Homework#2 (Math 342)

(due Wed Oct 3)

Linear Algebra: a Modern Introduction, by D. Poole (4th Edition)

Note: Detail your work to receive full credit.

Sec. 6.5: 2, 4

Sec. 6.7: 4, 6, 12 (solve these problems by reducing the scalar equation to a first-order system)

Additional problems:

1) Show that, if $P^{-1}AP = D$ for any diagonalizable matrix A where D is the corresponding diagonal matrix with the eigenvalues of A and P is a transition matrix, then $Q^{-1}A^\top Q = D^\top$ for some matrix Q to be determined in terms of P.

2) Use the method of diagonalization to compute A^6 for

$$A = \begin{pmatrix} 5 & 6 \\ -2 & -2 \end{pmatrix}$$

3) Solve the initial value problem $\boldsymbol{Y}' = A\boldsymbol{Y}$, $\boldsymbol{Y}(0) = \boldsymbol{Y}_0$ by computing $e^{tA}\boldsymbol{Y}_0$ for each of the following:

(a)

$$A = \begin{pmatrix} 1 & -2 \\ 0 & -1 \end{pmatrix}, \quad \boldsymbol{Y}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix}, \quad \boldsymbol{Y}_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

4) Find the general solution of each of the following systems:

(a)

$$\begin{aligned} y_1' &= 2y_1 + 4y_2 \\ y_2' &= -y_1 - 3y_2 \end{aligned}$$

(b)

$$\begin{aligned} y_1' &= y_1 - y_2 \\ y_2' &= y_1 + y_2 \end{aligned}$$

5) Solve each of the following initial value problems:

(a)

$$\begin{aligned} y_1' &= -y_1 + 2y_2 \\ y_2' &= 2y_1 - y_2 \end{aligned}$$

with $y_1(0) = 3$ and $y_2(0) = 1$.

(b)

$$\begin{aligned} y_1' &= y_1 - 2y_2 \\ y_2' &= 2y_1 + y_2 \end{aligned}$$

with $y_1(0) = 1$ and $y_2(0) = -2$.