MATH 426 HW2

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1 1.2.2(a)

1.2.6: $k_f(x) = |\frac{xf'(x)}{f(x)}|$, 1.2.9: $k_h(x) = k_f(g(x)) * k_g(x)$

$f(x) = \sqrt{x+5} = g(h(x))$ where $g(x) = \sqrt{x}$ and $h(x) = x+5$, $g'(x) = \frac{1}{2\sqrt{x}}$ and $h'(x) = 1$

$k_f(x) = k_g(h(x)) * k_h(x)$ (from 1.2.9)

$= |\frac{h(x)*g'(x)}{g(h(x))}| * |\frac{x*h'(x)}{h(x)}|$

$= |\frac{(x+5)*\frac{1}{2\sqrt{x+5}}}{\sqrt{x+5}}| * |\frac{x}{x+5}|$

$= \frac{1}{2} * |\frac{x}{x+5}|$

$k_f(x) = |\frac{xf'(x)}{f(x)}|$ (from 1.2.6) and $f(x) = \sqrt{x+5}$ and $h(x) = x+5$ and $g'(x) = \frac{1}{2\sqrt{x}}$

$|\frac{xf'(x)}{f(x)}| = |\frac{xg'(h(x))}{g(h(x))}| = |\frac{xf'(x)}{f(x)}| = |\frac{\frac{x}{2\sqrt{x+5}}}{\sqrt{x+5}}| = \frac{1}{2} * |\frac{x}{x+5}|$

2 1.2.2(b)

1.2.6: $k_f(x) = |\frac{xf'(x)}{f(x)}|$, 1.2.9: $k_h(x) = k_f(g(x)) * k_g(x)$

$f(x) = \cos(2\pi x) = g(h(x))$ where $g(x) = \cos(x)$ and $h(x) = 2\pi * x$, $g'(x) = -\sin(x)$ and $h'(x) = 2\pi$

$k_f(x) = k_g(h(x)) * k_h(x)$ (from 1.2.9)

$= |\frac{h(x)*g'(x)}{g(h(x))}| * |\frac{x*h'(x)}{h(x)}|$

$= |\frac{-2\pi x \sin(2\pi x)}{\cos(2\pi x)}| * 1$

$= 2\pi|x\tan(2\pi x)|$

$k_f(x) = |\frac{xf'(x)}{f(x)}|$ (from 1.2.6)

$f(x) = \cos(2\pi x)$ and $f'(x) = -2\pi \sin(2\pi x)$

$|\frac{xf'(x)}{f(x)}| = 2\pi|\tan(2\pi x)|$

3 1.2.2(c)

1.2.6: $k_f(x) = |\frac{xf'(x)}{f(x)}|$, 1.2.9: $k_h(x) = k_f(g(x)) * k_g(x)$
$f(x) = e^{-x^2} = g(h(x))$ where $g(x) = e^x$ and $h(x) = -x^2$, $g'(x) = -e^x$ and $h'(x) = -2x$

$k_f(x) = k_g(h(x)) * k_h(x)$ (from 1.2.9)

$= |\frac{h(x)*g'(h(x))}{g(h(x))}| * |\frac{xh'(x)}{h(x)}|$

$= |\frac{-x^2 * e^{-x^2}}{e^{-x^2}}| * |\frac{x(-2x)}{-x^2}|$

$= 2|x^2|$

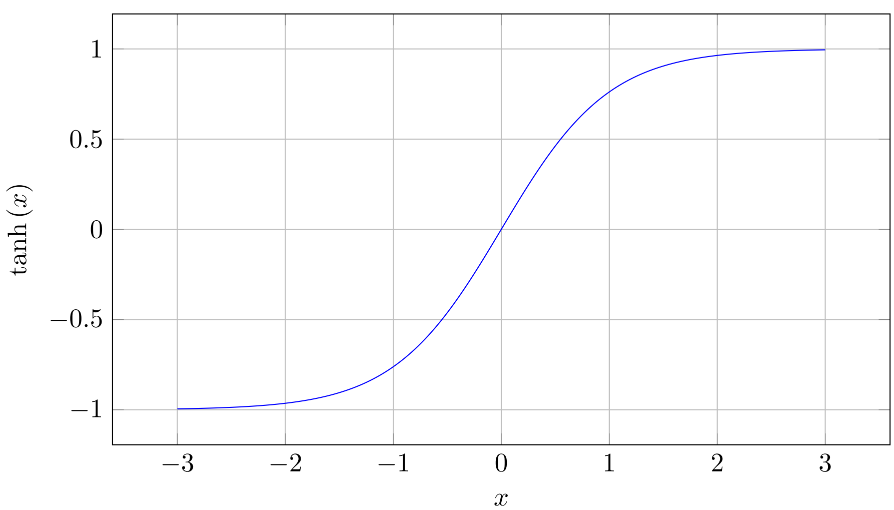
$k_f(x) = |\frac{xf'(x)}{f(x)}|$ (from 1.2.6)

$= |\frac{-2x^2 e^{-x^2}}{e^{-x^2}}|$

$= 2|x^2|$

4 1.2.3(a)

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$



As x gets large(∞), $\tanh(x)$ converges to 1 As x gets small($-\infty$), $\tanh(x)$ converges to -1

5 1.2.3(b)

$f(x) = \frac{e^x - 1}{x}$

$\lim_{x\to\infty} \frac{e^x - 1}{x}$

$= \lim_{x\to\infty} e^x$

$= \infty$

$\lim_{x \to -\infty} \frac{e^x - 1}{x}$

$= \lim_{x \to -\infty} e^x$

$= 0$
As x gets large(∞), $\frac{e^x-1}{x}$ converges to ∞. As x gets small($-\infty$), $\frac{e^x-1}{x}$ converges to 0

6 1.2.3(c)

$f(x) = \frac{1-\cos(x)}{x}$

$\lim_{x \to \infty} \frac{1-\cos(x)}{x}$

$= \lim_{x \to 0} \sin(x)$

$= 0$

As x gets small(0), $\frac{1-\cos(x)}{x}$ converges to 0

As x gets large(∞), $\frac{1-\cos(x)}{x}$ converges to ∞.

7 1.3.5(a)

$k_f(x) = |\frac{xf'(x)}{f(x)}|$ (from 1.2.6)

$f(x) = \frac{e^x-1}{x}$

$k_f(x) = |\frac{xe^x+1-e^x}{e^x-}|$

The maximum value for the condition number over the range (-1, 1) occurs at x = 1. The value at this point is $k_f(x) = 0.58$

8 1.3.5(b)

Done in Matlab

9 1.3.5(c)

Done in Matlab

10 1.3.5(d)

Done in Matlab