EXAM #2
ELEG 305 SIGNALS AND SYSTEMS
SPRING 2019

Name: __________________________ *(Family Name, Given Name)*
Major:__________________________

Read each problem carefully before you start.

There are five, equally-weighted, problems. There are also two Extra Credit questions worth a maximum of ten points total.

Closed book.
No calculators, no cellphones.
An extra blank page is included after Problem #5 in case you need more space.

Sheets with the sum of a geometric series, the defining equations, and the properties of the Fourier transform are attached after the grading rubric page. Feel free to detach those pages and discard them after the exam. (Please press down on the staple if you do detach the pages.) Some other useful equations:

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$
$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

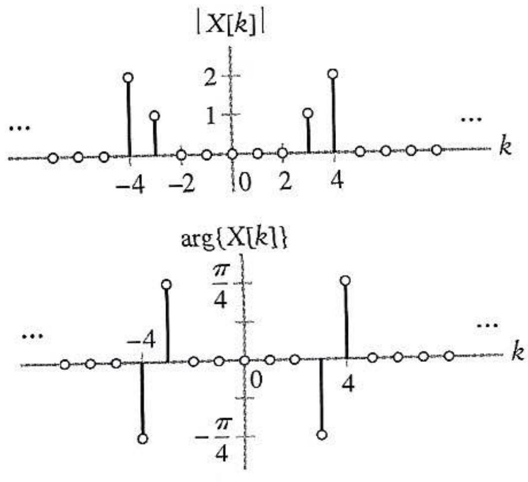
$$e^{-at}u(t) \leftrightarrow \frac{1}{j\omega + a}$$

$$x(t) = \begin{cases} 1, & |t| \le T_1 \\ 0, & elsewhere \end{cases} \leftrightarrow 2T_1 \frac{\sin\omega T_1}{\omega T_1}$$

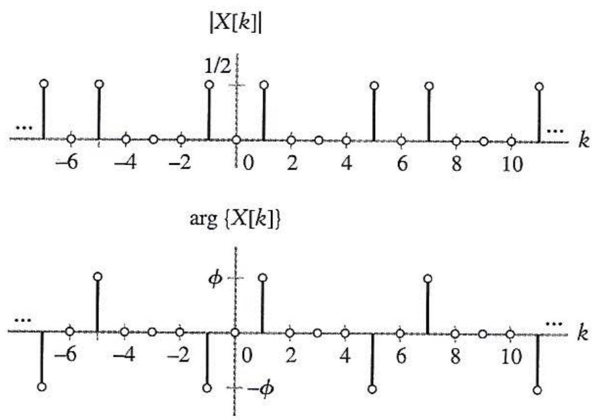
Problem #1 (20 points) (Questions are on the next page)

The following three spectra (I, II, and III) are the Fourier series coefficients, a_k, for *three different periodic time-domain signals*. The coefficients in (I) and (II) are complex, and the magnitude of a_k is represented as $|X(k)|$ and the phase as $\arg\{X(k)\}$. For (III), the coefficients are real, and simply labeled as $X(k)$.

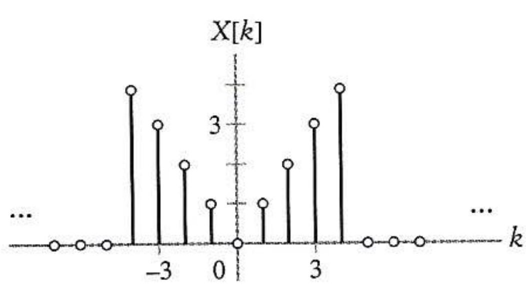
(I) $\omega_0 = \pi$



(II) $\omega_0 = \pi/3$



(III) $\omega_0 = \pi$



Problem #1 (Cont'd)

Please answer the following questions:

a.) (2 pts) Which spectrum could correspond to a discrete-time signal? Please explain.

b.) (2 pts) Which spectrum corresponds to a time-domain signal that is real and even? Please explain.

c.) (8 pts) What frequencies (in radians per second) are contained in each of the time-domain signals corresponding to each of the spectra in (I), (II), and (III)?

d.) (8 pts) Assume the time-domain signal corresponding to spectrum (III) is passed through an ideal lowpass filter that will not pass any frequency $|\omega| > 2\omega_0$. What is the resulting filtered time-domain signal?

Problem #2 (20 points) (Part b) is on the next page.)

a.) Determine the Fourier series coefficients for the following periodic, time-domain signals.

(i) (4 pts) $x(t) = \cos 6\pi t$

(ii) (8 pts) $x[n] = \sin\frac{\pi}{4}n$. Also, for this signal, plot the magnitude and phase of the coefficients as a function of k (the frequency index).

Problem #2 (cont'd)

b.) (8 pts) Determine the continuous-time, periodic signal that has the following Fourier series coefficients (assume $\omega_0 = \pi$):

$$a_k = \begin{cases} 2, & k = -3 \\ j, & k = -2 \\ -j, & k = 2 \\ 2, & k = 3 \\ 0, & otherwise \end{cases}$$

Problem #3 (20 points) (Part c) is on the next page.)

a.) (6 pts) Consider a continuous-time signal ($|\alpha| < 1$)

$$x(t) = \sum_{m=0}^{\infty} \alpha^m \delta(t - m)$$

Determine the frequency characteristic, $X(j\omega)$, for this signal.

b.) (6 pts) Consider a continuous-time signal given by

$$x(t) = \begin{cases} |t|, & -2 \le t \le 2 \\ 0, & otherwise \end{cases}$$

Compute the following functions of $X(j\omega)$, without explicitly computing the Fourier transform of $x(t)$.

(i) $\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega$

(ii) $\int_{-\infty}^{\infty} X(j\omega) e^{-j\omega} \, d\omega$

Problem #3 (cont'd)

c.) (8 pts) Compute the inverse Fourier transform of

$$X(j\omega) = j\frac{d}{d\omega}\left[\frac{e^{j2\omega}}{1 + j(\frac{\omega}{3})}\right]$$

Problem #4 (20 points)

Consider the following differential equation that describes a linear, time-invariant system with input $x(t)$ and output $y(t)$:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = \frac{dx(t)}{dt} + 3x(t)$$

a.) (10 pts) Derive the frequency response of this system, $H(j\omega)$.

b.) (10 pts) What is the corresponding impulse response, $h(t)$?

Problem #5 (20 points)

(NOTE: Very similar to Problem #5 from Sample Exam #2B)

Assume that we can model the nervous system of humans as a continuous-time, linear, time-invariant system. It is known from previous tests that if the system is excited by an impulse *(i.e., if the system input is an impulse),* the measured output is given by

$$h(t) = \frac{1}{2}e^{-6t}u(t) + \frac{1}{2}e^{-4t}u(t)$$

Today, the measured output signal is

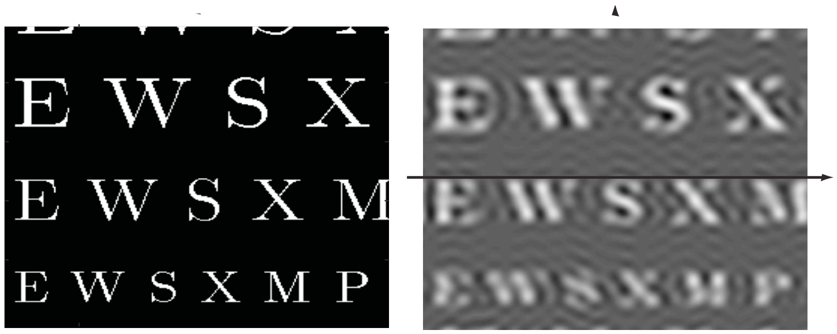
$$y(t) = \frac{1}{3}e^{-t}u(t) - \frac{1}{3}e^{-4t}u(t)$$

a.) (5 pts) What is the frequency response of the system?

b.) (15 pts) Determine the unknown input $x(t)$ that generated today's measured output?

Extra Credit #1 (5 points)

The image on the left is an "eye chart" used by doctors to crudely measure a person's vision.



Suppose, instead of this image, what you see with your eyes is the image on the right.
Is your visual system lowpass filtering or highpass filtering the original image? Please explain your answer.

Extra Credit #2 (5 points)

One major problem in real instrumentation systems is the electromagnetic interference caused by the 60-Hz power lines. A system with impulse response $h(t) = A[u(t) - u(t-t_0)]$ can reject 60 Hz and all its harmonics (i.e., 120 Hz, 180 Hz, …). Find the numerical value of t_0 that makes this happen.

Problem #1 (out of 20)	
Problem #2 (out of 20)	
Problem #3 (out of 20)	
Problem #4 (out of 20)	
Problem #5 (out of 20)	
Extra Credit #1 (out of 5)	
Extra Credit #2 (out of 5)	
TOTAL (out of 100)	

Geometric series

$$\sum_{n=0}^{M-1} \beta^n = \begin{cases} \dfrac{1-\beta^M}{1-\beta}, & \beta \neq 1 \\ M, & \beta = 1 \end{cases}$$

Continuous-time Fourier series

$$\boxed{\begin{aligned} x(t) &= \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}, \\ a_k &= \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}\,dt. \end{aligned}}$$

Discrete-time Fourier series

$$\boxed{\begin{aligned} x[n] &= \sum_{k=\langle N\rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}, \\ a_k &= \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\omega_0 n} = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk(2\pi/N)n}. \end{aligned}}$$

Continuous-time Fourier transform

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}\,d\omega}$$

$$\boxed{X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt.}$$

Properties of Continuous-Time Fourier Transform

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$	$X(j\omega)$
		$y(t)$	$Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re e\{X(j\omega)\} = \Re e\{X(-j\omega)\} \\ \Im m\{X(j\omega)\} = -\Im m\{X(-j\omega)\} \\ \lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\Re e\{X(j\omega)\}$ $j\Im m\{X(j\omega)\}$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$