SAMPLE EXAM #3B
ELEG 305 SIGNALS AND SYSTEMS
SPRING 2019

Note: Last year, the third exam included Chapter 10. So, there are some problems that will not be relevant for this year's Exam #3, which includes Chapters 5, 7, and 9. I have marked the problems that cover material that will NOT be on Exam #3 using # signs right next to the problem.

Problem #1 (30 points)
Consider a continuous-time LTI system, with input x(t) and output y(t), described by the following differential equation:

$$\frac{d^2y(t)}{dt^2} - 9y(t) = x(t) + \frac{dx(t)}{dt}$$

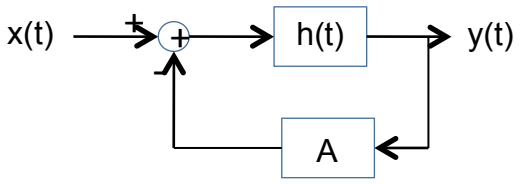
a.) (6 pts) Determine the transfer function, H(s), for this system.
b.) (6 pts) Determine the locations of the poles and zeros, and plot them in the s-plane.
c.) (6 pts) Draw the three different possibilities for the region of convergence (ROC), and label which one is for a right-sided signal, a left-sided signal, and a two-sided signal.
d.) (10 pts) Derive the impulse response, h(t), for a *stable* system.
e.) (2 pts) For which ROC is this system causal? Please justify your answer.

Problem #2 (25 points)
Consider the following impulse response for a discrete-time LTI system:

$$h[n] = \left(\frac{1}{4}\right)^n u[n] + \frac{4}{3}\left(\frac{1}{3}\right)^n u[-n-1]$$

a.) (10 pts) Determine the transfer function, H(z), and the region of convergence (ROC).
b.) (5 pts) Determine the poles and zeros, and plot them in the z-plane along with the ROC.
c.) (6 pts) Derive the difference equation relating the input and the output of this system.
d.) (2 pts) Does the Fourier transform of h[n] exist? Please justify your answer.
e.) (2 pts) Is this system causal? Please justify your answer

Problem #3 (35 points)
a.) (14 pts) Consider an impulse response $h(t) = e^t u(t)$; this system is clearly not stable.
(i) (6 pts) Compute the transfer function, H(s), and the region on convergence.
(ii) (8 pts) Then, consider the following feedback structure:



For what values of A is the system stable?
b.) (8 pts) Consider a linear time-invariant system with impulse response

$$h(t) = e^{-3t} \sin 4t \, u(t)$$

What is the region of convergence?

c.) (8 pts) Compute the Laplace transform of

$$x(t) = e^{-t} \frac{d}{dt}\left(e^{-(t+1)}u(t+1)\right)$$

d.) (5 pts) Consider the following transfer function for a causal LTI system.

$$H(s) = \frac{-3s^2 + 2}{s^3 + s^2 + 3s + 2}$$

Compute (i) h(t) at t=0 and (ii) h(t) as t → ∞.

Problem #4 (10 points)

Consider the following transfer function, H(z), for a discrete-time LTI system:

$$H(z) = \frac{z^2}{z^2 + \frac{8}{3}z - 1}$$

Draw the three possible regions of convergence. *Please make three separate plots.*

Extra Credit (10 points)

Assume that the Fourier transform of h(t) exists and that its Laplace transform is rational. If H(s) is known to have a pole at s=1/2, *could* h(t) be

a.) a left-sided signal?

b.) a right-sided signal?

c.) a two-sided signal?

Please give a reason with your answer *for each part*.