ELEG 310

2/21/19

Experiments –

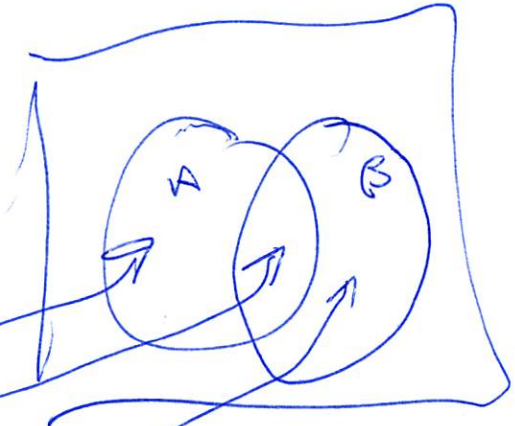
Outcomes

Events – sets of outcomes

union $A \cup B$ $A+B$

intersection $A \cap B$ $AB$

complement $\bar{A}$




$A \cup B = A\bar{B} \cup AB \cup \bar{A}B$

3 Axioms

1. $P(A) \geq 0$ 2. $P(S) = 1$ 3. if $AB = \phi$,

$P(A \cup B) = P(A) + P(B)$

Theorems

$0 \leq P(A) \leq 1$ for all $A$

$0 = P(\phi)$

$P(A \cup B) = P(A) + P(B) - P(AB)$

Independence — $A$ & $B$ are ind. iff $P(AB) = P(A)P(B)$

Ex. Roll die

$A$ = even $= \{2, 4, 6\}$ $\quad AB = \phi \quad P(AB) = 0$

$B$ = odd $= \{1, 3, 5\}$ $\quad P(A) = \frac{1}{2} \quad P(B) = \frac{1}{2}$

Disjoint $A$ & $B$ are disjoint if $AB = \phi$

$A = S = \{1, 2, 3, 4, 5, 6\}$ $AB = \{2, 3\}$ $P(AB) = \frac{2}{6} = \frac{1}{3}$

$B = \{2, 3\}$ $P(A) = 1$ $P(B) = \frac{2}{6} = \frac{1}{3}$ $\frac{1}{3} \stackrel{?}{=} 1 \times \frac{1}{3}$

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Ex. roll a 4-sided die

$B = \{2, 3\}$

B = TRUE if outcome is 2 or 3.

$A = \{1, 2\}$ $B = \{1, 2\}$

$AB = \{1, 2\}$ $P(A) = \frac{1}{2}$

$P(AB) = \frac{2}{4} = \frac{1}{2}$ $P(B) = \frac{1}{2}$

$\frac{1}{2} \stackrel{?}{=} \frac{1}{2} \times \frac{1}{2}$

$A = \{1, 2\}$ $B = \{2, 3\}$ $AB = \{2\}$ $P(AB) = \frac{1}{4}$

$P(A) = \frac{1}{2}$ $P(B)$ $\frac{1}{4} \stackrel{?}{=} \frac{1}{2} \times \frac{1}{2}$

$\Rightarrow$ A & B are ind. $\Leftarrow$

flip coin 3 times

P(2 heads) = ?

| $f_1$ $f_2$ $f_3$ | Exactly 2 heads | Prob |
|---|---|---|
| 0 0 0 | 0 | $q^3$ |
| 0 0 1 | 0 | $q^2p$ |
| 0 1 0 | 0 | $q^2p$ |
| 0 1 1 | 1 | $qp^2$ |
| 1 0 0 | 0 | |
| 1 0 1 | 1 | |
| 1 1 0 | 1 | |
| 1 1 1 | 0 | $p^3$ |

P(at least 2 H) = P(011, 101, 110, 111)

$= p^2q + p^2q + p^2q + p^3 = 3p^2q + p^3$

P(exactly 2 heads)

P(at least 2 heads)

P(2 heads in some order)

Assume ① P(heads) = p

P(tails) = 1 − p = q

② flips ind

P(exactly 2 H) = P(011, 101, 110)

$= P(011) + P(101) + P(110)$

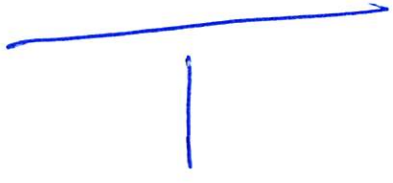
$= p^2q + p^2q + p^2q = 3p^2q$

$P(\text{exactly } 0\ H) = q^3$

$P(\text{exactly } 1\ H) = 3q^2p$

$P(\text{exactly } 2\ H) = 3qp^2$

$P(\text{exactly } 3\ H) = p^3$



$P(\text{exactly } 2H) = P(\{011, 101, 110\})$

$= P(\{011\}) + P(\{101\}) + P(\{110\})$

$= qp^2 + qp^2 + qp^2 = 3qp^2$

Roll a loaded die

$P(\{1,3\}) = P(1) = \frac{1}{2}$

$P(2) = 0$

$P(3) = \frac{1}{4}$

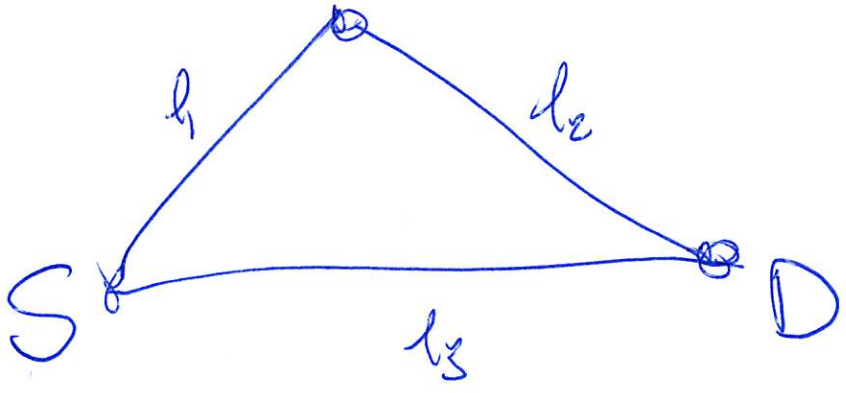
$P(4) = \frac{1}{8}$

$P(5) = \frac{1}{16}$

$P(6) = \frac{1}{16}$

$A = \{1, 2, 4\}$

$P(A) = \frac{1}{2} + 0 + \frac{1}{8} = \frac{5}{8}$




$P(l=1) = p$

links ind

| | $S \to D$ |
|---|---|
| 000 | 0 |
| 001 | 1 |
| 010 | 0 |
| 011 | 1 |
| 100 | 0 |
| 101 | 1 |
| 110 | 1 |
| 111 | 1 |

$$\{S \to D\} = \{001, 011, 101, 110, 111\}$$

$$P(S \to D) = q^2 p + q p^2 + q p^2 + q p^2 + p^3$$

$$= q_1 q_2 p_3 + q_1 p_2 q_3 + \cdots$$

let $A =$ 

$B =$ 

$\{S \to 0\} = A \cup B$

$P(S \to 0) = P(A) + P(B) - P(AB)$

$= p^2 + p - p^3$

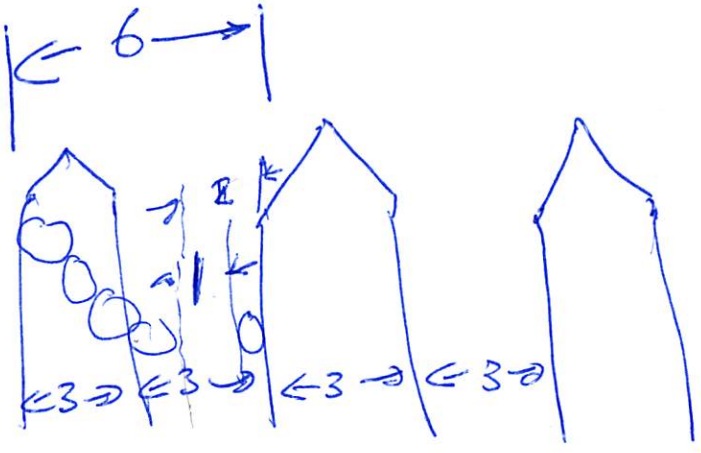
$\{11x\} = \{110, 111\}$

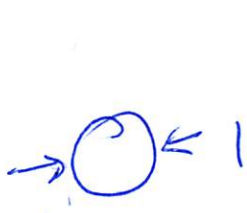
$P(11x) = P(110) + P(111) = p^2 q + p^3 = p^2(q + p) = p^2$

$B = \{x x 1\} = \{001, 011, 101, 111\}$

$P(\{xx1\}) =$ ---- $+ = p$

Picket Fence




$P(\text{hit}) = \frac{4}{6}$