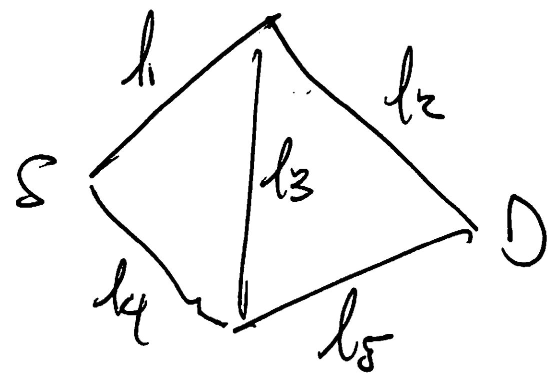
ELEG 310

22 Feb 2018

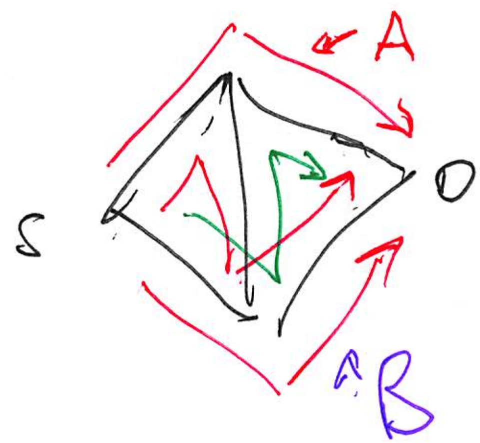


links iid, work with prob p

l_1	l_2	l_3	l_4	l_5	S→D	prob
0	0	0	0	0	0	$(1-p)^5$
0	0	0	0	1	0	$(1-p)^4 p$
0	0	0	1	0	0	
⋮						
1	1	1	1	1	1	p^5

$S \to D = \{ 11000, 11001, 11010, 11011,$
$11100, 11101, 11110, 11111,$
$00011, 00111, 00011, \ldots$
$10101, 10111, \ldots$
$01110, 01111, \ldots \}$

$P(S \to D) = P(11000) + P(11001) + \ldots$
$= p^2(1-p)^3 + p^3(1-p)^2 + \ldots$



$A = \{11000, 11001, \ldots 11111\}$

$P(A) = p^2(1-p)^3 + p^3(1-p)^2 + \cdots + p^5$

$\underbrace{p^2(1-p)^2(1-p+p)}_{p^2(1-p^2)}$

$= p^2(p^3 + p^2(1-p) + p^2(1-p) + p^2(1-p) + p(1-p)^2$
$+ p(1-p)^2 + p(1-p)^2 + (1-p)^3)$

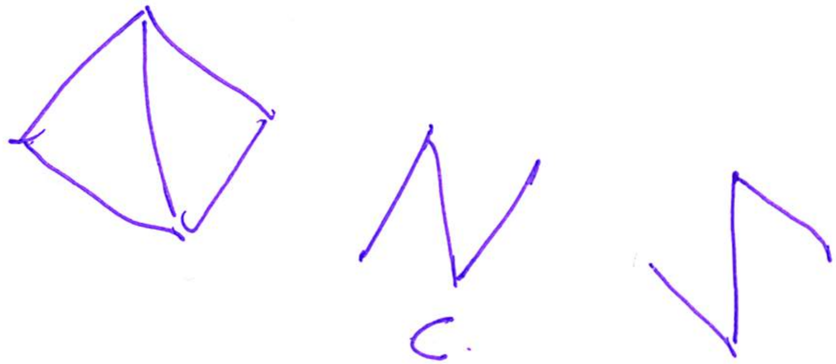
$= p^2$

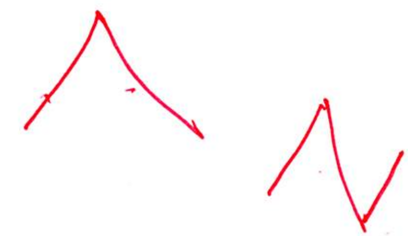
$A = \{l_1 = 1 \text{ and } l_2 = 1\} \Rightarrow P(A) = p^2$

$B = \{l_4 = 1 \text{ and } l_5 = 1\} \Rightarrow P(B) = p^2$

$AB = \{11011, 11111\}$ $P(AB) = p^4(1-p+p) = p^4$

are A + B ind? $P(AB) = p^4 = P(A)P(B) = p^2 p^2 = p^4$ ✓





$$= P(A \cup B \cup C \cup D)$$

$$P(S \to D) = P(A) + P(B) + P(C) + P(D)$$

$$- P(AB) - P(AC) - P(AD) - P(BC) - P(BD) - P(CD)$$

$$+ P(ABC) + P(ABD) + P(ACD) + P(BCD)$$

$$- P(ABCD)$$

$$= p^2 + p^2 + p^3 + p^3$$

$$- p^4 - p^4 - p^4 - p^4 - p^4 - p^5$$

$$+ p^5 + p^5 + p^5 + p^5$$

$$- p^5$$

$\#\text{pairs} = \binom{4}{2} = \frac{4!}{2!2!} = 6$

$\#\text{triples}\ \binom{4}{3} = \frac{4!}{3!1!} = 4$

Birthday Problem k people in room

birthday in common : $\left\{\begin{array}{lll} \text{a pair}, & \text{a triple}, & \text{pair + triple} \\ \binom{k}{2} & \binom{k}{3} & \text{2 pairs} \end{array}\right.$

$P(\text{no birthday in common}) = 1 - P(\text{common birthday})$

$= q(k)$

$$q(k) = 1 \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \frac{365-k+1}{365}$$

$q(1) = 1$

$q(2) = \frac{364}{365}$

$q(3) = q(2) \times \frac{363}{365}$

$q(4) = q(3) \times \frac{362}{365}$

generalize to n days in year

$$q(k) = \frac{n(n-1)\cdots(n-k+1)}{n \cdot n \cdots n} = \frac{(n)_k}{n^k}$$

Birthday Problem — k people in room

Birthday in common $\Rightarrow$ a pair, a triple, quadruple

$\binom{k}{2}$, $\binom{k}{3}$ ... groups

P(no birthday in common) $= 1 - P$(common birthday)

$g(1) = 1$

$g(2) = \frac{364}{365}$

$g(3) = g(2) \times \frac{363}{365}$

$g(4) = g(3) \times \frac{362}{365}$

$= g(k)$

$g(k) = 1 \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \cdots \times \frac{365-k+1}{365}$

generalize to n days in year

$g(k) = \frac{n(n-1)\cdots(n-k+1)}{n \cdot n \cdots n} = \frac{(n)_k}{n^k}$

$n! = n(n-1)(n-2)\cdots 2\cdot 1$ $\qquad 0! = 1$

$\# \text{perms} = \dfrac{n!}{(n-k)!} = n(n-1)\cdots(n-k+1)$

of k items selected from n

$\# \text{combs} = \binom{n}{k} = \dfrac{n!}{(n-k)!\,k!}$

\# ways of selecting k items ordered & w/ repl $= n^k$