ELEG 310

15 Feb 2015

Exp: rain this weekend $\qquad 1 = \text{rain} \quad 0 = \text{no rain}$

Sat	Sun	prob
0	0	p_1
0	1	p_2
1	0	p_3
1	1	p_4

Outcomes

Rain Sat $= \{10, 11\}$

Rain Sun $= \{01, 11\}$

Rain Weekend $= \{01, 10, 11\}$

Rain both days $= \{11\}$

$p_i \geq 0$

$p_1 + p_2 + p_3 + p_4 = 1$

① $P(\text{rain sat}) = 0.6 = p_3 + p_4$

② $P(\text{rain sun}) = 0.4 = p_2 + p_4$

③ $P(\text{rain this weekend}) = p_2 + p_3 + p_4 = 1$

$p_1 = p_4 = 0$

$p_3 = 0.6$

$p_2 = 0.4$

One solution:

$p_4 = 0.4$

$p_3 = 0.2$

$p_2 = 0$

$p_1 = 0.4$

P(rain sat) = 0.6

Prain (sun) = 0.4

~~P(rain weekend) = 0.~~

P(no rain weekend) = (1-0.6)(1-0.4) ⇐ ind

P(rain both days) = 0.6 × 0.4

---

if P(rain sat) = 1 ⇒ P(rain this weekend) = 1 + 1 = 2

P(rain Sun) = 1

A B

$P(A \cup B) = P(A) + P(B) - P(AB)$

AB = rain both days $P(AB) = 1$

~~Bay~~ Conditional Prob

$$P(A|B) = P(A \text{ given } B) = \frac{P(AB)}{P(B)}$$

Bayes Theorem

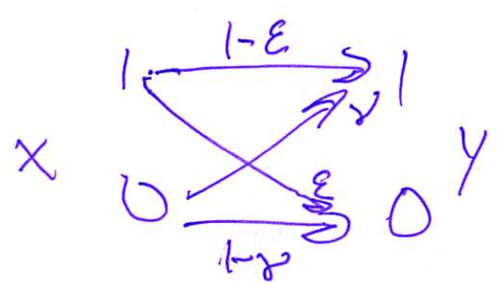
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

LTP

$$P(A) = \sum_{i=1}^{\infty} P(AB_i) = \sum_{i=1}^{\infty} P(A|B_i)P(B_i)$$

$$B_i B_j = \emptyset \quad i \neq j$$

$$\bigcup_{i=1}^{\infty} B_i = S$$



if $\varepsilon = \nu \Rightarrow$ binary symmetric channel

$P(Y=1 \mid X=1) = 1-\varepsilon$

$P(Y=0 \mid X=1) = \varepsilon$

$P(Y=1 \mid X=0) = \nu$

$P(Y=0 \mid X=0) = 1-\nu$

cross over probs

$P(X= \ ) = p$

$$P(Y=1) = P(Y=1 \mid X=0)\,P(X=0) + P(Y=1 \mid X=1)\,P(X=1)$$

$$= \nu(1-p) + (1-\varepsilon)\,p$$

$$P(X=1 \mid Y=1) = \frac{P(Y=1 \mid X=1)\,P(X=1)}{P(Y=1)} = \frac{(1-\varepsilon)p}{(1-\varepsilon)p + \nu(1-p)}$$

$$P(X=0 \mid Y=1) = \frac{P(Y=1 \mid X=0)\,P(X=0)}{P(Y=1)} = \frac{\nu(1-p)}{(1-\varepsilon)p + \nu(1-p)}$$

Receiver gets $Y=1$

Guess $\hat{X}=1$ if $\boxed{P(X=1 \mid Y=1) > P(X=0 \mid Y=1)}$

if $(1-\varepsilon)p > \nu(1-p)$

$\Rightarrow$ if $\dfrac{p}{1-p} > \dfrac{\nu}{1-\varepsilon}$

Maximum a posteriori Rule (MAP)

$P(X=1)$ = a priori prob

$P(X=1 \mid Y=1)$ = a posteriori prob