wei, Peng and Cao, Yi and Dong, Yizhe, Option Momentum: Further Evidence. Available at SSRN: https://ssrn.com/abstract=4657193/ or http://dx.doi.org/10.2139/ssrn.4657193

[2] Brunhuemer, Alexander, Lukas Larcher, Philipp Seidl, Sascha Desmettre, Johannes Kofler, and Gerhard Larcher. 2022. "Supervised Machine Learning Classification for Short Straddles on the S&P500" Risks 10, no. 12: 235. https://doi.org/10.3390/risks10120235

[3] Nafia, Abdellilah, Abdellah Yousfi, and Abdellah Echaoui. 2023. "Equity-Market-Neutral Strategy Portfolio Construction Using LSTM-Based Stock Prediction and Selection: An Application to S&P500 Consumer Staples Stocks" International Journal of Financial Studies 11, no. 2: 57. https://doi.org/10.3390/ijfs11020057

[4] Barratt, Shane and Tuck, Jonathan and Boyd, Stephen, Convex Optimization Over Risk-Neutral Probabilities (March 4, 2020). Available at SSRN: https://ssrn.com/abstract=3548956 or http://dx.doi.org/10.2139/ssrn.3548956

[5] Alessandro Fiori Maccioni, 2011. "Endogenous Bubbles in Derivatives Markets: The Risk Neutral Valuation Paradox," Papers 1106.5274, arXiv.org, revised Sep 2011.

[6] Madan, Dilip B., Momentum and Reversion in Risk Neutral Martingale Probabilities (March 8, 2014). Available at SSRN: https://ssrn.com/abstract=2251300 or http://dx.doi.org/10.2139/ssrn.2251300