Chapter 4 Electromagnetism: Electricity and Magnetism.

— Electricity generates a magnetic field.

— Magnetic field exerts a force on an electric current

Electricity $\xrightarrow{\text{in a field}}$ motion.

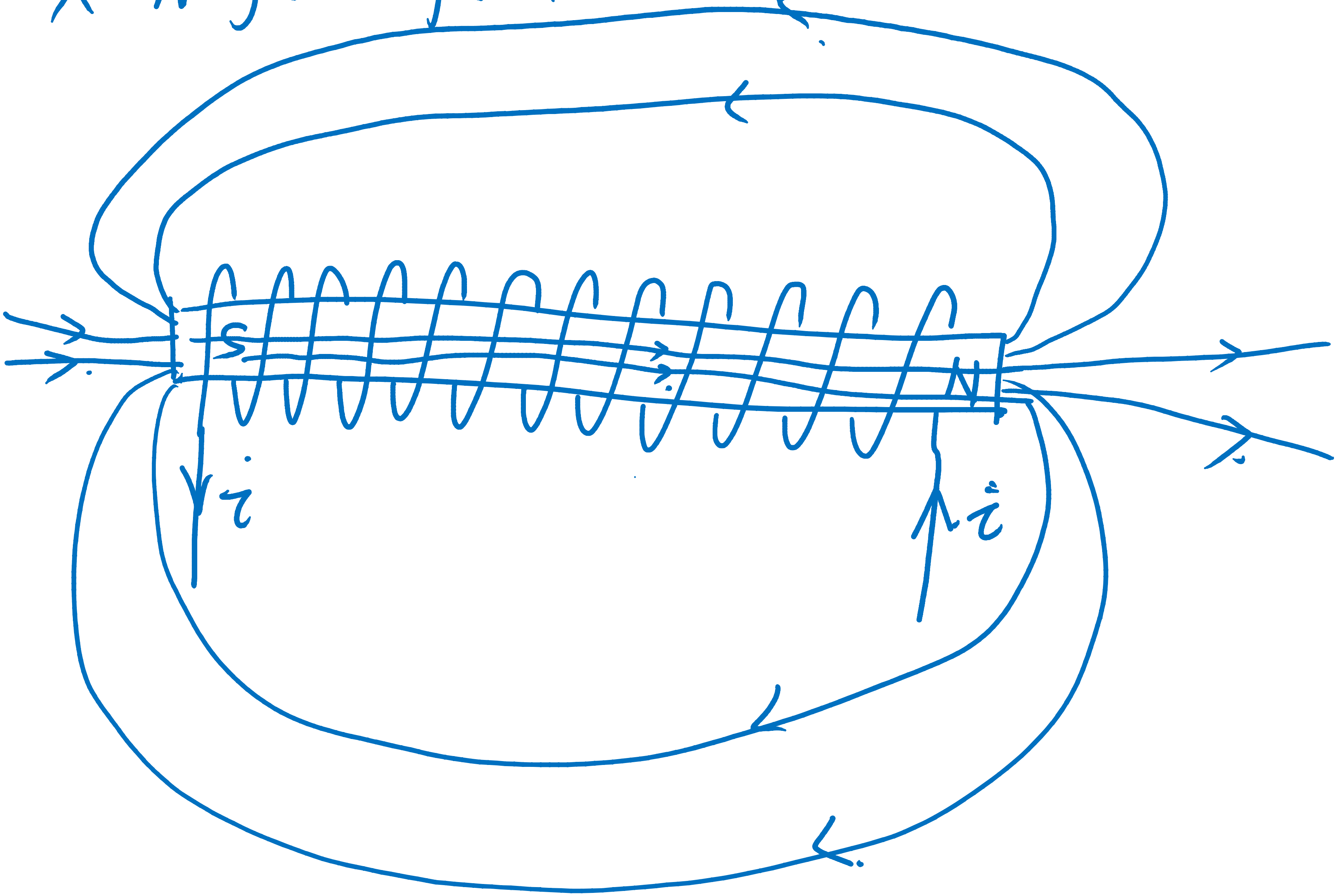
(how motor works).

— changing magnetic flux generates electricity

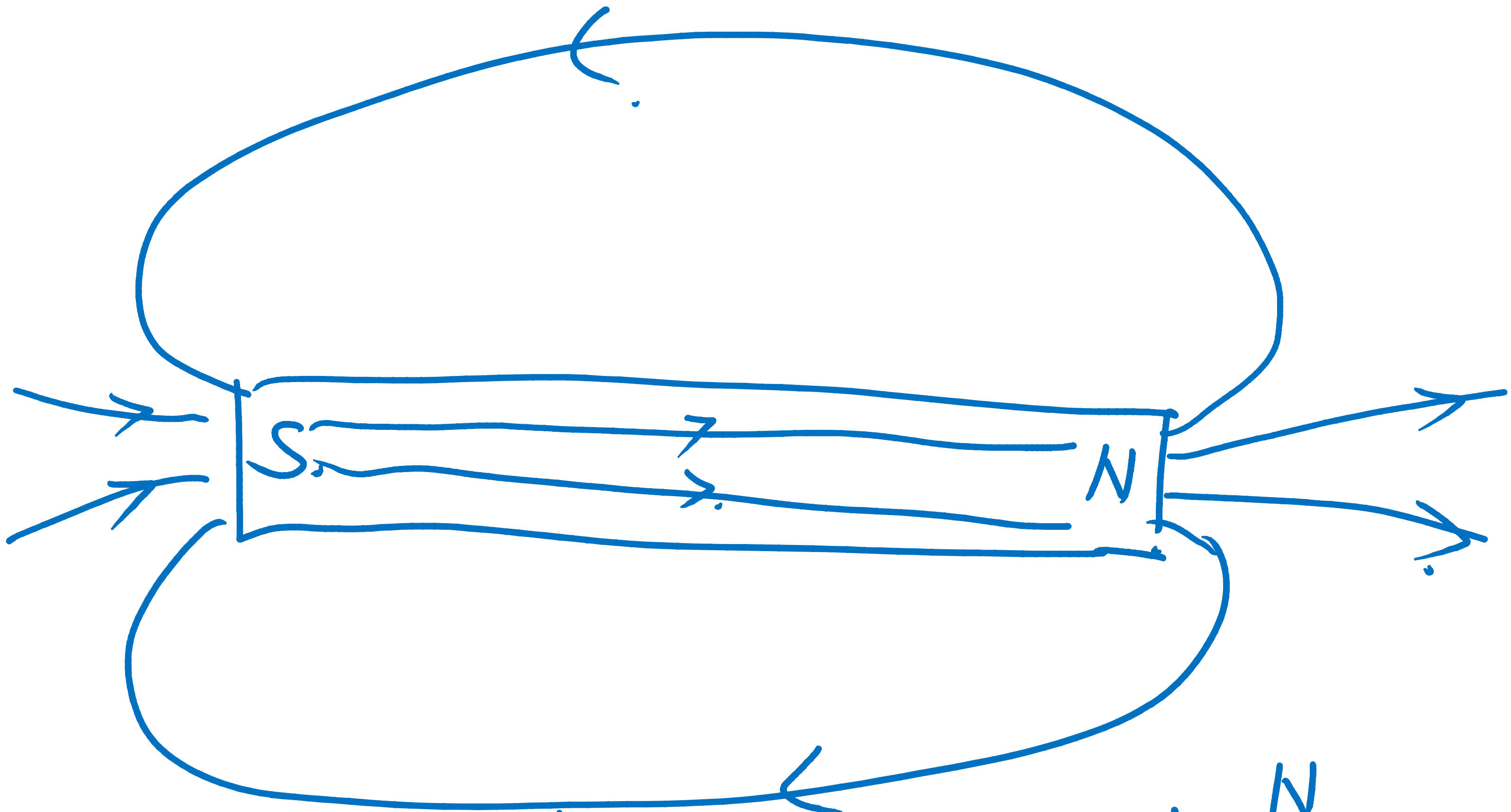
motion $\xrightarrow{\text{in a field}}$ electricity

(how generator works)

* Magnetic field.

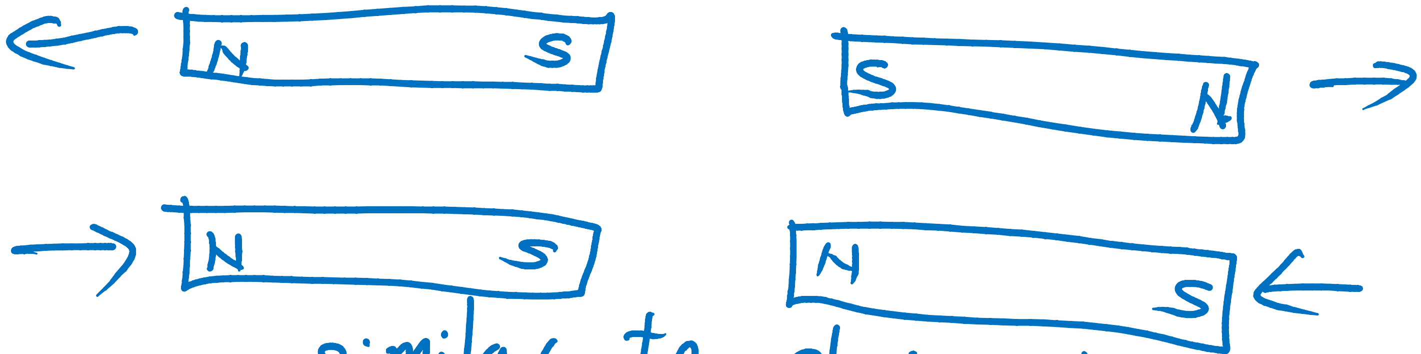


bar magnet (permanet).



B. unit. : Tesla (T) $1\,T = 1\,\frac{N}{A \cdot m}$

- like poles repel
- unlike poles attract.



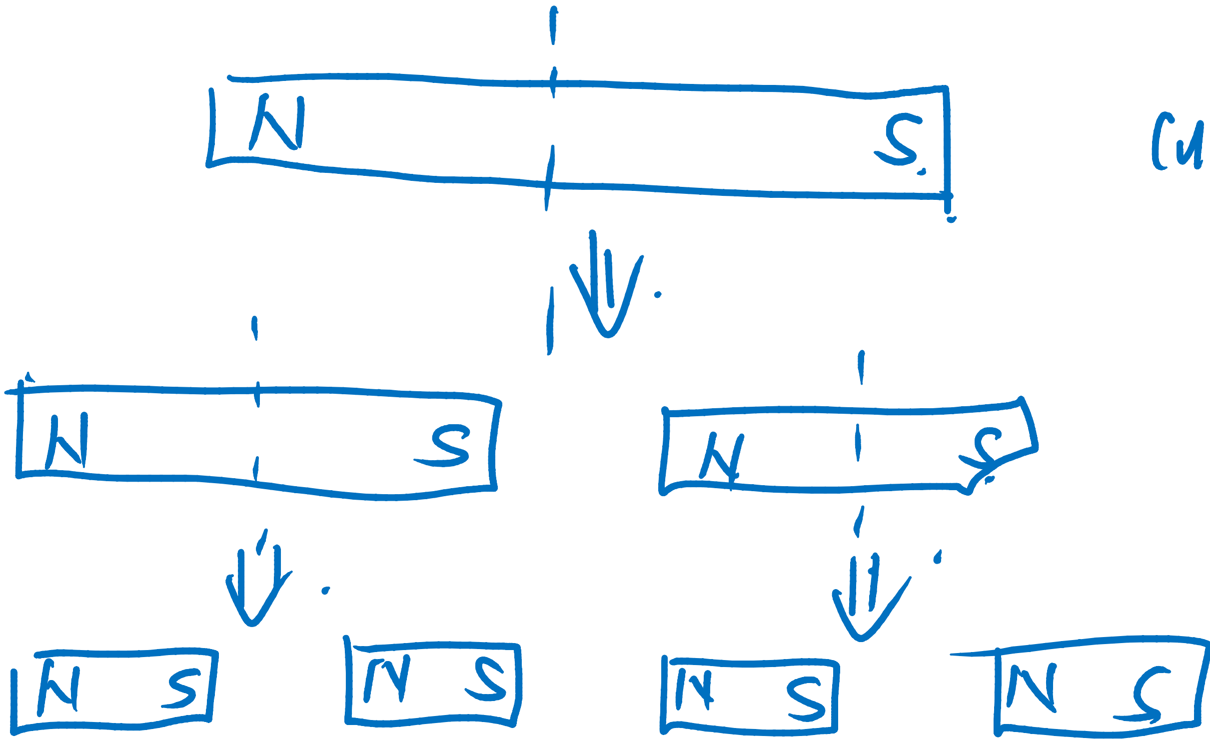
repel.

attract.

similar to electric charges.

where. like charges repel
unlike charges attract.

Difference: no magnetic monopoles
(isolated single N or S pole)

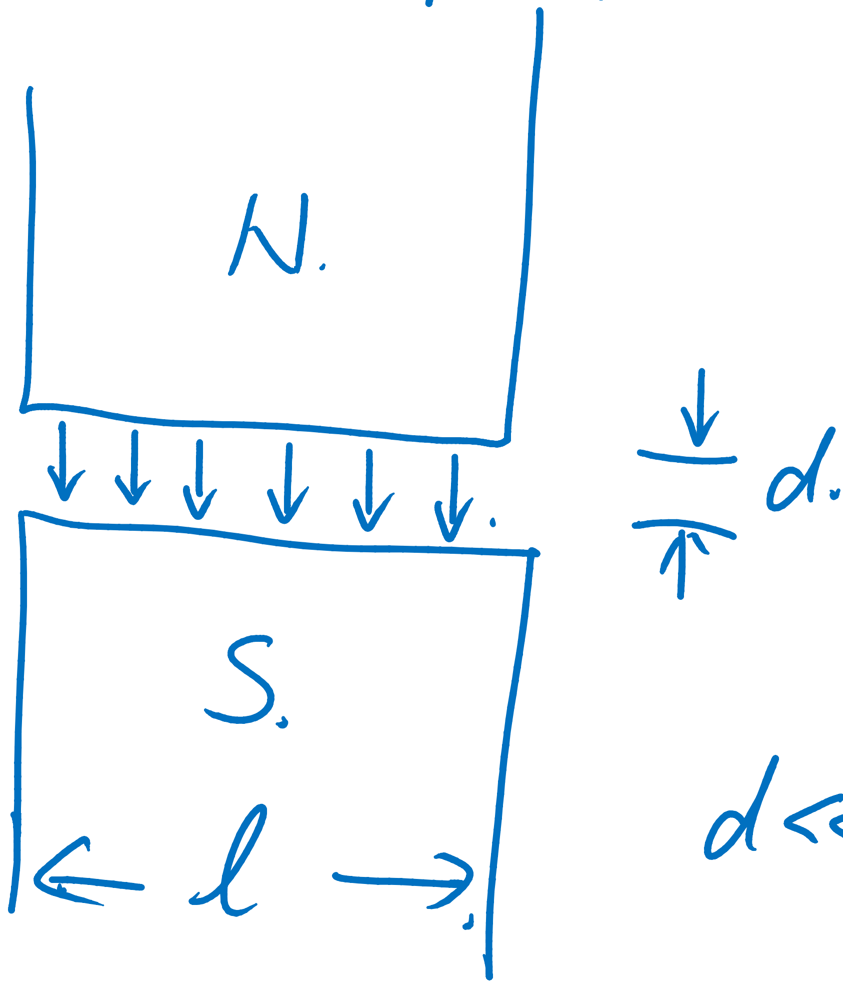


cut in half.

down. to atomic level.

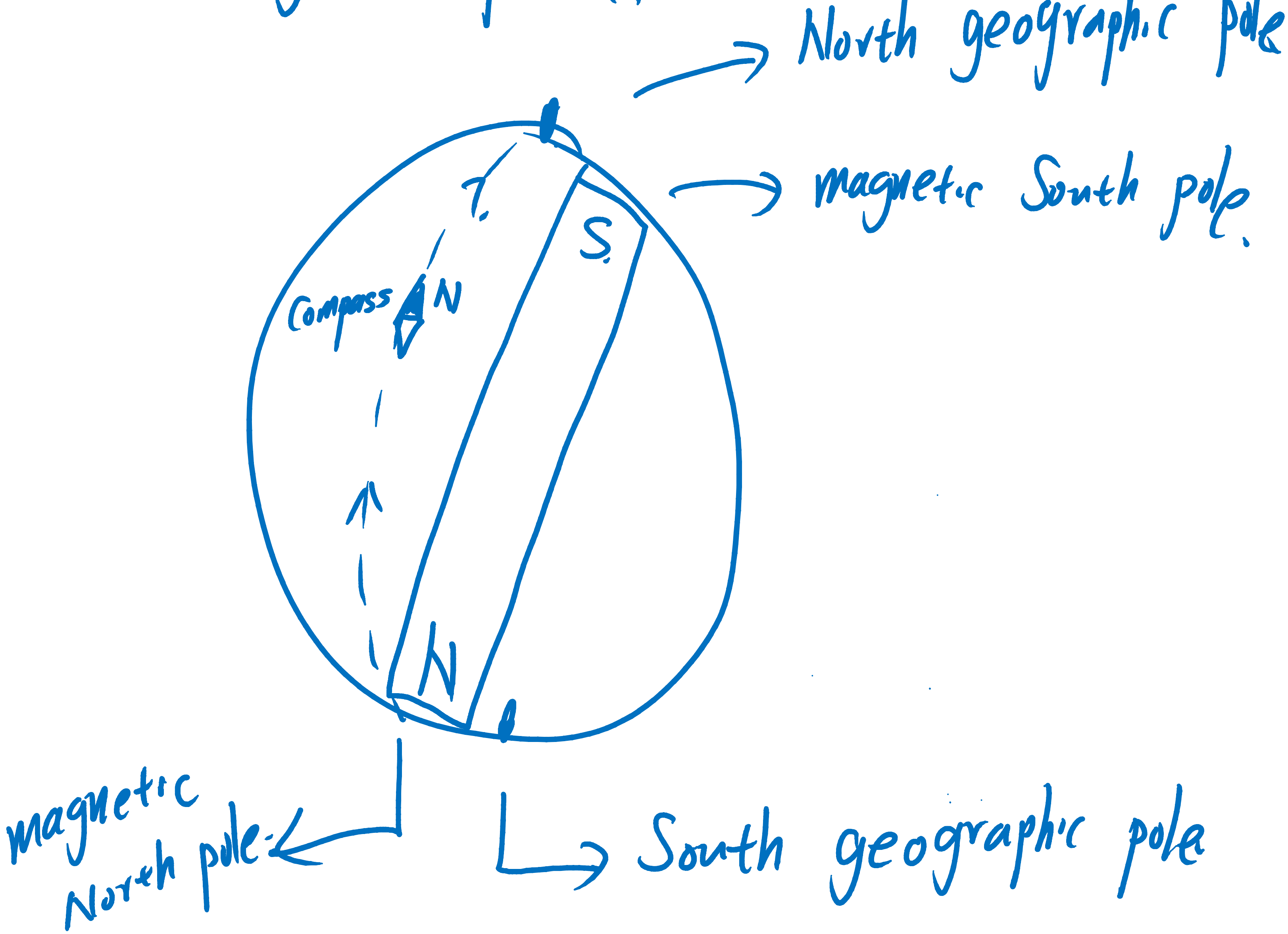
electron atomic orbital behaves like a single turn solonoid.
There is still a N pole and a S pole.

– How to generate a uniform field.



$d \ll l$.

— Earth magnetic field.

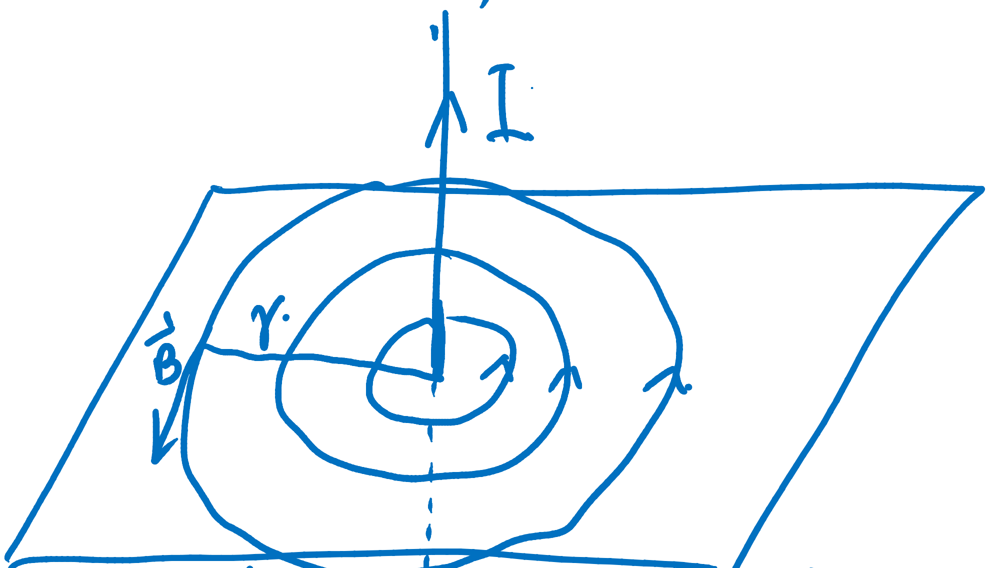


* Electric current produces magnetic field.

Straight wire carrying a current.

(wire infinitely long)

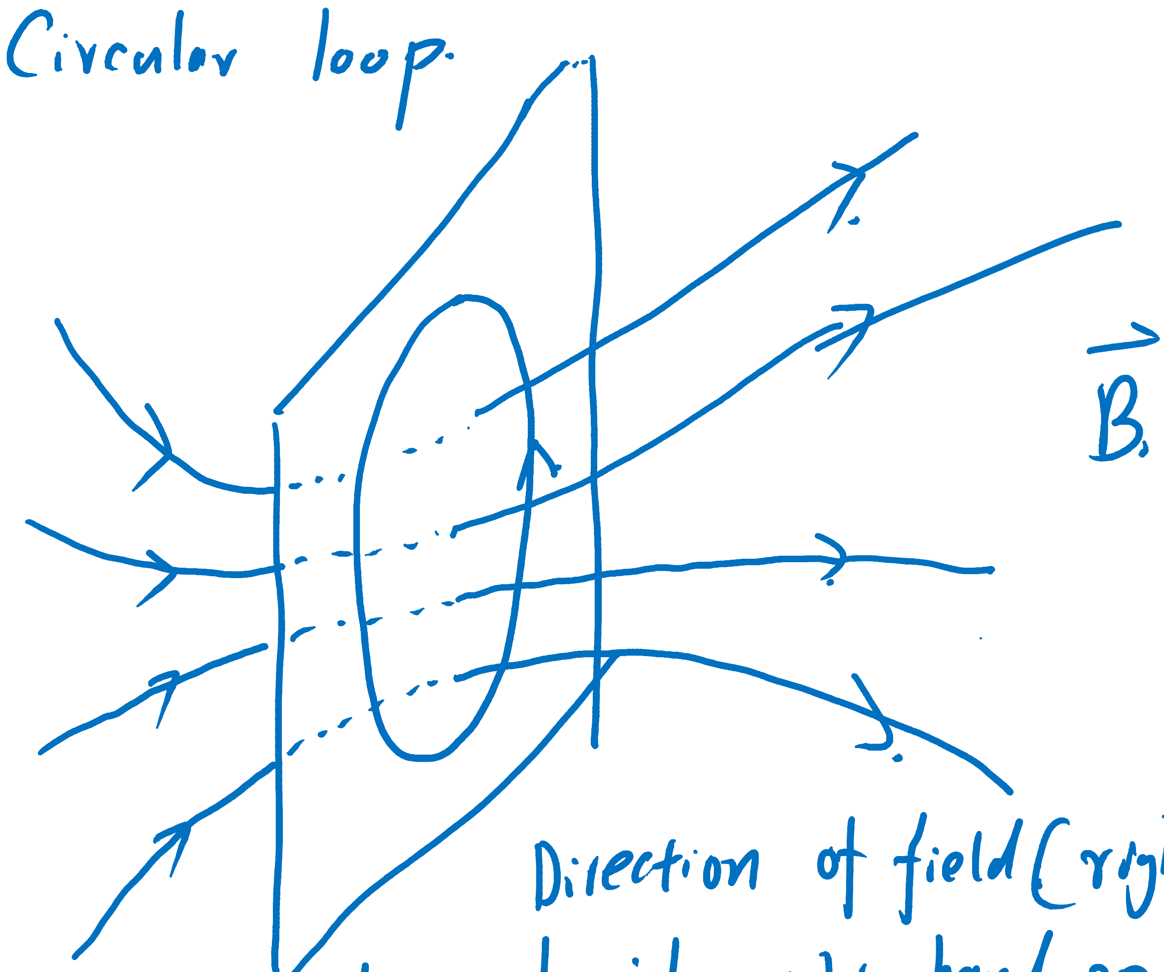
$$B = \frac{\mu_0 I}{2\pi r}$$



$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$$

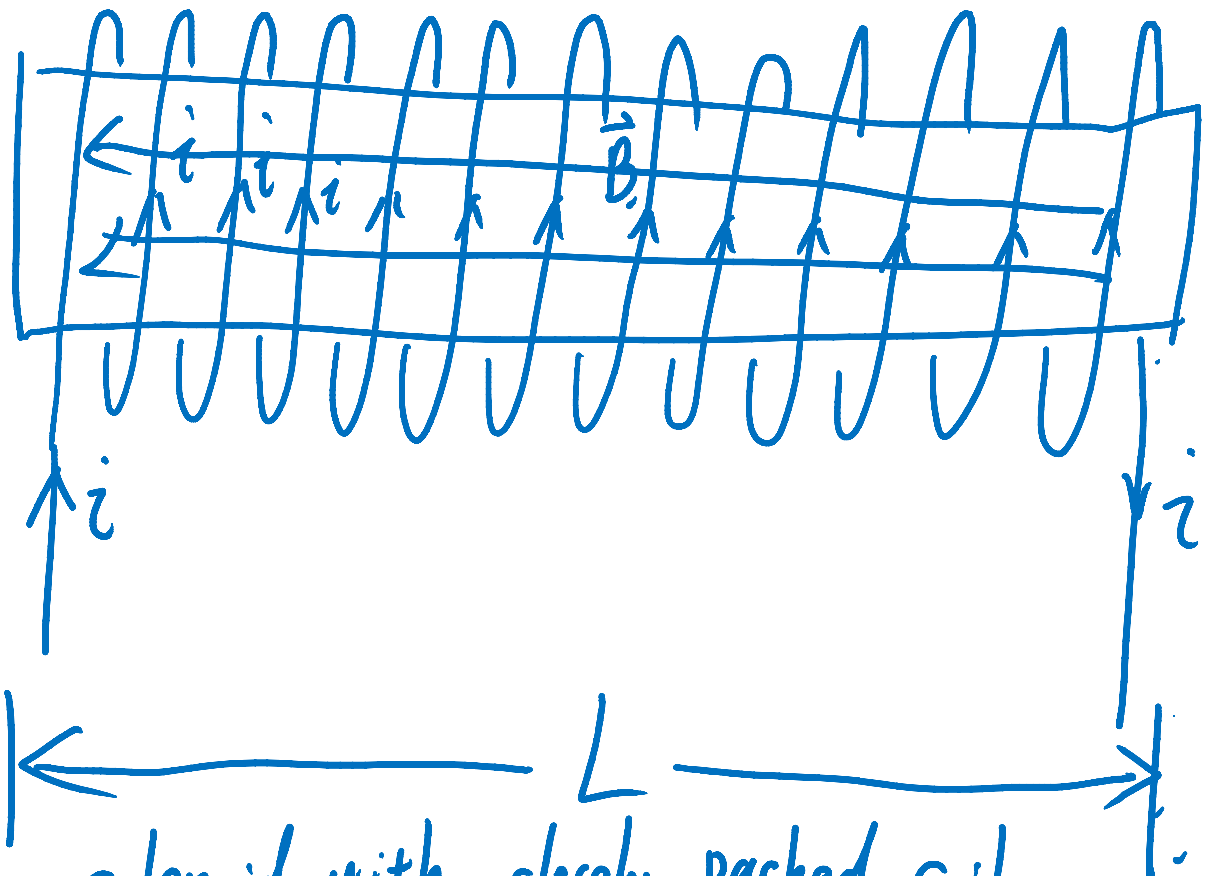
Direction of field (right hand rule)

Grasp the wire with your right hand so that your thumb points in the direction of current. Then your fingers will encircle the wire in the direction of magnetic field.



Direction of field (right hand rule)

Grasp the coil with right hand so that your fingers encircle the coil in the direction of current. Then your thumb will point in the direction of field.

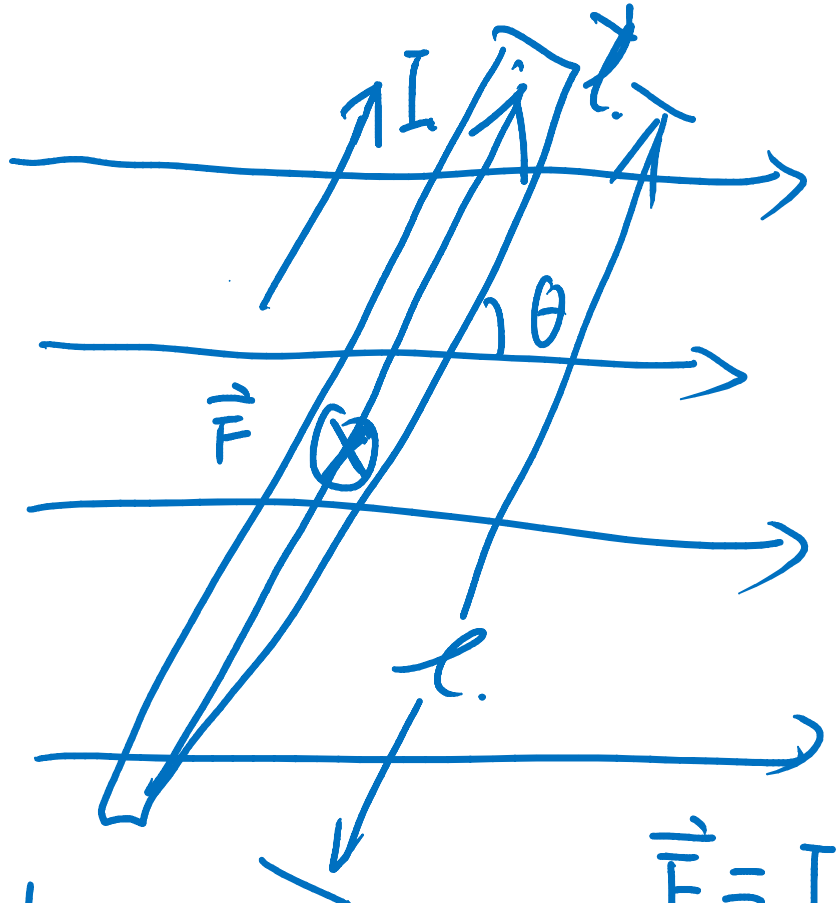


Long solenoid with closely packed coils.

Inside solenoid, uniform $\vec{B}$.

$B = \mu_0 i n$, $n = \dfrac{N}{L}$ $\begin{array}{l} \rightarrow \text{number of turns} \\ \rightarrow \text{length.} \end{array}$

* Force on electric current in a magnetic field.



$\vec{B}$

uniform field.

$\vec{F} = I \vec{l} \times \vec{B}$.

↓

vector: magnitude is length of wire. Direction is along the wire in the direction of current

$\vec{F}$ is perpendicular to both $\vec{l}$ and $\vec{B}$.

$\vec{F}$ is pointing into board.

$$|\vec{l} \times \vec{B}| = lB\sin\theta$$

$$F = IlB\sin\theta$$

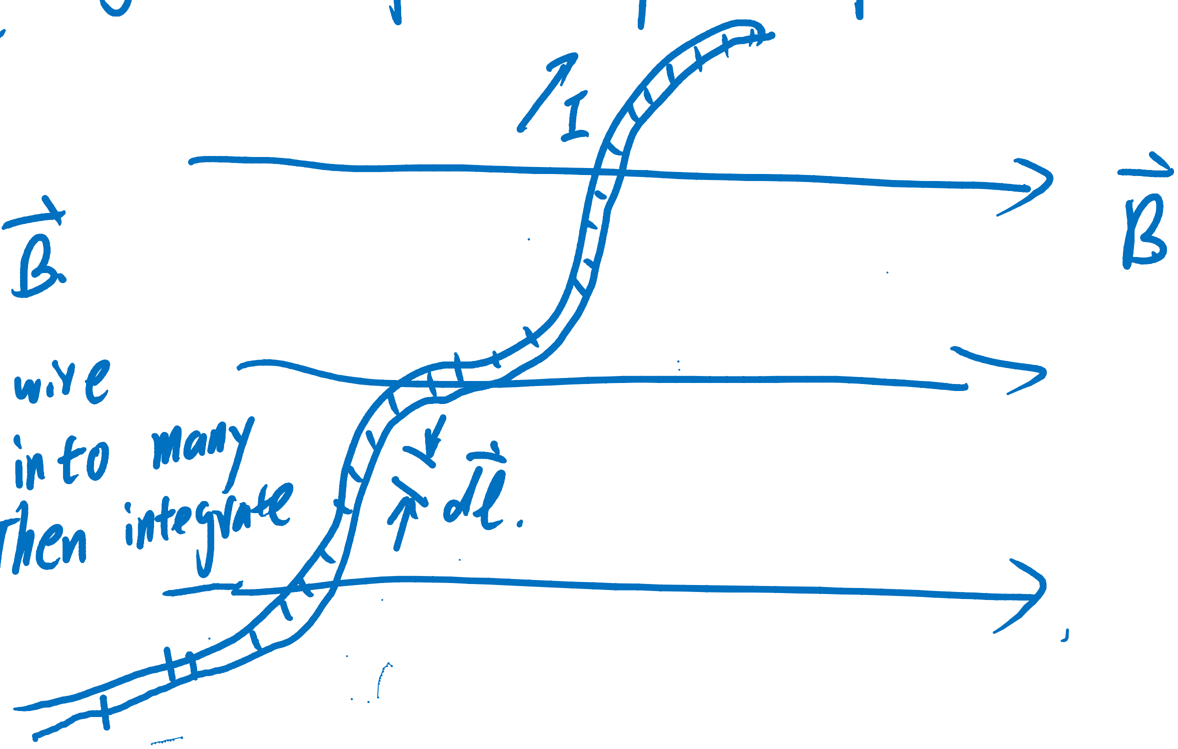
when $\theta = 0^\circ$ $\quad F = 0$

$\theta = 90^\circ$ $\quad F = F_{max} = IlB$

A more general form of the force.

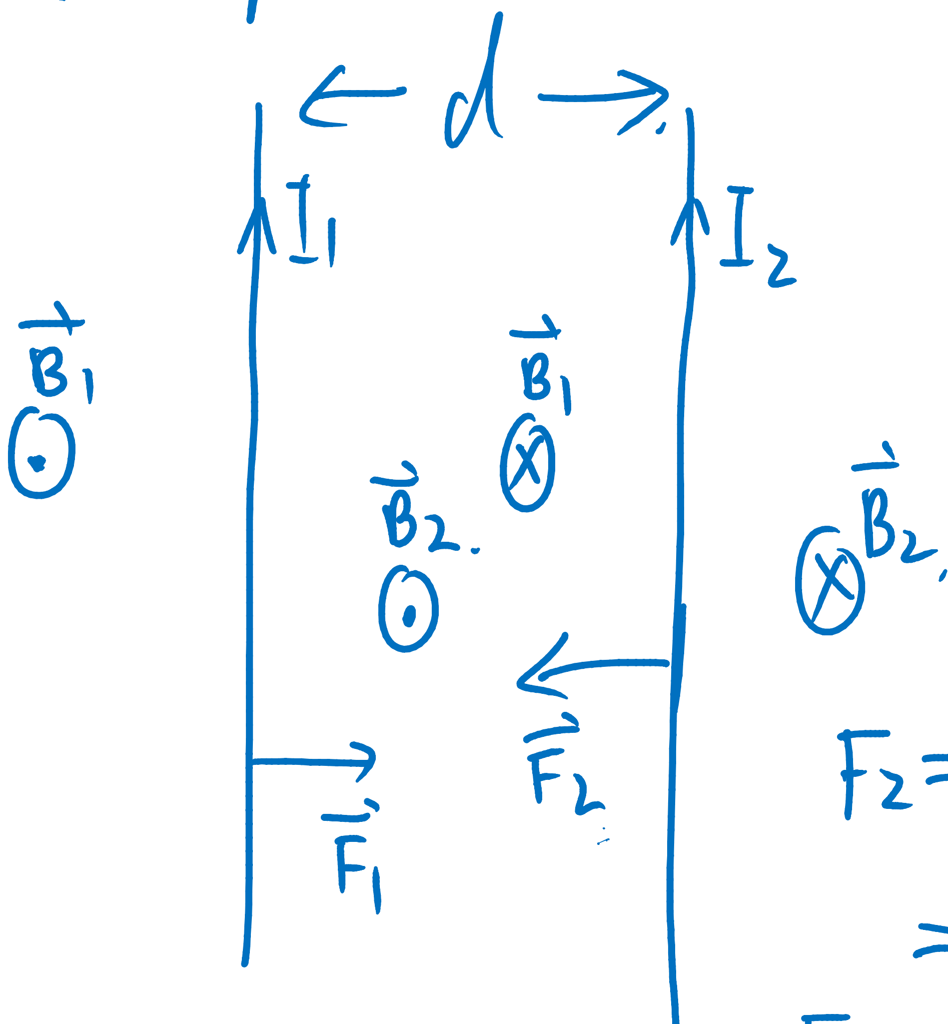
$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

$$\vec{F} = I \int_{\text{along the wire}} d\vec{l} \times \vec{B}$$



partition a wire into many segments.. $d\vec{l}$. Then integrate the forces.

Example. Forces between two parallel wires



$$\vec{F}_2 = I_2 \vec{l}_2 \times \vec{B}_1$$

$$\vec{F}_1 = I_1 \vec{l}_1 \times \vec{B}_2$$

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

$$F_2 = I_2 l_2 B_1 \underbrace{\sin\theta}_{\to 1}$$

$$= I_2 l_2 B_1$$

$$F_2 = I_2 l_2 \left(\frac{\mu_0 I_1}{2\pi d}\right) = \frac{\mu_0 I_1 I_2 l_2}{2\pi d}$$

Force per unit length: $F_2/l_2 = \dfrac{\mu_0 I_1 I_2}{2\pi d}$

$$F_1 = I_1 \cancel{B} l_1 B_2 \underbrace{\sin\theta}_{\hookrightarrow 1}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi d}.$$

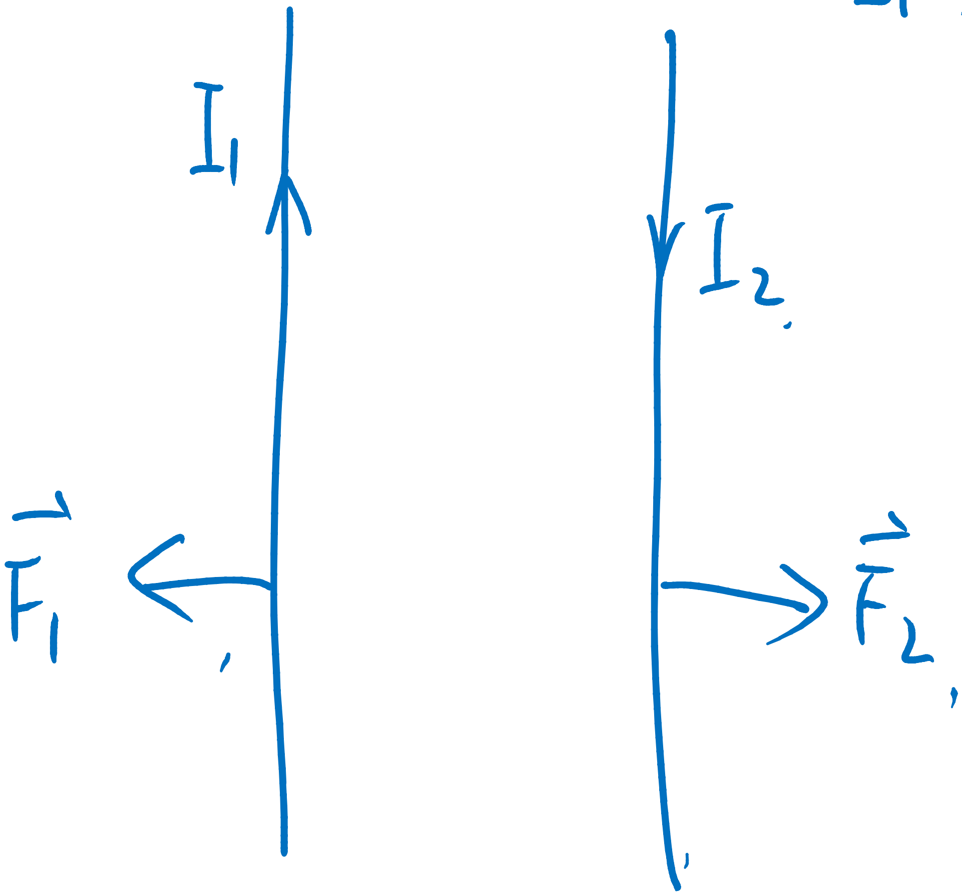
$$= I_1 l_1 B_2$$

$$= I_1 l_1 \left(\frac{\mu_0 I_2}{2\pi d} \right)$$

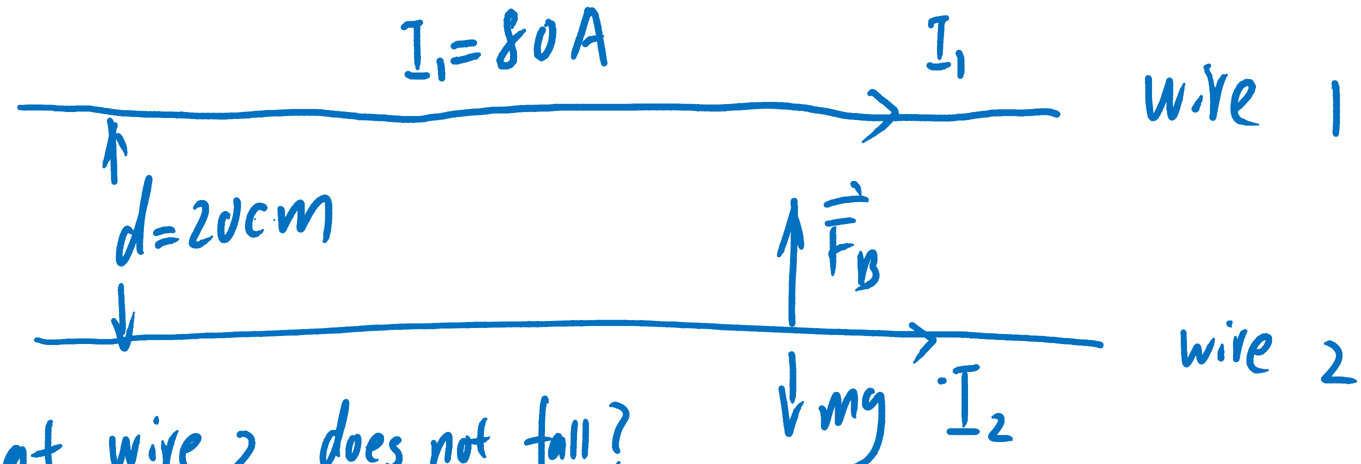
$$= \frac{\mu_0 I_1 I_2 l_1}{2\pi d}$$

$$\frac{F_1}{l_1} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{F_2}{l_2}$$

I_1, I_2 in opposite directions



Example



$I_2 = ?$ so that wire 2 does not fall?

wire 2: $\frac{m}{l_2} = 0.12\ g/m$

$$F_B = \frac{\mu_0 I_1 I_2 l_2}{2\pi d}$$

$$F_B = F_g = mg$$

$$\frac{\mu_0 I_1 I_2 l_2}{2\pi d} = mg$$

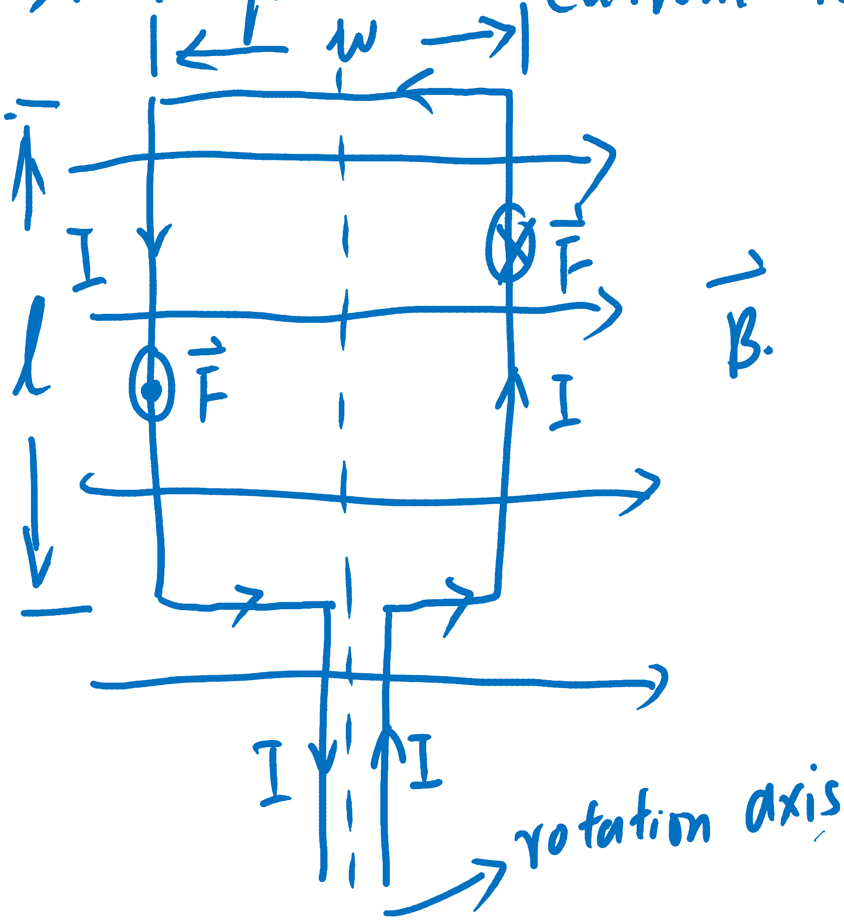
$$I_2 = \left(\frac{m}{l_2}\right) g \frac{2\pi d}{\mu_0 I_1}$$

$$= (0.12 \times 10^{-3})\, 9.8 \frac{2\pi \times 0.2}{(4\pi \times 10^{-7})(80)}$$

$$= 14.7\ (A)$$

* Torque on current loop : Motors

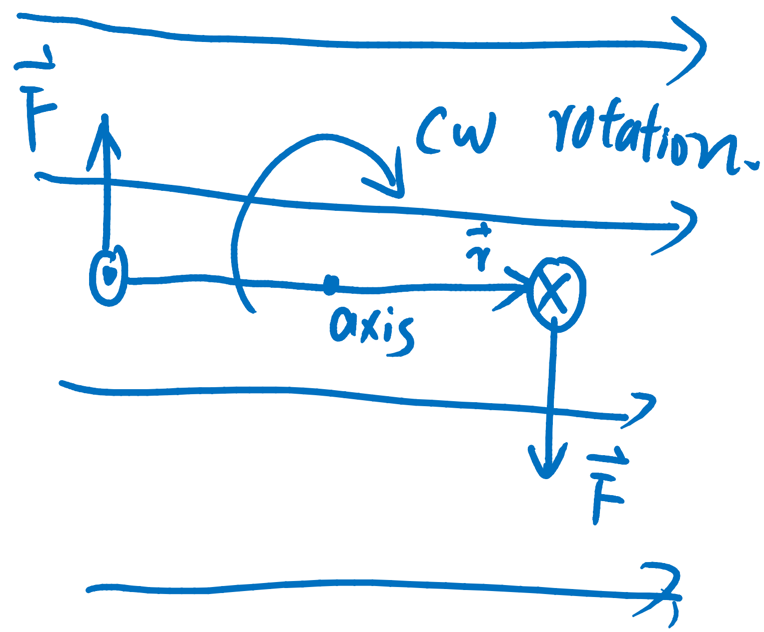
A: area of loop



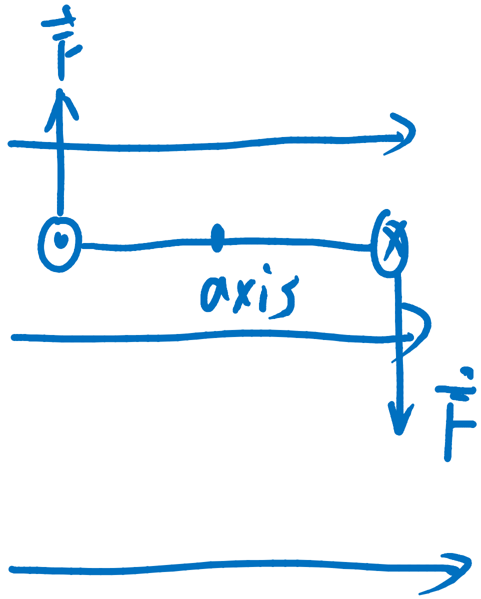
$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= 2rF$$
$$= 2\left(\frac{w}{2}\right)F = wF$$
$$= IB(lw) = IBA$$

$\vec{F} = I\vec{l} \times \vec{B}$!

bottom view

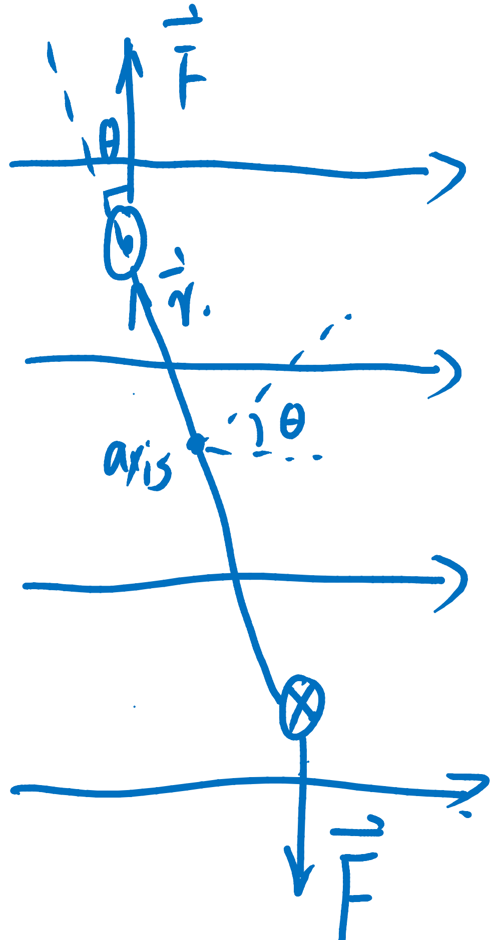


During the rotation of current loop



$\tau = IBA$

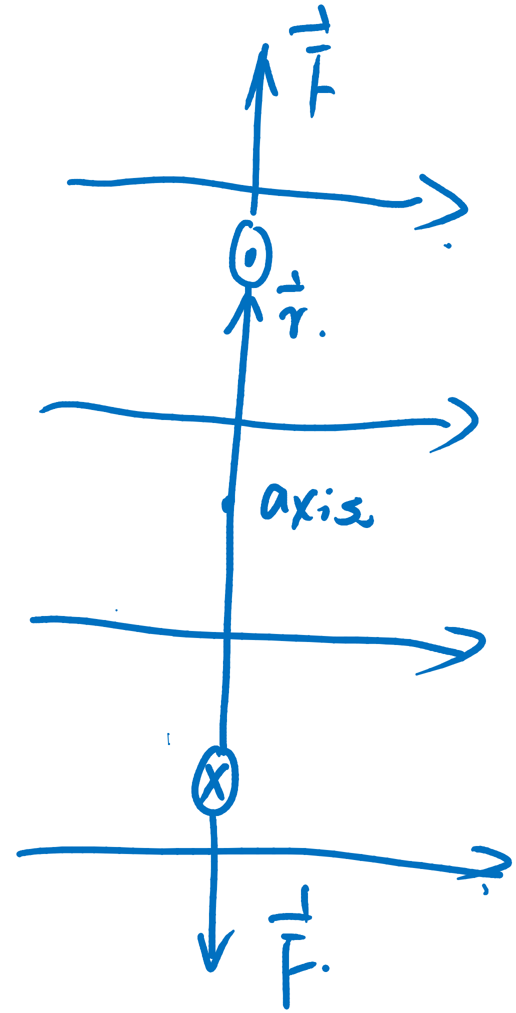
CW torque



$\tau = 2rF\sin\theta$

$= IBA\sin\theta$

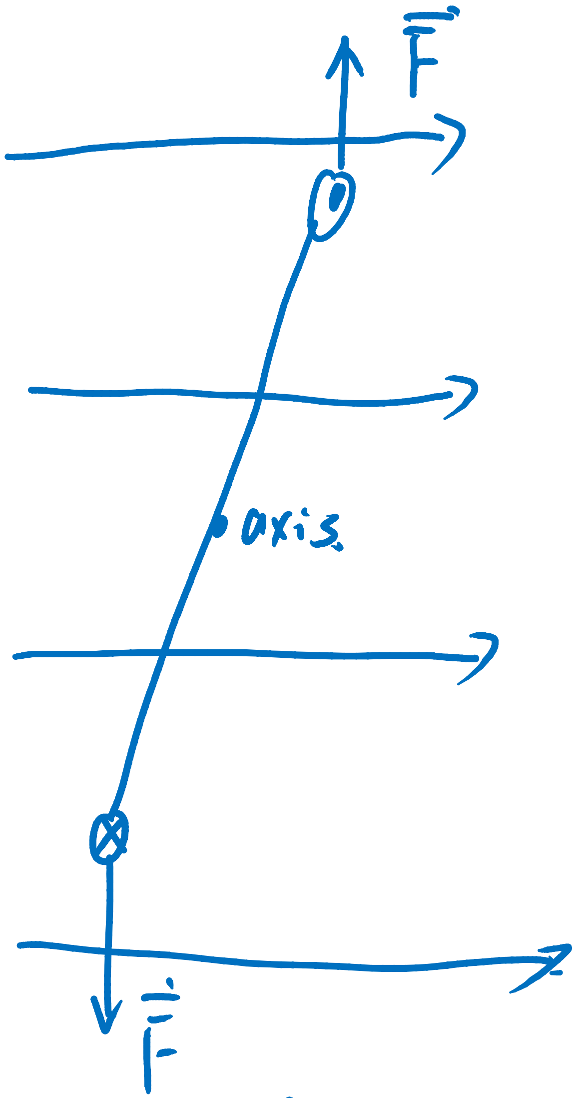
CW torque



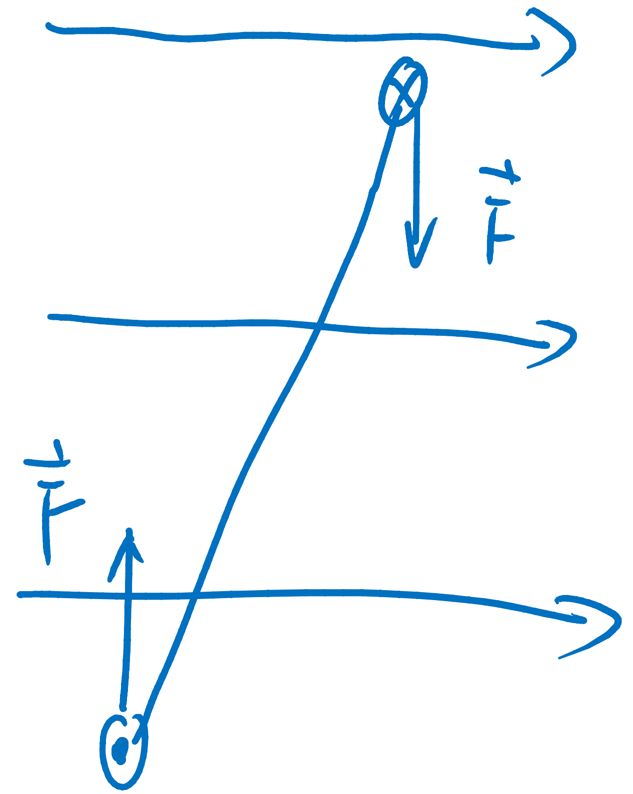
$\tau = 0$

but does not stop

(inertia)



CCW torque.
wrong direction

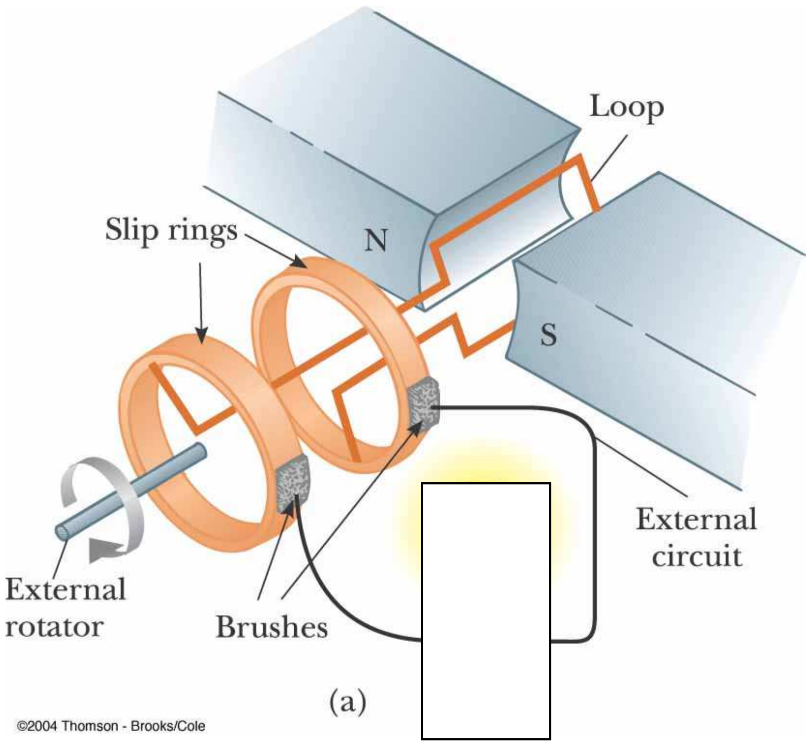


has to change the
polarity of current

a.c.

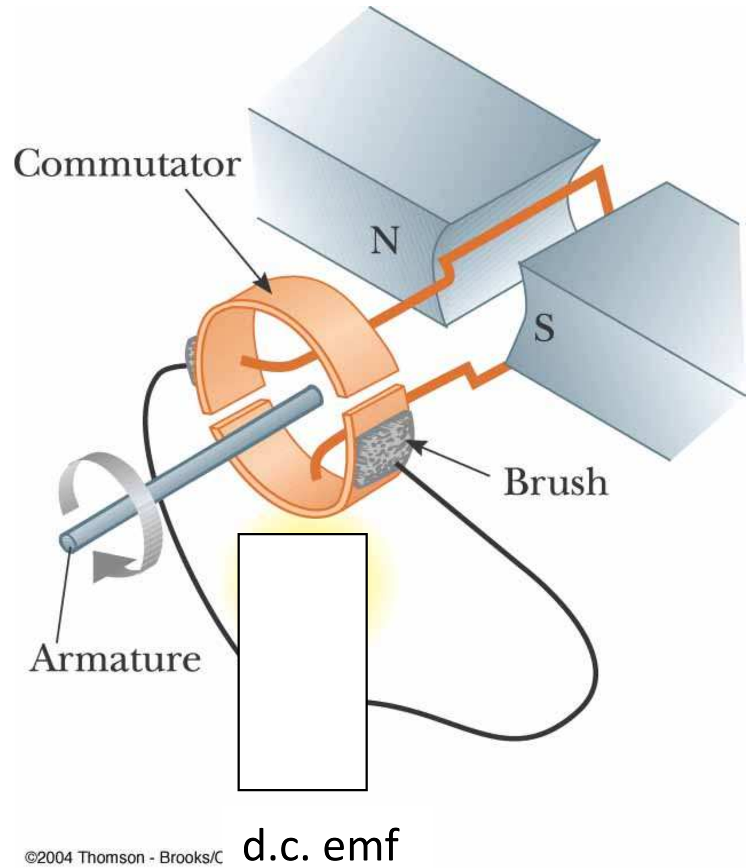
d.c.

Dual slip rings commutator



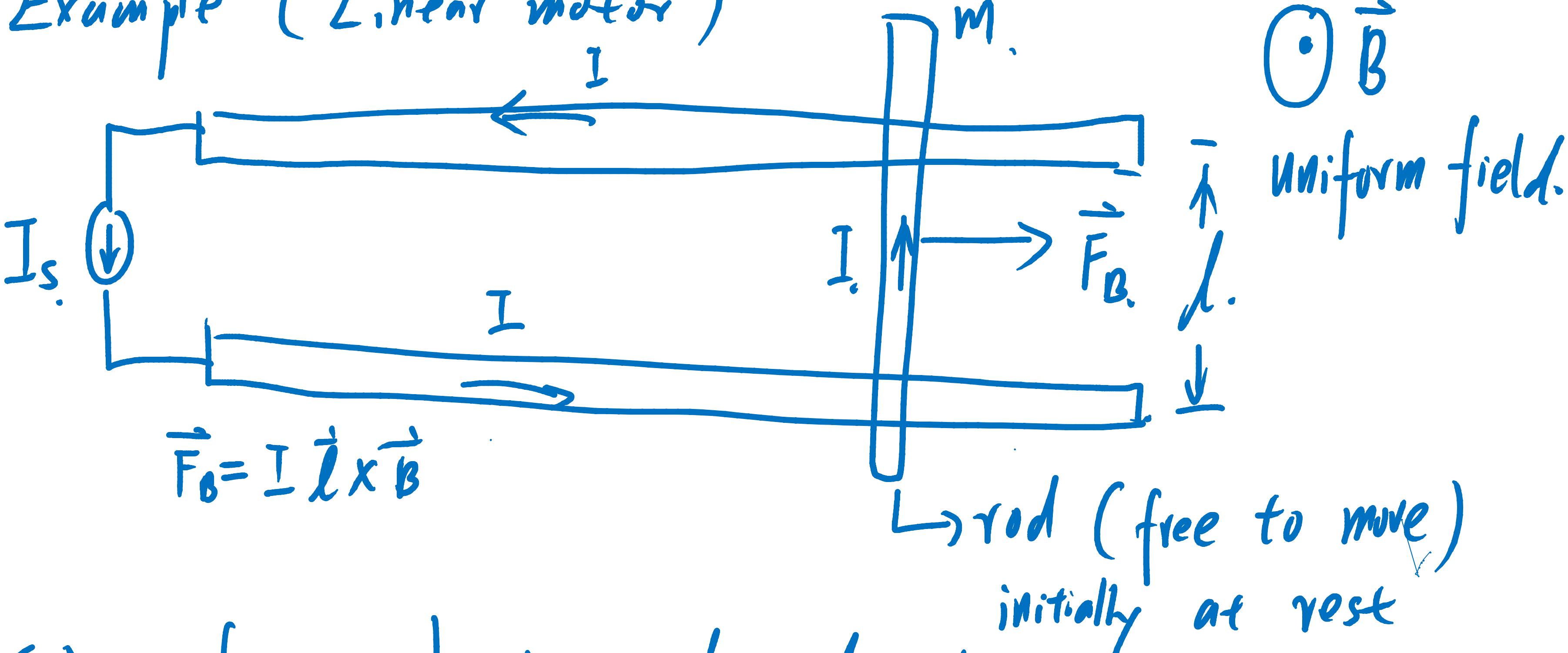
a.c. emf

Split ring commutator



d.c. emf

Example (Linear motor)



$\vec{F}_B = I\vec{l} \times \vec{B}$

(1) no friction between rod and rails. $v(t) = ?$

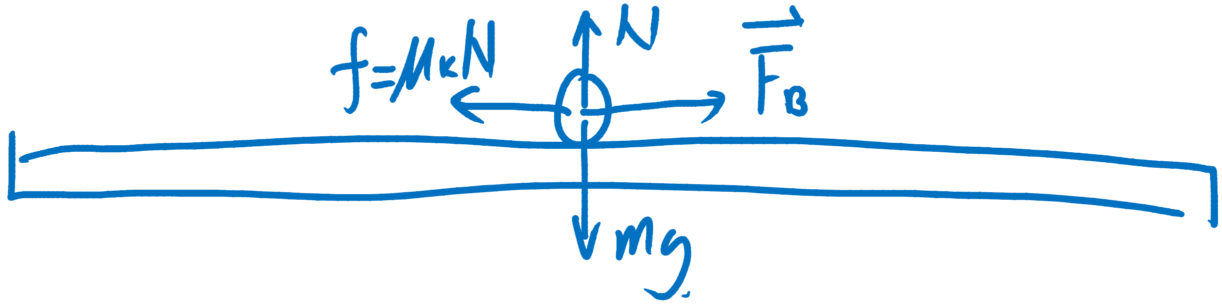
$$F_B = IlB$$

$$a = \frac{F_B}{m} = \frac{IlB}{m}$$

$$v(t) = \underset{\substack{\parallel \\ 0}}{v_0} + at = at = \frac{IlBt}{m}$$ moving to the right

(2) there is a friction between rod and rails.

coefficient of friction μ_k.



$$N = mg$$

$$f = \mu_k N = \mu_k mg$$

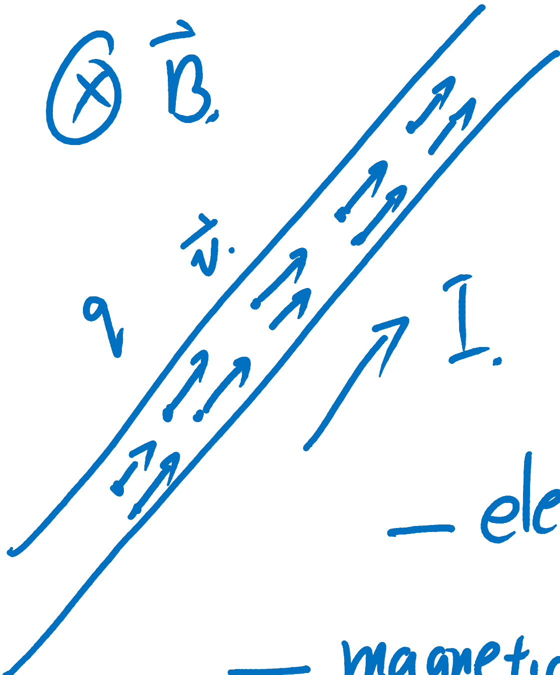
$$F_B - f = ma.$$

$$a = \frac{F_B - f}{m} = \left(\frac{IlB}{m} - \mu_k g\right)$$

$$v(t) = \underset{\overset{\parallel}{0}}{v_0} + at = \left(\frac{IlB}{m} - \mu_k g\right)t$$

* Force on electric charge.

(Microscopic origin of Ampere Force)



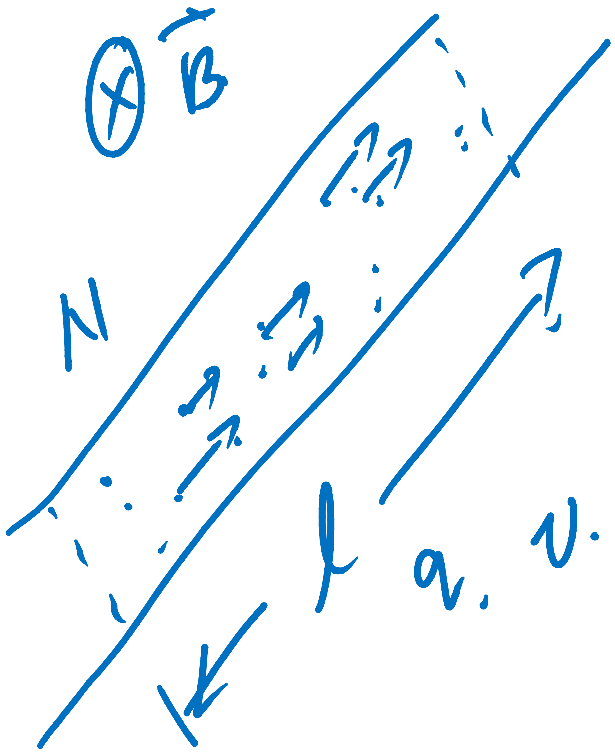
v

$$\vec{F} = I \vec{\ell} \times \vec{B}.$$

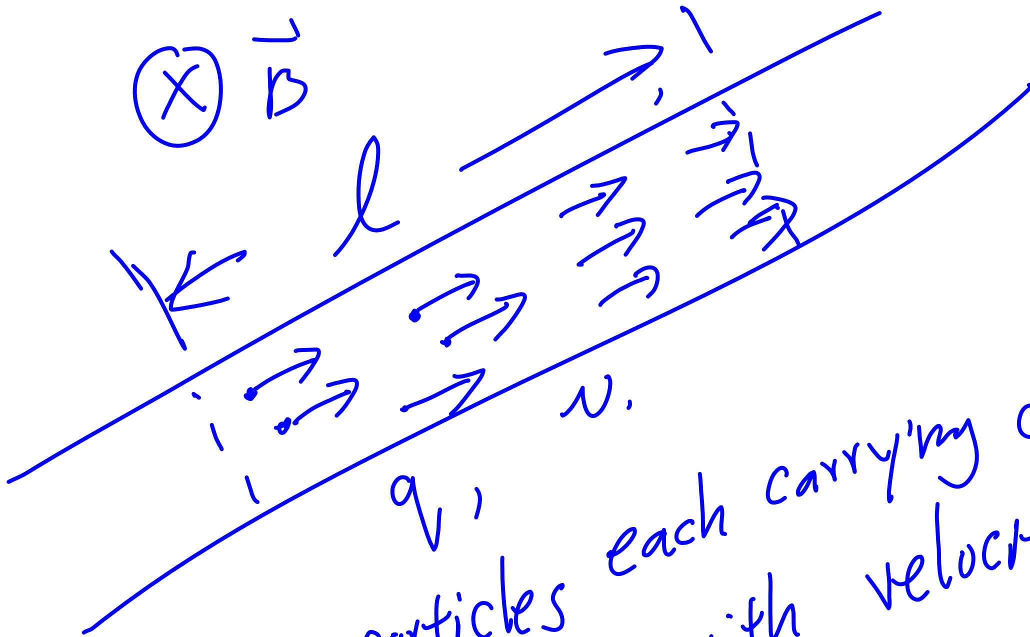
— electric current: moving electric charges

— magnetic field exerts a force on a moving charge

— Ampere Force is the collective result of the forces on moving charges in the wire.



N particles each carrying charge q within length l moving with velocity v.



N particles each carrying charge q within length l moving with velocity v

Find force ($\vec f$) on each charge q

$$N\vec f = \vec F_B = I\,\vec l \times \vec B$$

$I = ?$

correlate I to (q and $\vec v$)

macroscopic

microscopic

$$I = \frac{\Delta Q}{\Delta t} = \frac{Nq}{\left(\frac{\ell}{v}\right)} = \frac{Nqv}{\ell}$$

$$N\vec{f} = I\vec{\ell} \times \vec{B}$$

$$\cancel{N}\vec{f} = \frac{\cancel{N}qv}{\ell}\vec{\ell} \times \vec{B}$$

$$\vec{f} = qv\left(\frac{\vec{\ell}}{\ell}\right) \times \vec{B}$$

$\left(\frac{\vec{\ell}}{\ell}\right)$ → unit vector.

$$v\left(\frac{\vec{\ell}}{\ell}\right) = \vec{v}$$

$$\vec{f} = q\vec{v} \times \vec{B}$$

Lorentz force

$\vec{f} = q\,\vec{v} \times \vec{B}$

Lorentz force

$\vec{f}$ perpendicular to both $\vec{v}$ and $\vec{B}$

$\vec{v} \parallel \vec{B}$ $\quad \vec{f} = 0$

$\vec{F} = I\,\vec{l} \times \vec{B}$

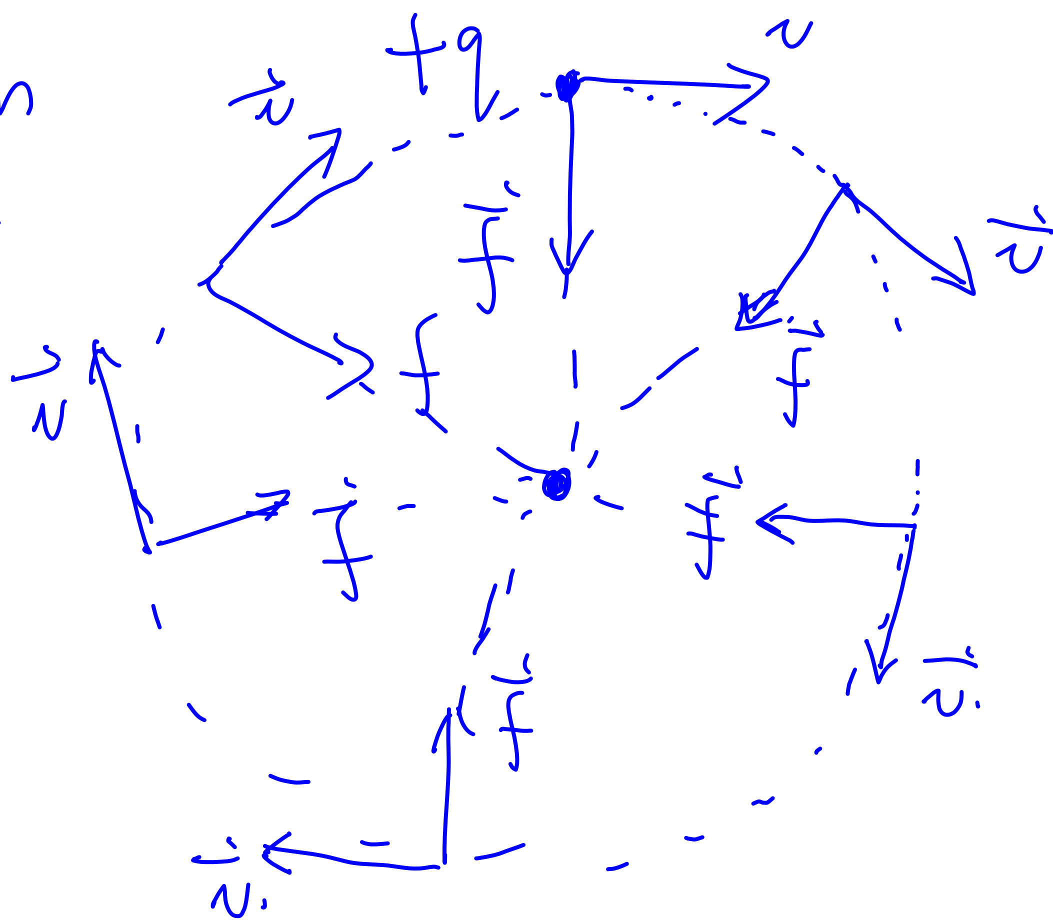
Ampere force

— Motion of electric charge in a uniform field.

positive charge $+q$ $\vec{f} = q\,\vec{v} \times \vec{B}$

$\vec{B}$ $\odot$

uniform field



$f = qvB\underline{\sin\theta}$ $\hookrightarrow 1$

$= qvB$

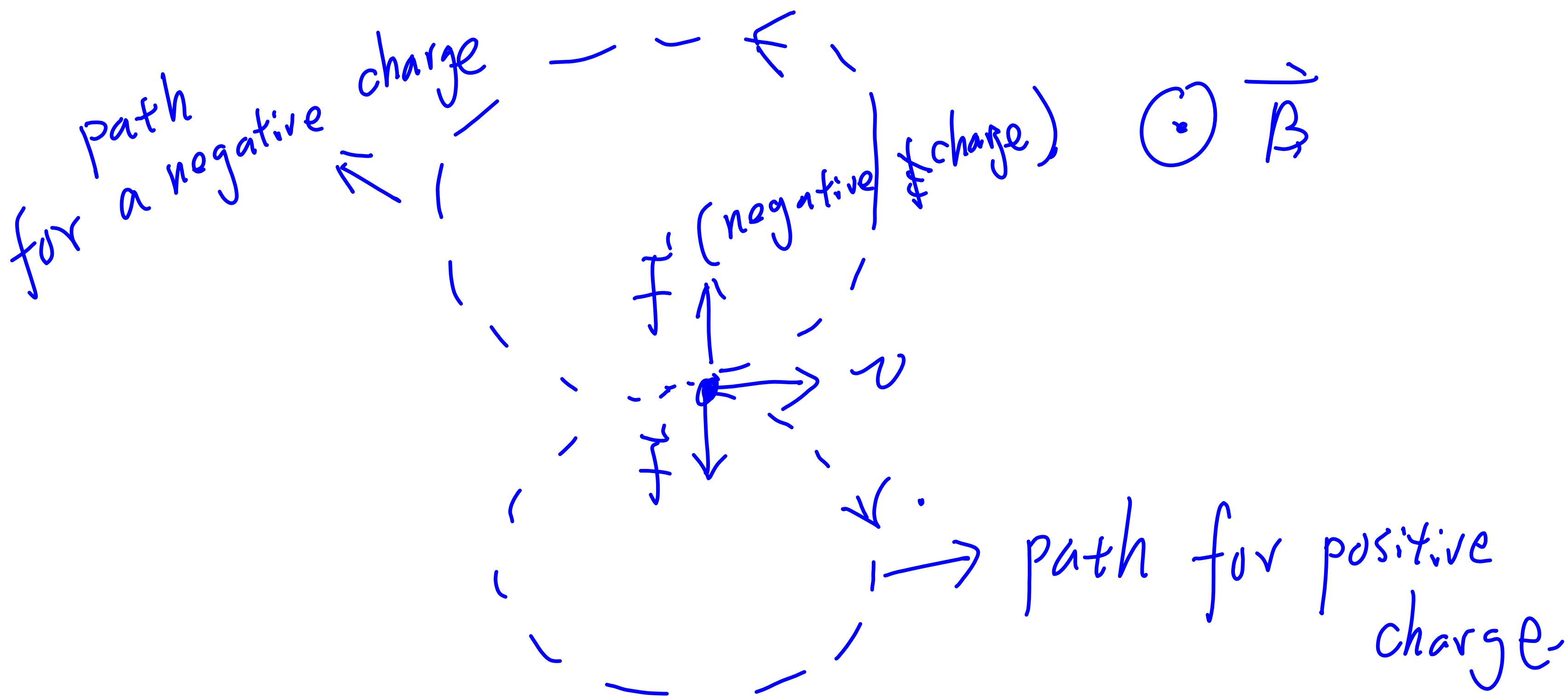
Circular motion

$\vec{f}$: centripetal force

$$f = q\cancel{v}B = m\frac{v^{\cancel{2}}}{r}$$

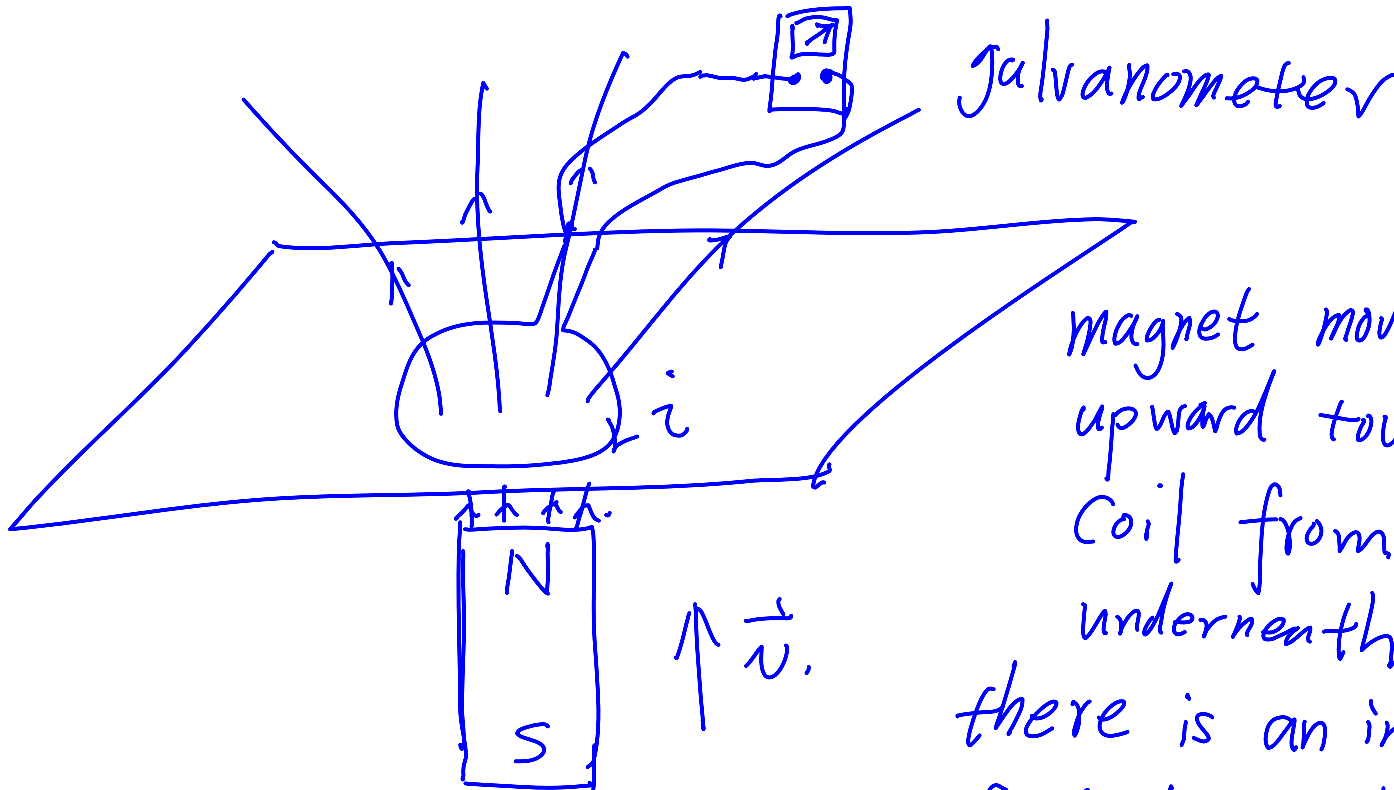
r: radius of circular motion.

$$\boxed{r = \frac{mv}{qB}}$$



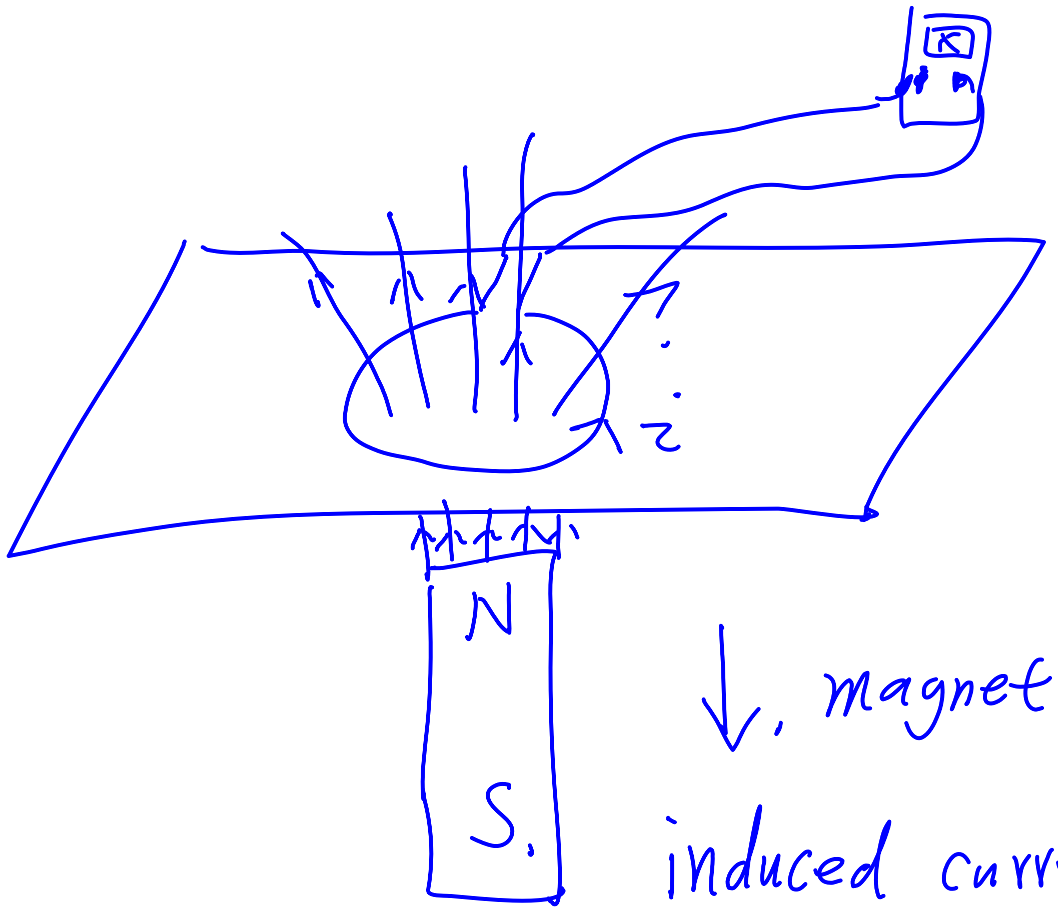
* Electromagnetic induction and Faraday's law.

— Changing magnetic flux induces an emf



magnet moving upward toward coil from underneath

there is an induced current in the loop.

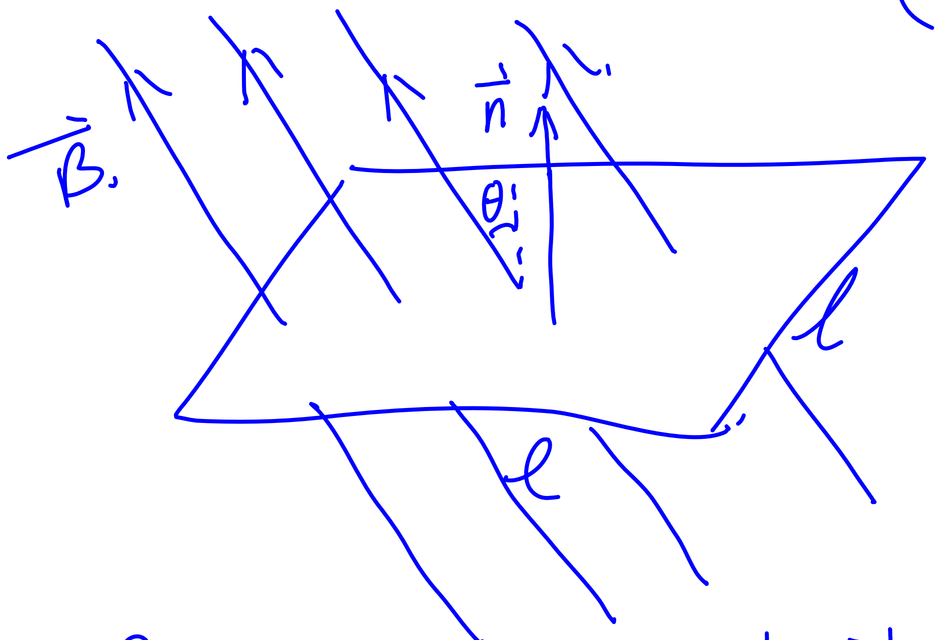


↓, magnet moving down

induced current changes direction.

hold the magnet still ; no induced current

— magnetic flux: ϕ_B, passing through a loop of area A for uniform field $\vec{B}$ is defined as $\phi_B = \vec{B}\cdot\vec{A} = (\vec{B}\cdot\vec{n})A = B_\perp A = BA\cos\theta$

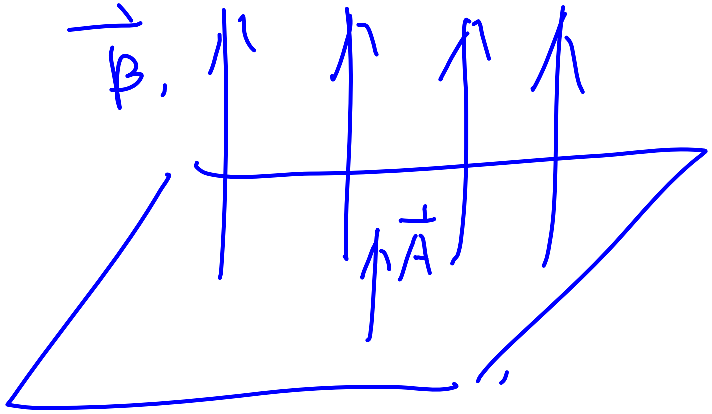


$A = \ell^2$

$\vec{A} = A\hat{n}$

$B_\perp = B\cos\theta$

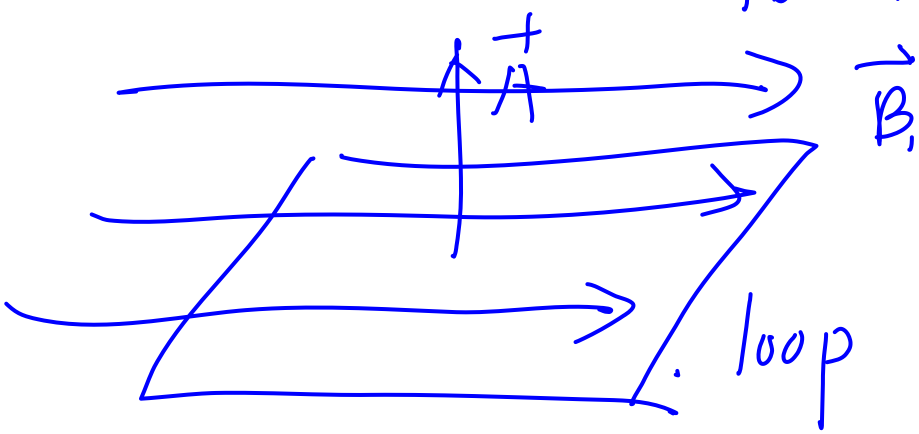
$B_\perp$: Component of $\vec{B}$ perpendicular to loop



If $\vec{B}$ is perpendicular to loop

$$\vec{B} \parallel \vec{A} \qquad \theta = 0$$

$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta = BA$$



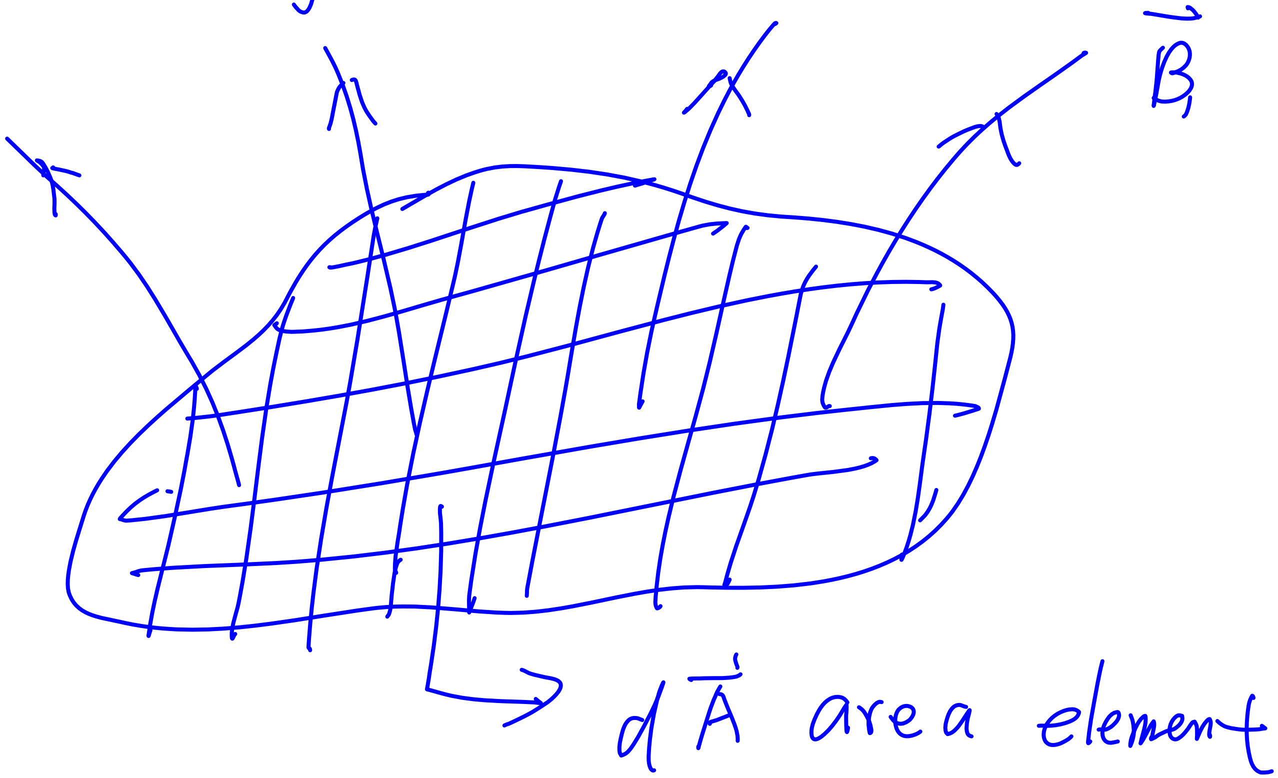
If $\vec{B} \parallel$ loop

$$\vec{B} \perp \vec{A} \qquad \theta = 90^{\circ}$$

$$\phi_B = BA\underbrace{\cos\theta}_{\to 0} = 0$$

If $\vec{B}$ is nonuniform

$$\phi_B = \int \vec{B} \cdot \vec{dA}$$



— Faraday's law

The emf induced in a loop is equal to the rate of change of magnetic flux through the loop

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

magnitude $|\mathcal{E}| = \left|\frac{d\phi_B}{dt}\right|$

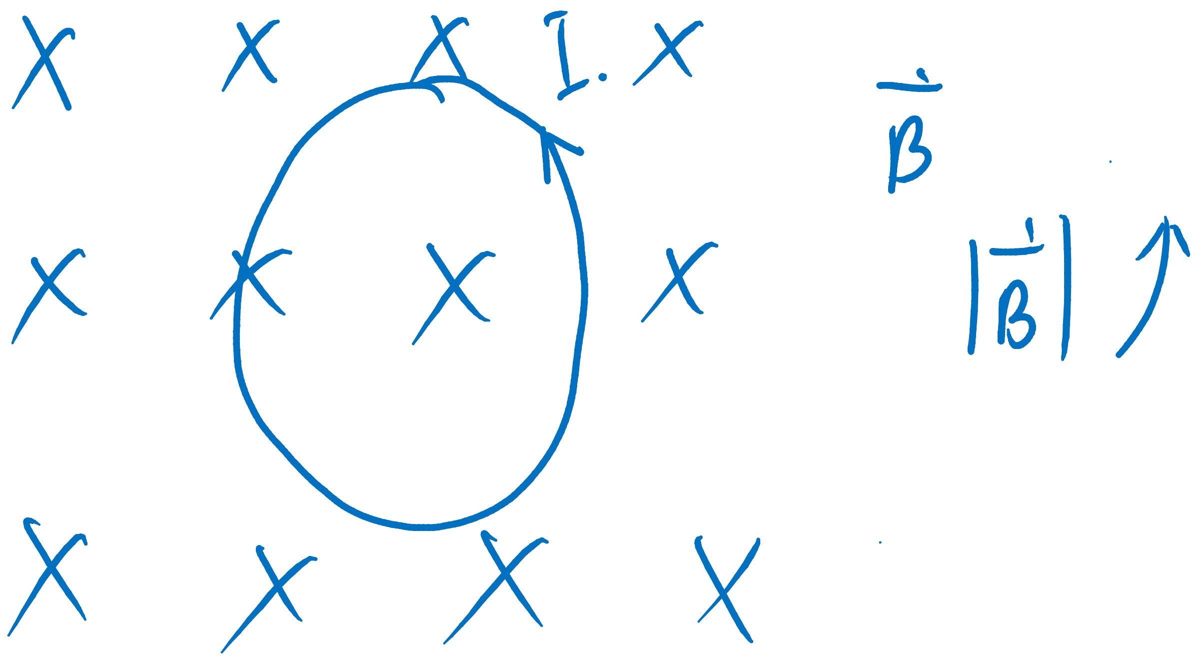
why minus sign?

To indicate direction of induced emf

Better summarized by Lentz's law

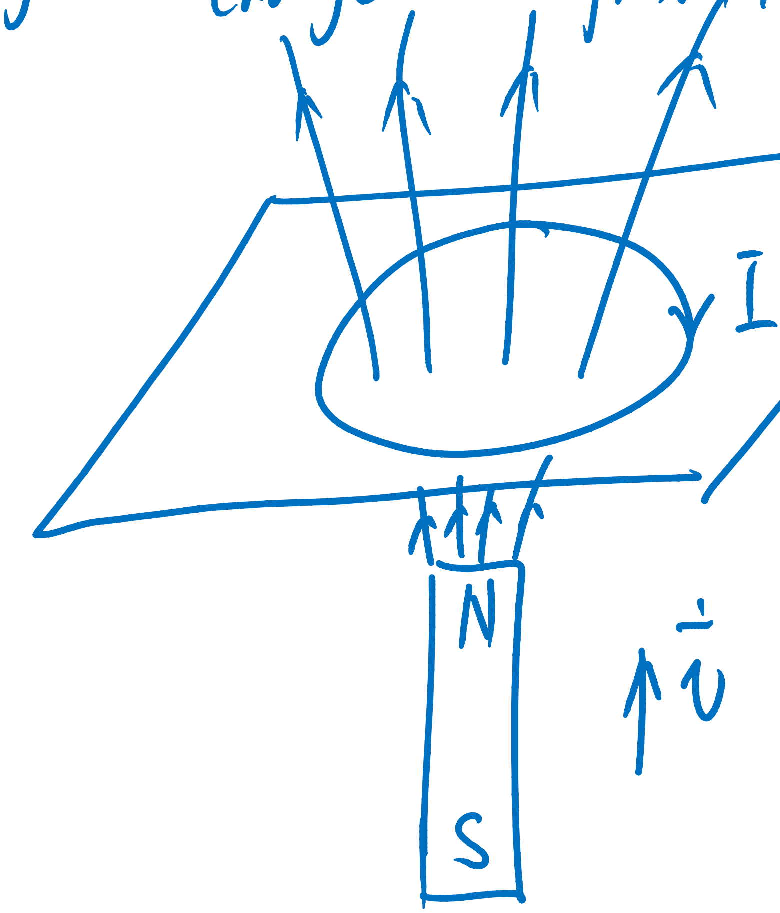
Lentz law: a current produced by an induced emf moves in a direction so that the magnetic field created by that induced current opposes the original change in flux.

↳ basis for minus sign.



Another form of Lenz law:

An induced emf is always in a direction that opposes the original change in flux that caused it.

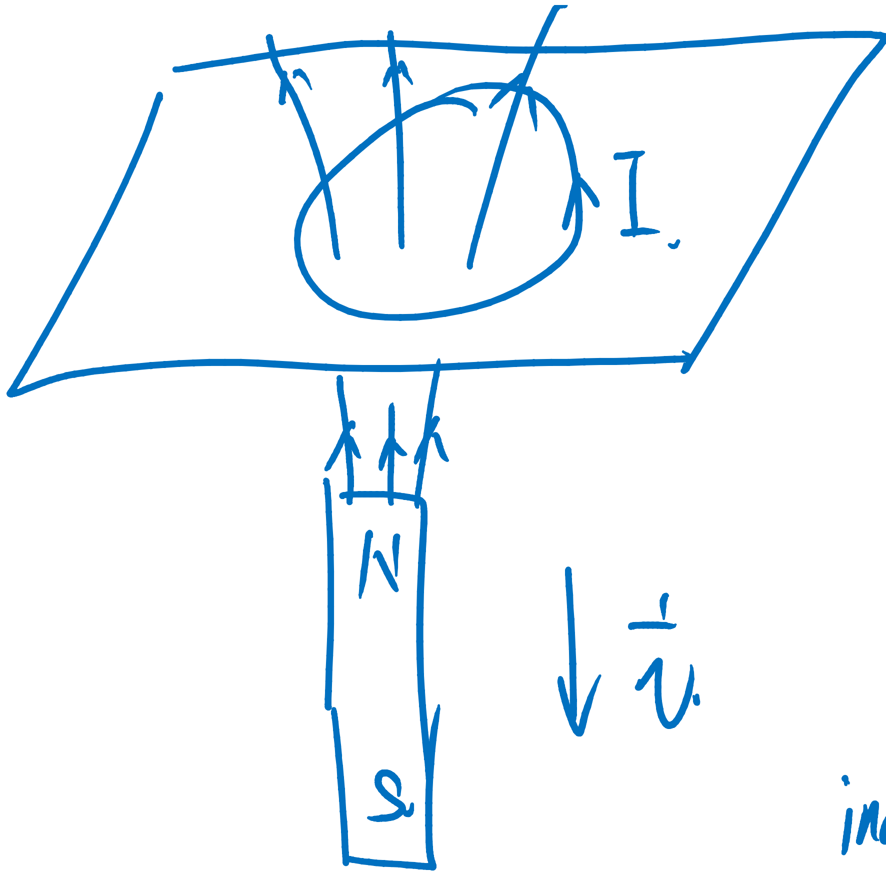


flux pointing up

magnitude of flux: $\uparrow$

change of flux: up

flux of induced current: down



flux : up

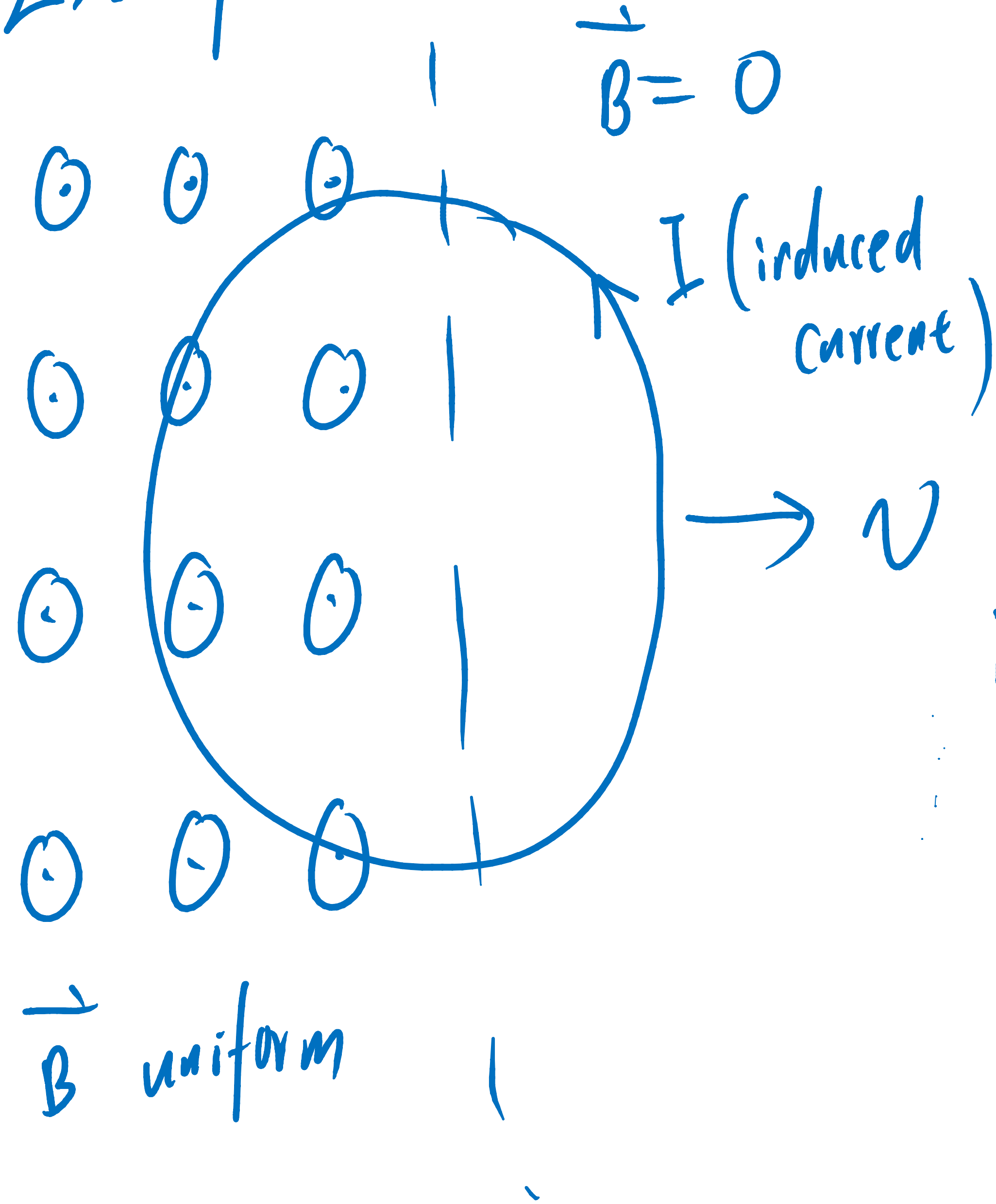
magnet moving down

magnitude of flux : ↓

change of flux : down

induced flux : up.

Example,



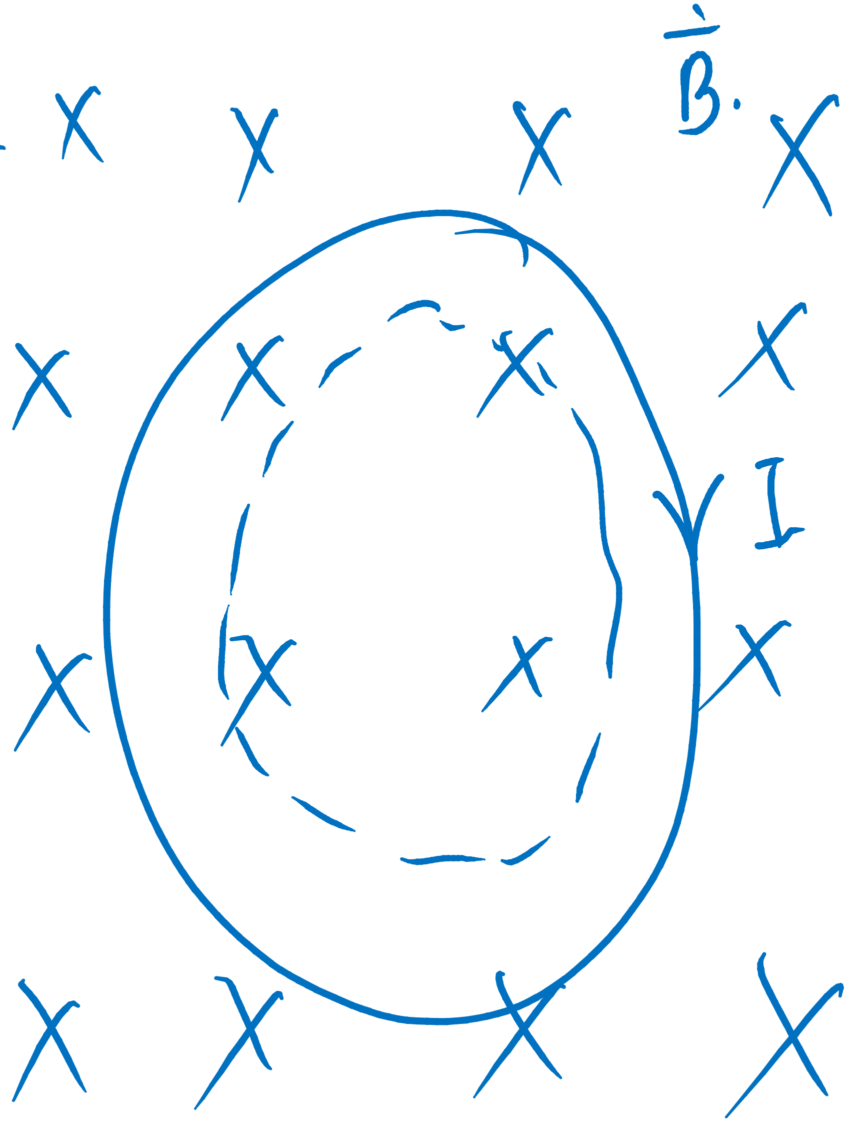
direction of flux: out of plane

magnitude of flux: ↓

change of flux: into plane

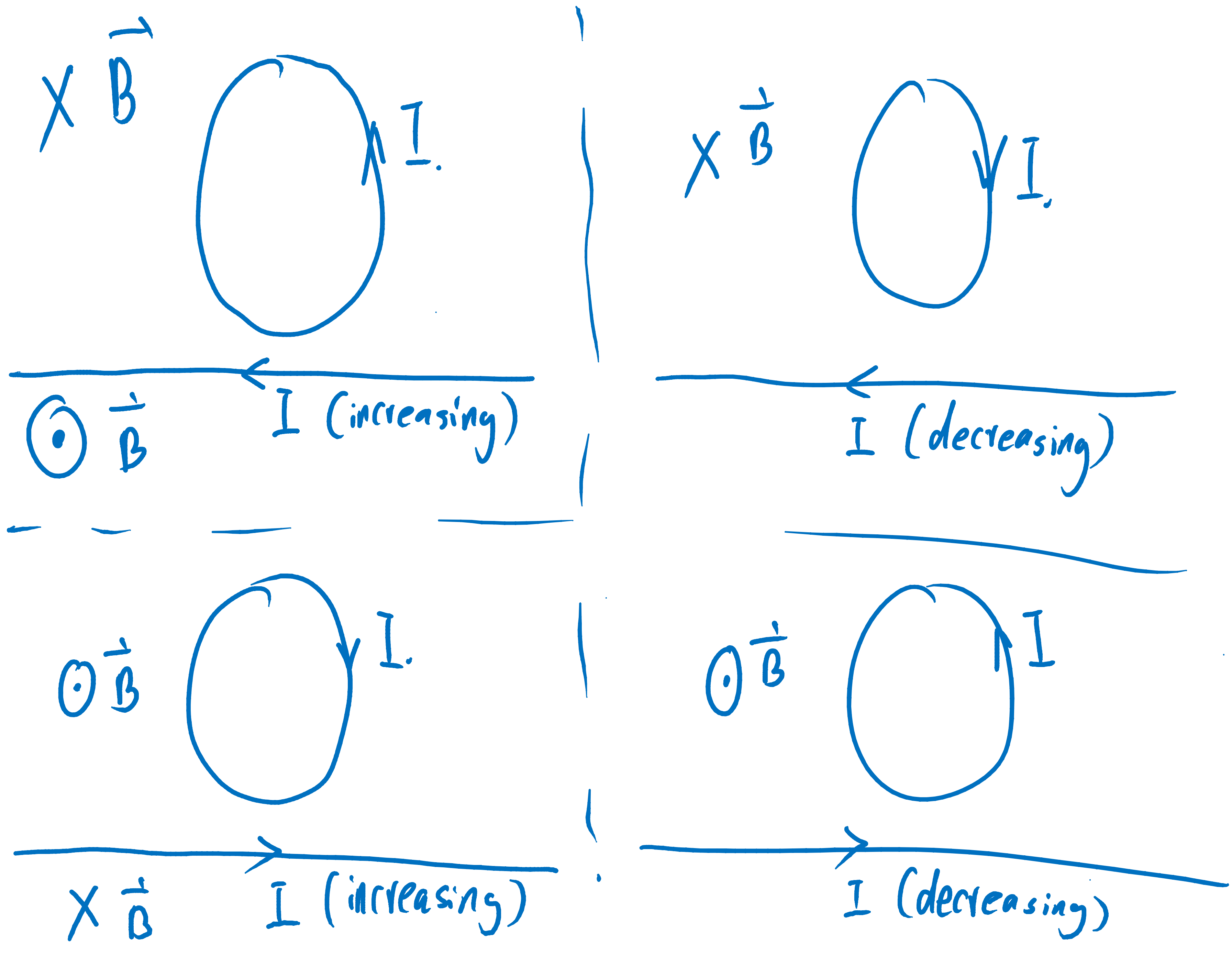
induced flux: out of plane

Example



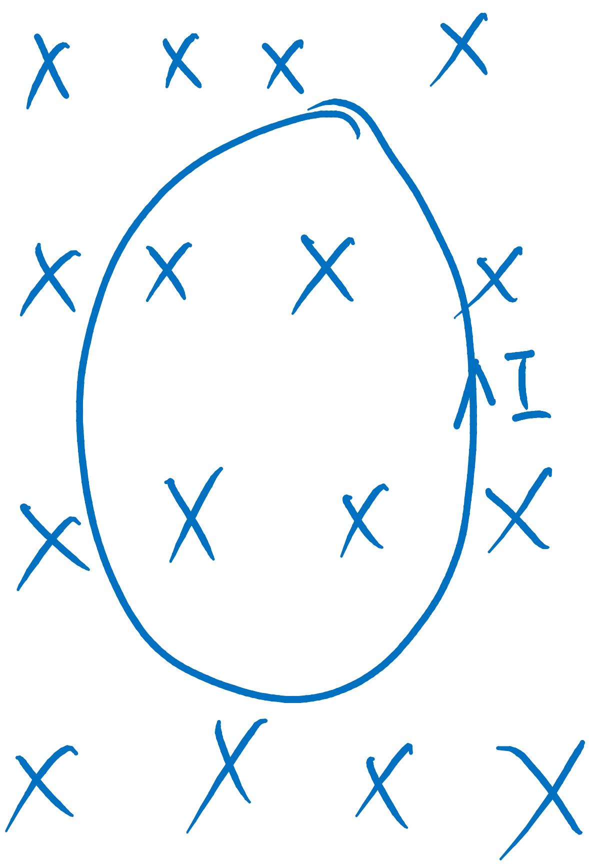
Shrinking loop

CW induced current



× $\vec{B}$
I.
⊙ $\vec{B}$
I (increasing)
× $\vec{B}$
I.
I (decreasing)
⊙ $\vec{B}$
I.
× $\vec{B}$
I (increasing)
⊙ $\vec{B}$
I
I (decreasing)

Example



$A = 8.0\ cm^2$

$B: 0.5T \longrightarrow 2.5T$

in 1.0 s stedily

resistance of loop : 2.0Ω

induced current = ?

$$\mathcal{E} = \frac{d\phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}$$

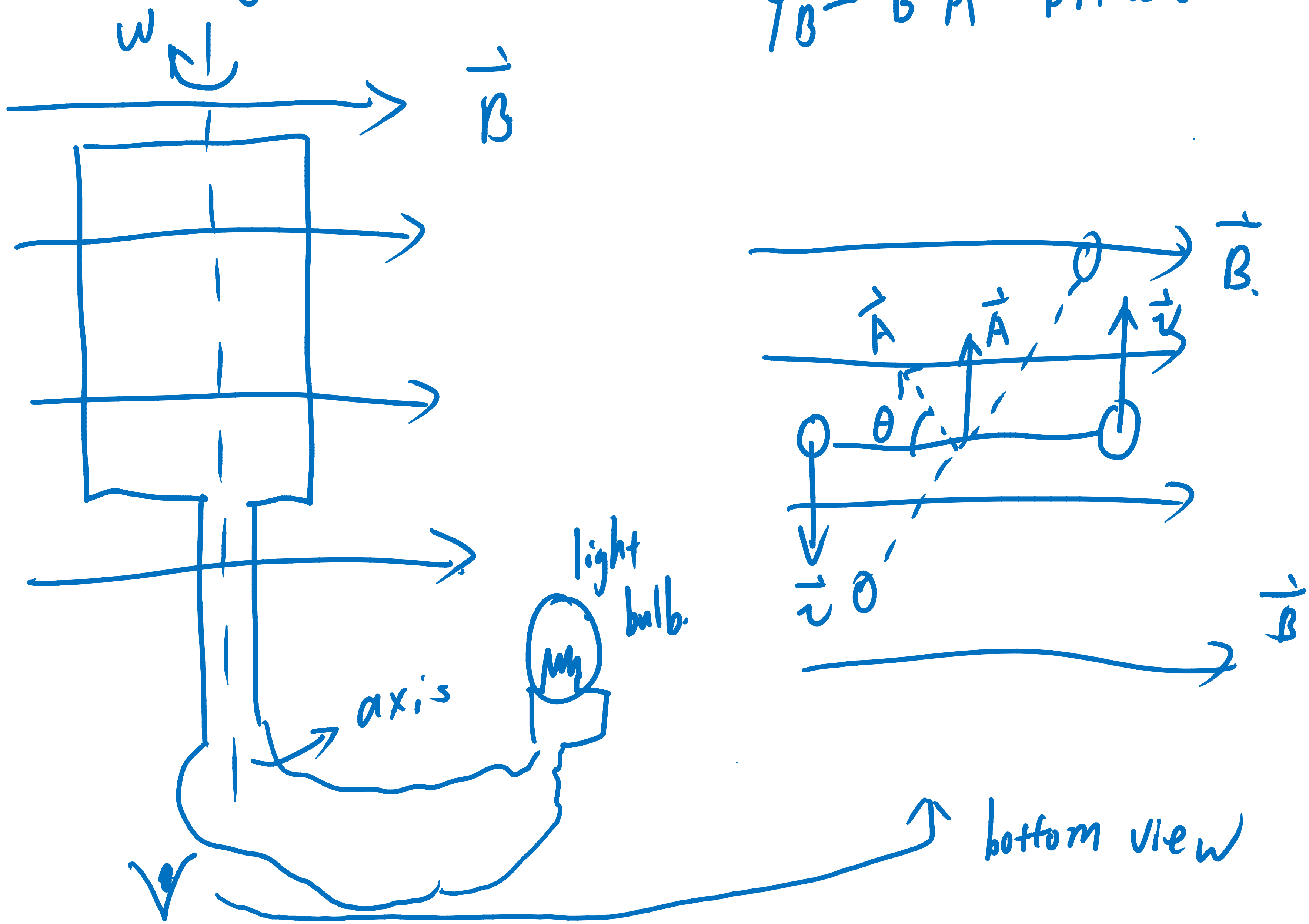
$$I = \frac{\mathcal{E}}{R} = \frac{A}{R}\frac{dB}{dt} = \frac{A}{R}\frac{\Delta B}{\Delta t}$$

$$= \frac{(8.0 \times 10^{-4}\ m^2)}{2.0\ \Omega} \cdot \frac{(2.5 - 0.5)T}{1.0s}$$

$$= 8.0 \times 10^{-4}\ (A) = 0.8\ mA$$

— How generator works

$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$



$$\varepsilon = \frac{d\phi_B}{dt} = \frac{d(BA\cos\theta)}{dt} = BA\,\frac{d(\cos\theta)}{dt}$$

$$= BA(-\sin\theta)\,\frac{d\theta}{dt}$$

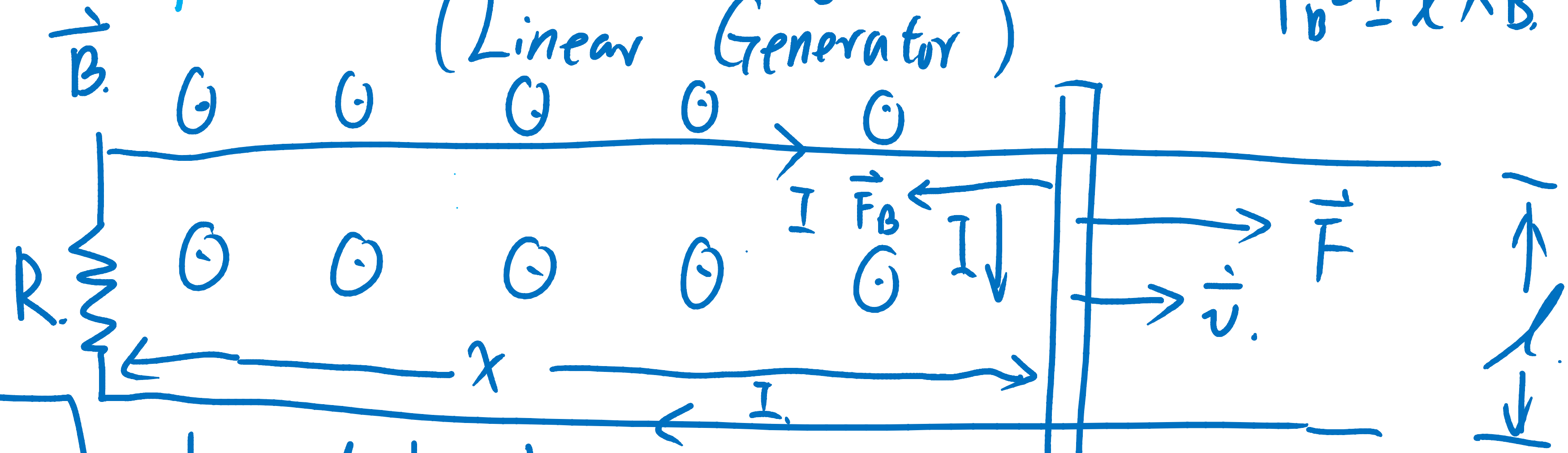
$$\frac{d\theta}{dt} = \omega \qquad \theta = \omega t + \varphi_0$$

$$\varepsilon = -BA\omega\sin(\omega t + \varphi_0)$$

$$I = \frac{\varepsilon}{R} = -\frac{BA\omega}{R}\sin(\omega t + \varphi_0)$$

— emf induced in moving conductor (Linear Generator)

$\vec{F}_B = I\vec{\ell}\times\vec{B}$



$\vec{B}$ uniform

rails and bar have zero resistance

no friction between rails and bar.

$$\mathcal{E} = \frac{d\phi_B}{dt} = \frac{d(BA)}{dt} = B\frac{dA}{dt} = B\frac{d(\ell x)}{dt}$$

$$= B\ell\frac{dx}{dt} = B\ell v$$

$$\boxed{\mathcal{E} = B\ell v}$$

assume $\vec{B}$, $\vec{\ell}$, $\vec{v}$ are perpendicular

$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}.$$

$$F_B = IlB = \frac{B^2 l^2 v}{R}.$$ Ampere force.

External force to sustain constant velocity v

$$F = F_B = \frac{B^2 l^2 v}{R}$$ (external force)

Power delivered by external force

$$W = F \cdot \Delta x \qquad P = \frac{W}{\Delta t} = F \cdot \frac{\Delta x}{\Delta t} = F \cdot v$$

$$P_{mechanicl} = F \cdot v = \left(\frac{B^2 l^2 v}{R}\right) v = \frac{B^2 l^2 v^2}{R}.$$

Electric Power

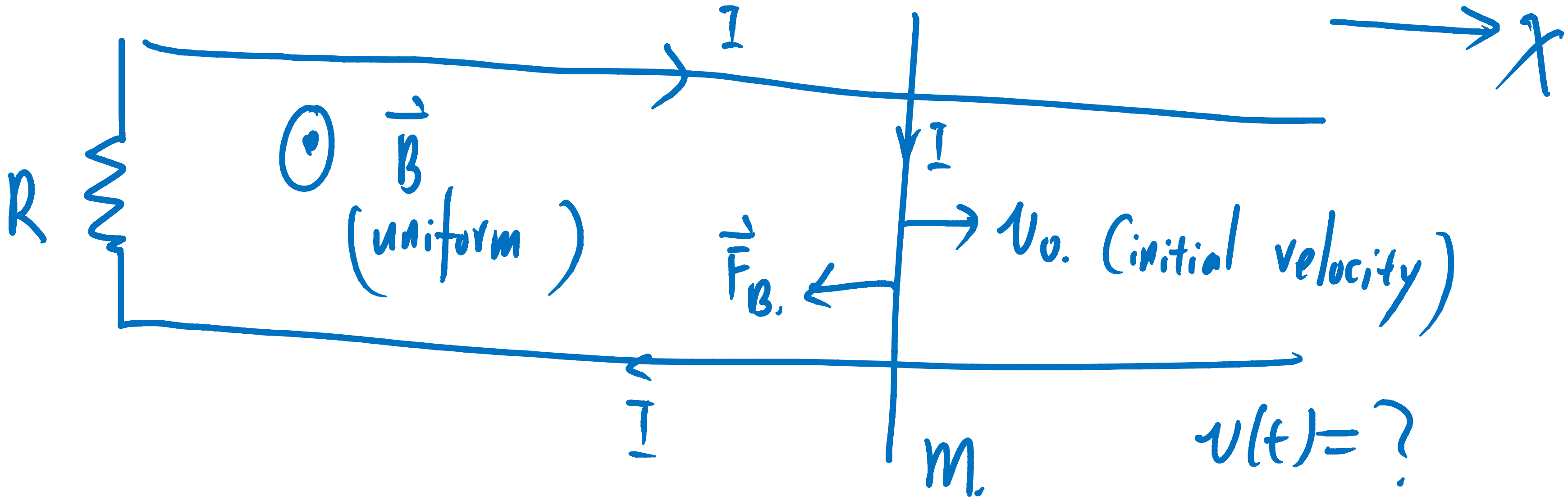
$$P_e = \mathcal{E} I = B l v \left(\frac{B l v}{R}\right) = \frac{B^2 l^2 v^2}{R}.$$

Thermal power

$$P_{thermal} = I^2 R.$$

$$= \left(\frac{B l v}{R}\right)^2 R = \frac{B^2 l^2 v^2}{R}.$$

Mechanical energy turns into electric energy and then into thermal energy



$$\mathcal{E} = B l v$$

$$i = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$

$$F_B = i l B = \frac{B^2 l^2 v}{R}$$

$$a = -\frac{F_B}{m} = \frac{-B^2 l^2 v}{mR}$$

$$a = \frac{dv}{dt} = -\frac{B^2 l^2 v}{Rm}$$

$$\frac{dv}{v} = -\frac{B^2 l^2}{Rm} dt$$

$$\ln v = -\frac{B^2 l^2}{Rm} t + C$$

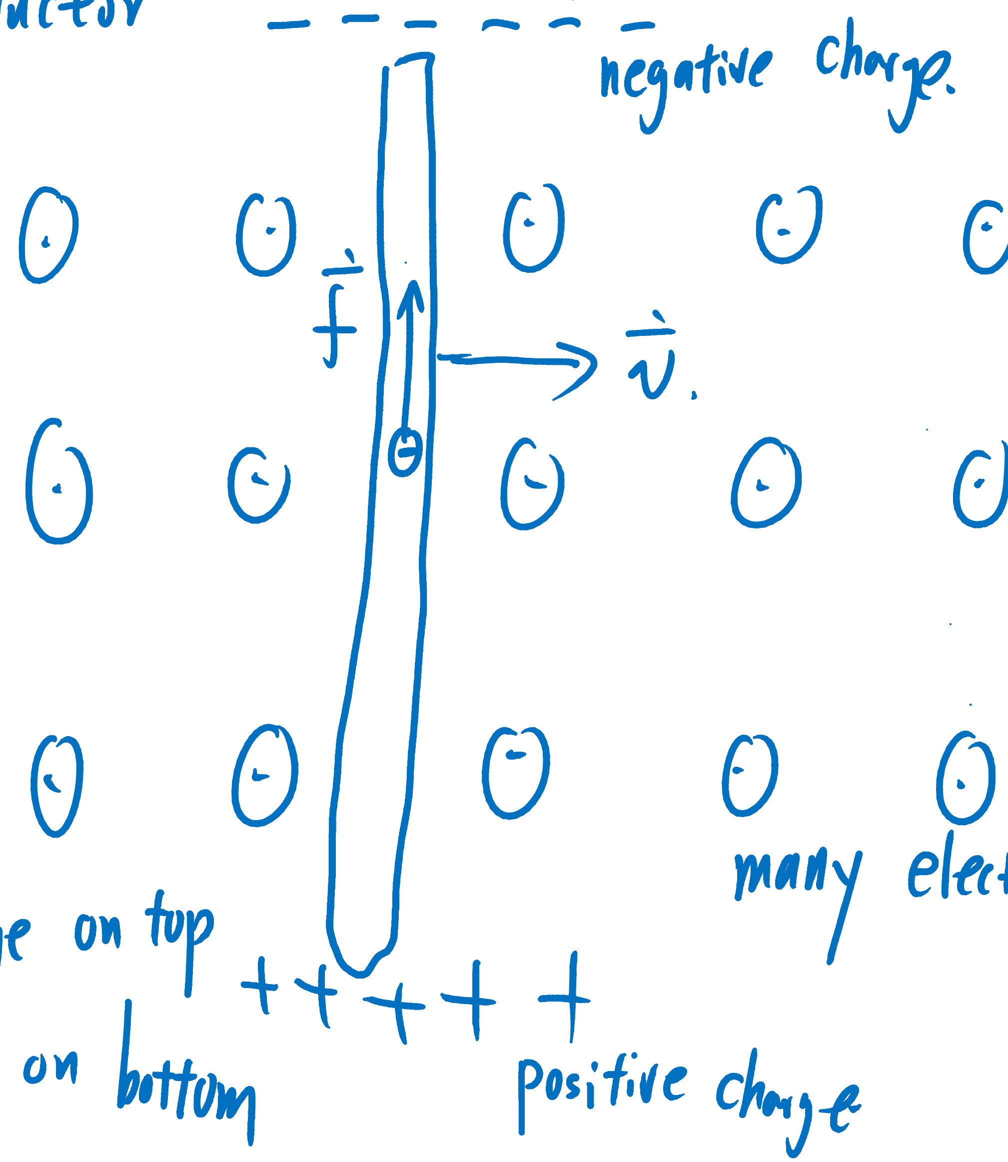
$$v = c' e^{-\frac{B^2 l^2}{Rm} t} \qquad c' = e^{C}.$$

$$t = 0 \qquad v = v_0.$$

$$v = v_0 e^{-\frac{B^2 l^2}{Rm} t}$$

— Microscropic origin of induced emf in moving conductor:

$\vec{B}$ uniform.



$$\vec{f} = q\vec{v} \times \vec{B}.$$

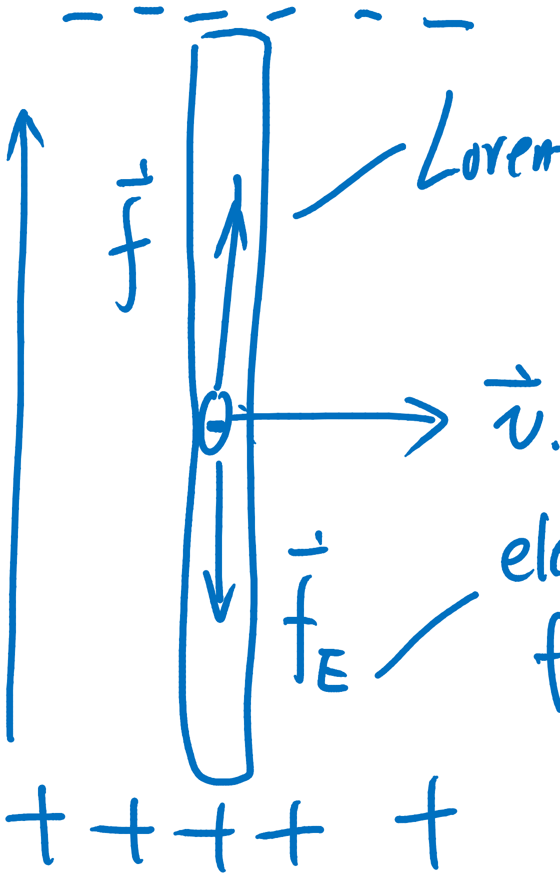
$$\vec{f} = (-e)\vec{v} \times \vec{B}.$$

negative charge on top

positive charge on bottom

$$\vec{F} = q\vec{v} \times \vec{B}$$

$\vec{B}$ ⊙



Lorentz force

$$f = evB$$

electric force

$$f_e = eE$$

$$= e\frac{\varepsilon}{\ell}$$

electric field due to charge accumulation.

induced emf exists without a closed loop

$$\vec{B} \perp \vec{\ell} \perp \vec{v}$$

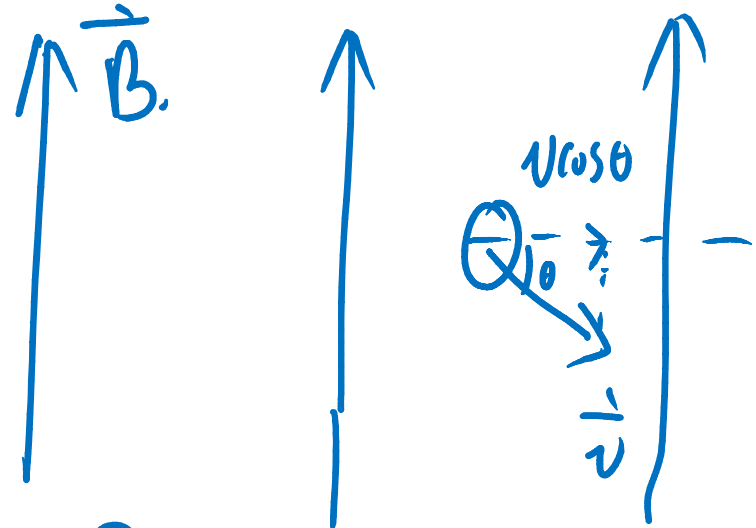
$$f = f_e.$$

$$\cancel{e}vB = \cancel{e}\frac{\varepsilon}{\ell}$$

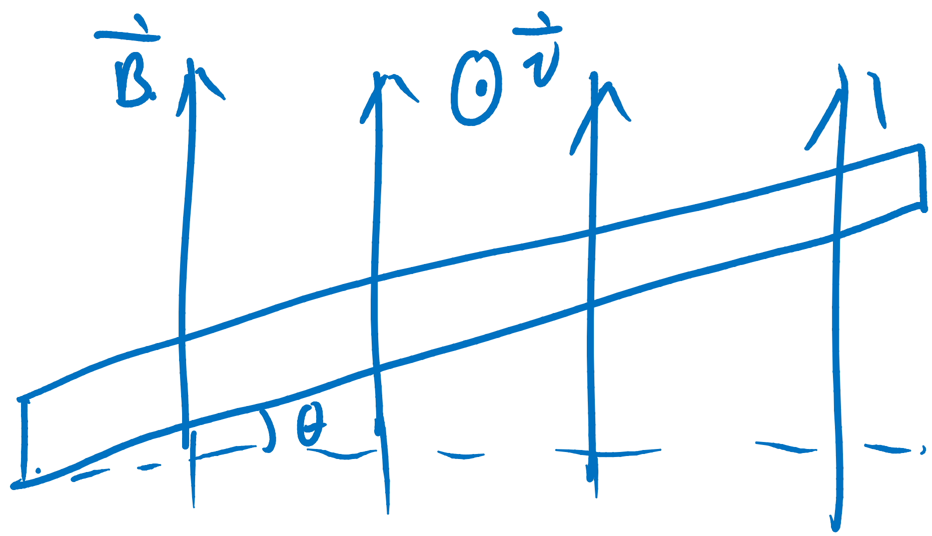
$$\boxed{\varepsilon = B\ell v}$$

side view of a metal bar (bar $\perp$ board)

project velocity to direction that is $\perp \vec{B}$



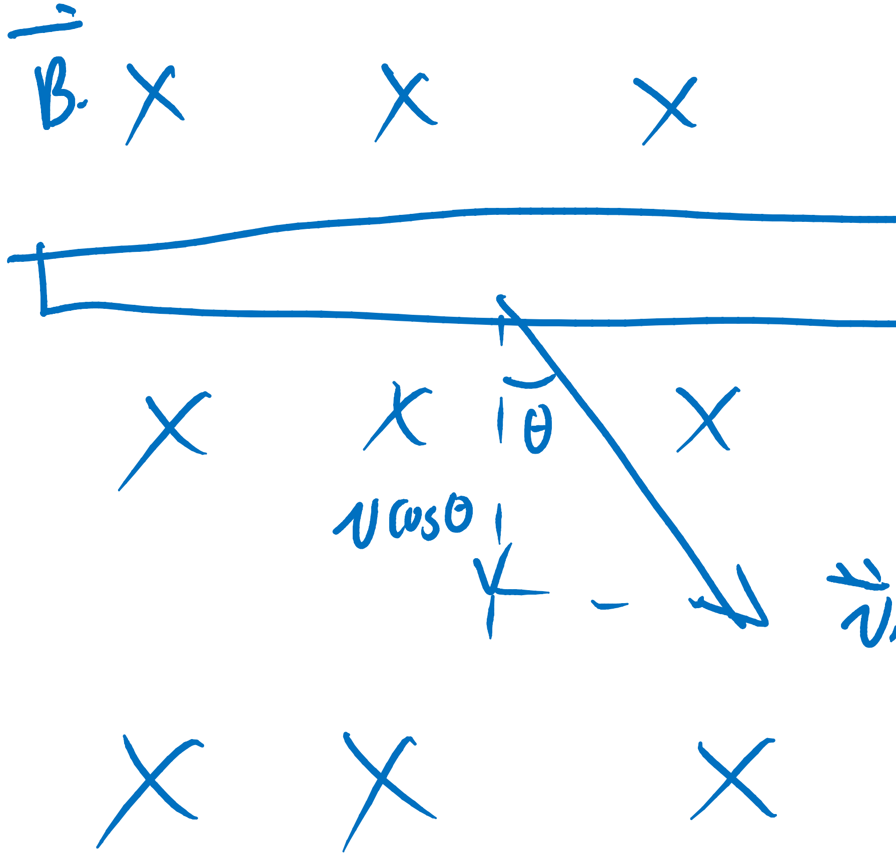
$$\mathcal{E} = Blv\cos\theta$$



bar moving out of plane

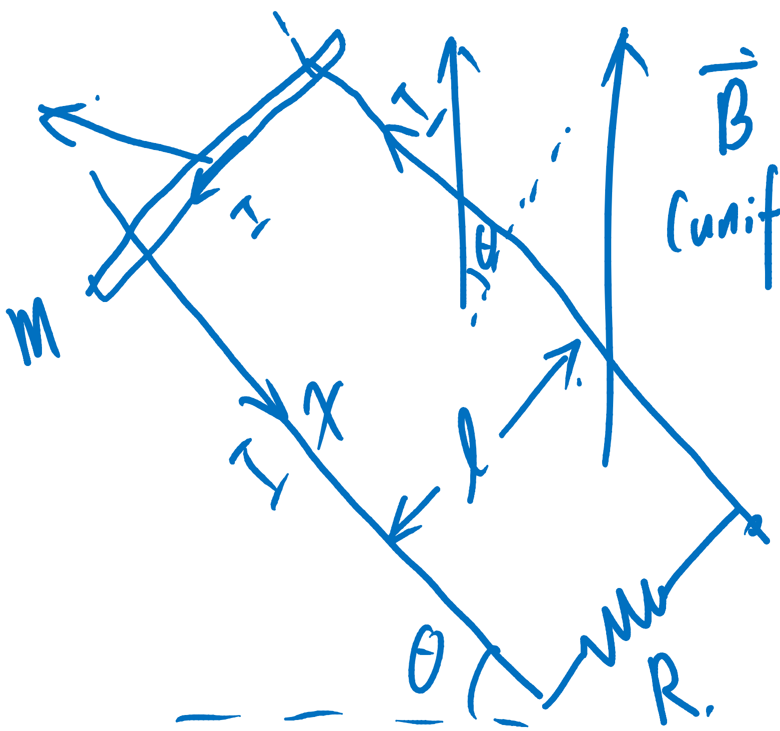
project length of bar to the direction $\perp \vec{B}$

$$\mathcal{E} = B(l\cos\theta)v$$



project velocity to direction $\perp$ rod

$\mathcal{E} = B\ell v\cos\theta$



rod and rails

frictionless

resistanceless

parallel rails positioned at angle θ with horizontal plane

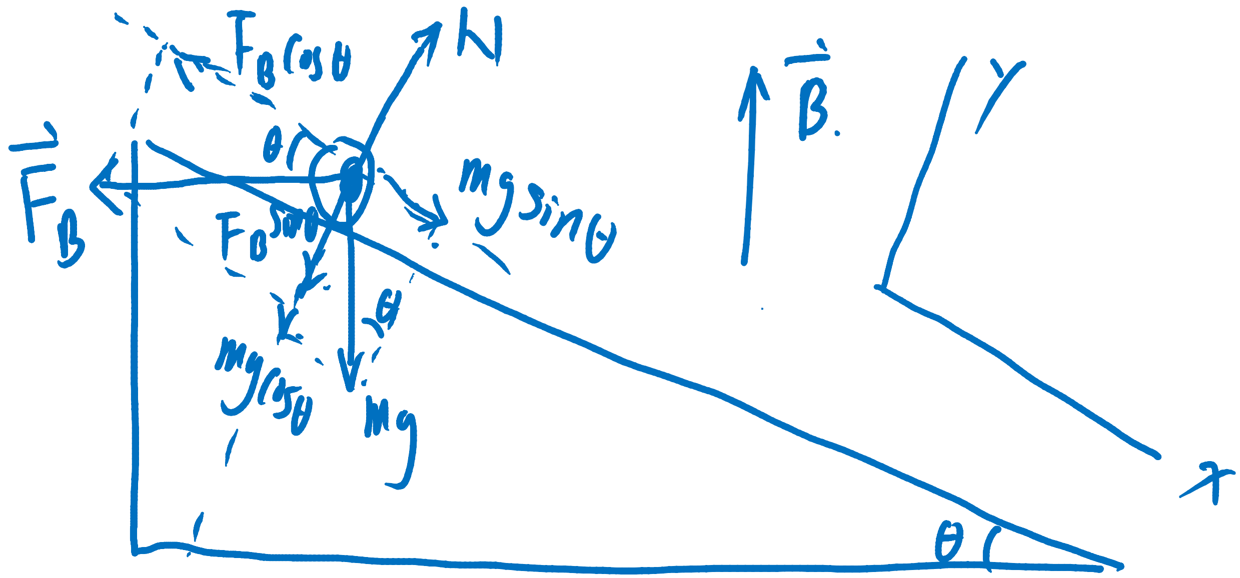
conducting bar slides down rails

Find terminal velocity $v_{term} = ?$

$$\mathcal{E} = B l v \cos\theta$$

$$\phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$
$$= B(lx)\cos\theta$$

$$\mathcal{E} = \left|\frac{d\phi}{dt}\right| = Bl\cos\theta \left|\frac{dx}{dt}\right| = Blv\cos\theta$$



$$\vec{F}_B = I\vec{\ell} \times \vec{B}$$

x: $$mg\sin\theta = F_B\cos\theta$$

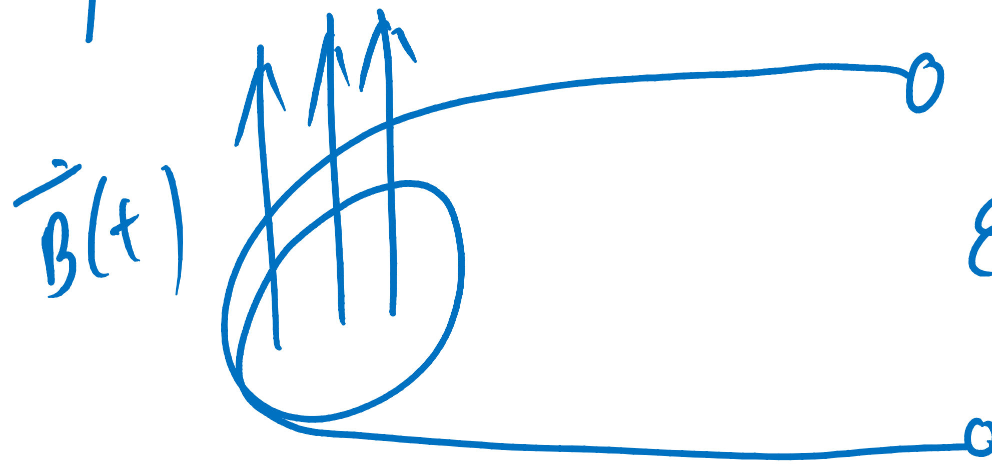
$$F_B = I\ell B \qquad I = \frac{\mathcal{E}}{R} \qquad \mathcal{E} = B\ell v\cos\theta$$

$$F_B = \frac{B\ell v\cos\theta}{R}(\ell B) = \frac{B^2\ell^2 v\cos\theta}{R}$$

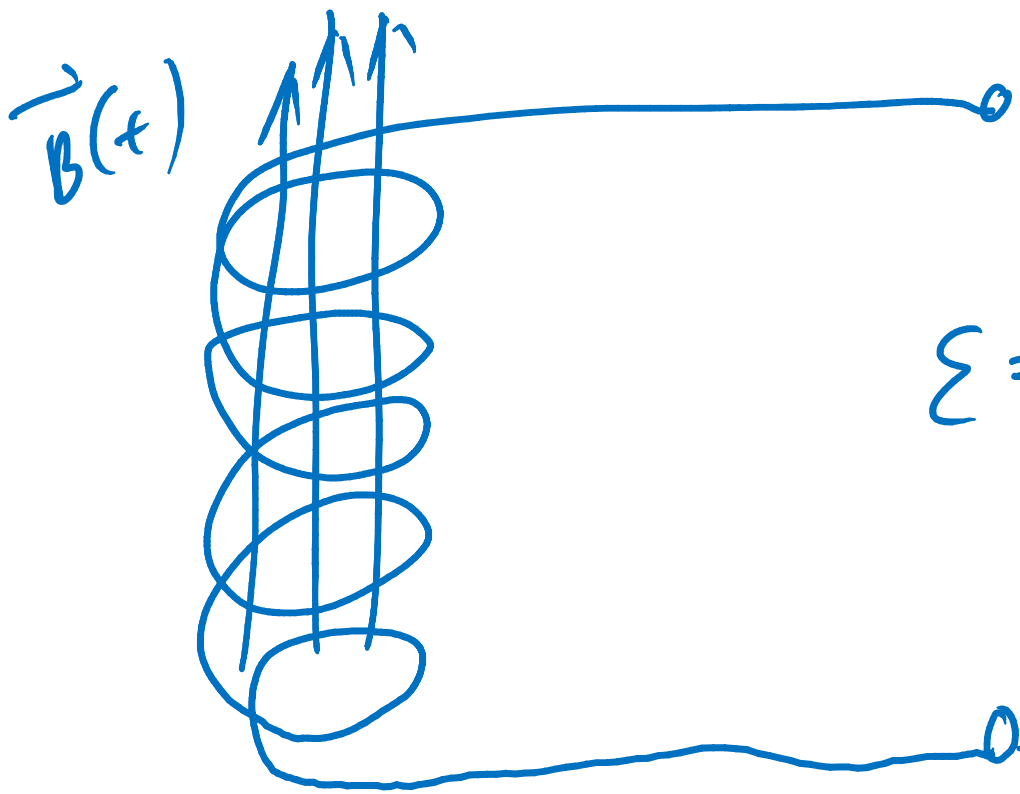
$$mg\sin\theta = \frac{B^2\ell^2 v\cos^2\theta}{R}$$

$$v_{term} = \frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}$$

— Loop with N turns



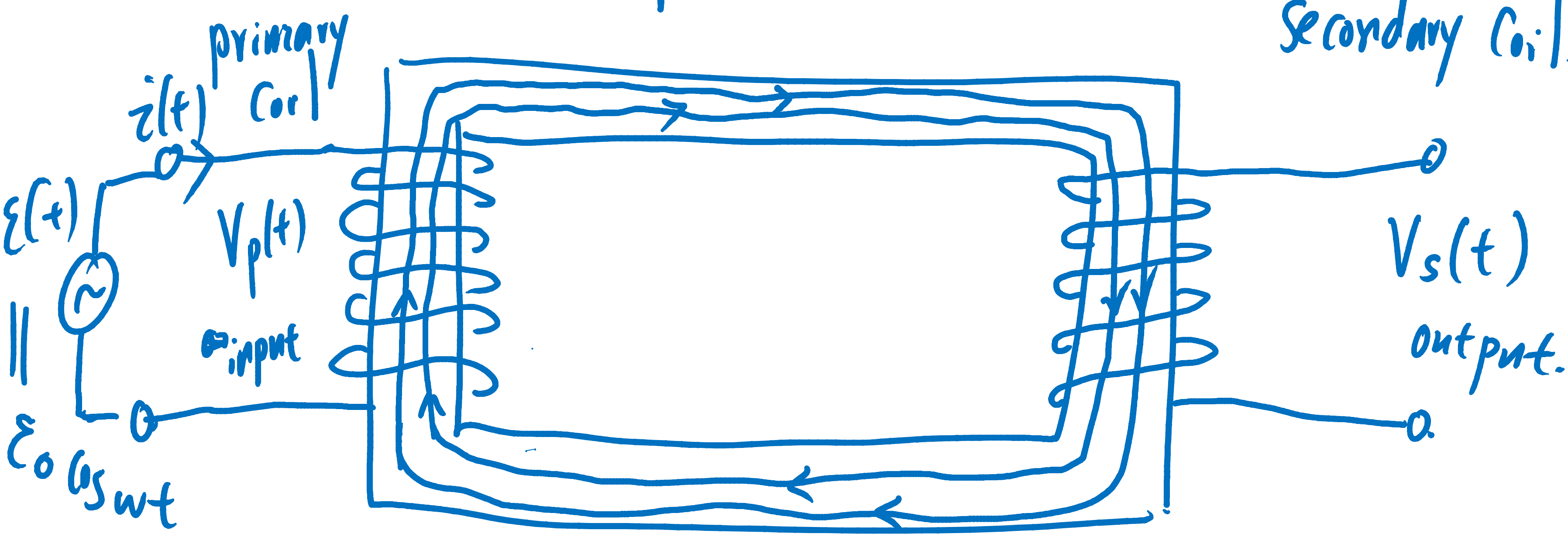
$$\mathcal{E} = \frac{d\phi}{dt}$$



number of turns

$$\mathcal{E} = N \frac{d\phi}{dt}$$

— How does transformer works



$V_p = \mathcal{E}(t)$ $\quad$ $\mathcal{E}(t)$ causes time varying current $i(t)$ in the primary.

$i(t)$ causes time varying field and flux in the iron core.

Iron core concentrate the flux in the core

primary and secondary coil see the same $\frac{d\phi}{dt} \propto \frac{di}{dt}$

$$V_p = N_p \frac{d\phi}{dt} \qquad V_s = N_s \frac{d\phi}{dt}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$V_s = \frac{N_s}{N_p} V_p = \frac{N_s}{N_p} \mathcal{E}(t)$$

If $N_p = 10$ $\quad N_s = 100$

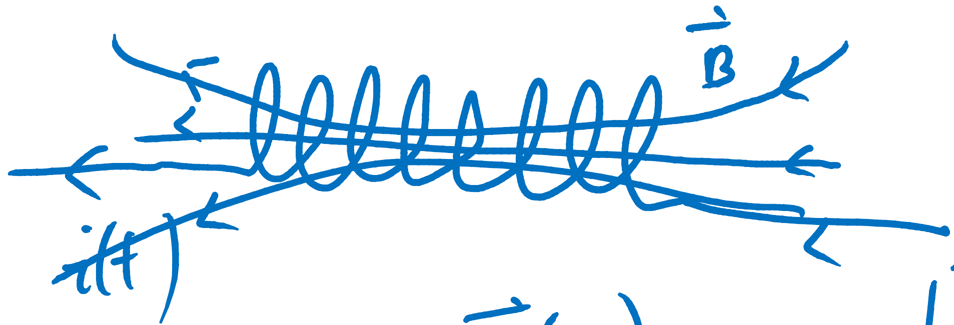
$V_s = 10 V_p$. $\quad$ step-up transformer

$(N_s > N_p)$

If $N_p = 100$ $\quad N_s = 10$

$V_s = \frac{1}{10} V_p$ $\quad$ step-down transformer

$(N_s < N_p)$

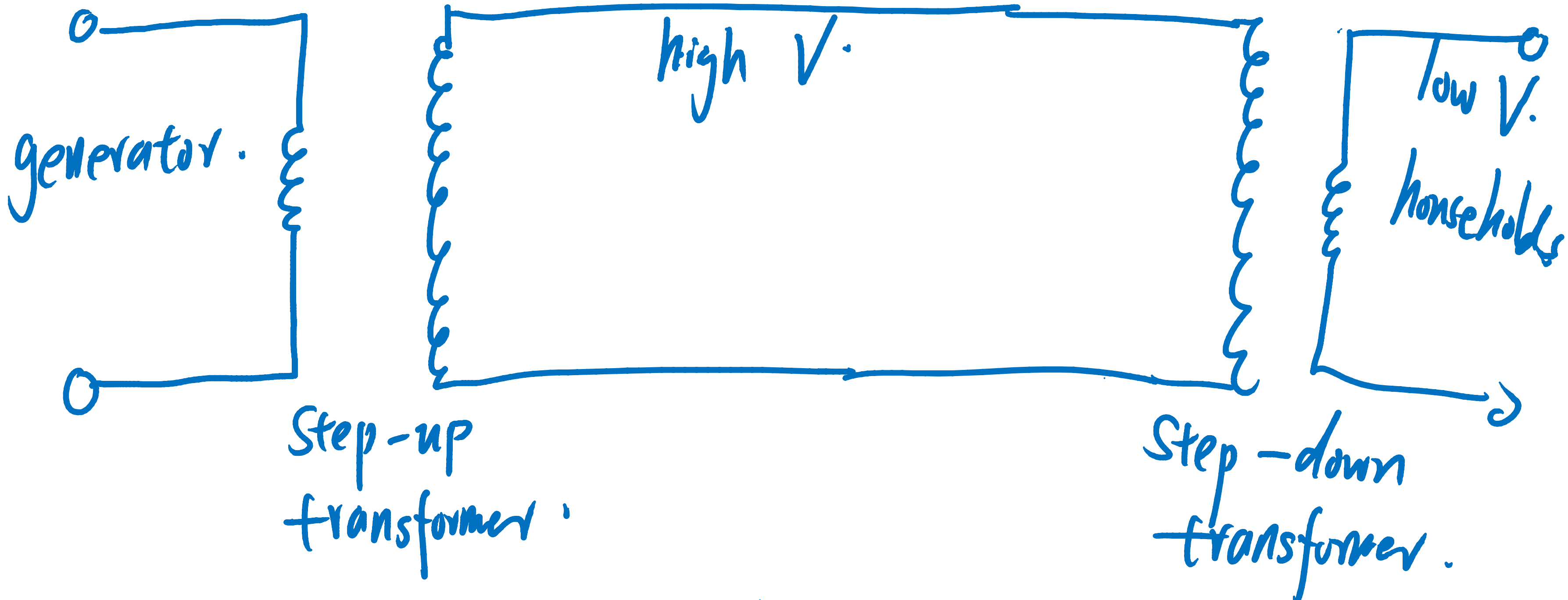


$\vec{B}(t)$ $\phi(t)$ $\frac{d\phi}{dt} \neq 0$

$$\frac{d\phi}{dt} \propto \frac{di}{dt}$$

$$v_L = N\frac{d\phi}{dt} \propto N\frac{di}{dt}$$

$$v_L = L\frac{di}{dt}$$



$$P = IV$$

$P_{loss} \ll P$ $\qquad$ $P_{loss} = I^2 r$. $\hookrightarrow$ resistance of lines

low I is favored

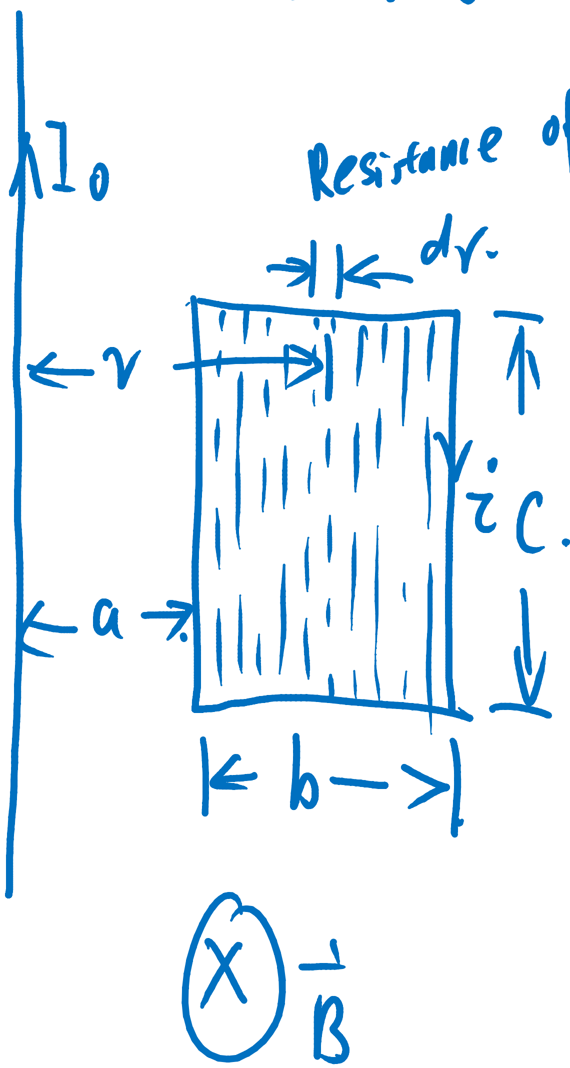
So high V is used for power transmission.

Example

I drops from I_0 to 0.

What is the charge going through the loop?

Resistance of loop: R.



$dA = c \cdot dr$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\phi = \int B \cdot dA$$

$$= \int_a^{a+b} \frac{\mu_0 I}{2\pi r} c \, dr$$

$$= \frac{\mu_0 I c}{2\pi} \int_a^{a+b} \frac{dr}{r}$$

$$\phi = \frac{\mu_0 I c}{2\pi} \ln \frac{a+b}{a}$$

$$\mathcal{E} = \frac{d\phi}{dt} = \frac{\mu_0 c}{2\pi} \ln \frac{a+b}{a} \frac{dI}{dt}$$

$$i = \frac{\varepsilon}{R} = \frac{\mu_0 c}{2\pi R} \ln\left(\frac{a+b}{a}\right) \frac{dI}{dt}$$

$$Q = \int_0^\infty i\, dt = \frac{\mu_0 c}{2\pi R} \ln\left(\frac{a+b}{a}\right) \int_0^\infty \left(\frac{dI}{\cancel{dt}}\right) \cancel{dt}$$

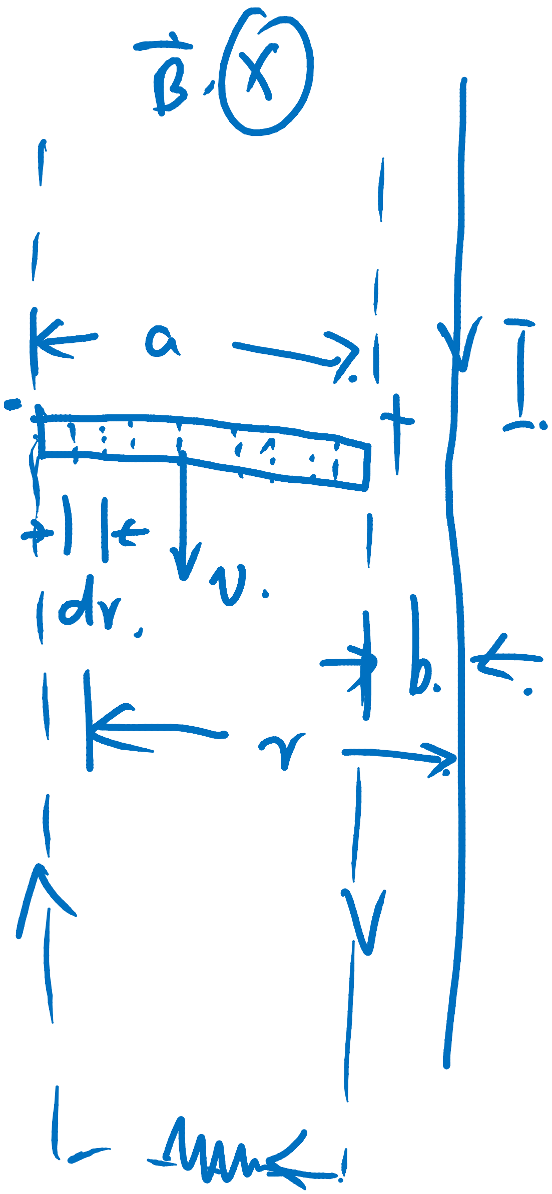
$$\Downarrow$$

$$\int_0^\infty dI$$

$$\| $$

$$I \Big|_{t=0}^{t=\infty} = (0 - I_0)$$

$$|Q| = \frac{\mu_0 c}{2\pi R} \ln\left(\frac{a+b}{a}\right) I_0$$



$\varepsilon = ?$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$d\varepsilon = B\,(dr)\,v$$

$$= \frac{\mu_0 I}{2\pi r}\, v\, dr$$

$$\varepsilon = \int_b^{a+b} \frac{\mu_0 I v}{2\pi r}\, dr$$

$$= \frac{\mu_0 I v}{2\pi} \int_b^{a+b} \frac{dr}{r}$$

$$\varepsilon = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{a+b}{b}\right)$$