Homework#1 (Math 342)
(due Wed Sep 19)

Linear Algebra: a Modern Introduction, by D. Poole (4th Edition)

Note: Detail your work to receive full credit.

Sec. 6.2: 19, 20, 22, 25

Sec. 6.4: 2, 26

Additional problems:

1) Determine whether the following vectors are linearly independent in $\mathbb{R}^3$:

(a)

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$$

(e)

$$\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

2) Determine whether the following matrices are linearly independent in $\mathbb{R}^{2\times 2}$:

(a)

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$$

3) Let $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$ be linearly independent vectors in $\mathbb{R}^n$ and let

$$\boldsymbol{y}_1 = \boldsymbol{x}_1 + \boldsymbol{x}_2 \ , \quad \boldsymbol{y}_2 = \boldsymbol{x}_2 + \boldsymbol{x}_3 \ , \quad \boldsymbol{y}_3 = \boldsymbol{x}_3 + \boldsymbol{x}_1$$

Are $\boldsymbol{y}_1$, $\boldsymbol{y}_2$ and $\boldsymbol{y}_3$ linearly independent? Prove your answer.

4) For each of the following, show that the given functions are linearly independent in $C[0,1]$:

(a) $\cos(\pi x)$, $\sin(\pi x)$

(b) $x^{3/2}$, $x^{5/2}$

(c) 1, $e^x + e^{-x}$, $e^x - e^{-x}$

(d) e^x, e^{-x}, e^{2x}

5) Prove that any finite set of vectors that contains the zero vector must be linearly dependent.

6) Let $\boldsymbol{a}$ be a fixed nonzero vector in $\mathbb{R}^2$. A mapping of the form

$$T(\boldsymbol{x}) = \boldsymbol{x} + \boldsymbol{a}$$

is called a translation. Show that a translation is not a linear transformation.

7) Determine whether the following are linear transformations from $\mathbb{R}^2$ into $\mathbb{R}^3$: for $\boldsymbol{x} = (x_1, x_2)^\top$

(a) $T(\boldsymbol{x}) = (x_1, x_2, 1)^\top$

(b) $T(\boldsymbol{x}) = (x_1, x_2, x_1 + 2x_2)^\top$

(c) $T(\boldsymbol{x}) = (x_1, 0, 0)^\top$

(d) $T(\boldsymbol{x}) = (x_1, x_2, x_1^2 + x_2^2)^\top$

8) Determine whether the following are linear transformations from $\mathbb{P}_2$ to $\mathbb{P}_3$:

(a) $T(p(x)) = x^2 + p(x)$

(b) $T(p(x)) = p(x) + xp(x) + x^2p'(x)$