Problem 1

6-3.3 (a) From $\Delta V = Ed$ we obtain $\Delta V = 25\times1.2\times10^{-3} = 0.03$ V. By (6.4),

$$C = \frac{A}{4\pi kd} = \frac{\pi(.05)^2}{(4\pi)(9\times10^9)(1.2\times10^{-3})} = 5.79\times10^{-11}\text{ F},$$

(b) so $Q = C\Delta V = 1.736\times10^{-12}$ C.

Problem 2

6-3.7 $C = 400$ pF $= 400\times10^{-12}$ F, $Q = \pm500$ nC $= \pm500\times10^{-9}$ C, $d = 2$ mm $= 2\times10^{-3}$ m,

(a) $\Delta V = \dfrac{Q}{\text{C}} = 1250$ V.

(b) $C = \dfrac{A}{4\pi kd}$, so $A = 4\pi kdC = .0905\text{ m}^2$.

(c) $E = \dfrac{\Delta\text{V}}{d} = 6.25\times10^5\ \dfrac{\text{N}}{\text{C}}$.

(d) $E = 4\pi k\sigma, \quad \sigma = \dfrac{E}{4\pi k} = 5.53\times10^{-6}\dfrac{\text{C}}{\text{m}^2}$.

Problem 3

6-4.5 (a) When placed in series the capacitance is

$$C_s = \left(C_1^{-1}+C_2^{-1}\right)^{-1} = \left((8\ \mu\text{F})^{-1}+(6\ \mu\text{F})^{-1}\right)^{-1} = 3.428\ \mu\text{F}.$$

When placed in parallel the capacitance is

$C_p = C_1 + C_2 = 8\ \mu\text{F} + 6\ \mu\text{F} = 14\ \mu\text{F}$.

(b) For series connection, the capacitors have the same charge

$$Q = C_s\Delta V = (3.428\ \mu\text{F})\,(12\ \mu\text{F}) = 41.14\ \mu C.$$

Now $\Delta V_1 = \dfrac{Q}{C_2} = \dfrac{41.14\ \mu\text{C}}{8\ \mu\text{F}} = 5.143$ V,

and $\Delta V_2 = \dfrac{Q}{C_2} = \dfrac{41.14\ \mu\text{C}}{6\ \mu\text{F}} = 6.857$ V.

Notice that $\Delta V = \Delta V_1 + \Delta V_2 = 12$ V.

(c) For parallel connection, the capacitors have the same voltage

$$\Delta V = 12\text{ V}.$$

Now $Q_1 = C_1\Delta\text{V} = (8\ \mu\text{F})\,(12\text{ V}) = 96\ \mu\text{F}$,

and $Q_2 = C_2\Delta\text{V} = (6\ \mu\text{F})\,(12\text{ V}) = 72\ \mu\text{F}$.

Problem 4

6-4.11 See Figure 6.28.

The pair of capacitors on the middle left are inactive, since $\Delta V = 0$ for them. The remaining capacitors are equivalent to, in the upper arm, two in series; and in the lower arm, one in series with two in parallel. Thus

$$C_{upper} = \left(C^{-1} + C^{-1}\right)^{-1} = \frac{1}{2}C.$$

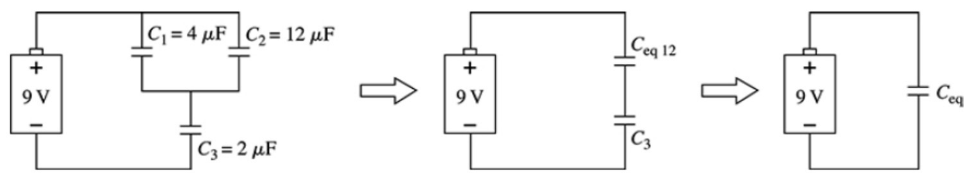
$$C_{lower} = \left(C^{-1} + (2C)^{-1}\right)^{-1} = C\left(1 + \frac{1}{2}\right)^{-1} = \frac{2}{3}C.$$

$$\text{Thus } C_{eff} = C_{upper} + C_{lower} = \frac{1}{2}C + \frac{2}{3}C = \frac{7}{6}C.$$

Since $C = 6\ \mu\text{F}$, we have $C_{eff} = 7\ \mu\text{F}$.

Problem 5

Visualize:



The pictorial representation shows how to find the equivalent capacitance of the three capacitors shown in the figure.

Solve: Because C_1 and C_2 are in parallel,

$$C_{eq\ 12} = C_1 + C_2 = 4\ \mu\text{F} + 12\ \mu\text{F} = 16\ \mu\text{F}$$

$C_{eq\ 12}$ and C_3 are in series, so

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq\ 12}} + \frac{1}{C_3} = \frac{1}{16\ \mu\text{F}} + \frac{1}{2\ \mu\text{F}} = \frac{18}{32}(\mu\text{F})^{-1} \Rightarrow C_{eq} = \tfrac{32}{18}\ \mu\text{F}$$

A potential difference of $\Delta V_C = 9$ V across a capacitor of equivalent capacitance $\frac{32}{18}\ \mu$F produces a charge

$$Q = C_{eq}\Delta V_C = (\tfrac{32}{18}\ \mu\text{F})9\ \text{V} = 16\ \mu\text{C}$$

Because C_{eq} is a series combination of two capacitors $C_{eq\ 12}$ and C_3, $Q_3 = Q_{eq\ 12} = 16\ \mu$C. The potential difference across C_3 is

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{16\ \mu\text{C}}{2\ \mu\text{F}} = 8.0\ \text{V}$$

Now, $Q_{eq\ 12} = 16\ \mu$C is the charge on the equivalent capacitor with $C_{eq\ 12} = 16\ \mu$F. So, the potential difference across the equivalent capacitor $C_{eq\ 12}$ is

$$\Delta V_{eq\ 12} = \frac{Q_{eq\ 12}}{C_{eq\ 12}} = \frac{16\ \mu\text{C}}{16\ \mu\text{F}} = 1.0\ \text{V}$$

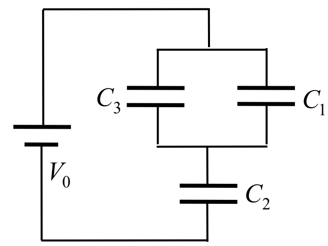
Parallel capacitors C_1 and C_2 have the same potential difference as the equivalent capacitor $C_{eq\ 12}$, so $\Delta V_1 = \Delta V_2 =$ 1.0 V. The charge on each is given by $Q = C\Delta V$, so $Q_1 = (4\ \mu\text{F})(1.0\ \text{V}) = 4.0\ \mu$C and $Q_2 = (12\ \mu\text{F})(1.0\ \text{V}) = 12.0\ \mu$C.

In summary, $Q_1 = 4.0\ \mu$C, $\Delta V_1 = 1.0$ V; $Q_2 = 12.0\ \mu$C, $\Delta V_2 = 1.0$ V; and $Q_3 = 16.0\ \mu$C, $\Delta V_3 = 8.0$ V.

Assess: Note that $\Delta V_3 + \Delta V_{eq\ 12} = 9.0\ \text{V} = \Delta V_{bat}$, as it should. Also that $Q_1 + Q_2 = 16.0\ \mu\text{C} = Q_{eq\ 12}$, as it should.

Problem 6

23. We want a small voltage drop across C_1. Since $V = Q/C$, if we put the smallest capacitor in series with the battery, there will be a large voltage drop across it. Then put the two larger capacitors in parallel, so that their equivalent capacitance is large and therefore will have a small voltage drop across them. So put C_1 and C_3 in parallel with each other, and then put that combination in series with C_2. See the diagram. To calculate the voltage across C_1, find the equivalent capacitance and the net charge. That charge is used to find the voltage drop across C_2, and then that voltage is subtracted from the battery voltage to find the voltage across the parallel combination.



Problem 7

87. Since the two capacitors are in series, they will both have the same charge on them.

$$Q_1 = Q_2 = Q_{\text{series}} \quad ; \quad \frac{1}{C_{\text{series}}} = \frac{V}{Q_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \rightarrow$$

$$C_2 = \frac{Q_{\text{series}} C_1}{C_1 V - Q_{\text{series}}} = \frac{\left(125 \times 10^{-12}\,\text{C}\right)\left(175 \times 10^{-12}\,\text{F}\right)}{\left(175 \times 10^{-12}\,\text{F}\right)\left(25.0\,\text{V}\right) - \left(125 \times 10^{-12}\,\text{C}\right)} = \boxed{5.15 \times 10^{-12}\,\text{F}}$$

Problem 8

89. The first capacitor is charged, and so has a certain amount of charge on its plates. Then, when the switch is moved, the capacitors are not connected to a source of charge, and so the final charge is equal to the initial charge. Initially treat capacitors C_2 and C_3 as their equivalent capacitance, $C_{23} = \frac{C_2 C_3}{C_2 + C_3} = \frac{\left(2.0\,\mu\text{F}\right)\left(2.4\,\mu\text{F}\right)}{4.4\,\mu\text{F}} = 1.091\,\mu\text{F}$. The final voltage across C_1 and C_{23} must be the same. The charge on C_2 and C_3 must be the same. Use Eq. 24-1.

$$Q_0 = C_1 V_0 = Q_1 + Q_{23} = C_1 V_1 + C_{23} V_{23} = C_1 V_1 + C_{23} V_1 \quad \rightarrow$$

$$V_1 = \frac{C_1}{C_1 + C_{23}} V_0 = \frac{1.0\,\mu\text{F}}{1.0\,\mu\text{F} + 1.091\,\mu\text{F}} \left(24\,\text{V}\right) = 11.48\,\text{V} = V_1 = V_{23}$$

$$Q_1 = C_1 V_1 = \left(1.0\,\mu\text{F}\right)\left(11.48\,\text{V}\right) = 11.48\,\mu\text{C}$$

$$Q_{23} = C_{23} V_{23} = \left(1.091\,\mu\text{F}\right)\left(11.48\,\text{V}\right) = 12.52\,\mu\text{C} = Q_2 = Q_3$$

$$V_2 = \frac{Q_2}{C_2} = \frac{12.52\,\mu\text{C}}{2.0\,\mu\text{F}} = 6.26\,\text{V} \quad ; \quad V_3 = \frac{Q_3}{C_3} = \frac{12.52\,\mu\text{C}}{2.4\,\mu\text{F}} = 5.22\,\text{V}$$

To summarize: $\boxed{Q_1 = 11\,\mu\text{C}\,,\ V_1 = 11\,\text{V} \ ;\ Q_2 = 13\,\mu\text{C}\,,\ V_2 = 6.3\,\text{V} \ ;\ Q_3 = 13\,\mu\text{C}\,,\ V_3 = 5.2\,\text{V}}$

Problem 9

$$i_C(t) = C\frac{dv_C(t)}{dt} = 100\times10^{-6}\frac{dv_C(t)}{dt} = 10^{-4}\frac{dv_C(t)}{dt}$$

a)

$$i_C(t) = 10^{-4}\left[-20\times40\sin\left(20t-\frac{\pi}{2}\right)\right] = -0.08\sin\left(20t-\frac{\pi}{2}\right)$$

$$= 0.08\sin\left(20t-\frac{\pi}{2}+\pi\right) = 0.08\sin\left(20t+\frac{\pi}{2}\right)\ \ \mathbf{A}$$

b)

$$i_C(t) = 10^{-4}\left[100\times20\cos100t\right] = 0.2\cos100t\ \ \mathbf{A}$$

c)

$$i_C(t) = 10^{-4}\left[-80\times60\cos\left(80t+\frac{\pi}{6}\right)\right] = -0.48\cos\left(80t+\frac{\pi}{6}\right)$$

$$= 0.48\cos\left(80t+\frac{\pi}{6}-\pi\right) = 0.48\cos\left(80t-\frac{5\pi}{6}\right)\ \ \mathbf{A}$$

d)

$$i_C(t) = 10^{-4}\left[-100\times30\sin\left(100t+\frac{\pi}{4}\right)\right] = -0.3\sin\left(100t+\frac{\pi}{4}\right)$$

$$= 0.3\sin\left(100t+\frac{\pi}{4}-\pi\right) = 0.3\sin\left(100t-\frac{3\pi}{4}\right)\ \ \mathbf{A}$$

Problem 10

$$v_L(t) = L\frac{di_L(t)}{dt} = 250\times10^{-3}\frac{di_L(t)}{dt} = 0.25\frac{di_L(t)}{dt}$$

a)

$$v_L(t) = 0.25\left[25\times5\cos25t\right] = 31.25\cos25t\ \ \mathbf{V}$$

b)

$$v_L(t) = 0.25\left[-50\times(-10\sin50t)\right] = 125\sin50t\ \ \mathbf{V}$$

c)

$$v_L(t) = 0.25\left[-100\times25\sin\left(100t+\frac{\pi}{3}\right)\right] = -625\sin\left(100t+\frac{\pi}{3}\right)$$

$$= 625\sin\left(100t+\frac{\pi}{3}-\pi\right) = 625\sin\left(100t-\frac{2\pi}{3}\right)\mathbf{V}$$

d)

$$v_L(t) = 0.25\left[10\times20\cos\left(10t-\frac{\pi}{12}\right)\right] = 50\cos\left(10t-\frac{\pi}{12}\right)\mathbf{V}$$