HW 3 - Problem 6-4

Link 1 = 4 Link 2 = 6 Link 3 = 10 Link 4 = 7 $\theta_2 = 88$

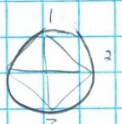
$\omega_2 = -80$ $R_{PA} = 10$ $\delta_3 = 330°$ From Last HW $\theta_{31} = 346.7°$ $\theta_{41} = 31.47°$

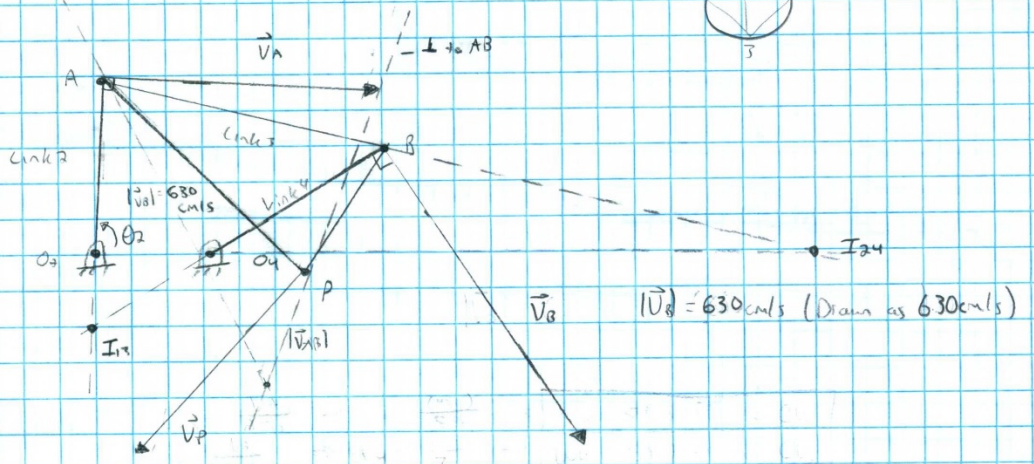
$\theta_{32} = 257.9°$ $\theta_{42} = 212.7°$

Open:

$\perp$ to BO_4

$V_A = AO_2 \omega_2 \Rightarrow V_A = (6)(80) \Rightarrow V_A = 480$ cm/s (Drawn as 4.8 cm/s)





$|\vec{V}_B| = 630$ cm/s (Drawn as 630 cm/s)

$\omega_3 = \frac{V_A}{AI_{13}} \Rightarrow \omega_3 = \frac{(480)}{(8.8)} \Rightarrow$ **$\omega_3 = 54.5$ rad/sec CW**

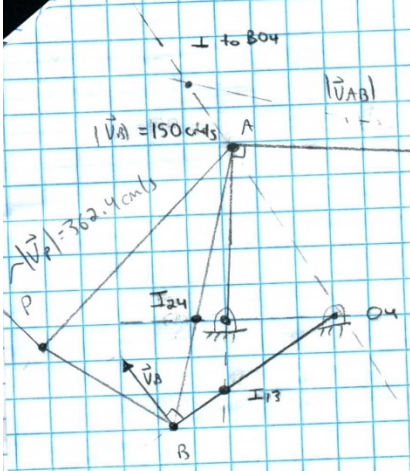
$\omega_4 = \frac{V_B}{BO_4} \Rightarrow \omega_4 = \frac{(630)}{7} \Rightarrow$ **$\omega_4 = 90.0$ rad/sec CW**

$V_P = \omega_3 PI_{13} \Rightarrow V_P = (54.5)(8) \Rightarrow$ **$|V_P| = 436$ cm/s**

HW3 - Problem 6-4

Crossed

$V_A = AO_3 \omega_3 \Rightarrow V_A = (6)(80) \Rightarrow V_A = 480 \text{ cm/s}$ (Drawn as 4.8 cm/s)



$\omega_3 = \frac{V_A}{AI_{13}} = \frac{(480)}{9.0} \Rightarrow$ **$\omega_3 = 53.3$ rad/sec CW**

$\omega_4 = \frac{V_B}{BO_4} = \frac{150}{7} \Rightarrow$ **$\omega_4 = 21.4$ rad/sec CW**

$V_P = \omega_3 P I_{13} \Rightarrow V_P = (53.3)(6.8) \Rightarrow$ **$|V_P| = 362.4$ cm/s**

HW3 - Problem 6-5

$r_1 = 4$ $r_2 = 6$ $r_3 = 10$ $r_4 = 7$ $\theta_2 = 88$ $\omega_2 = -80$

$r_{P4} = 10$ $\delta_3 = 330$ $\theta_{31} = 346.7$ $\theta_{41} = 31.47$ $\theta_{72} = 257.9$ $\theta_{42} = 212.7°$

Open: $\underbrace{\vec{R}_{A0_2} + \vec{R}_{BA} + \vec{R}_{0_4B_4} + \vec{R}_{0_2 0_4}}_{\text{All Diff points Same RB}} = 0$

Take derivative:

$r_2\omega_2 e^{j(\theta_2+90)} + r_3\omega_3 e^{j(\theta_3+90)} + r_4\omega_4 e^{j(\theta_4+90)} + \vec{0} = 0$

$\Rightarrow -480 e^{j178} + 10\omega_3 e^{j436.7} + 7\omega_4 e^{j121.47} = 0$

2 equations, 2 unknowns

$-480\sin 178 + 10\omega_3 \sin 436.7 + 7\omega_4 \sin 121.47 = 0$

$-480\cos 178 + 10\omega_3 \cos 436.7 + 7\omega_4 \sin 121.47 = 0$

$\omega_3 = \dfrac{-7\omega_4 \sin 121.47 + 480\sin 178}{10\sin 436.7} \Rightarrow \omega_3 = \dfrac{-5.97\omega_4 + 16.75}{9.73}$ (1)

$\omega_4 = \dfrac{480\cos 178 - 10\omega_3 \cos 436.7}{7\cos 121.4} \Rightarrow \omega_4 = \dfrac{-497.7 - 7.3\omega_3}{-3.65}$ (2)

Plug (1) into (2):

$\omega_4 = \dfrac{+479.7 + 1.41\omega_4 - 3.96}{-3.65} \Rightarrow 1.387\omega_4 = 132.48 \Rightarrow \boxed{\omega_4 = 95.6 \text{ rad/sec}}$

$\omega_3 = \dfrac{-5.97(95.6) + 16.75}{9.73} \Rightarrow \boxed{\omega_3 = 56.9 \text{ rad/sec}}$

$\frac{d}{dt}$

$\vec{R}_{P0_2} = \underbrace{\vec{R}_{A0_2} + \vec{R}_{PA}}_{\text{Diff point Same RB}}$

$\vec{V}_P = r_2\omega_2 e^{j(\theta_2+90)} + r_{PA}\omega_3 e^{j(\theta_3+\delta_3+90)}$

$= -480 e^{j178} - 569 e^{j(766.7)}$

$= -480\cos 178 + j480\sin(178) - 569\cos(766.7) - j569\sin(766.7$

$= -89.5 - j430.85$

$|\vec{V}_P| = \sqrt{(89.5)^2 + (-430.85)^2} \Rightarrow \boxed{|\vec{V}_P| = 440 \text{ cm/s}}$

	Graphical	Analytical	
ω_4	90 rad/sec	95.6 rad/sec	The values are very similar. May have had slight errors in graphing
ω_3	54.5 rad/sec	56.9 rad/sec	
$\lvert\vec{V}_P\rvert$	436 cm/s	440 cm/s	

HW3 - Problem 6-5

Crossed

$$\vec{R}_{AO_2} + \vec{R}_{BA} + \vec{R}_{O_4B} + \vec{R}_{O_2O_4} = 0$$

$$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} + r_4 e^{j\theta_4} + r_1 e^{j\theta_1} = 0$$

$\frac{d}{dt}$

$$r_2\omega_2 e^{j(\theta_2+90)} + r_3\omega_3 e^{j(\theta_3+90)} + r_4\omega_4 e^{j(\theta_4+90)} + \vec{0} = 0$$

$$-480 e^{j178} + 10\omega_3 e^{j(257.9+90)} + 7\omega_4 e^{j(212.7+90)} = 0$$

2 equations 2 unknowns

$$-480\sin 178 + 10\omega_3 \sin 347.9 + 7\omega_4 \sin 302.7 = 0$$

$$-480\cos 178 + 10\omega_3 \cos 347.9 + 7\omega_4 \cos 302.7 = 0$$

$$\omega_3 = \frac{480\sin(178) - 7\omega_4\sin(302.7)}{10\sin(347.9)} \Rightarrow \omega_3 = -7.99 - 2.81\omega_4 \quad (1)$$

$$\omega_4 = \frac{480\cos(178) - 10\omega_3\cos(347.9)}{7\cos(302.7)} \Rightarrow \omega_4 = -126.9 - 2.59\omega_3 \quad (2)$$

Plug (1) into (2):

$$\omega_4 = -126.9 - 2.59(-7.99 - 2.81\omega_4)$$

$$\Rightarrow \omega_4 = 106.2 + 7.28\omega_4 \Rightarrow \boxed{\omega_4 = -16.9 \text{ rad/sec}}$$

$$\omega_3 = -7.99 - 2.81(16.9) \Rightarrow \boxed{\omega_3 = -55.5 \text{ rad/sec}}$$

$$\vec{R}_{PO_2} = \vec{R}_{AO_2} + \vec{R}_{PA}$$

$$\vec{V}_P = (6)(-80) e^{j(178)} + 10(-55.5) e^{j677.9}$$

$$= +480\cos 178 - j480\sin 178 - 555\cos 677.9 - j555\sin 677.9$$

$$= -67.9 + j355.3$$

$$|\vec{V}_P| = \sqrt{(67.9)^2 + (355.3)^2} \Rightarrow \boxed{|\vec{V}_P| = 362 \text{ cm/s}}$$

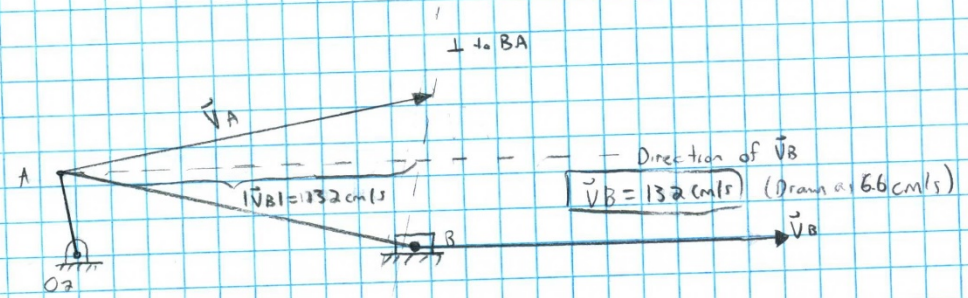
	Graphical	Analytical
ω_4	21.4 rad/sec	16.9 rad/sec
ω_3	53.3 rad/sec	55.5 rad/sec
$\lvert\vec{V}_P\rvert$	362 cm/s	362 cm/s

The values are very similar. May have had slight errors in graphing

HW3 - Problem 6-6

Link 2 = 3 Link 3 = 13 Offset = 0 $\theta_2 = 160°$ $\omega_2 = -45$
$d = 12.1$ $\theta_3 = 166.9°$

$V_A = \omega_2 \overline{AO_2} \Rightarrow$ **$V_A = 135$ cm/s** (Draw as 6.75 cm/s)



$\vec{V}_{slip} = \vec{V}_B$

$\vec{V}_B$ Parallel to slip direction

HW3 - Problem 6-7

$$\vec{R}_{AO_2} = r_2 e^{j\theta_2}$$

$$V_{AO_2} = (3)(-45) e^{j(90)}$$

$$V_{AO_2} = -135\, e^{j(90)} \Rightarrow \boxed{|\vec{V}_A| = 135 \text{ cm/s}}$$

$$\vec{R}_{BO_2} = \vec{R}_{AO_2} + \vec{R}_{BA}$$

$$V_{BO_2} = V_{AO_2} + V_{BA}$$

$$V_{BO_2} = -135\, e^{j(10)} + 13\omega_3\, e^{j(256.9)}$$

Need ω_3:

$$\omega_3 = \frac{3\cos(100)}{13\cos(166.9)}(-45) \Rightarrow \omega_3 = -1.9$$

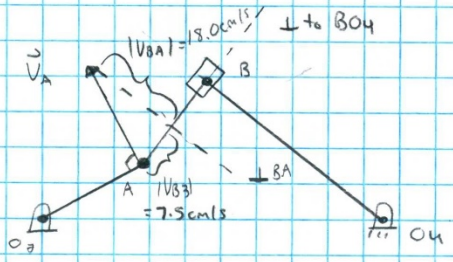
$$V_{BO_2} = -135\cos(10) + 13(-1.9)\cos(256.9) - 135j\sin(10) + j13(-1.9)\sin(256.9)$$

$$\boxed{|\vec{V}_{BO_2}| = 127.4 \text{ cm/s}} \qquad \boxed{\theta = 0^\circ}$$

$$\boxed{\vec{V}_{slip} = \vec{V}_B}$$

	Graphically	Analytical	
$\vec{V}_A$	135 cm/s	135 cm/s	The values are very similar. Differences may be from graphing errors.
$\vec{V}_B$	132 cm/s	127.4 cm/s	

HW3 - Problem 6.8



$V_A = \omega_2 AO_2 \Rightarrow V_A = (2)(10) \Rightarrow \boxed{|\vec{V}_A| = 20 \text{ cm/s}}$ (Drawn as 2 cm/s)

From graph: $V_{B3} = 7.5$ cm/s

$V_{BA} = 18.0$ cm/s

$V_{B4} = \omega_4 \overline{BO_4}$

Need ω_4

$\theta_3 = \theta_4 + \gamma \overset{d/dt}{\Rightarrow} \omega_3 = \omega_4$

$\omega_3 = \frac{V_{BA}}{\overline{AB}} \Rightarrow \omega_3 = \frac{(18.0)}{1.7} \Rightarrow \omega_3 = 10.6 \text{ rad/sec} = \omega_4$

$V_{B4} = (10.6)(4) \Rightarrow \boxed{|\vec{V}_{B4}| = 42.4 \text{ cm/s}}$

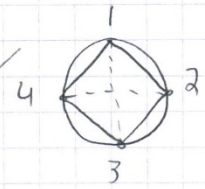
$\vec{V}_{slip} = V_3 - V_4 \Rightarrow \boxed{|\vec{V}_{slip}| = 35.4 \text{ cm/s}}$

	Graphical	Solutions
$\vec{V}_A$	20 cm/s	20 cm/s
$\vec{V}_B$	42.4 cm/s	41.2 cm/s
$\vec{V}_{slip}$	35.4 cm/s	33.5 cm/s

Answers were close to my graphical method

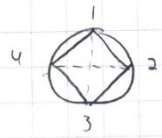
HW3 Problem 5

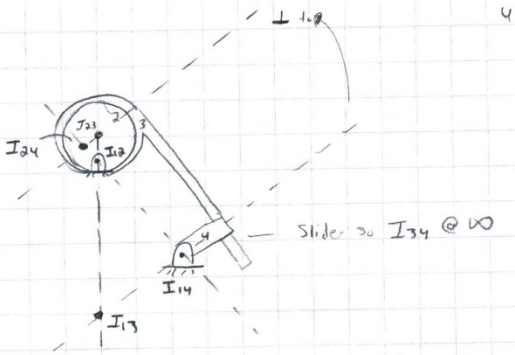
$V_A = \omega_2 \text{Link2} = 480 \text{ cm/s}$

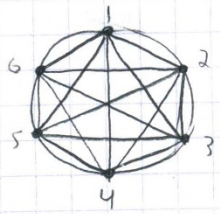


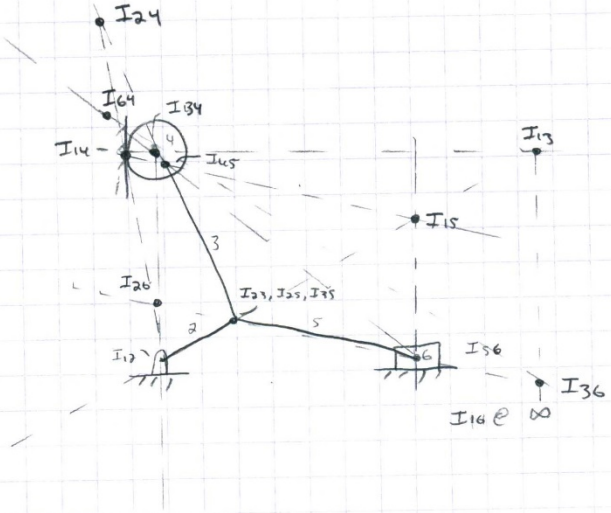
$|\vec{V}_B| = 640 \text{ cm/s}$
$|\vec{V}_P| = 420 \text{ cm/s}$
} Very close to matching Problem)

HW3- Problem 6









HW3 - Problem 7-2

$\omega = 100$ rad/sec $\quad \alpha = -500$ rad/sec^2

$r = 6.5$ in $\quad \theta_A = 45° \quad \theta_B = 120°$

$$\vec{A}_A = r_{OA}\alpha j e^{j\theta_A} - r_{OA}\omega^2 e^{j\theta_A}$$
$$= j(r_{OA}\alpha\cos\theta_A + j r_{OA}\alpha \sin\theta_A) - r_{OA}\omega^2\cos\theta_A - r_{OA}\omega^2\sin\theta_A$$
$$= 3250\sin 45 - 65000\cos 45 + j(-3250\cos 45 - 65000\sin 45)$$
$$= -43664 - 48760\, j$$

$$\vec{A}_B = 3750\sin 120 - 65000\cos 120 + j(-3750\cos 120 - 65000\sin 120)$$
$$= 35314 - 54667\, j$$

Need Relative acceleration:

$\vec{A}_{BA} = \vec{A}_B - \vec{A}_A$

$\vec{A}_{BA} = 78978 - 6407\, j \Rightarrow |\vec{A}_{BA}| = \sqrt{(78978)^2 + (-6407)^2}$

$\boxed{|\vec{A}_{BA}| = 79237 \text{ in/sec}^2}$

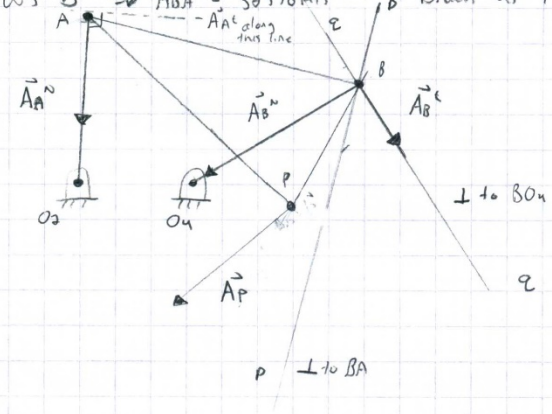
$\theta = \tan^{-1}\left(\frac{-6407}{78978}\right)$

$\boxed{\theta = -4.64°}$

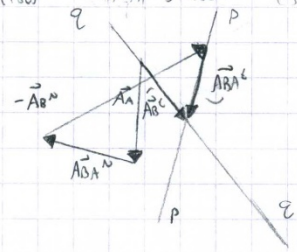
HW3- Problem 8

Given: Link1 = 4 Link2 = 6 Link3 = 10 Link4 = 7 $\theta_2 = 88$ $\omega_2 = -80$ $\alpha_2 = 30$
$R_{PA} = 16$ $\theta_{31} = 346.7$ $\theta_{41} = 31.47$ $\omega_3 = -56.9$ $\omega_4 = -95.6$ $\delta_3 = 330$

$\vec{A}_A^N = \omega_2^2\, a \Rightarrow \vec{A}_A^N = 38400\ m/s^2$ → Drawn as $1.92\ cm/s^2$
$\vec{A}_B^N = \omega_4^2\, c \Rightarrow \vec{A}_B^N = 63976\ m/s^2$ → Drawn as $3.2\ cm/s^2$
$\vec{A}_A^t = \alpha_2\, a \Rightarrow \vec{A}_A^t = 180\ m/s^2$ → will be very small @ new scale
$\vec{A}_{BA}^N = \omega_3^2\, b \Rightarrow \vec{A}_{BA}^N = 32376\ m/s^2$ → Drawn as $1.62\ cm/s^2$

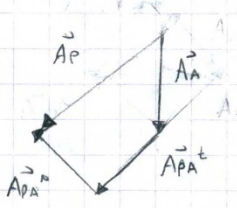


$\vec{A}_B = \vec{A}_A + \vec{A}_{BA} \Rightarrow \vec{A}_B^t + \vec{A}_B^N = A_A^t + A_A^N + A_{BA}^t + A_{BA}^N$
$|\vec{A}_A| = \sqrt{(38400)^2 + (180)^2} \Rightarrow |\vec{A}_A| = 38400.4$ essentially @ same angle of A_A^N since A_A^t is so small



$|\vec{A}_B^t| = 26000\ m/s^2$
$|\vec{A}_{BA}^t| = 30000\ m/s^2$
$\alpha_4 = \frac{26000}{7} \Rightarrow \boxed{\alpha_4 = 3714\ rad/s^2}$
$\alpha_3 = \frac{30000}{10} \Rightarrow \boxed{\alpha_3 = 3000\ rad/s^2}$

$\vec{A}_P = \vec{A}_A + \vec{A}_{PA} \Rightarrow \vec{A}_P = \vec{A}_A^t + \vec{A}_A^N + \vec{A}_{PA}^t + \vec{A}_{PA}^N$
$|\vec{A}_{PA}^t| = (10)(3000) \Rightarrow |\vec{A}_{PA}^t| = 30000\ cm/s^2$ (Drawn as $1.5\ cm/s^2$)
$|\vec{A}_{PA}^N| = (10)(56.9)^2 \Rightarrow |\vec{A}_{PA}^N| = 32376\ cm/s^2$ (Drawn as $1.62\ cm/s^2$)



$\boxed{|\vec{A}_P| = 52000\ cm/s^2}$

HW3 - Problem 9

$d=4 \quad a=6 \quad b=10 \quad c=7 \quad |R_{pa}|=10$

Equations from book:

$A = c\sin\theta_4 \Rightarrow A = 7\sin(31.47) \Rightarrow A = 3.65$

$B = b\sin\theta_3 \Rightarrow B = 10\sin(346.7) \Rightarrow B = -2.3$

$C = a\alpha_2\sin\theta_2 + a\omega_2^2\cos\theta_2 + b\omega_3^2\cos\theta_3 - c\omega_4^2\cos\theta_4$

$\Rightarrow C = (6)(30)(\sin 88) + (6)(80)^2(\cos 88) + (10)(56.9)^2(\cos 346.7) - (7)(95.6)^2(\cos 31.47)$

$\Rightarrow C = -21537.8$

$D = c\cos\theta_4 \Rightarrow D = 7\cos(31.47) \Rightarrow D = 5.97$

$E = b\cos\theta_3 \Rightarrow E = (10)\cos(346.7) \Rightarrow E = 9.73$

$F = a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\omega_3^2\sin\theta_3 + c\omega_4^2\sin\theta_4$

$\Rightarrow F = (6)(30)\cos 88 - (6)(80)^2\sin 88 - (10)(56.9)^2\sin(346.7) + (7)(95.6)^2\sin(31.47)$

$\Rightarrow F = 2476.3$

$$\alpha_3 = \frac{CD - AF}{AE - BD} \Rightarrow \alpha_3 = \frac{(-21537.8)(5.97) - (3.65)(2476.3)}{(3.65)(9.73) - (-2.3)(5.97)}$$

$$\boxed{\alpha_3 = -2795 \text{ rad/s}^2}$$

$$\alpha_4 = \frac{CE - BF}{AE - BD} \Rightarrow \alpha_4 = \frac{(-21537.8)(9.73) - (-2.3)(2476.3)}{(3.65)(9.73) - (-2.3)(5.97)}$$

$$\boxed{\alpha_4 = -4140 \text{ rad/s}^2}$$

$\vec{A}_P = \vec{A}_A + \vec{A}_{PA}$

$\vec{A}_P = \vec{A}_A^t + \vec{A}_A^N + \vec{A}_{PA}^t + \vec{A}_{PA}^N$

$\vec{A}_P = a\alpha_2 j e^{j\theta_2} - a\omega_2^2 e^{j\theta_2} + |R_{pa}|\alpha_3 j e^{j\theta_3+\delta} + |R_{pa}|\omega_3^2 e^{j\theta_3+\delta}$

$= 180 j e^{j88} - 38400 e^{j88} + 27950 j e^{j346.7} + 32376 e^{j346.7}$

$\vec{A}_P = 180j\cos 88 + 180j^2\sin 88 - 38400\cos 88 - 38400 j\sin 88$

$+ 27950 j\cos 676.7 + 27950 j^2\sin 676.7 + 32376\cos(676.7) + 32376 j\sin 876.7$

$\Rightarrow \vec{A}_P = (-180\sin 88 + 38400\cos 88 - 27950\sin 676.7 + 32376\cos 876.7)$

$+ j(180\cos 88 - 38400\sin 88 + 27950\cos 676.7 + 32376\sin 876.7)$

$\vec{A}_P = 4175.7 - j40733$

$$\boxed{|\vec{A}_P| = 40949 \text{ cm/s}^2}$$

	Graphically	Analytically
α_3	3000 rad/s²	2795 rad/s²
α_4	3714 rad/s²	4140 rad/s²
$\lvert\vec{A}_P\rvert$	52000 cm/s²	40949 cm/s²

These numbers aren't as similar as I would've liked, the numbers were so large scaling down made it so graphing couldn't be exact.