SAMPLE FINAL ELEG 305 SIGNALS AND SYSTEMS SPRING 2019

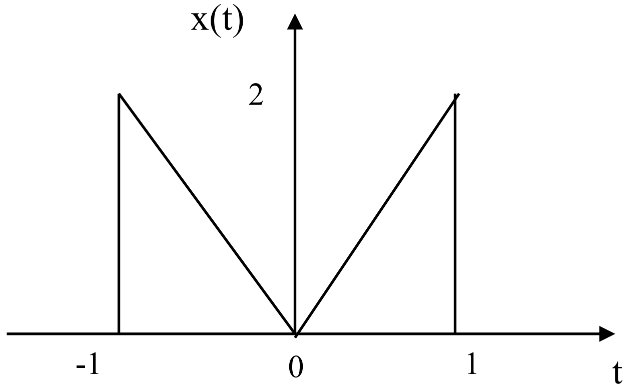
Problem #1 (15 points)

a.) (4 pts) Are the following functions periodic? Please explain your answer. If the function is periodic, determine the (i) period and (ii) the fundamental radian frequency.

(i) $x[n] = (-j)^n$

(ii) $x(t) = \dfrac{\sin \pi t}{\pi t}$

b.) (4 pts) Consider the following continuous-time signal, x(t):



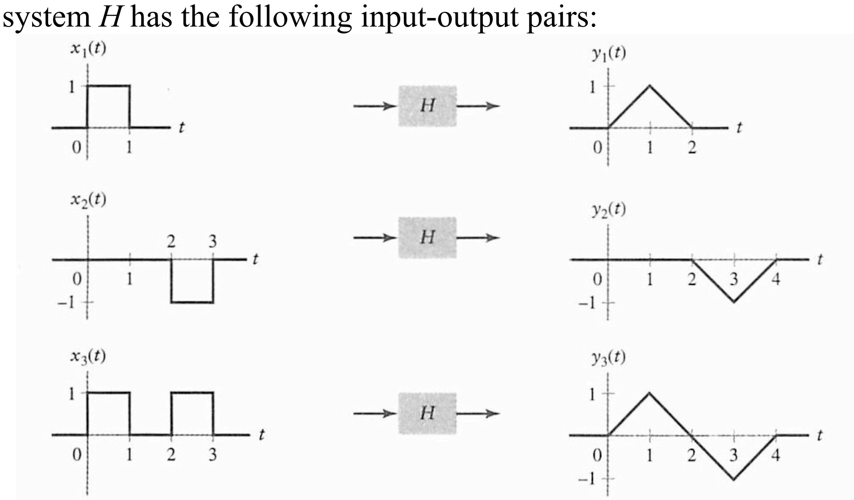
(i) Compute $x(t)\delta(t-0.5)$

(ii) Compute $\int_{-\infty}^{\infty} x(t)\delta(t-0.5)dt$

(iii) Assume x(t) is input to a linear, time invariant system. Compute or simply draw the output of this LTI system if the impulse response is $h(t) = 2\delta(t+3) - \delta(t-3)$.

c.) (3 pts) What is the impulse response for a discrete-time LTI system described by the equation $y[n] = \alpha x[n-1] + \beta x[n-2]$, where x[n] is the input and y[n] is the output?

d.) (4 pts) A system *H* has the following input-output pairs:



Answer the following questions, and please explain your answers.

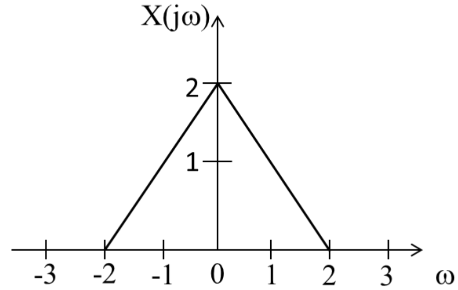
(i) Could this system be *causal*? (ii) Could this system be *linear*?

Problem #2 (10 points)

A unit impulse test signal is input to a simple model of the mammalian nervous system. We assume that the system is linear, time-invariant, and causal. The output for this input is a decaying exponential e^{-t}. Suppose a new test signal, x(t) = u(t-4), is used to determine the response of the nervous system to a sustained input. Using time-domain analysis **(i.e., convolution)**, determine the output for this input?

Problem #3 (10 points)

Consider a continuous-time signal, x(t), with the frequency characteristic shown below.



a.) (2 pts) What is the dc value of x(t)?

b.) (6 pts) What is the Nyquist rate? Suppose x(t) is sampled, at ***twice*** the Nyquist rate, using impulse-train sampling. Draw the spectrum of the resulting sampled signal.

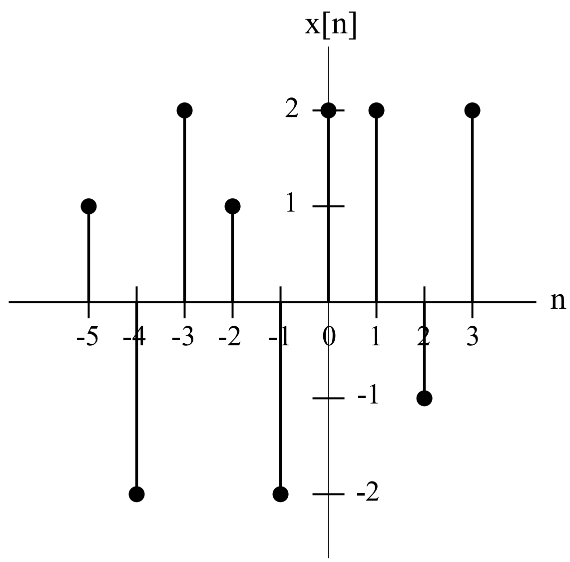
c.) (2 pts) Suppose x(t) is passed through an ideal lowpass filter with a bandwidth of 1 rad/sec. What is the minimum sampling rate for the filtered signal?

Problem #4 (20 points)

a.) (6 pts) Find the inverse Fourier transform of $X(j\omega) = \dfrac{2\sin(\omega-2)}{\omega-2} * \dfrac{e^{-2j\omega}\sin 2\omega}{\omega}$

b.) (4 pts) Consider the discrete-time sequence x[n] shown below. Evaluate the following:

(i) $X(e^{j0})$ (ii) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$



Problem #4 (cont'd)

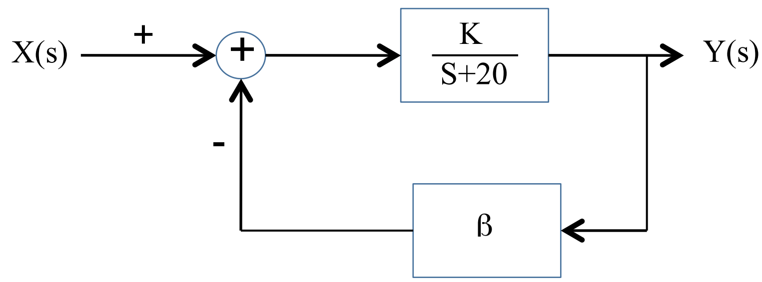
c.) (2 pts) Consider the following transfer function for a causal LTI system:

$$H(s) = \frac{10(2s+3)}{s(s^2+2s+5)}$$

Compute the values of the impulse response h(t) (i) at $t=0^+$ and (ii) in the limit as $t \to \infty$.

d.) (2 pts) Assume that the Fourier transform of x(t) exists and that its Laplace transform is rational. If X(s) has a pole at s = -1/2, could x(t) be (i) a left-sided signal? (ii) a right-sided signal? Please provide an answer for each part, along with an explanation.

e.) (4 pts) Consider the block diagram below for a causal system. What values of K will ensure that this system is stable? Express your result in terms of β.



f.) (2 pts) Consider the following continuous-time signal:

$$x(t) = \left(\frac{\sin(Wt)}{\pi t}\right)^4, \quad W = 100\pi \text{ rad/sec}$$

Impulse sampling is performed on x(t). What is the Nyquist rate for this signal?

Problem #5 (10 points)

Consider a continuous-time LTI system characterized by the following frequency response:

$$H(j\omega) = \frac{j\omega}{(j\omega+1)(j\omega-4)}$$

a.) (8 pts) Derive the impulse response of this system, h(t).

b.) (1 pt) Is this system memoryless? Please explain your answer.

c.) (1 pt) Is this system causal? Please explain your answer.

Problem #6 (10 points)

Consider a discrete-time LTI system characterized by the frequency response

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}}$$

a.) (2 pts) Find the difference equation relating the input and output of this system.

b.) (8 pts) Derive the impulse response of this system, h[n].

Problem #7 (10 points)

Consider a continuous-time LTI system with transfer function

$$H(s) = \frac{-s-4}{s^2+3s+2}$$

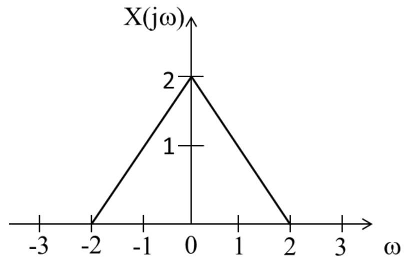
a.) (4 pts) Determine the poles and zeros for this system.

b.) (3 pts) List the three possibilities for the ROCs. (Just write the expressions; you do not have to draw them.)

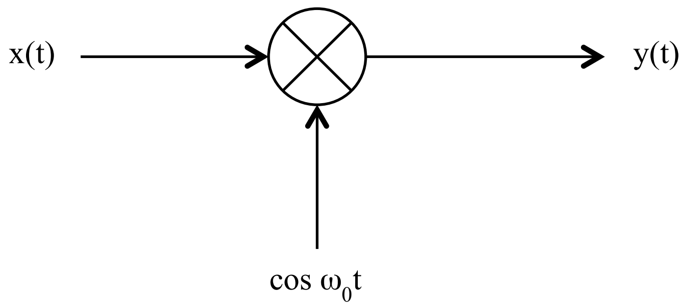
c.) (3 pts) Find the differential equation for a system with this transfer function.

Problem #8 (10 points)

Consider a message x(t), to be communicated to a distant receiver, with following spectrum:



One possible communication technique is called Amplitude Modulation (yes, the same AM that is in your car radio). In this scheme, the speech or music signal is multiplied by what is called a carrier wave (simply a cosine with radian frequency ω_0) as shown below:



a.) (5 pts) Compute and draw the spectrum (the frequency characteristic) of the output signal, $Y(j\omega)$ (assume $\omega_0 = 5$ radians/sec). Label each axis carefully.

b.) (5 pts) How might you recover the signal x(t) at the receiver?

Problem #9 (5 points)

A desired signal (such as a voice call on your cell phone, and image on your laptop, an EKG signal measured in the doctor's office, or many other possibilities) is often corrupted by noise and interference. This can come from many sources, including other people using the same spectrum at the same time, as happens in a cellular system. Assume that the desired signal has a flat spectrum equal to 2 for $|\omega_0| < 2$ radians/sec and 0 elsewhere. It is corrupted by noise that has a wide spectrum that goes from very low frequencies to very high frequencies. Assume the noise spectrum has amplitude 0.5 for $|\omega_0| < 20$ radians/sec, and 0 elsewhere.

(i) (2 pts) What is the power of the noise component of the signal?

(ii) (3 pts) What could we do to reduce the amount of noise in our desired signal?