Problem 1

8-8.1 See Figure 8.16.

(a) Initially, the charge on C is zero: $Q_2 = 0$. Hence the voltage drop across C is zero. Since this is in parallel with R, the voltage drop across R is zero, or $\Delta V = 0$. Hence the current through R is zero: $I_1 = 0$. Since $I = I_1 + I_2$, we have $I = I_2$ initially. By $I = \frac{(\mathcal{E} - \Delta V)}{r}$ we obtain $I = \frac{(12-0)}{2} = 6$ A.

(b) After a long time, C charges up, so $I_2 = 0$. Then $I = I_1$, and r and R are in series, so we can use $I = \frac{\mathcal{E}}{(r+R)} = \frac{12}{(2+6)} = 1.5$ A. The voltage drop across both R and C is thus $\Delta V = IR = (1.5)(6) = 9$ V, so $Q_2 = C\Delta V = 40.5\ \mu$C.

Problem 2

44. (*a*) From Eq. 26-7 the product RC is equal to the time constant.

$$\tau = RC \quad \rightarrow \quad C = \frac{\tau}{R} = \frac{24.0\times10^{-6}\text{s}}{15.0\times10^{3}\Omega} = \boxed{1.60\times10^{-9}\,\text{F}}$$

(*b*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \text{e}\left(1 - e^{-t/\tau}\right) \rightarrow e^{-t/\tau} = \left(1 - \frac{V_C}{\text{e}}\right) \rightarrow -\frac{t}{\tau} = \ln\left(1 - \frac{V_C}{\text{e}}\right) \rightarrow$$

$$t = -\tau\ln\left(1 - \frac{V_C}{\text{e}}\right) = -\left(24.0\times10^{-6}\text{s}\right)\ln\left(1 - \frac{8.0\,\text{V}}{24.0\,\text{V}}\right) = \boxed{9.73\times10^{-6}\text{s}}$$

Problem 3

46. Express the stored energy in terms of the charge on the capacitor, using Eq. 24-5. The charge on the
capacitor is given by Eq. 26-6a.

$$U = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}\frac{\left[C\text{e}\left(1-e^{-t/\tau}\right)\right]^2}{C} = \tfrac{1}{2}C\text{e}^2\left(1-e^{-t/\tau}\right)^2 = U_{\max}\left(1-e^{-t/\tau}\right)^2 \quad ;$$

$$U = 0.75U_{\max} \rightarrow U_{\max}\left(1-e^{-t/\tau}\right)^2 = 0.75U_{\max} \rightarrow \left(1-e^{-t/\tau}\right)^2 = 0.75 \rightarrow$$

$$t = -\tau\ln\left(1-\sqrt{0.75}\right) = \boxed{2.01\tau}$$

Problem 4

$\boxed{49.}$ (*a*) At $t = 0$, the capacitor is uncharged and so there is no voltage difference across it. The capacitor
is a "short," and so a simpler circuit can be drawn just by eliminating the capacitor. In this simpler circuit, the two resistors on the right are in parallel with each other, and then in series with the resistor by the switch. The current through the resistor by the switch splits equally when it reaches the junction of the parallel resistors.

$$R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \tfrac{3}{2}R \;\rightarrow\; I_1 = \frac{e}{R_{\text{eq}}} = \frac{e}{\tfrac{3}{2}R} = \boxed{\frac{2e}{3R}} \;;\; I_2 = I_3 = \tfrac{1}{2}I_1 = \boxed{\frac{e}{3R}}$$

(b) At $t = \infty$, the capacitor will be fully charged and there will be no current in the branch

containing the capacitor, and so a simpler circuit can be drawn by eliminating that branch. In this simpler circuit, the two resistors are in series, and they both have the same current.

$$R_{\text{eq}} = R + R = 2R \;\rightarrow\; I_1 = I_2 = \frac{e}{R_{\text{eq}}} = \boxed{\frac{e}{2R}} \;;\; I_3 = \boxed{0}$$

(c) At $t = \infty$, since there is no current through the branch containing the capacitor, there is no

potential drop across that resistor. Therefore the voltage difference across the capacitor equals the voltage difference across the resistor through which I_2 flows.

$$V_C = V_{R_2} = I_2 R = \left(\frac{e}{2R}\right)R = \boxed{\tfrac{1}{2}e}$$

Problem 5

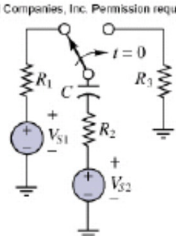
Solution:

Known quantities:

Circuit shown in Figure P5.22, $V_{S1} = 35V, V_{S2} = 130V, C = 11\mu F, R_1 = 17k\Omega, R_2 = 7k\Omega, R_3 = 23k\Omega$.

Find:

At $t = 0^+$ the initial current through R_3 just after the switch is changed.



Assumptions:

None.

Analysis:

To solve this problem, find the steady state voltage across the capacitor before the switch is thrown. Since the voltage across a capacitor cannot change instantaneously, this voltage will also be the capacitor voltage immediately after the switch is thrown. At that instant, the capacitor may be viewed as a DC voltage source.

At $t = 0^-$:

Determine the voltage across the capacitor. At steady state, the capacitor is modeled as an open circuit:

$i_{R1}(0^-) = i_{R2}(0^-) = 0$

Apply KVL:

$$V_{S1} + 0 - V_C(0^-) + 0 - V_{S2} = 0$$

$$V_C(0^-) = V_{S1} - V_{S2} = -95V$$

At $t = 0^+$:

$V_C(0^+) = V_C(0^-) = -95V$

$i_{R2}(0^+) = i_{R3}(0^+)$

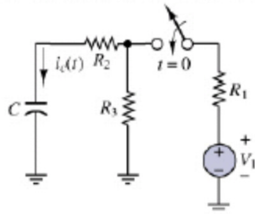
Apply KVL:

$$V_{S2} - i_{R3}(0^+)R_2 + V_C(0^+) - i_{R3}(0^+)R_3 = 0$$

$$i_{R3}(0^+) = \frac{V_{S2} + V_C(0^+)}{R_2 + R_3} = \frac{130 - 95}{7\times10^3 + 23\times10^3} = 1.167mA$$

Problem 6

Solution:



Known quantities:

Circuit shown in Figure P5.27,
$V_1 = 12\ V, R_1 = 0.68\ k\Omega, R_2 = 2.2\ k\Omega, R_3 = 1.8\ k\Omega, C = 0.47\ \mu F$.

Find:

The current through the capacitor at $t = 0^+$, just after the switch is closed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

For t < 0, the switch is open and no power source is connected to the left half of the circuit. In steady state, by definition, the voltage across the capacitor and the current out of it must be constant. However, without a power source to replenish the energy dissipated by the resistors, that constant must be zero. Otherwise, current would flow out of the capacitor, its voltage would drop as it lost charge, and the energy of that charge would be dissipated by the resistors. This process would continue until no net charge remained on the capacitor and its voltage was zero. At steady state, then, the voltage across the capacitor is zero.

At $t = 0^+$, the voltage across the capacitor is still zero since the voltage across a capacitor cannot change instantaneously. At that instant, the capacitor can be treated as a voltage source of strength zero (i.e. a short-circuit.) However, the current through the capacitor can change instantaneously (or relatively so) from 0 to a new value. In this problem it will change as the switch is closed because the voltage source V1 will drive current through R1and the parallel combination of R_2 and R_3. The current through R_2 is the capacitor current.

$$v_C(0^+) = v_C(0^-) = 0$$

$$\frac{V_{R3}(0^+) - 0}{R_2} + \frac{V_{R3}(0^+)}{R_3} + \frac{V_{R3}(0^+) - V_1}{R_1} = 0$$

Apply KCL:

$$V_{R3}(0^+) = \frac{\frac{V_1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{V_1}{1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}} = \frac{12}{1 + \frac{0.68}{2.2} + \frac{0.68}{1.8}} = 7.114V$$

Recall that the voltage across the capacitor (Volts = Joules/Coulomb) represents the energy stored in the electric field between the plates of the capacitor. The electric field is due to the amount of charge stored in the capacitor and it is not possible to instantaneously remove charge from the capacitor's plates. Therefore, the voltage across the capacitor cannot change instantaneously when the circuit is switched.

However, the <u>rate</u> at which charge is removed from the plates of the capacitor (i.e. the capacitor current) <u>can</u> change instantaneously (or relatively so) when the circuit is switched.

Note also that these conditions hold only at the instant $t = 0^+$. For $t > 0^+$, the capacitor is gaining charge, all voltages and currents exponentially approach their final or steady state values.

Apply KVL:

$$-V_C(0^+) + i_C(0^+)R_2 + V_{R3}(0^+) = 0 \Rightarrow i_C(0^+) = \frac{V_{R3}(0^+) - V_C(0^+)}{R_2} = \frac{7.114}{2.2 \times 10^3} = 3.234mA$$

Problem 7

89. (*a*) After the capacitor is fully charged, there is no current through it, and so it behaves like an "open" in the circuit. In the circuit diagram, this means that $I_5 = 0$, $I_1 = I_3$, and $I_2 = I_4$. Write loop equations for the leftmost loop and the outer loop in order to solve for the currents.

$$\mathcal{E} - I_2(R_2 + R_4) = 0 \rightarrow I_2 = \frac{\mathcal{E}}{R_2 + R_4} = \frac{12.0\,\text{V}}{10.0\,\Omega} = 1.20\,\text{A}$$

$$\mathcal{E} - I_1(R_1 + R_3) = 0 \rightarrow I_1 = \frac{\mathcal{E}}{R_1 + R_3} = \frac{12.0\,\text{V}}{15.0\,\Omega} = 0.800\,\text{A}$$

Use these currents to find the voltage at points c and d, which will give the voltage across the capacitor.

$$V_c = \mathcal{E} - I_2 R_2 = 12.0\,\text{V} - (1.20\,\text{A})(1.0\,\Omega) = 10.8\,\text{V}$$

$$V_d = \mathcal{E} - I_1 R_1 = 12.0\,\text{V} - (0.800\,\text{A})(10.0\,\Omega) = 4.00\,\text{V}$$

$$V_{cd} = 10.8\,\text{V} - 4.00\,\text{V} = \boxed{6.8\,\text{V}} \;;\; Q = CV = (2.2\,\mu\text{F})(6.8\,\text{V}) = 14.96\,\mu\text{C} \approx \boxed{15\,\mu\text{C}}$$

(*b*) When the switch is opened, the emf is taken out of the circuit. Then we have the capacitor discharging through an equivalent resistance. That equivalent resistance is the series combination of R_1 and R_2, in parallel with the series combination of R_3 and R_4. Use the expression for discharging a capacitor, Eq. 26-9a.

$$R_{eq} = \left(\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}\right)^{-1} = \left(\frac{1}{11.0\,\Omega} + \frac{1}{14.0\,\Omega}\right)^{-1} = 6.16\,\Omega$$

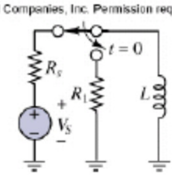
$$Q = Q_0 e^{-t/R_{eq}C} = 0.030 Q_0 \rightarrow$$

$$t = -R_{eq} C \ln(0.030) = -(6.16\,\Omega)(2.2 \times 10^{-6}\,\text{F}) \ln(0.030) = \boxed{4.8 \times 10^{-5}\,\text{s}}$$

Problem 8

Solution:

Known quantities:



Circuit shown in Figure P5.26,
$V_1 = 12V, R_S = 0.7\Omega, R_1 = 22k\Omega, L = 100mH$.

Find:

The voltage through the inductor just before and just after the switch is changed.

Assumptions:

The circuit is in steady-state conditions for $t < 0$.

Analysis:

In steady-state the inductor acts like a short-circuit so it has no voltage across it for t < 0. However, its current is non-zero and is equal to the current out of the source V_S and through R_S. At the instant the switch is changed the current through the inductor is unchanged since the current through an inductor cannot change instantaneously. Also notice that after the switch is changed the current through R_1 is always equal to the inductor current and the voltage across R_1 is always equal to the inductor voltage. Thus, at t = 0+ the voltage across the inductor must be non-zero. That's fine since the voltage across an inductor can change instantaneously (or relatively so.)

Assume a polarity for the voltage across the inductor.
$t = 0^-$: Steady state conditions exist. The inductor can be modeled as a short circuit with: $V_L(0^-) = 0$
Apply KVL;

$$-V_S + i_L(0^-)R_S + V_L(0^-) = 0$$

$$i_L(0^-) = \frac{V_S}{R_S} = \frac{12}{0.7} = 17.14A$$

At $t = 0^+$, the transient commences. Continuity requires: $i_L(0^+) = i_L(0^-)$
Apply KVL:

$$i_L(0^+)R_1 + V_L(0^+) = 0 \Rightarrow V_L(0^+) = -i_L(0^+)R_1 = -17.14 \times 22 \times 10^3 = -337.1kV$$

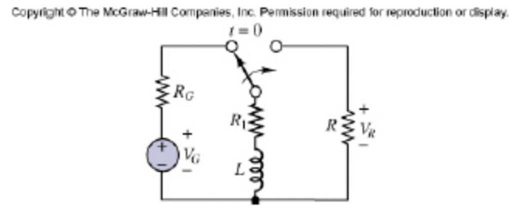
Problem 9

Solution:

Known quantities:

Circuit shown in Figure P5.35,

$V_G = 12V, R_G = 0.37\Omega, R = 1.7k\Omega$.



Find:

The value of L and R_1.

Assumptions:

The voltage across the spark plug gap V_R just after the switch is changed is $23kV$ and the voltage will change exponentially with a time constant $\tau = 13ms$.

Analysis:

At $t = 0^-$:

Assume steady state conditions exist. At steady state the inductor is modeled as a short circuit:

$$V_L(0^-) = 0$$

The current through the inductor at this point is given directly by Ohm's Law:

$$i_L(0^-) = \frac{V_G}{R_G + R_1}$$

At $t = 0^+$:

Continuity of the current through the inductor requires that:

$$i_L(0^+) = i_L(0^-) = \frac{V_G}{R_G + R_1}$$

$$V_R(0^+) = -i_L(0^+)R = -\frac{V_G R}{R_G + R_1}$$

$$R_1 = -\frac{V_G R}{V_R(0^+)} - R_G = -\frac{12 \times 1.7 \times 10^3}{-23 \times 10^3} - 0.37 = 0.5170\Omega$$

Note that the voltage across the gap V_R was written as –23 kV since the current from the inductor flows opposite to the polarity shown for V_R; that is, the actual polarity of the voltage across R is opposite that shown.

For $t > 0$:

Determine the Thevenin equivalent resistance as "seen" by the inductor, ie, with respect to the port or terminals of the inductor:

$$R_{eq} = R_1 + R$$

$$\tau = \frac{L}{R_{eq}} = \frac{L}{R_1 + R}$$

$$L = \tau(R_1 + R) = 13 \times 10^{-3} \times (0.5170 + 1.7 \times 10^3) = 22.11H$$