

# **Lecture12: Datapath Functional Units**

# Outline

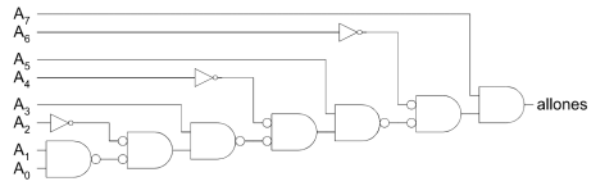
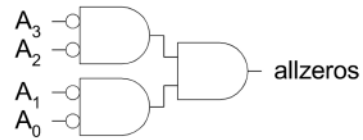
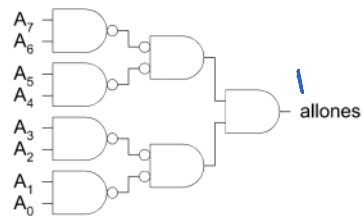
- ☐ Comparators
- ☐ Shifters
- ☐ Multi-input Adders
- ☐ Multipliers

# Comparators

- ☐ 0's detector:  $A = 00\dots000$
- ☐ 1's detector:  $A = 11\dots111$
- ☐ Equality comparator:  $A = B$
- ☐ Magnitude comparator:  $A < B$

# 1's & 0's Detectors

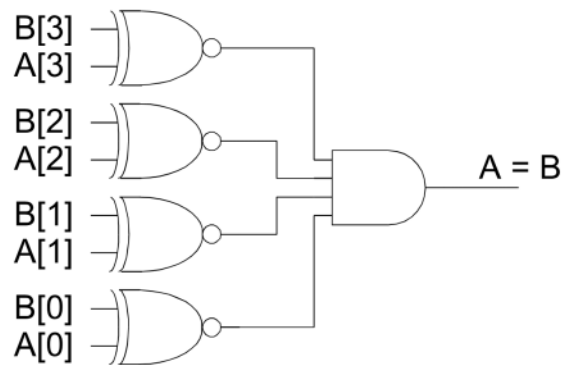
- ❑ 1's detector: N-input AND gate
- ❑ 0's detector: NOTs + 1's detector (N-input NOR)



0000  
0010

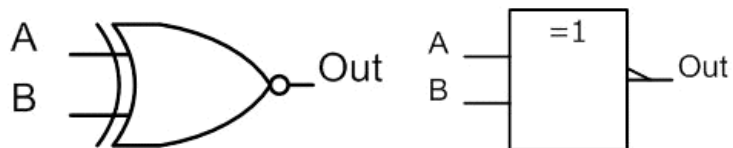
# Equality Comparator

- ❑ Check if each bit is equal (XNOR, aka equality gate)
- ❑ 1's detect on bitwise equality



# XNOR Gate

## ■ Symbols

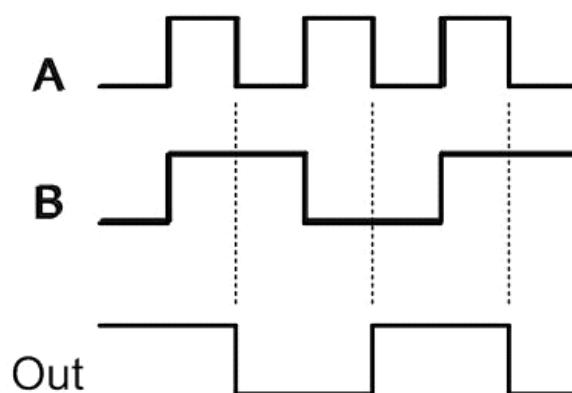


## ■ Truth Table

A	B	Out
0	0	1
0	1	0
1	0	0
1	1	1

242-208 CH2

## ■ Timing Diagram

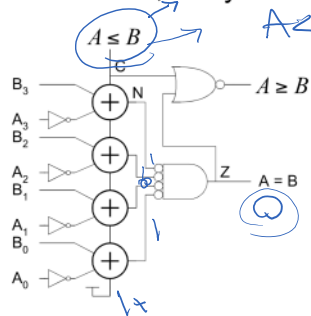


## ■ Logic Expression:

$$\text{Out} = \bar{A} \bar{B} + AB; \bar{A} \oplus \bar{B}$$

# Magnitude Comparator

- ❑ Compute  $B - A$  and look at sign
- ❑  $B - A = B + \bar{A} + 1$
- ❑ For unsigned numbers, carry out is sign bit



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CMOS VLSI Design 4th Ed.

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$$A = 1101 = 8 + 4 + 1 = 13$$

$$B = 1111 = 8 + 4 + 2 + 1 = 15$$

$$\begin{array}{r} \overline{A} = 0010 \\ + B = 1111 \\ \hline \end{array}$$

$$\begin{array}{r} 10010 \\ \hline \end{array}$$

# Shifters

## ❑ Logical Shift:

- Shifts number left or right and fills with 0's

•  $1011 \xrightarrow{\text{LSR } 1} 0101$        $1011 \xrightarrow{\text{LSL } 1} 0110$

## ❑ Arithmetic Shift:

- Shifts number left or right. Rt shift sign extends

•  $1011 \xrightarrow{\text{ASR } 1} 1101$        $1011 \xrightarrow{\text{ASL } 1} 0110$

## ❑ Rotate: $0111 \xrightarrow{\text{ASR } 1} 0011$

- Shifts number left or right and fills with lost bits

•  $1011 \xrightarrow{\text{ROR } 1} 1101$        $1011 \xrightarrow{\text{ROL } 1} 0111$

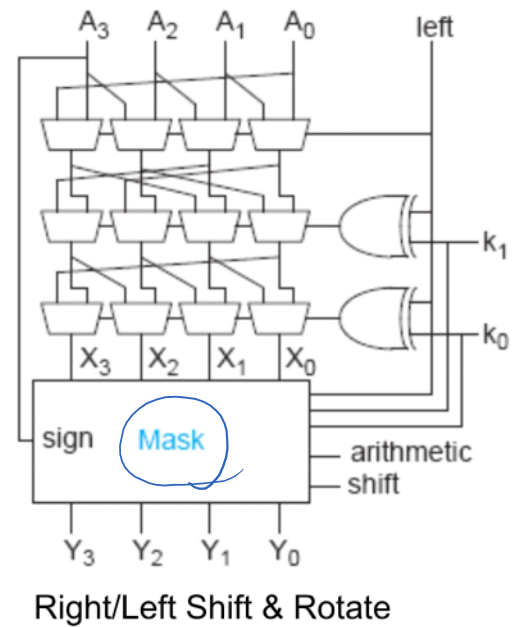
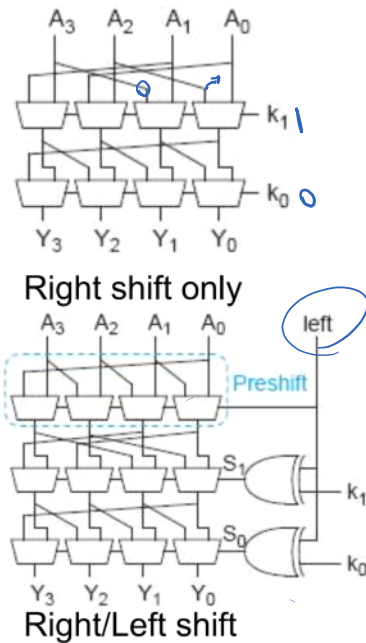
$\text{ROR } 1 \quad 1011 \rightarrow 1101 \Rightarrow 1110 = 0111$   
 $\text{ROL } 1 \quad 1011 \rightarrow 0111$



# Barrel Shifter

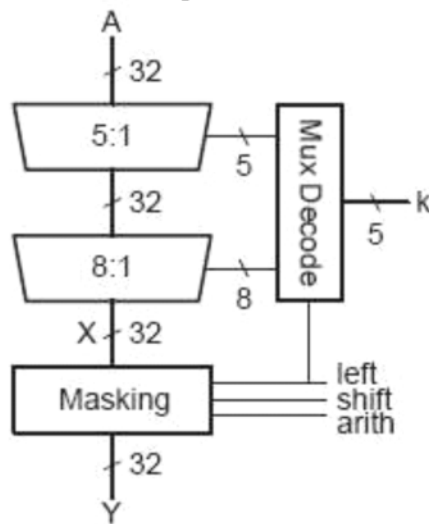
- ❑ Barrel shifters perform right rotations using wrap-around wires.
- ❑ Left rotations are right rotations by  $N - k = \bar{k} + 1$  bits.
- ❑ Shifts are rotations with the end bits masked off.

# Logarithmic Barrel Shifter



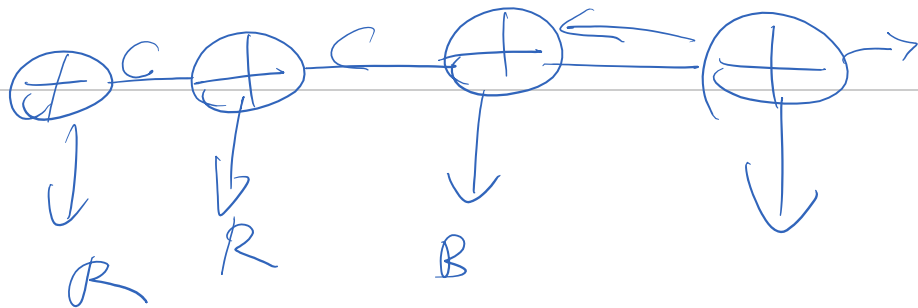
## 32-bit Logarithmic Barrel

- ❑ Datapath never wider than 32 bits
- ❑ First stage preshifts by 1 to handle left shifts



# Multi-input Adders

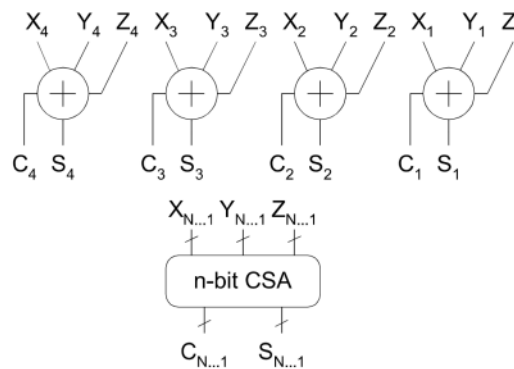
- ❑ Suppose we want to add  $k$   $N$ -bit words
  - Ex:  $0001 + 0111 + 1101 + 0010 = 10111$
- ❑ Straightforward solution:  $k-1$   $N$ -input CPAs
  - Large and slow



# Carry Save Addition

- ❑ A full adder sums 3 inputs and produces 2 outputs
  - Carry output has twice *weight* of sum output
- ❑ N full adders in parallel are called *carry save adder*
  - Produce N sums and N carry outs

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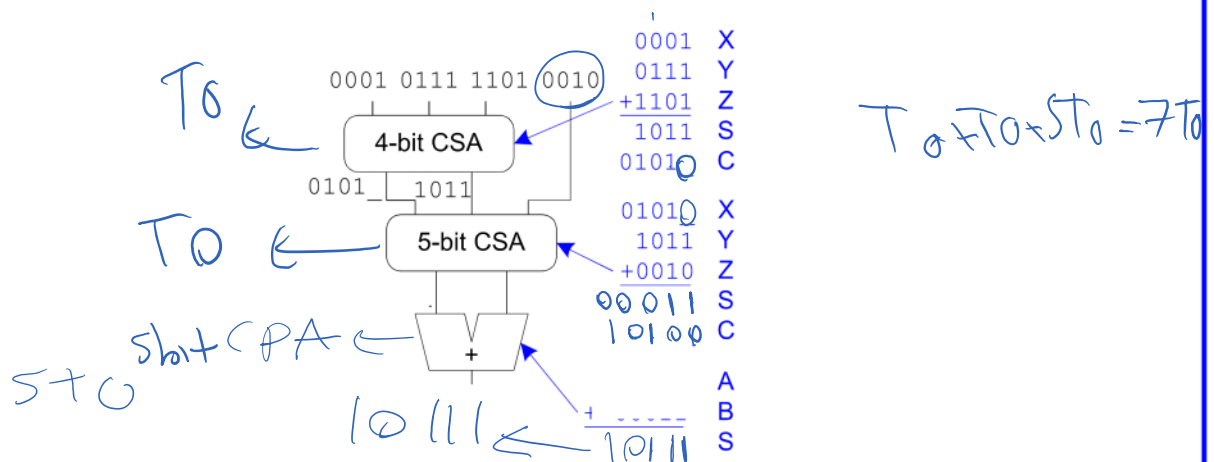
Handwritten binary addition example:

```

  1111
 1001
  ---
0110S
1001C
  
```

# CSA Application

- ❑ Use k-2 stages of CSAs
  - Keep result in carry-save redundant form
- ❑ Final CPA computes actual result



# Multiplication

## Example:

1100	:	$12_{10}$	multiplicand
0101	:	$5_{10}$	multiplier
<hr/>			
1100			partial products
0000			
1100			
0000			
<hr/>			
00111100	:	$60_{10}$	product

32 16 8 4 2 1

## M x N-bit multiplication

- Produce N M-bit partial products
- Sum these to produce M+N-bit product

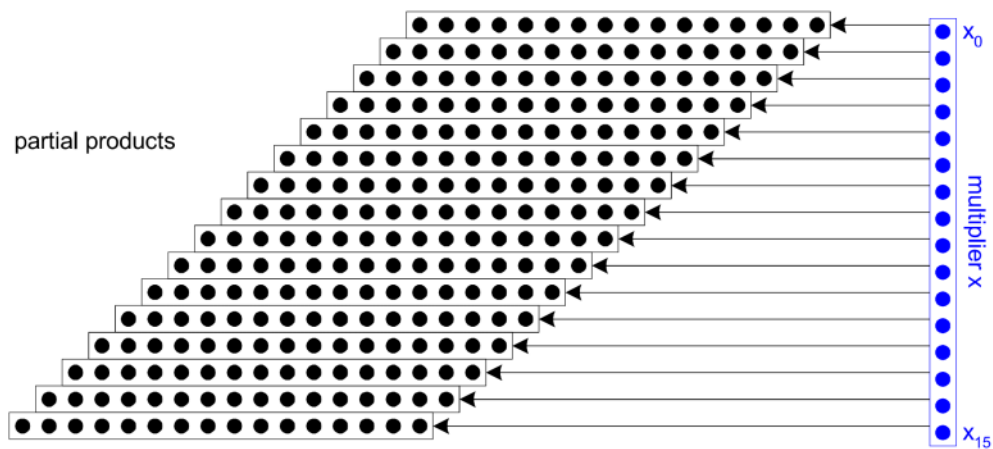
- ❑ Multiplicand:  $Y = (y_{M-1}, y_{M-2}, \dots, y_1, y_0)$
- ❑ Multiplier:  $X = (x_{N-1}, x_{N-2}, \dots, x_1, x_0)$
- ❑ Product: 
$$P = \left( \sum_{j=0}^{M-1} y_j 2^j \right) \left( \sum_{i=0}^{N-1} x_i 2^i \right) = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} x_i y_j 2^{i+j}$$

						$y_5$	$y_4$	$y_3$	$y_2$	$y_1$	$y_0$	multiplicand multiplier
						$x_5$	$x_4$	$x_3$	$x_2$	$x_1$	$x_0$	
						$x_0y_5$	$x_0y_4$	$x_0y_3$	$x_0y_2$	$x_0y_1$	$x_0y_0$	partial products
			$x_1y_5$			$x_1y_4$	$x_1y_3$	$x_1y_2$	$x_1y_1$	$x_1y_0$		
		$x_2y_5$	$x_2y_4$			$x_2y_3$	$x_2y_2$	$x_2y_1$	$x_2y_0$			
	$x_3y_5$	$x_3y_4$	$x_3y_3$			$x_3y_2$	$x_3y_1$	$x_3y_0$				
$x_4y_5$	$x_4y_4$	$x_4y_3$	$x_4y_2$			$x_4y_1$	$x_4y_0$					
$x_5y_5$	$x_5y_4$	$x_5y_3$	$x_5y_2$	$x_5y_1$	$x_5y_0$							product
$p_{11}$	$p_{10}$	$p_9$	$p_8$	$p_7$	$p_6$	$p_5$	$p_4$	$p_3$	$p_2$	$p_1$	$p_0$	

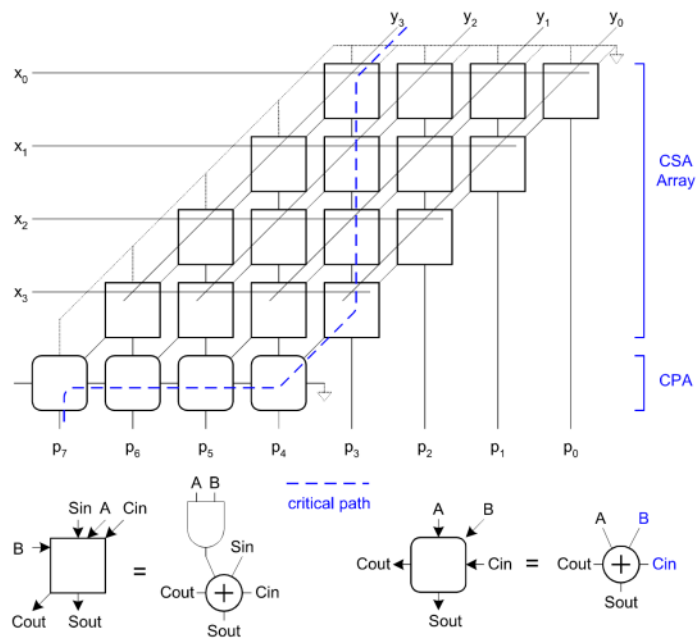


# Dot Diagram

- Each dot represents a bit

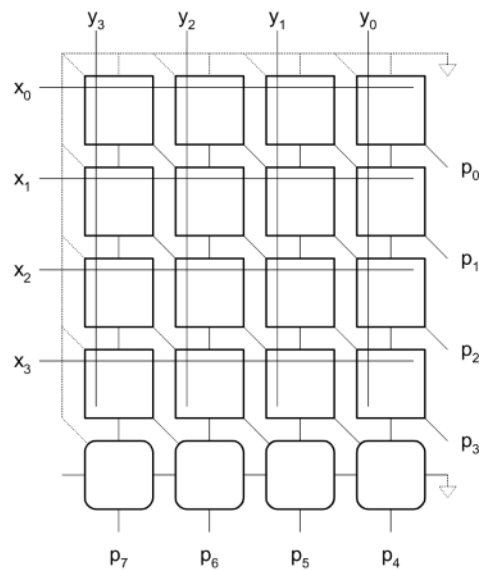


# Array Multiplier



# Rectangular Array

- ❑ Squash array to fit rectangular floorplan



# Advanced Multiplication

- ☐ Booth Encoding
- ☐ Signed vs. unsigned inputs
- ☐ Higher radix Booth encoding
- ☐ Array vs. tree CSA networks