ELEG 305 SIGNALS AND SYSTEMS
SPRING 2019

- **All Homeworks and Homework Quizzes are worth 25 points.**

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HOMEWORK #6 → *Hand-in on Thursday April 11 (Collected in Lecture)*

Read Chapter 5 in Oppenheim, Willsky, and Nawab (**O&W**)

Problem #1

Consider a continuous-time LTI system with input $x(t) = e^{-\alpha t}u(t)$, and impulse response $h(t) = e^{-\beta(t-2)}u(t-2)$. Assume that $\alpha \neq \beta$, $\alpha > 0$, and $\beta > 0$.

a.) Compute the output of the system, $y(t)$, using a time-domain derivation (i.e., convolution). Please do all the steps, even though you have done a very similar problem before.
b.) Compute the output spectrum, $Y(j\omega)$, using multiplication in the frequency domain.
c.) Compute the output signal, $y(t)$, by taking the inverse Fourier transform of the frequency characteristic of the output, $Y(j\omega)$. [Use partial fraction expansion to get your answer.]

Problem #2

The following problems involve the use of the properties of the Fourier transform.

a.) Find the inverse Fourier transform of

$$X(j\omega) = \frac{2\sin(\omega - 2)}{(\omega - 2)} * \frac{e^{-2j\omega}\sin 2\omega}{\omega}$$

b.) Compute the Fourier transform of $y(t)$ where

$$y(t) = x(t-2)$$

and

$$x(t) = \frac{\sin(t)}{\pi t} * \frac{d}{dt}\left[\frac{\sin(2t)}{\pi t}\right]$$

c.) Compute the value of the integral $\int_{-\infty}^{\infty} \left|\frac{2}{j\omega+2}\right|^2 d\omega$

Problem #3

Compute the Fourier transform of the signal $x(t)$; this function is called a *periodic impulse train* and is important in the sampling of continuous-time signals to generate discrete-time signals.

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

Problem #4

Consider a continuous-time LTI system characterized by a frequency response

$$H(j\omega) = \frac{2}{-\omega^2 + 3j\omega + 2}$$

a.) Derive the impulse response of this system, $h(t)$.
b.) Find the differential equation relating the input and output of this system.

Problem #5

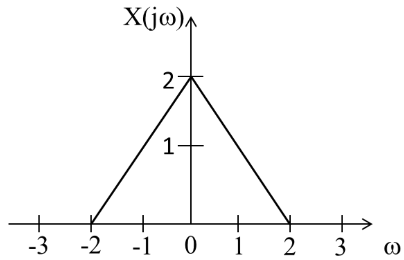
Consider a differential equation that describes an LTI system with input $x(t)$ and output $y(t)$

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt}$$

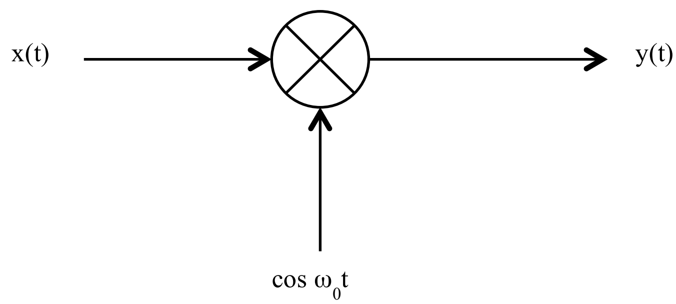
a.) Derive the frequency response of this system, $H(j\omega)$.
b.) What is the corresponding impulse response, $h(t)$?

Problem #6

Consider a message $x(t)$, to be communicated to a distant receiver, with following spectrum:



One possible communication technique is called *Amplitude Modulation* (yes, the same AM that is in your car radio). In this scheme, the speech or music signal is multiplied by what is called a carrier wave (simply a cosine with radian frequency ω_0) as shown below:



Compute and draw the spectrum (the frequency characteristic) of the output signal, $Y(j\omega)$ (assume $\omega_0 = 5$ radians/sec). Label each axis carefully.

Problem #7

A desired signal (such as a voice call on your cell phone, and image on your laptop, an EKG signal measured in the doctor's office, or many other possibilities) is often corrupted by noise and interference. This can come from many sources, including other people using the same spectrum at the same time, as happens in a cellular system. Assume that the desired signal has a flat spectrum equal to 2 for $|\omega_0| < 2$ radians/sec and 0 elsewhere. The signal is corrupted by noise that has a wide spectrum that goes from very low frequencies to very high frequencies. This noise simply adds with the desired signal. Assume the noise spectrum has amplitude 0.5 for $|\omega_0| < 20$ radians/sec, and 0 elsewhere.

(i) What is the energy of the noise component of the received signal?
(ii) What is the energy in the desired component of the received signal?
(iii) What could we do to reduce the amount of noise in our desired signal?

Conceptual: Consider a digital image that you have taken with your phone or camera. Suppose that image is passed through a lowpass filter. *Qualitatively*, what will happen to the image? If instead it is passed through a highpass filter, explain, *qualitatively*, what will happen to the image?

Math Review: Consider the following three integrals. For what values of α does the integral exist (i.e., the value of the integral is finite)? Assume that α is a real number.

$$\int_{\alpha}^{\infty} e^{-t}\,dt$$

$$\int_{0}^{\infty} e^{-(\alpha+j\omega)t}\,dt$$

$$\int_{-\infty}^{0} e^{-(\alpha+1)t}\,dt$$

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EXAM # 2 Tuesday April 16

- Closed everything: no calculators, cellphones, laptops, …
- Chapters 3 and 4
- Review on Monday April 15