ELEG 310 Chaps 5 & 6 3/28/19

Multiple RVs

$p_{XY}(k,l) = P(X=k \cap Y=l)$ $\quad p_{XY}(k,l) \geq 0$ $\quad \sum_k \sum_l p(k,l) = 1$

$$E(g(X,Y)) = \sum_k \sum_l g(k,l)\, p_{XY}(k,l)$$

Ind X & Y are ind $\Leftrightarrow$ $p_{XY}(k,l) = p_X(k)\, p_Y(l)$

Sums of ind RVs $X_1 \; X_2 \; X_3 \cdots X_n$

$$S = X_1 + X_2 + \cdots + X_n$$

$$E[S] = E[X_1] + E[X_2] + \cdots + E[X_n]$$

$$Var[S] = Var[X_1] + Var[X_2] + \cdots + Var[X_n]$$ if X_i ind

Also Assume X_i have same distribution

Ind & identically distributed IID

PMF of $S = X + Y$ $\quad X$ & Y ind

$$p_S(k) = P(S=k) = P(X+Y=k)$$

$$= \sum_{\ell} P(X+Y=k \mid Y=\ell)\, p_Y(\ell)$$

$$= \sum_{\ell} P(X+\ell=k \mid Y=\ell)\, p_Y(\ell)$$

$$= \sum_{\ell} P(X=k-\ell)\, p_Y(\ell)$$

(X & Y ind)

$$\boxed{p_S(k) = \sum_{\ell} p_X(k-\ell)\, p_Y(\ell)}$$

$$\boxed{p_S = p_X * p_Y} \Leftarrow X, Y \text{ ind}$$

Ex X, Y roll of a D6

$k = 1, \ldots 6$

$\ell = 1 \ldots 6$

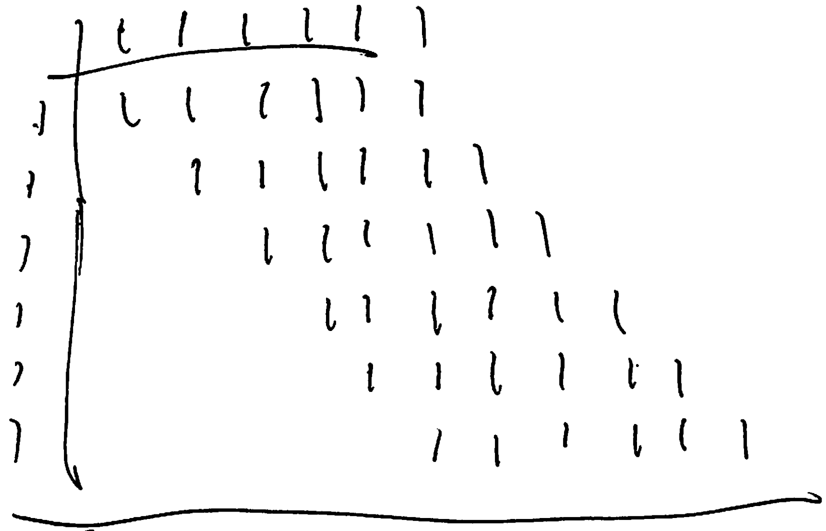
$$\begin{array}{ccccc} & h & * & x & \\ y = & \underset{\substack{\uparrow \\ n=0}}{[1\ 2\ 3\ -4]} & * & \underset{\substack{\uparrow \\ n=0}}{[5\ 6\ 7]} & = [5\ 6\ 7] * [1\ 2\ 3\ -4] \end{array}$$

		x[0]	x[1]				
		1	2	3	-4		
h[0] →	5	5	10	15	-20		
h[1]	6		6	12	18	-24	
	7			7	14	21	-28
		5	16	$\boxed{34}$ ← y[2]	12	-3	-28
		↑ n=0	↑ n=1				

$y[1] = x[0]h[1] + x[1]h[0]$

$y[2] = x[0]h[2] + x[1]h[1] + x[2]h[0]$

Ex Sum 2 Dice $\quad p_X(k) = \frac{1}{6} \quad p_Y(l) = \frac{1}{6}$



$$\left[\frac{1\ 2\ 3\ 4\ 5\ 6\ 5\ 4\ 3\ 2\ 1}{36}\right]$$

$P[S=2] = \frac{1}{36}$

$P[S=7] = \frac{6}{36}$

$P[S=8] = \frac{5}{36}$

$S = X + Y \qquad \left.\begin{array}{l} X \sim 1, 2, \ldots 6 \\ Y \sim 1, 2, \ldots 6 \end{array}\right\} \Rightarrow S = 2, 3, \ldots 12$

행0

	0	1	1	1	1	1	1	
열 → 0	0	0	0	0	0	0	0	
1		0	1	1	1	1	1	
1			0	1	1	1	1	1
1								
1								
1								
1								

0 0 1 2

↑ ↑ ↑

행0 열1 행2

Chap 6 Binomial Distribution

Q: flip coin n times (flips ind) (IID),

What is $P(k \text{ heads})$?

Ans: Let N = # heads observed = RV

n = # flips

k = # heads outcome

$$P(N=k) = \binom{n}{k} p^k q^{n-k} \qquad q = 1-p \qquad k = 0, 1, 2, \ldots, n$$

$$P_N(k) = b(n,k,p) = \binom{n}{k} p^k q^{n-k} \geq 0$$

$$\sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} = (p+q)^n = 1^n = 1$$

Binomial Theorem

Q: flip coin until get k heads. How many flips are needed?

Negative Binomial $\quad$ N = # flips required.

$$P(N=n) = \binom{n-1}{k-1} p^k q^{n-k} \qquad n = k, k+1, k+2, \ldots$$

sequence $k=3$ $\quad \underbrace{\overbrace{00110}1}_{n=6}$ $\quad \Rightarrow$ first $n-1$ flips has $k-1$ heads

Poisson $\quad P(N=k) = \frac{\lambda^k}{k!} e^{-\lambda} \qquad k = 0, 1, 2, \ldots \infty$

Interesting Result $\quad$ Binomial $\approx$ Poisson $\quad$ when n = large, p = small, $\lambda = np$ = medium

mean np

Var npq

$S = X_1 + X_2 \qquad X_1 \sim b(n_1, k, p) \qquad X_1 \,\&\, X_2$ ind

$X_2 \sim b(n_2, k, p)$

$S \sim b(n_1 + n_2, k, p)$