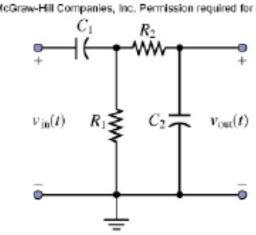
Problem 1

Solution:



Known quantities:

Resistance, inductance and capacitance values, in the circuit of Figure P6.5

Find:

a) The frequency response for the circuit of Figure P6.5

b) Plot magnitude and phase of the circuit using a linear scale for frequency.

c) Repeat part b., using semilog paper.

d) Plot the magnitude response using semilog paper with magnitude in dB.

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = Z_{R2} + (Z_{C1} \parallel Z_{R1}) = R_2 + \frac{R_1 / j\omega C_1}{1/j\omega C_1 + R_1} = R_2 + \frac{R_1}{1 + j\omega C_1 R_1}$$

and

$$v_{OC} = \frac{Z_{R1}}{Z_{R1} + Z_{C1}} v_{in} = \frac{R_1}{R_1 + 1/j\omega C_1} v_{in} = \frac{j\omega C_1 R_1}{1 + j\omega C_1 R_1} v_{in}$$

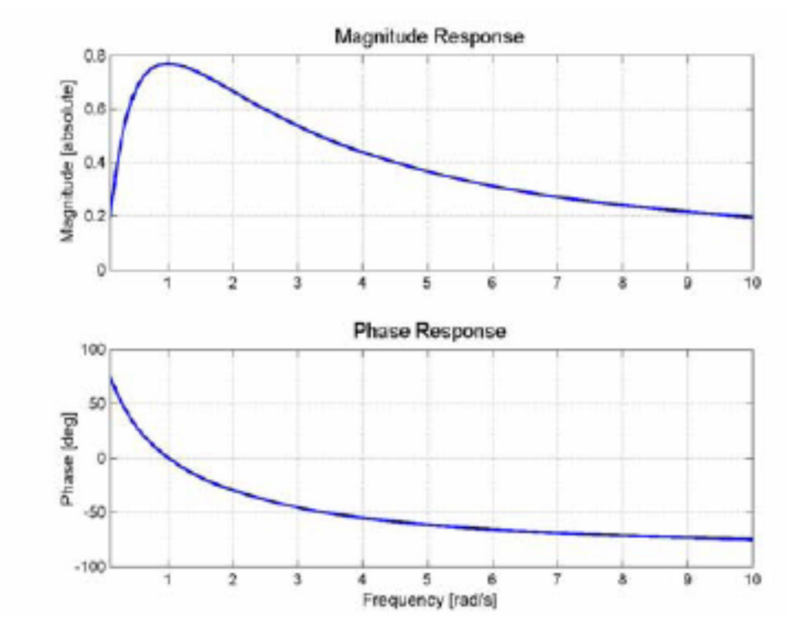
a)
$$\frac{v_{out}}{v_{OC}} = \frac{Z_{C2}}{Z_T + Z_{C2}} = \frac{1/j\omega C_2}{\left(R_2 + \frac{R_1}{1 + j\omega C_1 R_1}\right) + 1/j\omega C_2} = \frac{1}{1 + \left(R_2 + \frac{R_1}{1 + j\omega C_1 R_1}\right) j\omega C_2}$$

Therefore,

$$\frac{v_{out}}{v_{in}} = \frac{j\omega C_1 R_1}{1 + j\omega C_1 R_1} \cdot \frac{1}{1 + \left(R_2 + \frac{R_1}{1 + j\omega C_1 R_1}\right) j\omega C_2} = \frac{j\omega C_1 R_1}{1 + j\omega\left[C_1 R_1 + C_2 (R_1 + R_2)\right] + (j\omega)^2 C_1 C_2 R_1 R_2}$$

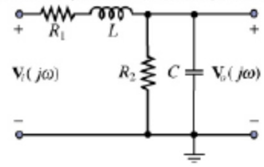
Substituting the numerical values:

$$\frac{v_{out}}{v_{in}} = \frac{j(2)\omega}{(1 - \omega^2) + j(2.6)\omega}$$



Problem 2

Problem 6.53



Solution:

Known quantities:

Figure P6.53.

Find:

a) If this is a low-pass, high-pass, band-pass, or band-stop filter.

b) Compute and plot the frequency response function if:

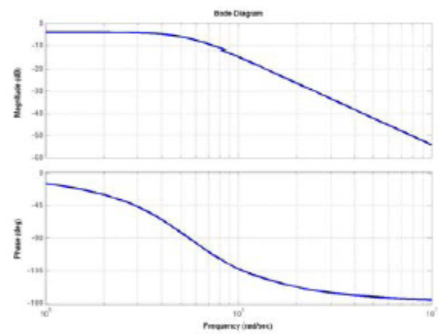
$L = 11\ \text{mH} \quad C = 0.47\ \text{nF} \quad R_1 = 2.2\ \text{k}\Omega \quad R_2 = 3.8\ \text{k}\Omega$

Analysis:

a)

$$As\ \omega \to 0: \quad Z_L \to 0 \Rightarrow Short$$
$$Z_C \to \infty \Rightarrow Open$$
$$\Rightarrow VD: \ H_v = \frac{V_o}{V_i} \to \frac{R_2}{R_1 + R_2}$$

$$As\ \omega \to \infty: \quad Z_L \to \infty \Rightarrow Open$$
$$Z_C \to 0 \Rightarrow Short$$
$$\Rightarrow H_v \to 0$$



The filter is a low pass filter.

b) First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = (Z_{R1} + Z_L) \| Z_{R2} = \left(\frac{1}{R_1 + j\omega L} + \frac{1}{R_2}\right)^{-1} = \frac{(R_1 + j\omega L)R_2}{R_1 + j\omega L + R_2}$$

and

$$\mathbf{V}_{OC} = \frac{Z_{R2}}{Z_{R1} + Z_L + Z_{R2}}\mathbf{V}_{in} = \frac{R_2}{R_1 + j\omega L + R_2}\mathbf{V}_{in}$$

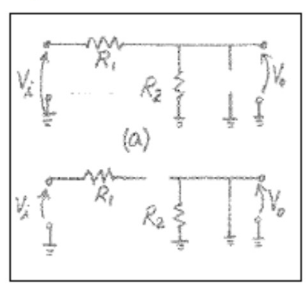
$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{OC}} = \frac{Z_C}{Z_T + Z_C} = \frac{1/j\omega C}{\frac{(R_1 + j\omega L)R_2}{R_1 + j\omega L + R_2} + 1/j\omega C} = \frac{R_1 + j\omega L + R_2}{R_1 + j\omega L + R_2 + (R_1 + j\omega L)j\omega CR_2}$$

Therefore,

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{R_2}{R_1 + j\omega L + R_2} \cdot \frac{R_1 + j\omega L + R_2}{R_1 + j\omega L + R_2 + (R_1 + j\omega L)j\omega CR_2} = \frac{1}{1 + \frac{R_1}{R_2} + j\omega\left(\frac{L}{R_2} + CR_1\right) + (j\omega)^2 LC}$$

Substituting the numerical values:

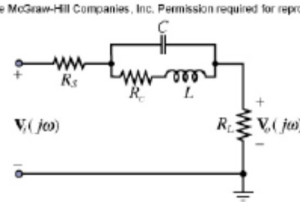
$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1}{\left(1.579 - 5.17 \times 10^{-12}\omega^2\right) + j\left(3.929 \times 10^{-6}\right)\omega}$$



The corresponding Bode diagrams are shown in the Figure.

Problem 3

Problem 6.58



Solution:

Known quantities:

The values of the resistors, of the capacitance and of the inductance in the circuit of Figure P6.58.

Find:

Compute and plot the voltage frequency response function. What type of filter is this?

Analysis:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$Z_T = Z_{R_s} + Z_C \parallel (Z_{Rc} + Z_L) = R_S + \left(j\omega C + \frac{1}{R_C + j\omega L} \right)^{-1}$$

$$= R_S + \frac{(R_C + j\omega L)}{j\omega C(R_C + j\omega L) + 1} = \frac{(R_C + j\omega L) + R_S\left[1 + j\omega C(R_C + j\omega L)\right]}{j\omega C(R_C + j\omega L) + 1}$$

and $\mathbf{V}_{OC} = \mathbf{V}_{in}$

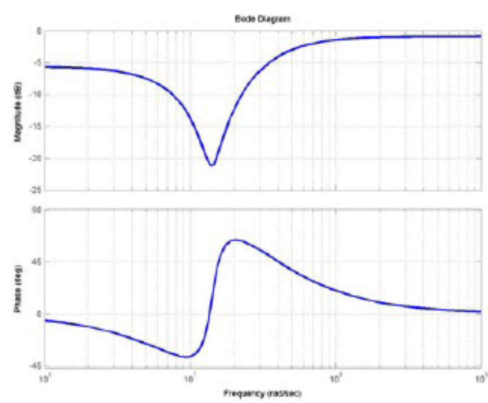
$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{Z_{R_L}}{Z_T + Z_{R_L}} = \frac{R_L}{Z_T + R_L} = \frac{1}{1 + \frac{Z_T}{R_L}} =$$

Therefore,
$$= \frac{j\omega C(R_C + j\omega L) + 1}{j\omega C(R_C + j\omega L) + 1 + \left\{(R_C + j\omega L) + R_S\left[1 + j\omega C(R_C + j\omega L)\right]\right\}/R_L} =$$

$$= \frac{1 + j\omega CR_C + (j\omega)^2 LC}{\left(1 + \frac{R_C + R_S}{R_L}\right) + j\omega\left[CR_C\left(1 + \frac{R_S}{R_L}\right) + \frac{L}{R_L}\right] + (j\omega)^2 LC\left(1 + \frac{R_S}{R_L}\right)}$$

Substituting the numerical values:
$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1 + j\left(2\times10^{-8}\right)\omega + (j\omega)^2 5\times10^{-15}}{(j\omega)^2 5.5\times10^{-15} + j\left(2.22\times10^{-7}\right)\omega + 1.9}$$

The corresponding Bode diagrams are shown in the figure.



The magnitude of the voltage transfer function is lowest at the resonant frequency and increases at higher and lower frequencies. Therefore, this is a band stop or "notch" filter.

At its resonant frequency, a parallel resonant circuit has a high equivalent resistance that is resistive. Connected here in series with the load, this high impedance reduces the magnitude of the voltage transfer function [or voltage gain or insertion loss] at the resonant frequency.

The loading due to the inductor losses, modeled here as an equivalent "coil" resistance, is fairly small giving a substantially lower gain at the resonant frequency compared with the gain at higher or lower frequencies. Therefore this is a high "Q" circuit with good performance and selectivity. The inductor losses also affect only slightly the resonant frequency.

The cutoff frequencies are difficult [but not impossible] to determine in circuits containing a parallel resonant circuit which includes inductor losses, so no attempt was made to do so.