ELEG 312
Homework #1
Solutions

Problems 8.1, 8.5, 8.9, 8.13, 8.14, and 8.17

Problem 8.1a

For V_{DD} = 1.3 V and using I_{REF} = 100 uA, it is required to design the circuit of Fig. 8.1 to obtain an output current whose nominal value is 100 uA. Find R if Q_1 and Q_2 are matched with channel lengths of 0.5 um, channel widths of 5 um, V_t = 0.4 V, and k'_n = 500 uA/V^2. What is the lowest possible value of V_O? Assuming that for this process technology the Early voltage V'_A = 5 V/um, find the output resistance of the current source. Also, find the change in output current resulting from +0.5-V change in V_O.

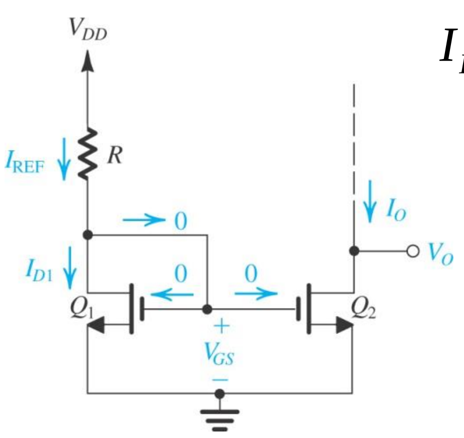


Figure 8.1 Circuit for a basic MOSFET constant-current source. For proper operation, the output terminal, that is, the drain of Q_2, must be connected to a circuit that ensures that Q_2 operates in saturation.

$$I_{D1} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_1 V_{OV}^2 \qquad V_{OV} = \sqrt{\frac{2I_{D1}}{k_n'\left(\frac{W}{L}\right)_1}} = \sqrt{\frac{2 \times 0.1\text{mA}}{500\frac{\text{uA}}{\text{V}^2} \times \frac{5\text{um}}{0.5\text{um}}}} = 0.2\text{V}$$

$$I_{D1} = I_{REF} = \frac{V_{DD} - V_{GS}}{R} = \frac{V_{DD} - V_{OV} - V_t}{R} = \frac{1.3 - 0.2 - 0.4\text{V}}{R} = \frac{0.7\text{V}}{R}$$

$$R = \frac{0.7\text{V}}{0.1\text{mA}} = 7\text{k}\Omega$$

Lowest value of V_o is when V_{DS} of Q_2 = V_{OV} = 0.2V

Problem 8.1b

For V_{DD} = 1.3 V and using I_{REF} = 100 uA, it is required to design the circuit of Fig. 8.1 to obtain an output current whose nominal value is 100 uA. Find R if Q_1 and Q_2 are matched with channel lengths of 0.5 um, channel widths of 5 um, V_t = 0.5 V, and k'_n = 500 uA/V^2. What is the lowest possible value of V_O? Assuming that for this process technology the Early voltage V'_A = 5 V/um, find the output resistance of the current source. Also, find the change in output current resulting from +0.5-V change in V_O.

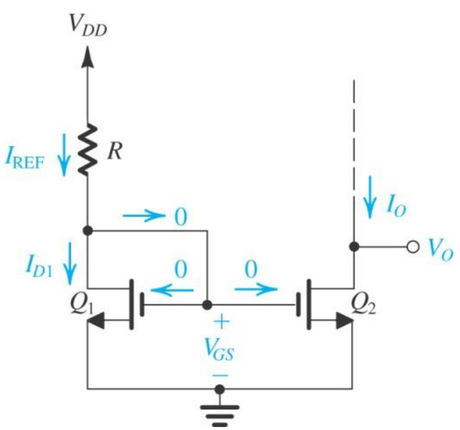


Figure 8.1 Circuit for a basic MOSFET constant-current source. For proper operation, the output terminal, that is, the drain of Q_2, must be connected to a circuit that ensures that Q_2 operates in saturation.

$$r_o = \frac{V_A}{I_D} = \frac{V_A' L}{I_D} = \frac{5\,\mathrm{V}/\mathrm{um} \times 0.5\mathrm{um}}{0.1\mathrm{mA}} = 25\mathrm{k}\Omega$$

$$\Delta V_o = \Delta I_o \times r_0$$

$$\Delta I_o = \frac{\Delta V_o}{r_0} = \frac{0.5\mathrm{V}}{25\mathrm{k}\Omega} = 0.02\mathrm{mA}$$

Problem 8.5

For the current-steering circuit of Fig. P8.5, find I_o in terms of I_{REF} and device W/L ratios.

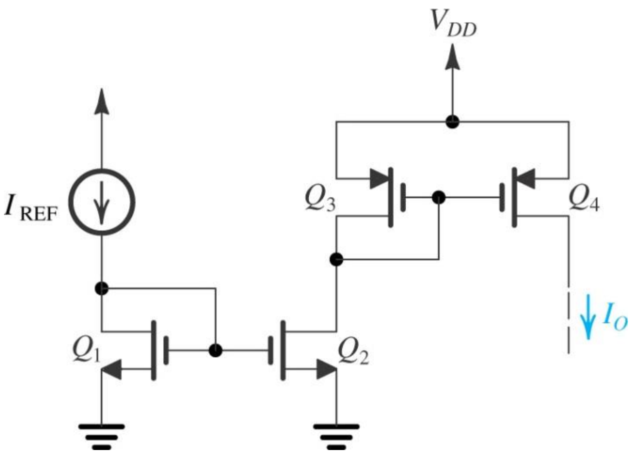


Figure P8.5

$$\frac{I_{Q2}}{I_{Q1}} = \frac{(W/L)_2}{(W/L)_1} \qquad \frac{I_{Q4}}{I_{Q3}} = \frac{(W/L)_4}{(W/L)_3}$$

$$I_{Q1} = I_{\text{REF}}\ ,\ I_{Q3} = I_{Q2} \text{ and } I_O = I_{Q4}$$

$$\frac{I_O}{I_{\text{REF}}} = \frac{I_{Q4}}{I_{Q1}} = \frac{I_{Q3}\left(\dfrac{(W/L)_4}{(W/L)_3}\right)}{I_{Q2}\left(\dfrac{(W/L)_1}{(W/L)_2}\right)} = \frac{(W/L)_2\,(W/L)_4}{(W/L)_1\,(W/L)_3}$$

Problem 8.9a

Consider the basic BJT current mirror of Fig. 8.7 for the case in which Q_2 has m times the area of Q_1. Show that the current transfer ratio is given by Eq. (8.19). If β is specified to be a minimum of 80, what is the largest current transfer ratio possible if the error introduced by the finite β is limited to10%?

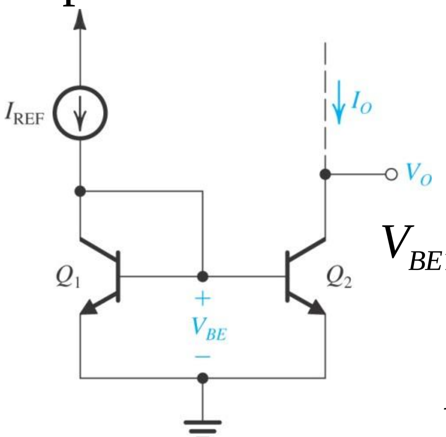


Figure 8.7: The basic BJT current mirror.

Eq. (8.19)

$$\frac{I_O}{I_{\mathrm{REF}}}=\frac{1}{\left(1+\frac{2}{\beta}\right)}$$

$$\beta=\frac{I_C}{I_B} \qquad I_{\mathrm{REF}}=I_C+\frac{I_C}{\beta}+\frac{I_O}{\beta} \qquad I_C=I_S e^{V_{BE}/V_T} \qquad I_{S2}=mI_{S1}$$

$$V_{BE1}=V_{BE2}=V_T\ln\left(\frac{I_{C1}}{I_{S1}}\right)=V_T\ln\left(\frac{I_{C2}}{I_{S2}}\right) \qquad \frac{I_{C1}}{I_{S1}}=\frac{I_{C2}}{I_{S2}}=\frac{I_O}{mI_{S1}} \qquad I_{C1}=I_C=\frac{I_O}{m}$$

$$I_{\mathrm{REF}}=I_C+\frac{I_C}{\beta}+\frac{I_O}{\beta}=\frac{I_0}{m}+\frac{I_0}{\beta m}+\frac{I_O}{\beta}=I_{\mathrm{O}}\left(\frac{1}{m}+\frac{1}{\beta m}+\frac{1}{\beta}\right)=I_{\mathrm{O}}\frac{\beta+1+m}{\beta m}$$

$$\frac{I_{\mathrm{O}}}{I_{\mathrm{REF}}}=\frac{\beta m}{\beta+1+m}=\frac{m}{1+\frac{1}{\beta}+\frac{m}{\beta}}=\frac{m}{1+\frac{m+1}{\beta}}$$

If the transistors are identical (matched) then m = 1 and we arrive at equation 8.19.

Problem 8.9b

Consider the basic BJT current mirror of Fig. 8.7 for the case in which Q_2 has m times the area of Q_1. Show that the current transfer ratio is given by Eq. (8.19). If β is specified to be a minimum of 80, what is the largest current transfer ratio possible if the error introduced by the finite β is limited to10%?

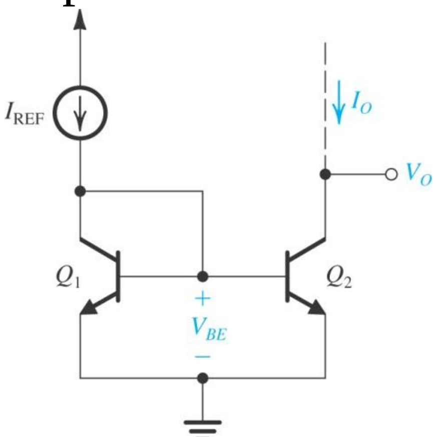


Figure 8.7: The basic BJT current mirror.

Eq. (8.19)

$$\frac{I_O}{I_{REF}} = \frac{1}{\left(1+\frac{2}{\beta}\right)}$$

$$\frac{I_O}{I_{REF}} = \frac{m}{1+\frac{m+1}{\beta}} \approx m$$

If we are to limit the error to 10% then the maximum current transfer ratio, m, can be found by:

$$\frac{I_O}{I_{REF}} = 0.9m \qquad \frac{I_O}{I_{REF}} = 0.9m = \frac{m}{1+\frac{m+1}{\beta}}$$

$$0.9 = \frac{1}{1+\frac{m+1}{80}} = \frac{80}{80+1+m} = \frac{80}{81+m} \qquad m = 7.888$$

Problem 8.13

Find the voltages at all nodes and the currents through all branches in the circuit of Fig. P8.13. Assume $|V_{BE}| = 0.7$ V and $\beta = \infty$.

$$I_{Q1} = \frac{8.6\text{V}}{20\text{k}\Omega} = 0.43\text{mA}$$

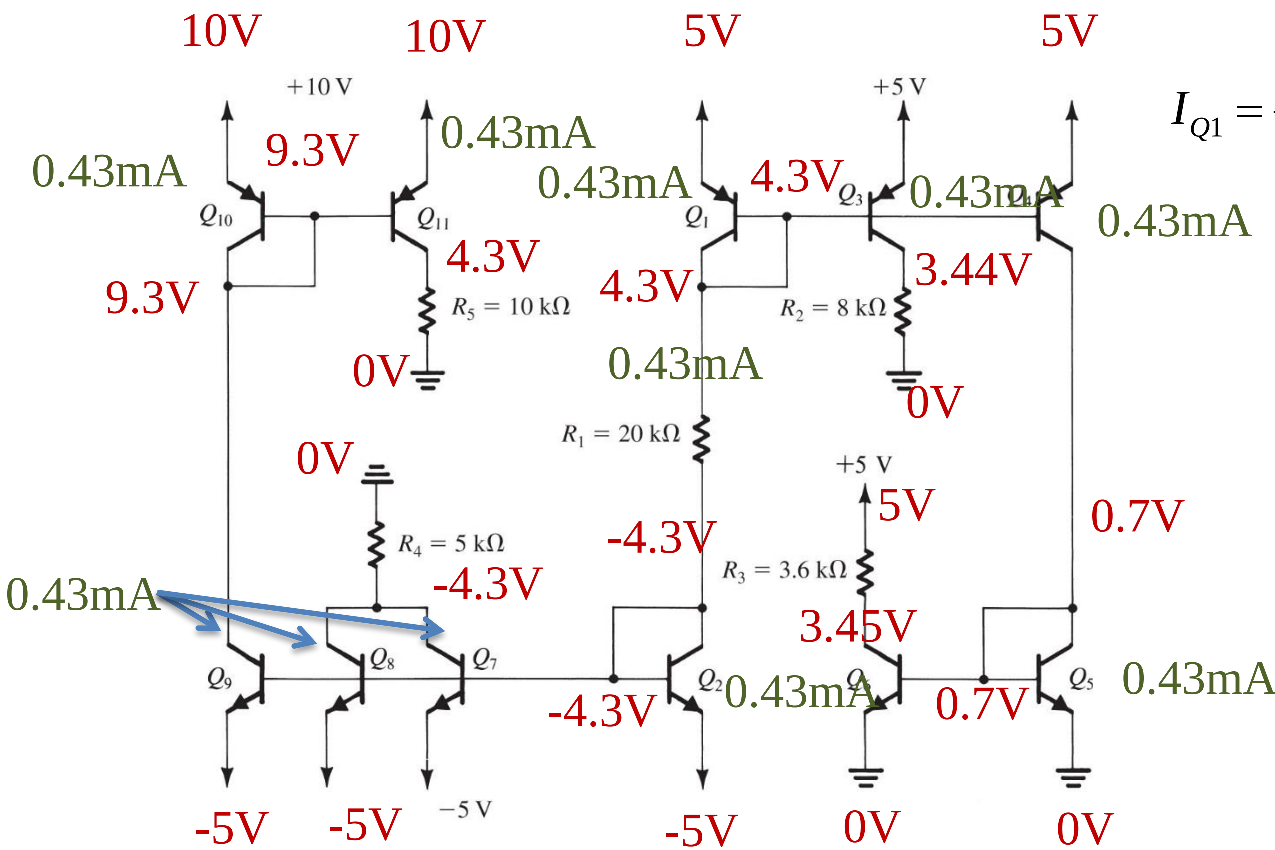


Figure P8.13

Problem 8.14

For the circuit in Fig. P8.14, let $|V_{BE}| = 0.7$ V and $\beta = \infty$. Find I, V_1, V_2, V_3, V_4, and V_5 for (a) $R = 10$ kΩ and (b) $R = 100$ kΩ.

$$V_1 = V_{B1} - V_{BE1} = 0\text{V} - 0.7\text{V} = -0.7\text{V}$$

$$I_{Q1} = \frac{V_1 - V_{EE}}{R} = \frac{-0.7\text{V} + 2.7\text{V}}{R} = \frac{2\text{V}}{R}$$

$$I = 2I_{Q1} = (2)\frac{2\text{V}}{R}$$

$$V_2 = V_{CC} - V_{EB} = 2.7\text{V} - 0.7\text{V} = 2.0\text{V}$$

$$V_3 = V_{B6} - V_{EB6} = 0\text{V} + 0.7\text{V} = 0.7\text{V}$$

$$V_4 = V_{EE} + \left(\frac{I}{2}\right)R = -2.7\text{V} + \frac{2\text{V}}{R}R = -0.7\text{V}$$

$$V_5 = V_{EE} + \left(\frac{I}{2}\right)\frac{R}{2} = -2.7\text{V} + \frac{2\text{V}}{R}\frac{R}{2} = -1.7\text{V}$$

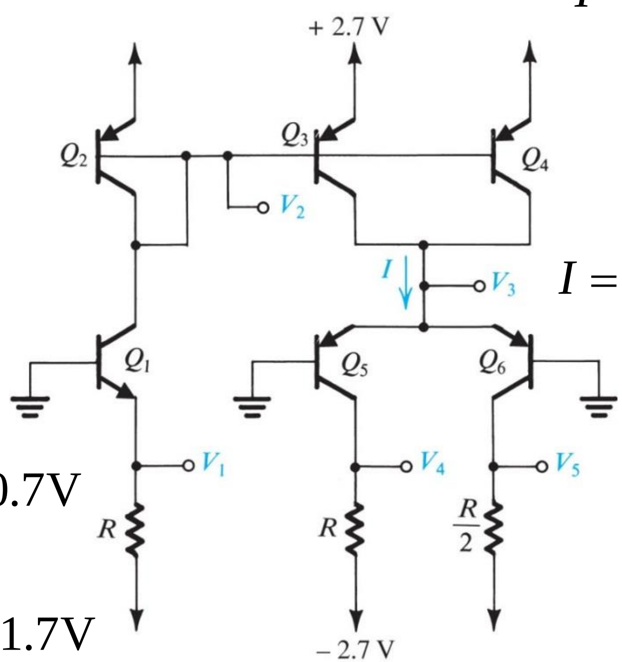


Figure P8.14

(a) $R = 10$ kΩ

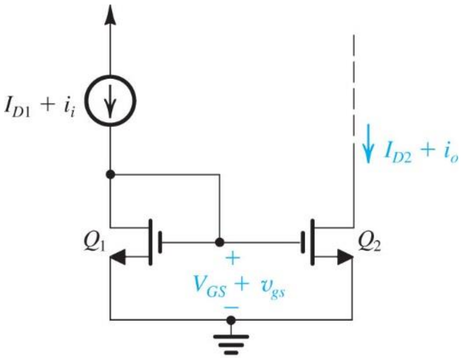
$$I = 2I_{Q1} = (2)\frac{2\text{V}}{10\text{k}\Omega} = 0.4\text{mA}$$

(b) $R = 100$ kΩ

$$I = 2I_{Q1} = (2)\frac{2\text{V}}{100\text{k}\Omega} = 0.04\text{mA}$$

Problem 8.17

The MOSFETs in the current mirror of Fig.8.12 (a) have equal channel lengths of 0.5 µm, W_1 = 10 µm, W_2 = 50 µm, $\mu_n C_{ox}$ = 500 µA/V², and V'_A = 10 V/µm. If the input bias current is 100 µA, find R_{in}, A_{is}, and R_o.



(a)

$$r_{o1} = \frac{V_A}{I_{D1}} = \frac{V_A' L}{I_{D1}} = \frac{10\frac{\text{V}}{\mu\text{m}}0.5\mu\text{m}}{0.1\text{mA}} = 50\text{k}\Omega$$

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}} = \sqrt{2\times 0.5\frac{\text{mA}}{\text{V}^2}\left(\frac{10}{0.5}\right)_1 0.1\text{mA}} = 1.414\text{mA/V}$$

$$R_{in} = r_{o1} \,||\, \frac{1}{g_{m1}} = 50\text{k}\Omega \,||\, \frac{1}{1.414} = 50\text{k}\Omega \,||\, 707\Omega = 697\Omega$$

$$A_{is} = \frac{(W/L)_2}{(W/L)_1} = 5\frac{\text{A}}{\text{A}} \quad \Rightarrow I_{D2} = A_{is} I_{D1} = 5\frac{\text{A}}{\text{A}}0.1\text{mA} = 0.5\text{mA}$$

$$R_o = r_{o2} = \frac{V_{A2}}{I_{D2}} = \frac{10\frac{\text{V}}{\mu\text{m}}0.5\mu\text{m}}{0.5\text{mA}} = 10\text{k}\Omega$$