ELEG 305
SOLUTIONS TO EXAM #1 (3/9/17)

#1. i.) Almost all signals can be represented as linear combinations of complex exponentials.

ii.) Complex exponentials are eigenfunctions of linear, time-invariant systems.

#2. a.) $x[n] = \cos 3n$

For a discrete-time signal to be periodic, $x[n+N] = x[n]$ for all n. For this to be true, as we demonstrated in class,

$$\frac{\omega_0}{2\pi} = \frac{m}{N} \Rightarrow \text{a ratio of integers (rational)}$$

In this case, $\omega_0 = 3$. So, it cannot be periodic since $\omega_0/2\pi = 3/2\pi$ which cannot be expressed as a ratio of integers.

b.) $$\int_{-2}^{2} (1+t)^2 \underbrace{\delta(t-1)}_{\text{impulse function located at } t=1 \text{ and in range of integration limits}} dt$$

$$= (1+t)^2 \Big|_{t=1} = 4$$

c.) $$y[n] = \sum_{k=0}^{\infty} \alpha^k x[n-k]$$

impulse response $h[n] = y[n] \Big|_{x[n]=\delta[n]}$

#2a.cont'd)

$$h[n] = \sum_{k=0}^{\infty} \alpha^k \underbrace{\delta[n-k]}$$

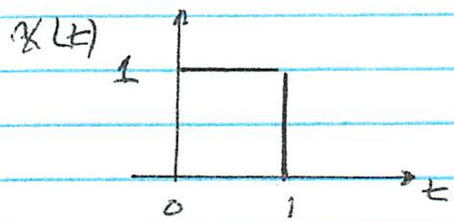
- only has value when $k=n$
- also n must be greater than k (which can never be negative)

$$\therefore\ h[n] = \alpha^n u[n]$$

#3.) LTI system with input $x(t) = u(t) - u(t-1)$ has response $y(t)$.

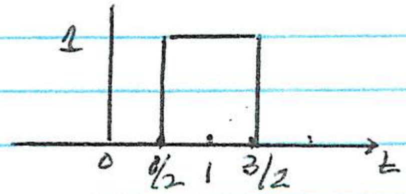
i.) New input $g(t) = u(t-1) - u(t-2)$
$= x(t-1)$
Therefore, since the system is time-invariant, the new output $= y(t-1)$

ii.)



$$x(t)\,\delta(t-0.5) = \underbrace{x(0.5)}_{1}\,\delta(t-0.5) = \delta(t-0.5)$$

iii.) $x(t) * \delta(t-0.5) = x(t-0.5)$

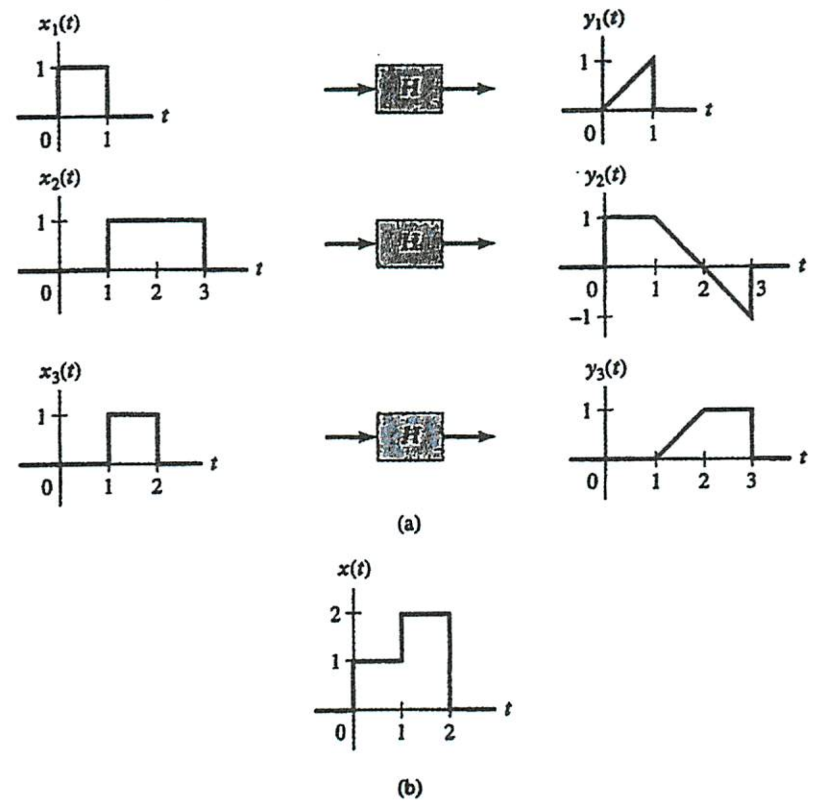


#3.

I.

II

III



a.) causal?

If the system is causal, there can be no output before the input is applied. However, the input $x_2(t)$ starts at $t=1$ and gives an output that starts at $t=0$.

Therefore, this system is <u>not causal</u>.

b.) time-invariant?

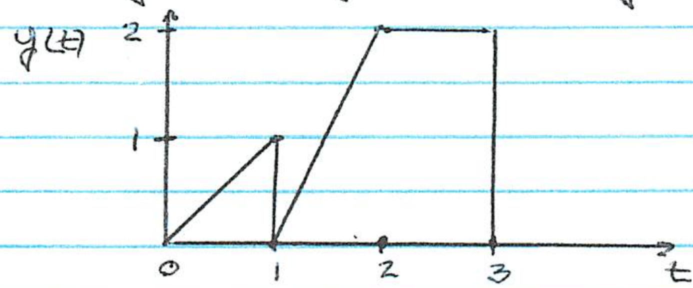
If the system is time-invariant, a shifted input will give the same output, shifted by the same amount. Notice that the input $x_3(t)$ is simply $x_1(t-1)$. However, $y_3(t) \neq y_1(t-1)$.

Therefore, this system is <u>not time-invariant</u>.

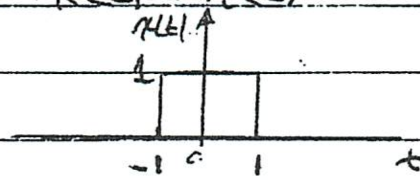
#3. a.) The system is linear. So, a linear combination of the inputs gives the same linear combination of the outputs.

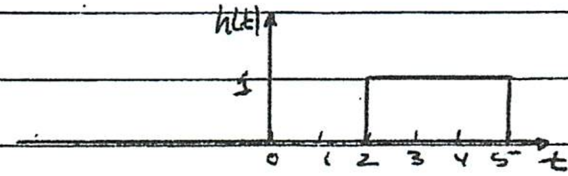
$$x(t) = x_1(t) + 2x_3(t)$$

$$\therefore \quad y(t) = y_1(t) + 2y_3(t)$$

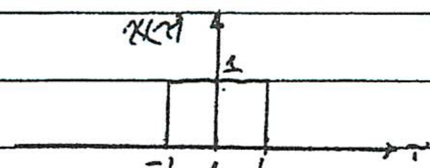


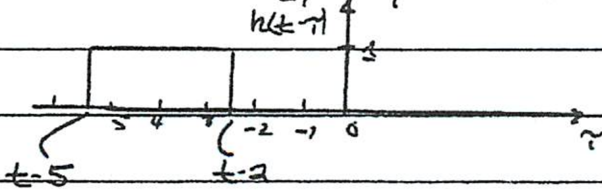
#4. $y(t) = x(t) * h(t)$





$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

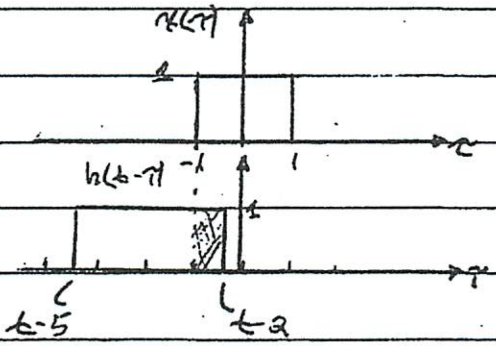




#4 cont'd

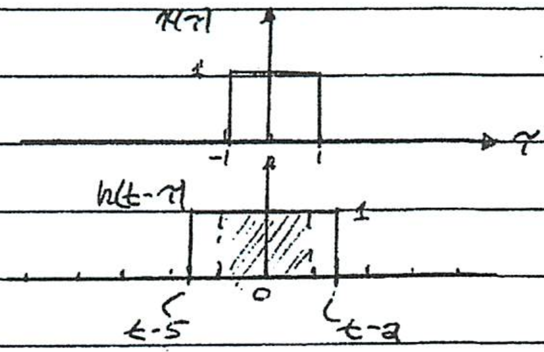
• $t-2 < -1 \Rightarrow t < 1$, $y(t) = 0$ no overlap

• $\underbrace{1 < t < 3}_{t-2>1}$



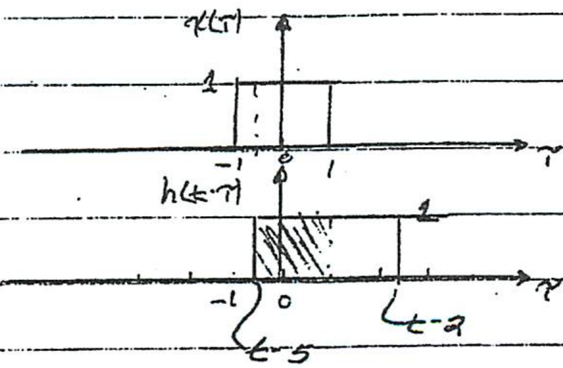
$$y(t) = \int_{-1}^{t-2} 1\,dt = t-2-(-1) = t-1$$

• $\underbrace{3 < t < 4}_{t-5>-1}$



$$y(t) = \int_{-1}^{1} 1\,dt = 2$$

• $\underbrace{4 < t < 6}_{t-5>1}$

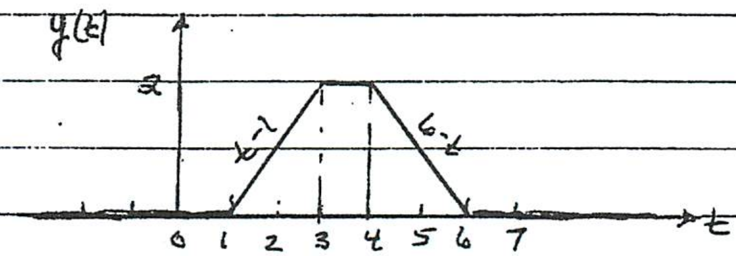


#4 cont'd)

$$y(t) = \int_{t-5}^{1} 1\,d\tau = 1-(t-5) = 6-t$$

- $t > 6$ $\quad y(t) = 0 \quad$ no overlap

$$y(t) = \begin{cases} 0 & t<1 \\ t-1 & 1<t<3 \\ 2 & 3<t<4 \\ 6-t & 4<t<6 \\ 0 & t>6 \end{cases}$$



#5. $x[n] \rightarrow$ info. retrieval in Prof. Cimini's brain $\rightarrow y[n]$

- linear
- time-invariant
- causal

- test signal $\quad x[n] = \delta[n]$

$y[n] =$ impulse response $= h[n] = \alpha^n$

because the system is causal, $h[n]$ must be zero for $n<0$

$\therefore \; h[n] = \alpha^n u[n]$

#5. cont'd] - Because the system is linear and time-invariant, the response to an arbitrary input can be found by convolving the input with the impulse response

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = x[n] * h[n]$$

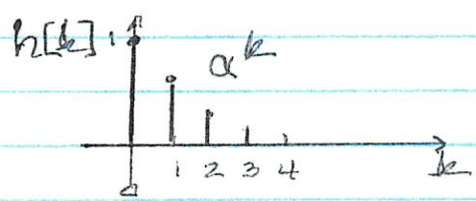
- What is the output for input $x[n] = u[n-3]$?

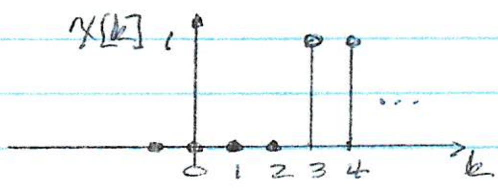
$$y[n] = x[n] * h[n] = u[n-3] * \alpha^n u[n]$$

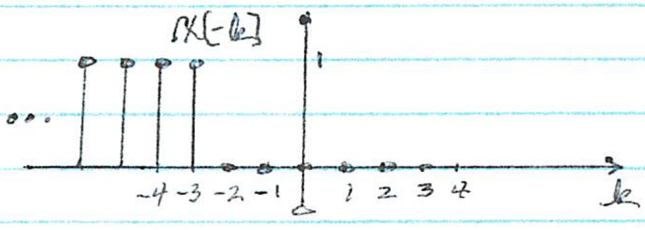
$$= h[n] * x[n] \rightarrow \text{do whichever is simpler}$$

Here, I will choose to flip and shift $x[n]$ because the limits will be simpler. However, you, of course, get the same answer if you flip and shift $h[n]$

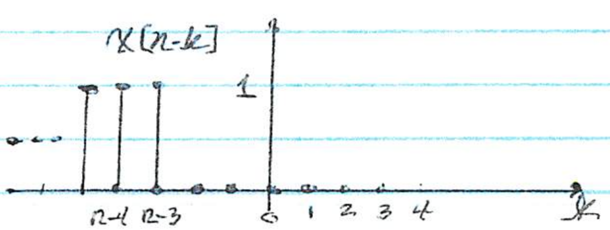
$$\therefore\ y[n] = \sum_{k=-\infty}^{\infty} h[k]\,h[n-k]$$







flip



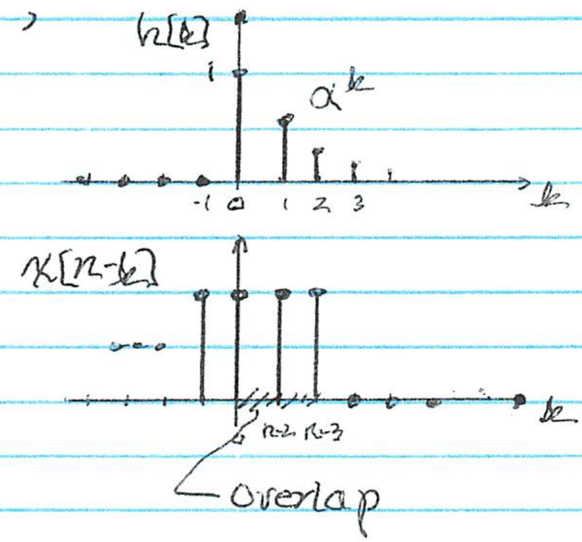
shift

$n-3 < 0$

$n < 3$

#5 cont'd) • $n < 3$, $y[n] = 0$ no overlap (see picture on previous page)

• $n \geq 3$,



The overlap occurs from $k=0$ to $k=n-3$. So,

$$y[n] = \sum_{k=0}^{n-3} \alpha^k \rightarrow \text{finite length geometric series}$$

$$= \frac{1-\alpha^{n-2}}{1-\alpha}$$

$$\therefore \quad y[n] = \frac{1-\alpha^{n-2}}{1-\alpha} u[n-3]$$

Extra Credit

$$h[n] = \text{composite impulse response}$$
$$= \underbrace{\underbrace{(h_1[n] + h_2[n])}_{\text{parallel connection}} * h_3[n]}_{\text{series interconnection}}$$

$$h_1[n] + h_2[n] = \cancel{u[n]} + u[n+2] - \cancel{u[n]}$$
$$= u[n+2]$$

$$h[n] = u[n+2] * h_3[n]$$
$$= u[n+2] * \delta[n-2]$$
$$= u[n]$$