ELEG 310 2/19/19

Probability = branch of mathematics

concerned with randomness

Statistics - science of learning from data

Experiment - thing we do

Outcomes - possible results

one outcome is TRUE

all other outcomes FALSE

Events = set of outcomes

Ex. roll a 6-sided die

S = sample space = set of all outcomes

$= \{1, 2, 3, 4, 5, 6\} = \{$⚀, ⚁, ⚂, ⚃, ⚄, ⚅$\}$

Random variable

$X($⚂$) = 3$ $\quad X($⚅$) = 6$

X = "outcome" of die roll

Bernoulli Trial $\quad X(\text{'tails'}) = 0 \quad X(\text{'heads'}) = 1$

Exp: roll die

define events $A = \{2, 4, 6\}$ = "roll is even"

$B = \{1, 3, 5\}$ = "odd"

$C = \{1, 2, 3\}$ = "small"

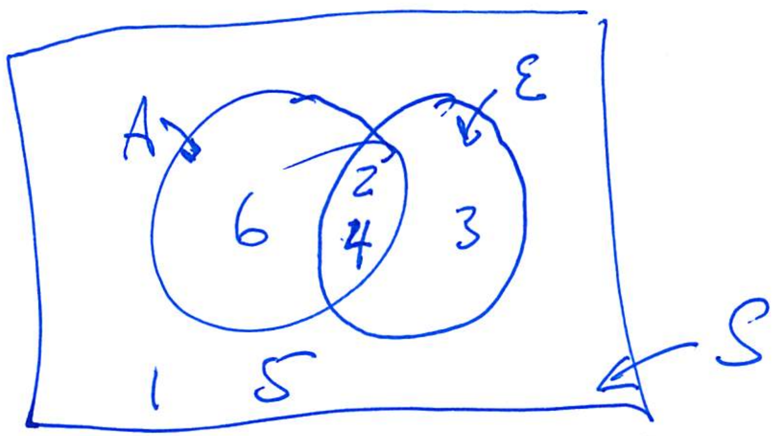
$D = \{1\}$ $E = \{2, 3, 4\}$ $F = \{\ \} = \phi$

$G = \{1, 2, 3, 4, 5, 6\} = S$ = sample space

say, die roll = 4 $\Rightarrow$ A, E, G are TRUE

Ex $A \cap B = \{\} = \emptyset$ $\quad A \cup B = S$

$A \cap E = \{2, 4\}$ $\quad A \cup E = \{2, 3, 4, 6\}$



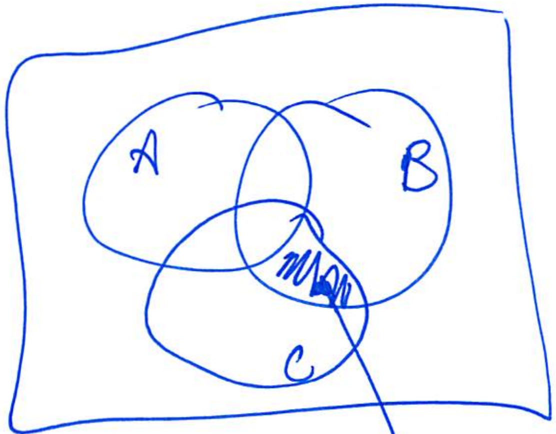
$\bar{A} = \{$ outcomes not in $A\}$

$= \{1, 3, 5\}$

De Morgan's Laws – $\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

$A \cap \bar{A} = \emptyset$ $\quad A \cup \bar{A} = S$



$\bar{A}BC = \bar{A} \cap B \cap C$

logical AND

$A \cup B = A + B$

logical OR

Probability of an event is the likelihood of the event.

$P(\underset{\text{event}}{A}) = \text{number}$ Ex. $P(\{3\}) = \text{number}$

3 Axioms —

1. $P(A) \geq 0$ for all events A
2. $P(S) = 1$
3. if $A \cap B = \emptyset$, $P(A \cup B) = P(A) + P(B)$

Question: if $P(A \cup B) = P(A) + P(B) \overset{?}{\Longrightarrow} A \cap B = \emptyset$

Theorems

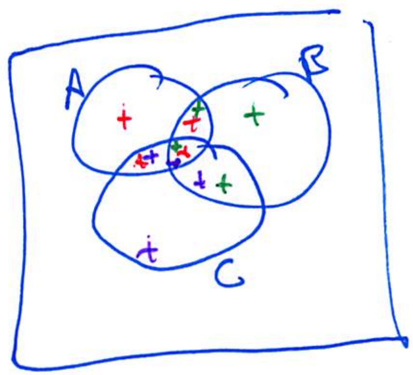
$0 \leq P(A) \leq 1$ for all events A

$P(\bar{A}) = 1 - P(A) \Rightarrow P(\bar{A}) + P(A) = 1$

$P(A \cup B) = P(A) + P(B) - P(AB)$

$P(\phi) = 0$

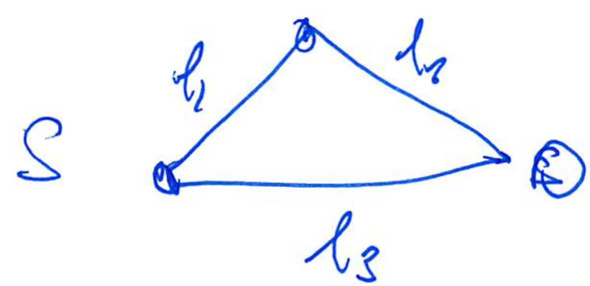
$P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$- P(AB) - P(AC) - P(BC)$
$+ P(ABC)$



Union Bound

$$\cancel{P(A)+} \quad P(A \cup B \cup C) \le P(A) + P(B) + P(C)$$

Ex.



$P(S \to D) = ?$

links are <u>independent</u>

$P(l_1 = 1 \cap l_2 = 1) = P(l_1 = 1)\,P(l_2 = 1)$

↑ working

$P(AB) = P(A)\,P(B)$

list all outcomes