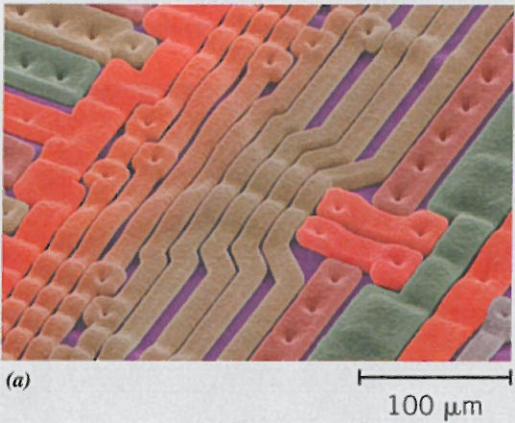


# Chapter 18 Electrical Properties

Andrew Syred/Photo Researchers, Inc.

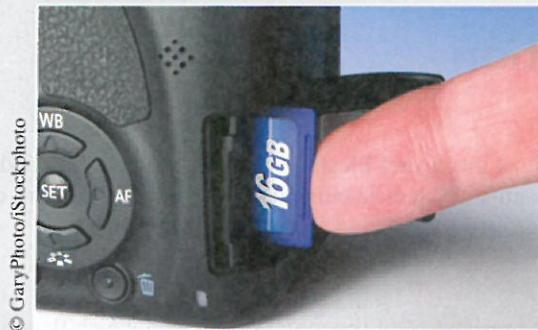


(a)

Courtesy SanDisk Corporation



(b)



(c)

The functioning of modern flash memory cards (and flash drives) that are used to store digital information relies on the unique electrical properties of silicon, a semiconducting material. (Flash memory is discussed in Section 18.15.)

(a) Scanning electron micrograph of an integrated circuit, which is composed of silicon and metallic interconnects. Integrated circuit components are used to store information in a digital format.

(b) Three different flash memory card types.

(c) A flash memory card being inserted into a digital camera. This memory card will be used to store photographic images (and in some cases GPS location).

## WHY STUDY the Electrical Properties of Materials?

Consideration of the electrical properties of materials is often important when materials selection and processing decisions are being made during the design of a component or structure. For example, when we consider an integrated circuit package, the

electrical behaviors of the various materials are diverse. Some need to be highly electrically conductive (e.g., connecting wires), whereas electrical insulativity is required of others (e.g., protective package encapsulation).

### Learning Objectives

After studying this chapter, you should be able to do the following:

1. Describe the four possible electron band structures for solid materials.
2. Briefly describe electron excitation events that produce free electrons/holes in (a) metals, (b) semiconductors (intrinsic and extrinsic), and (c) insulators.
3. Calculate the electrical conductivities of metals, semiconductors (intrinsic and extrinsic), and insulators given their charge carrier densities and mobilities.
4. Distinguish between *intrinsic* and *extrinsic* semiconducting materials.
5. (a) On a plot of logarithm of carrier (electron, hole) concentration versus absolute temperature, draw schematic curves for both intrinsic and extrinsic semiconducting materials.  
(b) On the extrinsic curve, note freeze-out, extrinsic, and intrinsic regions.
6. For a *p-n* junction, explain the rectification process in terms of electron and hole motions.
7. Calculate the capacitance of a parallel-plate capacitor.
8. Define dielectric constant in terms of permittivities.
9. Briefly explain how the charge storing capacity of a capacitor may be increased by the insertion and polarization of a dielectric material between its plates.
10. Name and describe the three types of polarization.
11. Briefly describe the phenomena of *ferroelectricity* and *piezoelectricity*.

### 18.1 INTRODUCTION

The prime objective of this chapter is to explore the electrical properties of materials—that is, their responses to an applied electric field. We begin with the phenomenon of electrical conduction: the parameters by which it is expressed, the mechanism of conduction by electrons, and how the electron energy band structure of a material influences its ability to conduct. These principles are extended to metals, semiconductors, and insulators. Particular attention is given to the characteristics of semiconductors and then to semiconducting devices. The dielectric characteristics of insulating materials are also treated. The final sections are devoted to the phenomena of ferroelectricity and piezoelectricity.

## Electrical Conduction

### 18.2 OHM'S LAW

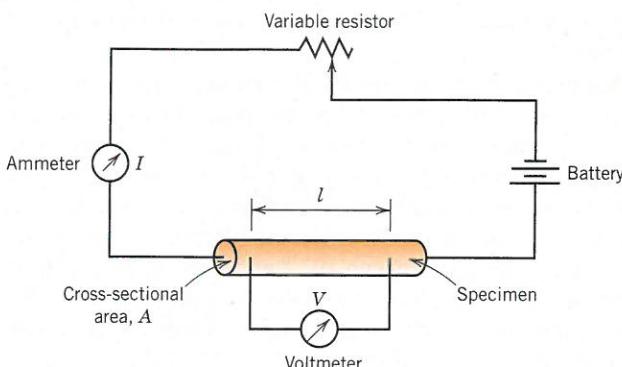
#### Ohm's law

One of the most important electrical characteristics of a solid material is the ease with which it transmits an electric current. **Ohm's law** relates the current  $I$ —or time rate of charge passage—to the applied voltage  $V$  as follows:

$$V = IR \quad (18.1)$$

#### Ohm's law expression

where  $R$  is the resistance of the material through which the current is passing. The units for  $V$ ,  $I$ , and  $R$  are, respectively, volts (J/C), amperes (C/s), and ohms (V/A). The value of  $R$  is influenced by specimen configuration and for many materials is independent of



**Figure 18.1** Schematic representation of the apparatus used to measure electrical resistivity.

### electrical resistivity

Electrical resistivity—dependence on resistance, specimen cross-sectional area, and distance between measuring points

Electrical resistivity—dependence on applied voltage, current, specimen cross-sectional area, and distance between measuring points

## 18.3 ELECTRICAL CONDUCTIVITY

### electrical conductivity

Reciprocal relationship between electrical conductivity and resistivity

Ohm's law expression—in terms of current density, conductivity, and applied electric field

Electric field intensity

current. The **electrical resistivity**  $\rho$  is independent of specimen geometry but related to  $R$  through the expression

$$\rho = \frac{RA}{l} \quad (18.2)$$

where  $l$  is the distance between the two points at which the voltage is measured and  $A$  is the cross-sectional area perpendicular to the direction of the current. The units for  $\rho$  are ohm-meters ( $\Omega \cdot \text{m}$ ). From the expression for Ohm's law and Equation 18.2,

$$\rho = \frac{VA}{Il} \quad (18.3)$$

Figure 18.1 is a schematic diagram of an experimental arrangement for measuring electrical resistivity.

### electrical conductivity

Sometimes, **electrical conductivity**  $\sigma$  is used to specify the electrical character of a material. It is simply the reciprocal of the resistivity, or

$$\sigma = \frac{1}{\rho} \quad (18.4)$$

and is indicative of the ease with which a material is capable of conducting an electric current. The units for  $\sigma$  are reciprocal ohm-meters [ $(\Omega \cdot \text{m})^{-1}$ ].<sup>1</sup> The following discussions on electrical properties use both resistivity and conductivity.

In addition to Equation 18.1, Ohm's law may be expressed as

$$J = \sigma \mathcal{E} \quad (18.5)$$

in which  $J$  is the current density—the current per unit of specimen area  $I/A$ —and  $\mathcal{E}$  is the electric field intensity, or the voltage difference between two points divided by the distance separating them—that is,

$$\mathcal{E} = \frac{V}{l} \quad (18.6)$$

<sup>1</sup>The SI units for electrical conductivity are siemens per meter (S/m), in which  $1 \text{ S/m} = 1 (\Omega \cdot \text{m})^{-1}$ . We opted to use  $(\Omega \cdot \text{m})^{-1}$  on the basis of convention—these units are traditionally used in introductory materials science and engineering texts.

The demonstration of the equivalence of the two Ohm's law expressions (Equations 18.1 and 18.5) is left as a homework exercise.

Solid materials exhibit an amazing range of electrical conductivities, extending over 27 orders of magnitude; probably no other physical property exhibits this breadth of variation. In fact, one way of classifying solid materials is according to the ease with which they conduct an electric current; within this classification scheme there are three groupings: *conductors*, *semiconductors*, and *insulators*. Metals are good conductors, typically having conductivities on the order of  $10^7 \text{ } (\Omega \cdot \text{m})^{-1}$ . At the other extreme are materials with very low conductivities, ranging between  $10^{-10}$  and  $10^{-20} \text{ } (\Omega \cdot \text{m})^{-1}$ ; these are electrical insulators. Materials with intermediate conductivities, generally from  $10^{-6}$  to  $10^4 \text{ } (\Omega \cdot \text{m})^{-1}$ , are termed **semiconductors**. Electrical conductivity ranges for the various material types are compared in the bar chart of Figure 1.8.

metal

insulator

semiconductor

ionic conduction

## 18.4 ELECTRONIC AND IONIC CONDUCTION

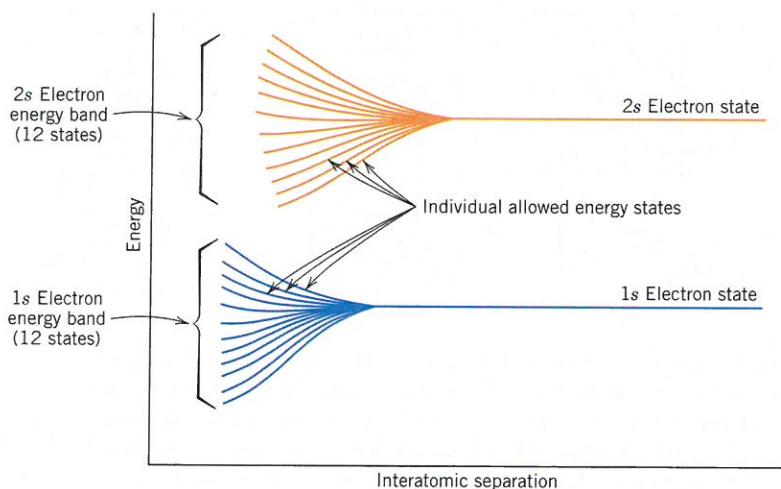
An electric current results from the motion of electrically charged particles in response to forces that act on them from an externally applied electric field. Positively charged particles are accelerated in the field direction, negatively charged particles in the direction opposite. Within most solid materials a current arises from the flow of electrons, which is termed *electronic conduction*. In addition, for ionic materials, a net motion of charged ions is possible that produces a current; this is termed **ionic conduction**. The present discussion deals with electronic conduction; ionic conduction is treated briefly in Section 18.16.

## 18.5 ENERGY BAND STRUCTURES IN SOLIDS

In all conductors, semiconductors, and many insulating materials, only electronic conduction exists, and the magnitude of the electrical conductivity is strongly dependent on the number of electrons available to participate in the conduction process. However, not all electrons in every atom accelerate in the presence of an electric field. The number of electrons available for electrical conduction in a particular material is related to the arrangement of electron states or levels with respect to energy and the manner in which these states are occupied by electrons. A thorough exploration of these topics is complicated and involves principles of quantum mechanics that are beyond the scope of this book; the ensuing development omits some concepts and simplifies others.

Concepts relating to electron energy states, their occupancy, and the resulting electron configurations for isolated atoms were discussed in Section 2.3. By way of review, for each individual atom there exist discrete energy levels that may be occupied by electrons, arranged into shells and subshells. Shells are designated by integers (1, 2, 3, etc.) and subshells by letters (*s*, *p*, *d*, and *f*). For each of *s*, *p*, *d*, and *f* subshells, there exist, respectively, one, three, five, and seven states. The electrons in most atoms fill only the states having the lowest energies—two electrons of opposite spin per state, in accordance with the Pauli exclusion principle. The electron configuration of an isolated atom represents the arrangement of the electrons within the allowed states.

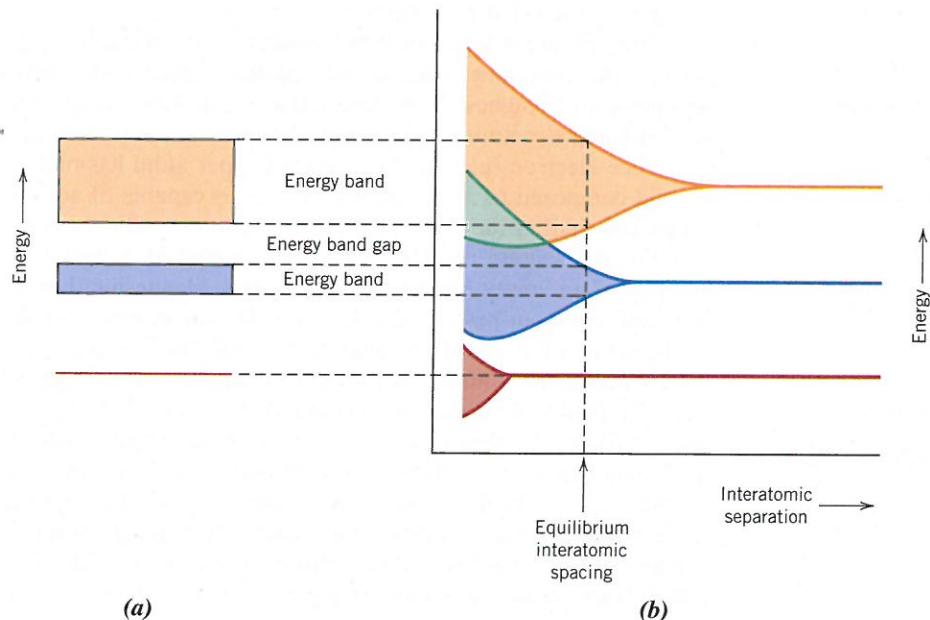
Let us now make an extrapolation of some of these concepts to solid materials. A solid may be thought of as consisting of a large number—say, *N*—of atoms initially separated from one another that are subsequently brought together and bonded to form the ordered atomic arrangement found in the crystalline material. At relatively large separation distances, each atom is independent of all the others and has the atomic energy levels and electron configuration as if isolated. However, as the atoms come within close proximity of one another, electrons are acted upon, or *perturbed*, by the electrons and nuclei of



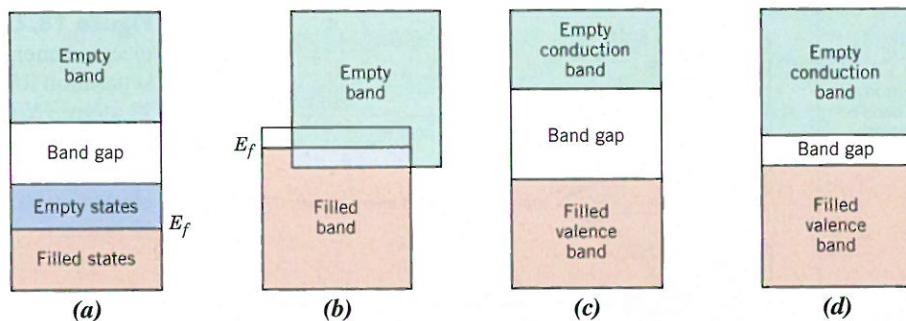
**Figure 18.2** Schematic plot of electron energy versus interatomic separation for an aggregate of 12 atoms ( $N = 12$ ). Upon close approach, each of the 1s and 2s atomic states splits to form an electron energy band consisting of 12 states.

### electron energy band

adjacent atoms. This influence is such that each distinct atomic state may split into a series of closely spaced electron states in the solid to form what is termed an **electron energy band**. The extent of splitting depends on interatomic separation (Figure 18.2) and begins with the outermost electron shells because they are the first to be perturbed as the atoms coalesce. Within each band, the energy states are discrete, yet the difference between adjacent states is exceedingly small. At the equilibrium spacing, band formation may not occur for the electron subshells nearest the nucleus, as illustrated in Figure 18.3b. Furthermore, gaps may exist between adjacent bands, as also indicated in the figure; normally, energies lying within these band gaps are not available for electron occupancy. The conventional way of representing electron band structures in solids is shown in Figure 18.3a.



**Figure 18.3** (a) The conventional representation of the electron energy band structure for a solid material at the equilibrium interatomic separation. (b) Electron energy versus interatomic separation for an aggregate of atoms, illustrating how the energy band structure at the equilibrium separation in (a) is generated.  
(From Z. D. Jastrzebski, *The Nature and Properties of Engineering Materials*, 3rd edition. Copyright © 1987 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.)



**Figure 18.4** The various possible electron band structures in solids at 0 K. (a) The electron band structure found in metals such as copper, in which there are available electron states above and adjacent to filled states, in the same band. (b) The electron band structure of metals such as magnesium, in which there is an overlap of filled and empty outer bands. (c) The electron band structure characteristic of insulators; the filled valence band is separated from the empty conduction band by a relatively large band gap ( $>2$  eV). (d) The electron band structure found in the semiconductors, which is the same as for insulators except that the band gap is relatively narrow ( $<2$  eV).

The number of states within each band is equal to the total of all states contributed by the  $N$  atoms. For example, an  $s$  band consists of  $N$  states and a  $p$  band of  $3N$  states. With regard to occupancy, each energy state may accommodate two electrons that must have oppositely directed spins. Furthermore, bands contain the electrons that resided in the corresponding levels of the isolated atoms; for example, a  $4s$  energy band in the solid contains those isolated atoms'  $4s$  electrons. Of course, there are empty bands and, possibly, bands that are only partially filled.

The electrical properties of a solid material are a consequence of its electron band structure—that is, the arrangement of the outermost electron bands and the way in which they are filled with electrons.

Four different types of band structures are possible at 0 K. In the first (Figure 18.4a), one outermost band is only partially filled with electrons. The energy corresponding to the highest filled state at 0 K is called the **Fermi energy**  $E_f$ , as indicated. This energy band structure is typified by some metals, in particular those that have a single  $s$  valence electron (e.g., copper). Each copper atom has one  $4s$  electron; however, for a solid composed of  $N$  atoms, the  $4s$  band is capable of accommodating  $2N$  electrons. Thus, only half of the available electron positions within this  $4s$  band are filled.

For the second band structure, also found in metals (Figure 18.4b), there is an overlap of an empty band and a filled band. Magnesium has this band structure. Each isolated Mg atom has two  $3s$  electrons. However, when a solid is formed, the  $3s$  and  $3p$  bands overlap. In this instance and at 0 K, the Fermi energy is taken as that energy below which, for  $N$  atoms,  $N$  states are filled, two electrons per state.

The final two band structures are similar; one band (the **valence band**) that is completely filled with electrons is separated from an empty **conduction band**, and an **energy band gap** lies between them. For very pure materials, electrons may not have energies within this gap. The difference between the two band structures lies in the magnitude of the energy gap; for materials that are insulators, the band gap is relatively wide (Figure 18.4c), whereas for semiconductors it is narrow (Figure 18.4d). The Fermi energy for these two band structures lies within the band gap—near its center.

### Fermi energy

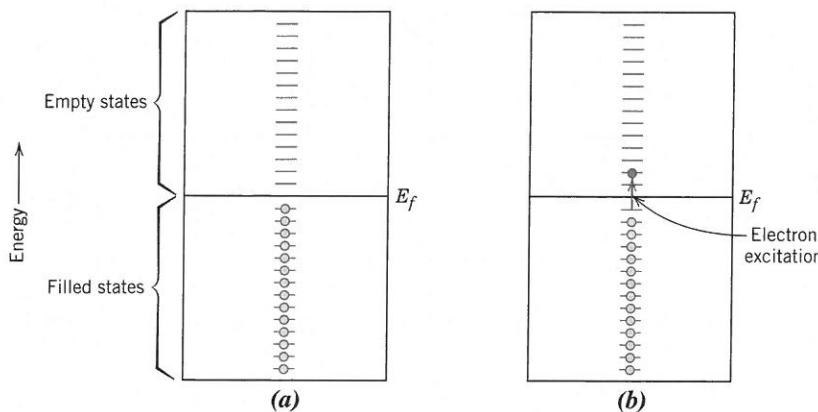
### valence band

### conduction band

### energy band gap

## 18.6 CONDUCTION IN TERMS OF BAND AND ATOMIC BONDING MODELS

At this point in the discussion, it is vital that another concept be understood—namely, that only electrons with energies greater than the Fermi energy may be acted on and accelerated in the presence of an electric field. These are the electrons that participate



**Figure 18.5** For a metal, occupancy of electron states (a) before and (b) after an electron excitation.

free electron  
hole

in the conduction process, which are termed **free electrons**. Another charged electronic entity called a **hole** is found in semiconductors and insulators. Holes have energies less than  $E_f$  and also participate in electronic conduction. The ensuing discussion shows that the electrical conductivity is a direct function of the numbers of free electrons and holes. In addition, the distinction between conductors and nonconductors (insulators and semiconductors) lies in the numbers of these free electron and hole charge carriers.

### Metals

For an electron to become free, it must be excited or promoted into one of the empty and available energy states above  $E_f$ . For metals having either of the band structures shown in Figures 18.4a and 18.4b, there are vacant energy states adjacent to the highest filled state at  $E_f$ . Thus, very little energy is required to promote electrons into the low-lying empty states, as shown in Figure 18.5. Generally, the energy provided by an electric field is sufficient to excite large numbers of electrons into these conducting states.

For the metallic bonding model discussed in Section 2.6, it was assumed that all the valence electrons have freedom of motion and form an *electron gas* that is uniformly distributed throughout the lattice of ion cores. Although these electrons are not locally bound to any particular atom, they must experience some excitation to become conducting electrons that are truly free. Thus, although only a fraction are excited, this still gives rise to a relatively large number of free electrons and, consequently, a high conductivity.

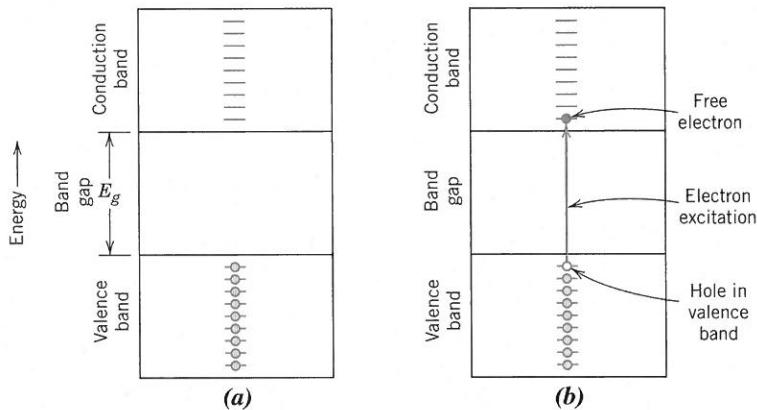
### Insulators and Semiconductors

For insulators and semiconductors, empty states adjacent to the top of the filled valence band are not available. To become free, therefore, electrons must be promoted across the energy band gap and into empty states at the bottom of the conduction band. This is possible only by supplying to an electron the difference in energy between these two states, which is approximately equal to the band gap energy  $E_g$ . This excitation process is demonstrated in Figure 18.6.<sup>2</sup> For many materials, this band gap is several electron volts wide. Most often the excitation energy is from a nonelectrical source such as heat or light, usually the former.

The number of electrons excited thermally (by heat energy) into the conduction band depends on the energy band gap width and the temperature. At a given temperature, the larger the  $E_g$ , the lower the probability that a valence electron will be promoted into an energy state within the conduction band; this results in fewer conduction electrons. In other words, the larger the band gap, the lower the electrical conductivity at a given temperature. Thus, the distinction between semiconductors and insulators lies

<sup>2</sup>The magnitudes of the band gap energy and the energies between adjacent levels in both the valence and conduction bands of Figure 18.6 are not to scale. Whereas the band gap energy is on the order of an electron volt, these levels are separated by energies on the order of  $10^{-10}$  eV.

**Figure 18.6** For an insulator or semiconductor, occupancy of electron states (a) before and (b) after an electron excitation from the valence band into the conduction band, in which both a free electron and a hole are generated.



in the width of the band gap; for semiconductors, it is narrow, whereas for insulating materials, it is relatively wide.

Increasing the temperature of either a semiconductor or an insulator results in an increase in the thermal energy that is available for electron excitation. Thus, more electrons are promoted into the conduction band, which gives rise to an enhanced conductivity.

The conductivity of insulators and semiconductors may also be viewed from the perspective of atomic bonding models discussed in Section 2.6. For electrically insulating materials, interatomic bonding is ionic or strongly covalent. Thus, the valence electrons are tightly bound to or shared with the individual atoms. In other words, these electrons are highly localized and are not in any sense free to wander throughout the crystal. The bonding in semiconductors is covalent (or predominantly covalent) and relatively weak, which means that the valence electrons are not as strongly bound to the atoms. Consequently, these electrons are more easily removed by thermal excitation than they are for insulators.

## 18.7 ELECTRON MOBILITY

When an electric field is applied, a force is brought to bear on the free electrons; as a consequence, they all experience an acceleration in a direction opposite to that of the field, by virtue of their negative charge. According to quantum mechanics, there is no interaction between an accelerating electron and atoms in a perfect crystal lattice. Under such circumstances, all the free electrons should accelerate as long as the electric field is applied, which would give rise to an electric current that is continuously increasing with time. However, we know that a current reaches a constant value the instant that a field is applied, indicating that there exist what might be termed *frictional forces*, which counter this acceleration from the external field. These frictional forces result from the scattering of electrons by imperfections in the crystal lattice, including impurity atoms, vacancies, interstitial atoms, dislocations, and even the thermal vibrations of the atoms themselves. Each scattering event causes an electron to lose kinetic energy and to change its direction of motion, as represented schematically in Figure 18.7. There is, however, some net electron motion in the direction opposite to the field, and this flow of charge is the electric current.

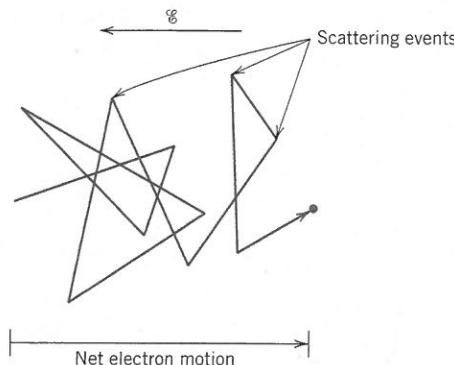
The scattering phenomenon is manifested as a resistance to the passage of an electric current. Several parameters are used to describe the extent of this scattering; these include the *drift velocity* and the **mobility** of an electron. The drift velocity  $v_d$  represents the average electron velocity in the direction of the force imposed by the applied field. It is directly proportional to the electric field as follows:

$$v_d = \mu_e \mathcal{E} \quad (18.7)$$

The constant of proportionality  $\mu_e$  is called the *electron mobility* and is an indication of the frequency of scattering events; its units are square meters per volt-second ( $\text{m}^2/\text{V}\cdot\text{s}$ ).

### mobility

Electron drift velocity—  
dependence on  
electron mobility  
and electric field  
intensity



**Figure 18.7** Schematic diagram showing the path of an electron that is deflected by scattering events.

Electrical conductivity—dependence on electron concentration, charge, and mobility

The conductivity  $\sigma$  of most materials may be expressed as

$$\sigma = n|e|\mu_e \quad (18.8)$$

where  $n$  is the number of free or conducting electrons per unit volume (e.g., per cubic meter) and  $|e|$  is the absolute magnitude of the electrical charge on an electron ( $1.6 \times 10^{-19}$  C). Thus, the electrical conductivity is proportional to both the number of free electrons and the electron mobility.



**Concept Check 18.1** If a metallic material is cooled through its melting temperature at an extremely rapid rate, it forms a noncrystalline solid (i.e., a metallic glass). Will the electrical conductivity of the noncrystalline metal be greater or less than its crystalline counterpart? Why?

[The answer may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site).]

## 18.8 ELECTRICAL RESISTIVITY OF METALS

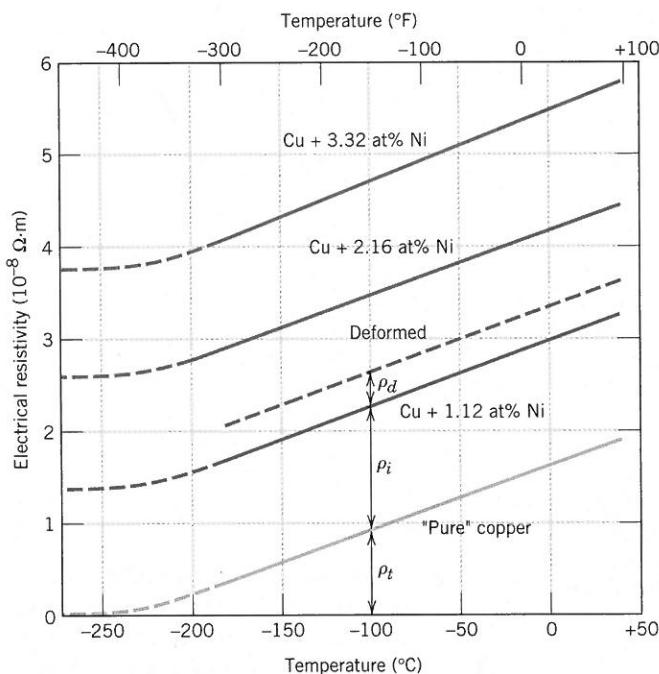
As mentioned previously, most metals are extremely good conductors of electricity; room-temperature conductivities for several of the more common metals are given in Table 18.1. (Table B.9 in Appendix B lists the electrical resistivities of a large number of

**Table 18.1**  
Room-Temperature  
Electrical  
Conductivities for  
Nine Common Metals  
and Alloys

Metal	Electrical Conductivity [( $\Omega \cdot m$ ) <sup>-1</sup> ]
Silver	$6.8 \times 10^7$
Copper	$6.0 \times 10^7$
Gold	$4.3 \times 10^7$
Aluminum	$3.8 \times 10^7$
Brass (70 Cu–30 Zn)	$1.6 \times 10^7$
Iron	$1.0 \times 10^7$
Platinum	$0.94 \times 10^7$
Plain carbon steel	$0.6 \times 10^7$
Stainless steel	$0.2 \times 10^7$

**Figure 18.8** The electrical resistivity versus temperature for copper and three copper–nickel alloys, one of which has been deformed. Thermal, impurity, and deformation contributions to the resistivity are indicated at  $-100^{\circ}\text{C}$ .

[Adapted from J. O. Linde, *Ann. Physik*, **5**, 219 (1932); and C. A. Wert and R. M. Thomson, *Physics of Solids*, 2nd edition, McGraw-Hill Book Company, New York, 1970.]



metals and alloys.) Again, metals have high conductivities because of the large numbers of free electrons that have been excited into empty states above the Fermi energy. Thus  $n$  has a large value in the conductivity expression, Equation 18.8.

At this point it is convenient to discuss conduction in metals in terms of the resistivity, the reciprocal of conductivity; the reason for this switch should become apparent in the ensuing discussion.

Because crystalline defects serve as scattering centers for conduction electrons in metals, increasing their number raises the resistivity (or lowers the conductivity). The concentration of these imperfections depends on temperature, composition, and the degree of cold work of a metal specimen. In fact, it has been observed experimentally that the total resistivity of a metal is the sum of the contributions from thermal vibrations, impurities, and plastic deformation—that is, the scattering mechanisms act independently of one another. This may be represented in mathematical form as follows:

$$\rho_{\text{total}} = \rho_t + \rho_i + \rho_d \quad (18.9)$$

in which  $\rho_t$ ,  $\rho_i$ , and  $\rho_d$  represent the individual thermal, impurity, and deformation resistivity contributions. Equation 18.9 is sometimes known as **Matthiessen's rule**. The influence of each  $\rho$  variable on the total resistivity is demonstrated in Figure 18.8, which is a plot of resistivity versus temperature for copper and several copper–nickel alloys in annealed and deformed states. The additive nature of the individual resistivity contributions is demonstrated at  $-100^{\circ}\text{C}$ .

### Influence of Temperature

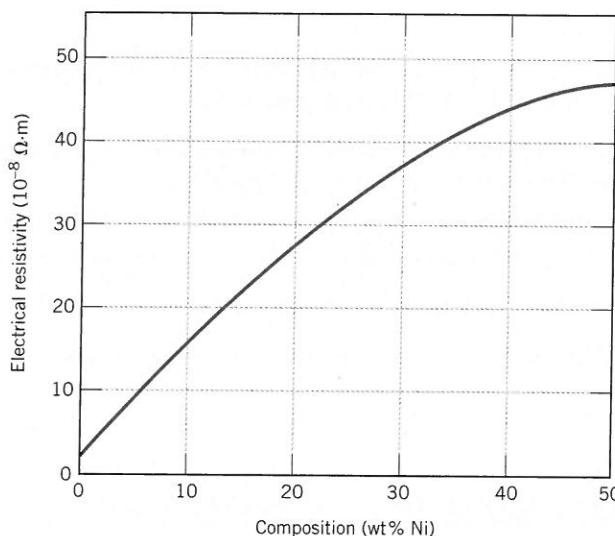
For the pure metal and all the copper–nickel alloys shown in Figure 18.8, the resistivity rises linearly with temperature above about  $-200^{\circ}\text{C}$ . Thus,

$$\rho_t = \rho_0 + aT \quad (18.10)$$

Matthiessen's rule—  
for a metal, total  
electrical resistivity  
equals the sum of  
thermal, impurity,  
and deformation  
contributions

### Matthiessen's rule

Dependence of  
thermal resistivity  
contribution on  
temperature



**Figure 18.9** Room-temperature electrical resistivity versus composition for copper–nickel alloys.

where  $\rho_0$  and  $a$  are constants for each particular metal. This dependence of the thermal resistivity component on temperature is due to the increase with temperature in thermal vibrations and other lattice irregularities (e.g., vacancies), which serve as electron-scattering centers.

### Influence of Impurities

For additions of a single impurity that forms a solid solution, the impurity resistivity  $\rho_i$  is related to the impurity concentration  $c_i$  in terms of the atom fraction (at%/100) as follows:

Impurity resistivity contribution (for solid solution)—dependence on impurity concentration (atom fraction)

$$\rho_i = Ac_i(1 - c_i) \quad (18.11)$$

where  $A$  is a composition-independent constant that is a function of both the impurity and host metals. The influence of nickel impurity additions on the room-temperature resistivity of copper is demonstrated in Figure 18.9, up to 50 wt% Ni; over this composition range nickel is completely soluble in copper (Figure 9.3a). Again, nickel atoms in copper act as scattering centers, and increasing the concentration of nickel in copper results in an enhancement of resistivity.

For a two-phase alloy consisting of  $\alpha$  and  $\beta$  phases, a rule-of-mixtures expression may be used to approximate the resistivity as follows:

$$\rho_i = \rho_\alpha V_\alpha + \rho_\beta V_\beta \quad (18.12)$$

where the  $V_s$  and  $\rho_s$  represent volume fractions and individual resistivities for the respective phases.

### Influence of Plastic Deformation

Plastic deformation also raises the electrical resistivity as a result of increased numbers of electron-scattering dislocations. The effect of deformation on resistivity is also represented in Figure 18.8. Furthermore, its influence is much weaker than that of increasing temperature or the presence of impurities.

Impurity resistivity contribution (for two-phase alloy)—dependence on volume fractions and resistivities of two phases



**Concept Check 18.2** The room-temperature electrical resistivities of pure lead and pure tin are  $2.06 \times 10^{-7}$  and  $1.11 \times 10^{-7} \Omega \cdot \text{m}$ , respectively.

- Make a schematic graph of the room-temperature electrical resistivity versus composition for all compositions between pure lead and pure tin.
- On this same graph, schematically plot electrical resistivity versus composition at 150°C.
- Explain the shapes of these two curves as well as any differences between them.

*Hint:* You may want to consult the lead–tin phase diagram, Figure 9.8.

[The answer may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site).]

## 18.9 ELECTRICAL CHARACTERISTICS OF COMMERCIAL ALLOYS

Electrical and other properties of copper render it the most widely used metallic conductor. Oxygen-free high-conductivity (OFHC) copper, having extremely low oxygen and other impurity contents, is produced for many electrical applications. Aluminum, having a conductivity only about one-half that of copper, is also frequently used as an electrical conductor. Silver has a higher conductivity than either copper or aluminum; however, its use is restricted on the basis of cost.

On occasion, it is necessary to improve the mechanical strength of a metal alloy without impairing significantly its electrical conductivity. Both solid-solution alloying (Section 7.9) and cold working (Section 7.10) improve strength at the expense of conductivity; thus, a trade-off must be made for these two properties. Most often, strength is enhanced by introducing a second phase that does not have so adverse an effect on conductivity. For example, copper–beryllium alloys are precipitation hardened (Section 11.9); even so, the conductivity is reduced by about a factor of 5 over that of high-purity copper.

For some applications, such as furnace heating elements, a high electrical resistivity is desirable. The energy loss by electrons that are scattered is dissipated as heat energy. Such materials must have not only a high resistivity, but also a resistance to oxidation at elevated temperatures and, of course, a high melting temperature. Nichrome, a nickel–chromium alloy, is commonly employed in heating elements.

## MATERIALS OF IMPORTANCE

### Aluminum Electrical Wires

Copper is normally used for electrical wiring in residential and commercial buildings. However, between 1965 and 1973, the price of copper increased significantly and, consequently, aluminum wiring was installed in many buildings constructed or remodeled during this period because aluminum was a less expensive electrical conductor. An inordinately high number

of fires occurred in these buildings, and investigations revealed that the use of aluminum posed an increased fire hazard risk over copper wiring.

When properly installed, aluminum wiring can be just as safe as copper. These safety problems arose at connection points between the aluminum and copper; copper wiring was used for connection

terminals on electrical equipment (circuit breakers, receptacles, switches, etc.) to which the aluminum wiring was attached.

As electrical circuits are turned on and off, the electrical wiring heats up and then cools down. This thermal cycling causes the wires to alternately expand and contract. The amounts of expansion and contraction for aluminum are greater than for copper—aluminum has a higher coefficient of thermal expansion than copper (Section 19.3).<sup>3</sup> Consequently, these differences in expansion and contraction between the aluminum and copper wires can cause the connections to loosen. Another factor that contributes to the loosening of copper-aluminum wire connections is creep (Section 8.12); mechanical stresses exist at these wire connections, and aluminum is more susceptible to creep deformation at or near room temperature than copper. This loosening of the connections compromises the electrical wire-to-wire contact, which increases the electrical resistance at the connection and leads to increased heating. Aluminum oxidizes more readily than copper, and this oxide coating further increases the electrical resistance at the connection. Ultimately, a connection may deteriorate to the point that electrical arcing and/or heat buildup can ignite any combustible materials in the vicinity of the junction. Inasmuch as most receptacles, switches, and other connections are concealed, these materi-

als may smolder or a fire may spread undetected for an extended period of time.

Warning signs that suggest possible connection problems include warm faceplates on switches or receptacles, the smell of burning plastic in the vicinity of outlets or switches, lights that flicker or burn out quickly, unusual static on radio/television, and circuit breakers that trip for no apparent reason.

Several options are available for making buildings wired with aluminum safe.<sup>4</sup> The most obvious (and also most expensive) is to replace all of the aluminum wires with copper. The next-best option is to install a crimp connector repair unit at each aluminum–copper connection. With this technique, a piece of copper wire is attached to the existing aluminum wire branch using a specially designed metal sleeve and powered crimping tool; the metal sleeve is called a “COPALUM parallel splice connector.” The crimping tool essentially makes a cold weld between the two wires. Finally, the connection is encased in an insulating sleeve. A schematic representation of a COPALUM device is shown in Figure 18.10. Only qualified and specially trained electricians are allowed to install these COPALUM connectors.

Two other less-desirable options are CO/ALR devices and pigtailing. A CO/ALR device is simply a switch or wall receptacle that is designed to be used with aluminum wiring. For pigtailing, a twist-

**Table 18.2** Compositions, Electrical Conductivities, and Coefficients of Thermal Expansion for Aluminum and Copper Alloys Used for Electrical Wiring

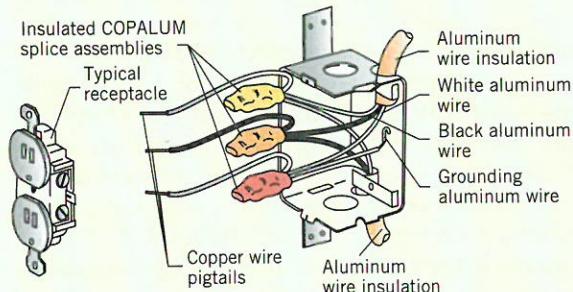
Alloy Name	Alloy Designation	Composition (wt%)	Electrical Conductivity [ $(\Omega \cdot m)^{-1}$ ]	Coefficient of Thermal Expansion ( $^{\circ}C$ ) $^{-1}$
Aluminum (electrical conductor grade)	1350	99.50 Al, 0.10 Si, 0.05 Cu, 0.01 Mn, 0.01 Cr, 0.05 Zn, 0.03 Ga, 0.05 B	$3.57 \times 10^7$	$23.8 \times 10^{-6}$
Copper (electrolytic touch pitch)	C11000	99.90 Cu, 0.04 O	$5.88 \times 10^7$	$17.0 \times 10^{-6}$

<sup>3</sup> Coefficient of thermal expansion values, as well as compositions and other properties of the aluminum and copper alloys used for electrical wiring, are presented in Table 18.2.

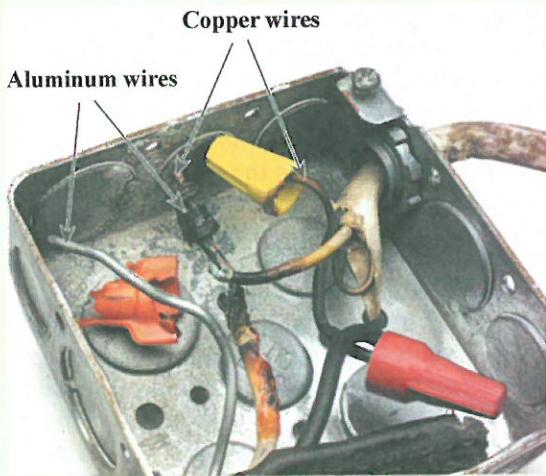
<sup>4</sup> A discussion of the various repair options may be downloaded from the following Web site: <http://www.cpsc.gov/cpscpub/pubs/516.pdf>. (Accessed May 2013.)

(continued)

on connecting wire nut is used, which uses a grease that inhibits corrosion while maintaining a high electrical conductivity at the junction.



**Figure 18.10** Schematic of a COPALUM connector device that is used in aluminum wire electrical circuits. (Reprinted by permission of the U.S. Consumer Product Safety Commission.)



Courtesy of John Fernez

Two copper wire–aluminum wire junctions (located in a junction box) that experienced excessive heating. The one on the right (within the yellow wire nut) failed completely.

## Semiconductivity

**intrinsic semiconductor**

**extrinsic semiconductor**

The electrical conductivity of semiconducting materials is not as high as that of metals; nevertheless, they have some unique electrical characteristics that render them especially useful. The electrical properties of these materials are extremely sensitive to the presence of even minute concentrations of impurities. **Intrinsic semiconductors** are those in which the electrical behavior is based on the electronic structure inherent in the pure material. When the electrical characteristics are dictated by impurity atoms, the semiconductor is said to be **extrinsic**.

### 18.10 INTRINSIC SEMICONDUCTION

Intrinsic semiconductors are characterized by the electron band structure shown in Figure 18.4d: at 0 K, a completely filled valence band, separated from an empty conduction band by a relatively narrow forbidden band gap, generally less than 2 eV. The two elemental semiconductors are silicon (Si) and germanium (Ge), having band gap energies of approximately 1.1 and 0.7 eV, respectively. Both are found in Group IVA of the periodic table (Figure 2.8) and are covalently bonded.<sup>5</sup> In addition, a host of compound semiconducting materials also display intrinsic behavior. One such group is formed between elements of Groups IIIA and VA, for example, gallium arsenide (GaAs) and indium antimonide (InSb); these are frequently called III–V compounds. The compounds composed of elements of Groups IIB and VIA also display semiconducting behavior; these include cadmium sulfide (CdS) and zinc telluride (ZnTe). As the two elements forming these compounds become more widely separated with respect to their relative positions in the periodic table (i.e., the electronegativities become more dissimilar, Figure 2.9), the atomic bonding becomes more ionic and the magnitude of the band gap energy increases—the materials tend to become more insulative. Table 18.3 gives the band gaps for some compound semiconductors.

<sup>5</sup>The valence bands in silicon and germanium correspond to  $sp^3$  hybrid energy levels for the isolated atom; these hybridized valence bands are completely filled at 0 K.

**Table 18.3**  
Band Gap Energies,  
Electron and Hole  
Mobilities, and  
Intrinsic Electrical  
Conductivities at  
Room Temperature  
for Semiconducting  
Materials

Material	Band Gap (eV)	Electron Mobility ( $m^2/V\cdot s$ )	Hole Mobility ( $m^2/V\cdot s$ )	Electrical Conductivity (Intrinsic) ( $\Omega \cdot m$ ) <sup>-1</sup>
<b>Elemental</b>				
Ge	0.67	0.39	0.19	2.2
Si	1.11	0.145	0.050	$3.4 \times 10^{-4}$
<b>III–V Compounds</b>				
AlP	2.42	0.006	0.045	—
AlSb	1.58	0.02	0.042	—
GaAs	1.42	0.80	0.04	$3 \times 10^{-7}$
GaP	2.26	0.011	0.0075	—
InP	1.35	0.460	0.015	$2.5 \times 10^{-6}$
InSb	0.17	8.00	0.125	$2 \times 10^4$
<b>II–VI Compounds</b>				
CdS	2.40	0.040	0.005	—
CdTe	1.56	0.105	0.010	—
ZnS	3.66	0.060	—	—
ZnTe	2.4	0.053	0.010	—

**Source:** This material is reproduced with permission of John Wiley & Sons, Inc.



**Concept Check 18.3** Which of ZnS and CdSe has the larger band gap energy  $E_g$ ? Cite reason(s) for your choice.

[The answer may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site).]

### Concept of a Hole

In intrinsic semiconductors, for every electron excited into the conduction band there is left behind a missing electron in one of the covalent bonds, or in the band scheme, a vacant electron state in the valence band, as shown in Figure 18.6b.<sup>6</sup> Under the influence of an electric field, the position of this missing electron within the crystalline lattice may be thought of as moving by the motion of other valence electrons that repeatedly fill in the incomplete bond (Figure 18.11). This process is expedited by treating a missing electron from the valence band as a positively charged particle called a *hole*. A hole is considered to have a charge that is of the same magnitude as that for an electron, but of opposite sign ( $+1.6 \times 10^{-19} \text{ C}$ ). Thus, in the presence of an electric field, excited electrons and holes move in opposite directions. Furthermore, in semiconductors both electrons and holes are scattered by lattice imperfections.

### Intrinsic Conductivity

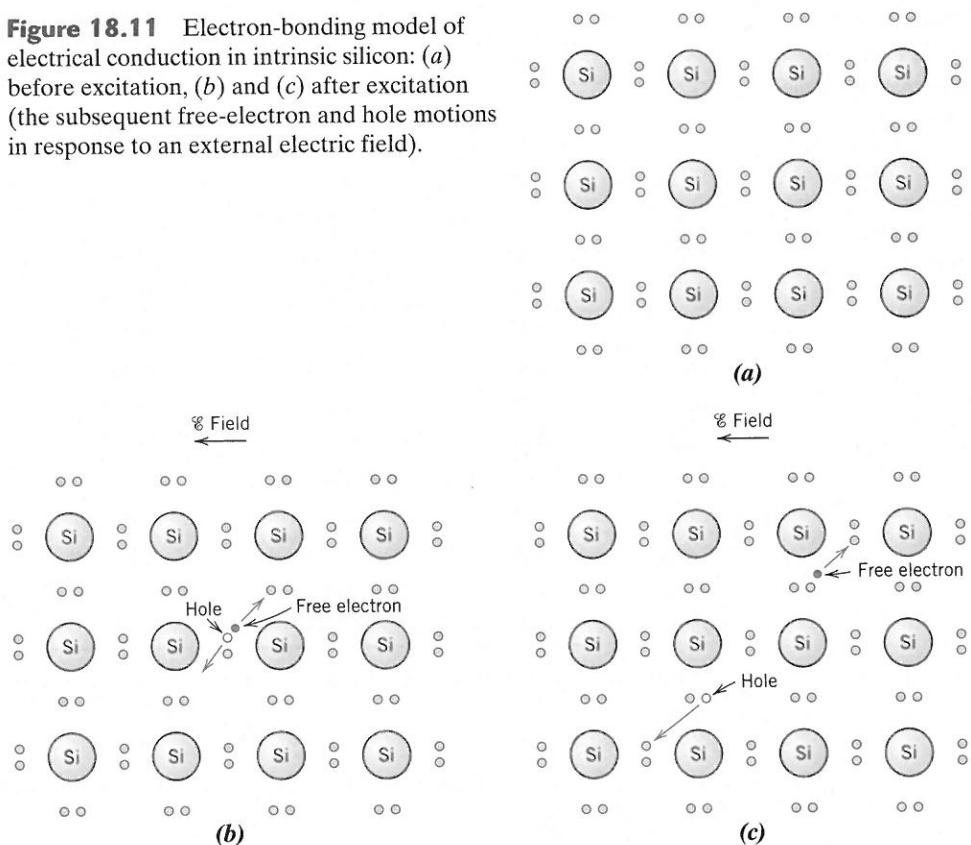
Because there are two types of charge carrier (free electrons and holes) in an intrinsic semiconductor, the expression for electrical conduction, Equation 18.8, must be modified to include a term to account for the contribution of the hole current. Therefore, we write

$$\sigma = n|e|\mu_e + p|e|\mu_h \quad (18.13)$$

Electrical conductivity for an intrinsic semiconductor—dependence on electron/hole concentrations and electron/hole mobilities

<sup>6</sup>Holes (in addition to free electrons) are created in semiconductors and insulators when electron transitions occur from filled states in the valence band to empty states in the conduction band (Figure 18.6). In metals, electron transitions normally occur from empty to filled states *within the same band* (Figure 18.5), without the creation of holes.

**Figure 18.11** Electron-bonding model of electrical conduction in intrinsic silicon: (a) before excitation, (b) and (c) after excitation (the subsequent free-electron and hole motions in response to an external electric field).



where  $p$  is the number of holes per cubic meter and  $\mu_h$  is the hole mobility. The magnitude of  $\mu_h$  is always less than  $\mu_e$  for semiconductors. For intrinsic semiconductors, every electron promoted across the band gap leaves behind a hole in the valence band; thus,

$$n = p = n_i \quad (18.14)$$

where  $n_i$  is known as the *intrinsic carrier concentration*. Furthermore,

$$\begin{aligned} \sigma &= n |e| (\mu_e + \mu_h) = p |e| (\mu_e + \mu_h) \\ &= n_i |e| (\mu_e + \mu_h) \end{aligned} \quad (18.15)$$

The room-temperature intrinsic conductivities and electron and hole mobilities for several semiconducting materials are also presented in Table 18.3.

### EXAMPLE PROBLEM 18.1

#### Computation of the Room-Temperature Intrinsic Carrier Concentration for Gallium Arsenide

For intrinsic gallium arsenide, the room-temperature electrical conductivity is  $3 \times 10^{-7} (\Omega \cdot m)^{-1}$ ; the electron and hole mobilities are, respectively, 0.80 and  $0.04 \text{ m}^2/\text{V}\cdot\text{s}$ . Compute the intrinsic carrier concentration  $n_i$  at room temperature.

For an intrinsic semiconductor, conductivity in terms of intrinsic carrier concentration

**Solution**

Because the material is intrinsic, carrier concentration may be computed using Equation 18.15 as

$$\begin{aligned} n_i &= \frac{\sigma}{|e|(\mu_e + \mu_h)} \\ &= \frac{3 \times 10^{-7} (\Omega \cdot m)^{-1}}{(1.6 \times 10^{-19} C)[(0.80 + 0.04) m^2/V \cdot s]} \\ &= 2.2 \times 10^{12} \text{ m}^{-3} \end{aligned}$$

## 18.11 EXTRINSIC SEMICONDUCTION

Virtually all commercial semiconductors are *extrinsic*—that is, the electrical behavior is determined by impurities that, when present in even minute concentrations, introduce excess electrons or holes. For example, an impurity concentration of 1 atom in  $10^{12}$  is sufficient to render silicon extrinsic at room temperature.

### **n-Type Extrinsic Semiconduction**

To illustrate how extrinsic semiconductor is accomplished, consider again the elemental semiconductor silicon. An Si atom has four electrons, each of which is covalently bonded with one of four adjacent Si atoms. Now, suppose that an impurity atom with a valence of 5 is added as a substitutional impurity; possibilities would include atoms from the Group VA column of the periodic table (i.e., P, As, and Sb). Only four of five valence electrons of these impurity atoms can participate in the bonding because there are only four possible bonds with neighboring atoms. The extra nonbonding electron is loosely bound to the region around the impurity atom by a weak electrostatic attraction, as illustrated in Figure 18.12a. The binding energy of this electron is relatively small (on the order of 0.01 eV); thus, it is easily removed from the impurity atom, in which case it becomes a free or conducting electron (Figures 18.12b and 18.12c).

The energy state of such an electron may be viewed from the perspective of the electron band model scheme. For each of the loosely bound electrons, there exists a single energy level, or energy state, which is located within the forbidden band gap just below the bottom of the conduction band (Figure 18.13a). The electron binding energy corresponds to the energy required to excite the electron from one of these impurity states to a state within the conduction band. Each excitation event (Figure 18.13b) supplies or donates a single electron to the conduction band; an impurity of this type is aptly termed a *donor*. Because each donor electron is excited from an impurity level, no corresponding hole is created within the valence band.

At room temperature, the thermal energy available is sufficient to excite large numbers of electrons from **donor states**; in addition, some intrinsic valence–conduction band transitions occur, as in Figure 18.6b, but to a negligible degree. Thus, the number of electrons in the conduction band far exceeds the number of holes in the valence band (or  $n \gg p$ ), and the first term on the right-hand side of Equation 18.13 overwhelms the second—that is,

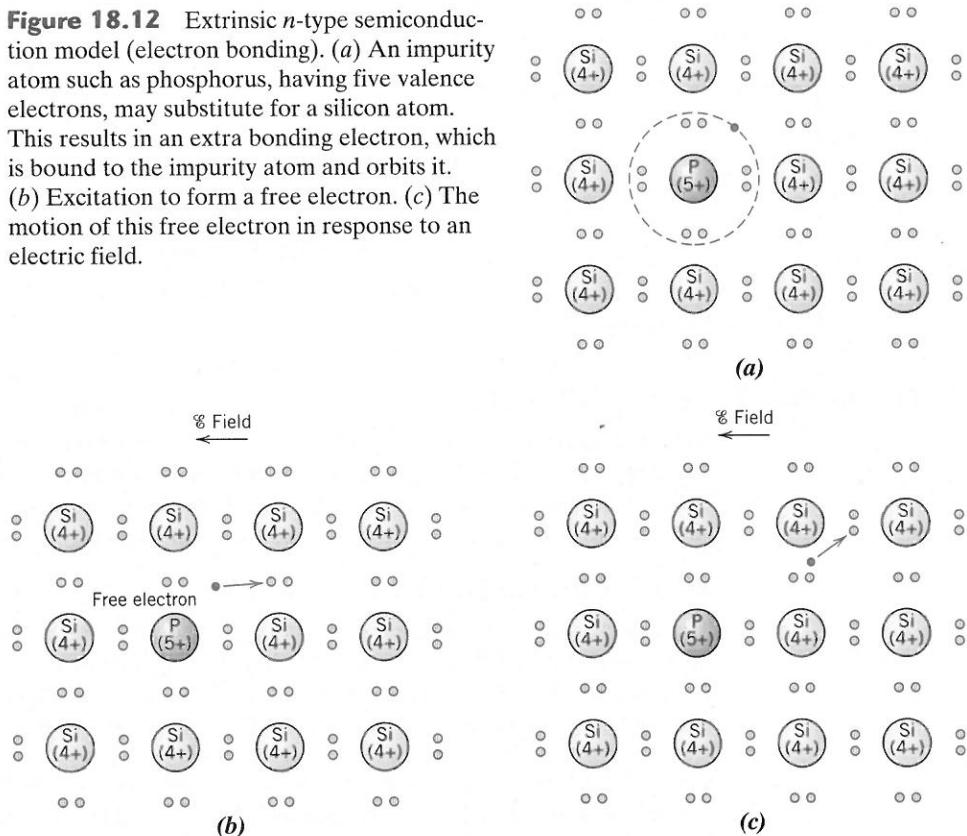
**donor state**

For an *n*-type extrinsic semiconductor, dependence of conductivity on concentration and mobility of electrons

$$\sigma \equiv n |e| \mu_e \quad (18.16)$$

A material of this type is said to be an *n-type* extrinsic semiconductor. The electrons are *majority carriers* by virtue of their density or concentration; holes, on the other hand, are the *minority charge carriers*. For *n*-type semiconductors, the Fermi level is shifted upward in the band gap, to within the vicinity of the donor state; its exact position is a function of both temperature and donor concentration.

**Figure 18.12** Extrinsic *n*-type semiconduction model (electron bonding). (a) An impurity atom such as phosphorus, having five valence electrons, may substitute for a silicon atom. This results in an extra bonding electron, which is bound to the impurity atom and orbits it. (b) Excitation to form a free electron. (c) The motion of this free electron in response to an electric field.

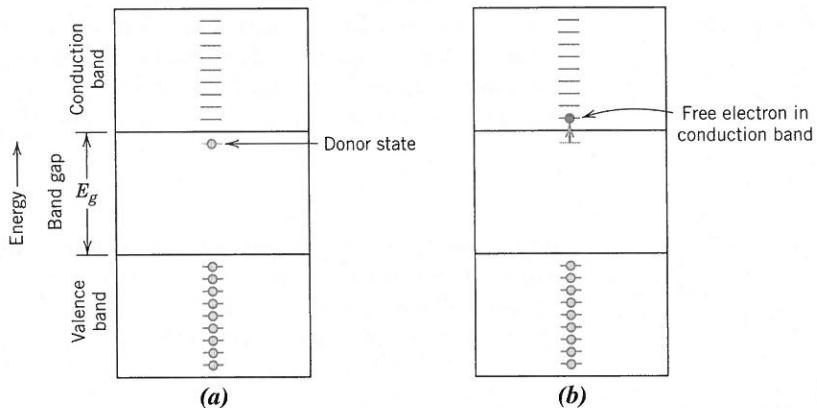


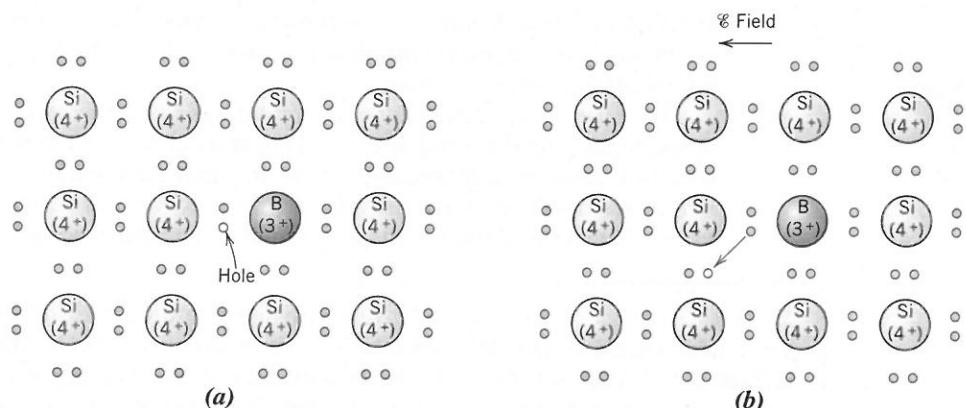
### p-Type Extrinsic Semiconduction

An opposite effect is produced by the addition to silicon or germanium of trivalent substitutional impurities such as aluminum, boron, and gallium from Group IIIA of the periodic table. One of the covalent bonds around each of these atoms is deficient in an electron; such a deficiency may be viewed as a hole that is weakly bound to the impurity atom. This hole may be liberated from the impurity atom by the transfer of an electron from an adjacent bond, as illustrated in Figure 18.14. In essence, the electron and the hole exchange positions. A moving hole is considered to be in an excited state and participates in the conduction process, in a manner analogous to an excited donor electron, as described earlier.

Extrinsic excitations, in which holes are generated, may also be represented using the band model. Each impurity atom of this type introduces an energy level within the

**Figure 18.13** (a) Electron energy band scheme for a donor impurity level located within the band gap and just below the bottom of the conduction band. (b) Excitation from a donor state in which a free electron is generated in the conduction band.





**Figure 18.14** Extrinsic *p*-type semiconductor model (electron bonding). (a) An impurity atom such as boron, having three valence electrons, may substitute for a silicon atom. This results in a deficiency of one valence electron, or a hole associated with the impurity atom. (b) The motion of this hole in response to an electric field.

acceptor state

For a *p*-type extrinsic semiconductor, dependence of conductivity on concentration and mobility of holes

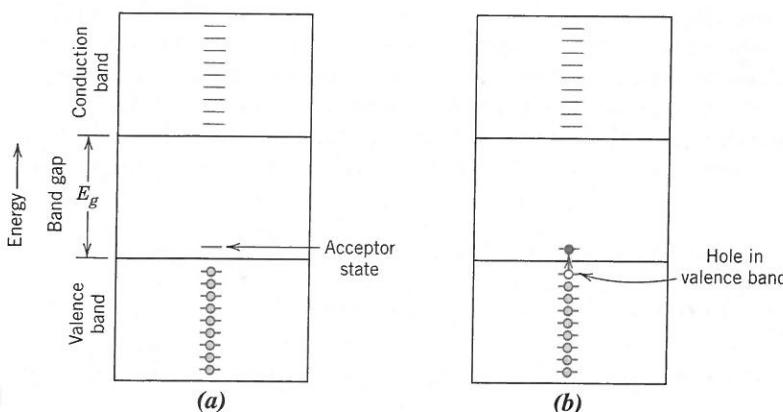
band gap, above yet very close to the top of the valence band (Figure 18.15a). A hole is imagined to be created in the valence band by the thermal excitation of an electron from the valence band into this impurity electron state, as demonstrated in Figure 18.15b. With such a transition, only one carrier is produced—a hole in the valence band; a free electron is *not* created in either the impurity level or the conduction band. An impurity of this type is called an *acceptor* because it is capable of accepting an electron from the valence band, leaving behind a hole. It follows that the energy level within the band gap introduced by this type of impurity is called an *acceptor state*.

For this type of extrinsic conduction, holes are present in much higher concentrations than electrons (i.e.,  $p \gg n$ ), and under these circumstances a material is termed *p*-type because positively charged particles are primarily responsible for electrical conduction. Of course, holes are the majority carriers, and electrons are present in minority concentrations. This gives rise to a predominance of the second term on the right-hand side of Equation 18.13, or

$$\sigma \approx p |e| \mu_h \quad (18.17)$$

For *p*-type semiconductors, the Fermi level is positioned within the band gap and near to the acceptor level.

Extrinsic semiconductors (both *n*- and *p*-type) are produced from materials that are initially of extremely high purity, commonly having total impurity contents on the



**Figure 18.15** (a) Energy band scheme for an acceptor impurity level located within the band gap and just above the top of the valence band. (b) Excitation of an electron into the acceptor level, leaving behind a hole in the valence band.

doping

order of  $10^{-7}$  at %. Controlled concentrations of specific donors or acceptors are then intentionally added, using various techniques. Such an alloying process in semiconducting materials is termed **doping**.

In extrinsic semiconductors, large numbers of charge carriers (either electrons or holes, depending on the impurity type) are created at room temperature by the available thermal energy. As a consequence, relatively high room-temperature electrical conductivities are obtained in extrinsic semiconductors. Most of these materials are designed for use in electronic devices to be operated at ambient conditions.

**Concept Check 18.4** At relatively high temperatures, both donor- and acceptor-doped semiconducting materials exhibit intrinsic behavior (Section 18.12). On the basis of discussions of Section 18.5 and this section, make a schematic plot of Fermi energy versus temperature for an *n*-type semiconductor up to a temperature at which it becomes intrinsic. Also note on this plot energy positions corresponding to the top of the valence band and the bottom of the conduction band.

**Concept Check 18.5** Will Zn act as a donor or as an acceptor when added to the compound semiconductor GaAs? Why? (Assume that Zn is a substitutional impurity.)

[The answers may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site).]

## 18.12 THE TEMPERATURE DEPENDENCE OF CARRIER CONCENTRATION

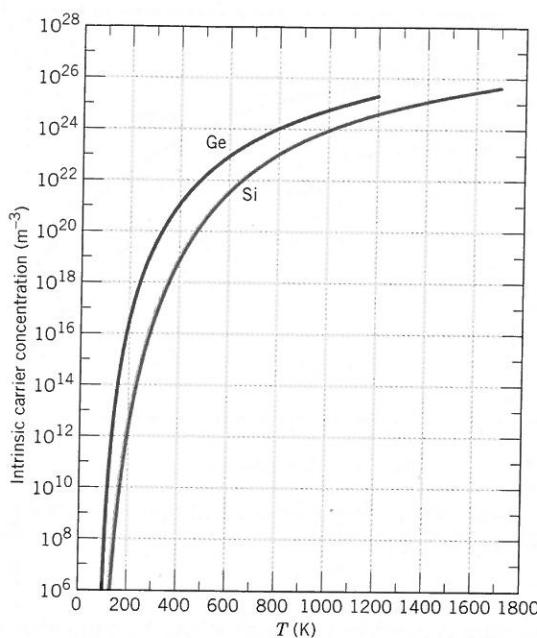
Figure 18.16 plots the logarithm of the *intrinsic* carrier concentration  $n_i$  versus temperature for both silicon and germanium. A couple of features of this plot are worth noting. First, the concentrations of electrons and holes increase with temperature because, with rising temperature, more thermal energy is available to excite electrons from the valence to the conduction band (per Figure 18.6b). In addition, at all temperatures, carrier concentration in Ge is greater than in Si. This effect is due to germanium's smaller band gap (0.67 vs. 1.11 eV, Table 18.3); thus, for Ge, at any given temperature, more electrons will be excited across its band gap.

However, the carrier concentration–temperature behavior for an *extrinsic* semiconductor is much different. For example, electron concentration versus temperature for silicon that has been doped with  $10^{21} \text{ m}^{-3}$  phosphorus atoms is plotted in Figure 18.17. [For comparison, the dashed curve shown is for intrinsic Si (taken from Figure 18.16)].<sup>7</sup> Noted on the extrinsic curve are three regions. At intermediate temperatures (between approximately 150 K and 475 K) the material is *n*-type (inasmuch as P is a donor impurity), and electron concentration is constant; this is termed the *extrinsic-temperature region*.<sup>8</sup> Electrons in the conduction band are excited from the phosphorus donor state (per Figure 18.13b), and because the electron concentration is approximately equal to the P content ( $10^{21} \text{ m}^{-3}$ ), virtually all of the phosphorus atoms have been ionized (i.e., have donated electrons). Also, intrinsic excitations across the band gap are insignificant in relation to these extrinsic donor excitations. The range of temperatures over which this extrinsic region exists depends on impurity concentration; furthermore, most solid-state devices are designed to operate within this temperature range.

At low temperatures, below about 100 K (Figure 18.17), electron concentration drops dramatically with decreasing temperature and approaches zero at 0 K. Over these temperatures, the thermal energy is insufficient to excite electrons from the P donor

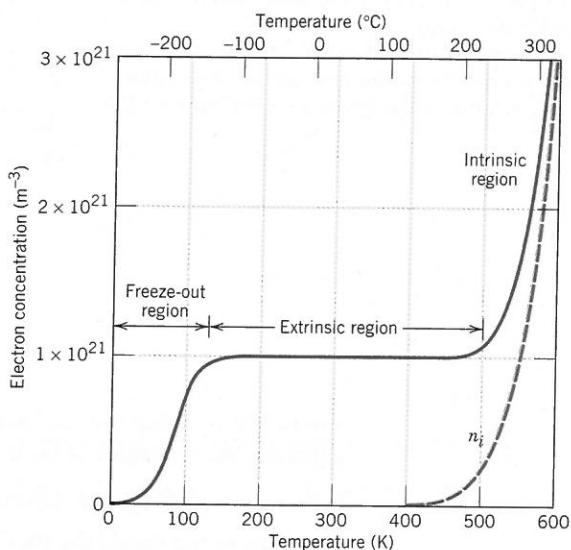
<sup>7</sup>Note that the shapes of the Si curve of Figure 18.16 and the  $n_i$  curve of Figure 18.17 are not the same, even though identical parameters are plotted in both cases. This disparity is due to the scaling of the plot axes: temperature (i.e., horizontal) axes for both plots are scaled linearly; however, the carrier concentration axis of Figure 18.16 is logarithmic, whereas this same axis of Figure 18.17 is linear.

<sup>8</sup>For donor-doped semiconductors, this region is sometimes called the *saturation* region; for acceptor-doped materials, it is often termed the *exhaustion* region.



**Figure 18.16** Intrinsic carrier concentration (logarithmic scale) as a function of temperature for germanium and silicon.

(From C. D. Thurmond, "The Standard Thermodynamic Functions for the Formation of Electrons and Holes in Ge, Si, GaAs, and GaP," *Journal of the Electrochemical Society*, **122**, [8], 1139 (1975). Reprinted by permission of The Electrochemical Society, Inc.)



**Figure 18.17** Electron concentration versus temperature for silicon ( $n$ -type) that has been doped with  $10^{21} m^{-3}$  of a donor impurity and for intrinsic silicon (dashed line). Freeze-out, extrinsic, and intrinsic temperature regimes are noted on this plot.

(From S. M. Sze, *Semiconductor Devices, Physics and Technology*. Copyright © 1985 by Bell Telephone Laboratories, Inc. Reprinted by permission of John Wiley & Sons, Inc.)

level into the conduction band. This is termed the *freeze-out temperature region* inasmuch as charged carriers (i.e., electrons) are “frozen” to the dopant atoms.

Finally, at the high end of the temperature scale of Figure 18.17, electron concentration increases above the P content and asymptotically approaches the intrinsic curve as temperature increases. This is termed the *intrinsic temperature region* because at these high temperatures the semiconductor becomes intrinsic—that is, charge carrier concentrations resulting from electron excitations across the band gap first become equal to and then completely overwhelm the donor carrier contribution with rising temperature.

**Concept Check 18.6** On the basis of Figure 18.17, as dopant level is increased, would you expect the temperature at which a semiconductor becomes intrinsic to increase, to remain essentially the same, or to decrease? Why?

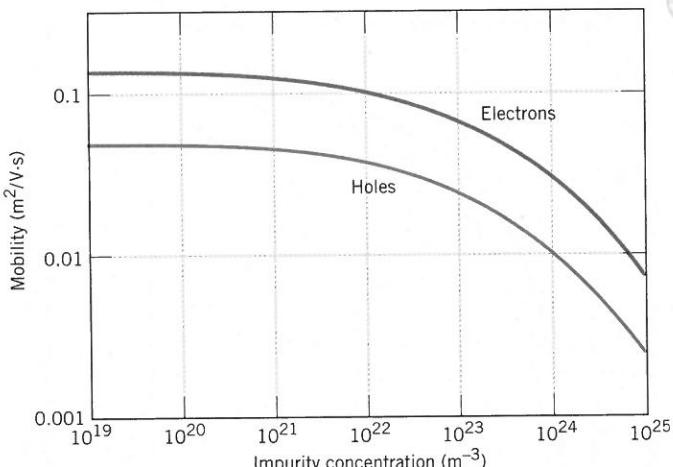
[The answer may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site).]

## 18.13 FACTORS THAT AFFECT CARRIER MOBILITY

The conductivity (or resistivity) of a semiconducting material, in addition to being dependent on electron and/or hole concentrations, is also a function of the charge carriers' mobilities (Equation 18.13)—that is, the ease with which electrons and holes are transported through the crystal. Furthermore, magnitudes of electron and hole mobilities are influenced by the presence of those same crystalline defects that are responsible for the scattering of electrons in metals—thermal vibrations (i.e., temperature) and impurity

**Figure 18.18** For silicon, dependence of room-temperature electron and hole mobilities (logarithmic scale) on dopant concentration (logarithmic scale).

(Adapted from W. W. Gärtner, "Temperature Dependence of Junction Transistor Parameters," *Proc. of the IRE*, **45**, 667, 1957. Copyright © 1957 IRE now IEEE.)



atoms. We now explore the manner in which dopant impurity content and temperature influence the mobilities of both electrons and holes.

### Influence of Dopant Content

Figure 18.18 represents the room-temperature dependence of electron and hole mobilities in silicon as a function of the dopant (both acceptor and donor) content; note that both axes on this plot are scaled logarithmically. At dopant concentrations less than about  $10^{20} m^{-3}$ , both carrier mobilities are at their maximum levels and independent of the doping concentration. In addition, both mobilities decrease with increasing impurity content. Also worth noting is that the mobility of electrons is always larger than the mobility of holes.

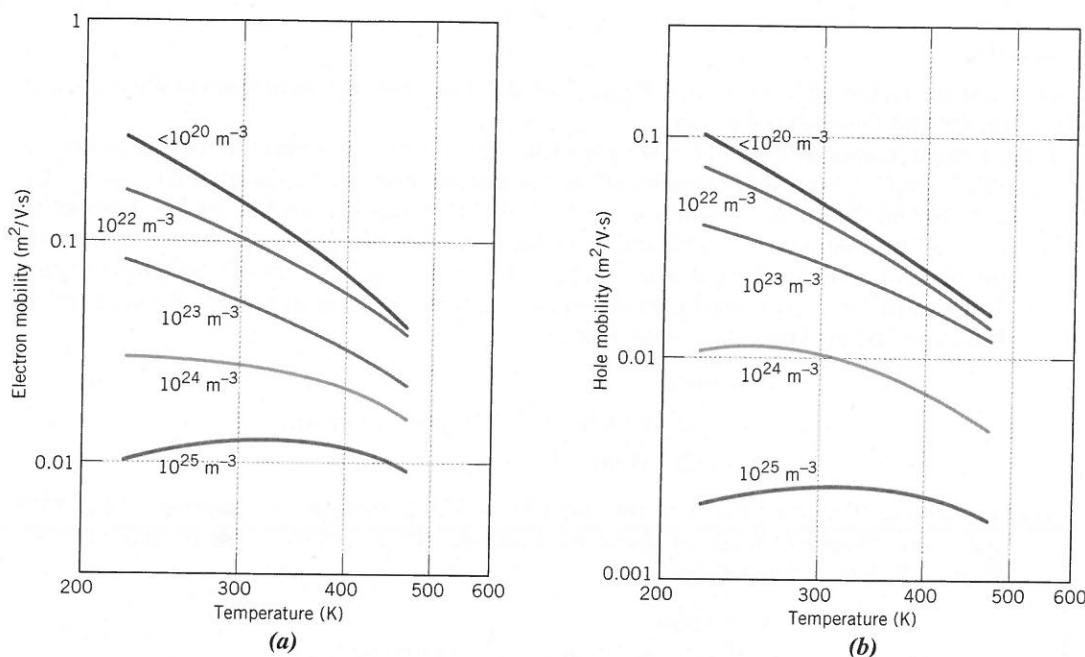
### Influence of Temperature

The temperature dependences of electron and hole mobilities for silicon are presented in Figures 18.19a and 18.19b, respectively. Curves for several impurity dopant contents are shown for both carrier types; note that both sets of axes are scaled logarithmically. From these plots, note that, for dopant concentrations of  $10^{24} m^{-3}$  and less, both electron and hole mobilities decrease in magnitude with rising temperature; again, this effect is due to enhanced thermal scattering of the carriers. For both electrons and holes and dopant levels less than  $10^{20} m^{-3}$ , the dependence of mobility on temperature is independent of acceptor/donor concentration (i.e., is represented by a single curve). Also, for concentrations greater than  $10^{20} m^{-3}$ , curves in both plots are shifted to progressively lower mobility values with increasing dopant level. These latter two effects are consistent with the data presented in Figure 18.18.

These previous treatments discussed the influence of temperature and dopant content on both carrier concentration and carrier mobility. Once values of  $n$ ,  $p$ ,  $\mu_e$ , and  $\mu_h$  have been determined for a specific donor/acceptor concentration and at a specified temperature (using Figures 18.16, through 18.19), computation of  $\sigma$  is possible using Equation 18.15, 18.16, or 18.17.

**Concept Check 18.7** On the basis of the electron-concentration-versus-temperature curve for  $n$ -type silicon shown in Figure 18.17 and the dependence of the logarithm of electron mobility on temperature (Figure 18.19a), make a schematic plot of logarithm electrical conductivity versus temperature for silicon that has been doped with  $10^{21} m^{-3}$  of a donor impurity. Now, briefly explain the shape of this curve. Recall that Equation 18.16 expresses the dependence of conductivity on electron concentration and electron mobility.

[The answer may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site).]



**Figure 18.19** Temperature dependence of (a) electron and (b) hole mobilities for silicon that has been doped with various donor and acceptor concentrations. Both sets of axes are scaled logarithmically.  
 (From W. W. Gärtner, "Temperature Dependence of Junction Transistor Parameters," *Proc. of the IRE*, **45**, 667, 1957. Copyright © 1957 IRE now IEEE.)

### EXAMPLE PROBLEM 18.2

#### Electrical Conductivity Determination for Intrinsic Silicon at 150°C

Calculate the electrical conductivity of intrinsic silicon at 150°C (423 K).

##### Solution

This problem may be solved using Equation 18.15, which requires specification of values for  $n_i$ ,  $\mu_e$ , and  $\mu_h$ . From Figure 18.16,  $n_i$  for Si at 423 K is  $4 \times 10^{19} \text{ m}^{-3}$ . Furthermore, intrinsic electron and hole mobilities are taken from the  $<10^{20} \text{ m}^{-3}$  curves of Figures 18.19a and 18.19b, respectively; at 423 K,  $\mu_e = 0.06 \text{ m}^2/\text{V}\cdot\text{s}$  and  $\mu_h = 0.022 \text{ m}^2/\text{V}\cdot\text{s}$  (realizing that both mobility and temperature axes are scaled logarithmically). Finally, from Equation 18.15, the conductivity is given by

$$\begin{aligned}\sigma &= n_i |e| (\mu_e + \mu_h) \\ &= (4 \times 10^{19} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})(0.06 \text{ m}^2/\text{V}\cdot\text{s} + 0.022 \text{ m}^2/\text{V}\cdot\text{s}) \\ &= 0.52 (\Omega\cdot\text{m})^{-1}\end{aligned}$$

### EXAMPLE PROBLEM 18.3

#### Room-Temperature and Elevated-Temperature Electrical Conductivity Calculations for Extrinsic Silicon

To high-purity silicon is added  $10^{23} \text{ m}^{-3}$  arsenic atoms.

- (a) Is this material *n*-type or *p*-type?
- (b) Calculate the room-temperature electrical conductivity of this material.
- (c) Compute the conductivity at 100°C (373 K).

**Solution**

- (a) Arsenic is a Group VA element (Figure 2.8) and, therefore, acts as a donor in silicon, which means that this material is *n*-type.
- (b) At room temperature (298 K), we are within the extrinsic temperature region of Figure 18.17, which means that virtually all of the arsenic atoms have donated electrons (i.e.,  $n = 10^{23} \text{ m}^{-3}$ ). Furthermore, inasmuch as this material is extrinsic *n*-type, conductivity may be computed using Equation 18.16. Consequently, it is necessary to determine the electron mobility for a donor concentration of  $10^{23} \text{ m}^{-3}$ . We can do this using Figure 18.18: at  $10^{23} \text{ m}^{-3}$ ,  $\mu_e = 0.07 \text{ m}^2/\text{V}\cdot\text{s}$  (remember that both axes of Figure 18.18 are scaled logarithmically). Thus, the conductivity is just

$$\begin{aligned}\sigma &= n|e|\mu_e \\ &= (10^{23} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})(0.07 \text{ m}^2/\text{V}\cdot\text{s}) \\ &= 1120 (\Omega\cdot\text{m})^{-1}\end{aligned}$$

- (c) To solve for the conductivity of this material at 373 K, we again use Equation 18.16 with the electron mobility at this temperature. From the  $10^{23} \text{ m}^{-3}$  curve of Figure 18.19a, at 373 K,  $\mu_e = 0.04 \text{ m}^2/\text{V}\cdot\text{s}$ , which leads to

$$\begin{aligned}\sigma &= n|e|\mu_e \\ &= (10^{23} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})(0.04 \text{ m}^2/\text{V}\cdot\text{s}) \\ &= 640 (\Omega\cdot\text{m})^{-1}\end{aligned}$$

**DESIGN EXAMPLE 18.1****Acceptor Impurity Doping in Silicon**

An extrinsic *p*-type silicon material is desired having a room-temperature conductivity of  $50 (\Omega\cdot\text{m})^{-1}$ . Specify an acceptor impurity type that may be used, as well as its concentration in atom percent, to yield these electrical characteristics.

**Solution**

First, the elements that, when added to silicon, render it *p*-type lie one group to the left of silicon in the periodic table. These include the Group IIIA elements (Figure 2.8): boron, aluminum, gallium, and indium.

Because this material is extrinsic and *p*-type (i.e.,  $p \gg n$ ), the electrical conductivity is a function of both hole concentration and hole mobility according to Equation 18.17. In addition, it is assumed that at room temperature, all the acceptor dopant atoms have accepted electrons to form holes (i.e., that we are in the *extrinsic region* of Figure 18.17), which is to say that the number of holes is approximately equal to the number of acceptor impurities  $N_a$ .

This problem is complicated by the fact that  $\mu_h$  is dependent on impurity content per Figure 18.18. Consequently, one approach to solving this problem is trial and error: assume an impurity concentration, and then compute the conductivity using this value and the corresponding hole mobility from its curve of Figure 18.18. Then, on the basis of this result, repeat the process, assuming another impurity concentration.

For example, let us select an  $N_a$  value (i.e., a *p* value) of  $10^{22} \text{ m}^{-3}$ . At this concentration, the hole mobility is approximately  $0.04 \text{ m}^2/\text{V}\cdot\text{s}$  (Figure 18.18); these values yield a conductivity of

$$\begin{aligned}\sigma &= p|e|\mu_h = (10^{22} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})(0.04 \text{ m}^2/\text{V}\cdot\text{s}) \\ &= 64 (\Omega\cdot\text{m})^{-1}\end{aligned}$$

which is a little on the high side. Decreasing the impurity content an order of magnitude to  $10^{21} \text{ m}^{-3}$  results in only a slight increase of  $\mu_h$  to about  $0.045 \text{ m}^2/\text{V}\cdot\text{s}$  (Figure 18.18); thus, the resulting conductivity is

$$\begin{aligned}\sigma &= (10^{21} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})(0.045 \text{ m}^2/\text{V}\cdot\text{s}) \\ &= 7.2 (\Omega\cdot\text{m})^{-1}\end{aligned}$$

With some fine tuning of these numbers, a conductivity of  $50 (\Omega\cdot\text{m})^{-1}$  is achieved when  $N_a = p \approx 8 \times 10^{21} \text{ m}^{-3}$ ; at this  $N_a$  value,  $\mu_h$  remains approximately  $0.04 \text{ m}^2/\text{V}\cdot\text{s}$ .

It next becomes necessary to calculate the concentration of acceptor impurity in atom percent. This computation first requires the determination of the number of silicon atoms per cubic meter,  $N_{\text{Si}}$ , using Equation 4.2, which is given as follows:

$$\begin{aligned}N_{\text{Si}} &= \frac{N_A \rho_{\text{Si}}}{A_{\text{Si}}} \\ &= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(2.33 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{28.09 \text{ g/mol}} \\ &= 5 \times 10^{28} \text{ m}^{-3}\end{aligned}$$

The concentration of acceptor impurities in atom percent ( $C'_a$ ) is just the ratio of  $N_a$  and  $N_a + N_{\text{Si}}$  multiplied by 100, or

$$\begin{aligned}C'_a &= \frac{N_a}{N_a + N_{\text{Si}}} \times 100 \\ &= \frac{8 \times 10^{21} \text{ m}^{-3}}{(8 \times 10^{21} \text{ m}^{-3}) + (5 \times 10^{28} \text{ m}^{-3})} \times 100 = 1.60 \times 10^{-5}\end{aligned}$$

Thus, a silicon material having a room-temperature *p*-type electrical conductivity of  $50 (\Omega\cdot\text{m})^{-1}$  must contain  $1.60 \times 10^{-5}$  at% boron, aluminum, gallium, or indium.

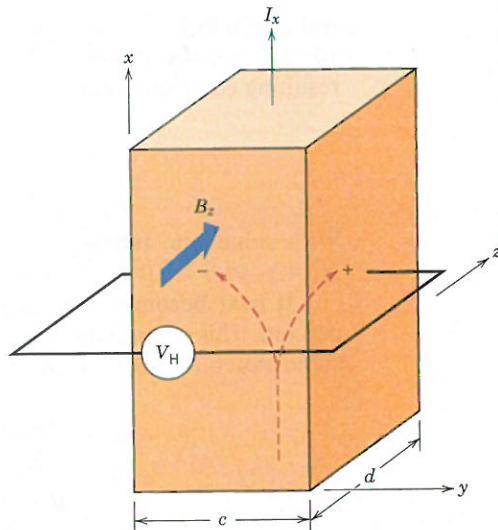
## 18.14 THE HALL EFFECT

### Hall effect

For some materials, it is on occasion desired to determine the material's majority charge carrier type, concentration, and mobility. Such determinations are not possible from a simple electrical conductivity measurement; a **Hall effect** experiment must also be conducted. This Hall effect is a result of the phenomenon by which a magnetic field applied perpendicular to the direction of motion of a charged particle exerts a force on the particle perpendicular to both the magnetic field and the particle motion directions.

To demonstrate the Hall effect, consider the specimen geometry shown in Figure 18.20—a parallelepiped specimen having one corner situated at the origin of a Cartesian coordinate system. In response to an externally applied electric field, the electrons and/or holes move in the  $x$  direction and give rise to a current  $I_x$ . When a magnetic field is imposed in the positive  $z$  direction (denoted as  $B_z$ ), the resulting force on the charge carriers causes them to be deflected in the  $y$  direction—holes (positively charged carriers) to the right specimen face and electrons (negatively charged carriers) to the left face, as indicated in the figure. Thus, a voltage, termed the *Hall voltage*  $V_H$ , is established in

**Figure 18.20** Schematic demonstration of the Hall effect. Positive and/or negative charge carriers that are part of the  $I_x$  current are deflected by the magnetic field  $B_z$  and give rise to the Hall voltage,  $V_H$ .



the  $y$  direction. The magnitude of  $V_H$  depends on  $I_x$ ,  $B_z$ , and the specimen thickness  $d$  as follows:

$$V_H = \frac{R_H I_x B_z}{d} \quad (18.18)$$

Dependence of Hall voltage on the Hall coefficient, specimen thickness, and current and magnetic field parameters shown in Figure 18.20

In this expression  $R_H$  is termed the *Hall coefficient*, which is a constant for a given material. For metals, in which conduction is by electrons,  $R_H$  is negative and is given by

Hall coefficient for metals

$$R_H = \frac{1}{n|e|} \quad (18.19)$$

Thus,  $n$  may be determined because  $R_H$  may be found using Equation 18.18 and the magnitude of  $e$ , the charge on an electron, is known.

Furthermore, from Equation 18.8, the electron mobility  $\mu_e$  is just

$$\mu_e = \frac{\sigma}{n|e|} \quad (18.20a)$$

or, using Equation 18.19,

$$\mu_e = |R_H| \sigma \quad (18.20b)$$

For metals, electron mobility in terms of the Hall coefficient and conductivity

Thus, the magnitude of  $\mu_e$  may also be determined if the conductivity  $\sigma$  has also been measured.

For semiconducting materials, the determination of majority carrier type and computation of carrier concentration and mobility are more complicated and are not discussed here.

**EXAMPLE PROBLEM 18.4****Hall Voltage Computation**

The electrical conductivity and electron mobility for aluminum are  $3.8 \times 10^7 \text{ } (\Omega \cdot \text{m})^{-1}$  and  $0.0012 \text{ m}^2/\text{V} \cdot \text{s}$ , respectively. Calculate the Hall voltage for an aluminum specimen that is 15 mm thick for a current of 25 A and a magnetic field of 0.6 tesla (imposed in a direction perpendicular to the current).

**Solution**

The Hall voltage  $V_H$  may be determined using Equation 18.18. However, it is first necessary to compute the Hall coefficient ( $R_H$ ) from Equation 18.20b as

$$\begin{aligned} R_H &= -\frac{\mu_e}{\sigma} \\ &= -\frac{0.0012 \text{ m}^2/\text{V} \cdot \text{s}}{3.8 \times 10^7 \text{ } (\Omega \cdot \text{m})^{-1}} = -3.16 \times 10^{-11} \text{ V} \cdot \text{m}/\text{A} \cdot \text{tesla} \end{aligned}$$

Now, use of Equation 18.18 leads to

$$\begin{aligned} V_H &= \frac{R_H I_x B_z}{d} \\ &= \frac{(-3.16 \times 10^{-11} \text{ V} \cdot \text{m}/\text{A} \cdot \text{tesla})(25 \text{ A})(0.6 \text{ tesla})}{15 \times 10^{-3} \text{ m}} \\ &= -3.16 \times 10^{-8} \text{ V} \end{aligned}$$

**18.15 SEMICONDUCTOR DEVICES**

The unique electrical properties of semiconductors permit their use in devices to perform specific electronic functions. Diodes and transistors, which have replaced old-fashioned vacuum tubes, are two familiar examples. Advantages of semiconductor devices (sometimes termed *solid-state devices*) include small size, low power consumption, and no warmup time. Vast numbers of extremely small circuits, each consisting of numerous electronic devices, may be incorporated onto a small silicon chip. The invention of semiconductor devices, which has given rise to miniaturized circuitry, is responsible for the advent and extremely rapid growth of a host of new industries in the past few years.

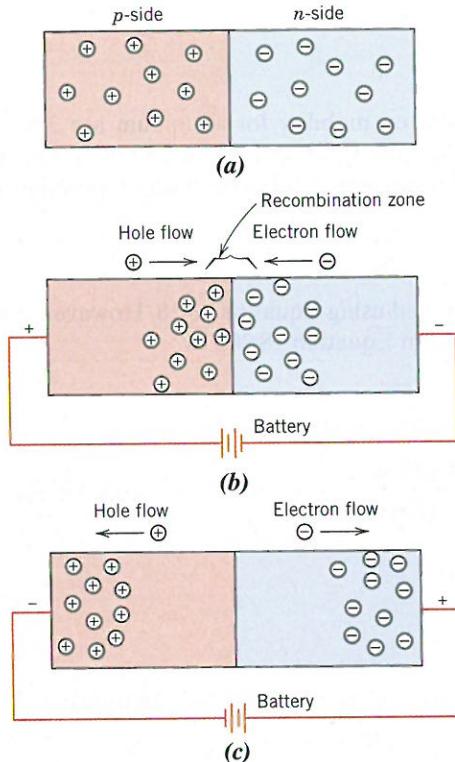
**The *p-n* Rectifying Junction**

**diode**

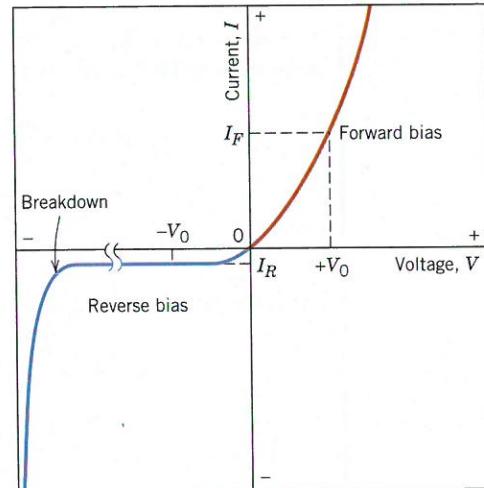
A rectifier, or **diode**, is an electronic device that allows the current to flow in one direction only; for example, a rectifier transforms an alternating current into direct current. Before the advent of the *p-n* junction semiconductor rectifier, this operation was carried out using the vacuum tube diode. The ***p-n* rectifying junction** is constructed from a single piece of semiconductor that is doped so as to be *n*-type on one side and *p*-type on the other (Figure 18.21a). If pieces of *n*- and *p*-type materials are joined together, a poor rectifier results because the presence of a surface between the two sections renders the device very inefficient. Also, single crystals of semiconducting materials must be used in all devices because electronic phenomena deleterious to operation occur at grain boundaries.

**rectifying junction**

Before the application of any potential across the *p-n* specimen, holes are the dominant carriers on the *p*-side, and electrons predominate in the *n*-region, as illustrated in Figure 18.21a. An external electric potential may be established across a *p-n*



**Figure 18.21** For a *p*-*n* rectifying junction, representations of electron and hole distributions for (a) no electrical potential, (b) forward bias, and (c) reverse bias.

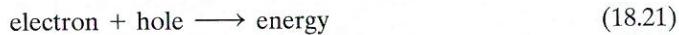


**Figure 18.22** The current–voltage characteristics of a *p*-*n* junction for forward and reverse biases. The phenomenon of breakdown is also shown.

**forward bias**  
**reverse bias**

junction with two different polarities. When a battery is used, the positive terminal may be connected to the *p*-side and the negative terminal to the *n*-side; this is referred to as a **forward bias**. The opposite polarity (minus to *p* and plus to *n*) is termed **reverse bias**.

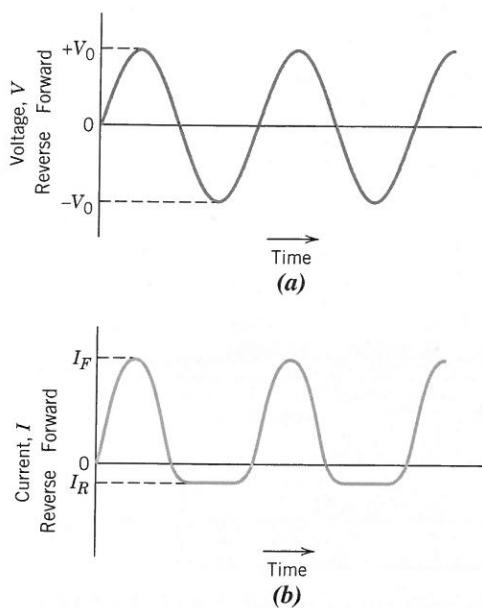
The response of the charge carriers to the application of a forward-biased potential is demonstrated in Figure 18.21*b*. The holes on the *p*-side and the electrons on the *n*-side are attracted to the junction. As electrons and holes encounter one another near the junction, they continuously recombine and annihilate one another, according to



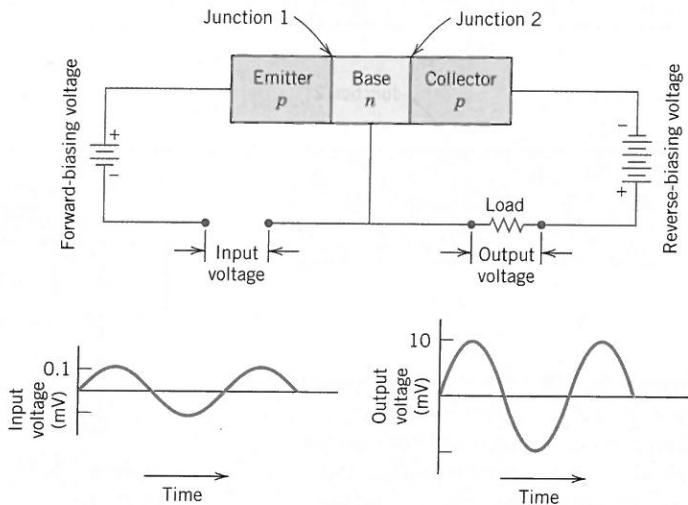
Thus for this bias, large numbers of charge carriers flow across the semiconductor and to the junction, as evidenced by an appreciable current and a low resistivity. The current–voltage characteristics for forward bias are shown on the right-hand half of Figure 18.22.

For reverse bias (Figure 18.21*c*), both holes and electrons, as majority carriers, are rapidly drawn away from the junction; this separation of positive and negative charges (or polarization) leaves the junction region relatively free of mobile charge carriers. Recombination does not occur to any appreciable extent, so that the junction is now highly insulative. Figure 18.22 also illustrates the current–voltage behavior for reverse bias.

The rectification process in terms of input voltage and output current is demonstrated in Figure 18.23. Whereas voltage varies sinusoidally with time (Figure 18.23*a*), maximum current flow for reverse bias voltage  $I_R$  is extremely small in comparison to



**Figure 18.23** (a) Voltage versus time for the input to a  $p-n$  rectifying junction. (b) Current versus time, showing rectification of voltage in (a) by a  $p-n$  rectifying junction having the voltage-current characteristics shown in Figure 18.22.



**Figure 18.24** Schematic diagram of a  $p-n-p$  junction transistor and its associated circuitry, including input and output voltage-time characteristics showing voltage amplification.  
(Adapted from A. G. Guy, *Essentials of Materials Science*, McGraw-Hill Book Company, New York, 1976.)

that for forward bias  $I_F$  (Figure 18.23b). Furthermore, correspondence between  $I_F$  and  $I_R$  and the imposed maximum voltage ( $\pm V_0$ ) is noted in Figure 18.22.

At high reverse bias voltages—sometimes on the order of several hundred volts—large numbers of charge carriers (electrons and holes) are generated. This gives rise to a very abrupt increase in current, a phenomenon known as *breakdown*, also shown in Figure 18.22; this is discussed in more detail in Section 18.22.

### The Transistor

Transistors, which are extremely important semiconducting devices in today's microelectronic circuitry, are capable of two primary types of function. First, they can perform the same operation as their vacuum-tube precursor, the triode—that is, they can amplify an electrical signal. In addition, they serve as switching devices in computers for the processing and storage of information. The two major types are the **junction** (or bimodal) **transistor** and the **metal-oxide-semiconductor field-effect transistor** (abbreviated as **MOSFET**).

#### Junction Transistors

The junction transistor is composed of two  $p-n$  junctions arranged back to back in either the  $n-p-n$  or the  $p-n-p$  configuration; the latter variety is discussed here. Figure 18.24 is a schematic representation of a  $p-n-p$  junction transistor along with its attendant circuitry. A very thin  $n$ -type *base* region is sandwiched between  $p$ -type *emitter* and *collector* regions. The circuit that includes the emitter-base junction (junction 1) is forward biased, whereas a reverse bias voltage is applied across the base-collector junction (junction 2).

Figure 18.25 illustrates the mechanics of operation in terms of the motion of charge carriers. Because the emitter is  $p$ -type and junction 1 is forward biased, large numbers of holes enter the base region. These injected holes are minority carriers in the  $n$ -type base,

junction transistor

MOSFET

### Microelectronic Circuitry

During the past few years, the advent of microelectronic circuitry, in which millions of electronic components and circuits are incorporated into a very small space, has revolutionized the field of electronics. This revolution was precipitated, in part, by aerospace technology, which needed computers and electronic devices that were small and had low power requirements. As a result of refinement in processing and fabrication techniques, there has been an astonishing depreciation in the cost of integrated circuitry. Consequently, personal computers have become affordable to large segments of the population in many countries. Also, the use of **integrated circuits** has become infused into many other facets of our lives—calculators, communications, watches, industrial production and control, and all phases of the electronics industry.

#### integrated circuit

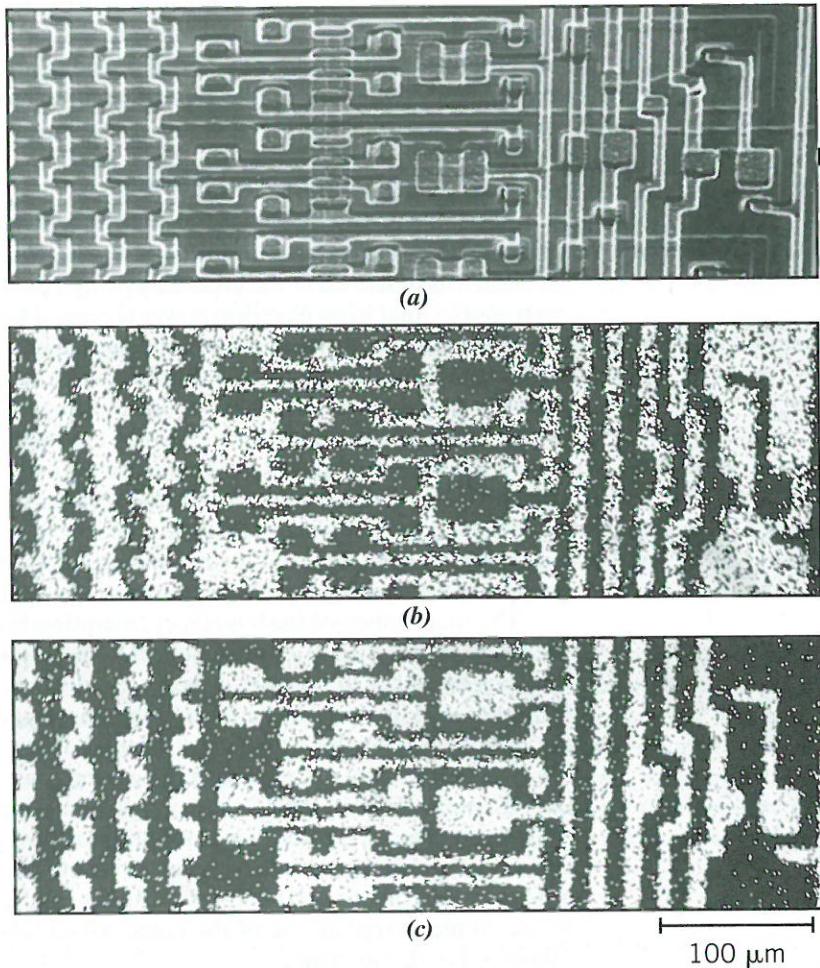
Inexpensive microelectronic circuits are mass produced by using some very ingenious fabrication techniques. The process begins with the growth of relatively large cylindrical single crystals of high-purity silicon from which thin circular wafers are cut. Many microelectronic or integrated circuits, sometimes called *chips*, are prepared on a single wafer. A chip is rectangular, typically on the order of 6 mm ( $\frac{1}{4}$  in.) on a side, and contains millions of circuit elements: diodes, transistors, resistors, and capacitors. Enlarged photographs and elemental maps of a microprocessor chip are presented in Figure 18.27;

**Figure 18.27** (a) Scanning electron micrograph of an integrated circuit.

(b) A silicon dot map for the integrated circuit above, showing regions where silicon atoms are concentrated. Doped silicon is the semiconducting material from which integrated circuit elements are made.

(c) An aluminum dot map. Metallic aluminum is an electrical conductor and, as such, wires the circuit elements together. Approximately 200 $\times$ .

Note: the discussion of Section 4.10 mentioned that an image is generated on a scanning electron micrograph as a beam of electrons scans the surface of the specimen being examined. The electrons in this beam cause some of the specimen surface atoms to emit x-rays; the energy of an x-ray photon depends on the particular atom from which it radiates. It is possible to selectively filter out all but the x-rays emitted from one kind of atom. When projected on a cathode ray tube, small white dots are produced that indicate the locations of the particular atom type; thus, a *dot map* of the image is generated.



these micrographs reveal the intricacy of integrated circuits. At this time, microprocessor chips with densities approaching 1 billion transistors are being produced, and this number doubles about every 18 months.

Microelectronic circuits consist of many layers that lie within or are stacked on top of the silicon wafer in a precisely detailed pattern. Using photolithographic techniques, very small elements for each layer are masked in accordance with a microscopic pattern. Circuit elements are constructed by the selective introduction of specific materials [by diffusion (Section 5.6) or ion implantation] into unmasked regions to create localized *n*-type, *p*-type, high-resistivity, or conductive areas. This procedure is repeated layer by layer until the total integrated circuit has been fabricated, as illustrated in the MOSFET schematic (Figure 18.26). Elements of integrated circuits are shown in Figure 18.27 and in chapter-opening photograph (a).

## Electrical Conduction in Ionic Ceramics and in Polymers

Most polymers and ionic ceramics are insulating materials at room temperature and, therefore, have electron energy band structures similar to that represented in Figure 18.4c; a filled valence band is separated from an empty conduction band by a relatively large band gap, usually greater than 2 eV. Thus, at normal temperatures, only very few electrons may be excited across the band gap by the available thermal energy, which accounts for the very small values of conductivity; Table 18.4 gives the room-temperature electrical conductivities of several of these materials. (The electrical resistivities of a large number of ceramic and polymeric materials are provided in Table B.9, Appendix B.) Many materials are used on the basis of their ability to insulate, and thus a high electrical resistivity is desirable. With rising temperature, insulating materials experience an increase in electrical conductivity, which may ultimately be greater than that for semiconductors.

**Table 18.4**  
Typical Room-Temperature Electrical Conductivities for Thirteen Nonmetallic Materials

Material	Electrical Conductivity [ $(\Omega \cdot m)^{-1}$ ]
Graphite	$3 \times 10^4$ – $2 \times 10^5$
<i>Ceramics</i>	
Concrete (dry)	$10^{-9}$
Soda-lime glass	$10^{-10}$ – $10^{-11}$
Porcelain	$10^{-10}$ – $10^{-12}$
Borosilicate glass	$\sim 10^{-13}$
Aluminum oxide	$< 10^{-13}$
Fused silica	$< 10^{-18}$
<i>Polymers</i>	
Phenol-formaldehyde	$10^{-9}$ – $10^{-10}$
Poly(methyl methacrylate)	$< 10^{-12}$
Nylon 6,6	$10^{-12}$ – $10^{-13}$
Polystyrene	$< 10^{-14}$
Polyethylene	$10^{-15}$ – $10^{-17}$
Polytetrafluoroethylene	$< 10^{-17}$

### 18.16 CONDUCTION IN IONIC MATERIALS

Both cations and anions in ionic materials possess an electric charge and, as a consequence, are capable of migration or diffusion when an electric field is present. Thus, an electric current results from the net movement of these charged ions, which are present in addition to current due to any electron motion. Anion and cation migrations are in opposite directions. The total conductivity of an ionic material  $\sigma_{\text{total}}$  is thus equal to the sum of electronic and ionic contributions, as follows:

$$\sigma_{\text{total}} = \sigma_{\text{electronic}} + \sigma_{\text{ionic}} \quad (18.22)$$

Either contribution may predominate, depending on the material, its purity, and temperature.

A mobility  $\mu_I$  may be associated with each of the ionic species as follows:

$$\mu_I = \frac{n_I e D_I}{kT} \quad (18.23)$$

where  $n_I$  and  $D_I$  represent, respectively, the valence and diffusion coefficient of a particular ion;  $e$ ,  $k$ , and  $T$  denote the same parameters as explained earlier in the chapter. Thus, the ionic contribution to the total conductivity increases with increasing temperature, as does the electronic component. However, in spite of the two conductivity contributions, most ionic materials remain insulative, even at elevated temperatures.

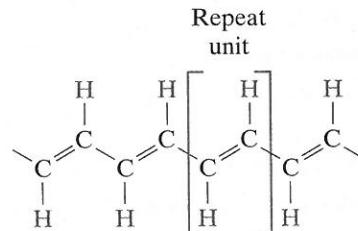
### 18.17 ELECTRICAL PROPERTIES OF POLYMERS

Most polymeric materials are poor conductors of electricity (Table 18.4) due to the unavailability of large numbers of free electrons to participate in the conduction process; electrons in polymers are tightly bound in covalent bonds. The mechanism of electrical conduction in these materials is not well understood, but it is believed that conduction in polymers of high purity is electronic.

#### Conducting Polymers

Polymeric materials have been synthesized that have electrical conductivities on par with those of metallic conductors; they are appropriately termed *conducting polymers*. Conductivities as high as  $1.5 \times 10^7 (\Omega \cdot \text{m})^{-1}$  have been achieved in these materials; on a volume basis, this value corresponds to one-fourth of the conductivity of copper, or twice its conductivity on the basis of weight.

This phenomenon is observed in a dozen or so polymers, including polyacetylene, polyparaphenylene, polypyrrole, and polyaniline. Each of these polymers contains a system of alternating single and double bonds and/or aromatic units in the polymer chain. For example, the chain structure of polyacetylene is as follows:



The valence electrons associated with the alternating single and double chain-bonds are delocalized, which means they are shared among the backbone atoms in the polymer chain—similar to the way that electrons in a partially filled band for a metal are shared by the ion cores. In addition, the band structure of a conductive polymer is characteristic

of that for an electrical insulator (Figure 18.4c)—at 0 K, a filled valence band separated from an empty conduction band by a forbidden energy band gap. In their pure forms, these polymers, which typically have band gap energies greater than 2 eV, are semiconductors or insulators. However, they become conductive when doped with appropriate impurities such as  $\text{AsF}_5$ ,  $\text{SbF}_5$ , or iodine. As with semiconductors, conducting polymers may be made either *n*-type (i.e., free-electron dominant) or *p*-type (i.e., hole dominant), depending on the dopant. However, unlike semiconductors, the dopant atoms or molecules do not substitute for or replace any of the polymer atoms.

The mechanism by which large numbers of free electrons and holes are generated in these conducting polymers is complex and not well understood. In very simple terms, it appears that the dopant atoms lead to the formation of new energy bands that overlap the valence and conduction bands of the intrinsic polymer, giving rise to a partially filled band, and the production at room temperature of a high concentration of free electrons or holes. Orienting the polymer chains, either mechanically (Section 15.7) or magnetically, during synthesis results in a highly anisotropic material having a maximum conductivity along the direction of orientation.

These conducting polymers have the potential to be used in a host of applications inasmuch as they have low densities and are flexible. Rechargeable batteries and fuel cells are being manufactured that use polymer electrodes. In many respects, these batteries are superior to their metallic counterparts. Other possible applications include wiring in aircraft and aerospace components, antistatic coatings for clothing, electromagnetic screening materials, and electronic devices (e.g., transistors, diodes). Several conductive polymers display the phenomenon of *electroluminescence*—that is, light emission stimulated by an electrical current. Electroluminescent polymers are being used in applications such as solar panels and flat panel displays (see the Materials of Importance piece on light-emitting diodes in Chapter 21).

## Dielectric Behavior

**dielectric**

**electric dipole**

A dielectric material is one that is electrically insulating (nonmetallic) and exhibits or may be made to exhibit an **electric dipole** structure—that is, there is a separation of positive and negative electrically charged entities on a molecular or atomic level. This concept of an electric dipole was introduced in Section 2.7. As a result of dipole interactions with electric fields, dielectric materials are used in capacitors.

### 18.18 CAPACITANCE

**capacitance**

Capacitance in terms of stored charge and applied voltage

When a voltage is applied across a capacitor, one plate becomes positively charged and the other negatively charged, with the corresponding electric field directed from the positive to the negative plates. The **capacitance**  $C$  is related to the quantity of charge stored on either plate  $Q$  by

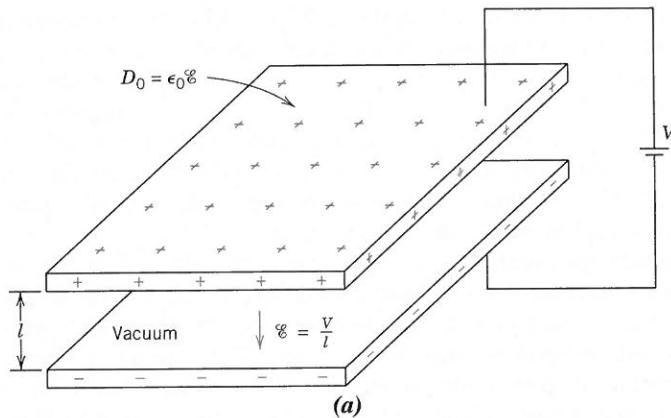
$$C = \frac{Q}{V} \quad (18.24)$$

where  $V$  is the voltage applied across the capacitor. The units of capacitance are coulombs per volt, or farads (F).

Now, consider a parallel-plate capacitor with a vacuum in the region between the plates (Figure 18.28a). The capacitance may be computed from the relationship

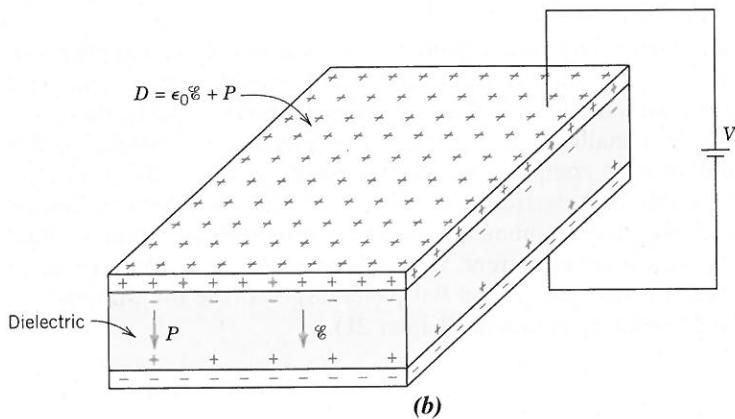
$$C = \epsilon_0 \frac{A}{l} \quad (18.25)$$

Capacitance for parallel-plate capacitor, in a vacuum



**Figure 18.28** A parallel-plate capacitor  
(a) when a vacuum is present and (b) when a dielectric material is present.

(From K. M. Ralls, T. H. Courtney, and J. Wulff, *Introduction to Materials Science and Engineering*. Copyright © 1976 by John Wiley & Sons, Inc. Reprinted by permission of John Wiley & Sons, Inc.)



### permittivity

where  $A$  represents the area of the plates and  $l$  is the distance between them. The parameter  $\epsilon_0$ , called the **permittivity** of a vacuum, is a universal constant having the value of  $8.85 \times 10^{-12} \text{ F/m}$ .

If a dielectric material is inserted into the region within the plates (Figure 18.28b), then

Capacitance for parallel-plate capacitor, with dielectric material

### dielectric constant

where  $\epsilon$  is the permittivity of this dielectric medium, which is greater in magnitude than  $\epsilon_0$ . The relative permittivity  $\epsilon_r$ , often called the **dielectric constant**, is equal to the ratio

### Definition of dielectric constant

$$C = \epsilon \frac{A}{l} \quad (18.26)$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} \quad (18.27)$$

which is greater than unity and represents the increase in charge-storing capacity upon insertion of the dielectric medium between the plates. The dielectric constant is one material property of prime consideration for capacitor design. The  $\epsilon_r$  values of a number of dielectric materials are given in Table 18.5.

**Table 18.5**  
Dielectric Constants  
and Strengths for  
Some Dielectric  
Materials

Material	Dielectric Constant		Dielectric Strength (V/mil) <sup>a</sup>
	60 Hz	1 MHz	
<i>Ceramics</i>			
Titanate ceramics	—	15–10,000	50–300
Mica	—	5.4–8.7	1000–2000
Steatite ( $\text{MgO-SiO}_2$ )	—	5.5–7.5	200–350
Soda-lime glass	6.9	6.9	250
Porcelain	6.0	6.0	40–400
Fused silica	4.0	3.8	250
<i>Polymers</i>			
Phenol-formaldehyde	5.3	4.8	300–400
Nylon 6,6	4.0	3.6	400
Polystyrene	2.6	2.6	500–700
Polyethylene	2.3	2.3	450–500
Polytetrafluoroethylene	2.1	2.1	400–500

<sup>a</sup>One mil = 0.001 in. These values of dielectric strength are average ones, the magnitude being dependent on specimen thickness and geometry, as well as the rate of application and duration of the applied electric field.

## 18.19 FIELD VECTORS AND POLARIZATION

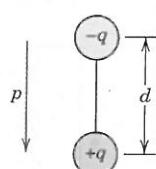
Perhaps the best approach to an explanation of the phenomenon of capacitance is with the aid of field vectors. To begin, for every electric dipole, there is a separation between a positive and a negative electric charge, as demonstrated in Figure 18.29. An electric dipole moment  $p$  is associated with each dipole as follows:

Electric dipole  
moment

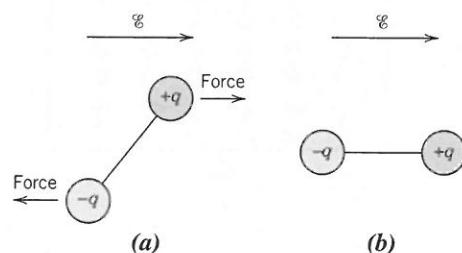
$$p = qd \quad (18.28)$$

polarization

where  $q$  is the magnitude of each dipole charge and  $d$  is the distance of separation between them. A *dipole moment* is a vector that is directed from the negative to the positive charge, as indicated in Figure 18.29. In the presence of an electric field  $\mathcal{E}$ , which is also a vector quantity, a force (or torque) comes to bear on an electric dipole to orient it with the applied field; this phenomenon is illustrated in Figure 18.30. The process of dipole alignment is termed polarization.



**Figure 18.29** Schematic representation of an electric dipole generated by two electric charges (of magnitude  $q$ ) separated by the distance  $d$ ; the associated polarization vector  $p$  is also shown.



**Figure 18.30** (a) Imposed forces (and torque) acting on a dipole by an electric field. (b) Final dipole alignment with the field.

Again, to return to the capacitor, the surface charge density  $D$ , or quantity of charge per unit area of capacitor plate ( $C/m^2$ ), is proportional to the electric field. When a vacuum is present, then

Dielectric displacement  
(surface charge density) in a vacuum

Dielectric displacement when a dielectric medium is present  
**dielectric displacement**

$$D_0 = \epsilon_0 \mathcal{E} \quad (18.29)$$

where the constant of proportionality is  $\epsilon_0$ . Furthermore, an analogous expression exists for the dielectric case—that is,

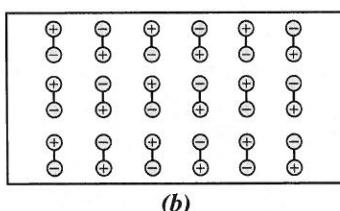
$$D = \epsilon \mathcal{E} \quad (18.30)$$

Sometimes,  $D$  is also called the **dielectric displacement**.

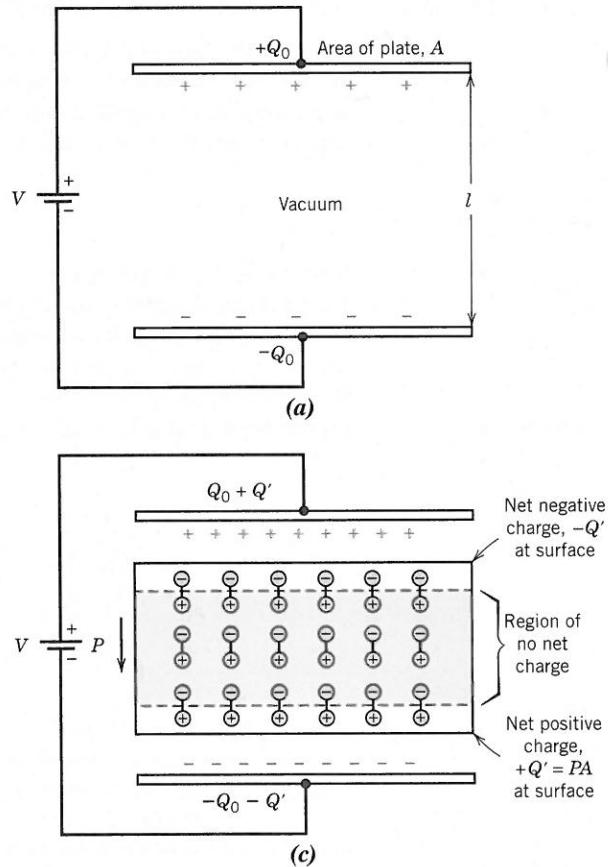
The increase in capacitance, or dielectric constant, can be explained using a simplified model of polarization within a dielectric material. Consider the capacitor in Figure 18.31a—the vacuum situation—where a charge of  $+Q_0$  is stored on the top plate and  $-Q_0$  on the bottom plate. When a dielectric is introduced and an electric field is applied, the entire solid within the plates becomes polarized (Figure 18.31c). As a result of this polarization, there is a net accumulation of negative charge of magnitude  $-Q'$  at the dielectric surface near the positively charged plate and, in a similar manner, a surplus

**Figure 18.31** Schematic representations of (a) the charge stored on capacitor plates for a vacuum, (b) the dipole arrangement in an unpolarized dielectric, and (c) the increased charge-storing capacity resulting from the polarization of a dielectric material.

(Adapted from A. G. Guy, *Essentials of Materials Science*, McGraw-Hill Book Company, New York, 1976.)



(b)



(c)

of  $+Q'$  charge at the surface adjacent to the negative plate. For the region of dielectric removed from these surfaces, polarization effects are not important. Thus, if each plate and its adjacent dielectric surface are considered to be a single entity, the induced charge from the dielectric ( $+Q'$  or  $-Q'$ ) may be thought of as nullifying some of the charge that originally existed on the plate for a vacuum ( $-Q_0$  or  $+Q_0$ ). The voltage imposed across the plates is maintained at the vacuum value by increasing the charge at the negative (or bottom) plate by an amount  $-Q'$  and that at the top plate by  $+Q'$ . Electrons are caused to flow from the positive to the negative plate by the external voltage source such that the proper voltage is reestablished. Thus, the charge on each plate is now  $Q_0 + Q'$ , having been increased by an amount  $Q'$ .

In the presence of a dielectric, the charge density between the plates, which is equal to the surface charge density on the plates of a capacitor, may also be represented by

Dielectric displacement—dependence on electric field intensity and polarization (of dielectric medium)

$$D = \epsilon_0 \mathcal{E} + P \quad (18.31)$$

where  $P$  is the *polarization*, or the increase in charge density above that for a vacuum because of the presence of the dielectric; or, from Figure 18.31c,  $P = Q'/A$ , where  $A$  is the area of each plate. The units of  $P$  are the same as for  $D$  ( $C/m^2$ ).

The polarization  $P$  may also be thought of as the total dipole moment per unit volume of the dielectric material, or as a polarization electric field within the dielectric that results from the mutual alignment of the many atomic or molecular dipoles with the externally applied field  $\mathcal{E}$ . For many dielectric materials,  $P$  is proportional to  $\mathcal{E}$  through the relationship

$$P = \epsilon_0(\epsilon_r - 1)\mathcal{E} \quad (18.32)$$

in which case  $\epsilon_r$  is independent of the magnitude of the electric field.

Table 18.6 lists dielectric parameters along with their units.

**Table 18.6**  
Primary and Derived Units for Various Electrical Parameters and Field Vectors

<i>Quantity</i>	<i>Symbol</i>	<i>SI Units</i>	
		<i>Derived</i>	<i>Primary</i>
Electric potential	<i>V</i>	volt	$kg \cdot m^2/s^2 \cdot C$
Electric current	<i>I</i>	ampere	C/s
Electric field strength	$\mathcal{E}$	volt/meter	$kg \cdot m/s^2 \cdot C$
Resistance	<i>R</i>	ohm	$kg \cdot m^2/s \cdot C^2$
Resistivity	$\rho$	ohm-meter	$kg \cdot m^3/s \cdot C^2$
Conductivity <sup>a</sup>	$\sigma$	$(ohm-meter)^{-1}$	$s \cdot C^2/kg \cdot m^3$
Electric charge	<i>Q</i>	coulomb	C
Capacitance	<i>C</i>	farad	$s^2 \cdot C^2/kg \cdot m^2$
Permittivity	$\epsilon$	farad/meter	$s^2 \cdot C^2/kg \cdot m^3$
Dielectric constant	$\epsilon_r$	dimensionless	dimensionless
Dielectric displacement	<i>D</i>	farad-volt/m <sup>2</sup>	$C/m^2$
Electric polarization	<i>P</i>	farad-volt/m <sup>2</sup>	$C/m^2$

<sup>a</sup>The derived SI units for conductivity are siemens per meters (S/m).

**EXAMPLE PROBLEM 18.5****Computations of Capacitor Properties**

Consider a parallel-plate capacitor having an area of  $6.45 \times 10^{-4} \text{ m}^2$  (1 in.<sup>2</sup>) and a plate separation of  $2 \times 10^{-3} \text{ m}$  (0.08 in.) across which a potential of 10 V is applied. If a material having a dielectric constant of 6.0 is positioned within the region between the plates, compute the following:

- The capacitance
- The magnitude of the charge stored on each plate
- The dielectric displacement  $D$
- The polarization

**Solution**

- (a) Capacitance is calculated using Equation 18.26; however, the permittivity  $\epsilon$  of the dielectric medium must first be determined from Equation 18.27, as follows:

$$\begin{aligned}\epsilon &= \epsilon_r \epsilon_0 = (6.0)(8.85 \times 10^{-12} \text{ F/m}) \\ &= 5.31 \times 10^{-11} \text{ F/m}\end{aligned}$$

Thus, the capacitance is given by

$$\begin{aligned}C &= \epsilon \frac{A}{l} = (5.31 \times 10^{-11} \text{ F/m}) \left( \frac{6.45 \times 10^{-4} \text{ m}^2}{20 \times 10^{-3} \text{ m}} \right) \\ &= 1.71 \times 10^{-11} \text{ F}\end{aligned}$$

- (b) Because the capacitance has been determined, the charge stored may be computed using Equation 18.24, according to

$$Q = CV = (1.71 \times 10^{-11} \text{ F})(10 \text{ V}) = 1.71 \times 10^{-10} \text{ C}$$

- (c) The dielectric displacement is calculated from Equation 18.30, which yields

$$\begin{aligned}D &= \epsilon E = \epsilon \frac{V}{l} = \frac{(5.31 \times 10^{-11} \text{ F/m})(10 \text{ V})}{2 \times 10^{-3} \text{ m}} \\ &= 2.66 \times 10^{-7} \text{ C/m}^2\end{aligned}$$

- (d) Using Equation 18.31, the polarization may be determined as follows:

$$\begin{aligned}P &= D - \epsilon_0 E = D - \epsilon_0 \frac{V}{l} \\ &= 2.66 \times 10^{-7} \text{ C/m}^2 - \frac{(8.85 \times 10^{-12} \text{ F/m})(10 \text{ V})}{2 \times 10^{-3} \text{ m}} \\ &= 2.22 \times 10^{-7} \text{ C/m}^2\end{aligned}$$

**18.20 TYPES OF POLARIZATION**

Again, polarization is the alignment of permanent or induced atomic or molecular dipole moments with an externally applied electric field. There are three types or sources of polarization: electronic, ionic, and orientation. Dielectric materials typically exhibit at least one of these polarization types, depending on the material and the manner of external field application.

electronic polarization

ionic polarization

Electric dipole moment for an ion pair

orientation polarization

### Electronic Polarization

**Electronic polarization** may be induced to one degree or another in all atoms. It results from a displacement of the center of the negatively charged electron cloud relative to the positive nucleus of an atom by the electric field (Figure 18.32a). This polarization type is found in all dielectric materials and exists only while an electric field is present.

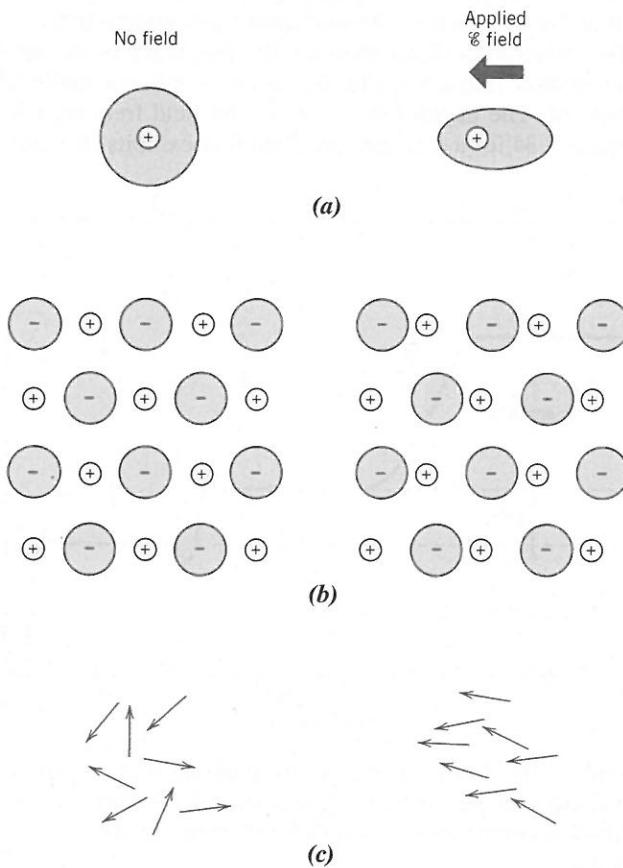
### Ionic Polarization

**Ionic polarization** occurs only in materials that are ionic. An applied field acts to displace cations in one direction and anions in the opposite direction, which gives rise to a net dipole moment. This phenomenon is illustrated in Figure 18.32b. The magnitude of the dipole moment for each ion pair  $p_i$  is equal to the product of the relative displacement  $d_i$  and the charge on each ion, or

$$p_i = qd_i \quad (18.33)$$

### Orientation Polarization

The third type, **orientation polarization**, is found only in substances that possess permanent dipole moments. Polarization results from a rotation of the permanent moments into the direction of the applied field, as represented in Figure 18.32c. This alignment tendency is counteracted by the thermal vibrations of the atoms, such that polarization decreases with increasing temperature.



**Figure 18.32** (a) Electronic polarization that results from the distortion of an atomic electron cloud by an electric field. (b) Ionic polarization that results from the relative displacements of electrically charged ions in response to an electric field. (c) Response of permanent electric dipoles (arrows) to an applied electric field, producing orientation polarization. (From O. H. Wyatt and D. Dew-Hughes, *Metals, Ceramics and Polymers*, Cambridge University Press, 1974. Reprinted with the permission of the Cambridge University Press.)

The total polarization  $P$  of a substance is equal to the sum of the electronic, ionic, and orientation polarizations ( $P_e$ ,  $P_i$ , and  $P_o$ , respectively), or

$$P = P_e + P_i + P_o \quad (18.34)$$

Total polarization of a substance equals the sum of electronic, ionic, and orientation polarizations

It is possible for one or more of these contributions to the total polarization to be either absent or negligible in magnitude relative to the others. For example, ionic polarization does not exist in covalently bonded materials in which no ions are present.

**Concept Check 18.9** For solid lead titanate ( $\text{PbTiO}_3$ ), what kind(s) of polarization is (are) possible? Why? Note: Lead titanate has the same crystal structure as barium titanate (Figure 18.35).

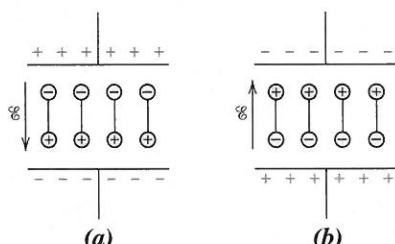
[The answer may be found at [www.wiley.com/college/callister](http://www.wiley.com/college/callister) (Student Companion Site).]

## 18.21 FREQUENCY DEPENDENCE OF THE DIELECTRIC CONSTANT

relaxation frequency

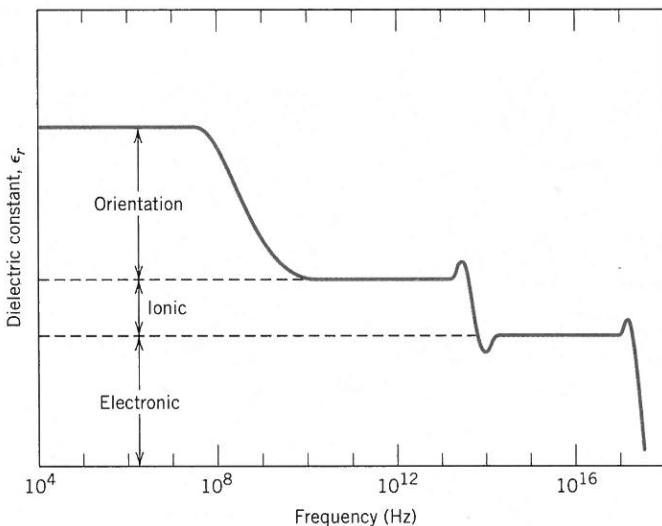
In many practical situations, the current is alternating (ac)—that is, an applied voltage or electric field changes direction with time, as indicated in Figure 18.23a. Consider a dielectric material that is subject to polarization by an ac electric field. With each direction reversal, the dipoles attempt to reorient with the field, as illustrated in Figure 18.33, in a process requiring some finite time. For each polarization type, some minimum reorientation time exists that depends on the ease with which the particular dipoles are capable of realignment. The relaxation frequency is taken as the reciprocal of this minimum reorientation time.

A dipole cannot keep shifting orientation direction when the frequency of the applied electric field exceeds its relaxation frequency and, therefore, it will not make a contribution to the dielectric constant. The dependence of  $\epsilon_r$  on the field frequency is represented schematically in Figure 18.34 for a dielectric medium that exhibits all three



**Figure 18.33** Dipole orientations for (a) one polarity of an alternating electric field and (b) the reversed polarity.

(From Richard A. Flinn and Paul K. Trojan, *Engineering Materials and Their Applications*, 4th edition. Copyright © 1990 by John Wiley & Sons, Inc. Adapted by permission of John Wiley & Sons, Inc.)



**Figure 18.34** Variation of dielectric constant with frequency of an alternating electric field. Electronic, ionic, and orientation polarization contributions to the dielectric constant are indicated.

types of polarization; note that the frequency axis is scaled logarithmically. As indicated in Figure 18.34, when a polarization mechanism ceases to function, there is an abrupt drop in the dielectric constant; otherwise,  $\epsilon_r$  is virtually frequency independent. Table 18.5 gave values of the dielectric constant at 60 Hz and 1 MHz; these provide an indication of this frequency dependence at the low end of the frequency spectrum.

The absorption of electrical energy by a dielectric material that is subjected to an alternating electric field is termed *dielectric loss*. This loss may be important at electric field frequencies in the vicinity of the relaxation frequency for each of the operative dipole types for a specific material. A low dielectric loss is desired at the frequency of utilization.

## 18.22 DIELECTRIC STRENGTH

When very high electric fields are applied across dielectric materials, large numbers of electrons may suddenly be excited to energies within the conduction band. As a result, the current through the dielectric by the motion of these electrons increases dramatically; sometimes localized melting, burning, or vaporization produces irreversible degradation and perhaps even failure of the material. This phenomenon is known as *dielectric breakdown*. The **dielectric strength**, sometimes called the *breakdown strength*, represents the magnitude of an electric field necessary to produce breakdown. Table 18.5 presents dielectric strengths for several materials.

**dielectric strength**

## 18.23 DIELECTRIC MATERIALS

A number of ceramics and polymers are used as insulators and/or in capacitors. Many of the ceramics, including glass, porcelain, steatite, and mica, have dielectric constants within the range of 6 to 10 (Table 18.5). These materials also exhibit a high degree of dimensional stability and mechanical strength. Typical applications include power line and electrical insulation, switch bases, and light receptacles. The titania ( $TiO_2$ ) and titanate ceramics, such as barium titanate ( $BaTiO_3$ ), can be made to have extremely high dielectric constants, which render them especially useful for some capacitor applications.

The magnitude of the dielectric constant for most polymers is less than for ceramics because the latter may exhibit greater dipole moments:  $\epsilon_r$  values for polymers generally lie between 2 and 5. These materials are commonly used for insulation of wires, cables, motors, generators, and so on and, in addition, for some capacitors.

# Other Electrical Characteristics of Materials

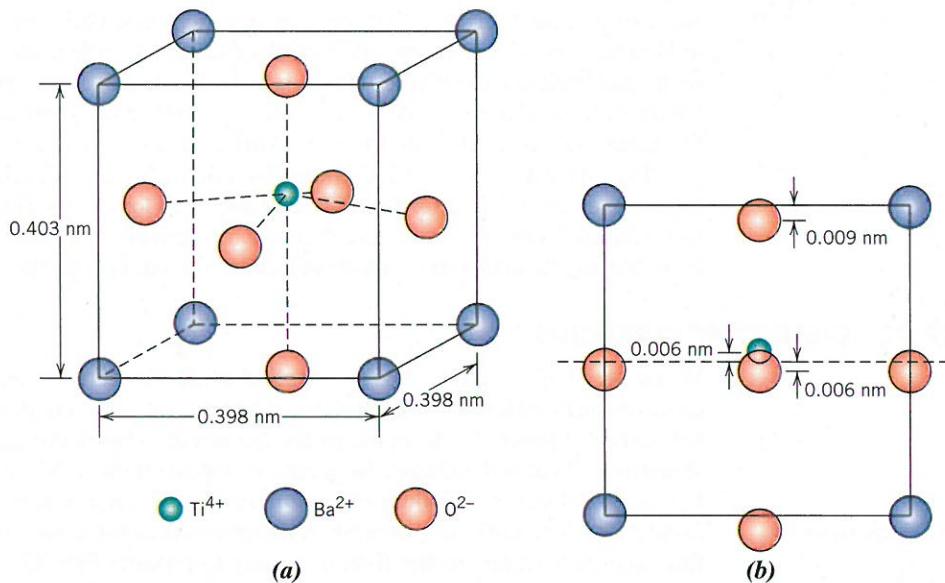
Two other relatively important and novel electrical characteristics that are found in some materials deserve brief mention—ferroelectricity and piezoelectricity.

## 18.24 FERROELECTRICITY

**ferroelectric**

The group of dielectric materials called **ferroelectrics** exhibit *spontaneous polarization*—that is, polarization in the absence of an electric field. They are the dielectric analogue of ferromagnetic materials, which may display permanent magnetic behavior. There must exist in ferroelectric materials permanent electric dipoles, the origin of which is explained for barium titanate, one of the most common ferroelectrics. The spontaneous polarization is a consequence of the positioning of the  $Ba^{2+}$ ,  $Ti^{4+}$ , and  $O^{2-}$  ions within the unit cell, as represented in Figure 18.35. The  $Ba^{2+}$  ions are located at the corners of the unit cell, which is of *tetragonal symmetry* (a cube that has been elongated slightly in one direction). The dipole moment results from the relative displacements of the  $O^{2-}$  and  $Ti^{4+}$  ions from their symmetrical positions, as shown in the side view of the unit cell. The  $O^{2-}$  ions are located near, but slightly below, the centers of each of the six faces, whereas the  $Ti^{4+}$  ion

**Figure 18.35** A barium titanate ( $\text{BaTiO}_3$ ) unit cell (a) in an isometric projection, and (b) looking at one face, which shows the displacements of  $\text{Ti}^{4+}$  and  $\text{O}^{2-}$  ions from the center of the face.



is displaced upward from the unit cell center. Thus, a permanent ionic dipole moment is associated with each unit cell (Figure 18.35b). However, when barium titanate is heated above its *ferroelectric Curie temperature* [120°C (250°F)], the unit cell becomes cubic, and all ions assume symmetric positions within the cubic unit cell; the material now has a perovskite crystal structure (Figure 12.6), and the ferroelectric behavior ceases.

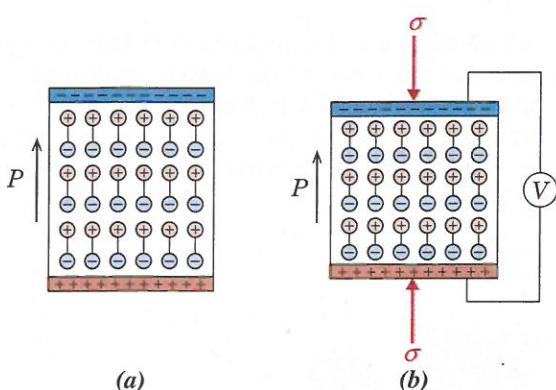
Spontaneous polarization of this group of materials results as a consequence of interactions between adjacent permanent dipoles in which they mutually align, all in the same direction. For example, with barium titanate, the relative displacements of  $\text{O}^{2-}$  and  $\text{Ti}^{4+}$  ions are in the same direction for all the unit cells within some volume region of the specimen. Other materials display ferroelectricity; these include Rochelle salt ( $\text{NaKC}_4\text{H}_4\text{O}_6 \cdot 4\text{H}_2\text{O}$ ), potassium dihydrogen phosphate ( $\text{KH}_2\text{PO}_4$ ), potassium niobate ( $\text{KNbO}_3$ ), and lead zirconate-titanate ( $\text{Pb}[\text{ZrO}_3, \text{TiO}_3]$ ). Ferroelectrics have extremely high dielectric constants at relatively low applied field frequencies; for example, at room temperature,  $\epsilon_r$  for barium titanate may be as high as 5000. Consequently, capacitors made from these materials can be significantly smaller than capacitors made from other dielectric materials.

## 18.25 PIEZOELECTRICITY

An unusual phenomenon exhibited by a few ceramic materials (as well as some polymers) is *piezoelectricity*—literally, pressure electricity. Electric polarization (i.e., an electric field or voltage) is induced in the piezoelectric crystal as a result of a mechanical strain (dimensional change) produced from the application of an external force (Figure 18.36). Reversing the sign of the force (e.g., from tension to compression) reverses the direction of the field. The inverse piezoelectric effect is also displayed by this group of materials—that is, a mechanical strain results from the imposition of an electrical field.

### piezoelectric

**Piezoelectric** materials may be used as transducers between electrical and mechanical energies. One of the early uses of piezoelectric ceramics was in sonar systems, in which underwater objects (e.g., submarines) are detected and their positions determined using an ultrasonic emitting and receiving system. A piezoelectric crystal is caused to oscillate



**Figure 18.36** (a) Dipoles within a piezoelectric material. (b) A voltage is generated when the material is subjected to a compressive stress.  
(© 1989 by Addison-Wesley Publishing Company, Inc.)

by an electrical signal, which produces high-frequency mechanical vibrations that are transmitted through the water. Upon encountering an object, the signals are reflected, and another piezoelectric material receives this reflected vibrational energy, which it then converts back into an electrical signal. Distance from the ultrasonic source and reflecting body is determined from the elapsed time between sending and receiving events.

More recently, the use of piezoelectric devices has grown dramatically as a consequence of increases in automation and consumer attraction to modern sophisticated gadgets. Piezoelectric devices are used in many of today's applications, including automotive—wheel balances, seat-belt buzzers, tread-wear indicators, keyless door entry, and airbag sensors; computer/electronic—microphones, speakers, microactuators for hard disks and notebook transformers; commercial/consumer—ink-jet printing heads, strain gauges, ultrasonic welders, and smoke detectors; medical—insulin pumps, ultrasonic therapy, and ultrasonic cataract-removal devices.

Piezoelectric ceramic materials include titanates of barium and lead ( $\text{BaTiO}_3$  and  $\text{PbTiO}_3$ ), lead zirconate ( $\text{PbZrO}_3$ ), lead zirconate-titanate (PZT) [ $\text{Pb}(\text{Zr},\text{Ti})\text{O}_3$ ], and potassium niobate ( $\text{KNbO}_3$ ). This property is characteristic of materials having complicated crystal structures with a low degree of symmetry. The piezoelectric behavior of a polycrystalline specimen may be improved by heating above its Curie temperature and then cooling to room temperature in a strong electric field.

## MATERIAL OF IMPORTANCE

### Piezoelectric Ceramic Ink-Jet Printer Heads

Piezoelectric materials are used in one kind of ink-jet printer head that has components and a mode of operation represented in the schematic diagrams in Figure 18.37a through 18.37c. One head component is a flexible, bilayer disk that consists of a piezoelectric ceramic (orange region) bonded to a nonpiezoelectric deformable material (green region); liquid ink

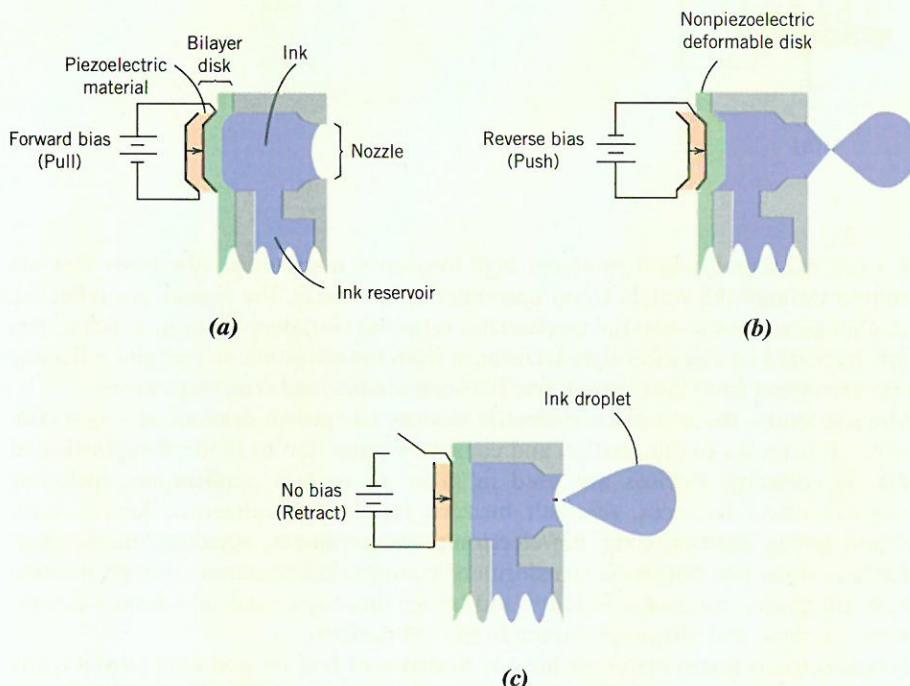
and its reservoir are represented by blue areas in these diagrams. Short, horizontal arrows within the piezoelectric note the direction of the permanent dipole moment.

Printer head operation (i.e., ejection of ink droplets from the nozzle) is a result of the inverse piezoelectric effect—that is, the bilayer disk is caused to flex

(continued)

back and forth by the expansion and contraction of the piezoelectric layer in response to changes in bias of an applied voltage. For example, Figure 18.37a shows how the imposition of forward bias voltage causes the bilayer disk to flex in such a way as to pull (or draw) ink from the reservoir into the nozzle chamber. Reversing

the voltage bias forces the bilayer disk to bend in the opposite direction—toward the nozzle—so as to eject a drop of ink (Figure 18.37b). Finally, removal of the voltage causes the disk to return to its unbent configuration (Figure 18.37c) in preparation for another ejection sequence.



**Figure 18.37** Operation sequence of a piezoelectric ceramic ink-jet printer head (schematic). (a) Imposing a forward-bias voltage draws ink into the nozzle chamber as the bilayer disk flexes in one direction. (b) Ejection of an ink drop by reversing the voltage bias and forcing the disk to flex in the opposite direction. (c) Removing the voltage retracts the bilayer disk to its unbent configuration in preparation for the next sequence. (Images provided courtesy of Epson America, Inc.)

## SUMMARY

### Ohm's Law

### Electrical Conductivity

- The ease with which a material is capable of transmitting an electric current is expressed in terms of electrical conductivity or its reciprocal, electrical resistivity (Equations 18.2 and 18.3).
- The relationship between applied voltage, current, and resistance is Ohm's law (Equation 18.1). An equivalent expression, Equation 18.5, relates current density, conductivity, and electric field intensity.
- On the basis of its conductivity, a solid material may be classified as a metal, a semiconductor, or an insulator.

### Electronic and Ionic Conduction

- For most materials, an electric current results from the motion of free electrons, which are accelerated in response to an applied electric field.
- In ionic materials, there may also be a net motion of ions, which also makes a contribution to the conduction process.

### Energy Band Structures in Solids

#### Conduction in Terms of Band and Atomic Bonding Models

- The number of free electrons depends on the electron energy band structure of the material.
- An electron band is a series of electron states that are closely spaced with respect to energy, and one such band may exist for each electron subshell found in the isolated atom.
- Electron energy band structure* refers to the manner in which the outermost bands are arranged relative to one another and then filled with electrons.

For metals, two band structure types are possible (Figures 18.4a and 18.4b)—empty electron states are adjacent to filled ones.

Band structures for semiconductors and insulators are similar—both have a forbidden energy band gap that, at 0 K, lies between a filled valence band and an empty conduction band. The magnitude of this band gap is relatively wide ( $>2$  eV) for insulators (Figure 18.4c) and relatively narrow ( $<2$  eV) for semiconductors (Figure 18.4d).

- An electron becomes free by being excited from a filled state to an available empty state at a higher energy.

Relatively small energies are required for electron excitations in metals (Figure 18.5), giving rise to large numbers of free electrons.

Greater energies are required for electron excitations in semiconductors and insulators (Figure 18.6), which accounts for their lower free electron concentrations and smaller conductivity values.

### Electron Mobility

- Free electrons being acted on by an electric field are scattered by imperfections in the crystal lattice. The magnitude of electron mobility is indicative of the frequency of these scattering events.
- In many materials, the electrical conductivity is proportional to the product of the electron concentration and the mobility (per Equation 18.8).

### Electrical Resistivity of Metals

- For metallic materials, electrical resistivity increases with temperature, impurity content, and plastic deformation. The contribution of each to the total resistivity is additive—per Matthiessen's rule, Equation 18.9.
- Thermal and impurity contributions (for both solid solutions and two-phase alloys) are described by Equations 18.10, 18.11, and 18.12.

### Intrinsic Semiconduction

#### Extrinsic Semiconduction

- Semiconductors may be either elements (Si and Ge) or covalently bonded compounds.
- With these materials, in addition to free electrons, holes (missing electrons in the valence band) may also participate in the conduction process (Figure 18.11).
- Semiconductors are classified as either intrinsic or extrinsic.

For intrinsic behavior, the electrical properties are inherent in the pure material, and electron and hole concentrations are equal. The electrical conductivity may be computed using Equation 18.13 (or Equation 18.15).

Electrical behavior is dictated by impurities for extrinsic semiconductors.

Extrinsic semiconductors may be either *n*- or *p*-type depending on whether electrons or holes, respectively, are the predominant charge carriers.

- Donor impurities introduce excess electrons (Figures 18.12 and 18.13); acceptor impurities introduce excess holes (Figures 18.14 and 18.15).
- The electrical conductivity on an *n*-type semiconductor may be calculated using Equation 18.16; for a *p*-type semiconductor, Equation 18.17 is used.
- With rising temperature, intrinsic carrier concentration increases dramatically (Figure 18.16).
- For extrinsic semiconductors, on a plot of majority carrier concentration versus temperature, carrier concentration is independent of temperature in the *extrinsic region*.

### The Temperature Dependence of Carrier Concentration

### Factors That Affect Carrier Mobility

(Figure 18.17). The magnitude of carrier concentration in this region is approximately equal to the impurity level.

- For extrinsic semiconductors, electron and hole mobilities (1) decrease as impurity content increases (Figure 18.18) and (2) in general, decrease with rising temperature (Figures 18.19a and 18.19b).

#### The Hall Effect

- Using a Hall effect experiment, it is possible to determine the charge carrier type (i.e., electron or hole), as well as carrier concentration and mobility.

#### Semiconductor Devices

- A number of semiconducting devices employ the unique electrical characteristics of these materials to perform specific electronic functions.
- The *p–n* rectifying junction (Figure 18.21) is used to transform alternating current into direct current.
- Another type of semiconductor device is the transistor, which may be used for amplification of electrical signals, as well as for switching devices in computer circuitries. Junction and MOSFET transistors (Figures 18.24, 18.25, and 18.26) are possible.

#### Electrical Conduction in Ionic Ceramics and in Polymers

- Most ionic ceramics and polymers are insulators at room temperature. Electrical conductivities range between about  $10^{-9}$  and  $10^{-18} (\Omega \cdot m)^{-1}$ ; by way of comparison, for most metals,  $\sigma$  is on the order of  $10^7 (\Omega \cdot m)^{-1}$ .

#### Dielectric Behavior

##### Capacitance

##### Field Vectors and Polarization

- A *dipole* is said to exist when there is a net spatial separation of positively and negatively charged entities on an atomic or molecular level.
- *Polarization* is the alignment of electric dipoles with an electric field.
- *Dielectric materials* are electrical insulators that may be polarized when an electric field is present.
- This polarization phenomenon accounts for the ability of the dielectrics to increase the charge-storing capability of capacitors.
- Capacitance is dependent on applied voltage and quantity of charge stored according to Equation 18.24.
- The charge-storing efficiency of a capacitor is expressed in terms of a dielectric constant or relative permittivity (Equation 18.27).
- For a parallel-plate capacitor, capacitance is a function of the permittivity of the material between the plates, as well as plate area and plate separation distance per Equation 18.26.
- The dielectric displacement within a dielectric medium depends on the applied electric field and the induced polarization according to Equation 18.31.
- For some dielectric materials, the polarization induced by an applied electric field is described by Equation 18.32.

#### Types of Polarization Frequency

##### Dependence of the Dielectric Constant

- Possible polarization types include electronic (Figure 18.32a), ionic (Figure 18.32b), and orientation (Figure 18.32c); not all types need be present in a particular dielectric.
- For alternating electric fields, whether a specific polarization type contributes to the total polarization and dielectric constant depends on frequency; each polarization mechanism ceases to function when the applied field frequency exceeds its relaxation frequency (Figure 18.34).

#### Other Electrical Characteristics of Materials

- Ferroelectric materials exhibit spontaneous polarization—that is, they polarize in the absence of an electric field.
- An electric field is generated when mechanical stresses are applied to a piezoelectric material.

## Equation Summary

Equation Number	Equation	Solving For	Page Number
18.1	$V = IR$	Voltage (Ohm's law)	726
18.2	$\rho = \frac{RA}{l}$	Electrical resistivity	727
18.4	$\sigma = \frac{1}{\rho}$	Electrical conductivity	727
18.5	$J = \sigma \mathcal{E}$	Current density	727
18.6	$\mathcal{E} = \frac{V}{l}$	Electric field intensity	727
18.8, 18.16	$\sigma = n e \mu_e$	Electrical conductivity (metal); conductivity for <i>n</i> -type extrinsic semiconductor	733, 741
18.9	$\rho_{\text{total}} = \rho_t + \rho_i + \rho_d$	For metals, total resistivity (Matthiessen's rule)	734
18.10	$\rho_t = \rho_0 + aT$	Thermal resistivity contribution	734
18.11	$\rho_i = A c_i (1 - c_i)$	Impurity resistivity contribution—single-phase alloy	735
18.12	$\rho_i = \rho_\alpha V_\alpha + \rho_\beta V_\beta$	Impurity resistivity contribution—two-phase alloy	735
18.13	$\sigma = n e \mu_e + p e \mu_h$	Conductivity for intrinsic semiconductor	739, 740
18.15	$= n_i e (\mu_e + \mu_h)$		
18.17	$\sigma \approx p e \mu_h$	Conductivity for <i>p</i> -type extrinsic semiconductor	743
18.24	$C = \frac{Q}{V}$	Capacitance	759
18.25	$C = \epsilon_0 \frac{A}{l}$	Capacitance for a parallel-plate capacitor in a vacuum	759
18.26	$C = \epsilon \frac{A}{l}$	Capacitance for a parallel-plate capacitor with a dielectric medium between plates	760
18.27	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$	Dielectric constant	760
18.29	$D_0 = \epsilon_0 \mathcal{E}$	Dielectric displacement in a vacuum	762
18.30	$D = \epsilon \mathcal{E}$	Dielectric displacement in a dielectric material	762
18.31	$D = \epsilon_0 \mathcal{E} + P$	Dielectric displacement	763
18.32	$P = \epsilon_0(\epsilon_r - 1)\mathcal{E}$	Polarization	763



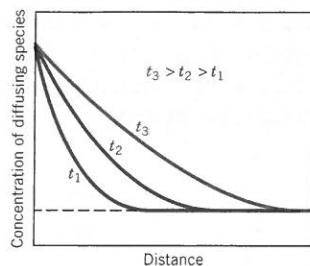
### List of Symbols

<i>Symbol</i>	<i>Meaning</i>
<i>A</i>	Plate area for a parallel-plate capacitor; concentration-independent constant
<i>a</i>	Temperature-independent constant
<i>c<sub>i</sub></i>	Concentration in terms of atom fraction
$ e $	Absolute magnitude of charge on an electron ( $1.6 \times 10^{-19}$ C)
<i>I</i>	Electric current
<i>l</i>	Distance between contact points that are used to measure voltage (Figure 18.1); plate separation distance for a parallel-plate capacitor (Figure 18.28a)
<i>n</i>	Number of free electrons per unit volume
<i>n<sub>i</sub></i>	Intrinsic carrier concentration
<i>p</i>	Number of holes per unit volume
<i>Q</i>	Quantity of charge stored on a capacitor plate
<i>R</i>	Resistance
<i>T</i>	Temperature
<i>V<sub>α</sub>, V<sub>β</sub></i>	Volume fractions of $\alpha$ and $\beta$ phases
$\epsilon$	Permittivity of a dielectric material
$\epsilon_0$	Permittivity of a vacuum ( $8.85 \times 10^{-12}$ F/m)
$\mu_e, \mu_h$	Electron, hole mobilities
$\rho_{\alpha}, \rho_{\beta}$	Electrical resistivities of $\alpha$ and $\beta$ phases
$\rho_0$	Concentration-independent constant



### Processing/Structure/Properties/Performance Summary

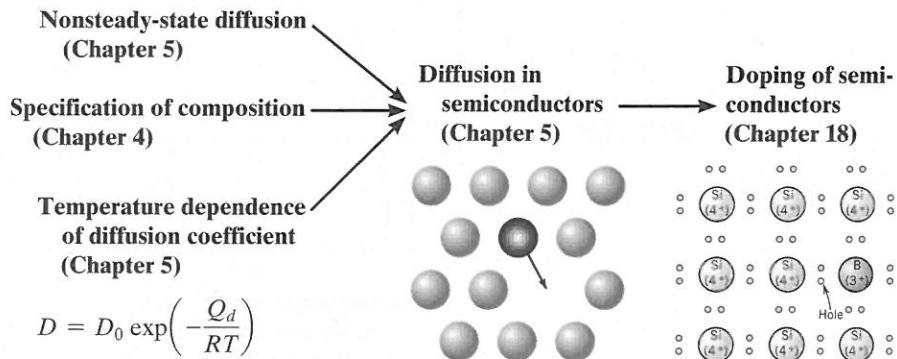
Relative to the processing, in Chapter 5 we discussed principles of diffusion as they apply to semiconductors (specifically silicon). In some instances, doping of impurity atoms (which makes the semiconductor extrinsic) is accomplished by diffusion. The following concept map illustrates these relationships:



### Silicon Semiconductors (Processing)

$$\frac{C_x - C_0}{C_s - C_0} = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$C_1 = \frac{m_1}{m_1 + m_2} \times 100$$

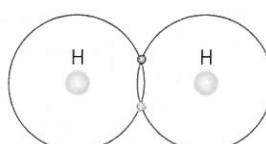


One of the important structural elements of semiconductors is electron band structure. We discussed this concept as well as band structures for both intrinsic and extrinsic materials. Band structure, to some degree, is due to the covalent (or predominantly covalent) interatomic bonding, which, in turn, is a consequence of the electron configuration of the semiconductor (Chapter 2). These relationships are noted in the following concept map:

### Silicon Semiconductors (Structure)

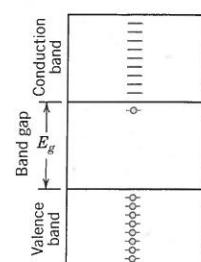
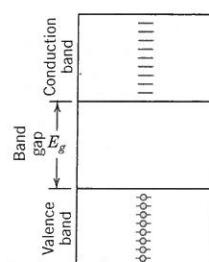
Electron configuration: Si  
(Chapter 2)  
 $1s^2 2s^2 2p^6 3s^2 3p^2$

Covalent interatomic bonding (Chapter 2)

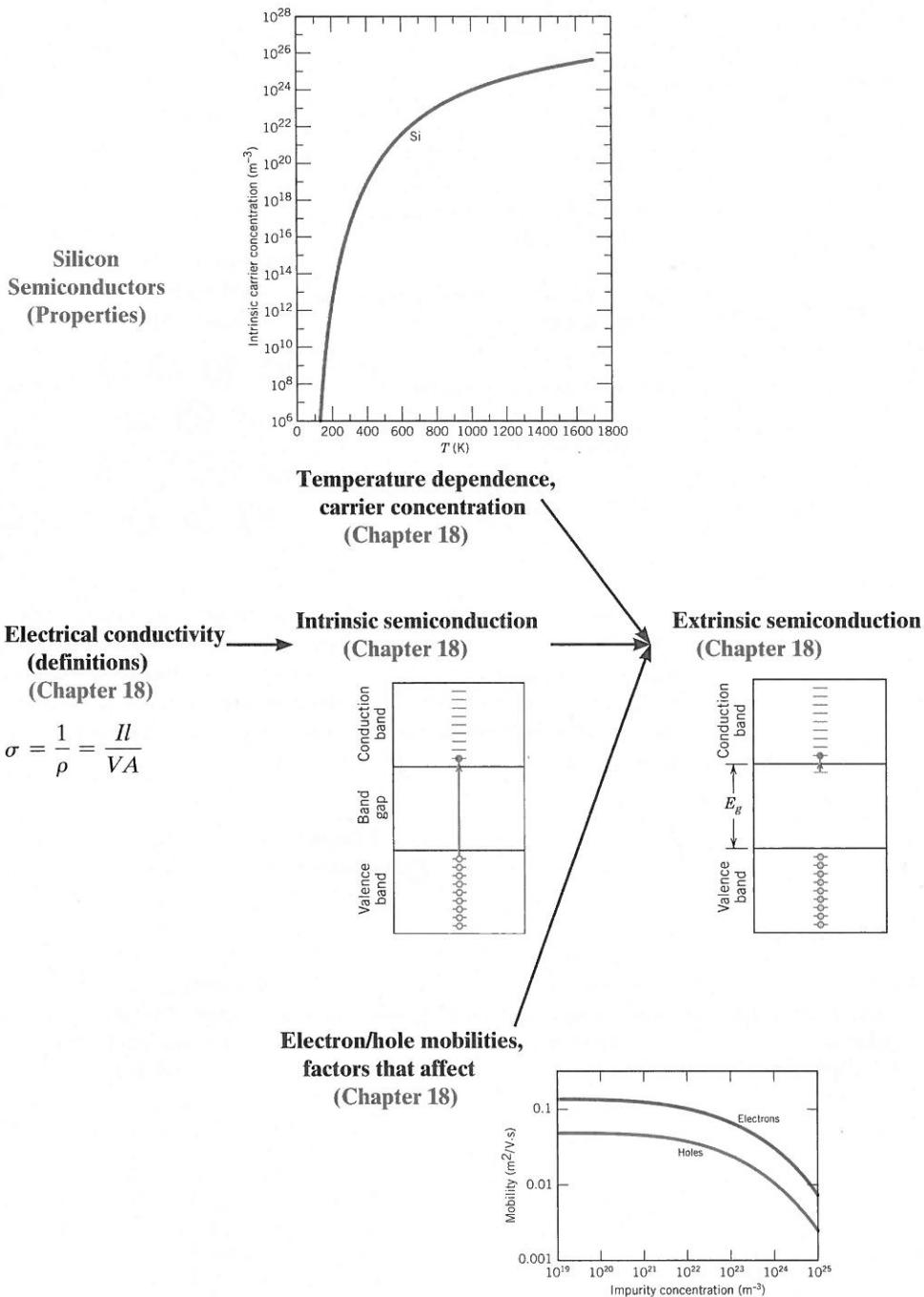


Intrinsic electron band structure (narrow band gap) (Chapter 18)

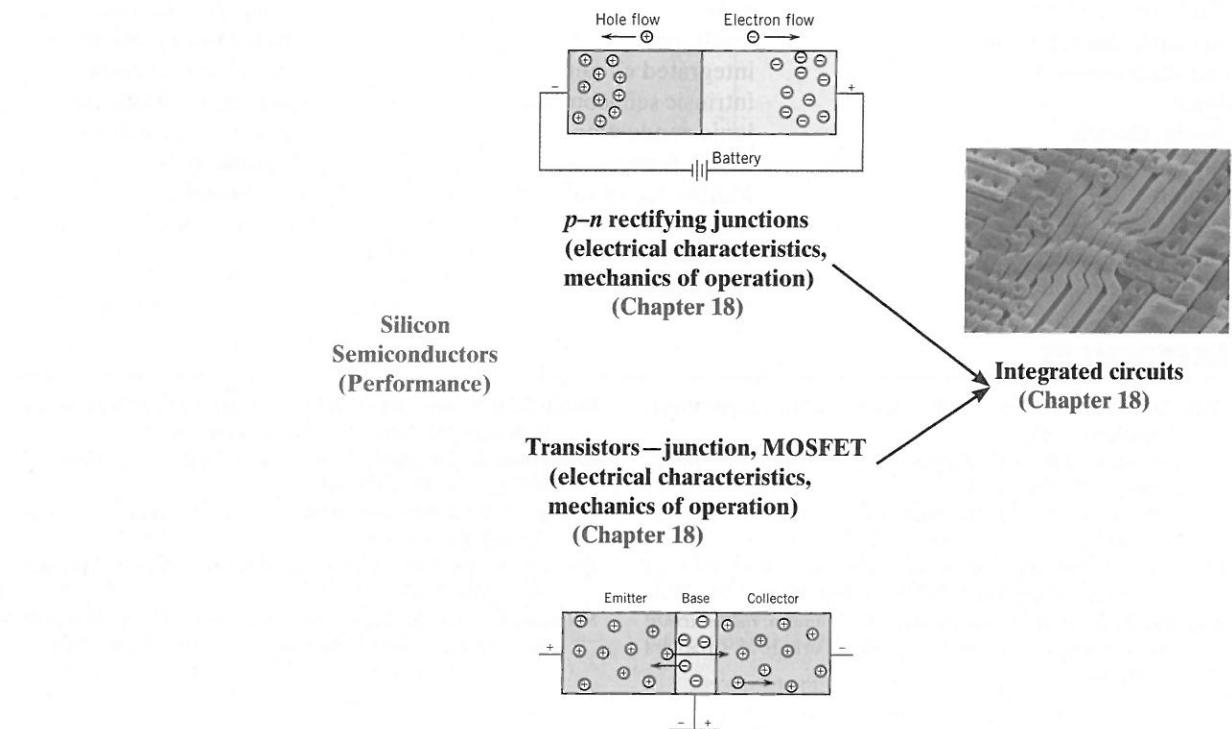
Extrinsic electron band structure (donor/acceptor levels) (Chapter 18)



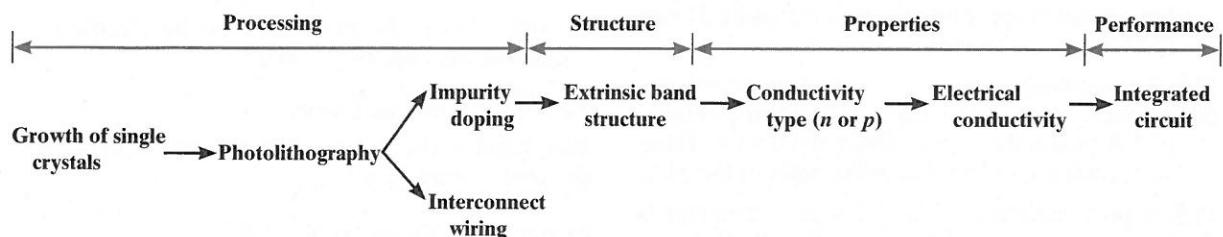
For both intrinsic and extrinsic semiconduction, electrical conductivity is a function of both carrier concentration and mobility of the carriers (electrons and/or holes). Both carrier concentration and mobility depend on temperature and impurity content. We note these relationships in the following concept map:



One common use for semiconducting materials is in integrated-circuit components. In our discussion of rectifying junctions and transistors—two of these components—we detailed their electrical characteristics and mechanics of operation, per the following concept map:



This concludes the processing/structure/properties/performance commentary for silicon semiconductors. For the most part, the individual components found in the preceding interrelationships are conceptual in nature—that is, they represent the scientific (as opposed to engineering) aspects of materials. A processing/structure/properties/performance relational diagram for these materials taken from the materials engineering perspective is presented in Figure 18.38.



**Figure 18.38** Schematic diagram that summarizes the elements of processing, structure, properties, and performance for silicon semiconductors, which includes components of materials engineering.

- 18.19** For intrinsic semiconductors, the intrinsic carrier concentration  $n_i$  depends on temperature, as follows:

$$n_i \propto \exp\left(-\frac{E_g}{2kT}\right) \quad (18.35a)$$

or, taking natural logarithms,

$$\ln n_i \propto -\frac{E_g}{2kT} \quad (18.35b)$$

Thus, a plot of  $\ln n_i$  versus  $1/T$  (K) $^{-1}$  should be linear and yield a slope of  $-E_g/2k$ . Using this information and the data presented in Figure 18.16, determine the band gap energies for silicon and germanium and compare these values with those given in Table 18.3.

- 18.20** Briefly explain the presence of the factor 2 in the denominator of Equation 18.35a.

- 18.21** At room temperature, the electrical conductivity of PbS is  $25 (\Omega \cdot m)^{-1}$ , whereas the electron and hole mobilities are 0.06 and  $0.02 \text{ m}^2/\text{V} \cdot \text{s}$ , respectively. Compute the intrinsic carrier concentration for PbS at room temperature.

- 18.22** Is it possible for compound semiconductors to exhibit intrinsic behavior? Explain your answer.

- 18.23** For each of the following pairs of semiconductors, decide which has the smaller band gap energy,  $E_g$ , and then cite the reason for your choice.

- (a) C (diamond) and Ge
- (b) AlP and InAs
- (c) GaAs and ZnSe
- (d) ZnSe and CdTe
- (e) CdS and NaCl

### Extrinsic Semiconduction

- 18.24** Define the following terms as they pertain to semiconducting materials: *intrinsic*, *extrinsic*, *compound*, *elemental*. Provide an example of each.

- 18.25** An *n*-type semiconductor is known to have an electron concentration of  $5 \times 10^{17} \text{ m}^{-3}$ . If the electron drift velocity is 350 m/s in an electric field of 1000 V/m, calculate the conductivity of this material.

- 18.26 (a)** In your own words, explain how donor impurities in semiconductors give rise to free electrons in numbers in excess of those generated by valence band-conduction band excitations.

- (b)** Also, explain how acceptor impurities give rise to holes in numbers in excess of those generated by valence band-conduction band excitations.

- 18.27 (a)** Explain why no hole is generated by the electron excitation involving a donor impurity atom.

- (b)** Explain why no free electron is generated by the electron excitation involving an acceptor impurity atom.

- 18.28** Predict whether each of the following elements will act as a donor or an acceptor when added to the indicated semiconducting material. Assume that the impurity elements are substitutional.

Impurity	Semiconductor
N	Si
B	Ge
S	InSb
In	CdS
As	ZnTe

- 18.29 (a)** The room-temperature electrical conductivity of a silicon specimen is  $500 (\Omega \cdot m)^{-1}$ . The hole concentration is known to be  $2.0 \times 10^{22} \text{ m}^{-3}$ . Using the electron and hole mobilities for silicon in Table 18.3, compute the electron concentration.

- (b)** On the basis of the result in part (a), is the specimen intrinsic, *n*-type extrinsic, or *p*-type extrinsic? Why?

- 18.30** Germanium to which  $10^{24} \text{ m}^{-3}$  As atoms have been added is an extrinsic semiconductor at room temperature, and virtually all the As atoms may be thought of as being ionized (i.e., one charge carrier exists for each As atom).

- (a)** Is this material *n*-type or *p*-type?

- (b)** Calculate the electrical conductivity of this material, assuming electron and hole mobilities of 0.1 and  $0.05 \text{ m}^2/\text{V} \cdot \text{s}$ , respectively.

- 18.31** The following electrical characteristics have been determined for both intrinsic and *p*-type extrinsic gallium antimonide (GaSb) at room temperature:

	$\sigma(\Omega \cdot m)^{-1}$	$n(\text{m}^{-3})$	$p(\text{m}^{-3})$
Intrinsic	$8.9 \times 10^4$	$8.7 \times 10^{23}$	$8.7 \times 10^{23}$
Extrinsic ( <i>p</i> -type)	$2.3 \times 10^5$	$7.6 \times 10^{22}$	$1.0 \times 10^{25}$

Calculate electron and hole mobilities.

### The Temperature Dependence of Carrier Concentration

- 18.32** Calculate the conductivity of intrinsic silicon at 80°C.

- 18.33** At temperatures near room temperature, the  
+ temperature dependence of the conductivity for intrinsic germanium is found to be given by

$$\sigma = CT^{-3/2} \exp\left(-\frac{E_g}{2kT}\right) \quad (18.36)$$

where  $C$  is a temperature-independent constant and  $T$  is in Kelvins. Using Equation 18.36, calculate the intrinsic electrical conductivity of germanium at 175°C.

- 18.34** Using Equation 18.36 and the results of Problem 18.33, determine the temperature at which the electrical conductivity of intrinsic germanium is  $40 \text{ } (\Omega \cdot \text{m})^{-1}$ .

- 18.35** Estimate the temperature at which GaAs has  
+ an electrical conductivity of  $1.6 \times 10^{-3} \text{ } (\Omega \cdot \text{m})^{-1}$ , assuming the temperature dependence for  $\sigma$  of Equation 18.36. The data shown in Table 18.3 may prove helpful.

- 18.36** Compare the temperature dependence of the conductivity for metals and intrinsic semiconductors. Briefly explain the difference in behavior.

#### Factors That Affect Carrier Mobility

- 18.37** Calculate the room-temperature electrical  
+ conductivity of silicon that has been doped with  $10^{23} \text{ m}^{-3}$  of arsenic atoms.

- 18.38** Calculate the room-temperature electrical  
+ conductivity of silicon that has been doped with  $2 \times 10^{24} \text{ m}^{-3}$  of boron atoms.

- 18.39** Estimate the electrical conductivity at 75°C  
+ of silicon that has been doped with  $10^{22} \text{ m}^{-3}$  of phosphorus atoms.

- 18.40** Estimate the electrical conductivity at 135°C  
+ of silicon that has been doped with  $10^{24} \text{ m}^{-3}$  of aluminum atoms.

#### The Hall Effect

- 18.41** A hypothetical metal is known to have an  
+ electrical resistivity of  $3.3 \times 10^{-8} \text{ } (\Omega \cdot \text{m})$ . A current of 25 A is passed through a specimen of this metal 15 mm thick. When a magnetic field of 0.95 tesla is simultaneously imposed in a direction perpendicular to that of the current, a Hall voltage of  $-2.4 \times 10^{-7} \text{ V}$  is measured. Compute the following:

- (a) the electron mobility for this metal

- (b) the number of free electrons per cubic meter

- 18.42** A metal alloy is known to have electrical conductivity and electron mobility values of  $1.2 \times 10^7 \text{ } (\Omega \cdot \text{m})^{-1}$  and  $0.0050 \text{ m}^2/\text{V} \cdot \text{s}$ , respectively. A current of 40 A is passed through a specimen of

this alloy that is 35 mm thick. What magnetic field would need to be imposed to yield a Hall voltage of  $-3.5 \times 10^{-7} \text{ V}$ ?

#### Semiconducting Devices

- 18.43** Briefly describe electron and hole motions in a *p-n* junction for forward and reverse biases; then explain how these lead to rectification.

- 18.44** How is the energy in the reaction described by Equation 18.21 dissipated?

- 18.45** What are the two functions that a transistor may perform in an electronic circuit?

- 18.46** State the differences in operation and application for junction transistors and MOSFETs.

#### Conduction in Ionic Materials

- 18.47** We note in Section 12.5 (Figure 12.20) that in FeO (wüstite), the iron ions can exist in both  $\text{Fe}^{2+}$  and  $\text{Fe}^{3+}$  states. The number of each of these ion types depends on temperature and the ambient oxygen pressure. Furthermore, we also note that in order to retain electroneutrality, one  $\text{Fe}^{2+}$  vacancy is created for every two  $\text{Fe}^{3+}$  ions that are formed; consequently, in order to reflect the existence of these vacancies, the formula for wüstite is often represented as  $\text{Fe}_{(1-x)}\text{O}$ , where  $x$  is some small fraction less than unity.

In this nonstoichiometric  $\text{Fe}_{(1-x)}\text{O}$  material, conduction is electronic and, in fact, it behaves as a *p*-type semiconductor—that is, the  $\text{Fe}^{3+}$  ions act as electron acceptors, and it is relatively easy to excite an electron from the valence band into an  $\text{Fe}^{3+}$  acceptor state with the formation of a hole. Determine the electrical conductivity of a specimen of wüstite with a hole mobility of  $1.0 \times 10^{-5} \text{ m}^2/\text{V} \cdot \text{s}$ , and for which the value of  $x$  is 0.040. Assume that the acceptor states are saturated (i.e., one hole exists for every  $\text{Fe}^{3+}$  ion). Wüstite has the sodium chloride crystal structure with a unit cell edge length of 0.437 nm.

- 18.48** At temperatures between 540°C (813 K) and

- + 727°C (1000 K), the activation energy and pre-exponential for the diffusion coefficient of  $\text{Na}^+$  in NaCl are 173,000 J/mol and  $4.0 \times 10^{-4} \text{ m}^2/\text{s}$ , respectively. Compute the mobility for an  $\text{Na}^+$  ion at 600°C (873 K).

#### Capacitance

- 18.49** A parallel-plate capacitor using a dielectric

- + material having an  $\epsilon_r$  of 2.2 has a plate spacing of 2 mm (0.08 in.). If another material having a dielectric constant of 3.7 is used and the capacitance is to be unchanged, what must the new spacing be between the plates?

- 18.50** A parallel-plate capacitor with dimensions of + 38 mm by 65 mm ( $1\frac{1}{2}$  in. by  $2\frac{1}{2}$  in.) and a plate separation of 1.3 mm (0.05 in.) must have a minimum capacitance of 70 pF ( $7 \times 10^{-11}$  F) when an ac potential of 1000 V is applied at a frequency of 1 MHz. Which of the materials listed in Table 18.5 are possible candidates? Why?

- 18.51** Consider a parallel-plate capacitor having an + area of  $3225 \text{ mm}^2$  (5 in.<sup>2</sup>), a plate separation of 1 mm (0.04 in.), and a material having a dielectric constant of 3.5 positioned between the plates.

- (a) What is the capacitance of this capacitor?  
 (b) Compute the electric field that must be applied for  $2 \times 10^{-8}$  C to be stored on each plate.

- 18.52** In your own words, explain the mechanism by which charge-storing capacity is increased by the insertion of a dielectric material within the plates of a capacitor.

### Field Vectors and Polarization

#### Types of Polarization

- 18.53** For CaO, the ionic radii for Ca<sup>2+</sup> and O<sup>2-</sup> ions are + 0.100 and 0.140 nm, respectively. If an externally applied electric field produces a 5% expansion of the lattice, compute the dipole moment for each Ca<sup>2+</sup>-O<sup>2-</sup> pair. Assume that this material is completely unpolarized in the absence of an electric field.

- 18.54** The polarization  $P$  of a dielectric material positioned within a parallel-plate capacitor is to be +  $4.0 \times 10^{-6}$  C/m<sup>2</sup>.

- (a) What must be the dielectric constant if an electric field of  $10^5$  V/m is applied?  
 (b) What will be the dielectric displacement  $D$ ?

- 18.55** A charge of  $2.0 \times 10^{-10}$  C is to be stored on + each plate of a parallel-plate capacitor having an area of  $650 \text{ mm}^2$  (1.0 in.<sup>2</sup>) and a plate separation of 4.0 mm (0.16 in.).

- (a) What voltage is required if a material having a dielectric constant of 3.5 is positioned within the plates?  
 (b) What voltage would be required if a vacuum were used?  
 (c) What are the capacitances for parts (a) and (b)?  
 (d) Compute the dielectric displacement for part (a).  
 (e) Compute the polarization for part (a).

- 18.56 (a)** For each of the three types of polarization, briefly describe the mechanism by which dipoles are induced and/or oriented by the action of an applied electric field.

- (b) For gaseous argon, solid LiF, liquid H<sub>2</sub>O, and solid Si, what kind(s) of polarization is (are) possible? Why?

- 18.57 (a)** Compute the magnitude of the dipole moment associated with each unit cell of BaTiO<sub>3</sub>, as illustrated in Figure 18.35.

- (b) Compute the maximum polarization possible for this material.

### Frequency Dependence of the Dielectric Constant

- 18.58** The dielectric constant for a soda-lime glass measured at very high frequencies (on the order of  $10^{15}$  Hz) is approximately 2.3. What fraction of the dielectric constant at relatively low frequencies (1 MHz) is attributed to ionic polarization? Neglect any orientation polarization contributions.

### Ferroelectricity

- 18.59** Briefly explain why the ferroelectric behavior of BaTiO<sub>3</sub> ceases above its ferroelectric Curie temperature.

### Spreadsheet Problem

- 18.1SS** For an intrinsic semiconductor whose electrical conductivity is dependent on temperature per Equation 18.36, generate a spreadsheet that allows the user to determine the temperature at which the electrical conductivity is some specified value, given values of the constant  $C$  and the band gap energy  $E_g$ .

### DESIGN PROBLEMS

#### Electrical Resistivity of Metals

- 18.D1** A 90 wt% Cu-10 wt% Ni alloy is known to + have an electrical resistivity of  $1.90 \times 10^{-7}$  Ω·m at room temperature (25°C). Calculate the composition of a copper-nickel alloy that gives a room-temperature resistivity of  $2.5 \times 10^{-7}$  Ω·m. The room-temperature resistivity of pure copper may be determined from the data in Table 18.1; assume that copper and nickel form a solid solution.

- 18.D2** Using information contained in Figures 18.8 and 18.39, determine the electrical conductivity of an 85 wt% Cu-15 wt% Zn alloy at -100°C (-150°F).

- 18.D3** Is it possible to alloy copper with nickel to achieve a minimum yield strength of 130 MPa (19,000 psi) and yet maintain an electrical conductivity of  $4.0 \times 10^6$  (Ω·m)<sup>-1</sup>? If not, why? If so, what concentration of nickel is required? See Figure 7.16b.

**Extrinsic Semiconduction****Factors That Affect Carrier Mobility**

**18.D4** Specify a donor impurity type and concentration (in weight percent) that will produce an *n*-type silicon material having a room-temperature electrical conductivity of 200 ( $\Omega \cdot \text{m}$ )<sup>-1</sup>.

**18.D5** One integrated circuit design calls for diffusing boron into very high-purity silicon at an elevated temperature. It is necessary that at a distance 0.2  $\mu\text{m}$  from the surface of the silicon wafer, the room-temperature electrical conductivity be 1000 ( $\Omega \cdot \text{m}$ )<sup>-1</sup>. The concentration of B at the surface of the Si is maintained at a constant level of  $1.0 \times 10^{25} \text{ m}^{-3}$ ; furthermore, it is assumed that the concentration of B in the original Si material is negligible, and that at room temperature, the boron atoms are saturated. Specify the temperature at which this diffusion heat treatment is to take place if the treatment time is to be 1 h. The diffusion coefficient for the diffusion of B in Si is a function of temperature as

$$D(\text{m}^2/\text{s}) = 2.4 \times 10^{-4} \exp\left(-\frac{347,000 \text{ J/mol}}{RT}\right)$$

**Semiconductor Devices**

**18.D6** One of the procedures in the production of integrated circuits is the formation of a thin insulating layer of  $\text{SiO}_2$  on the surface of chips (see Figure 18.26). This is accomplished by oxidizing the surface of the silicon by subjecting it to an oxidizing atmosphere (i.e., gaseous oxygen or water vapor) at an elevated temperature. The rate of growth of the oxide film is *parabolic*—that is, the thickness of the oxide layer ( $x$ ) is a function of time ( $t$ ) according to the following equation:

$$x^2 = Bt \quad (18.37)$$

Here, the parameter  $B$  is dependent on both temperature and the oxidizing atmosphere.

(a) For an atmosphere of  $\text{O}_2$  at a pressure of 1 atm, the temperature dependence of  $B$  (in units of  $\mu\text{m}^2/\text{h}$ ) is as follows:

$$B = 800 \exp\left(-\frac{1.24 \text{ eV}}{kT}\right) \quad (18.38a)$$

where  $k$  is Boltzmann's constant ( $8.62 \times 10^{-5}$  eV/atom) and  $T$  is in K. Calculate the time required to grow an oxide layer (in an atmosphere of  $\text{O}_2$ ) that is 100 nm thick at both 700°C and 1000°C.

(b) In an atmosphere of  $\text{H}_2\text{O}$  (1 atm pressure), the expression for  $B$  (again, in units of  $\mu\text{m}^2/\text{h}$ ) is

$$B = 215 \exp\left(-\frac{0.70 \text{ eV}}{kT}\right) \quad (18.38b)$$

Calculate the time required to grow an oxide layer that is 100 nm thick (in an atmosphere of  $\text{H}_2\text{O}$ ) at both 700°C and 1000°C, and compare these times with those computed in part (a).

**18.D7** The base semiconducting material used in virtually all modern integrated circuits is silicon. However, silicon has some limitations and restrictions. Write an essay comparing the properties and applications (and/or potential applications) of silicon and gallium arsenide.

**Conduction in Ionic Materials**

**18.D8** Problem 18.47 noted that  $\text{FeO}$  (wüstite) may

+ behave as a semiconductor by virtue of the transformation of  $\text{Fe}^{2+}$  to  $\text{Fe}^{3+}$  and the creation of  $\text{Fe}^{2+}$  vacancies; the maintenance of electroneutrality requires that for every two  $\text{Fe}^{3+}$  ions, one vacancy is formed. The existence of these vacancies is reflected in the chemical formula of this nonstoichiometric wüstite as  $\text{Fe}_{(1-x)}\text{O}$ , where  $x$  is a small number having a value less than unity. The degree of *nonstoichiometry* (i.e., the value of  $x$ ) may be varied by changing temperature and oxygen partial pressure. Compute the value of  $x$  required to produce an  $\text{Fe}_{(1-x)}\text{O}$  material having a *p*-type electrical conductivity of 1200 ( $\Omega \cdot \text{m}$ )<sup>-1</sup>; assume that the hole mobility is  $1.0 \times 10^{-5} \text{ m}^2/\text{V} \cdot \text{s}$ , the crystal structure for  $\text{FeO}$  is sodium chloride (with a unit cell edge length of 0.437 nm), and the acceptor states are saturated.

**FUNDAMENTALS OF ENGINEERING  
QUESTIONS AND PROBLEMS**

**18.1FE** For a metal that has an electrical conductivity of  $6.1 \times 10^7$  ( $\Omega \cdot \text{m}$ )<sup>-1</sup>, what is the resistance of a wire that is 4.3 mm in diameter and 8.1 m long?

- (A)  $3.93 \times 10^{-5} \Omega$
- (B)  $2.29 \times 10^{-3} \Omega$
- (C)  $9.14 \times 10^{-3} \Omega$
- (D)  $1.46 \times 10^{11} \Omega$

**18.2FE** What is the typical electrical conductivity + value/range for semiconducting materials?

- (A)  $10^7 (\Omega \cdot \text{m})^{-1}$
- (B)  $10^{-20}$  to  $10^7 (\Omega \cdot \text{m})^{-1}$
- (C)  $10^{-6}$  to  $10^4 (\Omega \cdot \text{m})^{-1}$
- (D)  $10^{-20}$  to  $10^{-10} (\Omega \cdot \text{m})^{-1}$

- 18.3FE** A two-phase metal alloy is known to be composed of  $\alpha$  and  $\beta$  phases that have mass fractions of 0.64 and 0.36, respectively. Using the room-temperature electrical resistivity and the following density data, calculate the electrical resistivity of this alloy at room temperature.

Phase	Resistivity ( $\Omega \cdot m$ )	Density ( $g/cm^3$ )
$\alpha$	$1.9 \times 10^{-8}$	8.26
$\beta$	$5.6 \times 10^{-7}$	8.60

- (A)  $2.09 \times 10^{-7} \Omega \cdot m$
- (B)  $2.14 \times 10^{-7} \Omega \cdot m$
- (C)  $3.70 \times 10^{-7} \Omega \cdot m$
- (D)  $5.90 \times 10^{-7} \Omega \cdot m$

- 18.4FE** For an *n*-type semiconductor, where is the Fermi level located?

- (A) In the valence band
- (B) In the band gap just above the top of valence band

(C) In the middle of the band gap

(D) In the band gap just below the bottom of the conduction band

- 18.5FE** The room-temperature electrical conductivity of a semiconductor specimen is  $2.8 \times 10^4 (\Omega \cdot m)^{-1}$ . If the electron concentration is  $2.9 \times 10^{22} m^{-3}$  and electron and hole mobilities are 0.14 and  $0.023 m^2/V \cdot s$ , respectively, calculate the hole concentration.

- (A)  $1.24 \times 10^{24} m^{-3}$
- (B)  $7.42 \times 10^{24} m^{-3}$
- (C)  $7.60 \times 10^{24} m^{-3}$
- (D)  $7.78 \times 10^{24} m^{-3}$