ELEG 312
Homework #3
Solutions

Problems 9.1, 9.3, 9.5, 9.12, 9.19, 9.26, 9.28, and 9.34

Problem 9.1a

For an NMOS differential pair with a common-mode voltage V_{CM} applied, as shown in Fig. 9.2, let $V_{DD} = V_{SS} = 1.0$ V, $k'_n = 0.4$ mA/V^2, $(W/L)_{1,2} = 10$, $V_{tn} = 0.4$ V, $I = 0.16$ mA, $R_D = 5$ kΩ, and neglect channel-length modulation.

(a) Find V_{OV} and V_{GS} for each transistor.

(b) For $V_{CM} = 0$, find V_S, I_{D1} , I_{D2}, V_{D1} , and V_{D2}.

(c) Repeat (b) for $V_{CM} = +0.4$ V.

(d) Repeal (b) for $V_{CM} = -0.1$ V.

(e) What is the highest value of V_{CM} for which Q_1 and Q_2 remain in saturation?

(f) If current source I requires a minimum voltage of 0.2 V to operate properly, what is the lowest value allowed for V_S and hence for V_{CM}?

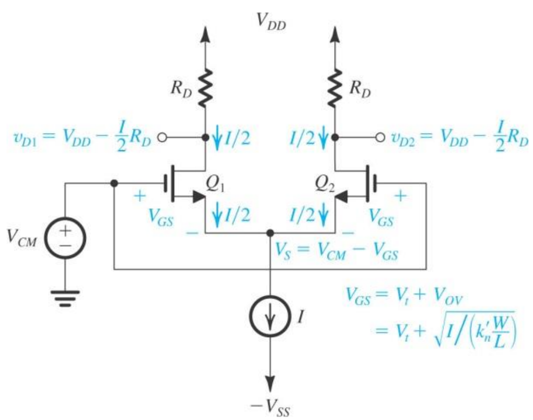


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$v_{G1} = v_{G2} = V_{CM} \qquad I_{D1} = I_{D2} = \frac{I}{2} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)_1 V_{OV}^2$$

$$V_{OV} = \sqrt{2I_D/\left[k_n'(W/L)\right]} = \sqrt{\frac{0.16\text{mA}}{0.4\frac{\text{mA}}{\text{V}^2}(10)}} = 0.2\text{V}$$

$$V_{GS} = V_t + V_{OV} = 0.4\text{V} + 0.2\text{V} = 0.6\text{V}$$

Problem 9.1b,c

For an NMOS differential pair with a common-mode voltage V_{CM} applied, as shown in Fig. 9.2, let $V_{DD} = V_{SS} = 1.0$ V, $k'_n = 0.4$ mA/V^2, $(W/L)_{1,2} = 10$, $V_{tn} = 0.4$ V, $I = 0.16$ mA, $R_D = 5$ kΩ, and neglect channel-length modulation.

(b) For $V_{CM} = 0$, find V_S, I_{D1}, I_{D2}, V_{D1}, and V_{D2}.

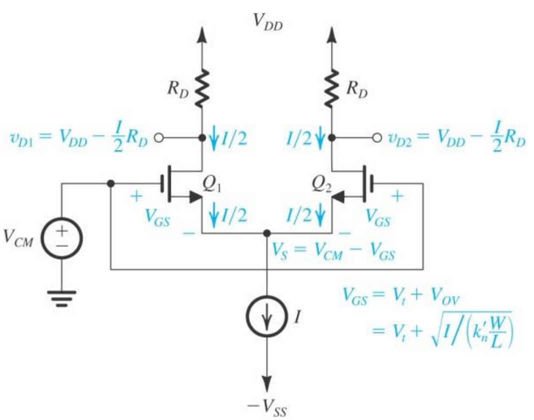


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$v_{G1} = v_{G2} = V_{CM} \qquad V_{OV} = 0.2\text{V} \qquad V_{GS} = 0.6\text{V}$$

$$V_S = V_G - V_{GS} = 0\text{V} - 0.6\text{V} = -0.6\text{V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = \frac{0.16\text{mA}}{2} = 0.08\text{mA}$$

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2}R_D = 1\text{V} - 0.08\text{mA} \times 5\text{k}\Omega = 0.6\text{V}$$

(c) Repeat (b) for $V_{CM} = +0.4$ V.

$$V_S = V_G - V_{GS} = 0.4\text{V} - 0.6\text{V} = -0.2\text{V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = \frac{0.16\text{mA}}{2} = 0.08\text{mA}$$

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2}R_D = 1\text{V} - 0.08\text{mA} \times 5\text{k}\Omega = 0.6\text{V}$$

Problem 9.1d,e,f

For an NMOS differential pair with a common-mode voltage V_{CM} applied, as shown in Fig. 9.2, let $V_{DD} = V_{SS} = 1.0$ V, $k'_n = 0.4$ mA/V^2, $(W/L)_{1,2} = 10$, $V_{tn} = 0.4$ V, $I = 0.16$ mA, $R_D = 5$ kΩ, and neglect channel-length modulation.

(d) Repeal (b) for V_{CM} = -0.1 V.

$$V_S = V_G - V_{GS} = -0.1\text{V} - 0.6\text{V} = -0.7\text{V}$$

$$I_{D1} = I_{D2} = \frac{I}{2} = \frac{0.16\text{mA}}{2} = 0.08\text{mA}$$

$$V_{D1} = V_{D2} = V_{DD} - \frac{I}{2}R_D = 1\text{V} - 0.08\text{mA} \times 5\text{k}\Omega = 0.6\text{V}$$

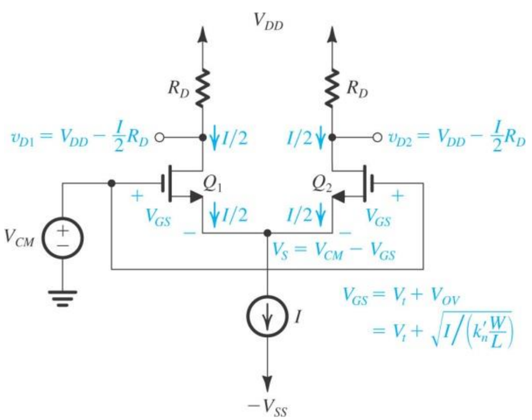


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

(e) What is the highest value of V_{CM} for which Q_1 and Q_2 remain in saturation?

$$V_{CM\,\max} = V_t + V_{DD} - \frac{I}{2}R_D = 0.4\text{V} + \left(1\text{V} - 0.08\text{mA} \times 5\text{k}\Omega\right) = 1.0\text{V}$$

(f) If current source I requires a minimum voltage of 0.2 V to operate properly, what is the lowest value allowed for V_S and hence for V_{CM}?

$$V_{CM\,\min} = -V_{SS} + V_{CS} + V_{GS} = -1\text{V} + 0.2\text{V} + 0.6\text{V} = -0.2\text{V}$$

Problem 9.3a

For the differential amplifier specified in Problem 9.1 let v_{G2} = 0 and v_{G1} = v_{id}. Find the value of v_{id} that corresponds to each of the following situations:

(a) i_{D1} = i_{D2} = 0.08 mA;

For each case, find v_s, v_{D1}, v_{D2}, and ($v_{D2} - v_{D1}$).

$$i_{D1} = i_{D2} = \frac{I}{2} \qquad v_{G1} = v_{G2} = 0\text{V} \qquad v_{id} = v_{G1} - v_{G2} = 0\text{V}$$

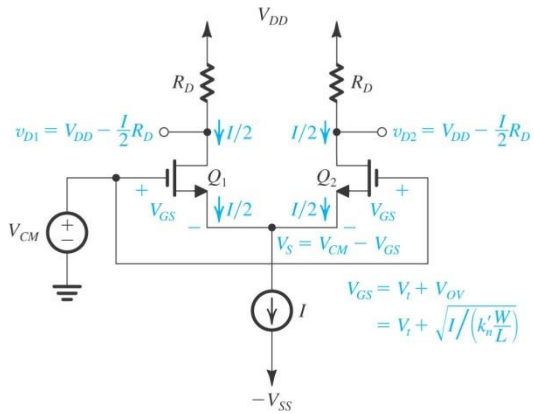


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$V_{OV} = \sqrt{2I_D/\left[k_n'(W/L)\right]} = \sqrt{\frac{0.16\text{mA}}{0.4\frac{\text{mA}}{\text{V}^2}(10)}} = 0.2\text{V}$$

$$v_s = v_G - v_{GS} = 0\text{V} - (0.4\text{V} + 0.2\text{V}) = -0.6\text{V}$$

$$v_{D1} = v_{D2} = V_{DD} - i_{D1,2}R_D = 1\text{V} - 0.08\text{mA} \times 5\text{k}\Omega = 0.6\text{V}$$

$$v_{D2} - v_{D1} = 0\text{V}$$

Problem 9.3b

For the differential amplifier specified in Problem 9.1 let v_{G2} = 0 and v_{G1} = v_{id}. Find the value of v_{id} that corresponds to each of the following situations: For each case, find v_s, v_{D1}, v_{D2}, and ($v_{D2} - v_{D1}$).

(b) i_{D1} = 0.12 mA and i_{D2} = 0.04 mA;

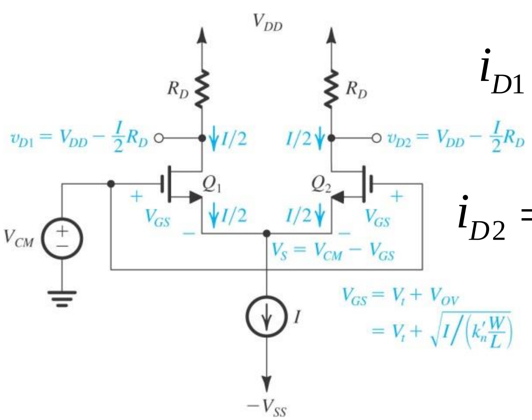


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2} = \frac{0.16\text{mA}}{2} + \left(\frac{0.16\text{mA}}{0.2\text{V}}\right)\frac{v_{id}}{2} = 0.12\text{mA} \qquad v_{id} = 0.10\text{V}$$

$$i_{D2} = \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2} = \frac{0.16\text{mA}}{2} - \left(\frac{0.16\text{mA}}{0.2\text{V}}\right)\frac{v_{id}}{2} = 0.04\text{mA} \qquad v_{id} = 0.10\text{V}$$

$$v_{OV} = \sqrt{2I_D/\left[k_n'(W/L)\right]} = \sqrt{\frac{2\times 0.12\text{mA}}{0.4\frac{\text{mA}}{\text{V}^2}(10)}} = 0.245\text{V}$$

$$v_{GS} = V_t + v_{OV} = 0.4\text{V} + 0.245\text{V} = 0.645\text{V}$$

$$v_S = v_G - v_{GS} = 0.1\text{V} - 0.645\text{V} = -0.545\text{V}$$

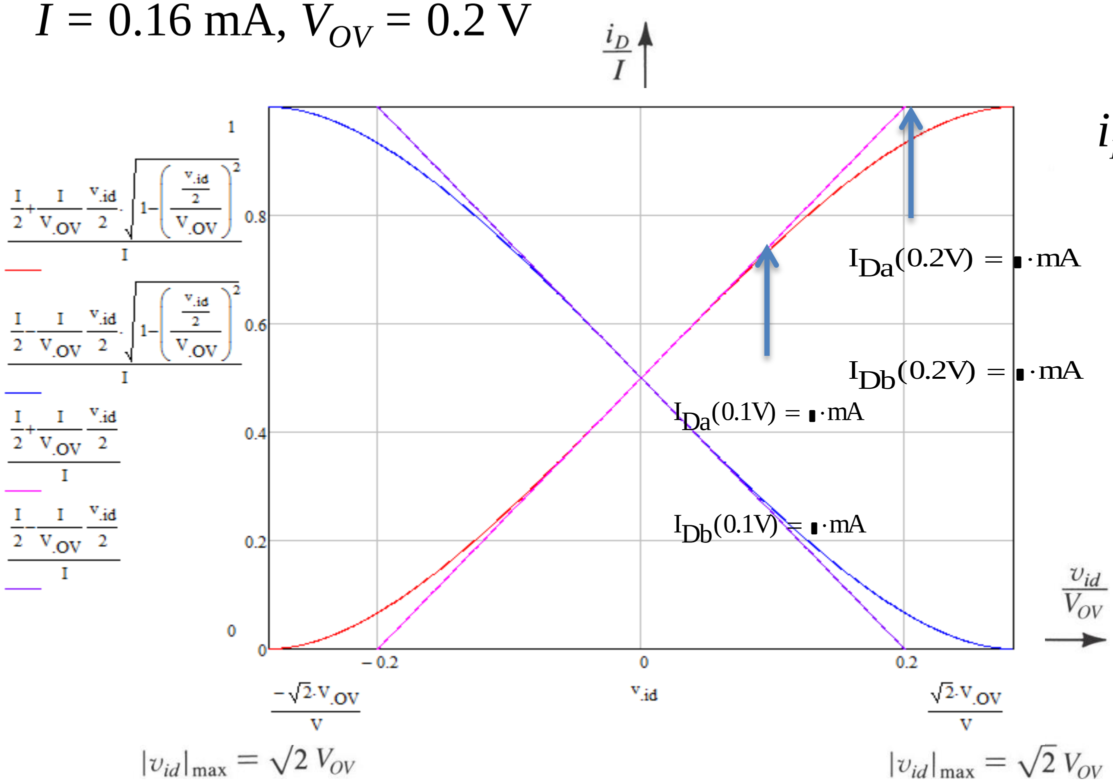
$$v_{D1} = V_{DD} - i_{D1}R_D = 1\text{V} - 0.12\text{mA}\times 5\text{k}\Omega = 0.4\text{V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D = 1\text{V} - 0.04\text{mA}\times 5\text{k}\Omega = 0.8\text{V}$$

$$v_{D2} - v_{D1} = 0.8\text{V} - (0.4\text{V}) = 0.4\text{V}$$

Problem 9.3

I = 0.16 mA, V_{OV} = 0.2 V



$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1-\left(\frac{v_{id}/2}{V_{OV}}\right)^2}$$

$$\frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1-\left(\frac{v_{id}/2}{V_{OV}}\right)^2}$$

Transfer characteristics are nonlinear and usually linear amplification is desirable.

If we can keep $v_{id}/2 << V_{OV}$ (small-signal) we can say that

$$i_{D1} \approx \frac{I}{2} + i_d \qquad i_{D2} \approx \frac{I}{2} - i_d \qquad i_d = \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2}$$

Problem 9.3c

For the differential amplifier specified in Problem 9.1 let v_{G2} = 0 and v_{G1} = v_{id}. Find the value of v_{id} that corresponds to each of the following situations: For each case, find v_s, v_{D1}, v_{D2}, and $(v_{D2} - v_{D1})$.

(c) i_{D1} = 0.16 mA and i_{D2} = 0 mA (Q_2 just cuts off);

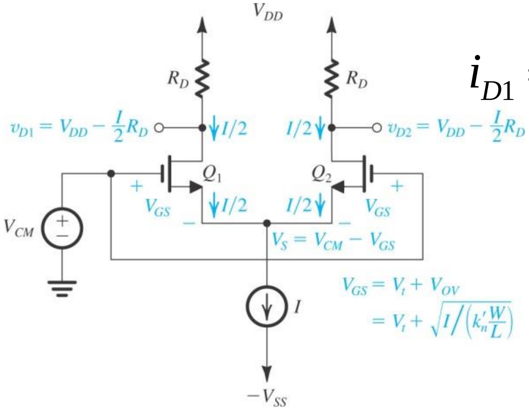


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2} = 0.16\text{mA} = I$$

$$v_{id} = \sqrt{2} \times V_{OV} = 1.414 \times 0.2\text{V} = 0.283\text{V}$$

$$v_{OV} = \sqrt{2I_D/\left[k_n'(W/L)\right]} = \sqrt{\frac{2 \times 0.16\text{mA}}{0.4\frac{\text{mA}}{\text{V}^2}(10)}} = 0.283\text{V}$$

$$v_{GS} = V_t + v_{OV} = 0.4\text{V} + 0.283\text{V} = 0.683\text{V}$$

$$v_S = v_G - v_{GS} = 0.283\text{V} - 0.683\text{V} = -0.4\text{V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D = 1\text{V} - 0.16\text{mA} \times 5\text{k}\Omega = 0.2\text{V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D = 1\text{V} - 0\text{mA} \times 5\text{k}\Omega = 1\text{V}$$

$$v_{D2} - v_{D1} = 1\text{V} - (0.2\text{V}) = 0.8\text{V}$$

Problem 9.3d

For the differential amplifier specified in Problem 9.1 let $v_{G2} = 0$ and $v_{G1} = v_{id}$. Find the value of v_{id} that corresponds to each of the following situations: For each case, find v_s, v_{D1}, v_{D2}, and $(v_{D1} - v_{D2})$.

(d) $i_{D1} = 0.04$ mA and $i_{D2} = 0.12$ mA;

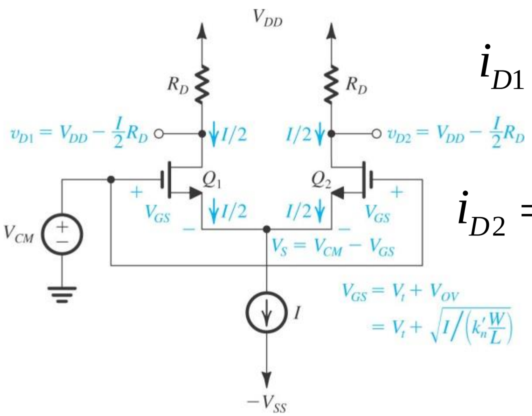


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2} = \frac{0.16\text{mA}}{2} + \left(\frac{0.16\text{mA}}{0.2\text{V}}\right)\frac{v_{id}}{2} = 0.04\text{mA} \qquad v_{id} = -0.10\text{V}$$

$$i_{D2} = \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2} = \frac{0.16\text{mA}}{2} - \left(\frac{0.16\text{mA}}{0.2\text{V}}\right)\frac{v_{id}}{2} = 0.12\text{mA} \qquad v_{id} = -0.10\text{V}$$

$$v_{OV} = \sqrt{2I_D/\left[k_n'(W/L)\right]} = \sqrt{\frac{2\times 0.08\text{mA}}{0.4\frac{\text{mA}}{\text{V}^2}(10)}} = 0.141\text{V}$$

$$v_{GS} = V_t + v_{OV} = 0.4\text{V} + 0.141\text{V} = 0.541\text{V}$$

$$v_S = v_G - v_{GS} = -0.1\text{V} - 0.541\text{V} = -0.641\text{V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D = 1\text{V} - 0.04\text{mA}\times 5\text{k}\Omega = 0.8\text{V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D = 1\text{V} - 0.12\text{mA}\times 5\text{k}\Omega = 0.4\text{V}$$

$$v_{D2} - v_{D2} = 0.4\text{V} - (0.8\text{V}) = -0.4\text{V}$$

Problem 9.3e

For the differential amplifier specified in Problem 9.1 let v_{G2} = 0 and v_{G1} = v_{id}. Find the value of v_{id} that corresponds to each of the following situations: For each case, find v_s, v_{D1}, v_{D2}, and (v_{D1} - v_{D2}).

(e) i_{D1} = 0 mA (Q_1 just cuts off) and i_{D2} = 0.16 mA.

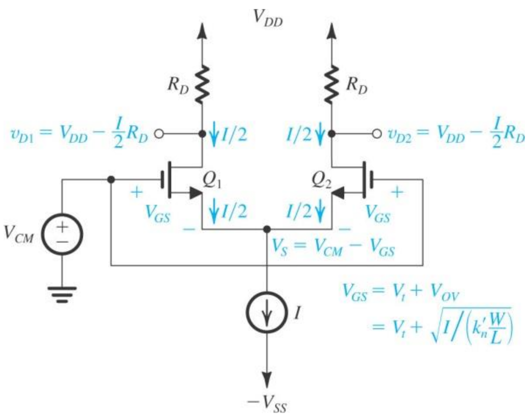


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_{id}}{2}\right)\sqrt{1 - \left(\frac{v_{id}/2}{V_{OV}}\right)^2} = 0\text{mA}$$

$$v_{id} = -\sqrt{2} \times V_{OV} = -1.414 \times 0.2\text{V} = -0.283\text{V}$$

$$v_{OV} = \sqrt{2I_D / \left[k_n'\left(W/L\right)\right]} = \sqrt{\frac{2 \times 0\text{mA}}{0.4\frac{\text{mA}}{\text{V}^2}(10)}} = 0\text{V}$$

$$v_{GS} = V_t + v_{OV} = 0.4\text{V} + 0\text{V} = 0.4\text{V}$$

$$v_S = v_{G2} - v_{GS2} = 0\text{V} - 0.683\text{V} = -0.683\text{V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D = 1\text{V} - 0\text{mA} \times 5\text{k}\Omega = 1\text{V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D = 1\text{V} - 0.16\text{mA} \times 5\text{k}\Omega = 0.2\text{V}$$

$$v_{D2} - v_{D2} = 0.2\text{V} - \left(1\text{V}\right) = -0.8\text{V}$$

Problem 9.5a

Consider the differential amplifier specified in Problem 9.1 with G_2 grounded and $v_{G1} = v_{id}$. Let v_{id} be adjusted to the value that causes $i_{D1} = 0.09$ mA and $i_{D2} = 0.07$ mA. Find the corresponding values of v_{GS2}, v_S, v_{GS1}, and hence v_{id}. What is the difference output voltage v_{D2}-v_{D1}? What is the voltage gain $(v_{D2}$-$v_{D1})/v_{id}$? What value of v_{id} results in $i_{D1} = 0.07$ mA and $i_{D2} = 0.09$ mA?

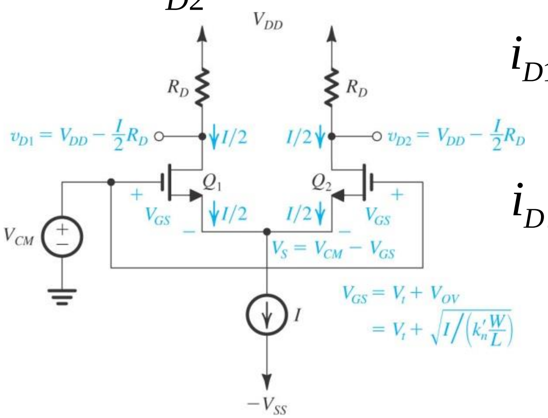


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2} = \frac{0.16\text{mA}}{2} + \left(\frac{0.16\text{mA}}{0.2\text{V}}\right)\frac{v_{id}}{2} = 0.09\text{mA} \qquad v_{id} = 0.025\text{V}$$

$$i_{D2} = \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2} = \frac{0.16\text{mA}}{2} - \left(\frac{0.16\text{mA}}{0.2\text{V}}\right)\frac{v_{id}}{2} = 0.07\text{mA} \qquad v_{id} = 0.025\text{V}$$

$$v_{OV} = \sqrt{2I_D/\left[k_n'(W/L)\right]} = \sqrt{\frac{2\times 0.09\text{mA}}{0.4\frac{\text{mA}}{\text{V}^2}(10)}} = 0.212\text{V}$$

$$v_{GS} = V_t + v_{OV} = 0.4\text{V} + 0.212\text{V} = 0.612\text{V}$$

$$v_S = v_G - v_{GS} = 0.025\text{V} - 0.612\text{V} = -0.587\text{V}$$

$$v_{D1} = V_{DD} - i_{D1}R_D = 1\text{V} - 0.09\text{mA}\times 5\text{k}\Omega = 0.55\text{V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D = 1\text{V} - 0.07\text{mA}\times 5\text{k}\Omega = 0.65\text{V}$$

$$v_{D2} - v_{D2} = 0.65\text{V} - (0.55\text{V}) = 0.1\text{V}$$

$$\frac{v_{D2} - v_{D2}}{v_{is}} = \frac{0.1\text{V}}{0.025\text{V}} = 4\text{V/V}$$

Problem 9.5b

Consider the differential amplifier specified in Problem 9.1 with G_2 grounded and $v_{G1} = v_{id}$. Let v_{id} be adjusted to the value that causes $i_{D1} = 0.09$ mA and $i_{D2} = 0.07$ mA. Find the corresponding values of v_{GS2}, v_S, v_{GS1}, and hence v_{id}. What is the difference output voltage v_{D2}-v_{D1}? What is the voltage gain $(v_{D2}$-$v_{D1})/v_{id}$? What value of v_{id} results in $i_{D1} = 0.07$ mA and $i_{D2} = 0.09$ mA?

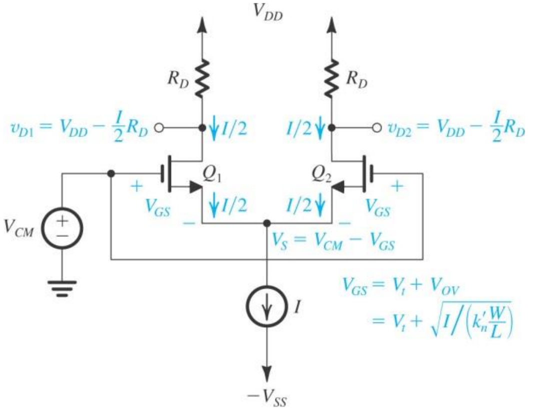


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2} = \frac{0.16\text{mA}}{2} + \left(\frac{0.16\text{mA}}{0.2\text{V}}\right)\frac{v_{id}}{2} = 0.07\text{mA}$$

$$v_{id} = -0.025\text{V}$$

$$i_{D2} = \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2} = \frac{0.16\text{mA}}{2} - \left(\frac{0.16\text{mA}}{0.2\text{V}}\right)\frac{v_{id}}{2} = 0.09\text{mA}$$

$$v_{id} = -0.025\text{V}$$

Problem 9.12

An NMOS differential amplifier is operated at a bias current I of 0.2 mA and has a W/L ratio of 32, $\mu_n C_{ox}$ = 200µA/V^2, V_A = 10 V, and R_D = 10 kΩ. Find V_{OV}, g_m, r_o, and A_d.

$$I_{D1} = I_{D2} = \frac{I}{2} = \frac{0.2\text{mA}}{2} = \frac{1}{2}\mu_n c_{ox}\left(\frac{W}{L}\right)_1 V_{OV}^2$$

$$V_{OV} = \sqrt{2I_D / \left[\mu_c c_{ox}\left(W/L\right)\right]} = \sqrt{\frac{0.2\text{mA}}{0.2\frac{\text{mA}}{\text{V}^2}(32)}} = 0.176\text{V}$$

$$g_m = \frac{2I_D}{V_{OV}} = \frac{0.2\text{mA}}{0.176\text{V}} = 1.14\text{mA/V}$$

$$r_o = \frac{V_A}{I_D} = \frac{10\text{V}}{0.1\text{mA}} = 100\text{k}\Omega$$

$$A_d = g_m\left(r_o \,||\, R_D\right) = 1.14\text{mA/V} \times \left(100\text{k}\Omega \,||\, 10\text{k}\Omega\right) = 10.36\text{V/V}$$

Problem 9.19a

Figure P9.19 shows a MOS differential amplifier with the drain resistors R_D implemented using diode connected PMOS transistors, Q_3 and Q_4. Let Q_1 and Q_2 be matched, and Q_3 and Q_4 be matched.

(a) Find the differential half-circuit and use it to derive an expression for A_d in terms of $g_{m1,2}$, $g_{m3,4}$, $r_{o1,2}$, and $r_{o3,4}$.

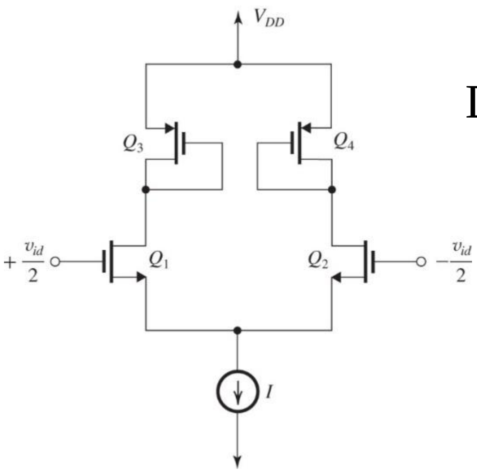
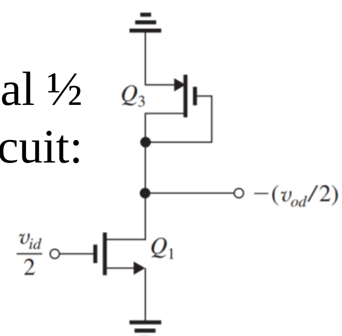
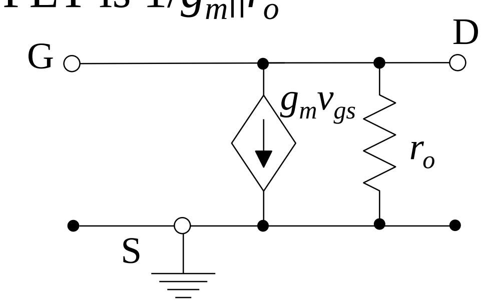


Figure P9.19

Differential ½ circuit:



The gate to source resistance of a "diode" connected FET is $1/g_m||r_o$



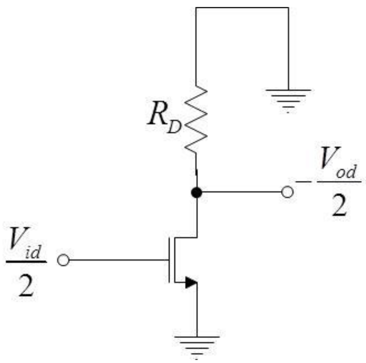
$$\Rightarrow R_D = \frac{1}{g_{m3}} \,||\, r_{o3}$$

$$A_d = g_{m1}\left(r_{o1} \,||\, R_D\right) = g_{m1}\left(r_{o1} \,||\, \frac{1}{g_{m3}} \,||\, r_{o3}\right)$$

Problem 9.19b

Figure P9.19 shows a MOS differential amplifier with the drain resistors R_D implemented using diode connected PMOS transistors, Q_3 and Q_4. Let Q_1 and Q_2 be matched, and Q_3 and Q_4 be matched.

(b) Neglecting the effect of the output resistances r_o, find A_d in terms of μ_n, μ_p, $(W/L)_{1,2}$, and $(W/L)_{3,4}$.



$$R_D = \frac{1}{g_{m3}} \,||\, r_{o3} \simeq \frac{1}{g_{m3}}$$

$$A_d = g_{m1}\left(r_{o1} \,||\, R_D\right) \simeq g_{m1} R_D = \frac{g_{m1}}{g_{m3}}$$

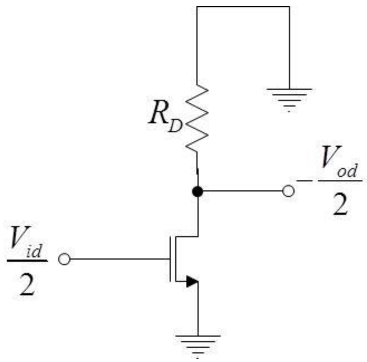
$$g_m = \sqrt{2\mu_{n,p} C_{ox} \left(W/L\right) I_D}$$

$$A_d = \frac{g_{m1}}{g_{m3}} = \frac{\sqrt{2\mu_n C_{ox} \left(W/L\right)_1 I_D}}{\sqrt{2\mu_p C_{ox} \left(W/L\right)_3 I_D}} = \sqrt{\frac{\mu_n \left(W/L\right)_1}{\mu_p \left(W/L\right)_3}}$$

Problem 9.19c

Figure P9.19 shows a MOS differential amplifier with the drain resistors R_D implemented using diode connected PMOS transistors, Q_3 and Q_4. Let Q_1 and Q_2 be matched, and Q_3 and Q_4 be matched.

(c) If $\mu_n = 4\mu_p$ and all four transistors have the same channel length, find $(W_{1,2}/W_{3,4})$ that results in $A_d = 10$ V/V.



$$A_d = \frac{g_{m1}}{g_{m3}} = \frac{\sqrt{2\mu_n C_{ox}(W/L)_1 I_D}}{\sqrt{2\mu_p C_{ox}(W/L)_3 I_D}} = \sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_3}}$$

$$A_d = 10\text{V/V} = \sqrt{\frac{4\mu_p (W/L)_1}{\mu_p (W/L)_3}}$$

$$\frac{(W/L)_1}{(W/L)_3} = 5^2 = 25$$

Problem 9.26

For the differential amplifier of Fig. 9.15(a) let I = 0.4 mA, V_{CC} = V_{EE} = 2.5V, V_{CM} = -1V, R_C = 5 kΩ, and β = 100. Assume that the BJTs have v_{BE} = 0.7 V at i_C = 1mA. Find the voltage at the emitters and at the outputs.

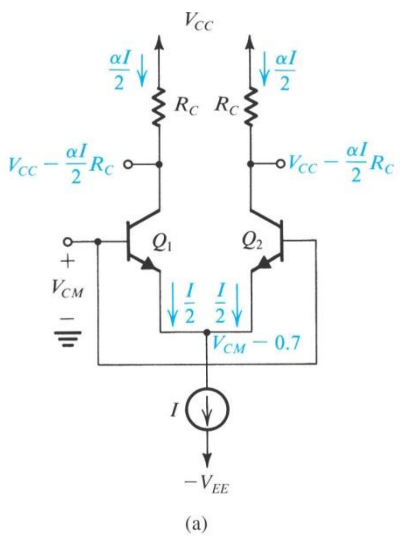


Figure 9.15: Different modes of operation of the BJT differential pair: **(a)** the differential pair with a common-mode input voltage V_{CM}.

$$i_E = \frac{I}{2} = \frac{0.4\text{ mA}}{2} = 0.2\text{mA} \qquad \alpha = \frac{\beta}{\beta+1} = \frac{100}{101} = 0.990$$

$$i_C = \alpha I_E = 0.99 \cdot 0.20\text{ mA} = 0.198\text{mA}$$

v_{BE} = 0.7 V at i_C = 1 mA, need v_{BE} at i_C = 0.198 mA

$$i_C = I_S e^{v_{BE}/V_T} \qquad \ln\left(\frac{i_{C1}}{i_{C2}}\right) = \frac{v_{BE1} - v_{BE2}}{V_T} \qquad v_{BE1} = v_{BE2} + V_T \ln\left(\frac{i_{C1}}{i_{C2}}\right)$$

$$v_{BE} = 0.7\text{V} + 0.025\text{V} \times \ln\left(\frac{0.198\text{mA}}{1\text{mA}}\right) = 0.660\text{V}$$

$$v_E = v_B - v_{BE} = -1\text{V} - 0.660\text{V} = -1.660\text{V}$$

$$v_{o1,2} = v_{C1,2} = V_{CC} - \alpha R_C \left(I/2\right) = 2.5\text{V} - (0.99)5\text{k}\Omega\left(\frac{0.4\text{mA}}{2}\right) = 1.51\text{V}$$

Problem 9.28

An *npn* differential amplifier with I = 0.4 mA, $V_{CC} = V_{EE}$ = 2.5 V, and R_C = 5 kΩ utilizes BJTs with β = 100 and v_{BE} = 0.7 V at i_C = 1 mA. Assuming that the bias current is obtained by a simple current source and that all transistors require a minimum of 0.3 V for operation in the active mode, find the input common-mode range.

$$V_{CM\max} \approx V_C + 0.4 = V_{CC} - \alpha \frac{I}{2} R_C + 0.3$$

$$V_{CM\max} = 2.5\text{V} - (0.99)\left(\frac{0.4\text{mA}}{2}\right)5\text{k}\Omega + 0.3\text{V} = 1.81\text{V}$$

$$V_{CM\min} = -V_{EE} + V_{CS} + V_{BE}$$

v_{BE} = 0.7 V at i_C = 1 mA

$$v_{BE} = 0.7\text{V} + 0.025\text{V} \times \ln\left(\frac{0.198\text{mA}}{1\text{mA}}\right) = 0.660\text{V}$$

$$V_{CM\min} = -2.5\text{V} + 0.3\text{V} + 0.660\text{V} = -1.54\text{V}$$

Problem 9.34

Design the circuit of Fig. 9.14 to provide a differential output voltage (i.e., one taken between the two collectors) of 1 V when the differential input signal is 10 mV. A current source of 1 mA and a positive supply of +5 V are available. What is the largest possible input common-mode voltage for which operation is as required? Assume α ≈ 1.

$$\frac{i_{E1}}{i_{E1}+i_{E2}}=\frac{1}{1+e^{(v_{B2}-v_{B1})/V_T}} \qquad i_{E1}=\frac{I}{1+e^{-v_{id}/V_T}}$$

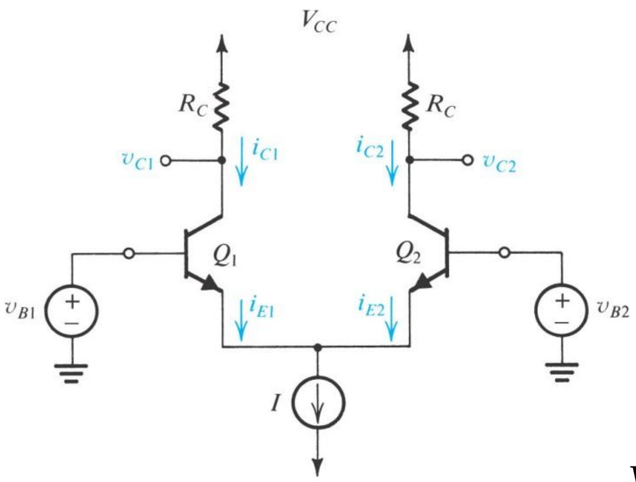


Figure 9.14: The basic BJT differential-pair configuration.

$$i_{E1}=\frac{1\text{mA}}{1+e^{-0.01\text{V}/0.025\text{V}}}=0.599\text{mA}$$

$$i_{E2}=I-i_{E1}=1\text{mA}-0.599\text{mA}=0.401\text{mA}$$

$$v_{od}=R_C(i_{C1}-i_{C2})\approx R_C(i_{E1}-i_{E2})=1\text{V}$$

$$R_C=\frac{1\text{V}}{(i_{E1}-i_{E2})}=\frac{1\text{V}}{0.198\text{mA}}=5.05\text{k}\Omega$$

$$v_{c1}=V_{CC}-i_{c1}R_C=5\text{V}-(0.599\text{mA})5.05\text{k}\Omega=1.975\text{V}$$

$$v_{c2}=V_{CC}-i_{c2}R_C=5\text{V}-(0.401\text{mA})5.05\text{k}\Omega=2.975\text{V}$$

$$V_{CM\max}=v_{C1}+0.4=2.375\text{V}$$