ELEG 310

4/5/18

Nontransitive Dice

A = 1, 1, 3, 5, 5, 6

B = 2, 3, 3, 4, 4, 5

C = 1, 2, 2, 4, 6, 6

$A > B$ 17 out of 36

$B > A$ 15 out of 36

$A = B$ 4 out of 36

Continuous RV's

$P(X \le u) = F_X(u)$ = cumulative distribution function

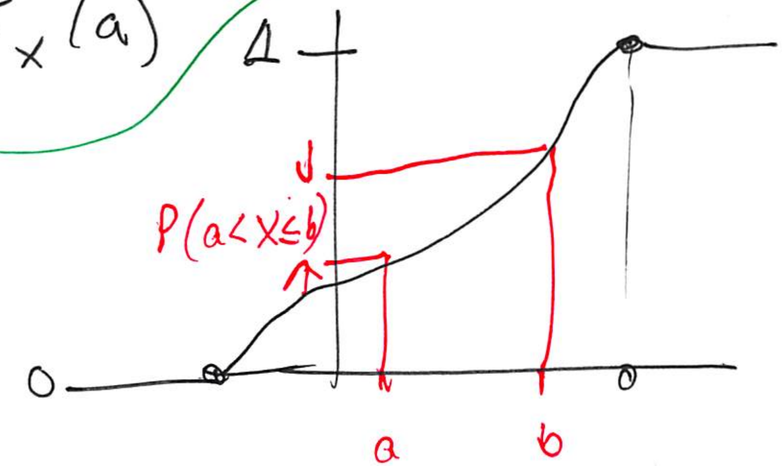
$f_X(u) = \dfrac{dF_X(u)}{du}$ = density $\Leftrightarrow$ pmf in discrete RV's

analogous

$$P(a < X \le b) = P(X \le b) - P(X \le a)$$

$$= F_X(b) - F_X(a)$$

Works for both continuous and discrete RVs.



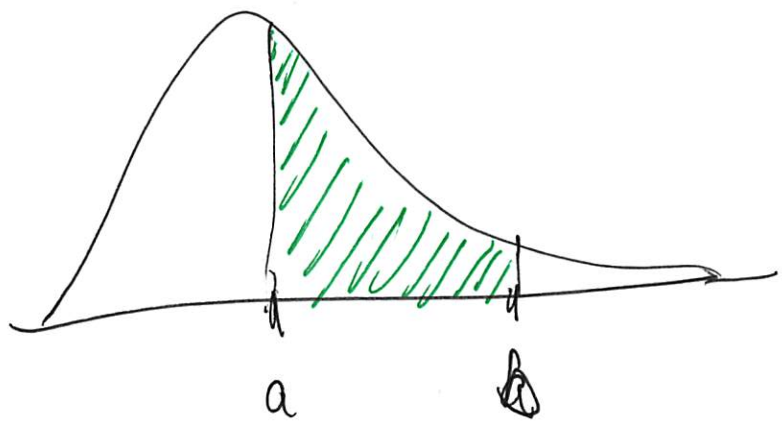
a RV X is continuous if $P(X = a) = 0$ for all a.

Don't ask

<u>Do ask:</u> $P(a < X \le b) = P(a < X < b) = P(a \le X \le b) = P(a \le X < b)$

$$P(a < X < b) = F(b) - F(a) = \int_a^b f(u)\,du$$

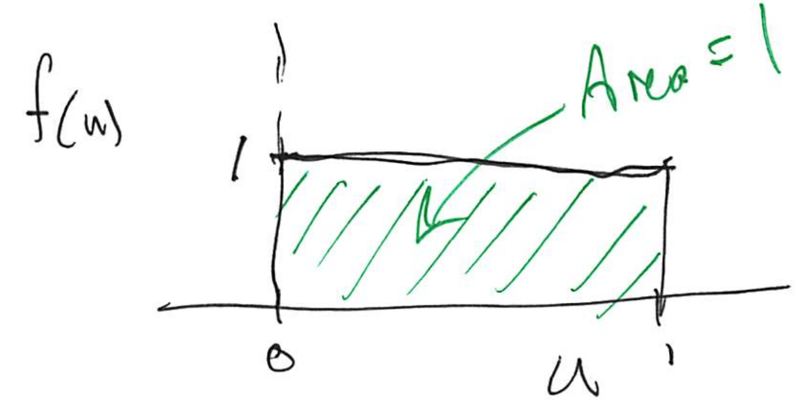
$$\left(= \sum_{k=a}^{b} p(k) \right)$$



$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx \qquad E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_{-\infty}^{\infty} u^2 f_X(u)\,du$$

$$\sigma^2 = Var(X) = E(X-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = E(X^2) - (EX)^2$$

Ex. $X \sim U(0,1)$ Uniform on $(0,1)$



$$\int_{-\infty}^{\infty} f(u)\,du = F(\infty) - F(-\infty) = 1 - 0 = 1$$

$f(u) \geq 0$

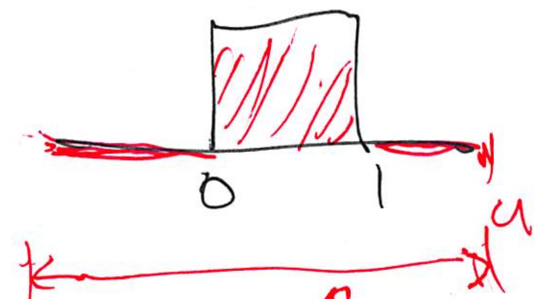
$$P\left(\frac{1}{4} < X < \frac{3}{4}\right) = \int_{1/4}^{3/4} 1\,du = \frac{1}{2}$$

$$E(X) = \int_{-\infty}^{\infty} u\,f(u)\,du = \int_0^1 u \cdot 1\,du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2}$$

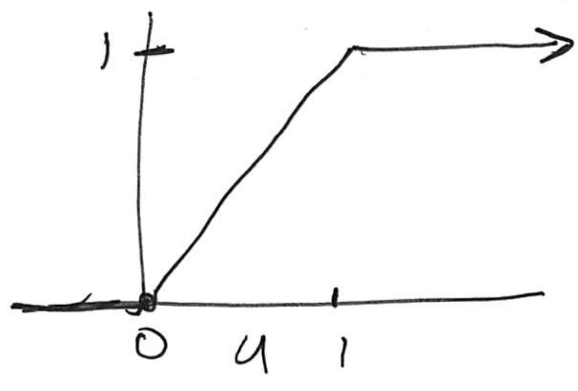
$$E(X^2) = \int_{-\infty}^{\infty} u^2 f(u)\,du = \int_0^1 u^2 \cdot 1\,du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$Var(X) = E(X^2) - (EX)^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12}$$

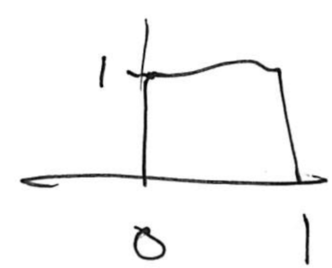
$$F_X(u) = P(X \le u) = \int_{-\infty}^{u} f(v)\,dv$$



$$= \begin{cases} \int_{-\infty}^{u} f(v)\,dv = 0 & u < 0 \\ \int_{-\infty}^{u} f(v)\,dv = u & 0 \le u < 1 \\ \int_{\infty}^{u} f(v)\,dv = 1 & u \ge 1 \end{cases}$$



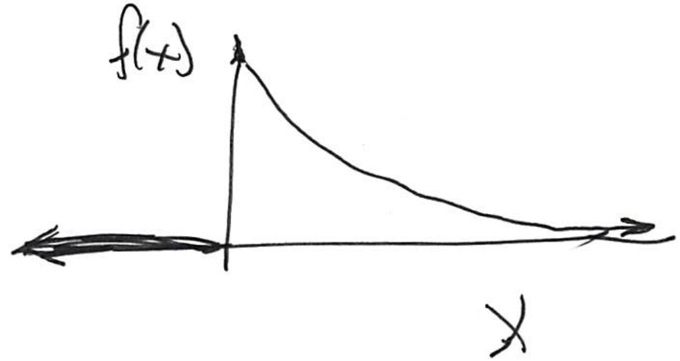
$$\Rightarrow \frac{d(\ )}{du} \Rightarrow$$



Ex. Exponential Distribution

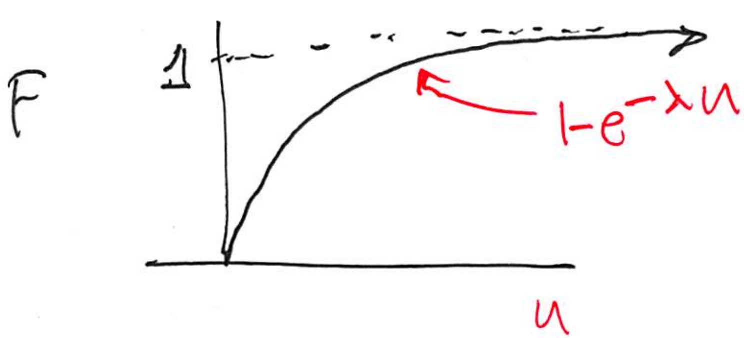
$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases} \qquad \lambda > 0$$

Analagous to geometric



$$F(u) = \int_{-\infty}^{u} f(v)\,dv$$

$$= \begin{cases} 0 & u < 0 \\ \int_0^u x e^{-\lambda v}\,dv = -e^{-\lambda v}\Big|_0^u = 1 - e^{-\lambda u} \end{cases}$$



$$P(a \leq X \leq b) = F(b) - F(a)$$

$a \geq 0$

$$= 1 - e^{-\lambda b} - (1 - e^{-\lambda a})$$

$$= e^{-\lambda a} - e^{-\lambda b}$$

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx = \int_0^\infty \underbrace{u\lambda}_{y} e^{-\overbrace{\lambda u}^{y}}\, du$$

let $y = u\lambda$

$dy = du\, \lambda$

$\Rightarrow du = \frac{1}{\lambda}\, dy$

$$= \int_0^\infty \frac{1}{\lambda} y e^{-y}\, dy$$

$$= \frac{1}{\lambda} \int_0^\infty y e^{-y}\, dy = 1$$

$$= \frac{1}{\lambda}$$

$$Var(X) = \frac{1}{\lambda^2}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \int_0^\infty \lambda x^2 e^{-\lambda x}\, dx$$

$$= \frac{1}{\lambda^2}$$