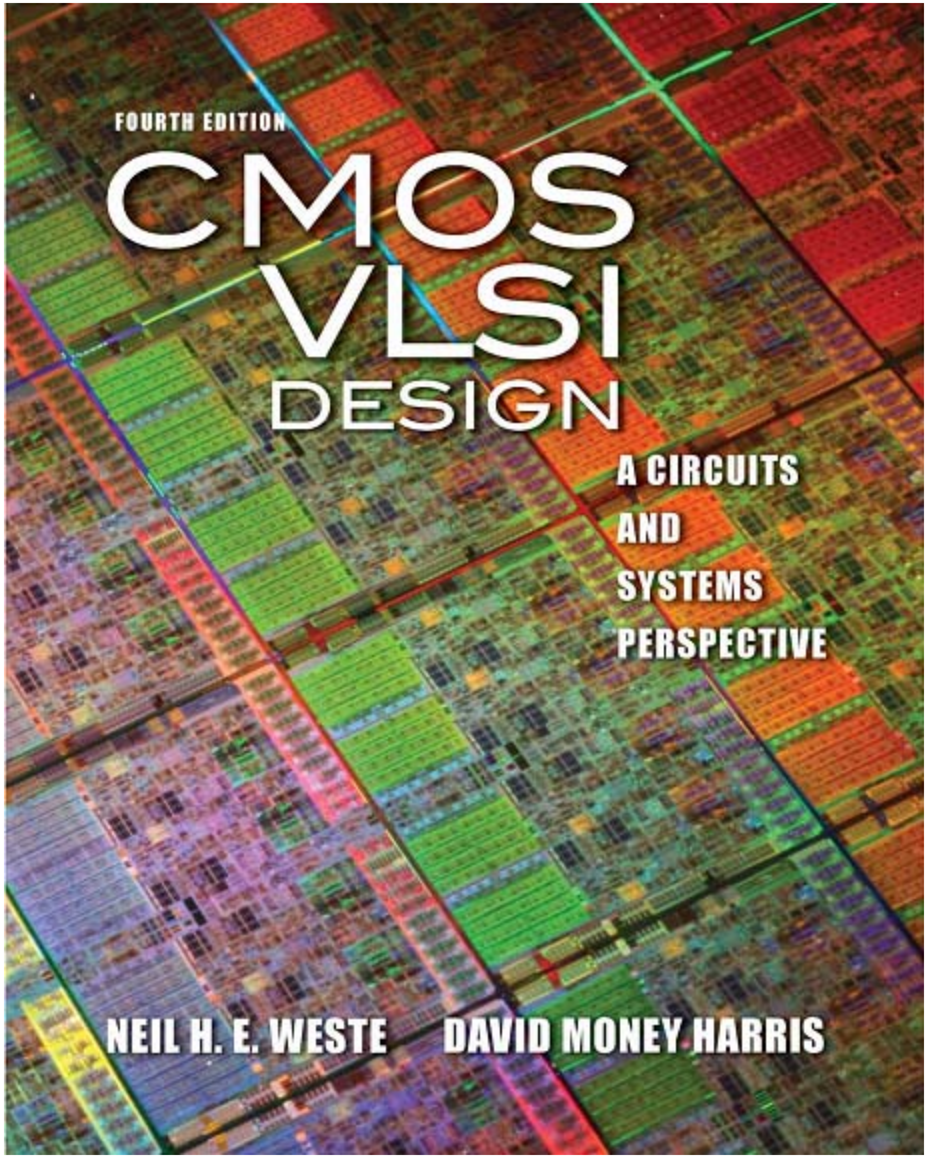



# Lecture12: Datapath Functional Units

# Outline

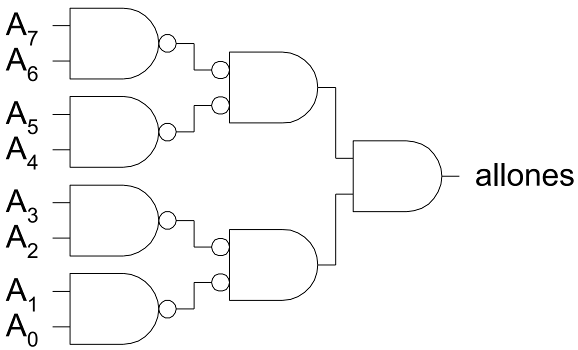
- Comparators
- Shifters
- Multi-input Adders
- Multipliers

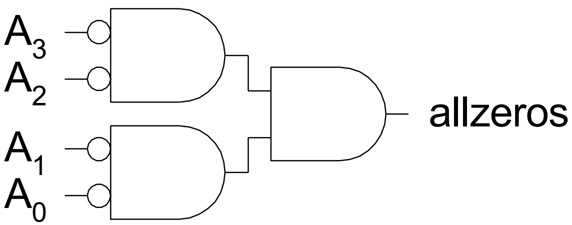
# Comparators

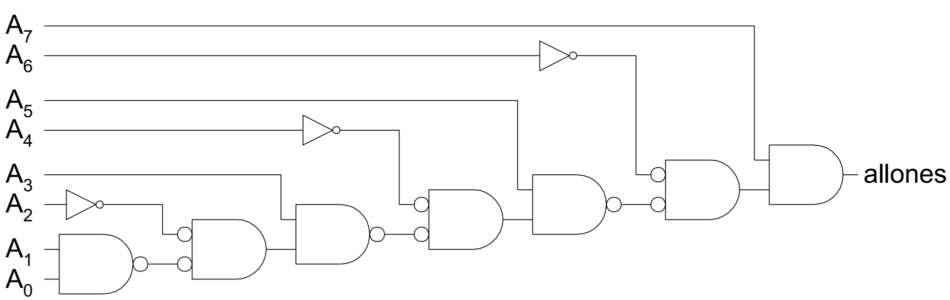
- 0's detector: A = 00…000
- 1's detector: A = 11…111
- Equality comparator: A = B
- Magnitude comparator: A < B

# 1’s & 0’s Detectors

- 1’s detector: N-input AND gate
- 0’s detector: NOTs + 1’s detector (N-input NOR)

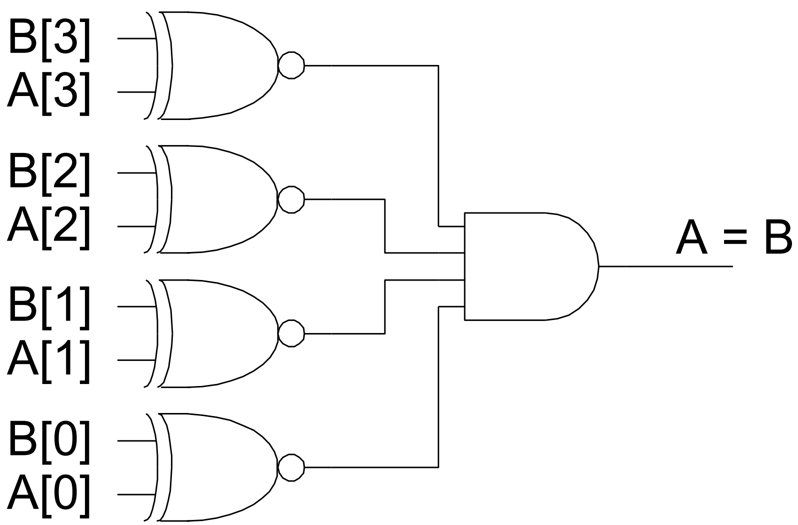


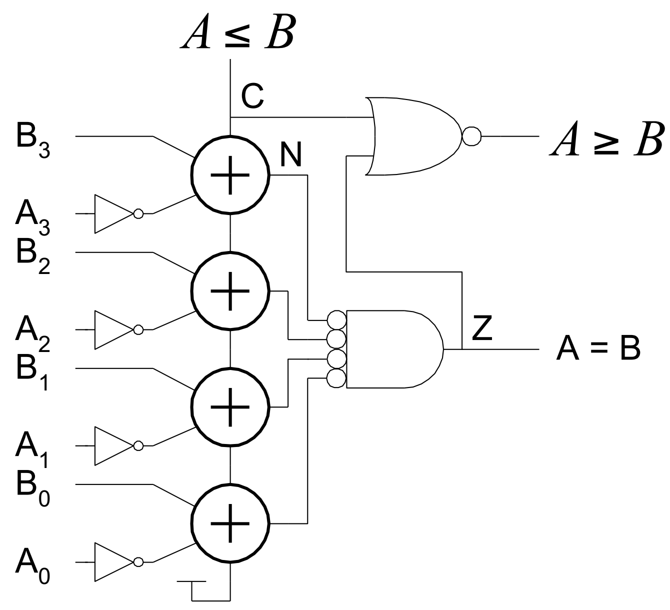
# Equality Comparator

- Check if each bit is equal (XNOR, aka equality gate)
- 1's detect on bitwise equality



# Magnitude Comparator

- Compute B – A and look at sign
- B – A = B + ~A + 1
- For unsigned numbers, carry out is sign bit



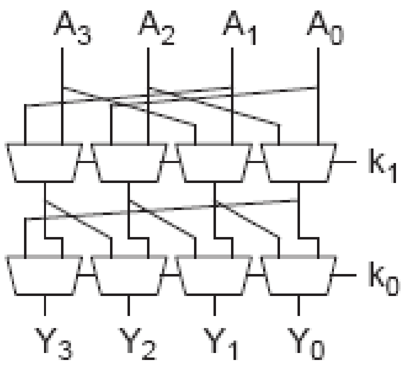
# Shifters

- Logical Shift:
  - Shifts number left or right and fills with 0's
    - 1011 LSR 1 = 0101 1011 LSL1 = 0110
- Arithmetic Shift:
  - Shifts number left or right. Rt shift sign extends
    - 1011 ASR1 = 1101 1011 ASL1 = 0110
- Rotate:
  - Shifts number left or right and fills with lost bits
    - 1011 ROR1 = 1101 1011 ROL1 = 0111

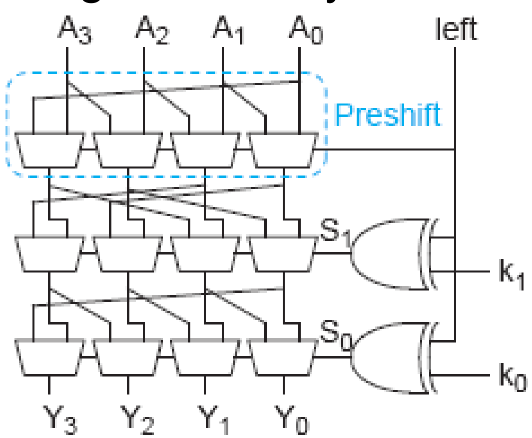
# Barrel Shifter

- Barrel shifters perform right rotations using wrap-around wires.
- Left rotations are right rotations by $N - k = \bar{k} + 1$ bits.
- Shifts are rotations with the end bits masked off.

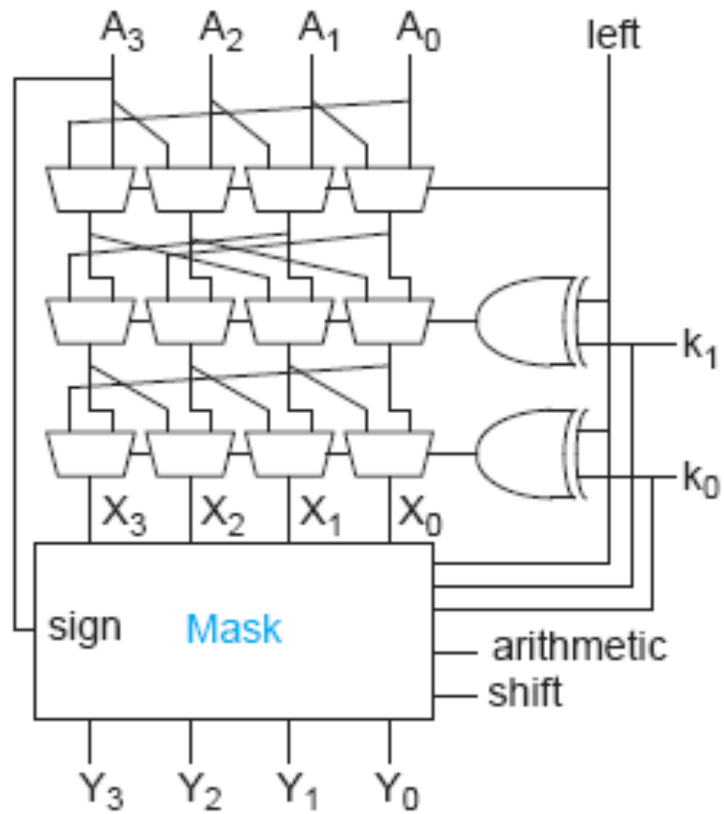
# Logarithmic Barrel Shifter




Right shift only



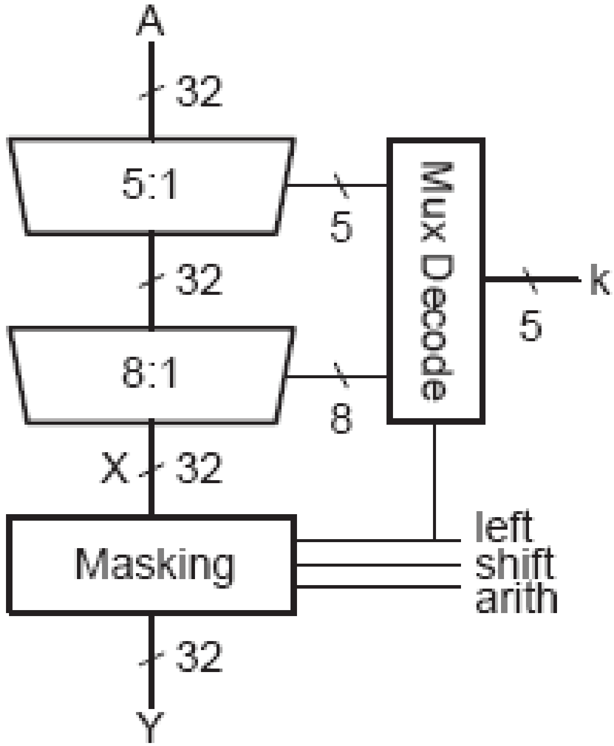

Right/Left shift




Right/Left Shift & Rotate

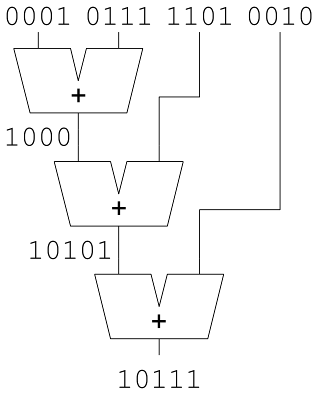
# 32-bit Logarithmic Barrel

- Datapath never wider than 32 bits
- First stage preshifts by 1 to handle left shifts



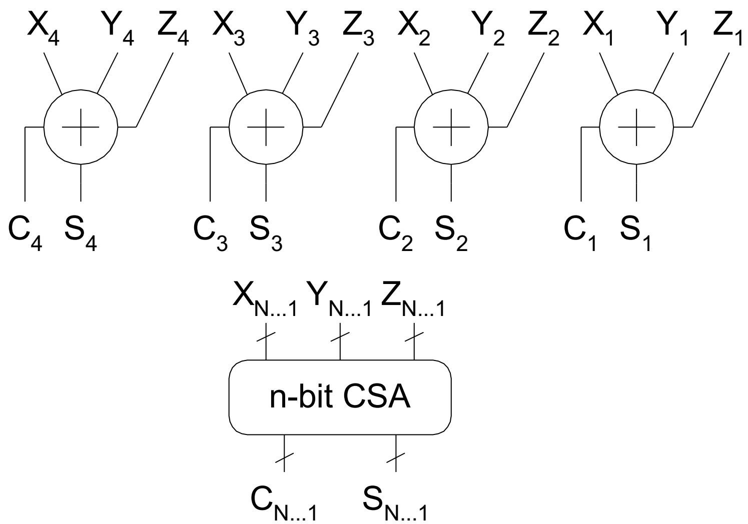
# Multi-input Adders

- Suppose we want to add k N-bit words
  - Ex: 0001 + 0111 + 1101 + 0010 = 10111
- Straightforward solution: k-1 N-input CPAs
  - Large and slow



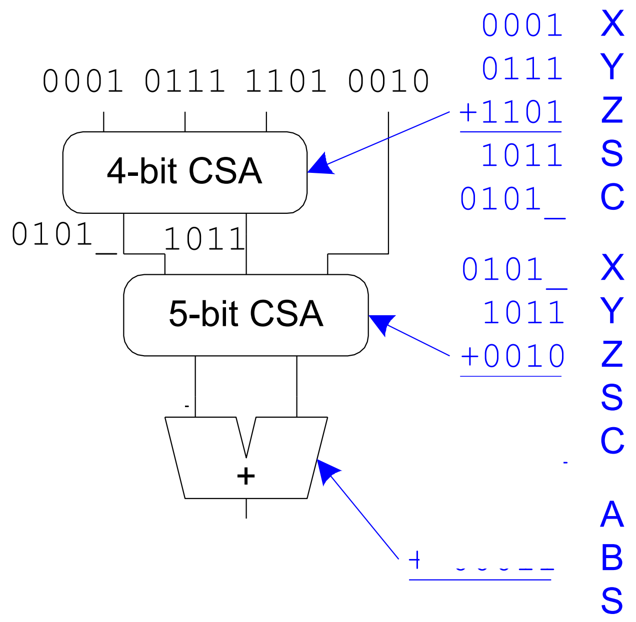
# Carry Save Addition

- A full adder sums 3 inputs and produces 2 outputs
  - Carry output has twice *weight* of sum output
- N full adders in parallel are called *carry save adder*
  - Produce N sums and N carry outs



# CSA Application

- Use k-2 stages of CSAs
  - Keep result in carry-save redundant form
- Final CPA computes actual result



# Multiplication

- Example:

```
      1100 : 12₁₀      multiplicand
      0101 : 5₁₀       multiplier
      ----
      1100
     0000             partial
    1100              products
   0000
  --------
  00111100 : 60₁₀      product
```

- M x N-bit multiplication
  - Produce N M-bit partial products
  - Sum these to produce M+N-bit product

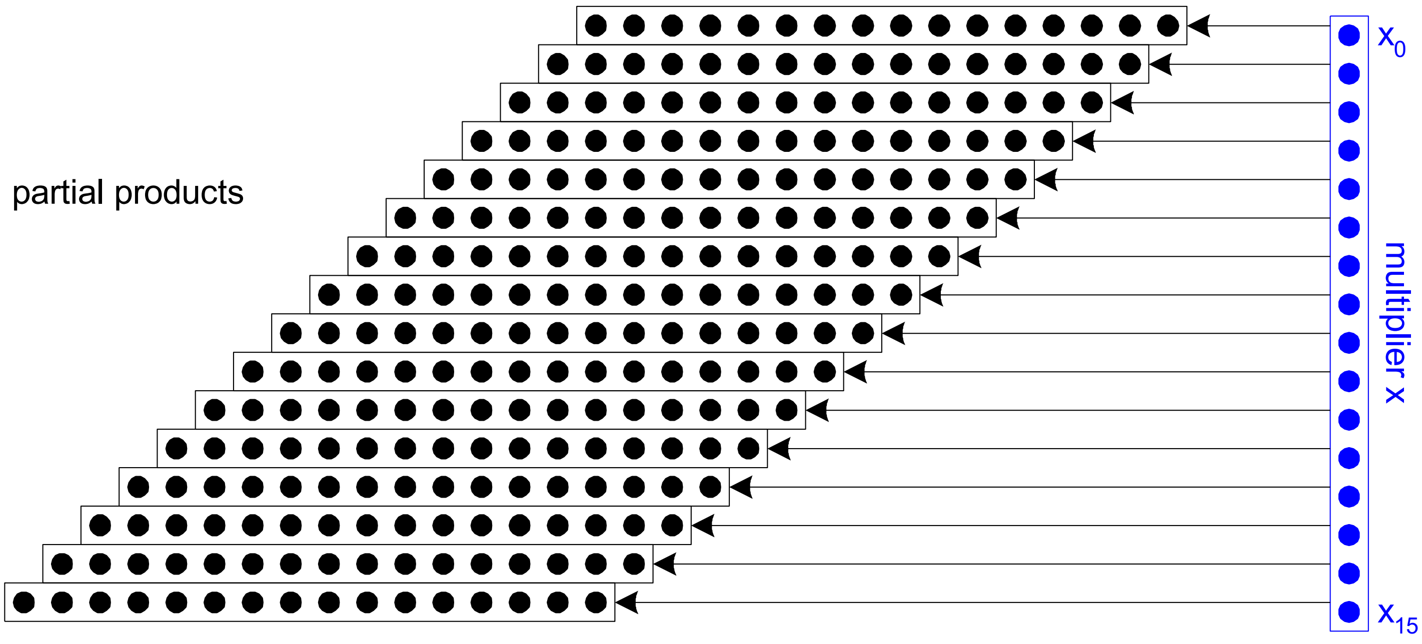
# General Form

- Multiplicand: $Y = (y_{M-1}, y_{M-2}, \ldots, y_1, y_0)$
- Multiplier: $X = (x_{N-1}, x_{N-2}, \ldots, x_1, x_0)$
- Product: $$P = \left(\sum_{j=0}^{M-1} y_j 2^j\right)\left(\sum_{i=0}^{N-1} x_i 2^i\right) = \sum_{i=0}^{N-1}\sum_{j=0}^{M-1} x_i y_j 2^{i+j}$$

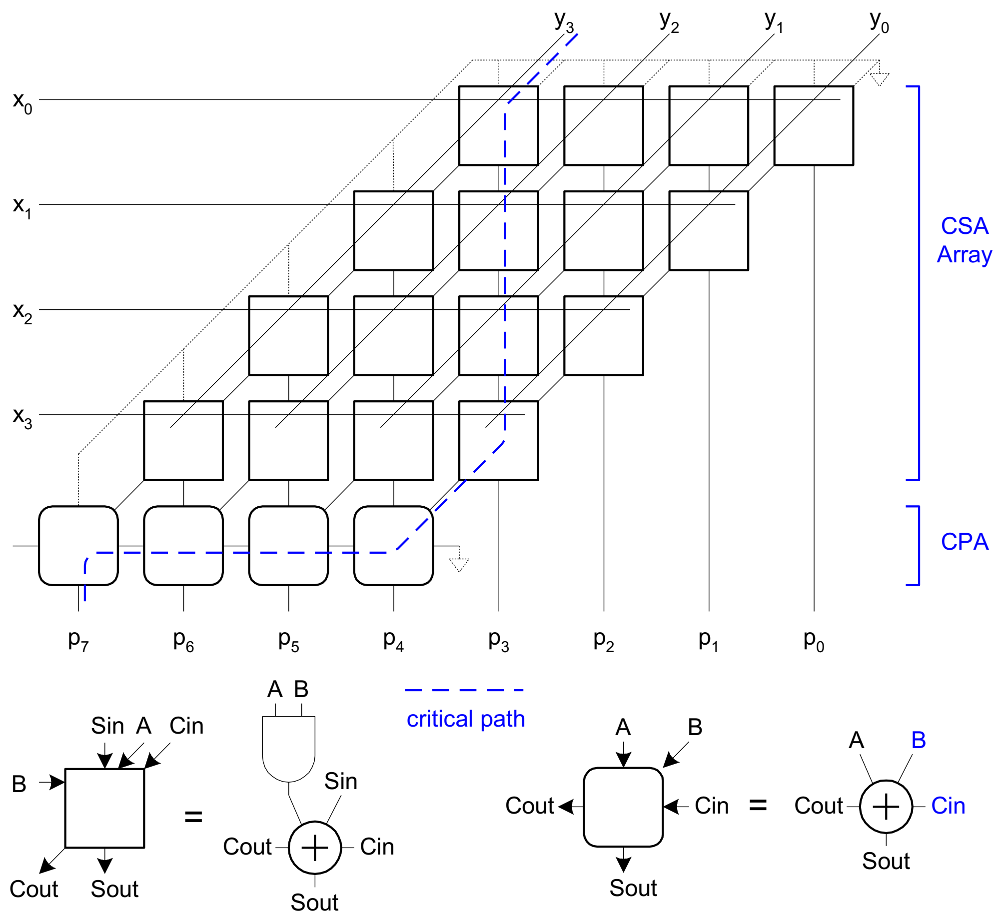
| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $y_5$ | $y_4$ | $y_3$ | $y_2$ | $y_1$ | $y_0$ | multiplicand |
| | | | | | | $x_5$ | $x_4$ | $x_3$ | $x_2$ | $x_1$ | $x_0$ | multiplier |
| | | | | | | $x_0y_5$ | $x_0y_4$ | $x_0y_3$ | $x_0y_2$ | $x_0y_1$ | $x_0y_0$ | partial products |
| | | | | | $x_1y_5$ | $x_1y_4$ | $x_1y_3$ | $x_1y_2$ | $x_1y_1$ | $x_1y_0$ | | |
| | | | | $x_2y_5$ | $x_2y_4$ | $x_2y_3$ | $x_2y_2$ | $x_2y_1$ | $x_2y_0$ | | | |
| | | | $x_3y_5$ | $x_3y_4$ | $x_3y_3$ | $x_3y_2$ | $x_3y_1$ | $x_3y_0$ | | | | |
| | | $x_4y_5$ | $x_4y_4$ | $x_4y_3$ | $x_4y_2$ | $x_4y_1$ | $x_4y_0$ | | | | | |
| | $x_5y_5$ | $x_5y_4$ | $x_5y_3$ | $x_5y_2$ | $x_5y_1$ | $x_5y_0$ | | | | | | |
| $p_{11}$ | $p_{10}$ | $p_9$ | $p_8$ | $p_7$ | $p_6$ | $p_5$ | $p_4$ | $p_3$ | $p_2$ | $p_1$ | $p_0$ | product |

# Dot Diagram

- Each dot represents a bit

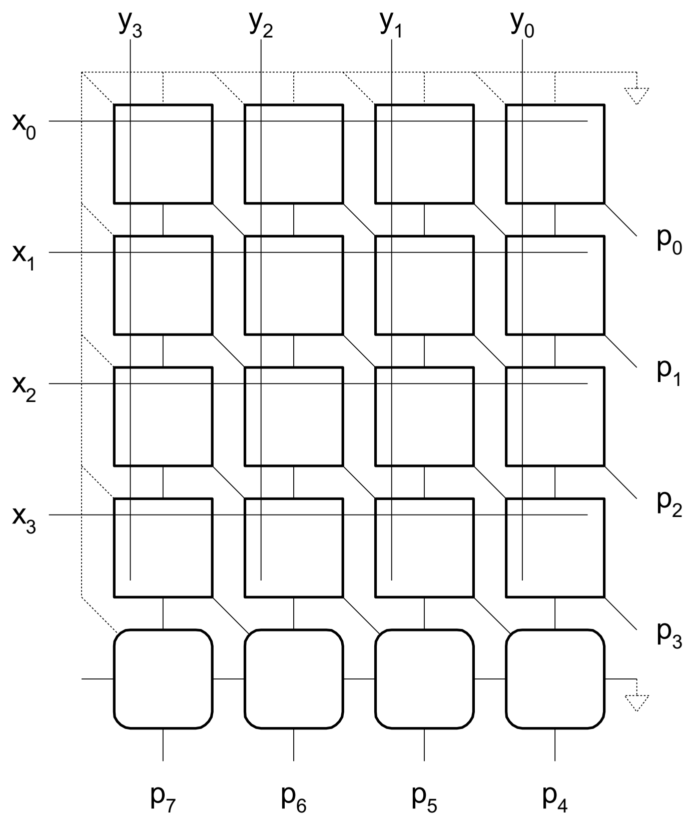


# Array Multiplier



# Rectangular Array

- Squash array to fit rectangular floorplan



# Advanced Multiplication

- Booth Encoding
- Signed vs. unsigned inputs
- Higher radix Booth encoding
- Array vs. tree CSA networks