HW9

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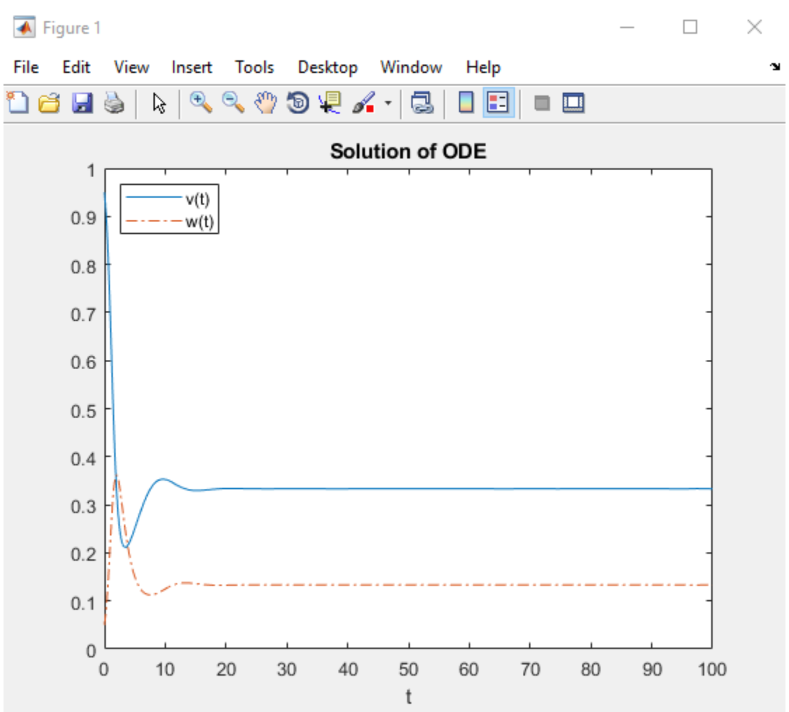
December 4, 2019

1 6.3.4



From the graph, the long-term steady state values are 0.3331 for $v(t)$ and 0.1334 for $w(t)$

2 6.4.3

a)

$\frac{du}{dt} = -2tu, 0 \leq t \leq 2, u(0) = 2, h = 0.2$

$t_{i+1} = t_i + h$

Initial Condition: $t_0 = 0$

$t_1 = t_0 + h$

$t_1 = 0.2$

$t_2 = t_1 + h$

$t_2 = 0.4$

The Predictor:

$u^*_{i+1} = u_i + h(f(u_i, t_i))$

$u^*_1 = u_0 + h(-2t_0u_0)$

$u^*_1 = 2 + (0.2)(-2(0)(2))$

$u^*_1 = 2$

$u^*_2 = u_1 + h(-2t_1u_1)$

$= 1.92 + (0.2)(-2)(0.2)(1.92)$

$= 1.77$

$u^*_2 = 1.77$

The Corrector:

$u_{i+1} = u_i + \frac{h}{2}(f(u_i, t_i) + f(u^*_{i+1}, t_{i+1}))$

$u_0 = 2$

$u_1 = u_0 + \frac{h}{2}(-2t_0u_0 - 2t_1u_1)$

$u_1 = 2 + \frac{0.2}{2}(-2(0)(2) - 2(0.2)(2))$

$u_1 = 1.92$

$u_2 = u_1 + \frac{h}{2}(-2t_1u_1 - 2t_2u^*_2)$

$u_2 = 1.92 + \frac{0.2}{2}(-0.768 - 1.416)$

$u_2 = 1.70$

Check Exact Solution:

$t_0 = 0, u_0 = 2, u_0^* = 2e^{-(0)} = 2$

$t_1 = 0.2, u_1 = 1.92, u_1^* = 2e^{-(0.2)} = 1.64$

$t_2 = 0.4, u_2 = 1.70, u_2^* = 2e^{-(0.4)} = 1.34$

Solution: $u^*(t) = 2e^{-t^2}$

b)

$\frac{du}{dt} = u + t, 0 \leq t \leq 1, u(0) = 2, h = 0.2$

$t_{i+1} = t_i + h$

Initial Condition: $t_0 = 0$

$t_1 = t_0 + 0.2$

$t_1 = 0.2$

$t_2 = t_1 + h$

$t_2 = 0.4$

The Predictor:

$u_{i+1}^* = u_i + h(f(u_i, t_i))$

$u_1^* = u_0 + h(-2t_0 u_0)$

$u_1^* = 2 + (0.2)(2)$

$u_1^* = 2.4$

$u_2^* = u_1 + h(t_1 + u_1)$

$= 2.46 + (0.2)(2.46 + 0.2)$

$= 2.992$

$u_2^* = 2.992$

The Corrector:

$u_{i+1} = u_i + \frac{h}{2}(f(u_i, t_i) + f(u_{i+1}^*, t_{i+1}))$

$u_0 = 2$

$u_1 = u_0 + \frac{h}{2}(u_0 + t_0 + u_1^* + t_1)$

$u_1 = 2 + \frac{0.2}{2}(2 + 0 + 2.4 + 0.2)$

$u_1 = 2.46$

$u_2 = u_1 + \frac{h}{2}(u_1 + t_1 + u_2^* + t_2)$

$u_2 = 2.46 + \frac{0.2}{2}(2.46 + 0.2 + 2.992 + 0.4)$

$u_2 = 3.07$

Check Exact Solution:

$t_0 = 0, u_0 = 2, u_0^* = -1 - 0 + 3e^0 = 2$

$t_1 = 0.2, u_1 = 2.46, u_1^* = -1 - 0.2 + 3e^{0.2} = 2.46$

$t_2 = 0.4, u_2 = 3.07, u_2^* = -1 - 0.4 + 3e^{0.4} = 3.075$

Solution: $u^*(t) = -1 - t + 3e^t$

c)

$(1 + x^3)uu' = x^2, 0 \leq x \leq 3, u(0) = 1, h = 0.2$

$u' = \frac{x^2}{(1+3x^3)u}$

$x_{i+1} = x_i + h$

Initial Condition: $x_0 = 0$

$x_1 = x_0 + h$

$x_1 = 0.2$

$x_2 = x_1 + h$

$x_2 = 0.4$

The Predictor:

$u^*_{i+1} = u_i + h(f(u_i, x_i))$

$u^*_1 = u_0 + h(\frac{x_0^2}{(1+x^3)u_0})$

$u^*_1 = 1 + 0.2(\frac{0}{(1+0)1})$

$u^*_1 = 1$

$u^*_2 = u_1 + h(\frac{x_1^2}{(1+x_1^3)u_1})$

$= 1.004 + 0.2(\frac{(0.2)^2}{(1+(0.2)^3)(1.004)})$

$= 1.012$

$u^*_2 = 1.012$

The Corrector:

$u_{i+1} = u_i + \frac{h}{2}(f(u_i, x_i) + f(u^*_{i+1}, x_{i+1}))$

$u_0 = 1$

$u_1 = u_0 + \frac{h}{2}(\frac{x_0^2}{(1+x_0^3)u_0} + \frac{x_1^2}{(1+x_1^3)u^*_1})$

$= 1 + \frac{0.2}{2}(\frac{0}{(1+0)1} + \frac{(0.2)^2}{[1+(0.2)^2]1})$

$u_1 = 1.004$

$u_2 = u_1 + \frac{h}{2}(\frac{x_1^2}{(1+x_1^3)u_1} + \frac{x_2^2}{(1+x_2^3)u^*_2})$

$u_2 = 1.004 + \frac{0.2}{2}(\frac{0.04}{0.012} + \frac{0.16}{1.077})$

$u_2 = 1.004 + 0.348$

$u_2 = 1.352$

Check Exact Solution:

$x_0 = 0, u_0 = 1, u^*_0 = [1 + \frac{2}{3}ln(1+0)]^{\frac{1}{2}}] = 1$

$x_1 = 0.2, u_1 = 1.004, u^*_1 = 1.0025$

$x_2 = 0.4, u_2 = 1.352, u_2^* = 1.0205$

Solution: $u^*(x) = [1 + (\frac{2}{3})ln(1 + x^3)]^{\frac{1}{2}}$

3 6.4.4

The formula for RK4 is

$u_{i+1} = u_i + \frac{K_1}{6} + \frac{K_2}{3} + \frac{K_3}{3} + \frac{K_4}{6}$

$K_1 = hf(t_i, u_i)$

$K_2 = hf(t_i + \frac{h}{2}, u_i + \frac{k_1}{2})$

$K_3 = hf(t_i + \frac{h}{2}, u_i + \frac{k_2}{2})$

$K_4 = hf(t_i + h, u_i + K_3)$

4 6.4.4.a

$f(t_i, u_i) = -2tu, h = 0.2$

$K_1 = 0.2f(0, 2) = 0$

$K_2 = 0.2f(0.1, 2) = -0.08$

$K_3 = 0.2f(0.1, 1.96) = -0.0784$

$K_4 = 0.2f(0.2, 1.9216) = -0.1537$

$u_{i+1} = 2 + 0 - \frac{0.08}{3} - \frac{0.0784}{3} - \frac{0.153728}{6} = 1.9215$

The true result is:

$u(0.2) = 2e^{-0.2^2} = 1.9215$

5 6.4.4.b

$f(t_i, u_i) = u + t, h = 0.2$

$K_1 = 0.2f(0, 2) = 0.4$

$K_2 = 0.2f(0.1, 2.2) = 0.46$

$K_3 = 0.2f(0.1, 2.23) = 0.466$

$K_4 = 0.2f(0.2, 2.466) = 0.5332$

$u_{i+1} = 2 + \frac{0.4}{6} + \frac{0.46}{3} + \frac{0.466}{3} + \frac{0.5332}{6} = 2.4642$

The true result is:

$u(0.2) = -1 - 0.2 + 3e^{0.2} = 2.4642...$

6 6.4.4.c

$f(x_i, u_i) = \frac{x^2}{u(x^3+1)}, h = 0.2$

$K_1 = 0.2f(0, 1) = 0$

$K_2 = 0.2f(0.1, 1) = 0.001998$

$K_3 = 0.2f(0.1, 1.000999) = 0.001996$

$K_4 = 0.2f(0.2, 1.001996) = 0.00792$

The true result is:

$u(0.2) = (1 + \frac{2}{3}\ln(x^3 + 1))^{0.5} = 1.0026$

7 6.4.5

8 6.4.11

Code in .m file