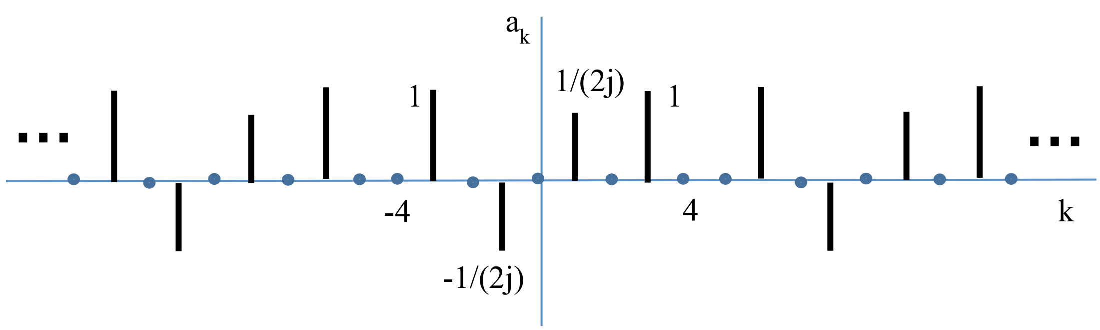
SAMPLE EXAM #2A
ELEG 305 SIGNALS AND SYSTEMS
SPRING 2019

Note: Last year, the second exam included Chapter 5 and so there are some problems that will not be relevant for this year's Exam #2 which will only cover Chapters 3 and 4. I have marked the problems that cover material that will not be on Exam #2 using # signs right next to the problem.

Problem #1 (20 points)

Consider a *periodic* time-domain signal (with period 9), with Fourier series coefficients



a.) (2 pts) Are these Fourier series coefficients representative of a continuous- or discrete-time signal? Explain your choice.
b.) (2 pts) Is the time-domain signal real? Please explain your answer.
c.) (4 pts) Compute the average power in the signal.
d.) (8 pts) Derive the periodic, time-domain, signal from the Fourier series coefficients.
e.) (4 pts) Assume that the signal is passed through an ideal lowpass filter with cutoff frequency $\omega_c = 2\omega_0$. What is the resulting filtered time-domain signal?

Problem #2 (20 points)

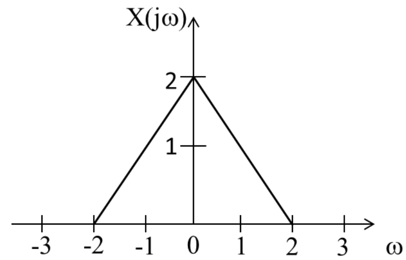
a.) ##### (4 pts) Consider a discrete-time signal given by

$$x[n] = \begin{cases} n^2, & -2 \le n \le 2 \\ 0, & otherwise \end{cases}$$

Compute the following integral where the Fourier transform of x[n] is $X(e^{j\omega})$

$$\int_{-\pi}^{\pi} \left| X(e^{j\omega}) \right|^2 d\omega$$

b.) (6 pts) Consider a continuous-time signal with the frequency characteristic, $X(j\omega)$, below



Compute and draw the frequency characteristic, $Y(j\omega)$, for the time-domain signal

$$y(t) = x(t)\cos\omega_0 t, \quad \omega_0 = 5\ radians/\sec$$

c.) (10 pts) Compute the Fourier transform of the following function (* denotes convolution)

$$x(t) = \frac{d}{dt}\left[\left(e^{-3t}u(t) * e^{-t}u(t-2)\right)\right]$$

Problem #3 (20 points)

a.) (6 pts) Consider a continuous-time, linear, time-invariant system with input

$$x(t) = e^{-3t}u(t)$$

and output

$$y(t) = e^{-3(t-2)}u(t-2)$$

Derive the system impulse response.

b.) ##### (8 pts) Consider a discrete-time, LTI system with frequency response

$$H(e^{j\omega}) = \frac{2 + \left(\frac{1}{4}\right)e^{-j\omega}}{\left(1 - \frac{1}{2}e^{-j\omega}\right)\left(1 + \frac{1}{4}e^{-j\omega}\right)}$$

Find the difference equation relating the input and output for this system.

c.) ##### (6 pts) Consider the following difference equation for a discrete-time, LTI system

$$y[n-2] + 6y[n] = 8x[n-1] + 18x[n]$$

Derive the frequency response for this system.

Problem #4 (20 points)

Consider the following differential equation that describes a linear time-invariant system

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 3\frac{dx(t)}{dt} + 8x(t)$$

a.) (10 pts) Derive the frequency response of this system, $H(j\omega)$.

b.) (10 pts) What is the corresponding impulse response, $h(t)$?

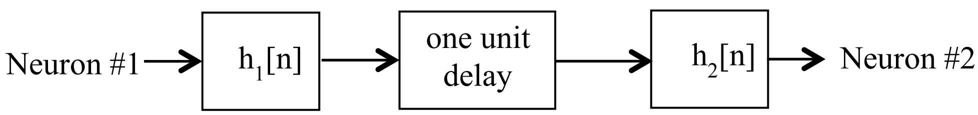
c.) (**Extra Credit** 4 pts) Is this system causal? Is this system stable? Please provide justifications for your answers.

Problem #5 (20 points) ##### (although a similar problem for continuous-time could be considered)

The nervous system of a chimpanzee (and all living things) is quite complicated. Here, we will consider a very simple model. We assume the system is *linear* and *time-invariant*. Moreover, we will focus on the fundamental neuron-neuron interaction across a synapse, modeled as a discrete-time system, as shown below. The model consists of three parts in cascade: The signal from the first neuron is filtered with impulse response $h_1[n]$, and then delayed through the synapse. At the next neuron, the signal is again filtered, this time using the impulse response $h_2[n]$. Assume that $h_1[n] = (1/2)^n u[n]$ and $h_2[n] = (1/4)^n u[n]$.

a.) (10 pts) Derive the <u>frequency response</u> for the end-to-end system.

b.) (10 pts) How does this system respond to an input that is an impulse?



c.) (**Extra Credit** 6 pts) How might you correct for the distortion caused by the overall frequency response?