Introduction to ANOVA

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Overview

- Our next set of lectures provides an introduction to **ANOVA** and **Experimental Design**
- This is a direct extension of the difference of means test we focused on earlier - ANOVA will do difference of means tests
- ANOVA is heavily used in Designed Experiements
- ANOVA will now allow us to compare multiple groups, not just two
- In order to do this, we have to be able to make some assumptions - equal variances and such
- This lecture will get us started with some terms and what is a designed experiment

ANOVA

- ANOVA provides a strategy to compare **two or more** population means associated with various treatments
- It is used when we have
  - A **Dependent Variable** (called a **Response Variable**) which is measured continuously
  - One or more **Factors** - levels of one or more categorical variables, thought of as **Independent Variables**
    - independent variables are thought to influence the dependent variable
    - E.g. treatment versus control; group membership; different levels of a treatment

ANOVA

- **ANOVA** is used heavily in experimental designs in the biological sciences
  - Treatment versus control groups
  - Levels of treatment of a drug
  - Levels of applications of fertilizers or pesticides
- It is possible to have both continuous and categorical independent variables
- Its origins were in agricultural studies, but it applications can now be found in biological and health related research; business; and economics

Designed Experiment versus an Observational Study

- **Designed Experiment**
  - The researcher manipulates the treatments and randomly assigns subjects to the treatments
  - The specification of the treatments and the way experimental units are assigned to treatments is under the control of the researcher
- **Observational Study**
  - The researcher observes the treatments and the response on a sample of experimental units
  - In essence we sample from a population where the treatments are already present

Elements of a Designed Experiment

- **Response Variable:** the variable of interest to be measured in the experiment. Also known as the dependent variable.
- **Factors:** variables which are thought to influence the response variable
  - Quantitative
  - Qualitative
- **Factor Levels:** the levels of the factor that are experimentally manipulated
- **In a single factor experiment, the factor levels can also be called the treatments**

Elements of a Designed Experiment

- **Treatments:** when two or more factors are utilized, the treatments are the combinations of factor levels used in the experiment.
- Factor 1: fertilizer (low; medium; high)
- Factor 2: water (low; high)
- The Treatments are:
  - Treatment 1: low fertilizer, low water
  - Treatment 2: low fertilizer, high water
  - Treatment 3: medium fertilizer, low water
  - And so forth……
- A special treatment that is used as a benchmark to compare the other treatments is called the **control treatment**.

Elements of a Designed Experiment

- **Experimental Unit:** the physical entity to which each treatment is randomly assigned
- **Measurement Unit:** the physical entity from which a measurement is taken.
- **Replications:** it is best to repeat the treatment on more than one experimental unit to get a better measurement of an effect.
  - If there is only one unit per treatment it is a single replication
  - We generally prefer to randomly assign the treatment to several experimental units.

Completely Randomized Design

- The treatments are **randomly assigned** to the experimental units
- **Or independent random samples** of experimental units are selected from target populations for each treatment
- Many texts refer to both designed and observational studies as being **randomized designs**
- **Most think of the best use of ANOVA for designed experiments**

An example of a designed experiment: shrimp weight gain

- A researcher is studying the conditions under which commercially raised shrimp reach maximum weight gain.
- Three water temperatures (25°, 30°, 35°) and four water salinity levels (10%, 20%, 30%, 40%) are examined
- While there are many other factors which influence shrimp growth, such as density of shrimp in a container, variety, and type of feeding - these two are the focus of this research.
- A specific variety and size of shrimp are selected for the study.
- The a fixed density of shrimp (40 per container) are randomly assigned to the treatments in 24 experimental containers, one shrimp to a container
- After 6 weeks the individual shrimp are weighed

Taken from Statistical Methods and Data Analysis, 5th Edition, Ott and Longnecker, 2001

An example of a designed experiment: shrimp weight gain

- The treatments are (temperature, salinity):
  - (25, 10%) (25, 20%) (25, 30%) (25, 40%)
  - (30, 10%) (30, 20%) (30, 30%) (30, 40%)
  - (35, 10%) (35, 20%) (35, 30%) (35, 40%)
- Replications: 2 for each treatment

An example of a designed experiment: shrimp weight gain

- Here is the experimental design
  - **Response Variable:** Weight of the shrimp in the container
  - **Factors:** 2 factors (temperature and salinity)
  - **Treatments:** 12
  - **Replication:** 2 per treatment
  - **Experimental Unit:** the container
  - **Measurement Unit:** the container
- Total sample size: 24
- This is an efficient design undertaken to estimate the effects of salinity and temperature on shrimp weight gain

What is ANOVA? Hint!!!

It is all about the variance!

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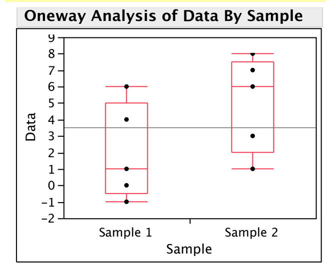
It is... ...all about the variance!

It is all about th
eVARIANCE!

Let's look at a simple problem

- I have a small data set of 5 observations for two variables.
- The difference between sample means is relatively small when compared to the variability within the sample observations
- We would say, **"The effect is small relative to the variability within each sample."**

Obs	Sample 1	Sample 2
1	6	8
2	-1	1
3	0	3
4	4	7
5	1	6
Sum	10.0	25.0
Mean	2.0	5.0
Var	8.5	8.5
Std Dev	2.9	2.9



Look at the result of the Difference of Means test from Excel

- The means are different
- The variances are similar so we pool the variance
- But we have a hard time rejecting a Null Hypothesis that the two means are equal to each other
- The t* = 1.627
- The p-value for a two-tailed test is .142
- We cannot reject Ho: $\mu_2 - \mu_1 = 0$

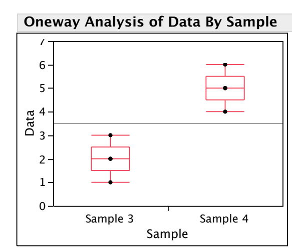
t-Test: Two-Sample Assuming Equal Variances

	Sample 2	*Sample 1*
Mean	5.00	2.00
Variance	8.50	8.50
Observations	5	5
Pooled Variance	8.5	
Hypothesized Mean Diff	0	
df	8	
t Stat	1.627	
P(T<=t) one-tail	0.071	
t Critical one-tail	1.860	
P(T<=t) two-tail	0.142	
t Critical two-tail	2.306	

Compare those results to this data

- I have the same mean differences
- But the spread of the data are much different - there is no overlapping of values
- The difference between sample means is relatively large when compared to the variability within the sample observations

Obs	Sample 3	Sample 4
1	2	5
2	3	5
3	2	5
4	2	4
5	1	6
Sum	10.0	25.0
Mean	2.0	5.0
Var	0.5	0.5
Std Dev	0.7	0.7



And when I do the difference of Means test in Excel

- Now it is easy to reject a Null Hypothesis that the two means are equal to each other
- The t* = 6.708
- The p-value for a two-tailed test is .000 or p < .001
- We can reject Ho: $\mu_2 - \mu_1 = 0$
- **When the effect is large relative to the variances within each group, I will be better able to reject the Null Hypothesis**

t-Test: Two-Sample Assuming Equal Variances

	Sample 4	*Sample 3*
Mean	5.00	2.00
Variance	0.50	0.50
Observations	5	5
Pooled Variance	0.5	
Hypothesized Mean Difference	0	
df	8	
t Stat	6.708	
P(T<=t) one-tail	0.000	
t Critical one-tail	1.860	
P(T<=t) two-tail	0.000	
t Critical two-tail	2.306	

Example of Contrasts of Two Samples

- One way to determine whether a difference exists between the population means is to examine the difference between the sample means and compare it to a measure of variability within the samples.
- This is what we do in a Difference of Means Test - the variability is accounted for in the standard error
  - The difference of the two means is only part of the story.
  - The other part is the variability and separation of the two samples
- ANOVA uses this strategy to compare and test the difference between two or more means
  - In essence it compares the variability across group means
  - To the variability within each group

ANOVA Strategy

- In ANOVA, we will decompose the variance of our dependent variable
  - Part due to the treatments or independent variables – this part is thought to be “explained” by our model.
  - Part that is “unexplained” or random “error”
- I will adjust these variances from different sources by dividing by degrees of freedom to get an average deviation
- We will decompose the Total Sum of Squares =
  - Sum of Squares for Treatment
  - Sum of Squares for Error

$$SS(Total) = \sum_{i=1}^{n} (y_i - \overline{Y})^2$$

ANOVA Model

- This is one way to look at the ANOVA model with i observations and j factor levels
- Each observation is a function of:
  - A Grand Mean
  - The deviations of each Factor Level Mean from the Grand Mean
  - Some random error

$$Y_{ij} = \mu + \tau_{\bullet j} + \varepsilon_{ij}$$

Another viewpoint

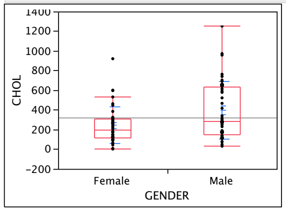
- The difference of an individual value from the Grand Mean is a function of: $Y_{ij} - \overline{Y} =$
- **The difference of the Factor Level Mean from the Grand Mean** — **Variability between Factor Levels**
- **The difference between the value and its Factor Level Mean** — **Variability within Factor Levels**

We want the variability between Factor Levels to be large relative to the variability within Factor Levels

Another Data Example

- This is some data comparing the cholesterol level of 40 males to 40 females
- We can see that the level for males is higher
- Since this a sample, I need a statistical tests to see if there is a difference.
- We can do a difference of means test from JMP
  - $t^* = 2.8169$
  - $p < .01$ for a 2-tailed test
  - We can conclude there is a difference in cholesterol levels between males and females

Oneway Analysis of CHOL By GENDER



Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
Female	40	240.875	185.982	29.406	181.39	300.36
Male	40	395.225	292.412	46.234	301.71	488.74

I can do the same analysis using ANOVA

- ANOVA - randomized design for an observational study
- **Response Variable:** cholesterol level
- **Factor:** gender
  - Females
  - Males
- **Experimental Units:** people
- We conduct an F-test with ANOVA

Excel: ANOVA, Single Factor

Mean, variance, and sample size for each group

Anova: Single Factor

SUMMARY

Groups	Count	Sum	Average	Variance
Females	40	9635.000	240.875	34589.446
Males	40	15809.000	395.225	85504.640

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	476478.450	1	476478.450	7.935	0.006	3.963
Within Groups	4683669.350	78	60047.043			
Total	5160147.8	79				

ANOVA Table with Sums of Squares breakdown

ANOVA

- ANOVA Table with Sums of Squares breakdown
  - SS Treatment **1,393.425** Between Groups
  - SS Error **41,846.879** Within Groups
  - SS Total **43,240.304**
- F-test

Anova: Single Factor

SUMMARY

Groups	Count	Sum	Average	Variance
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ANOVA

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Within Groups	4683669.350	78	60047.043			
Total	5160147.8	79				

Summary

- This is just an introduction to ANOVA. We will spend several lectures on the approach and some more complex models
- ANOVA comes with its own terms and jargon!
- There is a connection between the difference of means test and ANOVA - ANOVA will generate the same conclusion.
- But ANOVA will allow us to extend this to many means, not just two
- The next steps are looking at the F-distribution and the logic of the hypothesis test in ANOVA

Extra Problem

- An experiment is designed to examine protective coatings on frying pans.
- Four different coatings are being examined.
- Five frying pans are randomly assigned to each of the four coatings.
- A measure of abrasive resistance of the coatings is taken at three locations on each of the 20 pans.
- **Identify the following for this study:**
  - **Response variable**
  - **Treatments**
  - **Replications**
  - **Experimental Unit**
  - **Measurement Unit**
  - **Total sample size**