ELEG 312
Homework #6
Solutions

Problems F1, F2, F10, and F11

Problem F.1 original solution

Find the transfer function $T(s) = V_o(s)/V_i(s)$ of the circuit in Fig. PF.1. Is this an STC network? If so, of what type? For $C_1 = C_2 = 0.5\ \mu F$ and $R = 100\ k\Omega$, find the location of the pole(s) and zero(s), and sketch Bode plots for the magnitude response and the phase response.

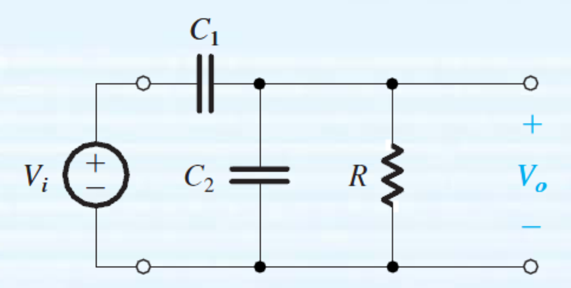


Figure PF.1

$$V_o(s) = V_i(s)\frac{\chi_{C2} \parallel R}{\chi_{C2} \parallel R + \chi_{C1}} \qquad \chi_{C2} \parallel R = \frac{\frac{1}{sC_2}R}{\frac{1}{sC_2} + R} = \frac{R}{1 + sC_2R}$$

$$T(s) \equiv \frac{V_o(s)}{V_i(s)} = \frac{s/\left(1 + C_2/C_1\right)}{s + 1/RC_1\left(1 + C_2/C_1\right)}$$

$$\boxed{T(s) = \frac{Ks}{s + \omega_0}}$$

STC highpass

For $C_1 = C_2 = 0.5\ \mu F$ and $R = 100\ k\Omega$

K = 1/(1+0.5uF/0.5uF) = 0.5

ω_o = 1/[100kΩ x 0.5uF(1+0.5uF/0.5uF)] = 10 rad/s

$f_o = \omega_o / 2\pi$ = 1.59 Hz

zeros: $s = 0$

poles: s = -10 rad/s

Problem F.1 worked through

Find the transfer function $T(s) = V_o(s)/V_i(s)$ of the circuit in Fig. PF.1. Is this an STC network? If so, of what type? For $C_1 = C_2 = 0.5$ μF and $R = 100$ kΩ, find the location of the pole(s) and zero(s), and sketch Bode plots for the magnitude response and the phase response.

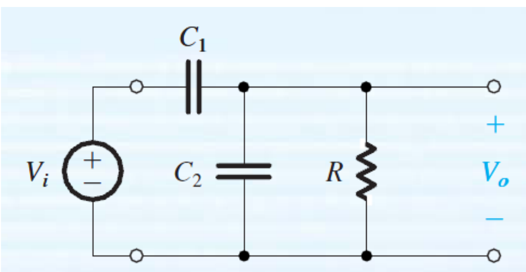


Figure PF.1

$$V_o(s) = V_i(s)\frac{R \parallel \chi_{C2}}{\chi_{C1} + \left(R \parallel \chi_{C2}\right)}$$

$$R \parallel \chi_{C2} = \frac{R \times \dfrac{1}{sC_2}}{R + \dfrac{1}{sC_2}}\left[\frac{sC_2}{sC_2}\right] = \frac{R}{1 + sRC_2}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R \parallel \chi_{C2}}{\chi_{C1} + \left(R \parallel \chi_{C2}\right)}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{\dfrac{R}{1 + sRC_2}}{\dfrac{1}{sC_1} + \dfrac{R}{1 + sRC_2}}\left[\frac{1 + sRC_2}{1 + sRC_2}\right] = \frac{R}{R + \dfrac{1 + sRC_2}{sC_1}}\left[\frac{sC_1}{sC_1}\right] = \frac{sRC_1}{1 + sRC_2 + sRC_1}$$

$$= \frac{sRC_1}{sR(C_2 + C_1) + 1}\left[\frac{\dfrac{1}{R(C_2 + C_1)}}{\dfrac{1}{R(C_2 + C_1)}}\right] = \frac{\dfrac{sRC_1}{R(C_2 + C_1)}}{s + \dfrac{1}{R(C_2 + C_1)}} = \frac{s\dfrac{C_1}{C_1 + C_2}}{s + \dfrac{1}{R(C_2 + C_1)}}$$

$$\boxed{T(s) = \frac{Ks}{s + \omega_0}}$$

STC highpass

Problem F.1 worked through

Find the transfer function $T(s) = V_o(s) / V_i(s)$ of the circuit in Fig. PF.1. Is this an STC network? If so, of what type? For $C_1 = C_2 = 0.5\ \mu$F and $R = 100$ kΩ, find the location of the pole(s) and zero(s), and sketch Bode plots for the magnitude response and the phase response.

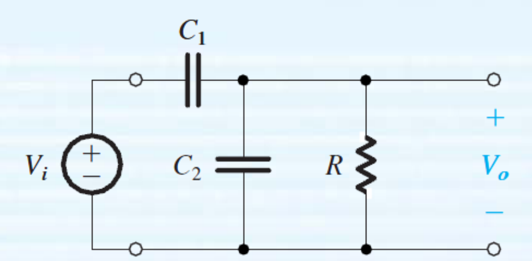


Figure PF.1

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{s\dfrac{C_1}{C_1+C_2}}{s+\dfrac{1}{R(C_2+C_1)}}$$

$$\boxed{T(s) = \frac{Ks}{s+\omega_0}}$$

STC highpass

For $C_1 = C_2 = 0.5\ \mu$F and $R = 100$ kΩ

K = 0.5uF/(0.5uF+0.5uF) = 0.5

ω_o = 1/[100kΩ x (0.5uF+0.5uF)] = 10 rad/s

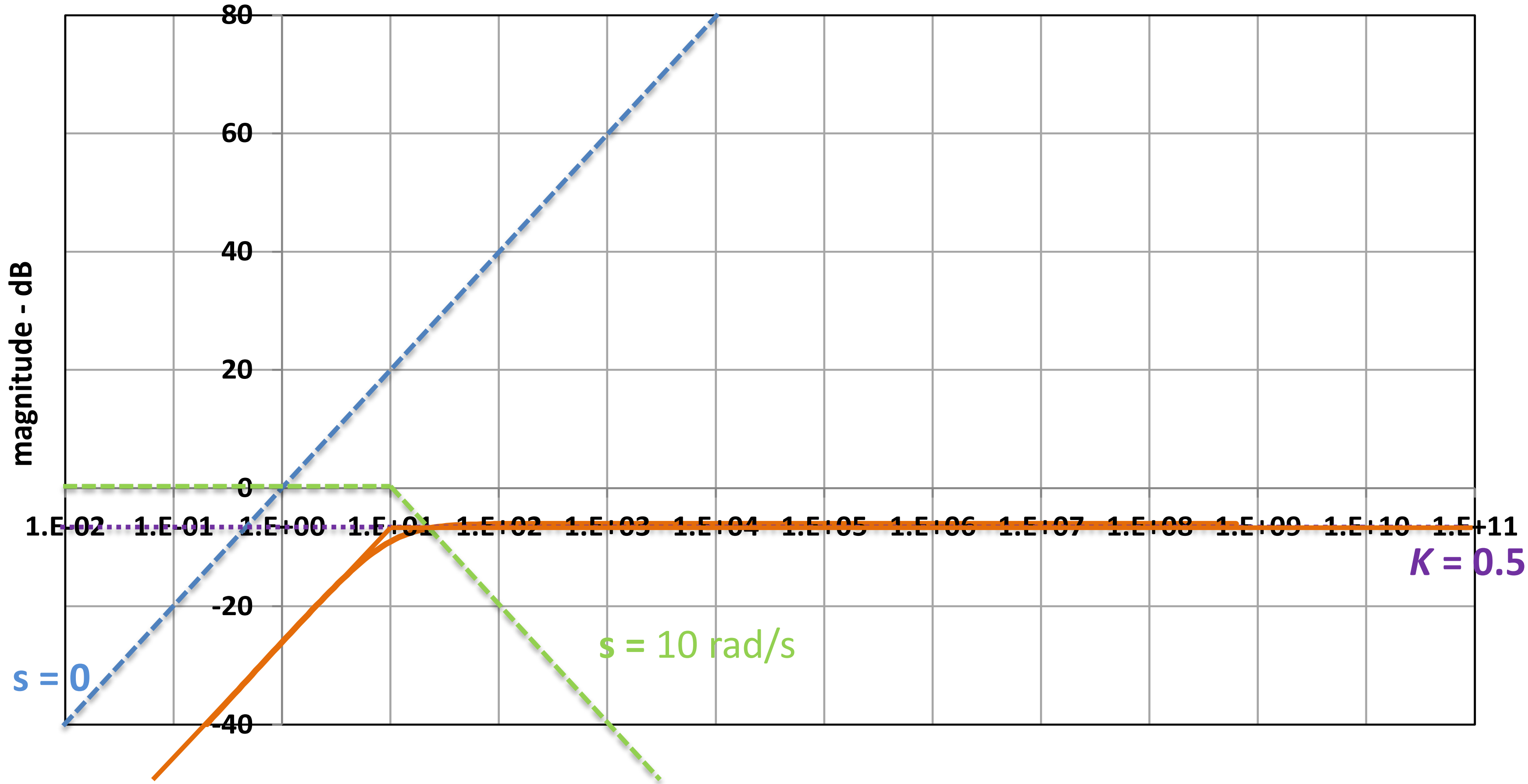
$f_o = \omega_o / 2\pi$ = 1.59 Hz

zeros: $s = 0$

poles: $s = -10$ rad/s

Problem F.1

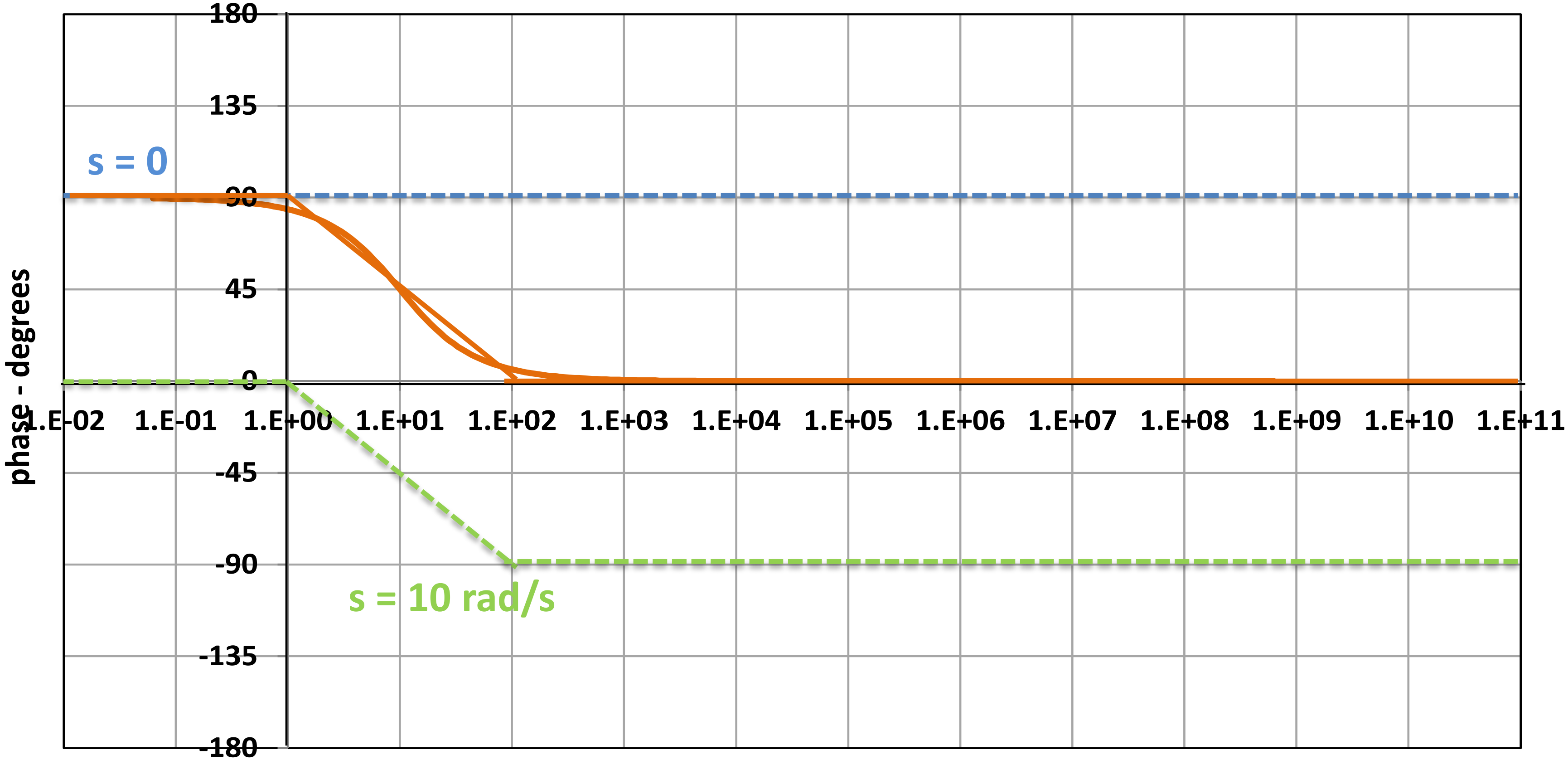
zeros: $s = 0$ poles: $s = 10$ rad/s



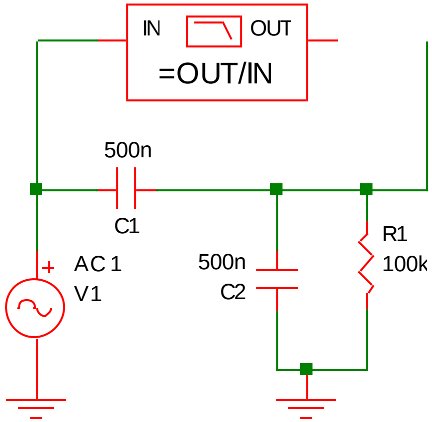
Problem F.1

zeros: $s = 0$

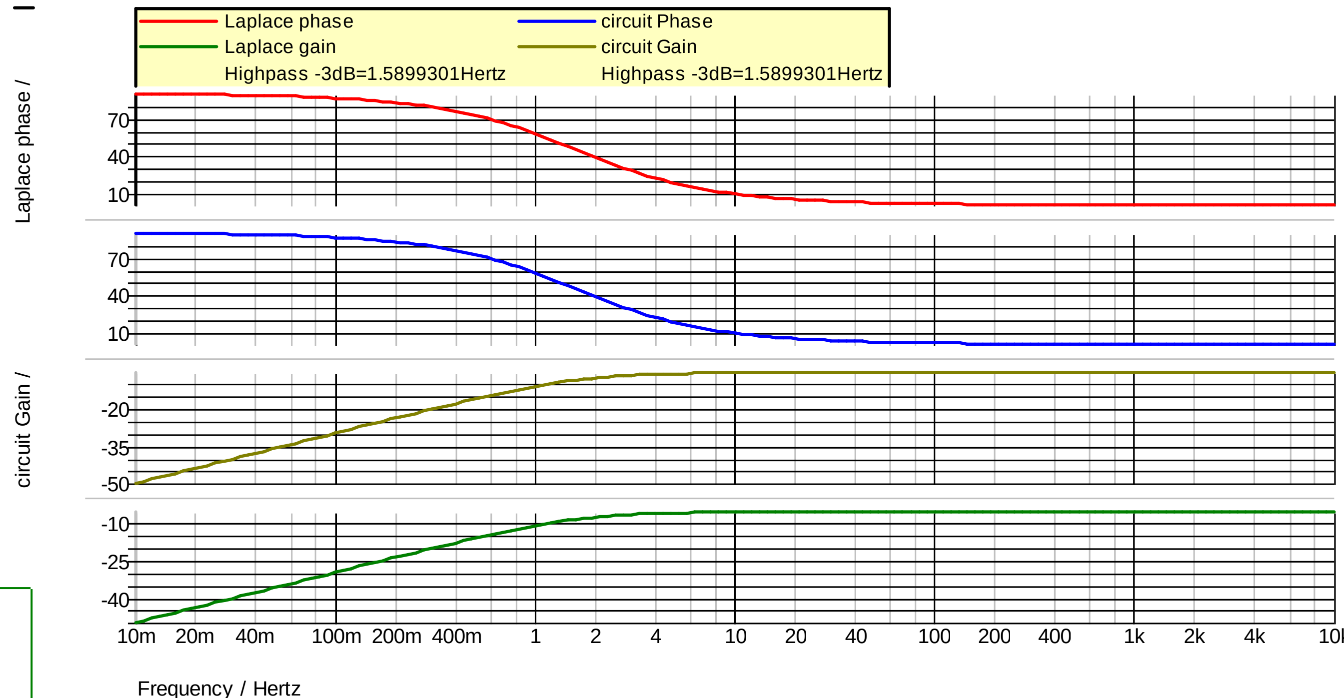
poles: $s = 10$ rad/s



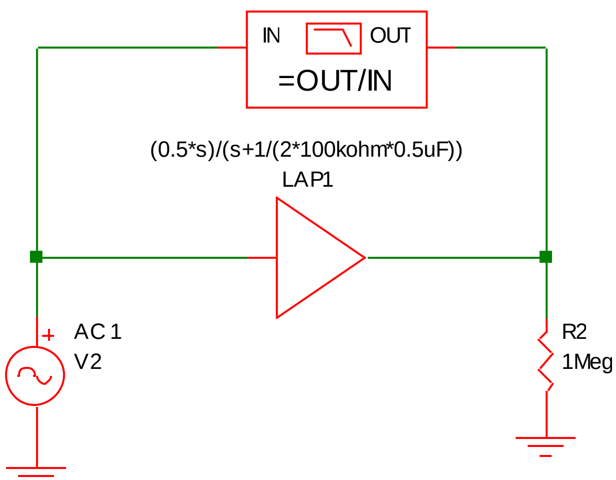
Problem F.1



IN
OUT
=OUT/IN
500n
C1
AC 1
V1
500n
C2
R1
100k



Laplace phase
circuit Phase
Laplace gain
circuit Gain
Highpass -3dB=1.5899301Hertz
Highpass -3dB=1.5899301Hertz
Laplace phase /
circuit Gain /
70
40
10
-20
-35
-50
-10
-25
-40
10m 20m 40m 100m 200m 400m 1 2 4 10 20 40 100 200 400 1k 2k 4k 10k
Frequency / Hertz



IN
OUT
=OUT/IN
(0.5*s)/(s+1/(2*100kohm*0.5uF))
LAP1
AC 1
V2
R2
1Meg

Problem F.2

(a) Find the voltage transfer function $T(s) = V_o(s) / V_i(s)$, for the STC network shown in Fig. PF.2.

(b) In this circuit, capacitor C is used to couple the signal source V_s having a resistance R_s to a load R_L. For R_s = 10 kΩ, design the circuit, specifying the values of R_L and C to only one significant digit to meet the following requirements:

(i) The load resistance should be as small as possible.

(ii) The output signal should be at least 70% of the input at high frequencies.

(iii) The output should be at least 10% of the input at 10 Hz.

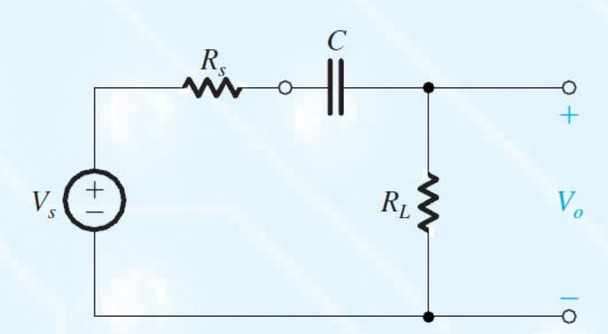


Figure PF.2

$$V_o = \frac{R_L}{R_L + \chi_C} V_i$$

$$T(s) \equiv \frac{V_o}{V_i} = \frac{R_L s C}{s C R_L + 1}$$

Problem F.2

(b) In this circuit, capacitor C is used to couple the signal source V_s having a resistance R_s to a load R_L. For R_s = 10 kΩ, design the circuit, specifying the values of R_L and C to only one significant digit to meet the following requirements:

(i) The load resistance should be as small as possible.

(ii) The output signal should be at least 70% of the input at high frequencies.

(iii) The output should be at least 10% of the input at 10 Hz.

$$T(s) = \frac{R_L sC}{1 + R_L sC}$$

$$\frac{R_L}{R_L + R_S} = \frac{R_L}{R_L + 10\text{k}\Omega} \geq 0.70 \qquad R_L \geq 23.3\text{k}\Omega$$

R_L to one significant digit is $R_L = 30\text{k}\Omega$

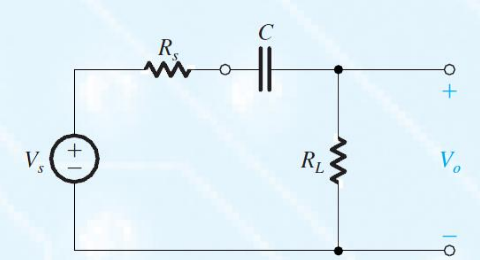


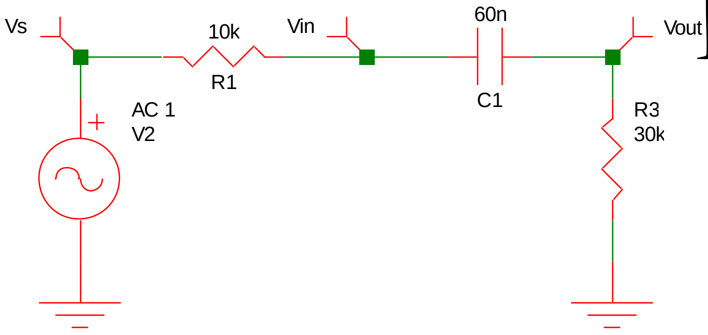
Figure PF.2

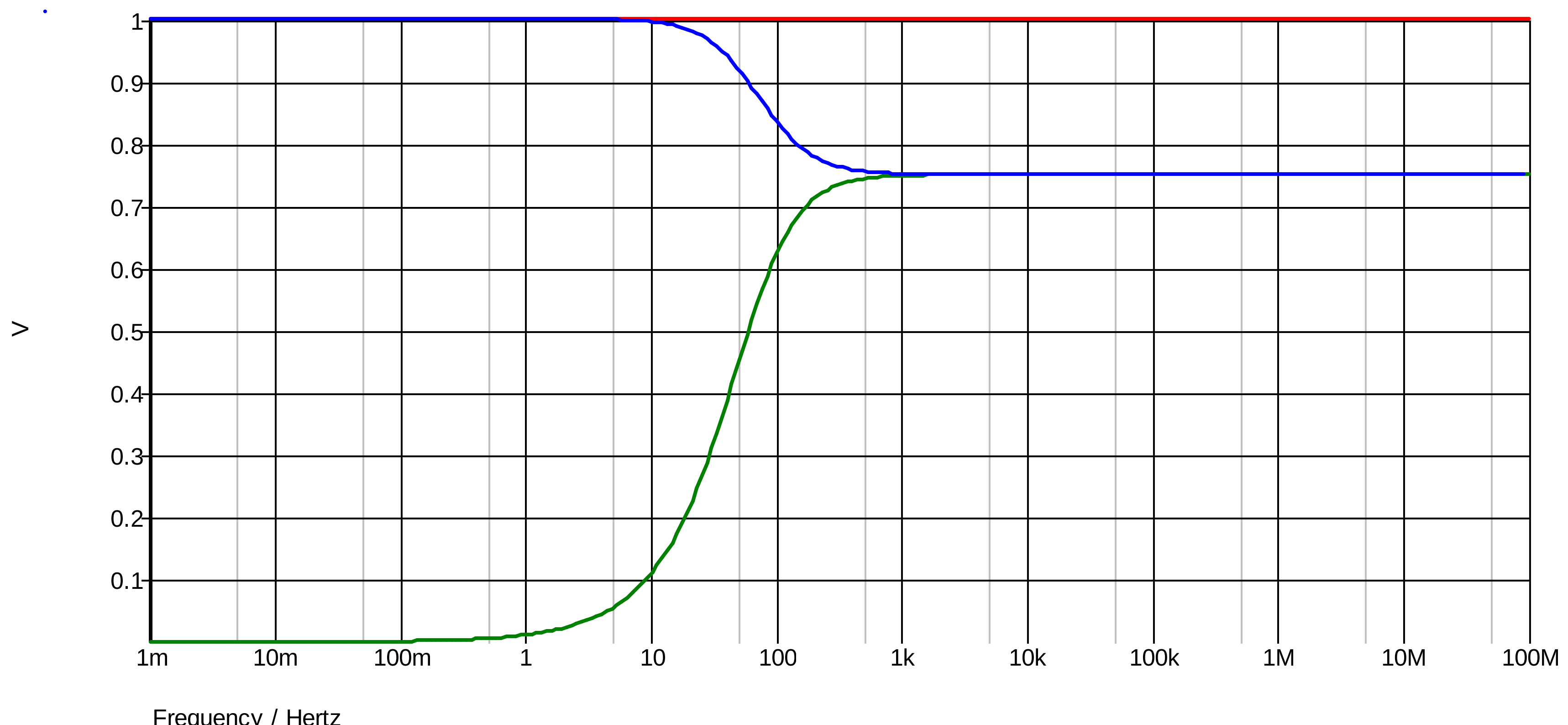
$$T(2\pi 10\text{Hz}) = \frac{(30\text{k}\Omega)(2\pi 10\text{Hz})C}{1 + (30\text{k}\Omega)(2\pi 10\text{Hz})C} \geq 0.10$$

$$C \geq 58.9\text{nF}$$

C to one significant digit is $C = 60\text{nF}$

Problem F.2


Vs
10k
R1
Vin
60n
C1
Vout
AC 1
V2
R3
30k


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
V
1m
10m
100m
1
10
100
1k
10k
100k
1M
10M
100M
Frequency / Hertz

Problem F.10

Sketch Bode plots for the magnitude and phase of the transfer function

$$T(s) = \frac{10^4\left(1 + {}^{s}/_{10^5}\right)}{(1 + s/10^3)(1 + s/10^4)}$$

From your sketches, determine approximate values for the magnitude and phase at ω = 10^6 rad/s. What are the exact values determined from the transfer function?

zeros: s = -10^5 rad/s , ∞

poles: s = -10^3 rad/s, -10^4 rad/s

determine approximate values for the magnitude and phase at ω = 10^6 rad/s.

Mag from plot on next slide at 10^6 rad/s ~ 0 dB, Phase ~ -90deg

Solving the equations (in MathCAD) for s = $j10^6$ rad/s yields mag = 0.043 dB and phase = -95.08 deg

MathCAD

$$T(s) := \frac{10^4\cdot\left(1 + \frac{s}{10^5}\right)}{\left(1 + \frac{s}{10^3}\right)\cdot\left(1 + \frac{s}{10^4}\right)}$$

$s1 := i\cdot 10^6$

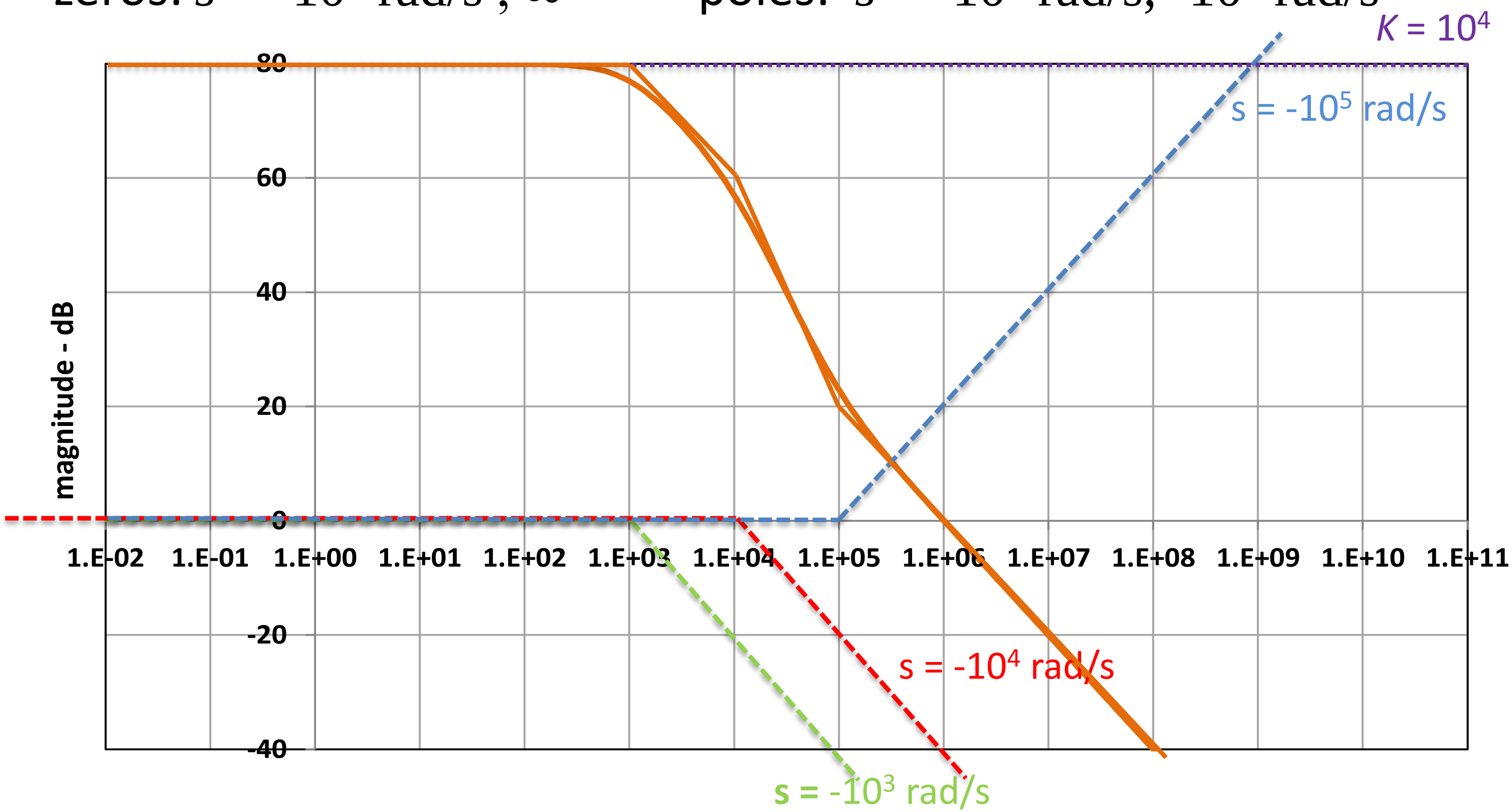
$T(s1) = -0.089 - 1.001j$

$20\cdot\log(|T(s1)|) = 0.043$

$\arg(T(s1)) = -95.08\,\text{deg}$

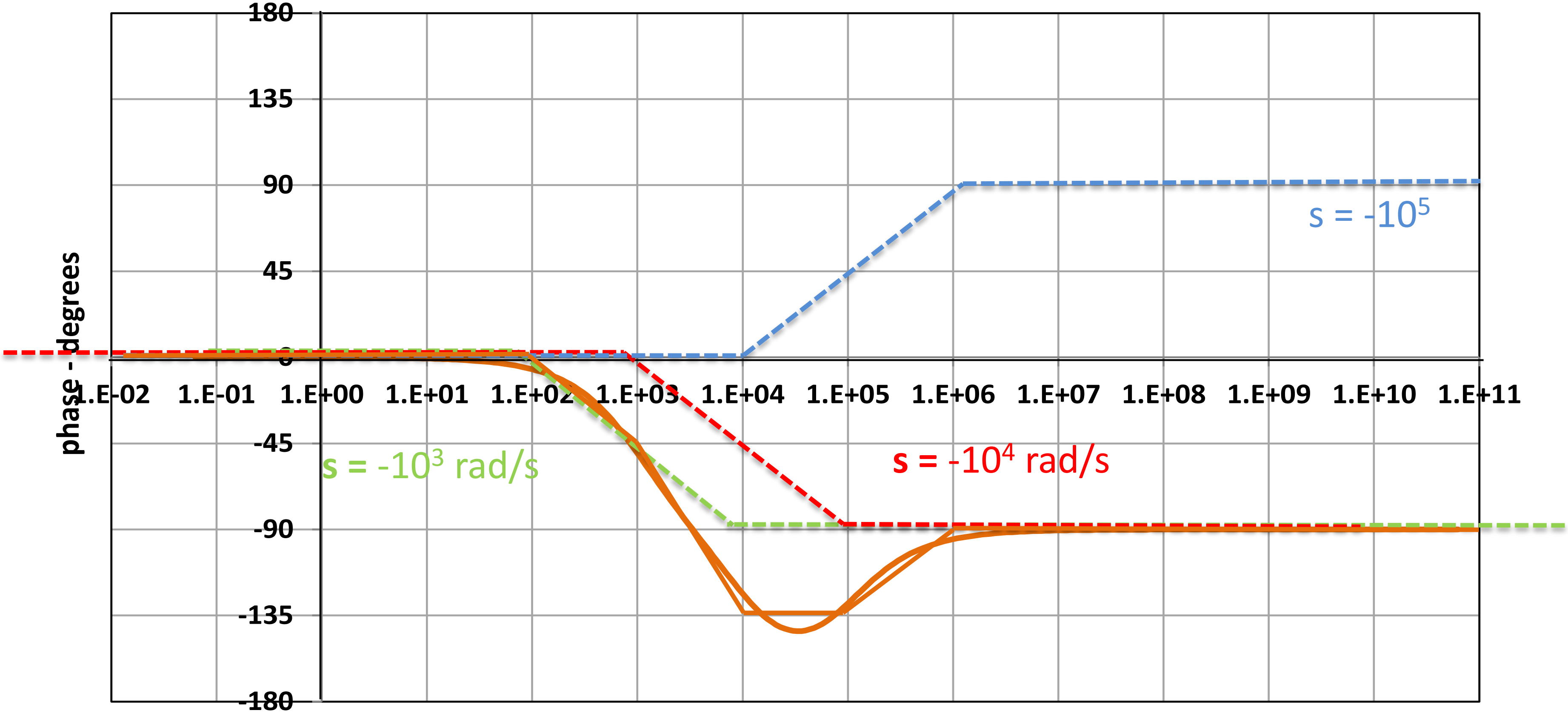
Problem F.10 Mag

zeros: $s = -10^5$ rad/s , ∞ poles: $s = -10^3$ rad/s, -10^4 rad/s



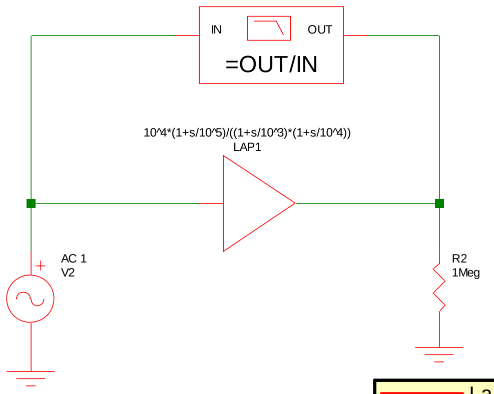
Problem F.10 Phase

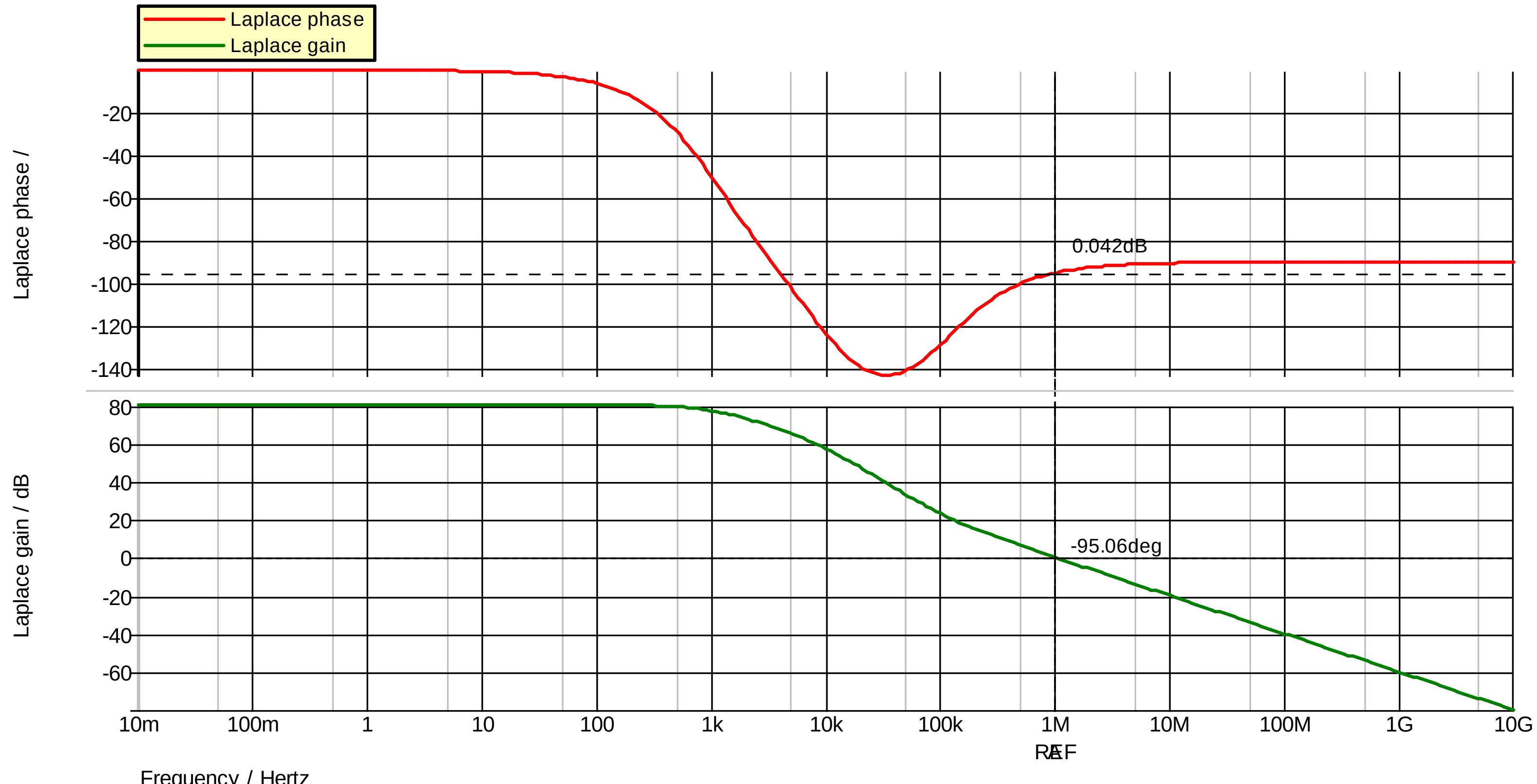
zeros: $s = -10^5$ rad/s , ∞ poles: $s = -10^3$ rad/s, -10^4 rad/s



determine approximate values for the magnitude and phase at $\omega = 10^6$ rad/s.

Problem F.10


IN
OUT
=OUT/IN
10^4*(1+s/10^5)/((1+s/10^3)*(1+s/10^4))
LAP1
AC 1
V2
R2
1Meg


Laplace phase
Laplace gain
Laplace phase /
Laplace gain / dB
-20
-40
-60
-80
-100
-120
-140
80
60
40
20
0
-20
-40
-60
0.042dB
-95.06deg
10m
100m
1
10
100
1k
10k
100k
1M
10M
100M
1G
10G
REF
Frequency / Hertz

Problem F.11

A particular amplifier has a voltage transfer function

$$T(s) = \frac{10s^2}{(1 + s/10)(1 + s/100)(1 + s/10^6)}$$

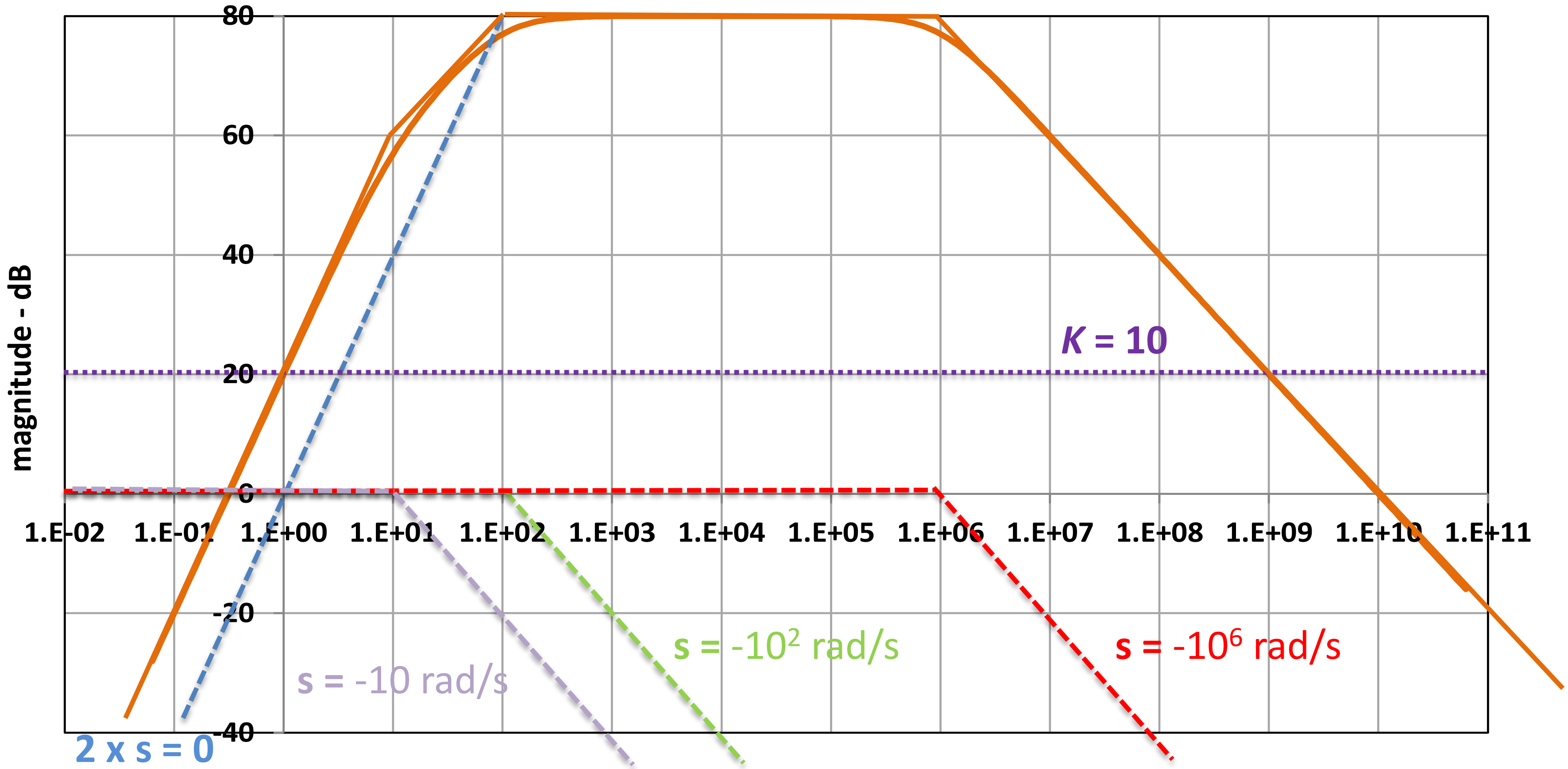
Find the poles and zeros. Sketch the magnitude of the gain in dB versus frequency on a logarithmic scale. Estimate the gain at 10^0, 10^3, 10^5, and 10^7 rad/s.

zeros: $s = 0, 0, \infty$

poles: s = -10 rad/s, -100 rad/s, -10^6 rad/s

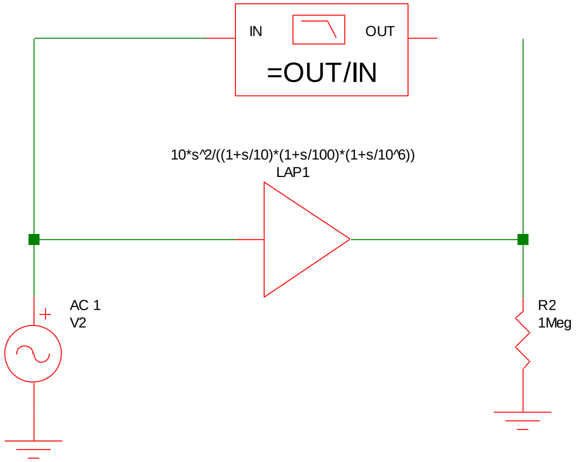
Problem F.11

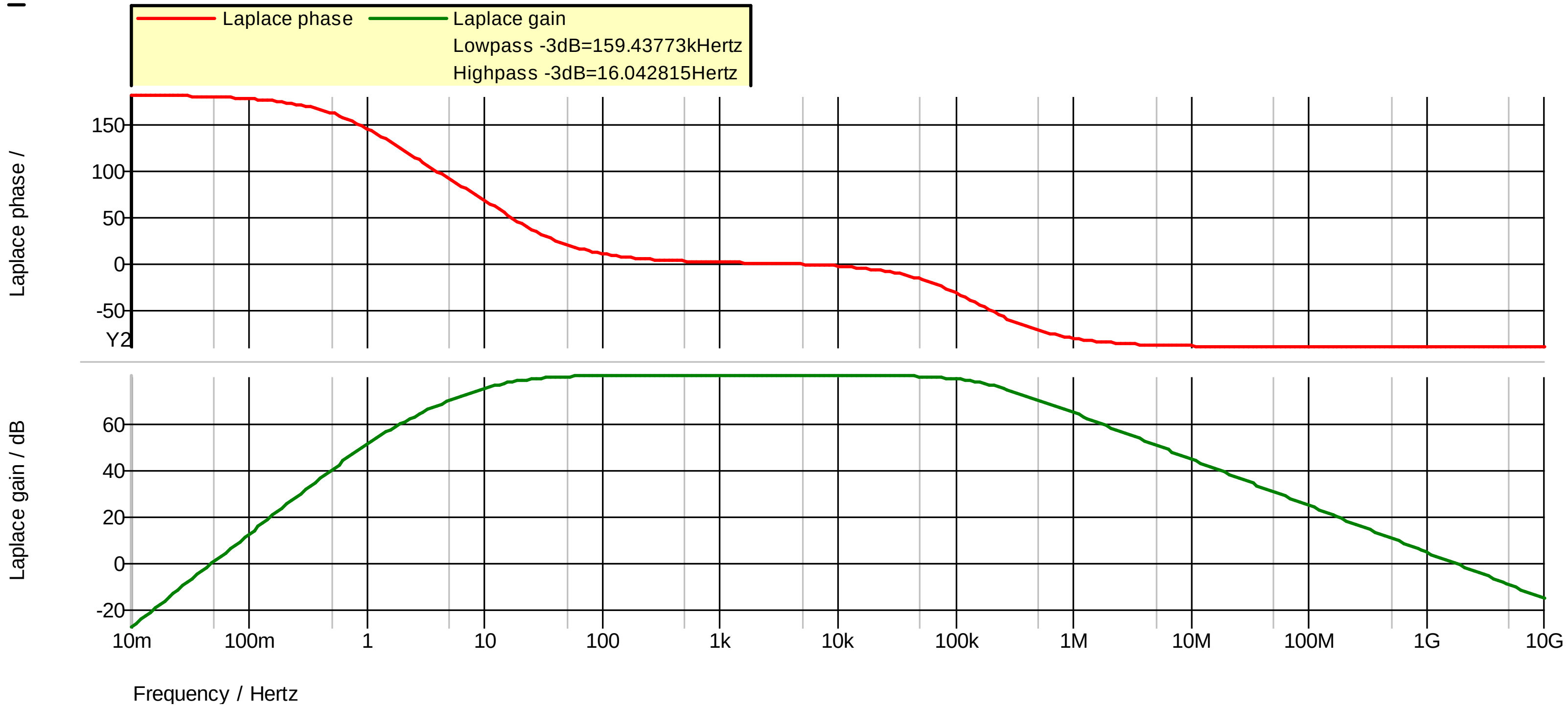
zeros: $s = 0$, $s = \infty$ poles: s =-10 rad/s, -10^2 rad/s, $s = -10^6$ rad/s



Estimate the gain at 10^0, 10^3, 10^5, and 10^7 rad/s

Problem F.11


IN
OUT
=OUT/IN
10*s^2/((1+s/10)*(1+s/100)*(1+s/10^6))
LAP1
AC 1
V2
R2
1Meg


Laplace phase
Laplace gain
Lowpass -3dB=159.43773kHertz
Highpass -3dB=16.042815Hertz
Laplace phase /
150
100
50
0
-50
Y2
Laplace gain / dB
60
40
20
0
-20
10m
100m
1
10
100
1k
10k
100k
1M
10M
100M
1G
10G
Frequency / Hertz