ELEG 310

13 Feb 2018

Bernoulli RV

heads $\Rightarrow 1$

tails $\Rightarrow 0$

X = result of flip
= random variable

$\boxed{\text{Flip coin until heads}}$

$P(X=1)$ = prob exp results in heads
$= p$

$P(X=0) = 1-p = q$ = prob get tails

Outcomes : 1, 01, 001, 0001, ...

Define N = # flips required to get a heads

Outcomes

$N = 1, 2, 3, 4, \ldots$

$P(N=1)$

$$P(N=k) = (1-p)^{k-1} p$$

$k = 0, 1, 2, 3$

= nonnegative integer

Outcome sequence	N	Prob
1	1	p
01	2	$(1-p)p$
001	3	$(1-p)^2 p$
0001	4	$(1-p)^3 p$
⋮	⋮	⋮

$P(N=k) = (1-p)^{k-1} p \geq 0$ for all k

$$1 = \sum_{k=1}^{\infty} P(N=k) = \left(\sum_{k=1}^{\infty} (1-p)^{k-1}\right) p$$

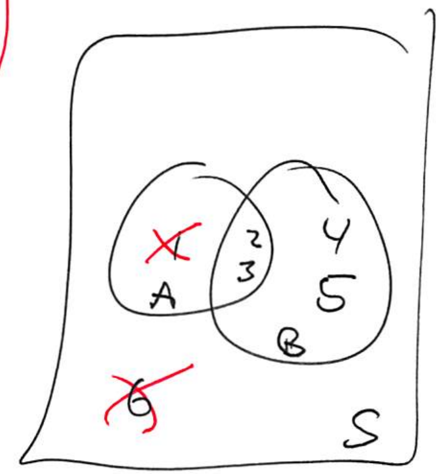
$$= p \sum_{l=0}^{\infty} (1-p)^l = \frac{p}{1-(1-p)} = 1 \quad l = k-1$$

Assume: flips are independent

$$P(AB) = P(A)P(B)$$

$$P(A|B) = P(A \text{ given } B)$$

$$= \frac{P(AB)}{P(B)} \leftarrow \text{definition}$$



$AB = \{2, 3\}$

$P(AB) = P(\{2, 3\})$

$= P(2) + P(3) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

$P(B) = P(2, 3, 4, 5) = \frac{4}{6} = \frac{2}{3}$

$$P(A|B) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$P(N \geq \ell)$ $\quad \ell = 1, 2, 3, \ldots$

$P(N \leq \ell)$ $\quad \ell = 1, 2, 3, \ldots$

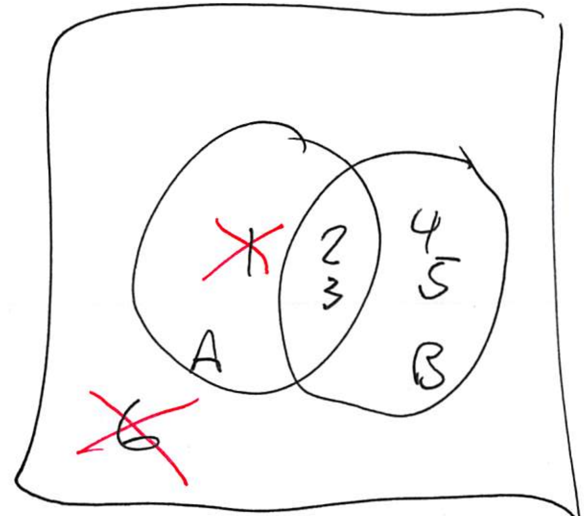
Conditional Probability

$P(A \text{ given } B \text{ is True})$

Exp: roll a fair 6-sided die

$A = \{1, 2, 3\}$

$B = \{2, 3, 4, 5\}$



$S' = \{2, 3, 4, 5\}$ $\quad P(A') = \frac{1}{2}$

$A' = \{2, 3\}$ $\quad P(A') = \frac{1}{4} + \frac{1}{4}$

Bayes Theorem

$$P(B|A) = \frac{P(AB)}{P(A)} \cdot \frac{P(B)}{P(B)} = \frac{P(A|B)\,P(B)}{P(A)}$$

Ex. $A=\{1,2,3\}$ $B=\{2,3,4,5\}$

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(2,3)}{P(1,2,3)} = \frac{\frac{1}{6}+\frac{1}{6}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{6}} = \frac{2}{3}$$

$$= \frac{P(A|B)P(B)}{P(A)} = \frac{\frac{1}{2}\cdot\frac{4}{6}}{\frac{1}{2}} = \frac{2}{3}$$

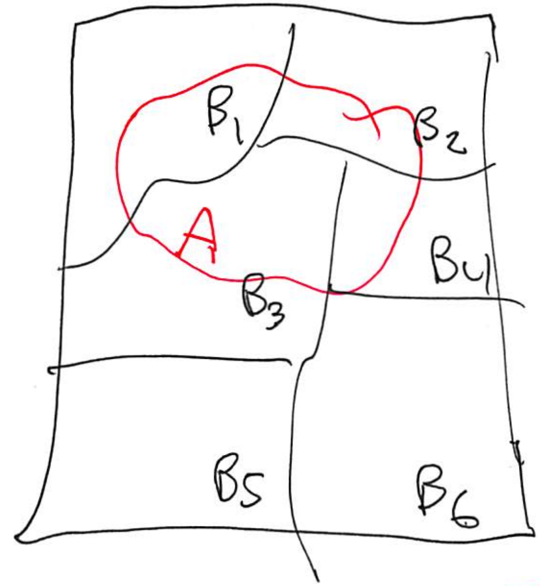
in general

$$P(A|B) \neq P(B|A)$$

Law of Total Probability (LTP)

$B_i B_j = \phi$

$\bigcup_{i=1}^{\infty} B_i = S$



B_i are a partition of S

$$\boxed{P(A)} = P(AS) = P\left(A \bigcup_{i=1}^{\infty} B_i\right) = P\left(\bigcup_{i=1}^{\infty} AB_i\right) = \boxed{\sum_{i=1}^{\infty} P(AB_i)}$$

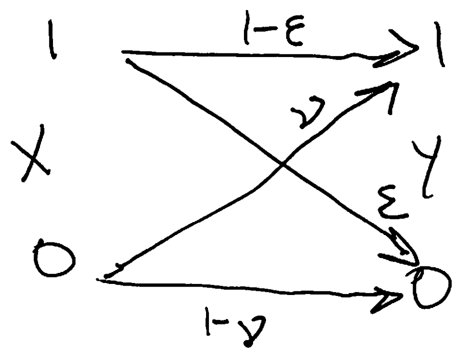
$AS = A$

~~Bayes Theorem~~ $P(A|B) = \dfrac{P(AB)}{P(B)}$

$$\boxed{P(AB) = P(A|B)P(B)}$$

$$\boxed{P(A) = \sum_{i=1}^{\infty} P(A|B_i)P(B_i)}$$

Ex. Binary Channel



$$P(Y=1 \mid X=1) = 1-\varepsilon$$
$$P(Y=0 \mid X=1) = \varepsilon$$
$$P(Y=1 \mid X=0) = \nu$$
$$P(Y=0 \mid X=0) = 1-\nu$$

$$P(X=1) = p$$
$$P(X=0) = 1-p.$$

$$P(Y=1) = P($$