Problem 1

In phasor form:

a) $V(jw) = 155\angle\text{-}25°$ V

b) $V(jw) = 5\angle\text{-}130°$ V

c) $I(jw) = 10\angle 63° + 15\angle\text{-}42° = (4.54 + j8.91) + (11.15 - j\,10.04) = 15.69 - j1.13 = 15.73\angle\text{-}4.12°$ A

d) $I(jw) = 460\angle\text{-}25° - 220\angle 75° = (416.90 - j194.40) - (56.94 - j212.50) = 359.96 + j18.10 = 360.4\angle 2.88°$ A

Problem 2

a) $4 + j4 = 4\sqrt{2}\angle 45° = 5.66\angle 45°$

b) $-3 + j\,4 = 5\ \angle 126.9°$

c) $j + 2 - j4 - 3 = -1 - j\,3 = 3.16\angle\text{-}108.4°$

Problem 3

a) $(50 + j10)(4 + j8) = (50.99\angle 11.30°)(8.94\angle 63.43°) = 456.1\angle 74.7°$

$(50 + j10)(4 + j8) = 200 + j400 + j40 + j^2\ 80 = 120 + j440 = 456.1\angle 74.7°$

b) $(j2 - 2)(4 + j5)(2 + j7) = (2.82\angle 135°)(6.40\angle 51.34°)(7.28\angle 74.05°) = 131.8\angle 260.4° = 131.8\angle\text{-}99.6°$

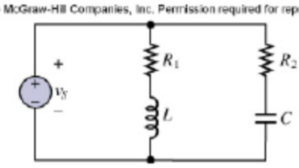
$(j2 - 2)(4 + j5)(2 + j7) = -36 - j126 - j4 - j^2\ 14 = -22 - j130 = 131.8\angle\text{-}99.6°$

Problem 4

Solution:

Known quantities:

The values of the impedance, $R_1 = 2.3\ \mathbf{k\Omega}$, $R_2 = 1.1\ \mathbf{k\Omega}$, $L = 190\ \mathbf{mH}$, $C = 55\ \mathbf{nF}$ and the voltage applied to the circuit shown in Figure P4.47, $v_s(t) = 7\cos\left(3000t + 30^o\right)\mathbf{V}$.



Find:

The equivalent impedance of the circuit.

Analysis:

$$X_L = \omega L = \left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(190\ \mathbf{mH}) = 0.57\ \mathbf{k\Omega} \Rightarrow Z_L = +j \cdot X_L = +j \cdot 0.57\ \mathbf{k\Omega}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3\ \mathbf{k}\frac{\mathbf{rad}}{\mathbf{s}}\right)(55\ \mathbf{nF})} = 6.061\ \mathbf{k\Omega} \Rightarrow Z_C = -j \cdot X_C = -j \cdot 6.061\ \mathbf{k\Omega}$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 2.3 + j \cdot 0.57\ \mathbf{k\Omega} = 2.37\angle 13.92^o\ \mathbf{k\Omega}$$

$$Z_{eq2} = Z_{R1} + Z_C = R_1 - jX_C = 1.1 - j \cdot 6.061\ \mathbf{k\Omega} = 6.16\angle -79.71^o\ \mathbf{k\Omega}$$

$$Z_{eq} = \frac{Z_{eq1} \cdot Z_{eq2}}{Z_{eq1} + Z_{eq2}} = \frac{\left(2.37\angle 13.92^o\ \mathbf{k\Omega}\right)\left(6.16\angle -79.71^o\ \mathbf{k\Omega}\right)}{(2.3 + j \cdot 0.57\ \mathbf{k\Omega}) + (1.1 - j \cdot 6.061\ \mathbf{k\Omega})} =$$

$$= \frac{14.60\angle -65.79^o\ \mathbf{k\Omega}^2}{3.4 - j \cdot 5.491\ \mathbf{k\Omega}} = \frac{14.60\angle -65.79^o\ \mathbf{k\Omega}^2}{6.458\angle -58.23^o\ \mathbf{k\Omega}} = 2.261\angle -7.56^o\ \mathbf{k\Omega}$$

Problem 5

$$V_{s_1}(j\omega) = V_{s_2}(j\omega) = 170\angle 0° \quad V$$

$$V_s(j\omega) = V_{s_1}(j\omega) + V_{s_2}(j\omega) = 340\angle 0°$$

$$I_{Z_3}(j\omega) = \frac{V_s(j\omega)}{Z_3}$$

$$Z_3 = 17\angle 0.192 = 17\angle 11° \ \Omega$$

$$I_{Z_3}(j\omega) = \frac{340\angle 0°}{17\angle 11°}$$

$$= 20\angle -11°$$

$$i_{Z_3}(t) = 20\cos(377t - 11°)$$

Problem 6

To have $I_i(jw)$ and $V_o(jw)$ in phase

$$Z_{CRL} = \frac{V_o(jw)}{I_i(jw)}$$ has to be a real number.

$$Z_{CRL} = Z_C // (Z_R + Z_L)$$

$$= \frac{Z_C(Z_R + Z_L)}{Z_C + Z_R + Z_L}$$

$$Z_C = -\frac{j}{wC} = -\frac{j}{w \times (220 \times 10^{-12})} = -\frac{4.54 \times 10^9}{w} j \ \Omega$$

$$Z_L = jwL = jw(19 \times 10^{-3}) = 0.019 jw \ (\Omega)$$

$$Z_R = 120 \ \Omega.$$

$$Z_{CRL} = \frac{-\frac{4.54}{w} \times 10^9 j(120 + 0.019 jw)}{120 + 0.019 wj - \frac{4.54}{w} \times 10^9 j}$$

$$= \frac{8.63 \times 10^7 - \frac{5.45}{w} \times 10^{11} j}{120 + j(0.019w - \frac{4.54 \times 10^9}{w})}.$$

For Z_{CRL} to be real, the phasor angles of denominator and numerator has to be the same.

$$\therefore \frac{-\frac{5.45}{w} \times 10^{11}}{8.63 \times 10^7} = \frac{0.019w - \frac{4.54}{w} \times 10^9}{120}$$

$$-\frac{6.32 \times 10^3}{w} = \frac{0.019w - \frac{4.54}{w} \times 10^9}{120}$$

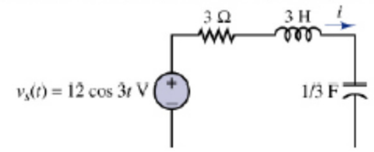
$$w^2 = \frac{4.54 \times 10}{0.019} \times 10^8$$

$$w = \sqrt{\frac{4.54 \times 10}{0.019}} \times 10^4 \ rad/s$$

$$w = 4.89 \times 10^5 \ rad/s$$

Problem 7

Solution:



Known quantities:

The values of the impedance and the voltage applied to the circuit shown in Figure P4.54.

Find:

The current in the circuit.

Analysis:

Assume clockwise currents:

$$\omega = 3\ \frac{\mathbf{rad}}{\mathbf{s}},\ V_S = 12\angle 0^o\ \mathbf{V}$$

$$Z_C = \frac{1}{j\omega C} = -j\ \Omega,\ Z_L = j\omega L = j9\ \Omega \Rightarrow Z_{total} = 3 + j9 - j = 3 + j8\ \ \Omega$$

$$I = \frac{12}{3 + j8} = 0.4932 - j1.3151\ \mathbf{A} = 1.4045\angle -69.44^o\ \mathbf{A},\ i(t) = 1.4\cos\left(\omega t - 69.4^o\right)\ \mathbf{A}$$

Problem 8

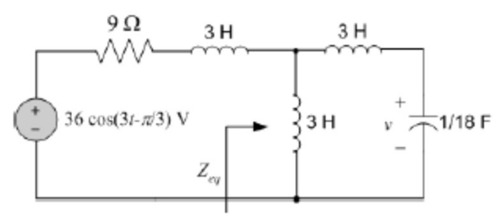
Solution:

Known quantities:

Circuit shown in Figure P4.68, the values of the resistance, $R = 9\ \Omega$, capacitance, $C = 1/18\ \mathbf{F}$, inductance, $L_1 = 3\ \mathbf{H}$, $L_2 = 3\ \mathbf{H}$, $L_3 = 3\ \mathbf{H}$, and the voltage source $v_s(t) = 36\cos\left(3t - \frac{\pi}{3}\right)\ \mathbf{V}$.

Find:

The voltage across the capacitance v using phasor tehcniques.



Analysis:

$$\omega = 3\ \frac{\mathbf{rad}}{\mathbf{s}},\ V_s = 36\angle -60°\ \ \mathbf{V}$$

$$Z_{L_2} = j\omega L_2 = j3\cdot 3 = j9\ \Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j3\cdot(1/18)} = -j6\ \Omega$$

$$Z_{L_3} = j\omega L_3 = j3\cdot 3 = j9\ \Omega$$

$$Z_{eq} = \frac{1}{Z_{L_3}\left\|\left(Z_{L_2} + Z_C\right)\right.} = \frac{1}{\dfrac{1}{Z_{L_3}} + \dfrac{1}{\left(Z_{L_2} + Z_C\right)}} = \frac{1}{\dfrac{1}{j9} + \dfrac{1}{j3}} = \frac{j9}{4} = 2.25\angle 90^o\ \Omega$$

$$Z_T = Z_R + Z_{L_1} + Z_{eq} = 9 + j3\cdot 3 + j2.25 = 9 + j11.25 = 14.407\angle 51.34^o\ \Omega$$

$$I = \frac{V_S}{Z_T} = \frac{36\angle -60^o\ \mathrm{V}}{14.407\angle 51.34^0\ \Omega} = 2.499\angle -111.34^o\ \mathrm{A}$$

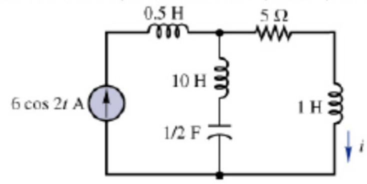
$$V_{eq} = IZ_{eq} = (2.499\angle -111.34^o)(2.25\angle 90^o) = 5.623\angle -21.34^o\ \mathrm{V}$$

$$V = \frac{Z_C}{\left(Z_{L_2} + Z_C\right)}V_{eq} = \frac{-j6}{j3}5.623\angle -21.34^o = 11.25\angle 158.66^o\ \mathrm{V}$$

$$v = 11.25\cos\left(3t - 158.66^o\right)\ \mathrm{V}$$

Problem 9

Solution:



Known quantities:

Circuit shown in Figure P4.69, the values of the resistance, $R = 5\ \Omega$, capacitance, $C = 1/2\ \mathbf{F}$, inductance, $L_1 = 0.5\ \mathbf{H}$, $L_2 = 1\ \mathbf{H}$, $L_2 = 10\ \mathbf{H}$, and the current source $i_s(t) = 6\cos(2t)\ \mathbf{A}$.

Find:

The current through the inductance i_{L_2}.

Analysis:

$$\omega = 2\ \frac{\mathbf{rad}}{\mathbf{s}},\ Z_{L_2} = j\omega L_2 = j2\Omega,\ Z_C = \frac{1}{j\omega C} = -j\ \Omega,\ Z_{L_3} = j\omega L_3 = j20\ \Omega$$

$$I = \frac{Z_{L_3} + Z_C}{(Z_{L_3} + Z_C) + (R + Z_{L_2})} I_S = \frac{j20 - j}{(j20 - j) + (5 + j2)} 6\angle 0^o = \frac{j19}{5 + j21} 6\angle 0^o = 5.28\angle 13.4^o\ \mathbf{A}$$

$$i = 5.28\cos\left(2t + 13.4^o\right)\ \mathbf{A}$$