ELEG 310

4/12/2018

$f_X(u)$ = density $\quad f_X(u) \geq 0 \quad \int_{-\infty}^{\infty} f(u)\,du = 1$

$$P(a \leq X \leq b) = \int_a^b f(u)\,du = F(b) - F(a)$$

$$P(a \leq X \leq a + \Delta a) = \int_a^{a+\Delta a} f_X(u)\,du \approx \Delta a\, f_X(a)$$

$$P(X \approx a) \approx \Delta a\, f(a)$$

$$E(e^{uX}) = M(u) = \int_{-\infty}^{\infty} e^{ux} f(x)\,dx$$

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

$$P(X \le \cancel{u} \mid X \le v) = \frac{P(X \le u \cap X \le v)}{P(X \le v)}$$

$$= \begin{cases} \frac{P(X \le v)}{P(X \le \cancel{u})} = 1 & v < u \\ \frac{P(X \le u)}{P(X \le v)} = \frac{F(u)}{F(v)} & u < v \end{cases}$$

let $$F_X(u \mid X \le v) = P(X \le u \mid X \le v) = \begin{cases} 1 & u > v \\ \frac{F(u)}{F(v)} & u \le v \end{cases}$$

$$f_X(u \mid X \le v) = \frac{d}{du} F_X(u \mid X \le v) = \begin{cases} 0 & u > v \\ \frac{f(u)}{F(v)} & u < v \end{cases}$$

$$\int_{-\infty}^{\infty} f_X(u \mid X \le v)\, du = \int_{-\infty}^{v} \frac{f(u)}{F(v)}\, du = \frac{\int_{-\infty}^{v} f(u)\, du}{F(v)} = \frac{F(\cancel{v})}{F(v)} = 1$$

$$P(X \le u \mid X \ge v) = \frac{P(v \le X \le u)}{P(X \ge v)} = \begin{cases} 0 & u < v \\ \dfrac{F(u) - F(v)}{1 - F(v)} & u > v \end{cases}$$

$$= F_X(u \mid X \ge v)$$

$$f_X(u \mid X \ge v) = \begin{cases} 0 & u < v \\ \dfrac{f(u)}{1 - F(v)} & u \ge v \end{cases}$$

$$E(X \mid X \ge v) = \int_v^\infty \frac{x f(x)}{1 - F(v)}\, dx = \text{function of } v$$

Multiple Continuous RV's

X & Y

$X_1, X_2, X_3, \ldots X_n$

Joint Distribution Function

$$F_{XY}(x,y) = P(X \le x \cap Y \le y)$$ 2nd order distribution

$$F(-\infty,-\infty) = 0 \qquad F(\infty,\infty) = 1$$

$$F_{XY}(x,\infty) = P(X \le x \cap Y \le \infty) = P(X \le x) = F_X(x)$$

$$F_{XY}(\infty,y) = F_Y(y)$$ ← first order distribution

$$f_{XY}(x,y) = \frac{\partial}{\partial x}\frac{\partial}{\partial y}F_{XY}(x,y)$$

$$E(g(X,Y)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(v,w)\, f_{XY}(v,w)\, dv\, dw$$

$$E(X) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} v\, f_{XY}(v,w)\, dw\right] dv = \int_{-\infty}^{\infty} v\, f_X(v)\, dv = E(X)$$

$$F_{XY}(x,y) = P(X \le x \cap Y \le y) = \int_{-\infty}^{x}\left[\int_{-\infty}^{y} f_{XY}(v,w)\,dw\right] dv$$

$$F_X(x) = F_{XY}(x,\infty) = \int_{-\infty}^{x}\left[\int_{-\infty}^{\infty} f_{XY}(v,w)\,dw\right] dv = \int_{-\infty}^{x} \left[f_X(v)\right] dv$$

$$f_X(v) = \int_{-\infty}^{\infty} f_{XY}(v,w)\,dw$$

correlación $E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} vw\, f_{XY}(v,w)\,dv\,dw$

covarianci $\sigma_{XY} = Cov(X,Y) = E\left((X-\mu_X)(Y-\mu_Y)\right) = E(XY) - \mu_X \mu_Y$

Independence X & Y are ind if $F_{XY}(x,y) = F_X(x)\, F_Y(y)$

or $f_{XY}(x,y) = f_X(x)\, f_Y(y)$

for all x & y

ind $\Rightarrow$ unc

$\Rightarrow E(XY) = E(X)\ E(Y)$

$= \mu_X \mu_Y$

$X_1 X_2 \dots X_n$ are independent and identically distributed

(IID) $\Rightarrow F_{X_1, X_2 \dots X_n}(x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} F(x_i)$

Same distribution for $X_1\ X_2, \dots X_n$

$X_1 X_2 \dots X_n$ are n $\overset{\text{ind}}{\wedge}$ repetitions of same exp.

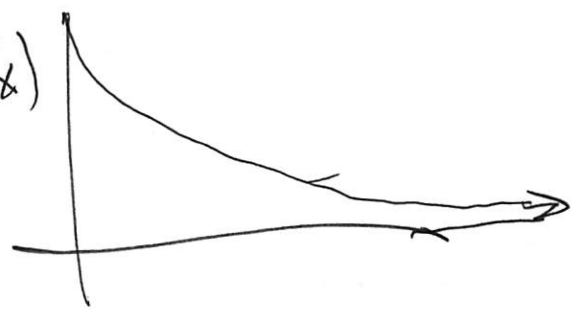
Ex. Lightbulb exp. $f(x) = \lambda e^{-\lambda x}$ $x \geq 0$

$$f(x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f(x_i) \Leftarrow \text{ind}$$

$$= \begin{cases} \lambda^n e^{-\lambda \underbrace{(x_1 + x_2 + \cdots + x_n)}_{\text{sum of } x_i}} & \\ 0 & \text{o.w} \end{cases}$$

$x_1 \geq 0$
$x_2 \geq 0$
$\vdots$
$x_n \geq 0$

$f(x)$



$F(x)$