ELE 6310

4/17/2018

X is continuous RV $\quad P(X=x)=0$ for all x

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(v)\,dv$$

$$P(a < X \le b) = F_X(b) - F_X(a) = \int_a^b f_X(v)\,dv$$

$$P(a < X \le a+\Delta) = \frac{F_X(a+\Delta) - F_X(a)}{\Delta}\,\Delta \approx f_X(a)\,\Delta$$

$$\approx \frac{dF}{da}$$

Multiple RVs

$$F_{XY}(x,y) = P(X \le x \cap Y \le y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(v,w)\, dw\, dv$$

$$E(g(X,Y)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(v,w) f_{XY}(v,w)\, dw\, dv$$

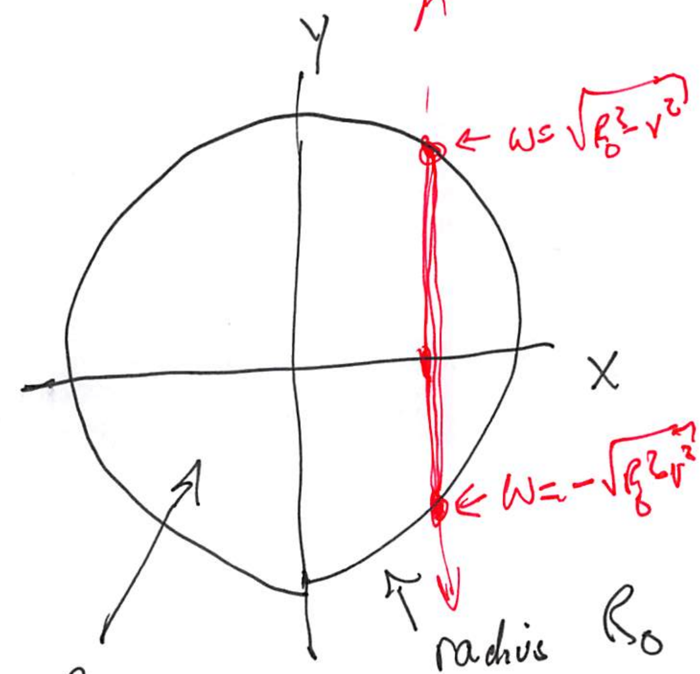
$X_1, X_2 .. X_n$ are IID independent and identically distributed

repeat same exp n times, repetitions ind.

$$F_{X_1 X_2 \cdots X_n}(x_1, x_2, \ldots, x_n) = F_{X_1}(x_1) F_{X_2}(x_2) \cdots F_{X_n}(x_n)$$

$$f_{X_1 \cdots X_n}(x_1, x_2, \ldots, x_n) = f_{X_1}(x_1) f_{X_2}(x_2) \cdots f_{X_n}(x_n)$$

X, Y is point chosen uniformly inside circle.



$$f_{XY}(v,w) = \begin{cases} C & v^2 + w^2 \le R_0^2 \\ 0 & \text{o.w.} \end{cases}$$

$f_{XY}(v,w)$ $\rightarrow$ 0

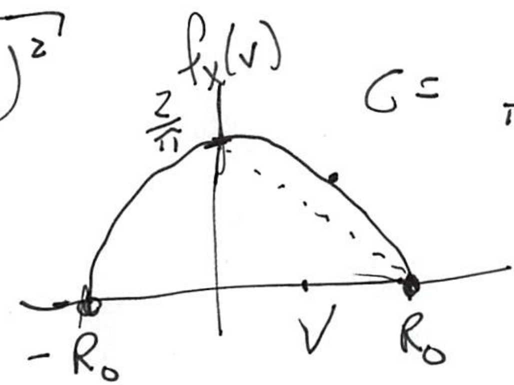
$$f_X(v) = \int_{-\infty}^{\infty} f_{XY}(v,w)\,dw = \int_{-\sqrt{R_0^2 - v^2}}^{\sqrt{R_0^2 - v^2}} c\,dw$$

$$= \frac{2\sqrt{R_0^2 - v^2}}{\pi R_0^2} = \frac{2}{\pi} \sqrt{1 - \left(\frac{v}{R_0}\right)^2}$$

$$1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(v,w)\,dw\,dv$$

$$1 = C \pi R_0^2$$

$$C = \frac{1}{\pi R_0^2}$$



$$EX = \int_{-R_0}^{R_0} v \frac{2}{\pi} \sqrt{1-\left(\frac{v}{R_0}\right)^2}\, dv = 0 \qquad EY = 0$$

$$EX^2 = \int_{-R_0}^{R_0} v^2 \frac{2}{\pi} \sqrt{1-\left(\frac{v}{R_0}\right)^2}\, dv = \int_{-R_0}^{R_0} \int_{-\sqrt{R_0^2-w^2}}^{\sqrt{R_0^2-w^2}} v^2 \frac{1}{\pi R_0^2}\, dv\, dw$$

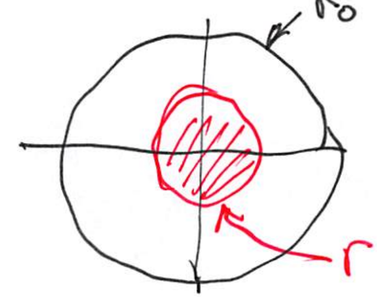
$$= \int_{-R_0}^{R} \int_{-\sqrt{R_0^2-v^2}}^{\sqrt{R_0^2-v^2}} v^2 \frac{1}{\pi R_0^2}\, dw\, dv = \int_{-R_0}^{R_0} v^2 \frac{1}{\pi R_0^2}\, 2\sqrt{R_0^2-v^2}\, dv$$

Change of Variable

$0 \le r \le R_0$

$$R = \sqrt{X^2+Y^2} \qquad F_R(r) = P(R \le r) = P(\sqrt{X^2+Y^2} \le r)$$

$$F_R(r) = \iint_{x^2+y^2 \le r^2} c\, dx\, dy = \int_0^{2\pi}\int_0^r c\, s\, ds\, d\theta$$



$$F_R(r) = \int_0^{2\pi}\int_0^{r} \frac{1}{\pi R_0^2}\, s\, ds\, d\theta = \frac{1}{\cancel{\pi} R_0^2}\,\frac{r^2}{\cancel{2}}\cdot \cancel{2\pi} = \frac{r^2}{R_0^2} \qquad \underline{0 < r \leq R_0}$$

$$f_R(r) = \frac{2r}{R_0^2} \qquad 0 < r \leq R_0$$

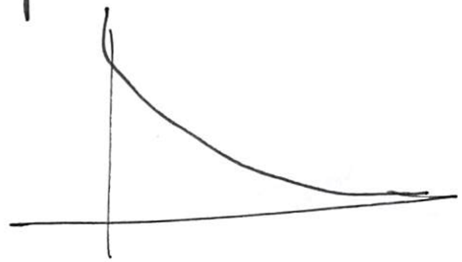
$$E(R) = \int_0^{R_0} r\,\frac{2r}{R_0^2}\, dr = \frac{2}{R_0^2}\,\frac{r^3}{3}\Big|_0^{R_0} = \frac{2}{3}R_0$$

$$E(R^2) = \int_0^{R_0} r^2\,\frac{2r}{R_0^2}\, dr = \frac{2}{R_0^2}\,\frac{r^4}{4}\Big|_0^{R_0} = \frac{R_0^2}{2}$$

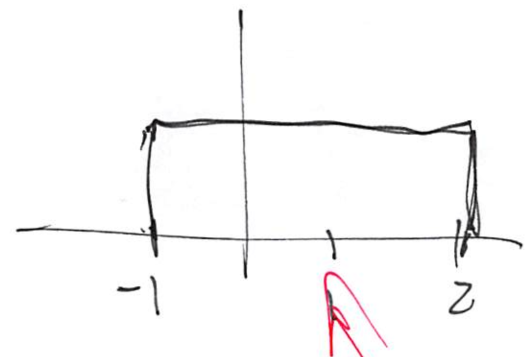
$$Var(R) = \frac{R_0^2}{2} - \left(\frac{2R_0}{3}\right)^2 = R_0^2\left(\frac{1}{2} - \frac{4}{9}\right) = \frac{R_0^2}{18}$$

$$\frac{R_0^2}{2} = ER^2 = E(X^2+Y^2) = E(X^2) + E(Y^2) \Rightarrow EX^2 = \frac{R_0^2}{4}$$

$X \sim$ exponential



$X \sim U(-1, 2)$



X, Y uniform $0 < X < 1$ and $-1 < Y < 1$