SOLUTION TO HOMEWORK #5

3.34

A continuous time LTI system has impulse response $h(t)$, $h(t) = e^{-4|t|}$. A signal x(t) is input to the system and the output is y(t). As shown in Section 3.8, the Fourier series representation of the output y(t) is

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t},$$

where a_k are the Fourier series coefficients of the input, b_k are the Fourier series coefficients of the output, and $b_k = a_k H(jk\omega_0)$.
Using Eq. 3.121, we first compute the frequency response of the system

$$\begin{aligned}
H(j\omega) &= \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt \\
&= \int_{-\infty}^{\infty} e^{-4|t|} e^{-j\omega t} dt \\
&= \int_{-\infty}^{0} e^{4t} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-4t} e^{-j\omega t} dt \\
&= \frac{1}{4-j\omega} e^{(4-j\omega)t}\Big|_{-\infty}^{0} + \frac{1}{-4-j\omega} e^{(-4-j\omega)t}\Big|_{0}^{\infty} \\
&= \frac{1}{4-j\omega} + \frac{1}{4+j\omega} \\
&= \frac{8}{16+\omega^2}
\end{aligned}$$

(a) This input $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$ is a periodic signal as shown in Fig. 1.

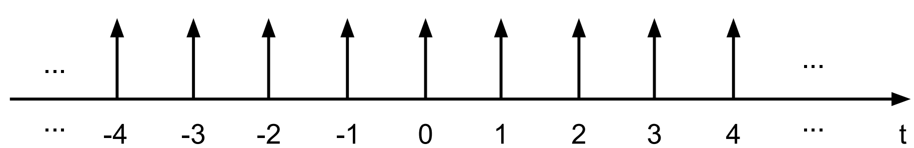


Figure 1: Problem 3.34 (a)

The period of x(t) is T=1, so $\omega_0 = 2\pi$. We can use the period from t=-1/2 to t=1/2; in this case, x(t)=$\delta(t)$. Thus,

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt = \int_{-1/2}^{1/2} \delta(t) e^{-jk2\pi t} dt = e^{-jk2\pi 0} = 1.$$

The Fourier series coefficients of the output y(t) are then

$$b_k = a_k H(jk\omega_0) = a_k H(jk2\pi) = \frac{8}{16+(k2\pi)^2} = \frac{2}{4+(k\pi)^2}$$

(b) This input $x(t) = \sum_{n=-\infty}^{\infty}(-1)^n\delta(t-n)$ is a periodic signal as shown in Fig. 2.

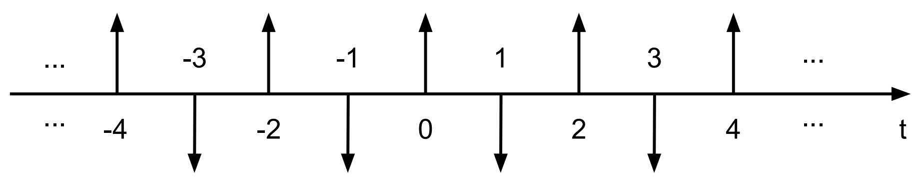


Figure 2: Problem 3.34 (b)

The period of x(t) is T=2, so $\omega_0 = \pi$. We can use the period from t=-1/2 to t=3/2; in this case, x(t)=$\delta(t) - \delta(t-1)$. Thus,

$$a_k = \frac{1}{2}\int_{-1/2}^{3/2}[\delta(t)-\delta(t-1)]e^{-jk\pi t}dt = \frac{1}{2}(1-e^{-jk\pi}) = \frac{1}{2}(1-(-1)^k) = \begin{cases} 0, & k \text{ even} \\ 1, & k \text{ odd} \end{cases}$$

The Fourier series coefficients of the output y(t) are

$$b_k = a_k H(jk\omega_0) = a_k H(jk2\pi) = \begin{cases} 0, & k \text{ even} \\ \dfrac{8}{16+\pi^2k^2}, & k \text{ odd} \end{cases}$$

3.37

A discrete-time LTI system has an impulse response $h[n] = (1/2)^{|n|}$. A signal x[n] is input to the system and the output is y[n]. As shown in Section 3.8, the Fourier series representation of the output y[n] is

$$y[n] = \sum_{k=<N>} b_k e^{jk(2\pi/N)n} = \sum_{k=<N>} a_k H(e^{j2\pi k/N})e^{jk(2\pi/N)n},$$

where a_k are the Fourier series coefficients of the input, b_k are the Fourier series coefficients of the output, and $b_k = a_k H(e^{jk\omega_0})$.
Using Eq. 3.122, we first compute the frequency response of the system

$$
\begin{aligned}
H(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} (\frac{1}{2})^{|n|} e^{-j\omega n} \\
&= \sum_{n=-\infty}^{-1} (\frac{1}{2})^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n} e^{-j\omega n} \\
&= \sum_{n=1}^{\infty} (\frac{1}{2})^{n} e^{j\omega n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n} e^{-j\omega n} \\
&= \frac{e^{j\omega}/2}{1 - e^{j\omega}/2} + \frac{1}{1 - e^{-j\omega}/2} \\
&= \frac{3/4}{5/4 - \cos\omega}
\end{aligned}
$$

(a) This input $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$ is a periodic signal as shown in Fig. 3.

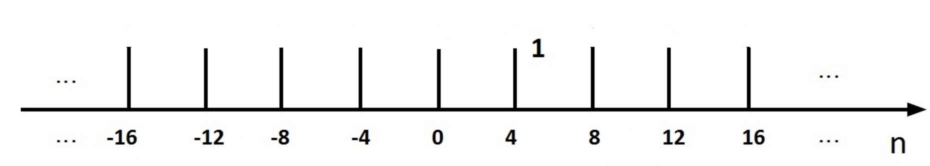


Figure 3: Problem 3.37 (a)

The period of x[n] is N=4, so $\omega_0 = \pi/2$. We can use the period from n=0 to n=3; over this range $x[n] = \delta[n]$. Thus

$$
a_k = \frac{1}{4}\sum_{n=0}^{3} x[n] e^{-jk(2\pi/4)n} = \frac{1}{4}\sum_{n=0}^{3} \delta[n] e^{-jk(2\pi/4)n} = \frac{1}{4} e^{-jk(2\pi/4)0} = \frac{1}{4}
$$

The Fourier series coefficients of the output y[n] are then,

$$
b_k = a_k H(e^{jk\omega_0}) = a_k H(e^{j2\pi k/4}) = \frac{1}{4}(\frac{3/4}{5/4 - \cos(\pi k/2)})
$$

4.1

(a) Let $x(t) = e^{-2(t-1)}u(t-1)$. Then, using Eq. 4.9, the Fourier transform $X(j\omega)$ of $x(t)$ is

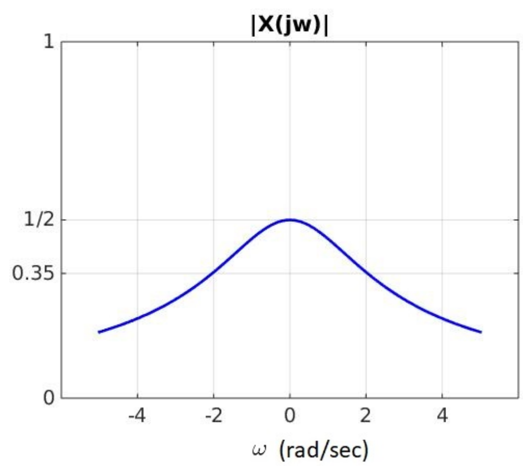
$$
\begin{aligned}
X(j\omega) &= \int_{-\infty}^{\infty} e^{-2(t-1)} u(t-1) e^{-j\omega t} dt \\
&= \int_{1}^{\infty} e^{-2(t-1)} e^{-j\omega t} dt
\end{aligned}
$$

Let $u = t - 1$, then $t = u + 1$ and

$$\begin{aligned}
X(j\omega) &= \int_0^{\infty} e^{-2u} e^{-j\omega(u+1)} du \\
&= e^{-j\omega} \int_0^{\infty} e^{-(2+j\omega)u} du \\
&= \frac{e^{-j\omega}}{-(2+j\omega)} \left. e^{-(2+j\omega)u} \right|_0^{\infty} \\
&= \frac{e^{-j\omega}}{2+j\omega}.
\end{aligned}$$

The magnitude of $X(j\omega)$ is (Remember $|x| = \sqrt{(Re(x))^2 + (Im(x))^2}$)

$$|X(j\omega)| = \left| \frac{e^{-j\omega}}{2+j\omega} \right| = \frac{|e^{-j\omega}|}{|2+j\omega|} = \frac{1}{\sqrt{4+\omega^2}}.$$



(b) Let $x(t) = e^{-2|t-1|}$. Then, using Eq. 4.9, the Fourier transform $X(jw)$ of $x(t)$ is

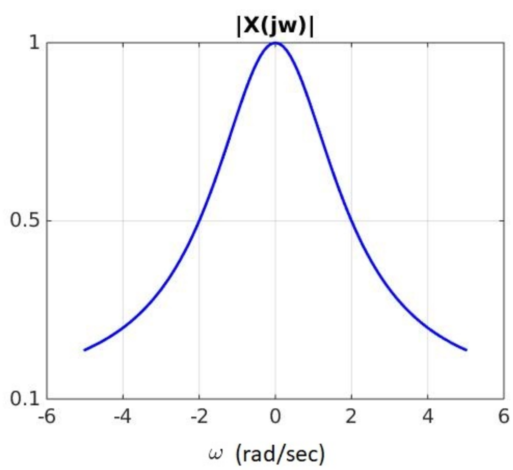
$$\begin{aligned}
X(j\omega) &= \int_{-\infty}^{\infty} e^{-2|t-1|} e^{-j\omega t} dt \\
&= \int_1^{\infty} e^{-2(t-1)} e^{-j\omega t} dt + \int_{-\infty}^{1} e^{2(t-1)} e^{-j\omega t} dt.
\end{aligned}$$

Let $u = t - 1$, then $t = u + 1$ and

$$\begin{aligned}
X(j\omega) &= \int_0^\infty e^{-2u} e^{-j\omega(u+1)} du + \int_{-\infty}^0 e^{2u} e^{-j\omega(u+1)} du \\
&= e^{-j\omega} \int_0^\infty e^{-(2+j\omega)u} du + e^{-j\omega} \int_{-\infty}^0 e^{(2-j\omega)u} du \\
&= \frac{e^{-j\omega}}{-(2+j\omega)} e^{-(2+j\omega)u} \Big|_0^\infty + \frac{e^{-j\omega}}{2-j\omega} e^{(2-j\omega)u} \Big|_{-\infty}^0 \\
&= \frac{e^{-j\omega}}{2+j\omega} + \frac{e^{-j\omega}}{2-j\omega} \\
&= e^{-j\omega} \frac{2 - j\omega + 2 + j\omega}{(2+j\omega)(2-j\omega)} \\
&= \frac{4e^{-j\omega}}{4+\omega^2}.
\end{aligned}$$

The magnitude of $X(j\omega)$ is

$$|X(j\omega)| = \left| \frac{4e^{-j\omega}}{4+\omega^2} \right| = \frac{4}{4+\omega^2}.$$



4.2

(a) The signal $x(t) = \delta(t+1) + \delta(t-1)$ is the sum of two time-shifted impulse functions. Using Eq. 4.9, the Fourier transform $X(j\omega)$ of $x(t)$ is

$$\begin{aligned}
X(j\omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \\
&= \int_{-\infty}^{\infty} (\delta(t+1) + \delta(t-1)) e^{-j\omega t} dt \\
&= \int_{-\infty}^{\infty} \delta(t+1) e^{-j\omega t} dt + \int_{-\infty}^{\infty} \delta(t-1) e^{-j\omega t} dt
\end{aligned}$$

Using $x(t_0) = \int_{-\infty}^{\infty} x(t)\delta(t-t_0)dt$, we get

$$X(j\omega) = e^{j\omega} + e^{-j\omega}$$

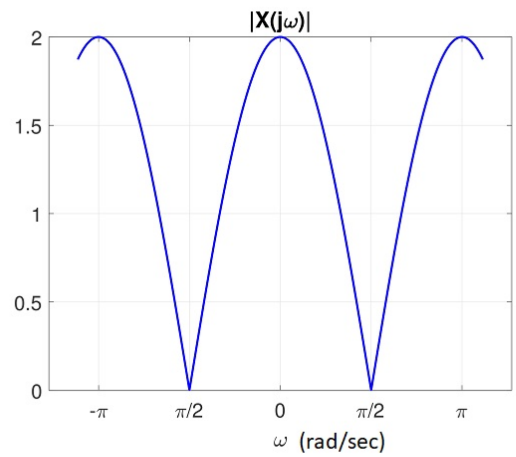
Then using the equation $\cos\theta = (e^{j\theta} + e^{-j\theta})/2$, we get

$$X(j\omega) = 2\cos\omega$$

The magnitude of $X(j\omega)$ is

$$|X(j\omega)| = 2|\cos\omega|,$$

and is shown below



(b) Note that the signal $x(t)$ is

$$x(t) = \frac{d}{dt}\{u(-2-t) + u(t-2)\} = -\delta(-2-t) + \delta(t-2).$$

If this step is not clear to you, simply draw the step functions and graphically compute the derivative. Then, using Eq. 4.9, the Fourier transform $X(j\omega)$ of $x(t)$ is

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \\ &= -\int_{-\infty}^{\infty} \delta(-2-t)e^{-j\omega t}dt + \int_{-\infty}^{\infty} \delta(t-2)e^{-j\omega t}dt \\ &= -e^{2j\omega} + e^{-2j\omega} \end{aligned}$$

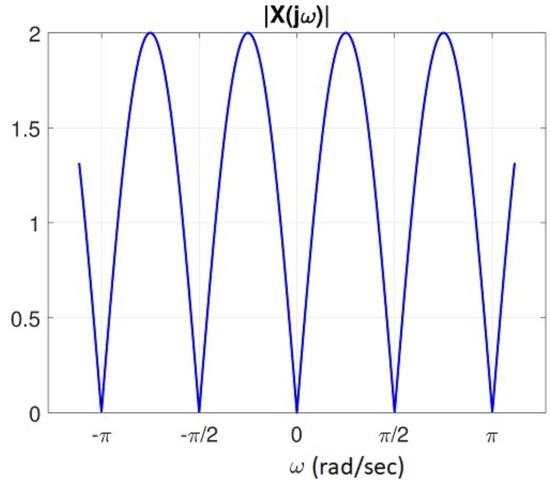
Then using the equation $\sin\theta = (e^{j\theta} - e^{-j\theta})/2j$, we get

$$X(j\omega) = -2j\sin 2\omega$$

The magnitude of $X(j\omega)$ is

$$|X(j\omega)| = 2|\sin 2\omega|,$$

and is shown below



4.6

Note that you could determine the Fourier transforms in this problem by simply putting x(t) int Eq. 4.9 and performing the integration. We will find the Fourier transforms by using the property.

(a) We know from Table 4.1 that $x(-t) \overset{FT}{\rightarrow} X(-j\omega)$.
Therefore, $x(1-t) = x(-(t-1)) \overset{FT}{\rightarrow} e^{-j\omega}X(-j\omega)$. Also, $x(-1-t) = x(-(t+1)) \overset{FT}{\rightarrow} e^{j\omega}X(-j\omega)$. So

$$x_1(t) \overset{FT}{\rightarrow} e^{-j\omega}X(-j\omega) + e^{j\omega}X(-j\omega) = X(-j\omega)[e^{-j\omega} + e^{j\omega}] = 2X(-j\omega)\cos(\omega)$$

(b) From Table 4.1, we have $x(at) \overset{FT}{\rightarrow} \frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$. Let $x_2'(t) = x(t-6)$, then $x_2(t) = x_2'(3t)$. Then, $X_2'(j\omega) = e^{-6j\omega}X(j\omega)$ Therefore

$$\begin{aligned} x_2(t) &\overset{FT}{\rightarrow} \frac{1}{3}X_2'\left(\frac{j\omega}{3}\right) \\ &\overset{FT}{\rightarrow} \frac{1}{3}e^{-6\frac{j\omega}{3}}X\left(\frac{j\omega}{3}\right) \\ &\overset{FT}{\rightarrow} \frac{1}{3}e^{-2j\omega}X\left(\frac{j\omega}{3}\right) \end{aligned}$$

To solve directly,

$$\begin{aligned} x_2(j\omega) &= \int_{-\infty}^{\infty} x_2(t)e^{-jwt}dt \\ &= \int_{-\infty}^{\infty} x(3t-6)e^{-jwt}dt \end{aligned}$$

$u = 3t - 6,\ t = \frac{u+6}{3},\ du = 3dt$

$$\begin{aligned} =&\frac{1}{3}\int_{-\infty}^{\infty} x(u)e^{-j\omega(\frac{u+6}{3})}du \\ =&\frac{1}{3}e^{-2j\omega}\int_{-\infty}^{\infty} x(u)e^{-j\frac{u+6}{3}}du \\ =&\frac{1}{3}e^{-2j\omega}X(\frac{j\omega}{3}) \end{aligned}$$

(c) From Table 4.1, we have $\frac{d}{dt}x(t) \overset{FT}{\rightarrow} j\omega X(j\omega)$. We can easily show that

$$\frac{d^2}{dt^2}x(t) \overset{FT}{\rightarrow} -\omega^2 X(j\omega)$$

So

$$\frac{d^2}{dt^2}x(t-1) \overset{FT}{\rightarrow} -e^{-j\omega}\omega^2 X(j\omega)$$

4.21

(a) For this problem, you are asked to compute the Fourier transform of $x(t) = [e^{-\alpha t}\cos\omega_0 t]u(t),\ \alpha > 0$. You could simply plug this into Eq. 4.9 and derive the answer. Or you could recognize that this signal can be rewritten as (using $\cos\theta = (e^{j\theta} + e^{-j\theta})/2$)

$$x(t) = [e^{-\alpha t}\cos\omega_0 t]u(t) = \frac{1}{2}e^{-\alpha t}e^{j\omega_0 t}u(t) + \frac{1}{2}e^{-\alpha t}e^{-j\omega_0 t}u(t)$$

You can either re-derive it, remember it from class, or notice from Table 4.2 in the textbook that the Fourier transform of $e^{-at}u(t)$ is $\dfrac{1}{a + j\omega}$. Using the frequency-shifting property (4.41) on p. 311 of the textbook for the two terms, the Fourier transform of the signal is

$$\begin{aligned} X(j\omega) &= \frac{1}{2}[\frac{1}{\alpha + j(\omega - \omega_0)} + \frac{1}{\alpha + j(\omega + \omega_0)}] \\ &= \frac{1}{2}(\frac{1}{\alpha + j\omega - j\omega_0} + \frac{1}{\alpha + j\omega + j\omega_0}) \end{aligned}$$

This could be simplified by combining the two terms but it is fine to leave it like this.

(b)

$$x(t) = e^{-3|t|}\sin 2t = \begin{cases} e^{-3t}\sin 2t, & t > 0 \\ e^{3t}\sin 2t, & t < 0 \end{cases}$$

Then, simply plugging this into Eq. 4.9, we get

$$X(j\omega) = \int_0^{\infty} e^{-3t}\left(\sin\ 2t\right)e^{-j\omega t}dt + \int_{-\infty}^{0} e^{3t}\left(\sin\ 2t\right)e^{-j\omega t}dt.$$

This is easiest to integrate if you first convert $\sin\theta$ to $\frac{e^{j\theta}-e^{-j\theta}}{2j}$. Then the integrals are simply integration of exponential functions. If we do this, we get

$$\begin{aligned}
X(j\omega) =& \int_0^{\infty} e^{-3t}\frac{e^{j2t}-e^{-j2t}}{2j}e^{-j\omega t}dt + \int_{-\infty}^{0} e^{3t}\frac{e^{j2t}-e^{-j2t}}{2j}e^{-j\omega t}dt \\
=&\frac{1}{2j}\int_0^{\infty} e^{-(3t-2j+j\omega)t}dt - \frac{1}{2j}\int_0^{\infty} e^{-(3t+2j+j\omega)t}dt \\
&+ \frac{1}{2j}\int_{-\infty}^{0} e^{(3+2j-j\omega)t}dt - \frac{1}{2j}\int_{-\infty}^{0} e^{(3-2j-j\omega)t}dt \\
=&\frac{1/2j}{-(3-2j+j\omega)}e^{-(3-2j+j\omega)t}\Big|_0^{\infty} - \frac{1/2j}{-(3+2j+j\omega)}e^{-(3t+2j+j\omega)t}\Big|_0^{\infty} \\
&+ \frac{1/2j}{3+2j-j\omega}e^{(3+2j-j\omega)t}\Big|_{-\infty}^{0} - \frac{1/2j}{3-2j-j\omega}e^{(3-2j-j\omega)t}\Big|_{-\infty}^{0} \\
=&\frac{1/2j}{3-2j+j\omega} - \frac{1/2j}{3+2j+j\omega} + \frac{1/2j}{3+2j-j\omega} - \frac{1/2j}{3-2j-j\omega}.
\end{aligned}$$

Let $a = 3+2j$, then $a^* = 3-2j$. Using these, $X(j\omega)$ can be written as

$$\begin{aligned}
X(j\omega) =&\frac{1/2j}{a^*+j\omega} - \frac{1/2j}{a+j\omega} + \frac{1/2j}{a-j\omega} - \frac{1/2j}{a^*-j\omega} \\
=&\frac{1}{2j}\left[\frac{1}{a^*+j\omega} - \frac{1}{a^*-j\omega}\right] + \frac{1}{2j}\left[-\frac{1}{a+j\omega} + \frac{1}{a-j\omega}\right] \\
=&\frac{1}{2j}\left[\frac{-2j\omega}{a^{*2}+\omega^2}\right] + \frac{1}{2j}\left[\frac{2j\omega}{a^2+\omega^2}\right] \\
=&\frac{-\omega}{a^{*2}+\omega^2} + \frac{\omega}{a^2+\omega^2} \\
=&\frac{(a^{*2}-a^2)\omega}{a^2a^{*2}+(a^2+a^{*2})\omega^2+\omega^4}.
\end{aligned}$$

Since $a = 3 + 2j$ and $a^* = 3 - 2j$, we have that

$$\begin{aligned} a^2 &= 5 + 12j, \\ a^{*2} &= 5 - 12j, \\ aa^* &= |a|^2 = 3^2 + 2^2 = 13. \end{aligned}$$

Therefore,

$$X(j\omega) = \frac{-24j\omega}{169 + 10\omega^2 + \omega^4}.$$

(f) For this problem, you are asked to compute the Fourier transform of $x(t) = x_1(t)x_2(t)$, where

$$x_1(t) = \frac{\sin \pi t}{\pi t}, \quad x_2(t) = \frac{\sin 2\pi(t-1)}{\pi(t-1)}.$$

You can plug this into Eq. 4.9 and try to compute the resulting integral; however, this is very difficult. It is much easier to recognize that this is the product of two time functions, each one of which has a very simple Fourier transform,

$$\mathcal{F}\{x_1(t)\} = X_1(j\omega) = \begin{cases} 1, & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}$$

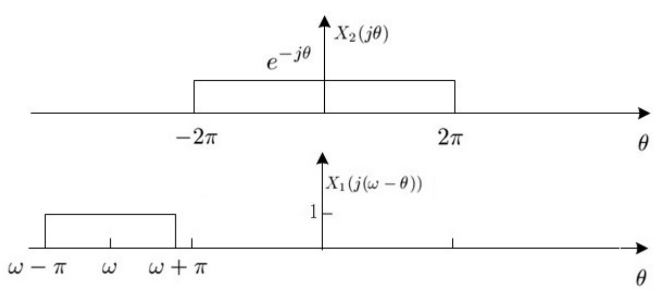
$$\mathcal{F}\{x_2(t)\} = X_2(j\omega) = \begin{cases} e^{-j\omega}, & |\omega| < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

Therefore,

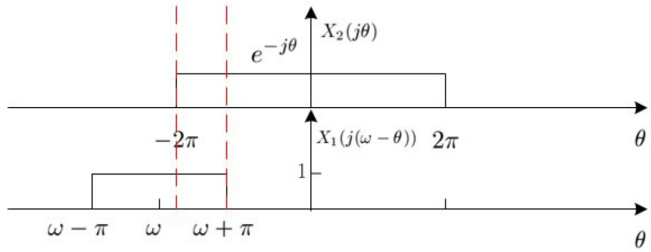
$$X(j\omega) = \frac{1}{2\pi}\left(X_1(j\omega) * X_2(j\omega)\right)$$

The computation of the convolution $X_1(j\omega) * X_2(j\omega)$, and the various regimes of ω are shown in the figures below. This is very similar to the convolution you performed in Chapter 2.

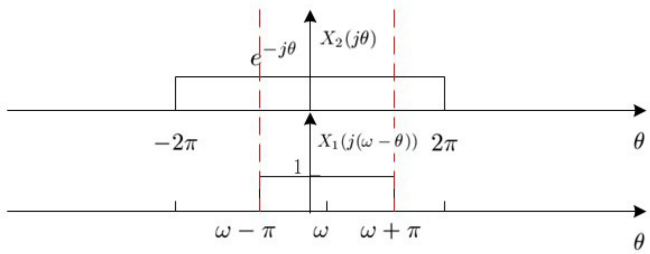
- $\omega < -3\pi$,



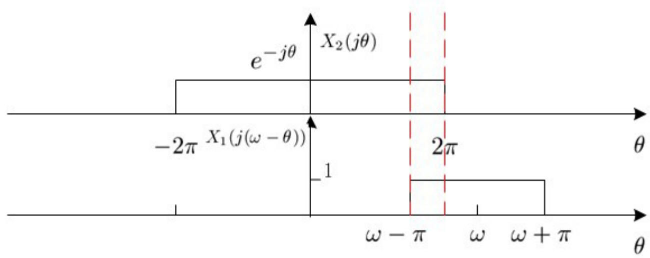
- $-3\pi < \omega < -\pi$,



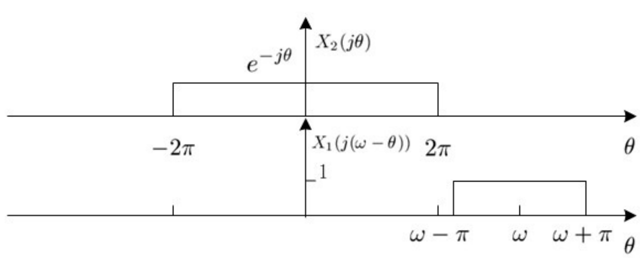
- $-\pi < \omega < \pi$,



- $\pi < \omega < 3\pi$,



- $\omega > 3\pi$,



Therefore, $X(j\omega) = \frac{1}{2\pi}\left(X_1(j\omega) * X_2(j\omega)\right)$ can be calculated as

$$X(j\omega) = \begin{cases} \frac{1}{2\pi}\int_{-2\pi}^{\omega+\pi} e^{-j\theta}d\theta = \frac{1}{-2\pi j}\left(e^{-j(\omega+\pi)} - e^{-j(-2\pi)}\right) & \\ \qquad = \frac{1}{2\pi j}\left(e^{-j\omega}+1\right), & -3\pi < \omega < -\pi \\ \frac{1}{2\pi}\int_{\omega-\pi}^{\omega+\pi} e^{-j\theta}d\theta = \frac{1}{-2\pi j}\left(e^{-j(\omega+\pi)} - e^{-j(\omega-\pi)}\right) = 0, & -\pi < \omega < \pi \\ \frac{1}{2\pi}\int_{\omega-\pi}^{2\pi} e^{-j\theta}d\theta = \frac{1}{-2\pi j}\left(e^{-j(2\pi)} - e^{-j(\omega-\pi)}\right) & \\ \qquad = -\frac{1}{2\pi j}\left(e^{-j\omega}+1\right), & \pi < \omega < 3\pi \\ 0, & \text{otherwise} \end{cases}$$

which reduces to (using the fact that $1 = e^{\frac{j\omega}{2}}e^{\frac{-j\omega}{2}}$ and $\cos\theta = (e^{j\theta}+e^{-j\theta})/2$)

$$X(j\omega) = \begin{cases} \frac{1}{2\pi j}\left(e^{-j\omega}+1\right) = \frac{1}{\pi}e^{-\frac{j}{2}(\pi-\omega)}\cos\frac{\omega}{2}, & -3\pi < \omega < -\pi \\ -\frac{1}{2\pi j}\left(1+e^{-j\omega}\right) = -\frac{1}{\pi}e^{-\frac{j}{2}(\pi-\omega)}\cos\frac{\omega}{2}, & \pi < \omega < 3\pi \\ 0, & \text{otherwise} \end{cases}$$

(h) The function $x(t)$ in the figure is the sum of two periodic sets of impulse functions, each with period $T = 2$. Let

$$x_1(t) = \sum_{n=-\infty}^{\infty} \delta(t-2n).$$

Then, $x(t) = 2x_1(t) + x_1(t-1)$. Using the linearity and time-shift properties

$$X(j\omega) = 2X_1(j\omega) + X_1(j\omega)e^{-j\omega} = (2+e^{-j\omega})X_1(j\omega).$$

We can determine $X_1(j\omega)$ using Example 4.8 or Table 4.2,

$$X_1(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi}{T}k\right) = \pi\sum_{k=-\infty}^{\infty}\delta(\omega - k\pi).$$

Therefore,

$$\begin{aligned}
X(j\omega) &= (2+e^{-j\omega})X_1(j\omega) \\
&= \pi(2+e^{-j\omega})\sum_{k=-\infty}^{\infty}\delta(\omega-k\pi) \\
&= \pi\sum_{k=-\infty}^{\infty}\left(\delta(\omega-k\pi)(2+e^{-jk\pi})\right) \\
&= \pi\sum_{k=-\infty}^{\infty}\left(\delta(\omega-k\pi)(2+(-1)^k)\right).
\end{aligned}$$

4.25

We are asked to find these answers without explicitly deriving $X(j\omega)$.

(a) $x(t)$ is given as the continuous time function in Fig. 4.25. You are instructed to perform these calculations without explicitly evaluating $X(j\omega)$. Let $y(t) = x(t+1)$, then $y(t)$ is even and real (since $x(t)$ is real). Using Property 4.31, then $Y(j\omega)$ is also even and real, which implies $\sphericalangle Y(j\omega) = 0$. Since $y(t)$ is just a shifted version of $x(t)$, $Y(j\omega) = e^{j\omega}X(j\omega) \Rightarrow X(j\omega) = e^{-j\omega}Y(j\omega)$, therefore $\sphericalangle X(j\omega) = -\omega$.

(b) Using the figure and the definition of the Fourier Transform, we have

$$\begin{aligned}
X(j0) &= X(j\omega)\Big|_{\omega=0} \\
&= \left[\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt\right]_{\omega=0} \\
&= \int_{-\infty}^{\infty} x(t)dt \\
&= 7 \text{ (area under the curve).}
\end{aligned}$$

(c)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

This implies that

$$\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega = 2\pi x(t).$$

So,

$$\int_{-\infty}^{\infty} X(j\omega)d\omega = 2\pi\, x(t)\Big|_{t=0} = 4\pi.$$

(e) From Parseval's relation,

$$
\begin{aligned}
&\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt \\
&= 2\pi \left[\int_{-1}^{0} 4dt + \int_{0}^{1} (2-t)^2 dt + \int_{1}^{2} t^2 dt + \int_{2}^{3} 4dt \right] \\
&= 2\pi \left[2 \cdot 4 + \frac{2 \cdot 7}{3} \right] = 2\pi \frac{38}{3} = \frac{76\pi}{3}
\end{aligned}
$$