HW8

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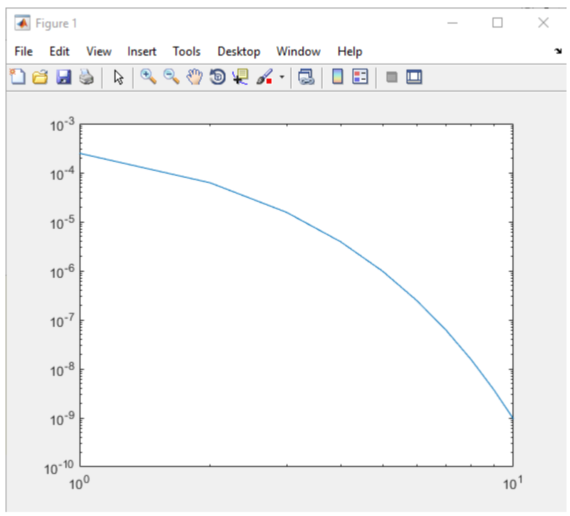
November 22, 2019

1 5.6.1

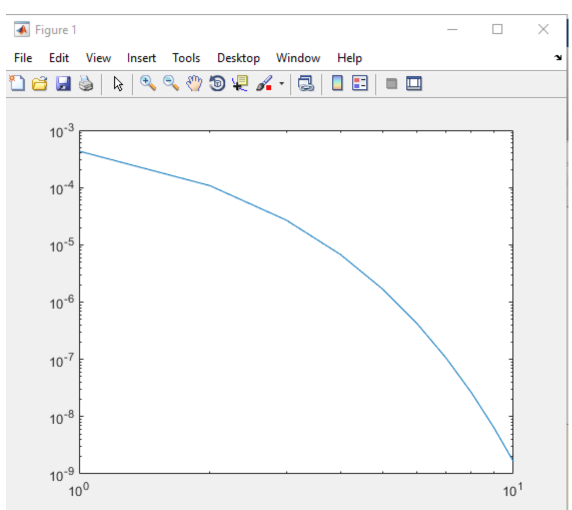
Code is in matlab file.

a)



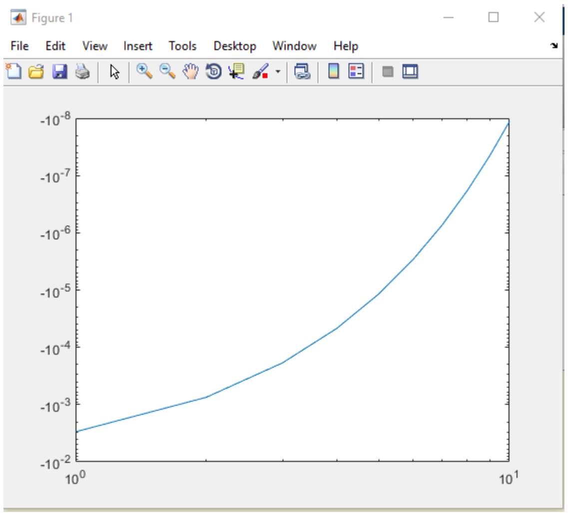
Second order convergence implies that each time the node count is doubled, the error will decrease by a factor of 4. Using the observed error values and the loglog plot, one can see that this refutes second order convergence as the error doesn't decrease by a factor of 4 with each iteration.

b)



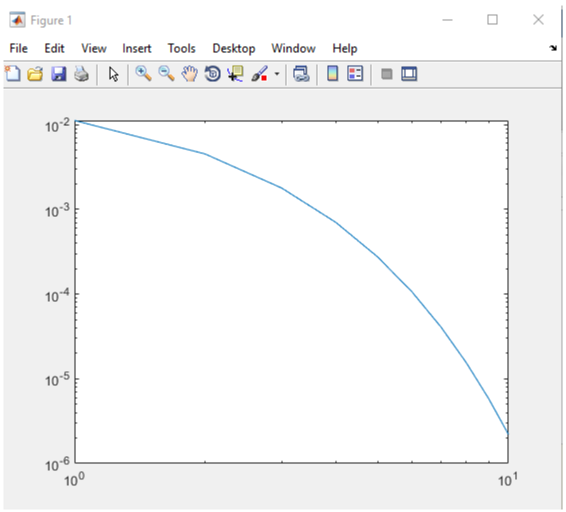
c)

Second order convergence implies that each time the node count is doubled, the error will decrease by a factor of 4. Using the observed error values and the loglog plot, one can see that this refutes second order convergence as the error doesn't decrease by a factor of 4 with each iteration.



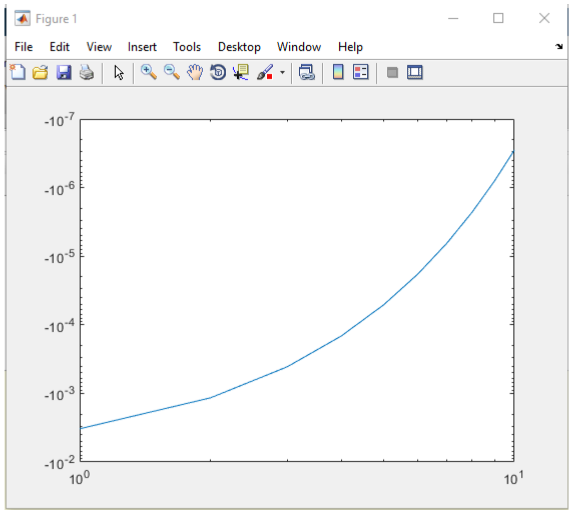
d)

Second order convergence implies that each time the node count is doubled, the error will decrease by a factor of 4. Using the observed error values and the loglog plot, one can see that this refutes second order convergence as the error doesn't decrease by a factor of 4 with each iteration.



e)

Second order convergence implies that each time the node count is doubled, the error will decrease by a factor of 4. Using the observed error values and the loglog plot, one can see that this refutes second order convergence as the error doesn't decrease by a factor of 4 with each iteration.



2 5.6.4a

Let $-h = x_0, 0 = x_1, h = x_2, \alpha = f(x_0), \beta = f(x_1), \gamma = f(x_2)$ and $p(x) = a + bx + cx^2$ be the approximating polynomial.

We create a system of equations to solve for the unknowns,

$p(x_0) = p(-h) = a - hb + ch^2 = \alpha$

$p(x_1) = p(0) = a = \beta$

$p(x_2) = p(h) = a + hb + ch^2 = \gamma$

Solving the system of equations results in,

$a = \beta$

$b = \frac{\gamma - \alpha}{2h}$

$c = \frac{\alpha - 2\beta + \gamma}{2h^2}$

And thus

$p(x) = \beta + \frac{\gamma - \alpha}{2h} + \frac{\alpha - 2\beta + \gamma}{2h^2}$

3 5.6.4b

We start with the integral

$\int_{-h}^{h} p(s)ds = \int_{-h}^{h} \beta + \frac{\gamma-\alpha}{2h} s + \frac{\alpha - 2\beta + \gamma}{2h^2} s^2 ds$

After integrating, we have,

$h\beta + \frac{\gamma - \alpha}{4} + \frac{\alpha - 2\beta + \gamma}{6} + h\beta - \frac{\gamma - \alpha}{4} h + \frac{\alpha - 2\beta + \gamma}{6} h$ Which simplifies to $\frac{4\beta}{3} h + \frac{\alpha}{3} h + \frac{\gamma}{3} h = \frac{h}{3}(4\beta + \alpha + \gamma)$

4 5.6.4c

Since our sub-interval is $[t_{i-1}, t_{i+1}]$, we use the values of x at $x_0 = t_{i-1}, x_1 = t_i, x_2 = t_{i+1}$.

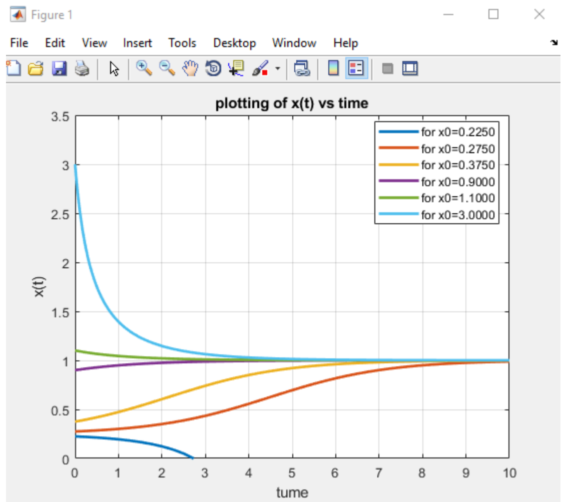
Using the equations found in part b, we can construct the integral $\int_{x_0}^{x_2} p(s)ds = \frac{h}{3}(4(fx_0) + f(x_1) + f(x_2))$.

Which is equivalent to $\int_{t_{i-1}}^{t_{i+1}} p(s)ds = \frac{h}{3}(4(f(t_{i-1}) + f(t_i) + f(t_{i+1}))$

Thus $\int_{t_{i-1}}^{t_{i+1}} p(s)ds \approx \int_{t_{i-1}}^{t_{i+1}} f(x)dx \approx \frac{h}{3}(4(f(t_{i-1}) + f(t_i + f(t_{i+1}))$

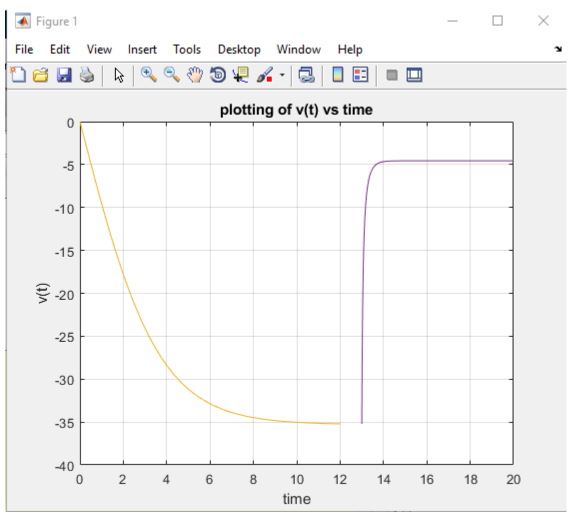
5 6.1.5

Code in matlab file.



6 6.1.8

Code in matlab file

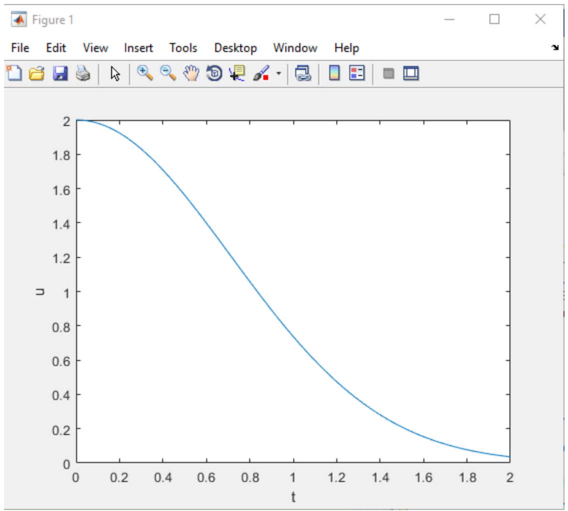


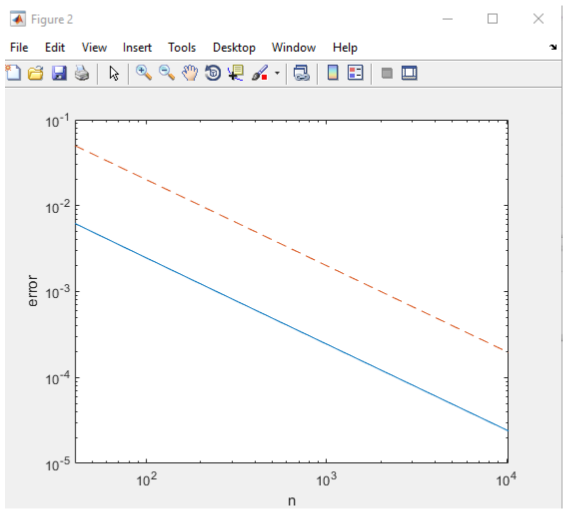
According to my matlab code, the skydiver hits the ground at time 1.6005 and has traveled a total distance of 1399m.

7 6.2.2a

Code in matlab file

The red dotted line is a reference line.





8 6.2.2c

Code in matlab file

9 6.2.4a

Given that

$v_{i+1} = u_i + hf(t_i, u_i)$

$u_{i+1} = u_i + hf(t_i + h, v_i + 1)$

Then,

$= u_i + hf(t_i + h, u_i + hf(t_i, u_i))$

Comparing this with

$u_{i+1} = u_i + h\phi(t_i, u_i, h)$

We get $\phi(t_i, u_i, h) = f(t_i + h, u_i + hf(t_i, u_i))$

10 6.2.4b

We need to show that $\phi(t_i, u_i, 0) = f(t_i, u_i)$

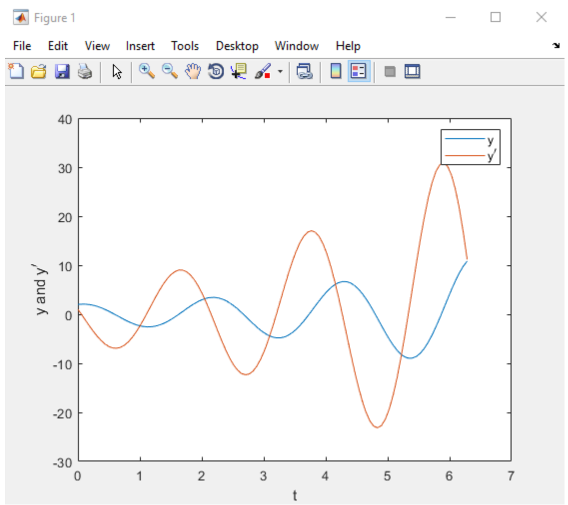
$\phi(t_i, u_i, 0) = f(t_i + 0, u_i + 0, f(t_i, u_i))$

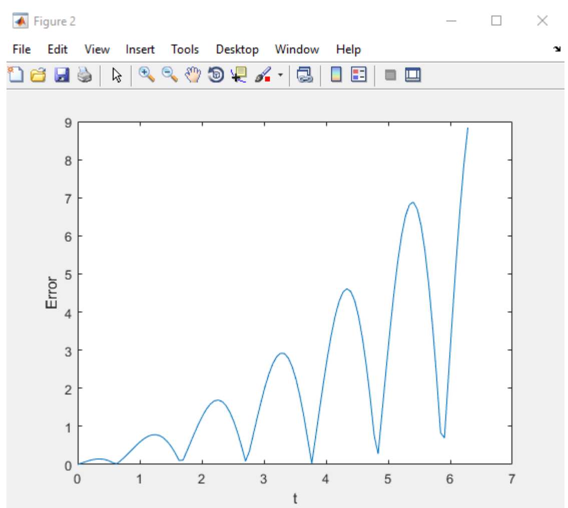
$= f(t_i, u_i)$

So the method is consistent.

11 6.3.3a

Code in matlab file





12 6.3.3b

Code in matlab file

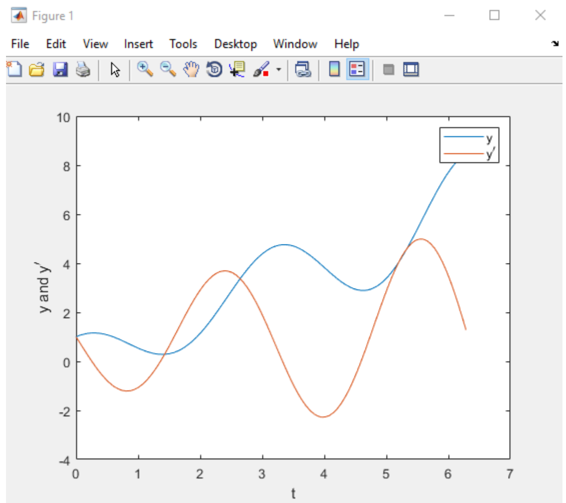


Figure 1
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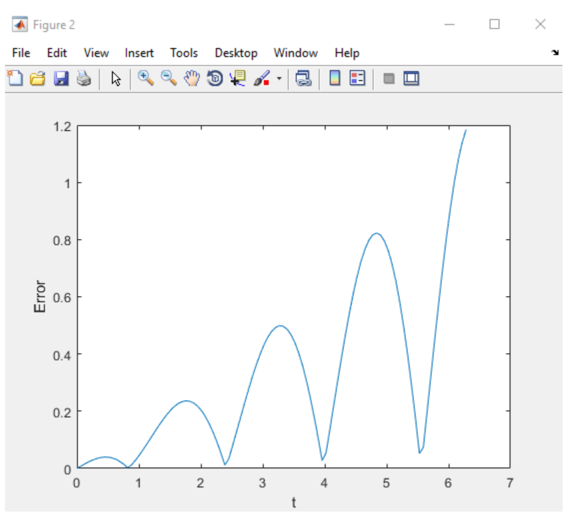
Figure 2
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