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Axiomatic Approaches to IR

(CPEG 456/657: Search and Data Mining)

Hui Fang
Department of Electrical and Computer Engineering
University of Delaware

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What is axiomatic thinking?

- A thinking process that helps you identify the optimal solution to problem (X)
  - Identify **desirable properties** of a solution
  - **Formalize** the properties as **axioms**.
  - **Use** the **axioms** to **guide** the search for the **optimal solution**.
- You might not have heard it, but it is likely that you used it.

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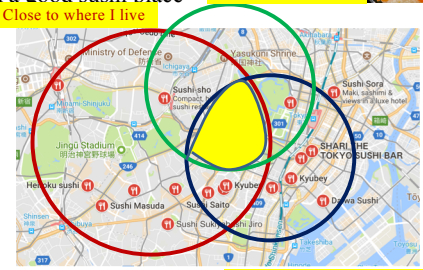
X = "Find a good sushi place"

Good reviews

Close to where I live



Open for lunch on Saturday

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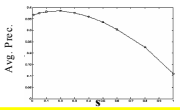
Empirical Observations

1. Retrieval performance is sensitive to parameter setting.



Parameter sensitivity of s in Pivoted Normalization Method

2. Optimal parameter setting depends on both queries and collections.

Optimal *s* in Pivoted Normalization Method

	AP	DOE	FR	ADF	Web	Trec 7	Trec 8
LK	0.2	0.2	0.05	0.2	---	---	---
SK	0.01	0.2	0.01	0.05	0.01	0.05	0.05
LV	0.3	0.3	0.1	0.2	0.2	0.2	0.2
SV	0.2	0.3	0.1	0.2	0.1	0.1	0.2

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Problems with Coarse Modeling of Relevance

- Vector-space model
  - RelScore(Q,D) → sim(Q,D) → ?
  - Slight variation of the TF normalization formula can cause significant difference in performance
- Probabilistic model (classic & language models)
  - RelScore(Q,D) → p(Rel=1|Q,D) →?
  - Slight variation in smoothing causes significant difference in performance

We need to model relevance directly at a finer granularity level of terms.

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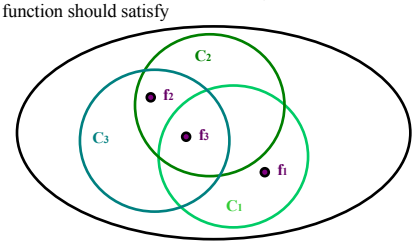
Basic Idea of Axiomatic Approach

A set of retrieval constraints that any reasonable retrieval function should satisfy



The search space for all possible retrieval functions

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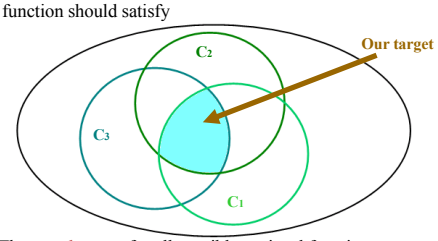
Basic Idea of Axiomatic Approach

A set of retrieval constraints that any reasonable retrieval function should satisfy



The search space for all possible retrieval functions

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An Axiomatic Framework for Retrieval Functions

- Component 1: Constraints to be satisfied by an effective retrieval function
- Component 2: Function space that allows us to efficiently search for an effective function

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Implementation of Component 1:

Question:
How do we define retrieval constraints?

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The 3 Major Heuristics

- Pivoted Normalization Method

$$\sum_{w\in q\cap d} \frac{1+\ln(1+\ln c(w,d))}{(1-s)+s\frac{|d|}{avdl}} \cdot c(w,q) \ln\frac{N+1}{df(w)}$$

- Dirichlet Prior Method

$$\sum_{w\in q\cap d} c(w,q)\times\ln\left(1+\frac{c(w,d)}{\mu\cdot p(w|C)}\right)+|q|\ln\frac{\mu}{\mu+|d|}$$

- Okapi Method

$$\sum_{w\in q\cap d} \ln\frac{N-df(w)+0.5}{df(w)+0.5} \frac{(k_1+1)\times c(w,d)}{k_1((1-b)+b\frac{|d|}{avdl})+c(w,d)} \frac{(k_3+1)\times c(w,q)}{k_3+c(w,q)}$$

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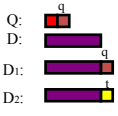
Term Frequency Constraints (TFC1)

TF weighting heuristic I:
Give a higher score to a document with more occurrences of a query term.

Let Q be a query and D be a document.
If $q \in Q$ and $t \notin Q$,
then $S(Q, D\cup\{q\}) > S(Q, D\cup\{t\})$



$S(Q, D_1) > S(Q, D_2)$

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Term Frequency Constraints (TFC2)

TF weighting heuristic II:
Require that the amount of increase in the score due to adding a query term must decrease as we add more terms.

- ***TFC2***

Let Q be a query with only one query term q.
Let D_1 be a document.
then $S(D_1\cup\{q\},Q)-S(D_1,Q) > S(D_1\cup\{q\}\cup\{q\},Q)-S(D_1\cup\{q\},Q)$

$S(D_2,Q)-S(D_1,Q) > S(D_3,Q)-S(D_2,Q)$

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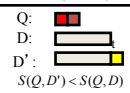
Length Normalization Constraints (LNCs)

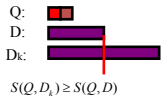
Document length normalization heuristic:
Penalize long documents(LNC1);
Avoid over-penalizing long documents (LNC2) .

- ***LNC1***
  Let Q be a query and D be a document.
  If t is a non-query term,
  then $S(D\cup\{t\},Q)<S(D,Q)$



$S(Q,D')<S(Q,D)$

- ***LNC2***
  Let Q be a query and D be a document.
  If $D\cap Q\neq\phi$, and D_k is constructed by concatenating D with itself k times,
  then $S(D_k,Q)\geq S(D,Q)$



$S(Q,D_k)\geq S(Q,D)$

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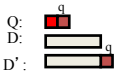
TF-LENGTH Constraint (TF-LNC)

TF-LN heuristic:
Regularize the interaction of TF and document length.

- ***TF-LNC***
  Let Q be a query and D be a document.
  If q is a query term,
  then $S(D\cup\{q\},Q)>S(D,Q)$.



$S(Q,D')>S(Q,D)$

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Is constraint analysis related to the performance of a retrieval function?

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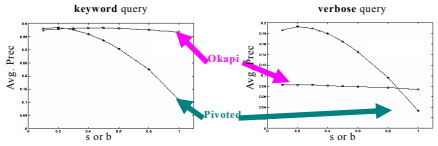
Violation of Constraints → Poor Performance

- Okapi Method

$$\sum_{w\in q\cap d}\ln\frac{N-df(w)+0.5}{df(w)+0.5}\cdot\frac{(k_1+1)\times c(w,d)}{k_1((1-b)+b\frac{|d|}{avdl})+c(w,d)}\cdot\frac{(k_3+1)\times c(w,q)}{k_3+c(w,q)}$$

Negative when df(w) is large → Violate many constraints



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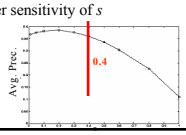
Conditional Satisfaction of Constraints → Parameter Bounds

- Pivoted Normalization Method LNC2 → s<0.4

Optimal s (for average precision)

	AP	DOE	FR	ADF	Web	Tree 7	Tree 8
LK	0.2	0.2	0.05	0.2	---	---	---
SK	0.01	0.2	0.01	0.05	0.01	0.05	0.05
LV	0.3	0.3	0.1	0.2	0.2	0.2	0.2
SV	0.2	0.3	0.1	0.2	0.1	0.1	0.2

Parameter sensitivity of s



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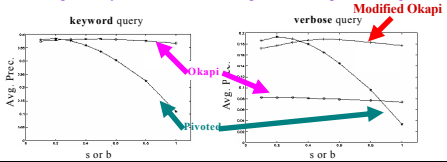
Constraints Analysis → Guidance for Improving an Existing Retrieval Function

- Modified Okapi Method

$$\sum_{w\in q\cap d}\ln\frac{N-df(w)+0.5}{df(w)+0.5}\cdot\ln\frac{N+1}{df}\cdot\frac{(k_1+1)\times c(w,d)}{k_1((1-b)+b\frac{|d|}{avdl})+c(w,d)}\cdot\frac{(k_3+1)\times c(w,q)}{k_3+c(w,q)}$$

Make Okapi satisfy more constraints; expected to help verbose queries



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Implementation of Component 2:

Question:
How can we define a function space that can be searched efficiently?

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Component 2: Function Space

$Q = \{q_1, q_2, ..., q_m\}$
$D = \{d_1, d_2, ..., d_n\}$ Bag of terms

$S : Q \times D \rightarrow \Re$ Function Space

Define the function space *inductively*

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Inductive Definition of Function Space

Primitive weighting function

Q: q
D: q
$S(Q, D) = weight(q)$

Q: q
D: d
$S(Q, D) = penalty(q, d)$

Query growth function

Q: q'
D:
$S(Q', D) = S(Q \cup \{q'\}, D) = h(S(Q, D), S(\{q'\}, D), q', D, Q)$

Document growth function

Q:
D': d'
$S(Q, D') = S(Q, D \cup \{d'\}) = g(S(Q, D), S(Q, \{d'\}), d', D, Q)$

Need to ensure that this indeed defines a function

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Derivation of New Retrieval Functions

Decompose the existing retrieval functions

⇩

Generalize each component function

⇩

Use constraints to find some alternative implementation of a component function

⇩

Assemble the new component functions to form a new retrieval function.

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Component Function 1: Primitive Weighting Function

Decompose the existing retrieval functions

⇩ Roughly IDF

Generalize the component function

⇩ Choice 1: $weight(q) = P(rel \mid occ)$

Choice 2: $weight(q) = \log \frac{P(occ \cap rel)}{P(occ)P(rel)}$

Use constraints to find some alternative implementation

Choice 1: $weight(q) = (\frac{N}{df(q)})^k$ EXP
Choice 2: $weight(q) = \log \frac{N}{df(q)}$ LOG

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Component Function 2: Query Growth Function

Follow the existing retrieval functions:

$$S(Q \cup \{q\}, D) = S(Q, D) + \alpha S(\{q\}, D)$$

$\alpha = 1$ (*same as Pivoted & Dirichlet*)

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Component Function 3: Document Growth Function

Decompose the existing functions Pivoted Normalization

$$\frac{1-s+s\frac{|D|}{avdl}}{1-s+s\frac{|D|+1}{avdl}}\cdot S(Q,D)+\frac{1-s+s\frac{1}{avdl}}{1-s+s\frac{|D|+1}{avdl}}\cdot[\ln(1+\ln(c(q,D)+1))-\ln(1+\ln(c(q,D)))]\cdot S(\{q\},D)$$

Generalize the component function of existing functions

$$\lambda_1(|D|)\cdot S(Q,D)+\lambda_2(|D|)\cdot z(c(q,D))\cdot S(\{q\},D)$$

Use constraints to find some alternative implementation

$$\lambda_1(k)=\frac{k+avdl/s}{k+1+avdl/s},\ \lambda_2(k)=\frac{1+avdl/s}{k+1+avdl/s}$$

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Summary of Derived Formulas

		Primitive Weighting Function	
		LOG	EXP
Doc. Growth Func.	PN-variation	$\sum_{i\in Q\cap D} c(t,Q)\cdot TF(c(t,D))\cdot LN(\lvert D\rvert)\cdot LW(t)$	$\sum_{i\in Q\cap D} c(t,Q)\cdot TF(c(t,D))\cdot LN(\lvert D\rvert)\cdot EW(t)$
	Okapi-variation	$\sum_{i\in Q\cap D} c(t,Q)\cdot TF\_LN(c(t,D),\lvert D\rvert)\cdot LW(t)$	$\sum_{i\in Q\cap D} c(t,Q)\cdot TF\_LN(c(t,D),\lvert D\rvert)\cdot EW(t)$
	DP-variation	$\sum_{i\in Q\cap D} c(t,Q)\cdot TF(c(t,D))\cdot LW(t)-\gamma(\lvert D\rvert,\lvert Q\rvert)$	$\sum_{i\in Q\cap D} c(t,Q)\cdot TF(c(t,D))\cdot EW(t)-\gamma(\lvert D\rvert,\lvert Q\rvert)$

$$S(Q,D)=\sum_{i\in Q\cap D} c(t,Q)\cdot\left(\frac{N}{df(t)}\right)^{\gamma}\frac{c(t,D)}{c(t,D)+s+\frac{s\cdot|D|}{avdl}}$$

Re-parameterization of Okapi TF with one parameter

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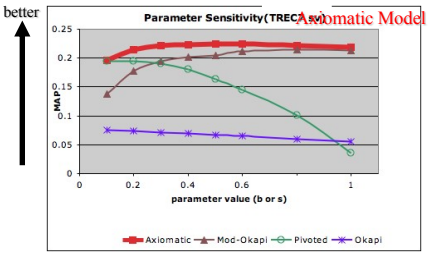
The derived function is less sensitive to the parameter setting



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Any limitation?

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Semantic Term Matching is Missing

Q: car

D1: vehicle $S(D_1,Q)$? $S(D_2,Q)$

D2: fish

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Semantic Term Matching Constraints (STMC1)

Semantic Term Matching Heuristic I:
Give a higher score to a document with a term that is more semantically related to a query term

- ***STMC1***

Let $Q=\{q\}$ be a query.
Let $D_1=\{d_1\}$, $D_2=\{d_2\}$ be two documents.
If $s(q,d_1)>s(q,d_2)$,
then $S(Q,D_1)>S(Q,D_2)$

Q: q
D1: d1 $s(q,d_1)>s(q,d_2)$
D2: d2
$S(Q,D_1)>S(Q,D_2)$

s(q,d) is any given semantic similarity function between two terms q and d.

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Semantic Term Matching Constraints

Semantic Term Matching heuristic II:
Favor semantically similar terms(SMTC2);
Avoid over-favoring semantic terms (SMTC3) .

- ***SMTC2***

Let $Q=\{q_1,q_2\}$ be a query and d be non-query term such that $s(q_2,d_2)>0$. $S(\{q_1\},\{q_1\})=S(\{q_2\},\{q_2\})$

If $|D_1|=|D_2|>1$, $c(q_1,D_1)=|D_1|$ and $c(q_1,D_2)=|D_2|-1$ then $S(Q,D_1)<S(Q,D_2)$

Q: D1: D2: $S(Q,D_1)<S(Q,D_2)$

- ***SMTC3***

Let $Q=\{q\}$ be a query and d be non-query term such that $s(q,d)>0$.
If $|D_1|=1$, $c(q,D_1)=1$, $|D_2|=k$, and $c(d,D_2)=k$

then $S(Q,D_1)>S(Q,D_2)$

Q: D1: D2: $S(Q,D_1)>S(Q,D_2)$

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Analyze Derived Function with SMTCs

$$S(Q,D)=\sum_{t\in Q\cap D} c(t,Q)\cdot\left(\frac{N}{df(t)}\right)^{0.35}\cdot\frac{c(t,D)}{c(t,D)+s+\frac{s\cdot|D|}{avdl}}$$

Primitive Weighting Function:

$$S(\{q\},\{d\})=\begin{cases}weight(q) & q=d\\ 0 & q\neq d\end{cases}$$

$$weight(q)=\left(\frac{N}{df(q)}\right)^{0.35}$$

Violate STMC1 constraint!

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Incorporate Semantic Term Matching

$$f(q,d)=\begin{cases}weight(q) & q=d\\ 0 & q\neq d\end{cases}$$

⇩ Generalization

$$f(q,d)=weight(q)\times s(q,d)$$

s(q,d) is any given semantic similarity function between two terms q and d.

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What You Should Know

- The key difference between the axiomatic approach and traditional retrieval models is that it directly models relevance with term-based constraints.
- The axiomatic framework makes it possible to
  - Predict the performance of a formula analytically without doing experiments
  - Derive completely new retrieval formulas with some guarantee of optimality
- Some new retrieval formulas derived tend to be more robust than existing models
- **Challenge**: deriving a formula that is consistently better (more effective and more robust) than all the existing formulas