Computational Fluid Dynamics (CFD) A Brief Overview

MEEG 332

February 25, 2019

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Navier-Stokes Equations

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Energy:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$
$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\,\tau_{xx} + v\,\tau_{xy} + w\,\tau_{xz}) + \frac{\partial}{\partial y}(u\,\tau_{xy} + v\,\tau_{yy} + w\,\tau_{yz}) + \frac{\partial}{\partial z}(u\,\tau_{xz} + v\,\tau_{yz} + w\,\tau_{zz})\right]$$

[NASA, 2015, https://www.grc.nasa.gov/www/k-12/airplane/nseqs.html]

Common Discretization Methods

- Finite Element
  - Mostly used for structural / solid mechanics
- Finite Difference
  - Based on Numerical Differentiation
    - Common & intuitive to implement
    - Can be numerically unstable
- Finite Volume
  - Calculates Fluxes
  - Based on Numerical Integration
    - Think control volume
    - Difficult to implement higher order schemes

Recall… Taylor Series

$$let\ f_i = f(x_i),\ f_{i+1} = f(x_{i+1})$$

$$f_{i+1} = \sum_{n=0}^{\infty} \frac{\partial^n f_i}{\partial x^n} \frac{\Delta x^n}{n!}$$

$$f_{i+1} = f_i + \frac{\partial f_i}{\partial x} \Delta x + \frac{\partial^2 f_i}{\partial x^2} \frac{\Delta x^2}{2!} + \frac{\partial^3 f_i}{\partial x^3} \frac{\Delta x^3}{3!} + \frac{\partial^4 f_i}{\partial x^4} \frac{\Delta x^4}{4!} + \cdots$$

[Kundu, Cohen, Dowling, Tryggvason, "Fluid Mechanics", 2016]

Forward Difference Example

$$f_{i+1} = \sum_{n=0}^{\infty} \frac{\partial^n f_i}{\partial x^n} \frac{\Delta x^n}{n!}$$

$$\frac{\partial f_i}{\partial x} = \frac{1}{\Delta x}\left[f_{i+1} - f_i - \frac{\partial^2 f_i}{\partial x^2}\frac{\Delta x^2}{2!} - \frac{\partial^3 f_i}{\partial x^3}\frac{\Delta x^3}{3!} - \frac{\partial^4 f_i}{\partial x^4}\frac{\Delta x^4}{4!} - \cdots\right]$$

$$\frac{\partial f_i}{\partial x} \approx \frac{f_{i+1} - f_i}{\Delta x}, \mathrm{Error} = \mathrm{O}(\Delta \mathrm{x})$$

[Kundu, Cohen, Dowling, Tryggvason, “Fluid Mechanics”, 2016]

Finite Difference Method (FDM) – Central Difference

$$\frac{\partial f_i}{\partial x} \approx \frac{f_{i+1}^n - f_{i-1}^n}{2\Delta x}, \mathrm{Error} = \mathrm{O}(\Delta \mathrm{x}^2)$$

$$\frac{\partial^2 f_i}{\partial x^2} \approx \frac{f_{i+1}^n - 2f_i^n + f_{i-1}^n}{\Delta x^2}, \mathrm{Error} = \mathrm{O}(\Delta \mathrm{x}^2)$$

$$\frac{\partial f_i}{\partial t} \approx \frac{f_i^{n+1} - f_i^n}{\Delta t}, \mathrm{Error} = \mathrm{O}(\Delta \mathrm{t})$$

[Kundu, Cohen, Dowling, Tryggvason, “Fluid Mechanics”, 2016]

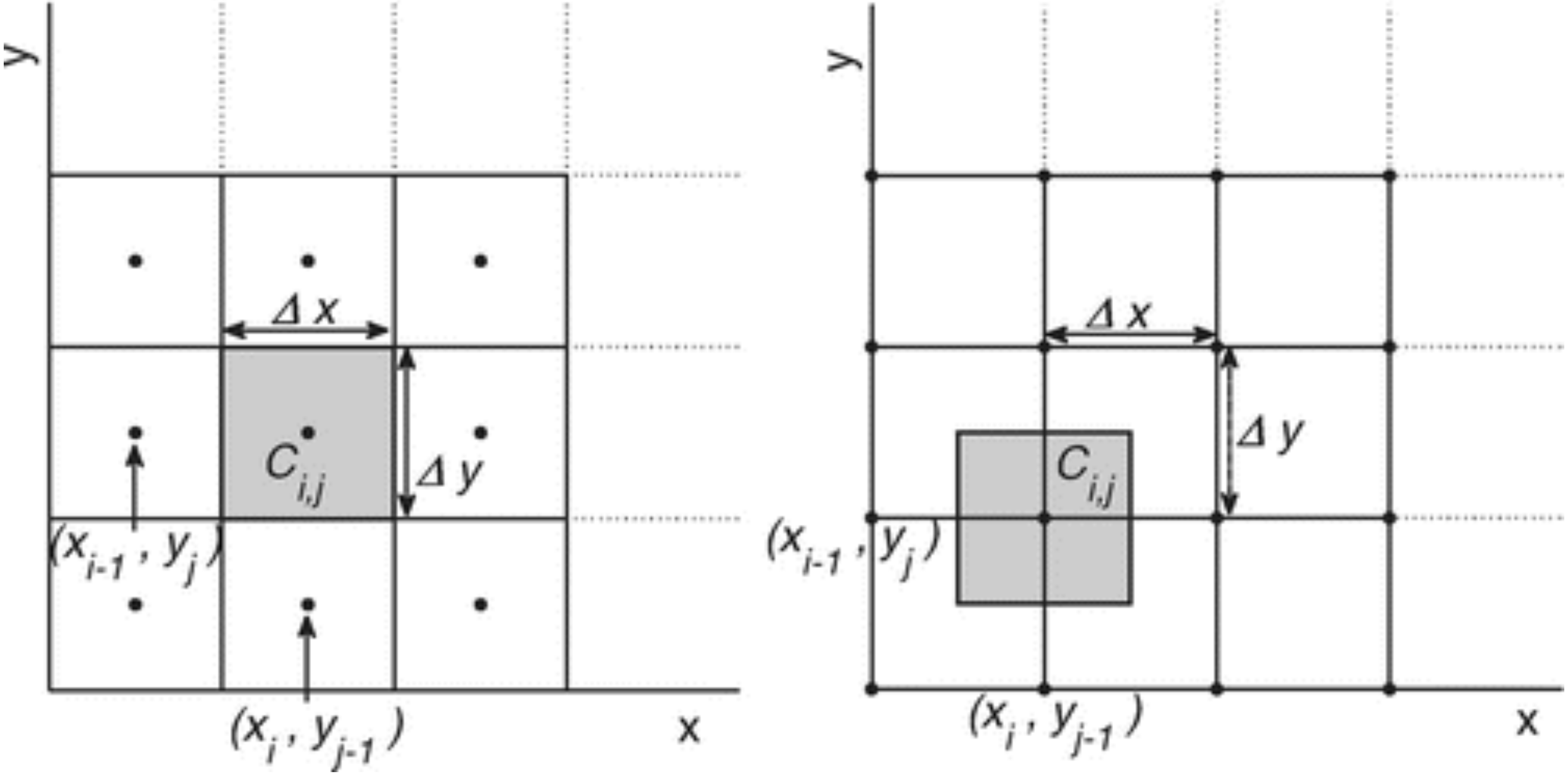
1D Advection-Diffusion Equation Example

$$\frac{\partial f_i}{\partial t} + U\frac{\partial f_i}{\partial x} = D\frac{\partial^2 f_i}{\partial x^2}$$

$$\frac{f_i^{n+1} - f_i^n}{\Delta t} + U\frac{f_{i+1}^n - f_{i-1}^n}{2\Delta x} = D\frac{f_{i+1}^n - 2f_i^n + f_{i-1}^n}{\Delta x^2}$$

[Kundu, Cohen, Dowling, Tryggvason, “Fluid Mechanics”, 2016]

Finite Volume vs Finite Difference



[ascelibrary.org, 2019]

Finite Volume Method – Numerical Integration

$$\frac{f_i^{n+1} - f_i^n}{\Delta t} = \frac{F_{in} - F_{out}}{\Delta x}$$

$$\frac{f_i^{n+1} - f_i^n}{\Delta t} = \frac{F_{i+1/2}^n - F_{i-1/2}^n}{\Delta x}$$

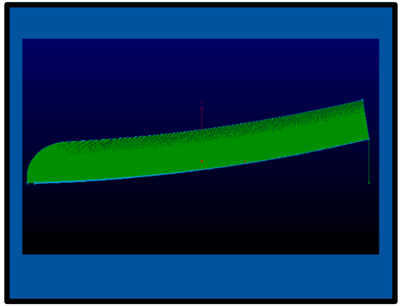
$$F_{i+1/2} = \frac{U}{2}(f_{i+1} + f_i) - D\left(\frac{f_{i+1} - f_i}{\Delta x}\right)$$

[Kundu, Cohen, Dowling, Tryggvason, “Fluid Mechanics”, 2016]

Overall CFD Process

CAD

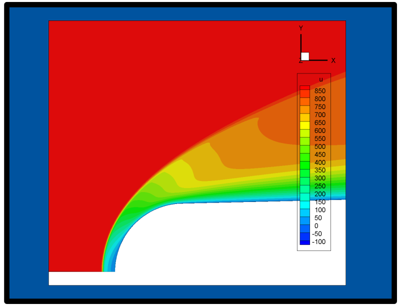
Mesh



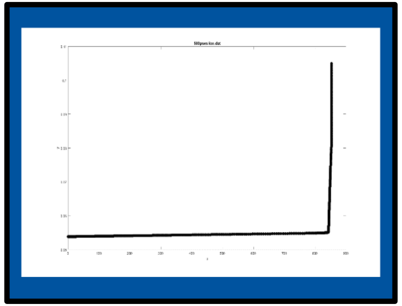
CFD Solver

```
[batistaa@login001 605k]$ tail -n 20 runflared3m1M.sh.o3452363
 604986   2.936121847E-06   3.105849856E-03   1.155971097E-08   1.000E+03
 604987   2.917241888E-06   3.105861416E-03   1.155971097E-08   1.000E+03
 604988   2.927833089E-06   3.105872976E-03   1.155971097E-08   1.000E+03
 604989   2.927572008E-06   3.105884535E-03   1.155971097E-08   1.000E+03
 604990   2.922228242E-06   3.105896095E-03   1.155971097E-08   1.000E+03
 604991   2.926211548E-06   3.105907655E-03   1.155971097E-08   1.000E+03
 604992   2.904072324E-06   3.105919214E-03   1.155971097E-08   1.000E+03
 604993   2.926039853E-06   3.105930774E-03   1.155971097E-08   1.000E+03
 604994   2.906479468E-06   3.105942334E-03   1.155971097E-08   1.000E+03
 604995   2.923306169E-06   3.105953894E-03   1.155971097E-08   1.000E+03
 604996   2.924719374E-06   3.105965453E-03   1.155971097E-08   1.000E+03
 604997   2.917505015E-06   3.105977013E-03   1.155971097E-08   1.000E+03
 604998   2.899340355E-06   3.105988573E-03   1.155971097E-08   1.000E+03
 604999   2.909199329E-06   3.106000132E-03   1.155971097E-08   1.000E+03
 605000   2.905225744E-06   3.106011692E-03   1.155971097E-08   1.000E+03
 == Writing HDF5 solution file to: /data/us3d/cone/flaredrough/3m1_25M/data.h5
 -- Time :    35.6890 mins
```

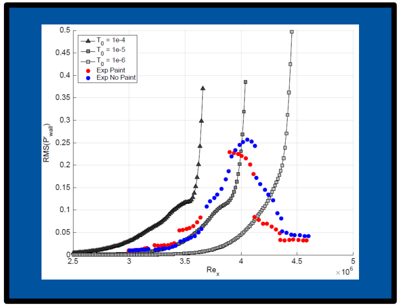
Visualize



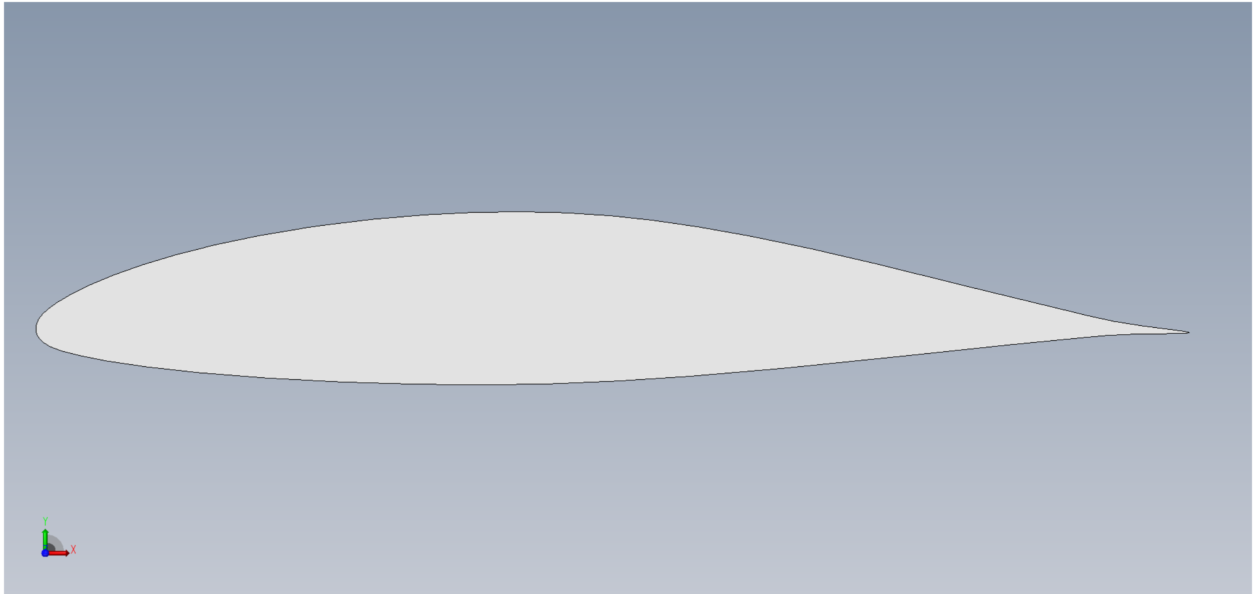
Data Extraction



Analysis

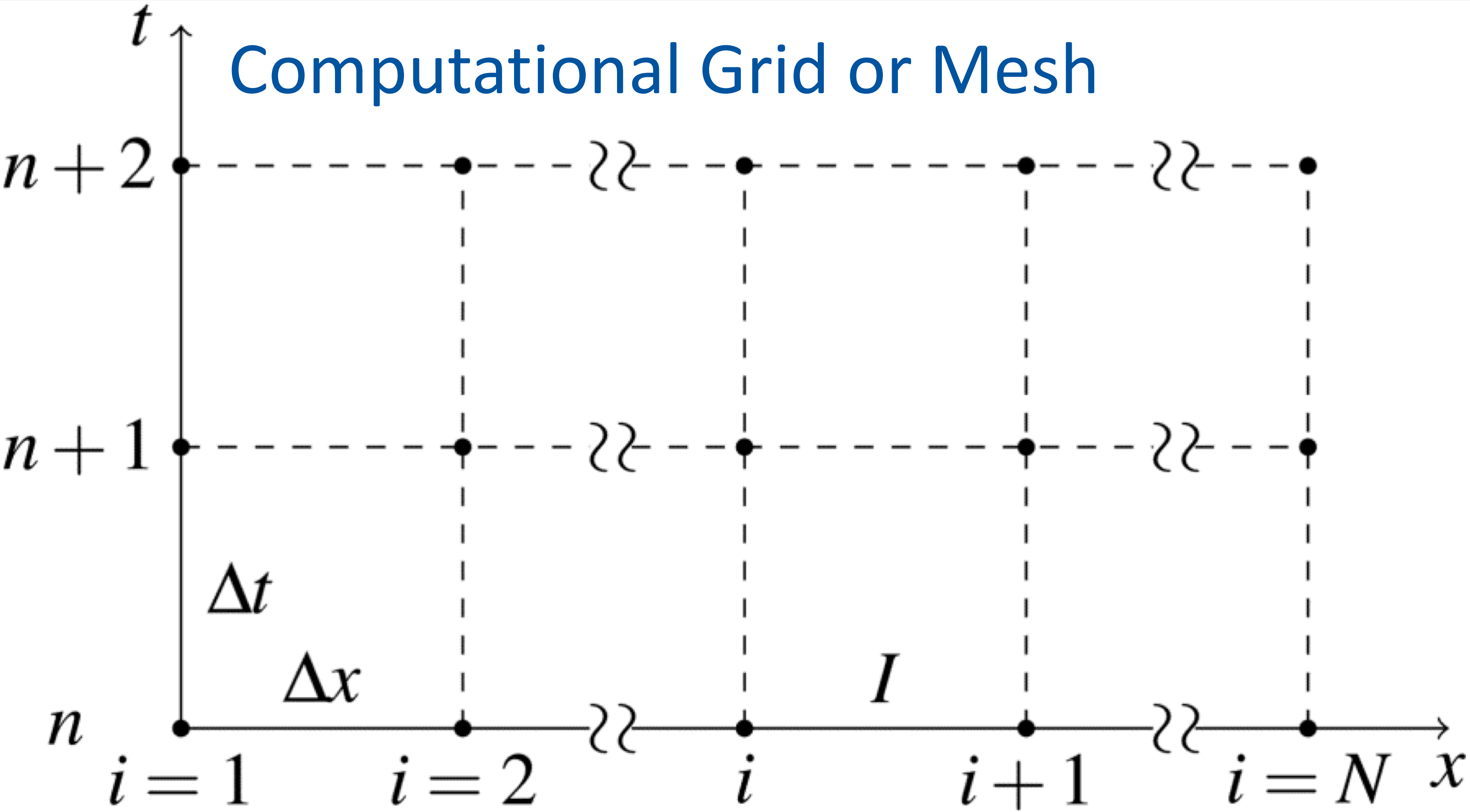


Solidworks CAD – NACA 65(2)-415 Airfoil



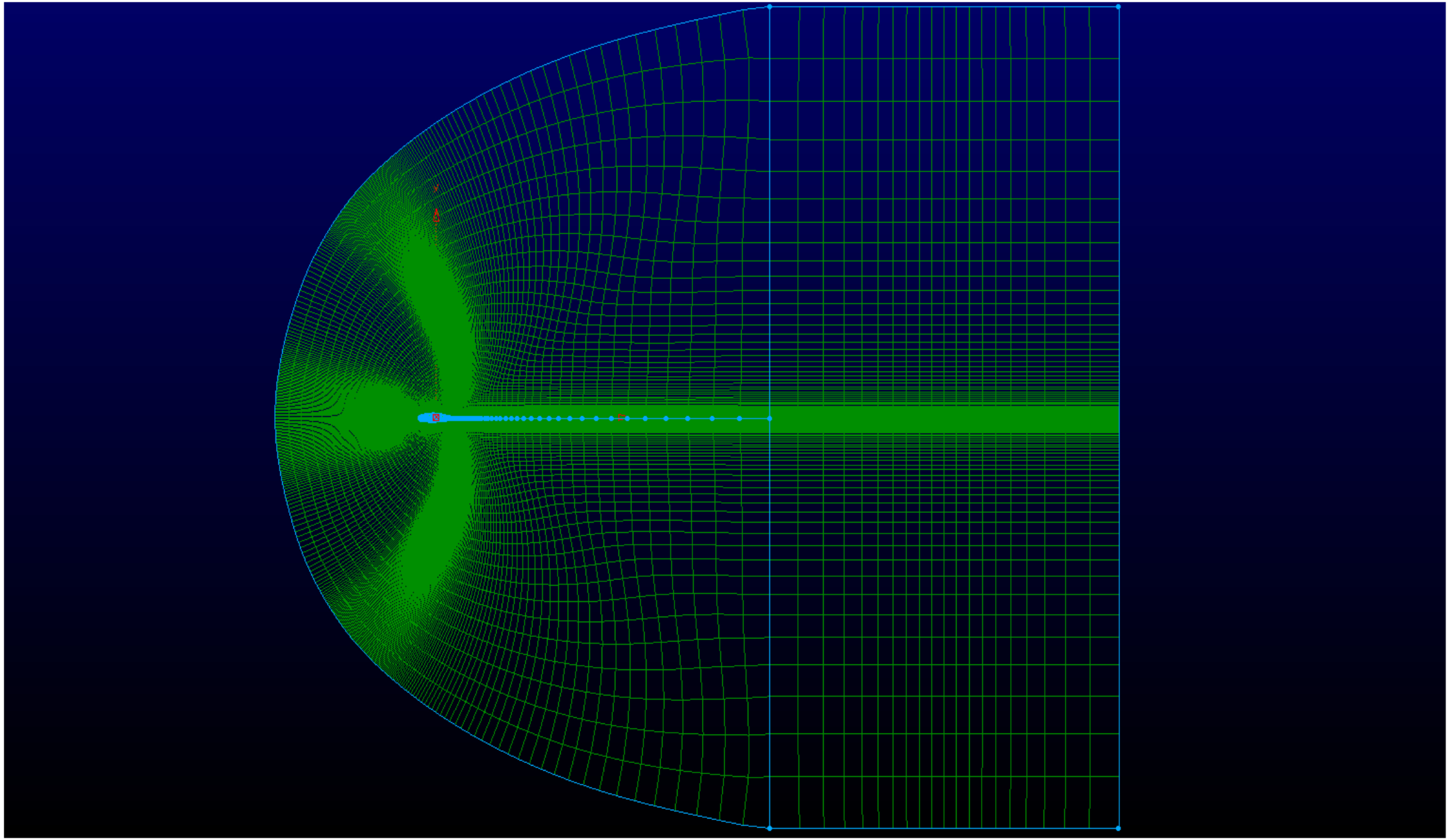
http://airfoiltools.com/airfoil/details?airfoil=naca652415a05-il

Computational Grid or Mesh

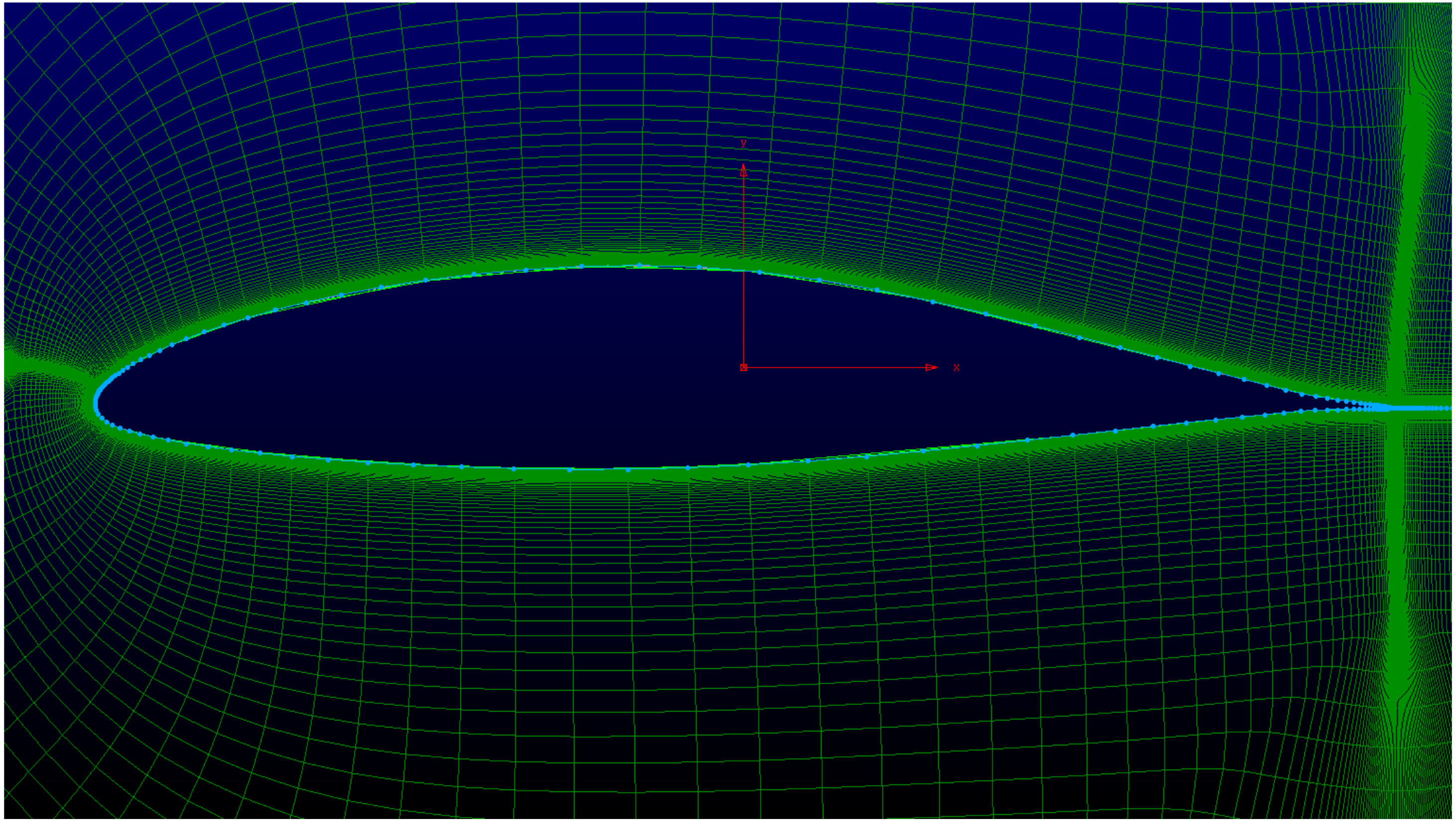


[Helqaker, Muller, Ytrehus. “Transient Flow in Natural Gas Pipelines Using Implicit Finite Difference Schemes”. ASME, 2014]

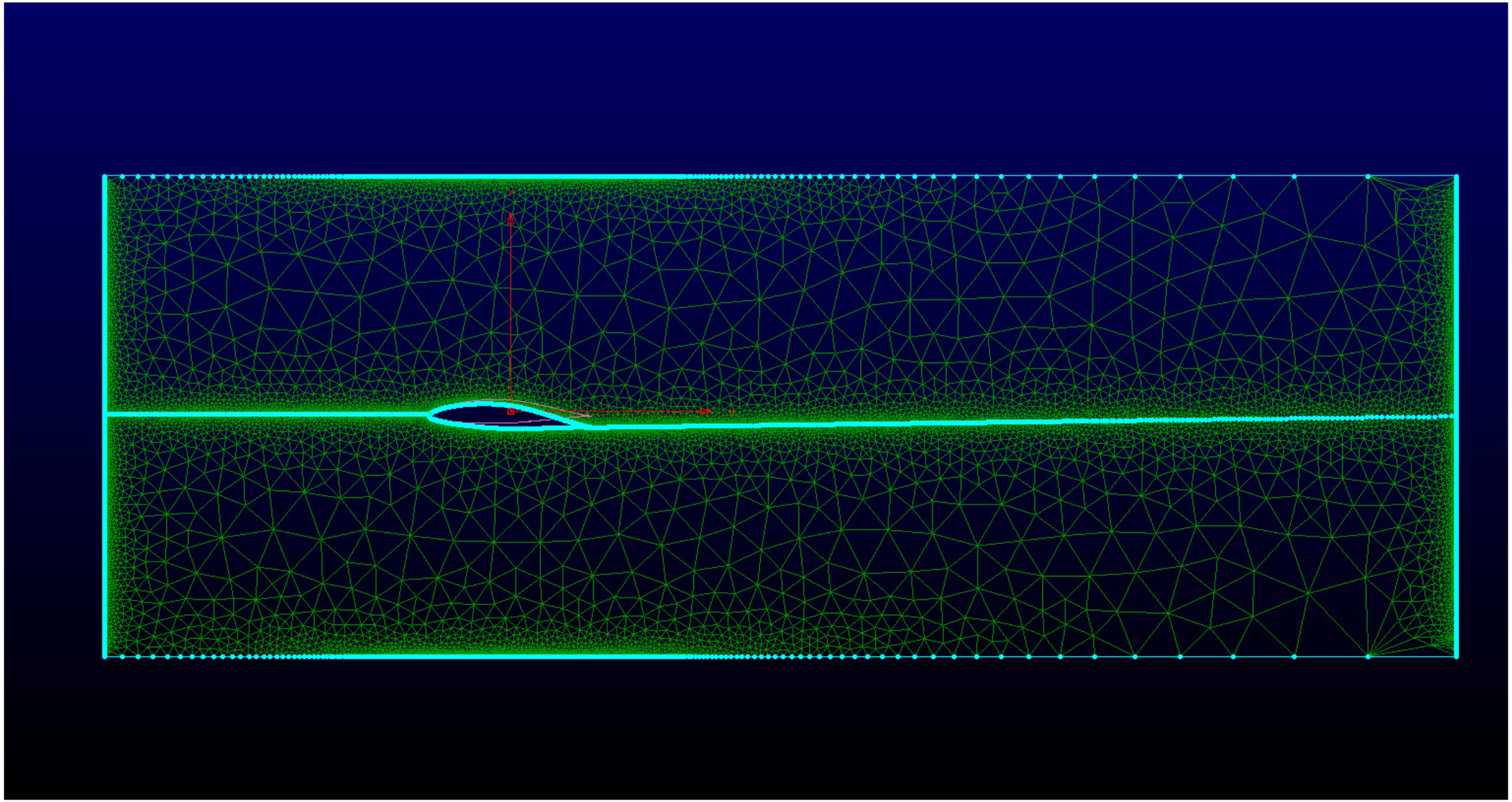
Airfoil C-Grid – Pointwise



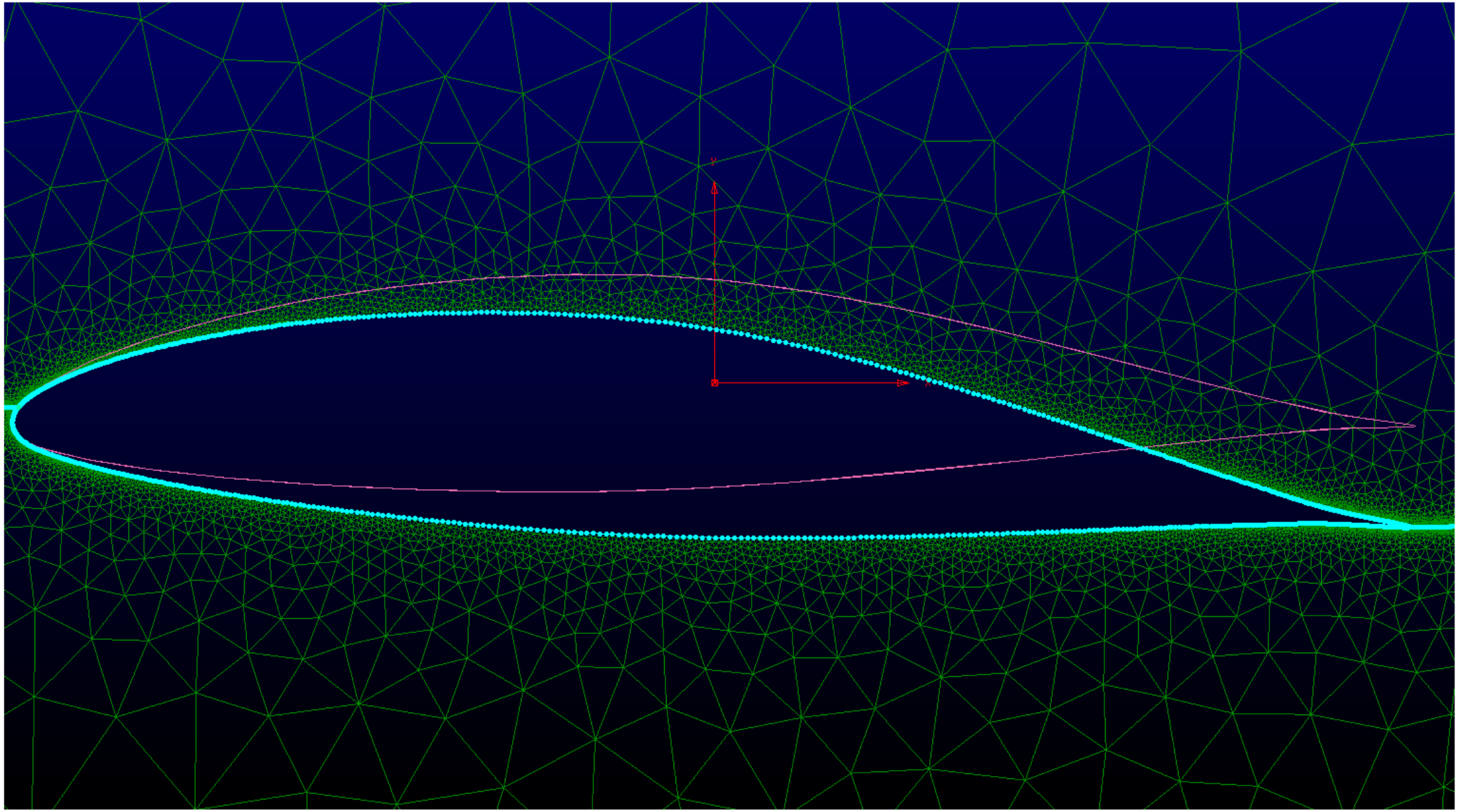
Structured Quadrilateral Mesh Wall Normal Extrusion



Airfoil in Wind Tunnel Mesh



Unstructured Triangular Mesh

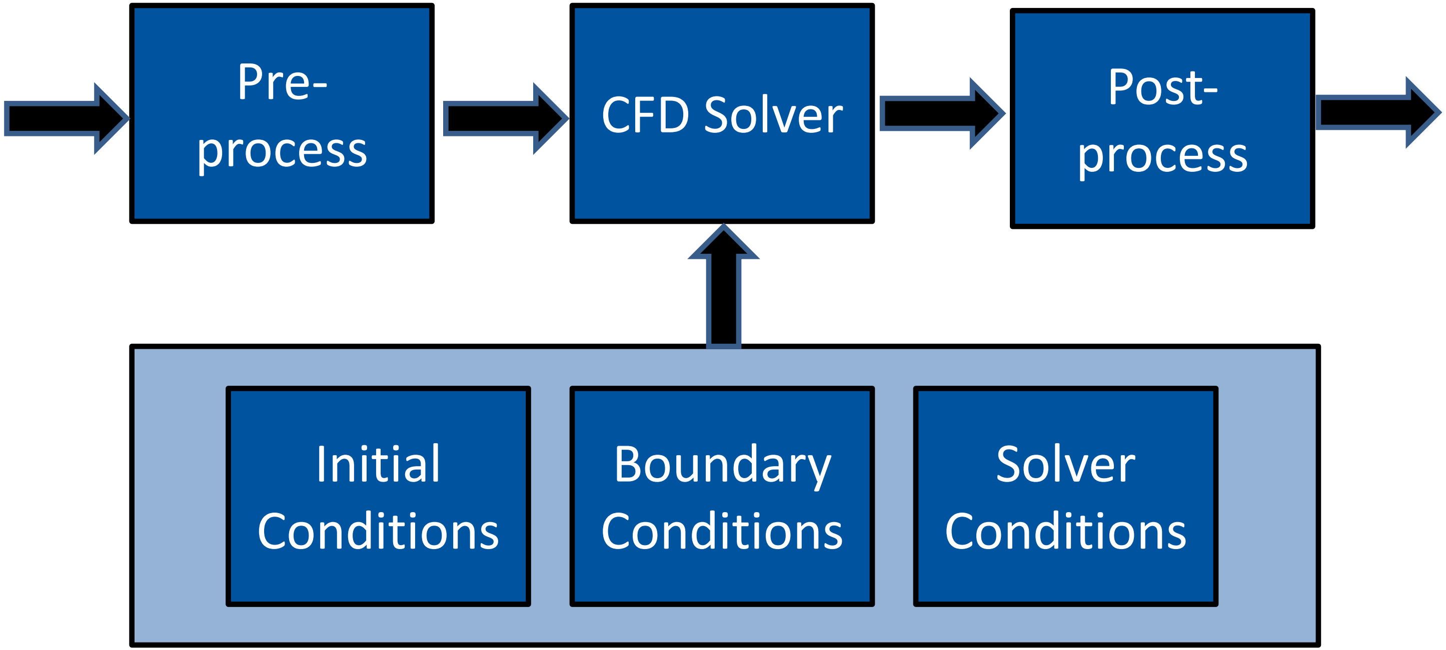


Parameters & Conditions

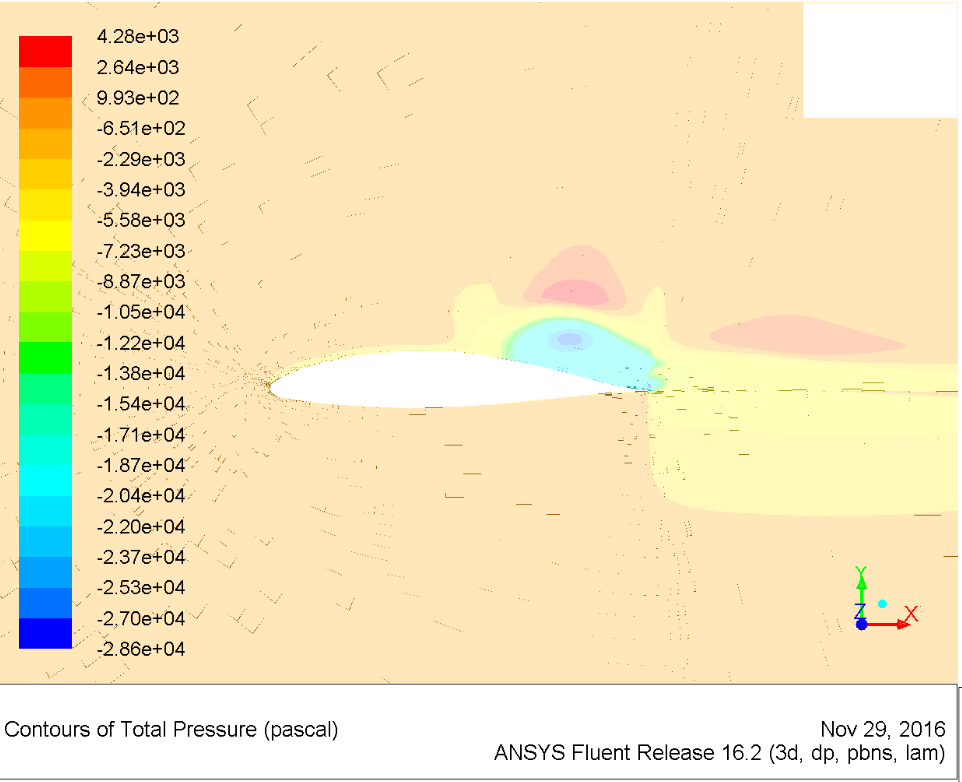
Mesh	Type	Shape	# Grid Points	# Test Article Wall Points Top/Bottom
Airfoil C-Grid	Structured	Quadrilaterls	126,390	100
Airfoil WT	Unstructured	Triangles	131,550	1000

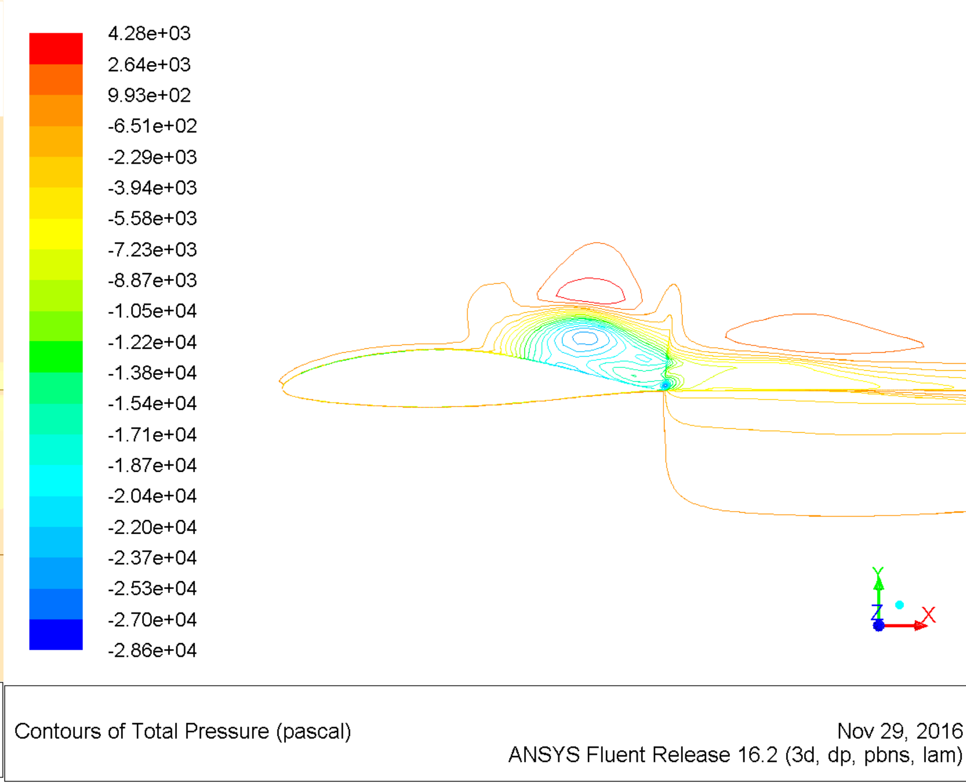
Geometry	ρ_{sea} [kg/m^3]	T [K]	ν [kg/m-s]	γ_{air}	solution scheme	Flow Type	Iterations	u [m/s]	v [m/s]	AoA [°]
Airfoil C-Grid	1.225	288.16	1.7894×10^{-5}	1.4	SimpleC	Laminar	10,000	124.6955	8.719559	4
Airfoil WT	1.225	288.16	1.7894×10^{-5}	1.4	SimpleC	Laminar	10,000	125	0	4

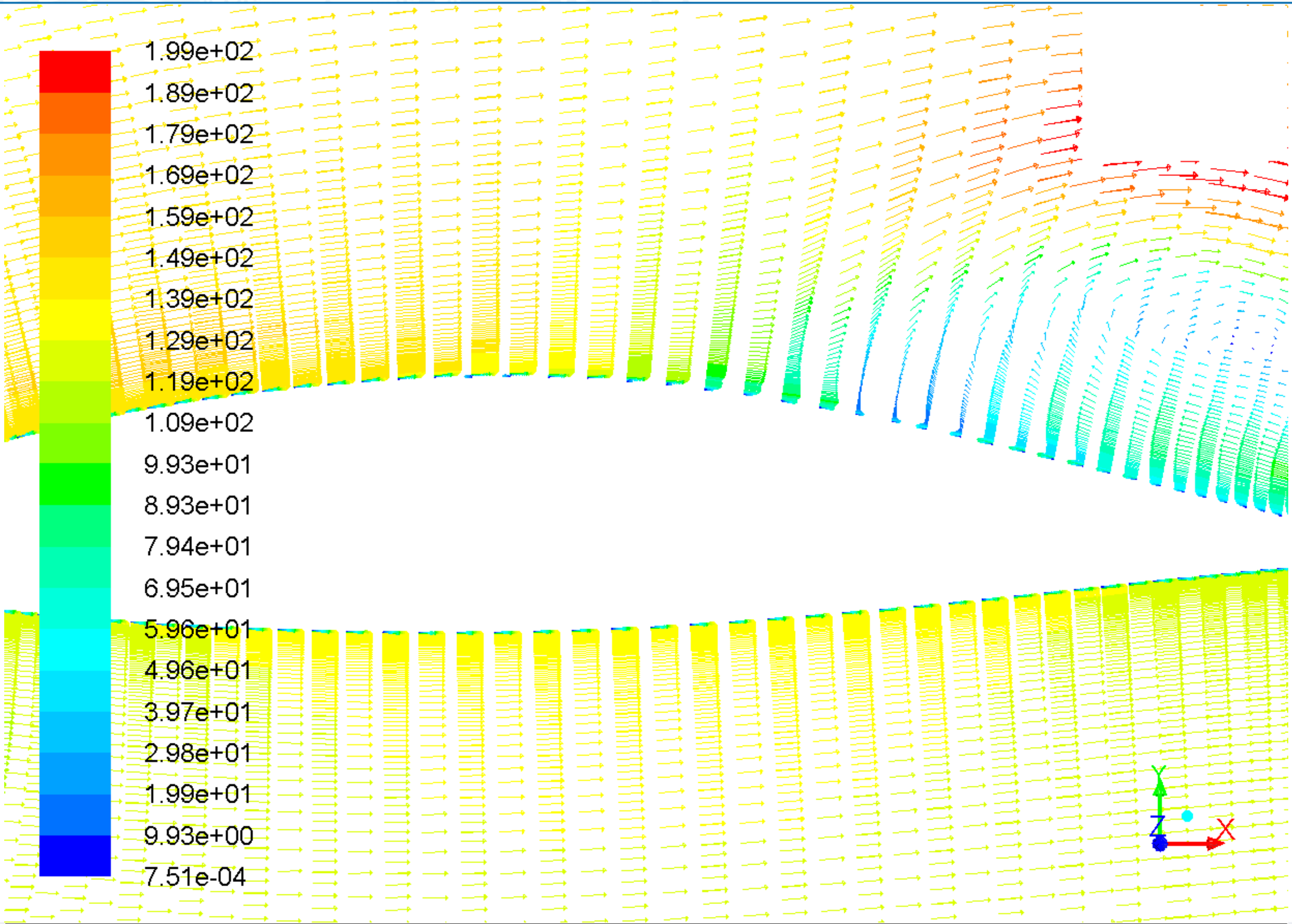
Solving Process – US3D, Fluent, ESI, etc…

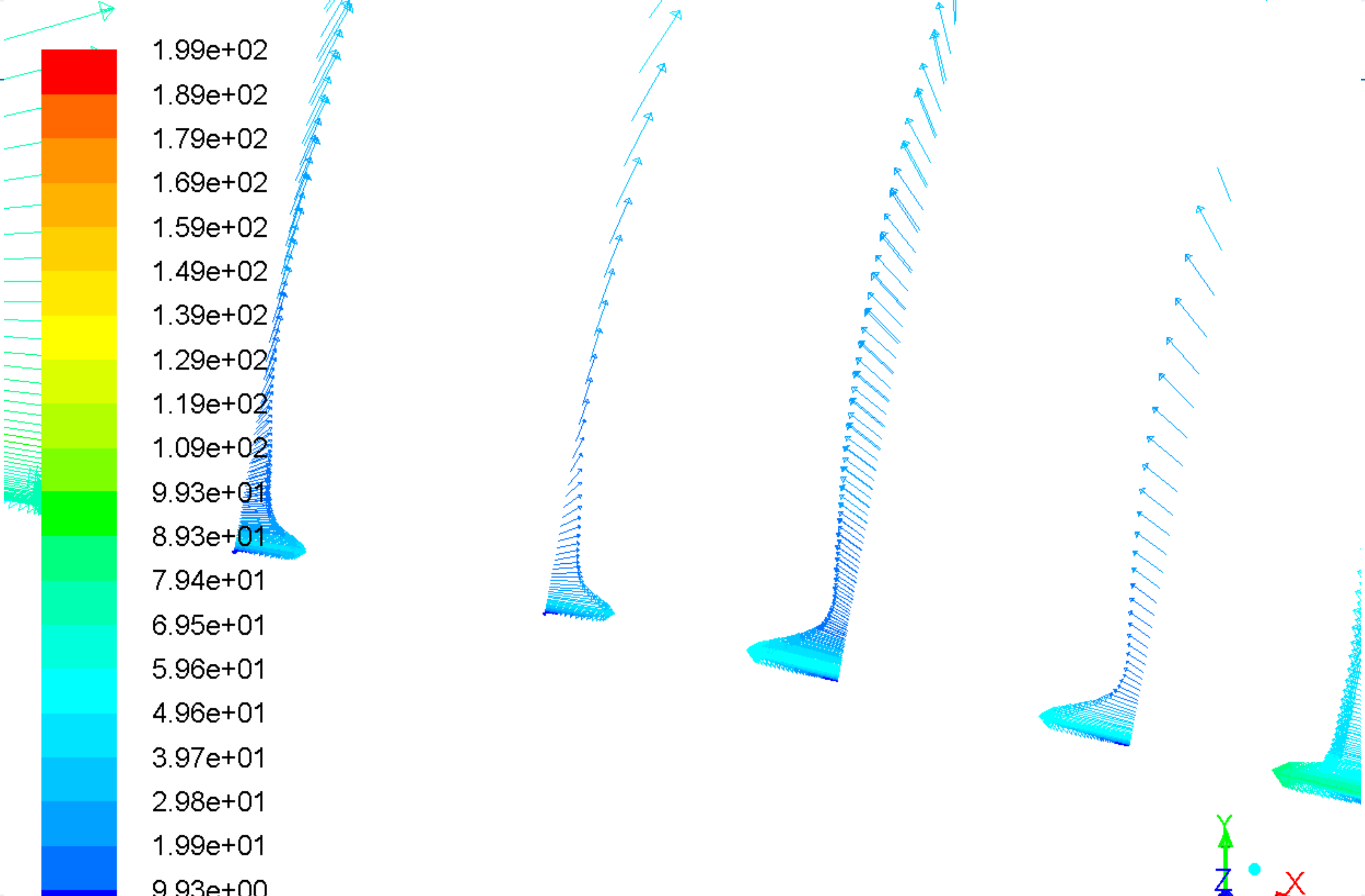


Visualization – Total Pressure Contour

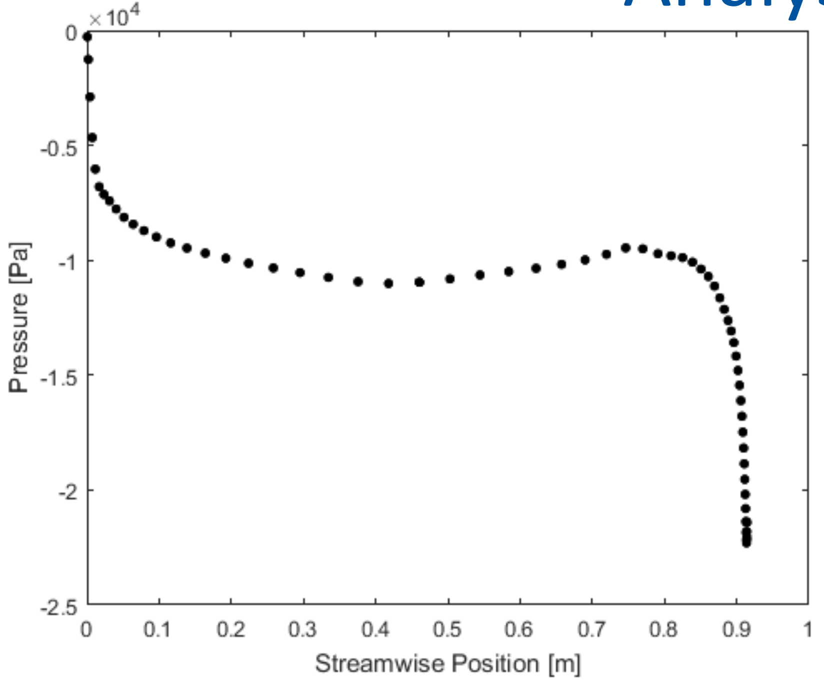


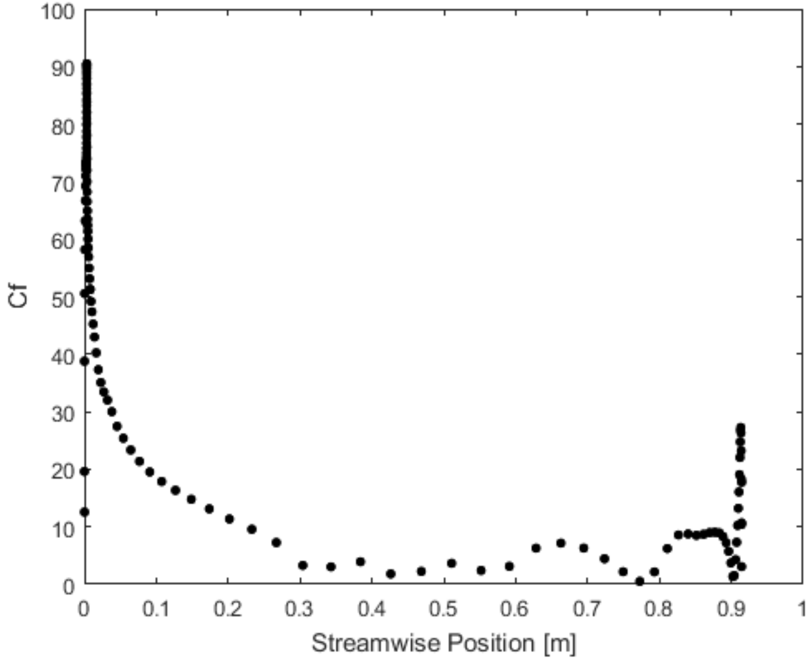



1.99e+02
1.89e+02
1.79e+02
1.69e+02
1.59e+02
1.49e+02
1.39e+02
1.29e+02
1.19e+02
1.09e+02
9.93e+01
8.93e+01
7.94e+01
6.95e+01
5.96e+01
4.96e+01
3.97e+01
2.98e+01
1.99e+01
9.93e+00
7.51e-04
Y
Z
X


1.99e+02
1.89e+02
1.79e+02
1.69e+02
1.59e+02
1.49e+02
1.39e+02
1.29e+02
1.19e+02
1.09e+02
9.93e+01
8.93e+01
7.94e+01
6.95e+01
5.96e+01
4.96e+01
3.97e+01
2.98e+01
1.99e+01
9.93e+00
Y
Z
X

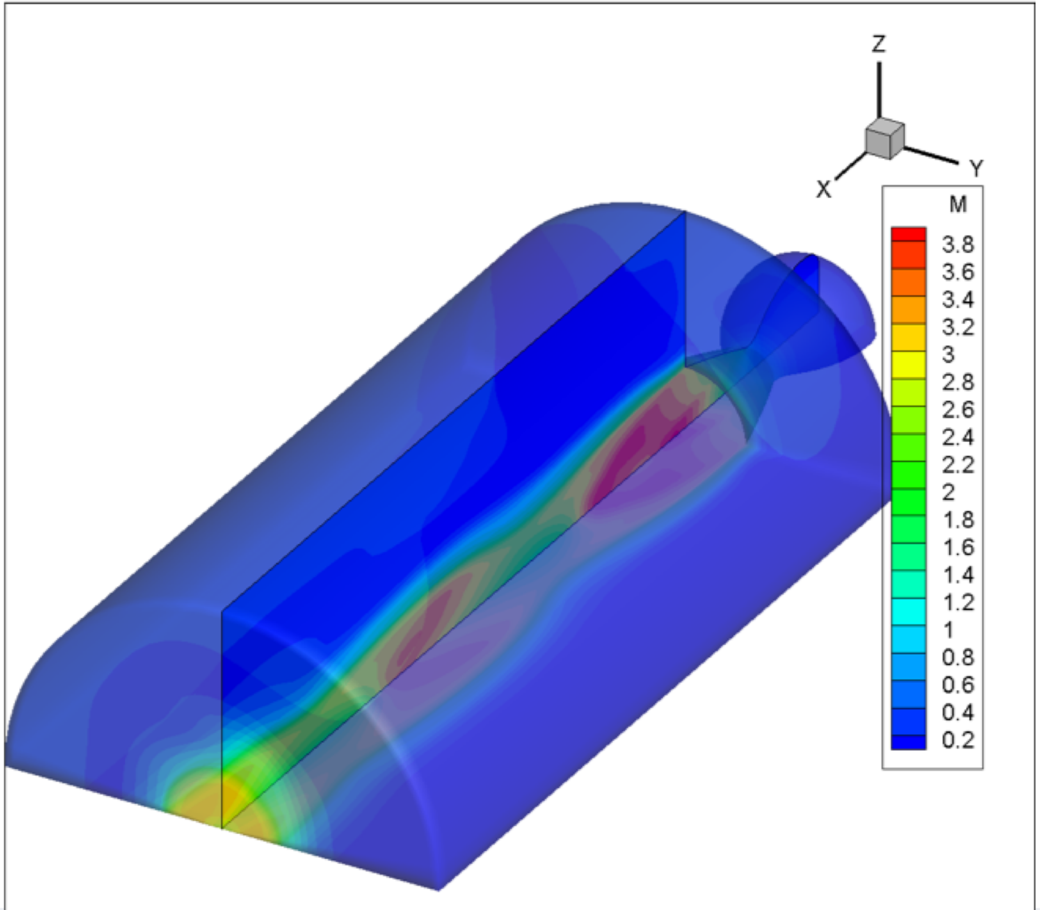
Analysis Results





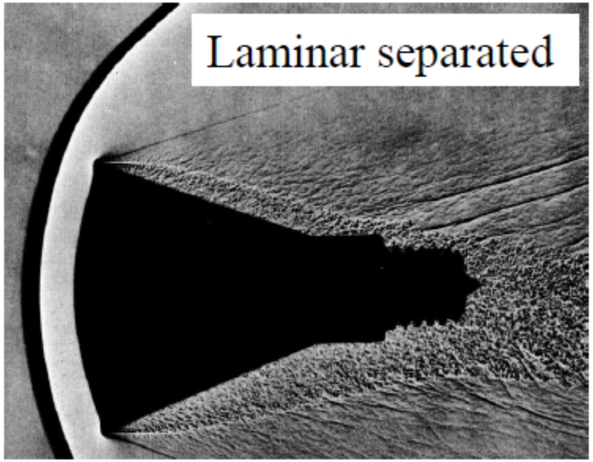
Test Case	Cl	Cd	Lift [N]	Drag [N]	%Error L	%Error D
C-Grid	0.776129	0.006835	13583.95	119.6225	2.46%	4.51%
Benchmark	0.7575	0.00654	13257.91	114.4643		

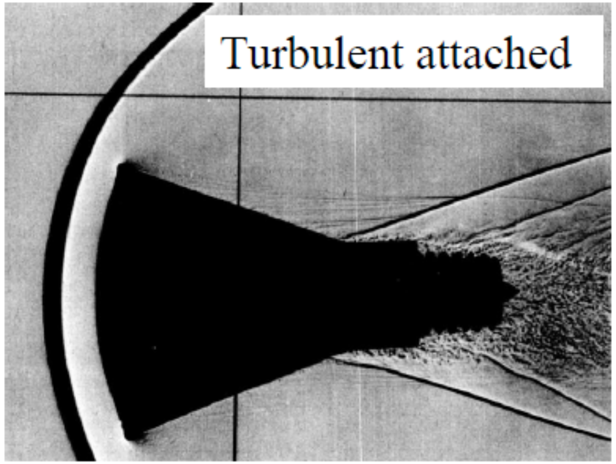
Questions So Far?



High-Speed CFD

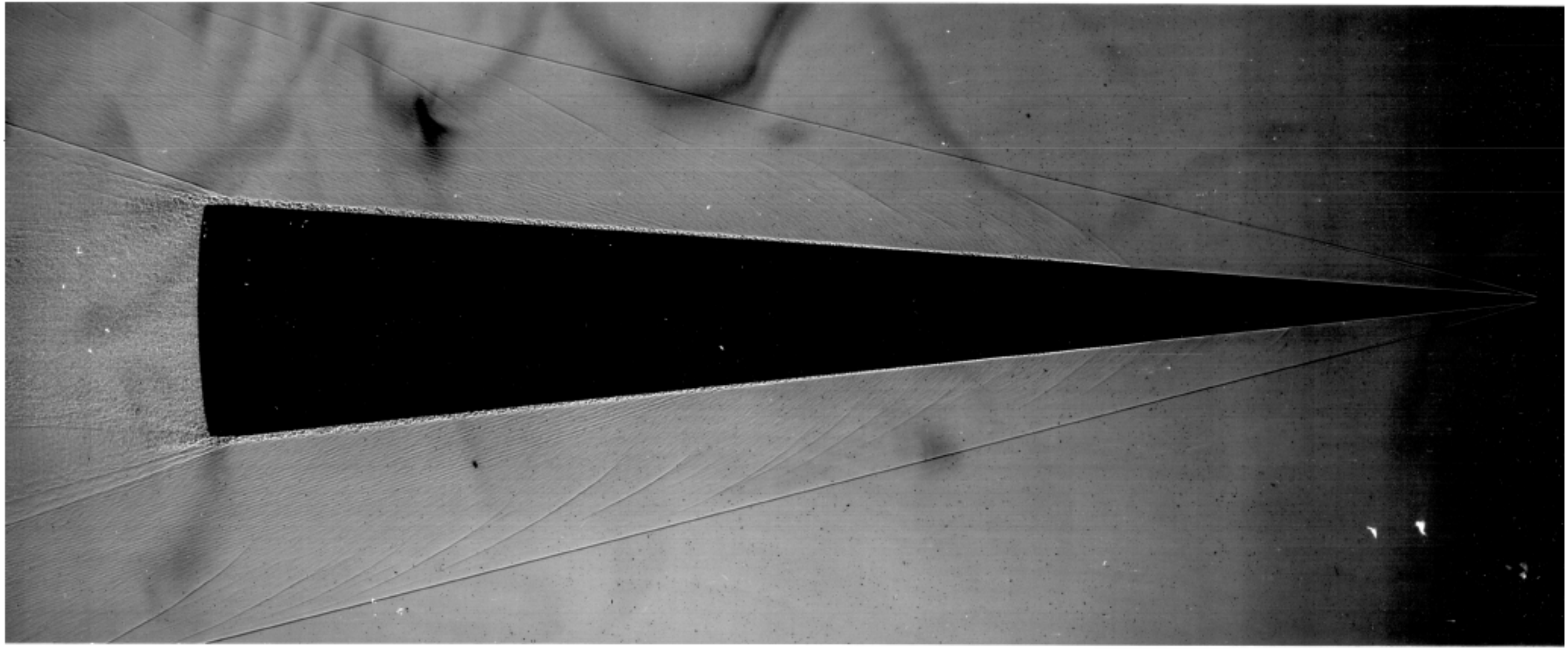
- Goals
  - Scramjets / Rockets
  - Re-entry vehicles
  - High-Speed Flight
- Effects
  - Aerothermodynamic Heating, Flight Characteristics, Control, Propulsion,…
- Methods
  - Flight Tests and Wind Tunnels
  - Navier-Stokes Equations
  - Direct Numerical Simulations





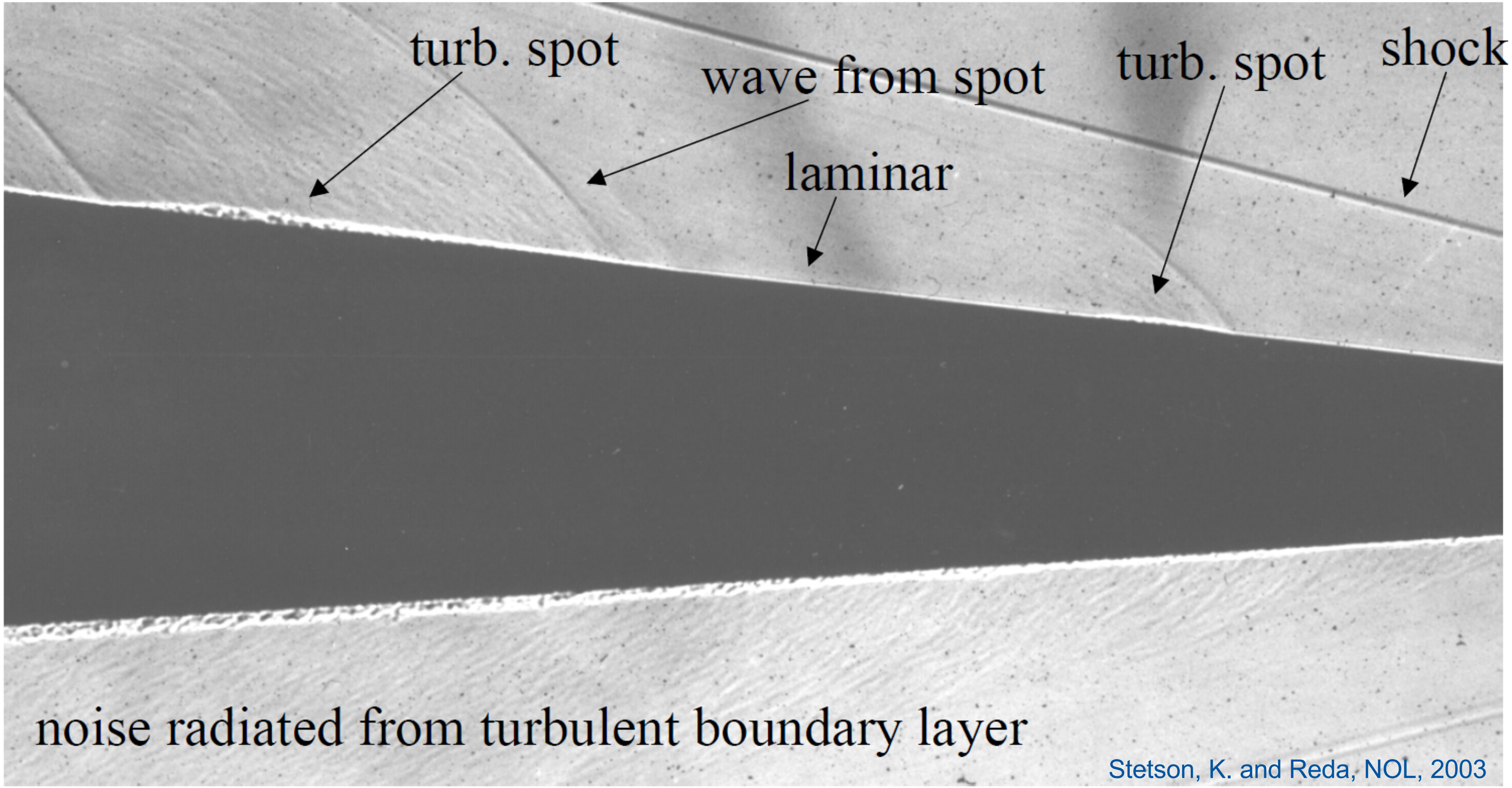
Mercury Capsule, NASA, Schneider, S. P., 2016

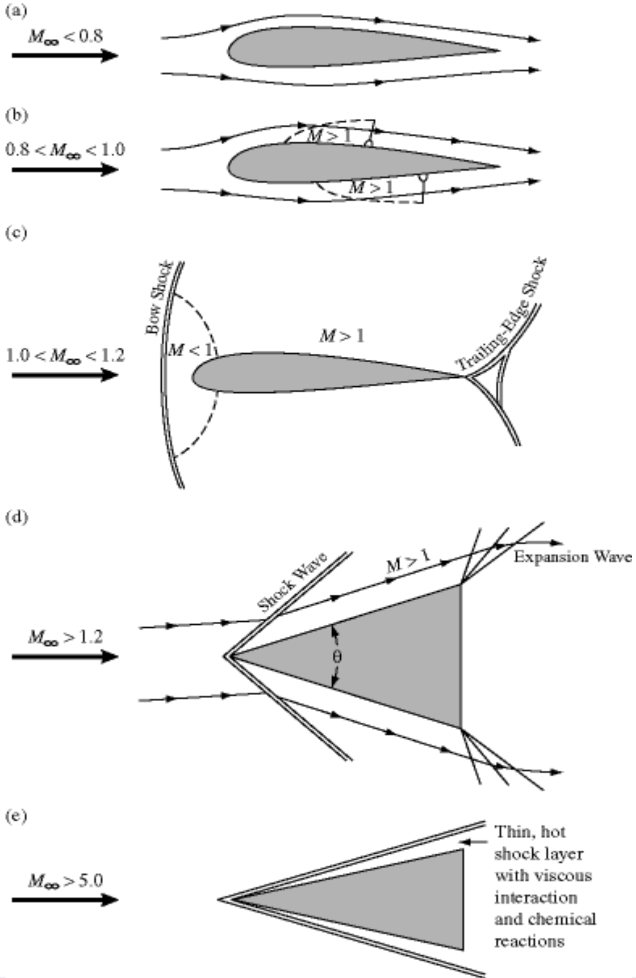
Mach 4 Cone Ballistics Range Test



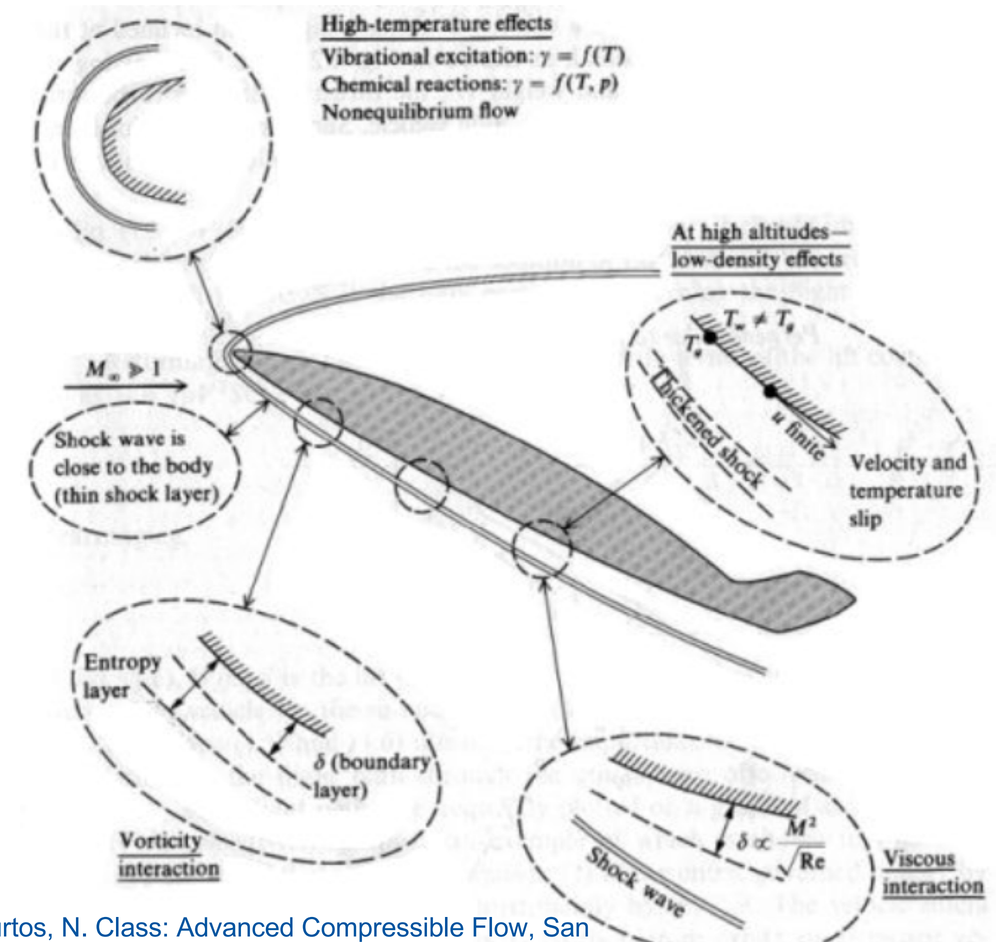
Stetson, K. and Reda,NOL, 2003

Mach 4 Cone Ballistics Range Test



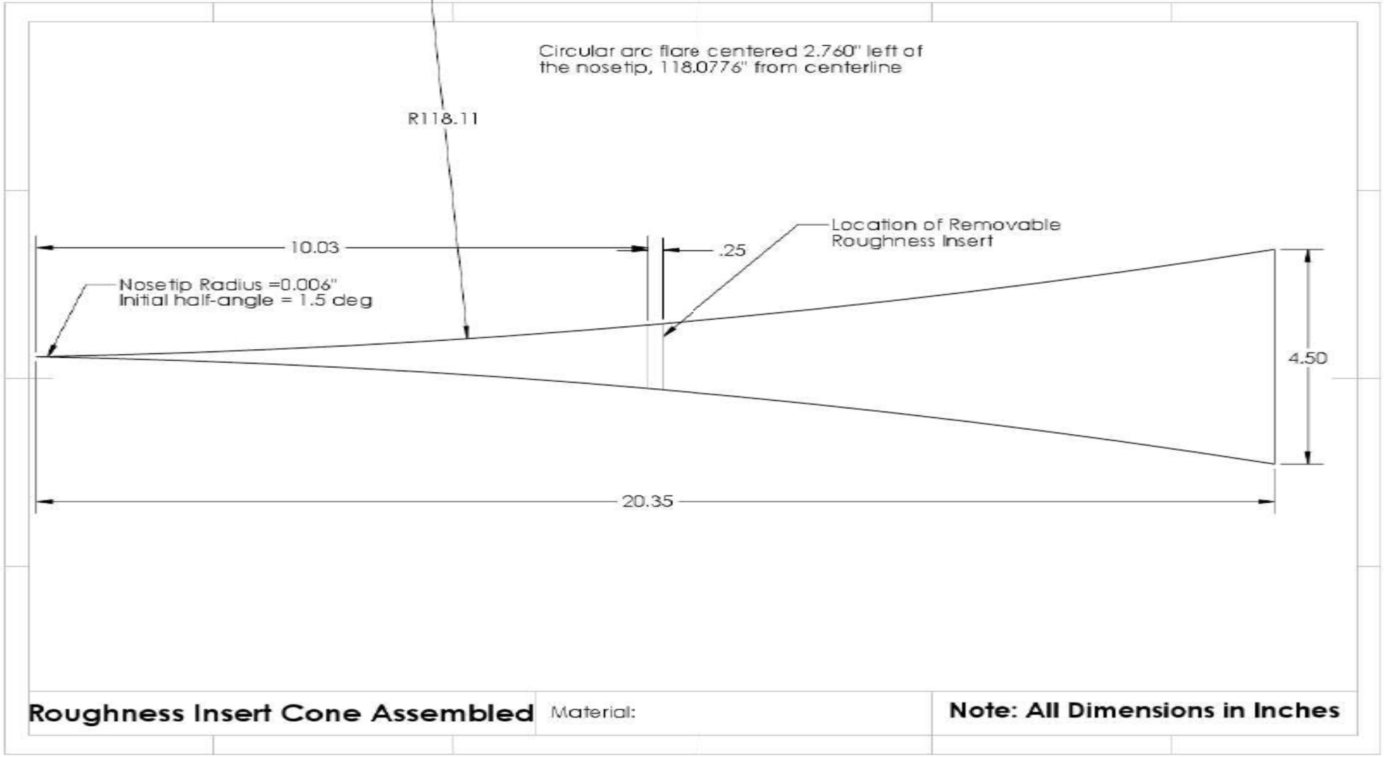


[Anderson, J. Intro. to Aerodynamics, 2014]



[Mourtos, N. Class: Advanced Compressible Flow, San Jose State University, 2015]

Purdue Flared Cone Dimensions

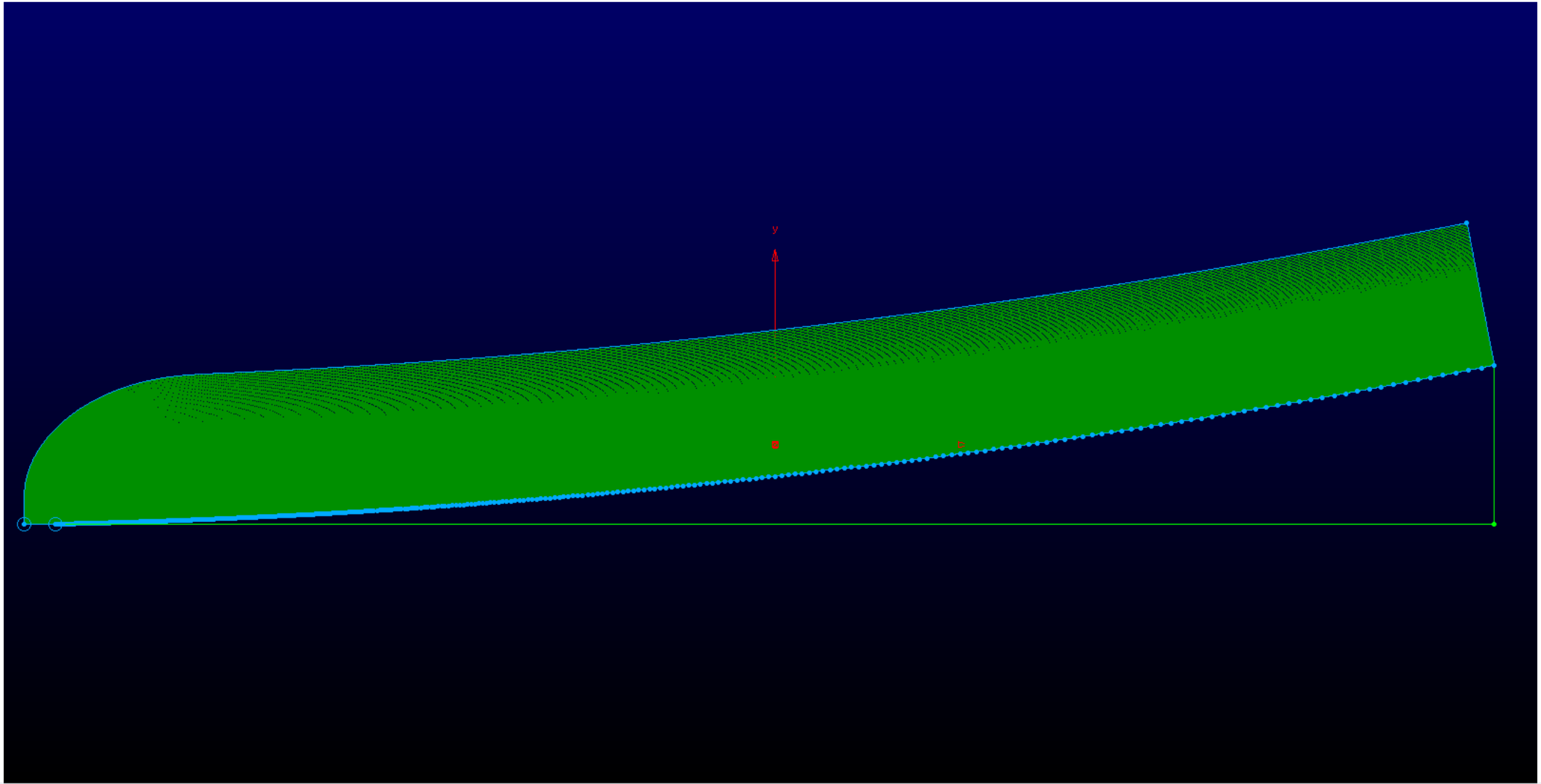


Chynoweth, B., Master's Thesis, Purdue University

Geometry – Solidworks CAD

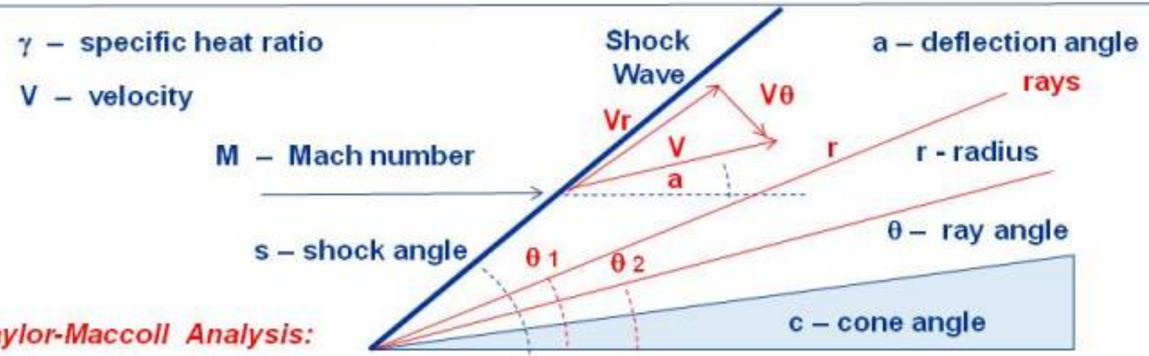
Chynoweth, B., Master's Thesis, Purdue University

Mesh – 1.25M Grid Points



National Aeronautics and Space Administration

Flow Past a Cone



Taylor-Maccoll Analysis:

1) For given Mach number and cone angle, guess a shock angle and use oblique shock relations to determine velocity V downstream of shock.
Resolve into Vr (radial) and Vθ (angular).

2) Numerically integrate the Taylor-Maccoll differential equations for Vr and $V\theta = f(\theta)$ from the shock wave until $V\theta = 0$

$$\frac{\gamma-1}{2}\left[1-Vr^2-\left(\frac{dVr}{d\theta}\right)^2\right]\left[2Vr+\cot\theta\,\frac{dVr}{d\theta}+\frac{d^2Vr}{d\theta^2}\right]-\frac{dVr}{d\theta}\left[Vr\,\frac{dVr}{d\theta}+\frac{dVr}{d\theta}\,\frac{d^2Vr}{d\theta^2}\right]=0$$

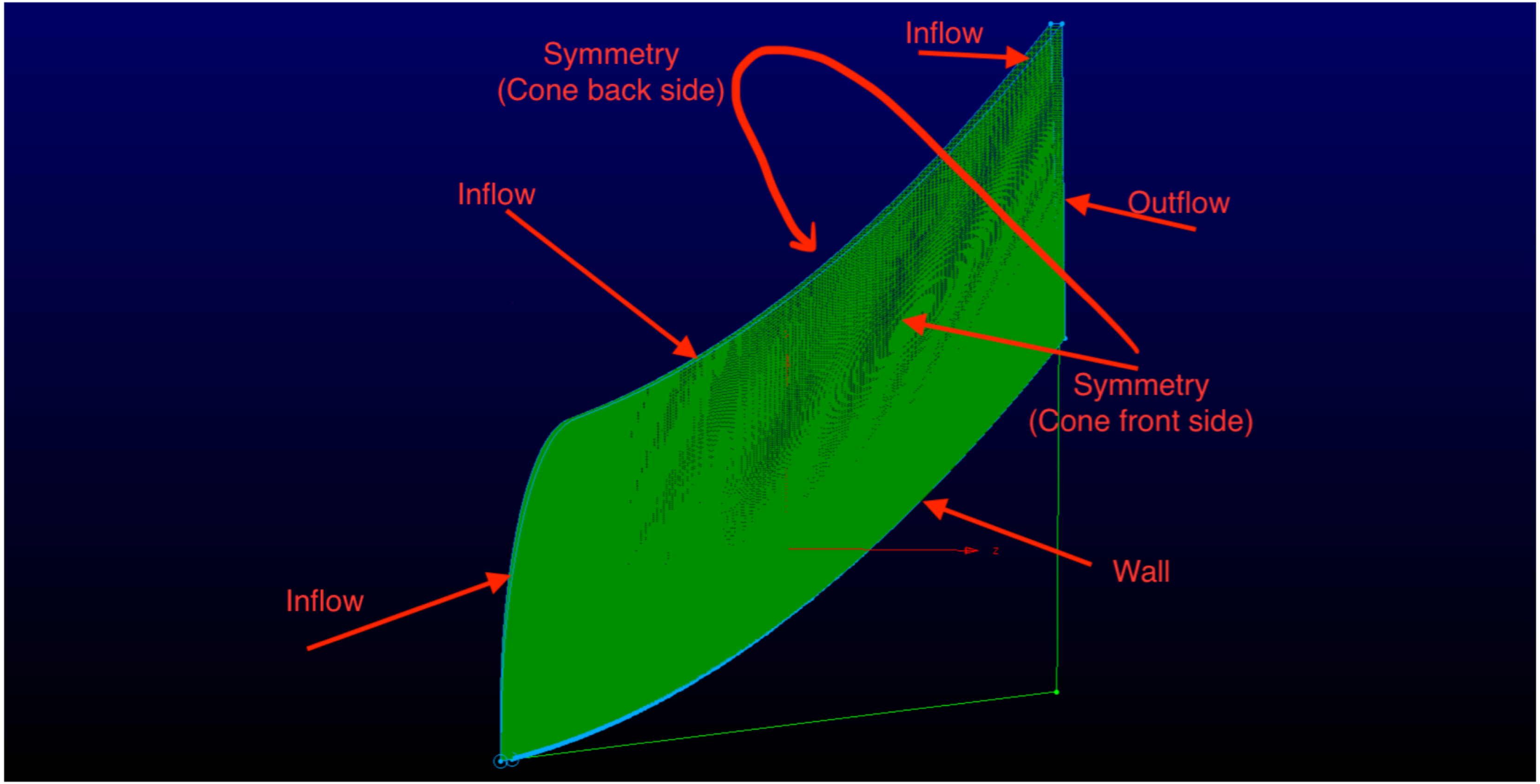
$$V\theta = \frac{dVr}{d\theta}$$

3) $V\theta = 0$ at $\theta = \theta$ surf. Adjust shock angle and repeat process until θ surf = cone angle.

4) Use isentropic relations to determine flow variables along each ray.

[NASA, 2018, https://www.grc.nasa.gov/www/k-12/airplane/coneflow.html]

Boundary Conditions



Flow Initial Conditions

$$\frac{P_0}{P} = \left(\frac{\rho_0}{\rho}\right)^{\gamma} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

Tunnel Conditions:

$P_{0,i}$ [kPa]	$T_{0,i}$ [K]	P_0 [kPa]	$Re \times 10^{-6}/m$	γ_{air}
8.9676	434.85	8.361	9.2	1.4

US3D Run Conditions for Base and Perturbation Cases:

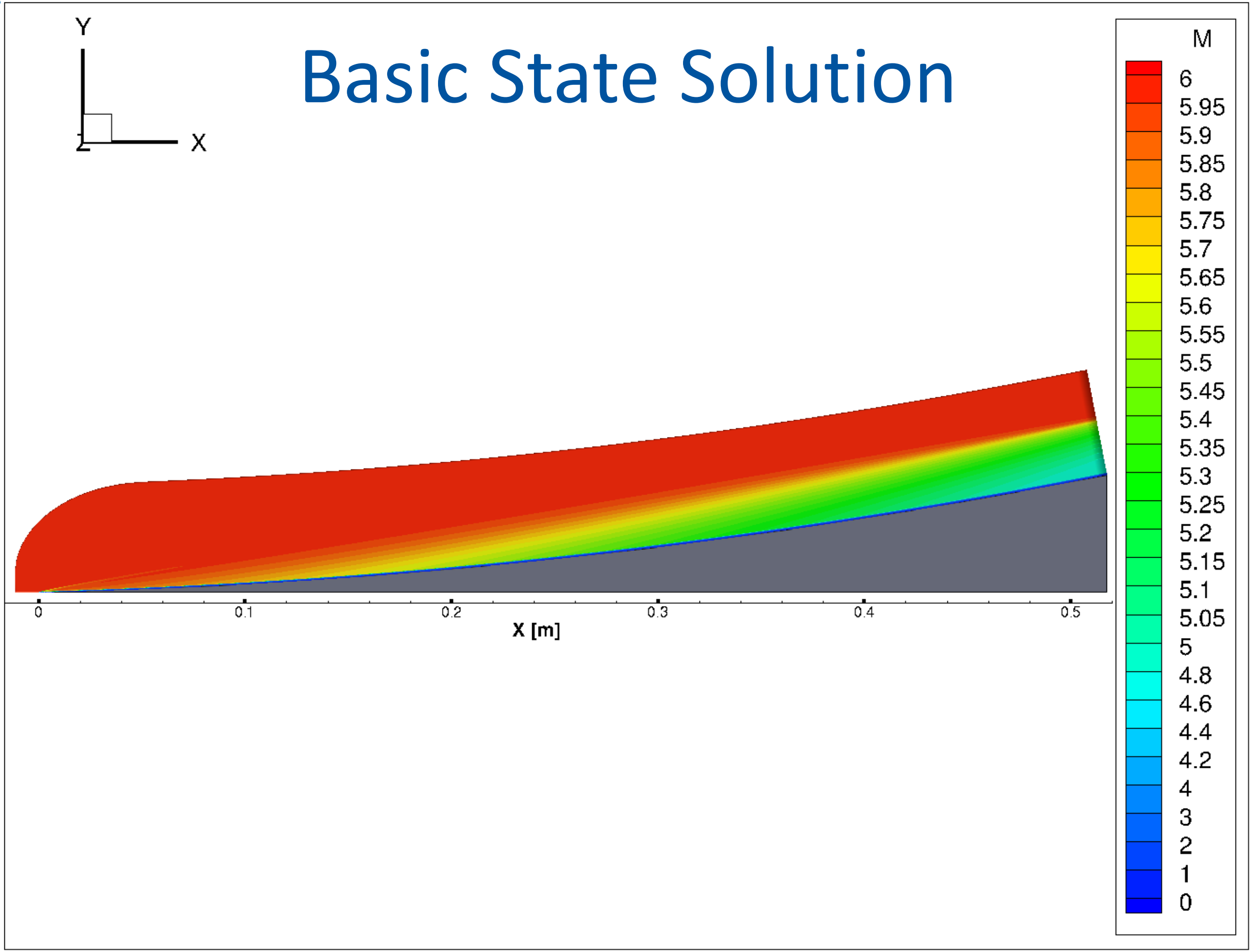
Run Case	Boundary	ρ [kg/m^3]	T [K]	u [m/s]
Base	Inflow	0.0355	51.9799	867.1092
Re +10%	Inflow	0.0355	51.9799	953.8201
Re -10%	Inflow	0.0355	51.9799	780.3983
Re/m = 10.8x10^6	Inflow	0.0417	51.6448	864.3102
All Above	Wall	N/A	300	N/A

Chynoweth, B., Master's Thesis, Purdue University

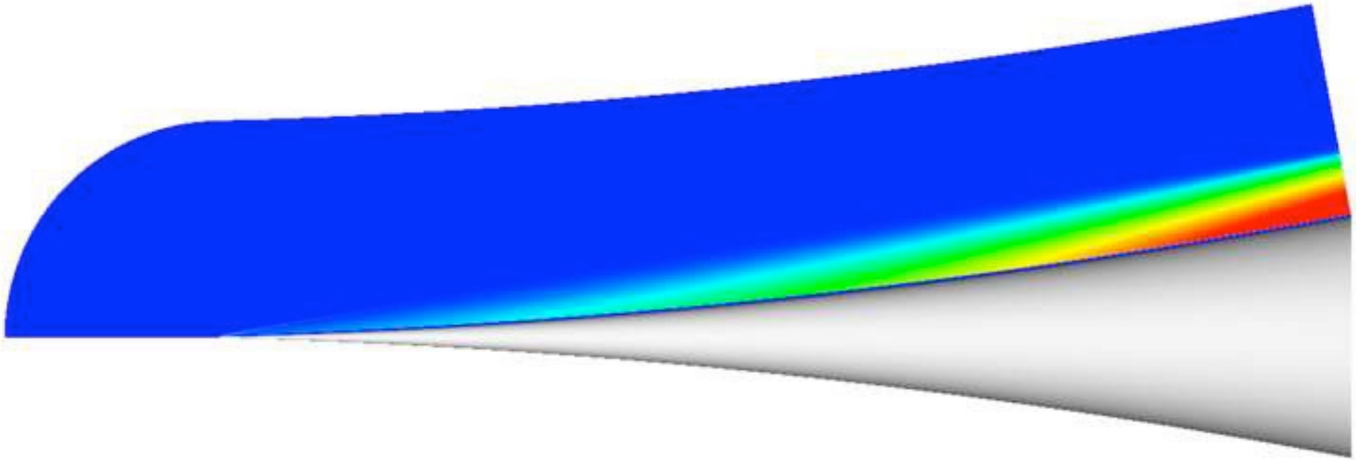
Solver Output Example – US3D

iter	residual	time	timestep	CFL
601001	2.087407450E-05	3.059888341E-03	1.155971097E-11	1.000E+00
601002	2.085491745E-05	3.059888353E-03	1.155971097E-11	1.000E+00
601003	2.081072436E-05	3.059888364E-03	1.155971097E-11	1.000E+00
601004	2.076873646E-05	3.059888376E-03	1.155971097E-11	1.000E+00
601005	2.071328239E-05	3.059888388E-03	1.155971097E-11	1.000E+00
601006	2.067556544E-05	3.059888399E-03	1.155971097E-11	1.000E+00
601007	2.062350991E-05	3.059888411E-03	1.155971097E-11	1.000E+00
601008	2.057505684E-05	3.059888422E-03	1.155971097E-11	1.000E+00

Basic State Solution

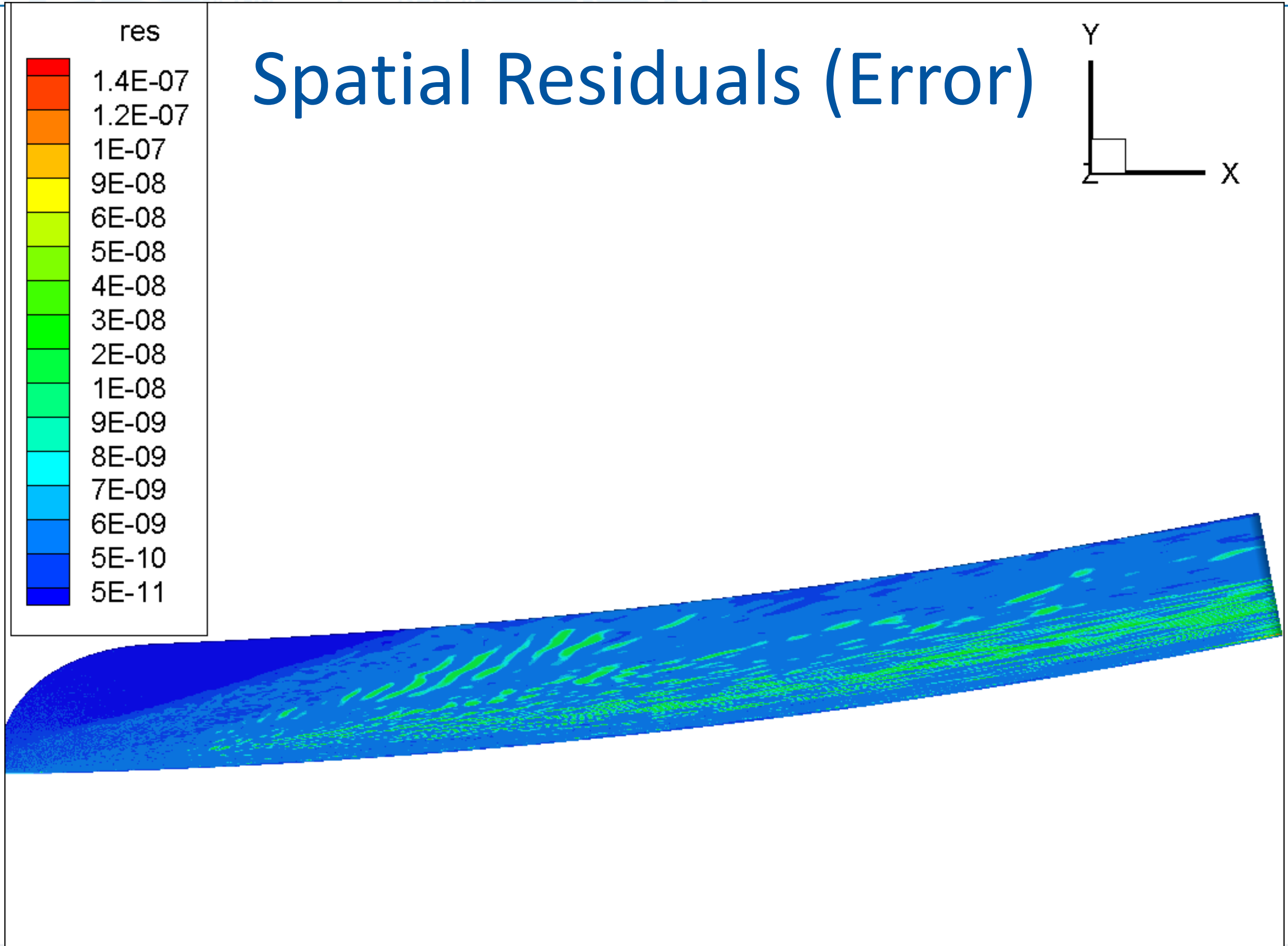


CFD Benchmark

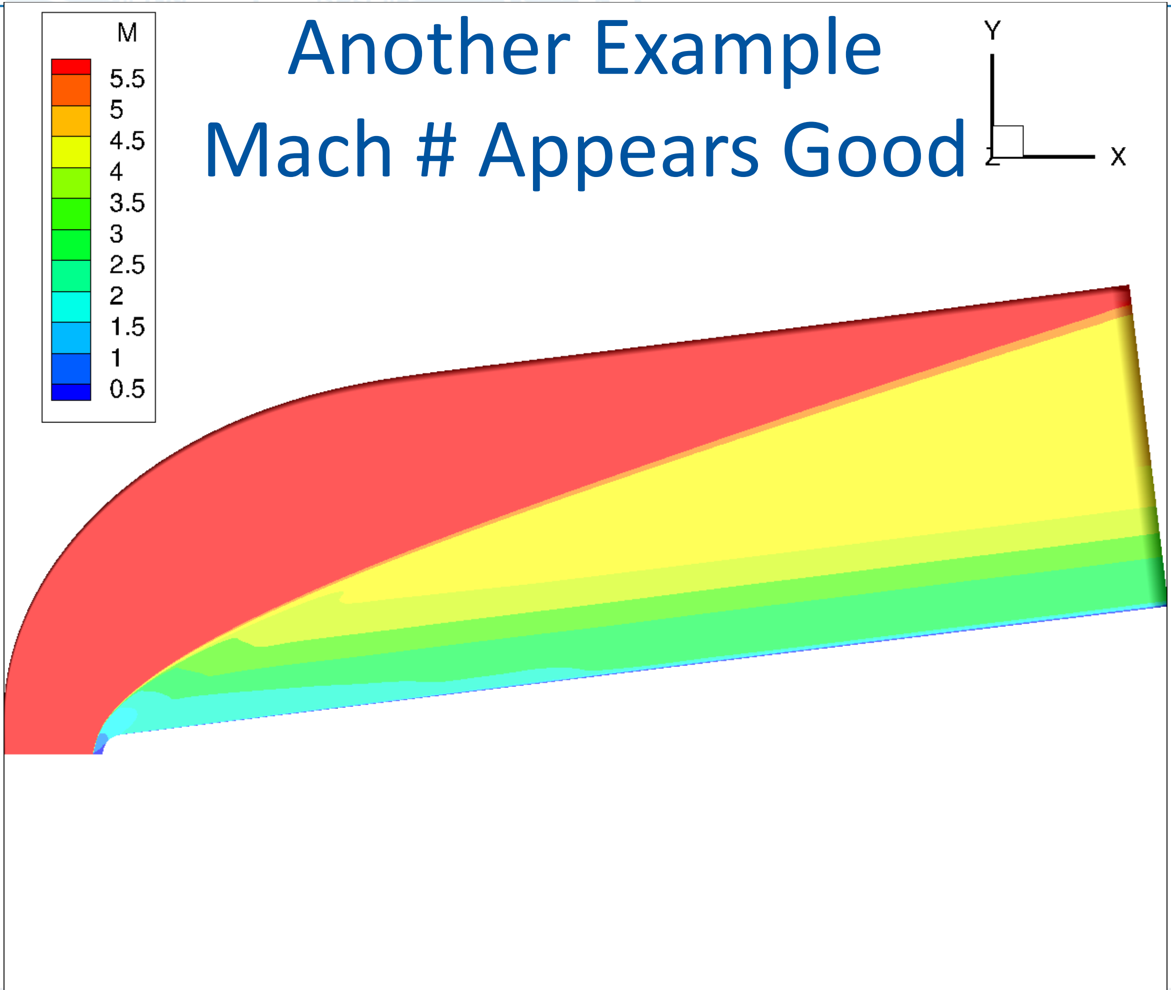


Fasel, et al, Flared Cone CFD Solutions

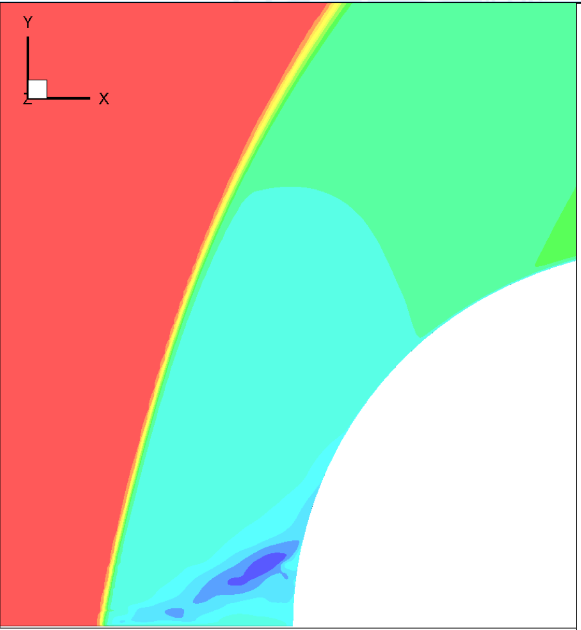
Spatial Residuals (Error)

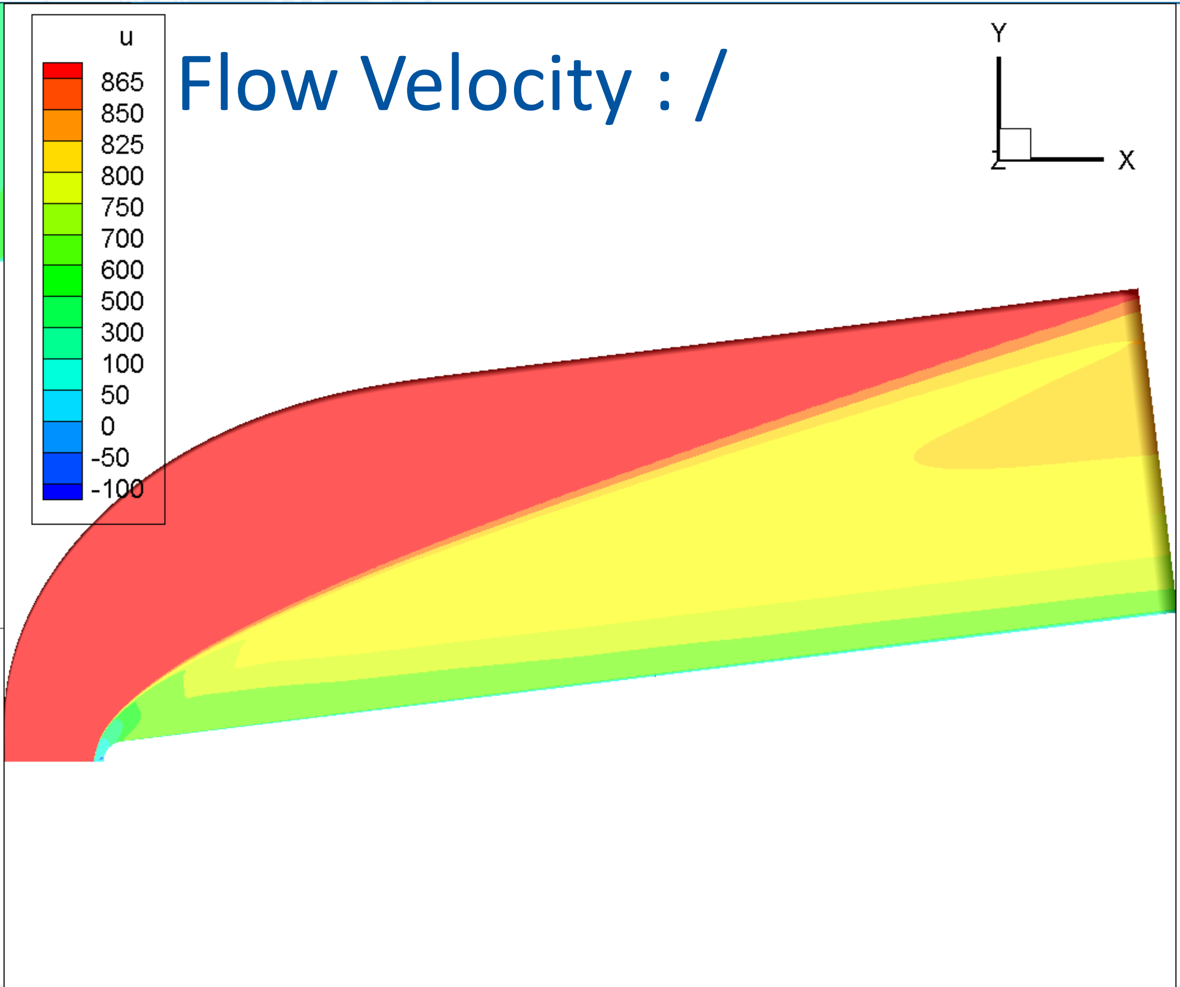


Another Example
Mach # Appears Good

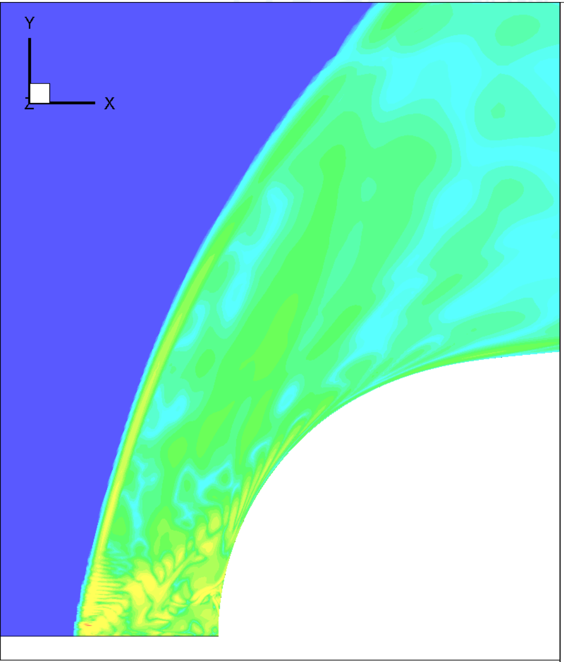


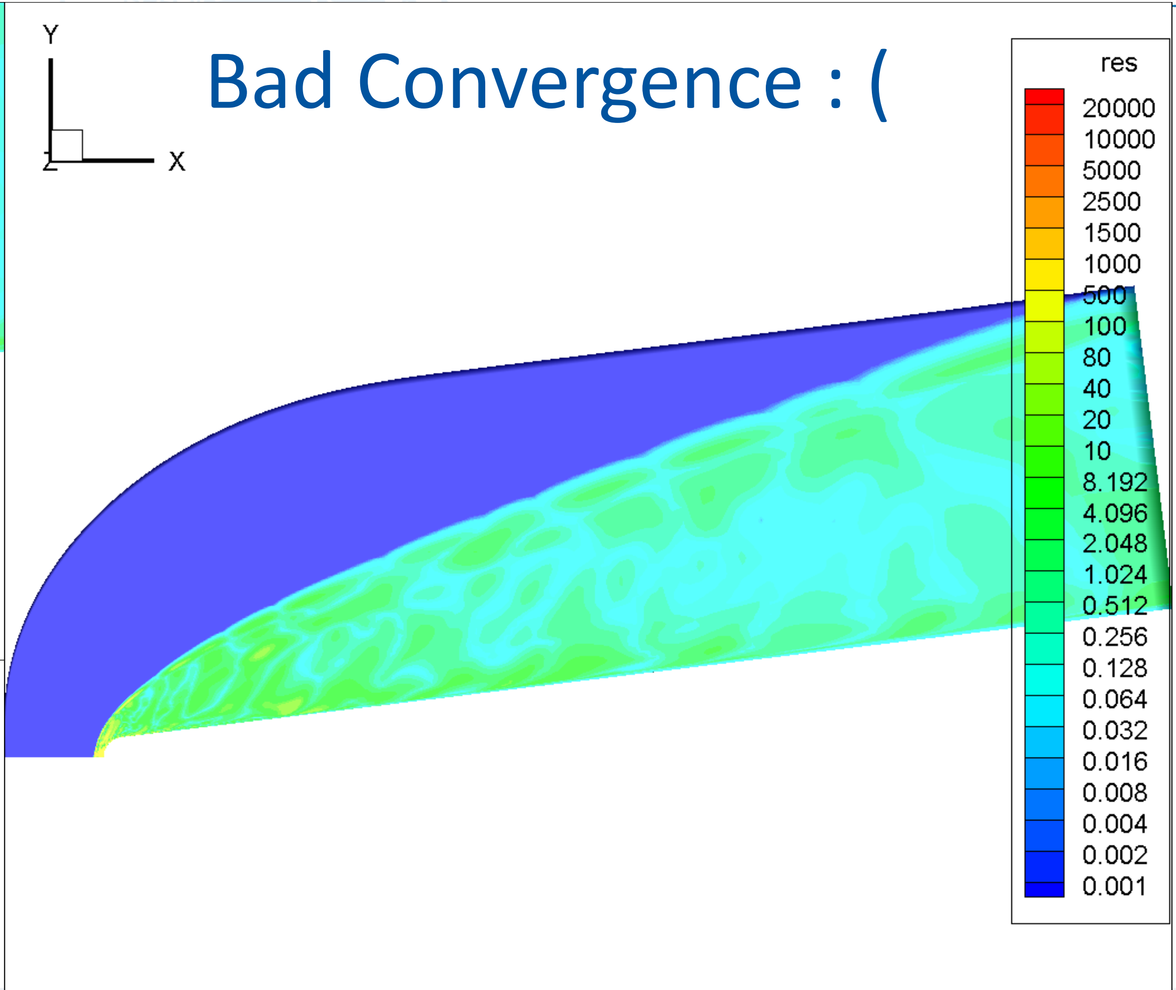
Flow Velocity : /



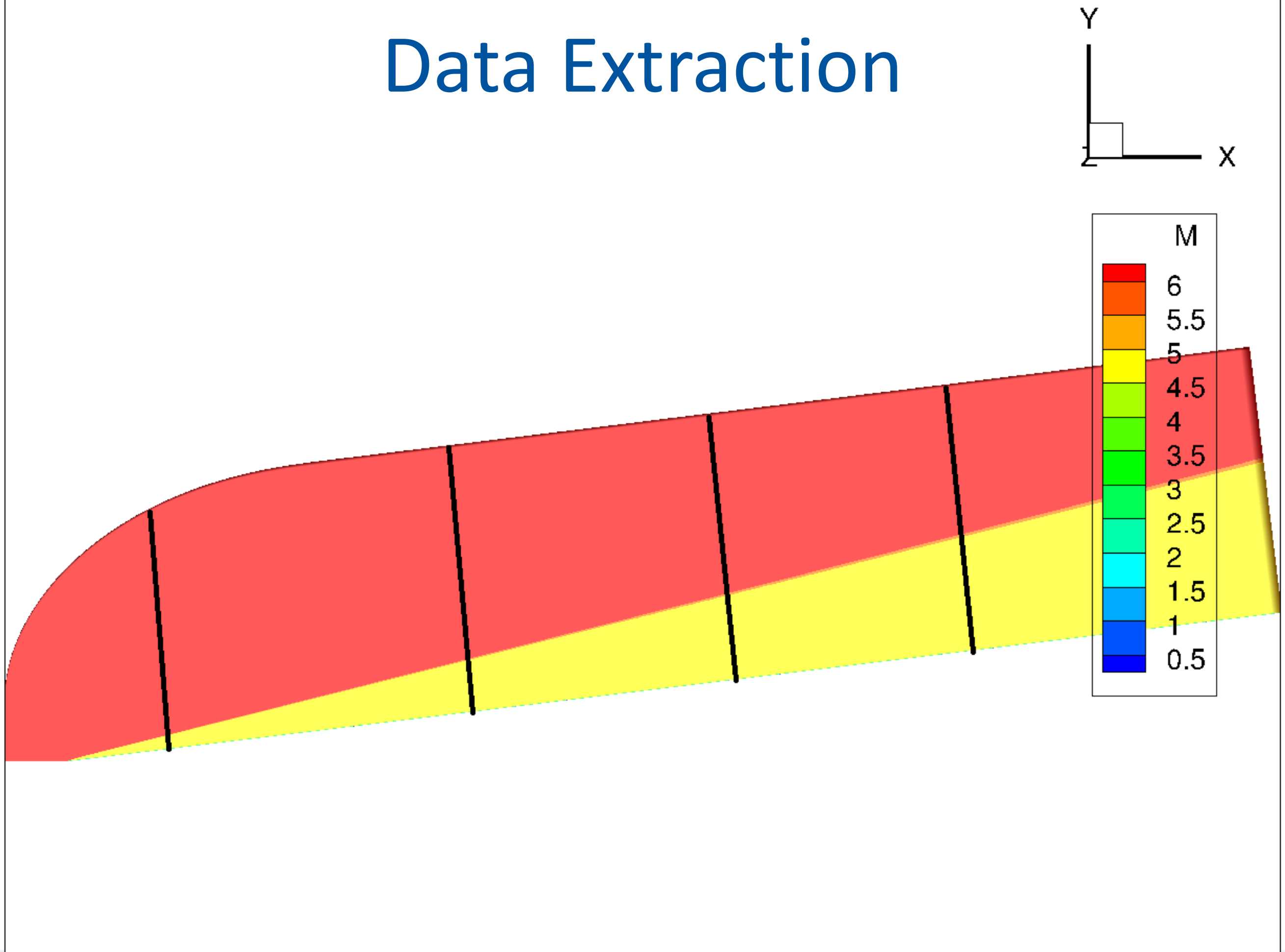


Bad Convergence : (

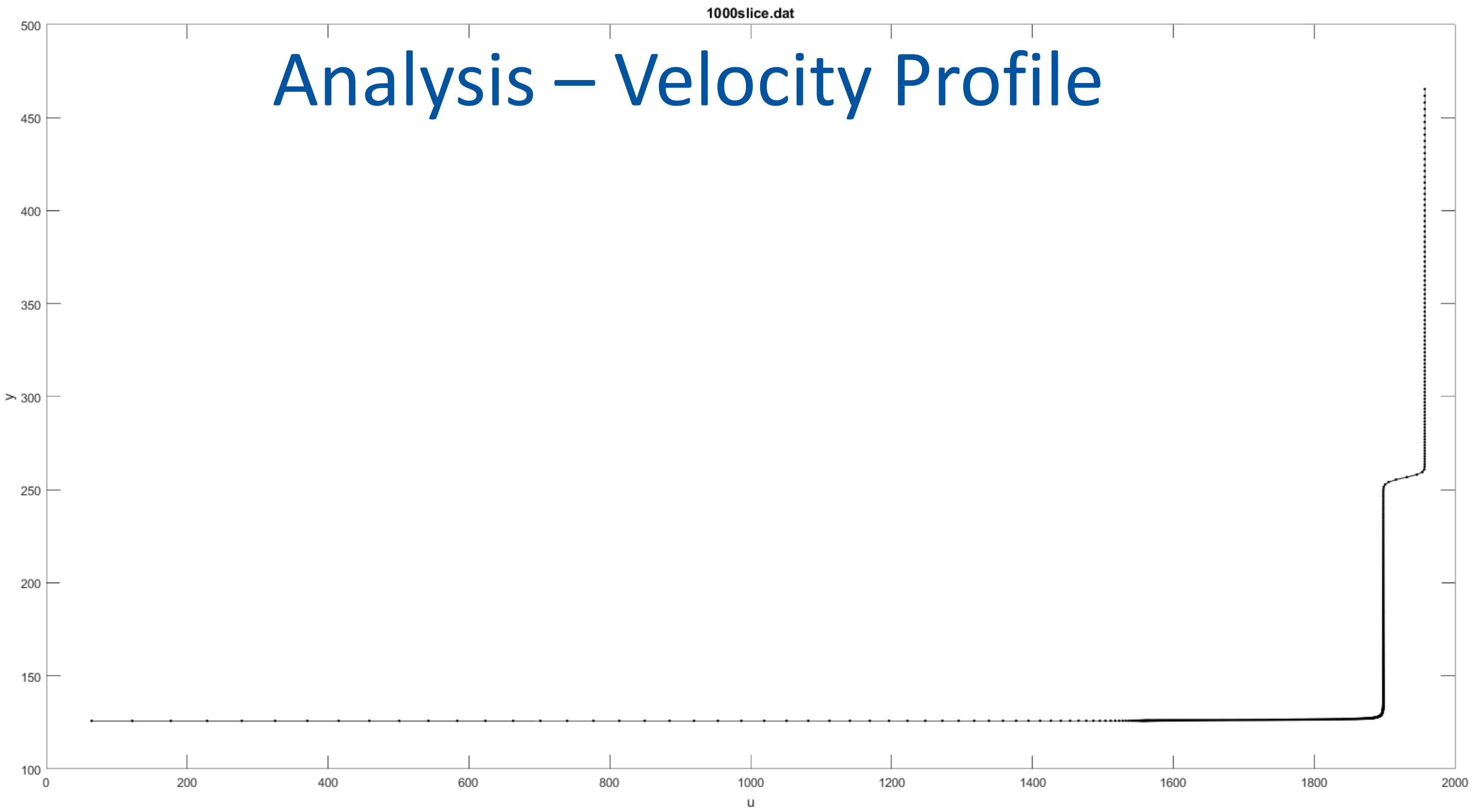




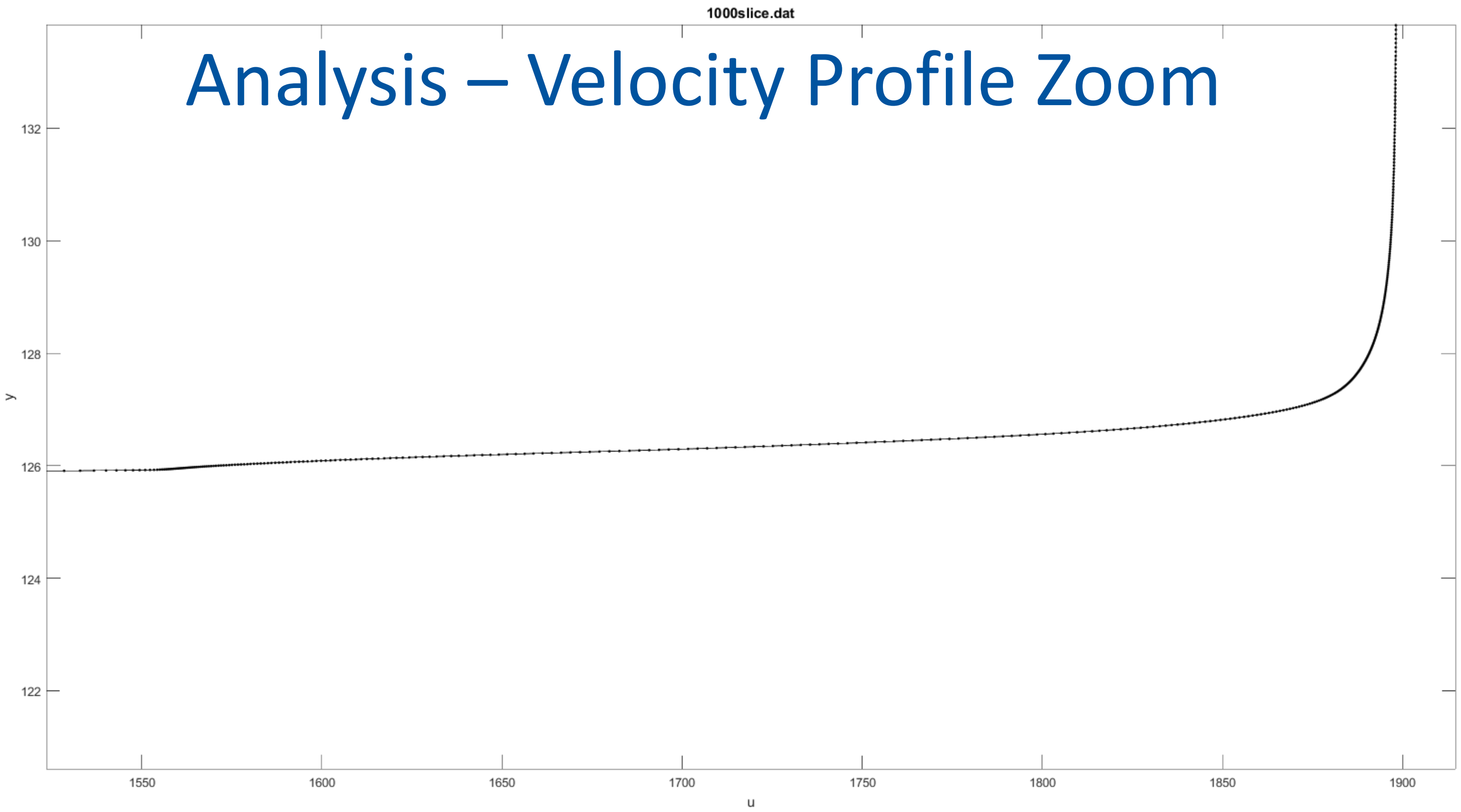
Data Extraction



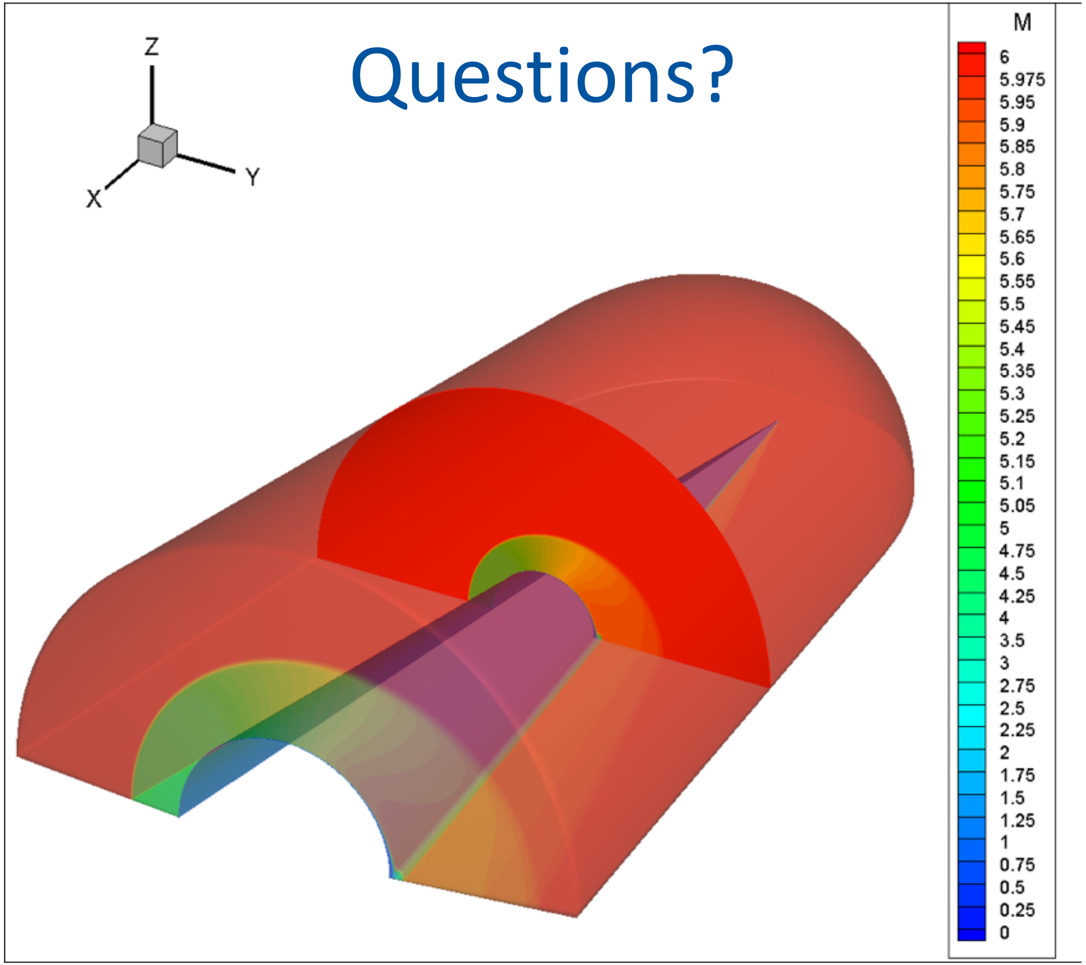
Analysis – Velocity Profile



Analysis – Velocity Profile Zoom



Questions?


Z
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2.25
2
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1
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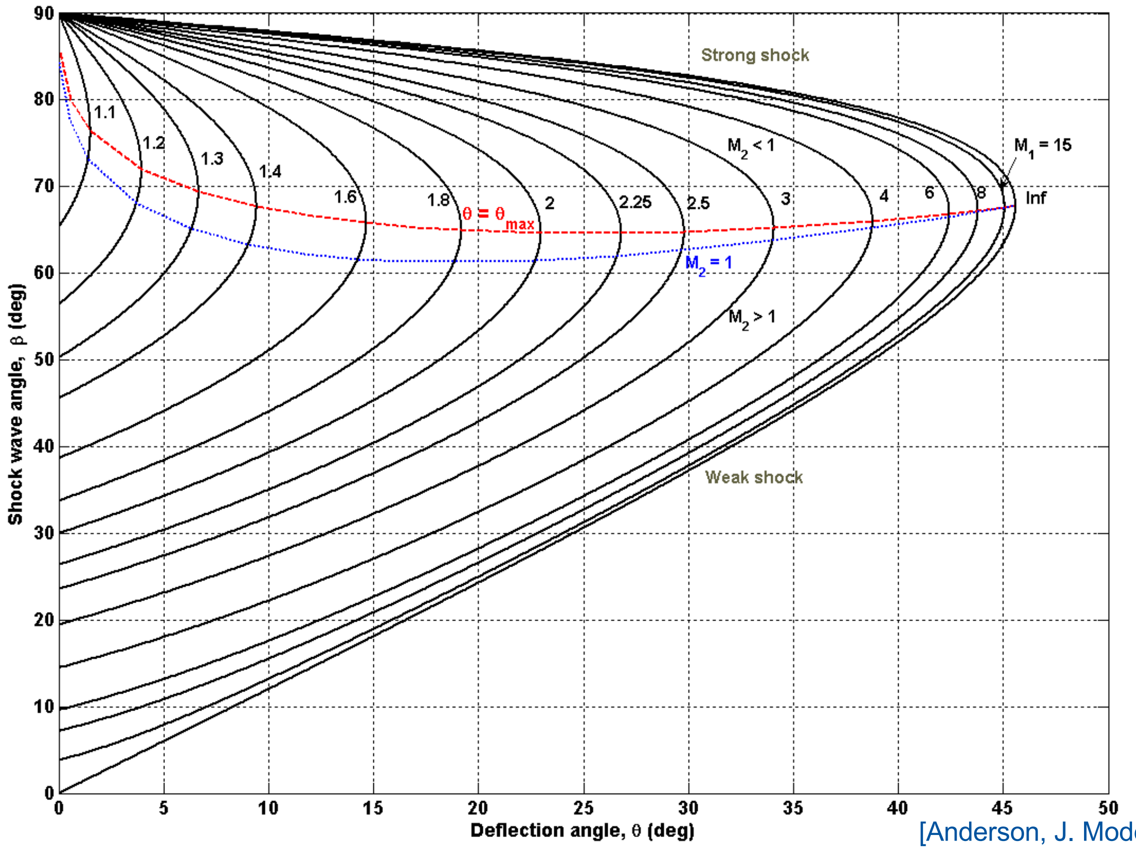
0.5
0.25
0

What’s Next

- ANSYS Fluent:
  - Free student download: https://www.ansys.com/academic/free-student-products/
- Tutorials: https://confluence.cornell.edu/display/SIMULATION/FLUENT+Learning+Modules
- Coding Challenge 1 is to perform a CFD Calculation
  - Assigned: W 2/27/2019
  - Due: 5/20/2019

Θ-β-Mach Diagram



[Anderson, J. Modern Compressible Flow, 20]