MEEG333 FLUIDS LABORATORY

X3. Free Jet Flow from an Orifice

Objectives

Test validity of Bernoulli's equation for flow from a tank through an orifice Determine a correction factor to account for non-ideal effects Design Objective: Apply results to a practical problem.

Apparatus

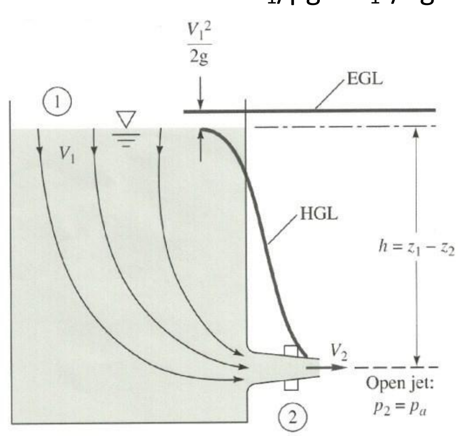
The apparatus consists of a tank of water whose depth can be controlled, with an orifice on the side near the bottom from which water will jet out. The actual jet flow will be measured by two different methods and compared to prediction from the Bernoulli equation. A correction factor, commonly called the Discharge Coefficient will be determined.

Theory

Measuring a fluid flow rate by measuring the pressure loss through an orifice or similar device is easily implemented , low cost methodology. However, calibration is required for accuracy. Because of its common use, orifices in pipes have been studied extensively, and we did also in X2. In this experiment, we are looking at an orifice where the upstream flow is from a relatively large tank, and discharges into the atmosphere. Example 3.13 in the textbook (7th ed) is the situation that we will evaluate.

Bernoulli's equation is given by (see textbook Section 3.5, eqn 3.54):

$$P_1/\rho g + V_1^2/2g + z_1 = P_2/\rho g + V_2^2/2g + z_2$$



V_1 and V_2 are assumed to be an average velocity across the flow cross section, such that the conservation of flow equation is satisfied:

$$\text{Flow } Q\ (m^3/sec) = A_1V_1 = A_2V_2 \qquad (1)$$

Where A_2 is the orifice area and A_1 is the tank cross section area.
Applying Bernoulli's equation to this situation, we assume $p_1 = p_2 = p_{atm}$ so that the pressure terms cancel.

In addition, if $A_2 << A_1$, (big tank, small orifice so $V_1 \sim 0$) Bernoulli's equation simplifies to

$$V_2 = (2gh)^{1/2} \qquad \text{(Torricelli equation)}$$

However, the flow through the orifice is not uniform, in fact it contracts slightly from both flow pattern effects and friction, so that that the actual exit flow area is slightly less than the orifice area A_2, and consequently the area to be used for A_2 is different. This effect is can be corrected by applying a coefficient of velocity C_v such that

$$V_2 = C_v\,(2gh)^{1/2} \qquad (2)$$

1. Measuring C_v

The value of C_v generally cannot be derived theoretically and must be measured. One method is as follows.

From dynamics, we know know that the trajectory of a ball tossed horizontally is given by

$$x= v*t \qquad (3)$$

$$y= gt^2/2 \qquad (4)$$

And eliminating t from (3) & (4) gives the theoretical trajectory:

$$y= -(1/2)(g/v^2)x^2$$

Substituting (2) into (3), and combining with (4) to eliminate t, we get

$$x = 2\,C_v\,(yh)^{1/2}$$

This expression estimates the coordinates of the jet stream trajectory, with x the horizontal distance and y the drop, correcting for the flow area.

Taking data for (x,y) by observation of the jet stream, and plotting x vs $2(yh)^{1/2}$, the slope of the resulting line will approximate C_v. Note: $2(yh)^{1/2}$ is the abscissa and x is the ordinate. Logically, C_v must be less than 1. If it is not, then you probably did not use "x" as the ordinate.

But, correcting for the vena contracta will still not match theory with actual flows because losses from friction between fluid and edges are present.

2. Measuring C_d

The way out of this conumdrum is the common practice of combining both effects into one number called the Discharge Coefficient C_d.

$$Q = C_d A_2 (2gh)^{1/2} \qquad (6)$$

Q = the actual volumetric flow rate in m^3/sec. This will generally be a more reliable relationship. Tables of Discharge Coefficients for different types and sizes of orifices are widely available. However, C_d can be measured directly for a given orifice by taking data for Q and h. As before a plot of Q (as ordinate) vs $A_2(2gh)^{1/2}$ should be linear with a slope of C_d. The value will be approximately the same in other systems with similar orifice geometry. C_d must logically be less than 1, and less than C_v since it includes both the vena contracta and friction effects.

3. Time to empty a tank

Calculating the time required to empty a tank, either by intent or by accident, is a common practical problem. In this situation, the fluid tank level is continuously decreasing, in other words the flow is not steady state as assumed in the Bernoulli equation.

For a transient case, conservation of mass in the tank leads to the following differential equation (A_1 is the cross section area of the tank). Note: Area $A_1 = 1.832\text{x}10^{-2}\ m^2$

$$Q\,dt = -A_1\,dh$$

Integrate this first order ordinary differential equation by separation of variables, assuming that A_1 is constant, and show that the time to drain a tank between two levels (h_1 and h_2).

$$t = (1/C_d)(2/g)^{1/2}(d_1/d_2)^2\,(h_1^{1/2} - h_2^{1/2}) \quad \text{seconds}$$

Experimental Procedure

The equipment has the following dimensions for use in the appropriate calculations.

Diameter of small orifice	.003 m
Diameter of large orifice	.006 m
Surface area of reservoir	$1.832\text{x}10^{-2}\ m^2$

(be sure you know which orifice you are using)

1. Measurement of C_v

Position the reservoir across the channel on the top of the hydraulic bench and level the reservoir by the adjustable feet using a spirit level on the base. Note the orifice size provided (different on each bench). Take care that O-ring seal is not lost. Connect the reservoir inflow tube to the bench flow connector. Position the overflow connecting tube so that it will discharge into the volumetric tank, and is out of the way of the jet trajectory. Turn on the pump and open the bench valve gradually. As the water level rises in the reservoir towards the top of the overflow tube, adjust the bench valve to give a water level 2 to 3 mm above the overflow level. This will insure a constant head and produce a steady flow through the orifice.

Note the value of the head, h. The jet trajectory is obtained by using the needles mounted on the vertical backboard to follow the profile of the jet. Adjust each needle so that its point is just above the jet. Attach a sheet of paper to the back-board between the needle and the board and secure it in place with the clamp provided. Mark the paper to show points in the trajectory.

The x-coordinates of the needles are approximately as shown in the data table below, measured from the orifice, which is x=0.

Needle	Orifice d	**h(constant)**	x(cm)	2(yh)^1/2	
1	Record!		4.5		
2			9.5		
3			14.5		
4			19.5		
5			24.5		
6			29.5		
7			34.5		
8			39.5		

Slope of x vs 2(yh)^1/2 = C_v

2. Measurement of C_d

For this part, data will be taken to calculate C_d using Equation(6).

Measure the flow rate by timed collection, using the measuring cylinder provided. Adjust the water level h to five different positions, from highest to low.

Run	Orifice d	**h(varies)**	Volume	Time (sec)	Flow (m^3/sec

Slope of Flow vs $A_2(2gh)^{1/2}$ = C_d

3. Measurement of time to empty a Tank

The overflow pipe is raised to obtain the maximum h, the header tank is filled to just below the top, the bench flow control valve closed and the pump stopped. Start a stopwatch when the level reaches the first convenient scale mark. You will need to take readings of the falling head (h) at 5 second intervals. The easiest way to do this accurately is to hold a piece of paper immediately adjacent to the scale on the reservoir and at 5 second intervals mark the position of the falling level. (You can make a mark quicker than you can read and write a number down). At the end of this procedure, you can then read off the head position marks corresponding to the known times.

Analysis

1. First, use the big sheet of paper on which you recorded the x & y coordinates of the stream. With a straight edge, add axes for convenience. On the same sheet of paper, plot the theoretical curve for the stream (Simplified Bernoulli eqn, or same as a ball tossed horizontally assuming no air friction).
2. Using computer and normal paper, plot x vs $2(yh)^{1/2}$. The slope should estimate C_v for the flow conditions. Calculate the average value.
3. For the value of h used in part 1 above plot y vs x using this C_v on the big sheet of paper to compare to the theoretical Bernoulli expression.
4. Reduce and plot (normal paper) your data for Flow rate vs $A_o(2gh)^{1/2}$. The curve should be linear with a slope of C_d. Calculate a least squares fit to the data to obtain a statistically valid estimate of the slope.

5. Similar to step 3 above, plot y vs x using this C_d on the big sheet of paper to compare to the theoretical Bernoulli expression, the C_v correction, and your plotted data.

6. Plot the transient data (normal paper) from draining the tank. Show the derivation of equation (7). Using your measured discharge coefficient C_d, calculate from the derived transient equation the corresponding curve of h vs t.

7. Use your data to answer the Design Objective problem posed below. Report the result as your Summary in the lab report.

Uncertainty Analysis

There are a lot of opportunities for measurement uncertainty In this experiment. The final result was to obtain a C_d correction such that future calculations matched your data.

Make a chart with a 45 degree line, plotting your measurements versus the points using your C_d value. Calculate the Root Mean Square uncertainty.

In the Discussion and Conclusions you can describe the several measurements that contributed to observed uncertainties.

Design Objective.

You are part of the engineering staff of a large chemical processing plant. A noxious petrochemical nitromethane, used for certain race car fuels, is stored in a large cylindrical tank, 10 meters tall and 6 meters in diameter. A leak from this tank would not be good, so an earthen berm is built around it to catch small spills, but it is only big enough to hold one half the contents of a full tank. This is not safe. The neighbors are concerned. You are asked to help design an emergency pump and piping system that would kick in and rapidly transfer material out of the berm before it overflowed, and into another tank.

Your task is to (1)calculate the time it would take to fill the berm (remember: berm capacity =1/2 tank capacity) if a .3m (12-inch) diameter leak began at the bottom (approx the size of a flanged inspection port that could pop off). Assume the tank is initially full, and that the material properties approximate water. In addition, (2) calculate the flow capacity required of a pump that would keep ahead of the spill before the berm filled (Hint: what is the maximum flow rate from the leak?) Report your findings by letter to plant owners and recommendations (running for high ground is not a valid recommendation).

(2)If you were doing this experiment for a serious issue as suggested by the Design Objective, what would you tell the plant management to plan for? That is, should they buy a pump 20% larger than you calculate? 100% larger? Close to your calculated value? Say why.

Letter:

Manager
Funny Car Fuels Refining and Manufacturing
Nowhere, DE

Summarize your findings and recommendations for emergency backup pump.

Sincerely