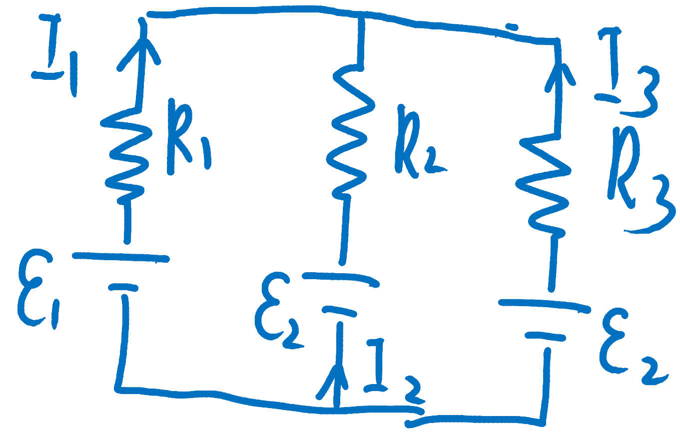
* Mesh analysis

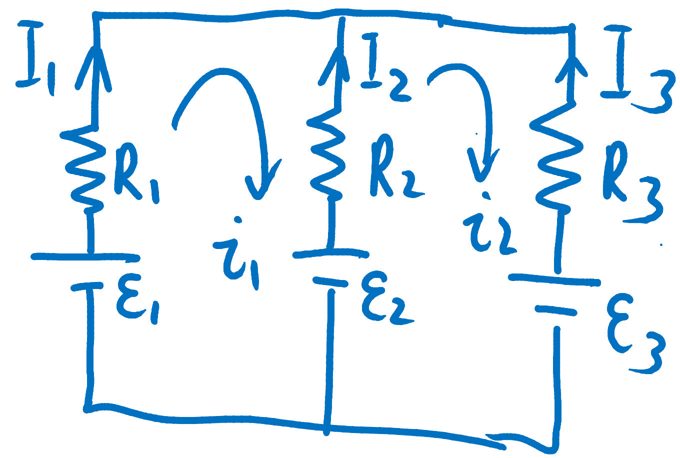
In Kirchhoff's laws, we use branch currents as variable.



3 variables

3 equations $\quad I_1 + I_2 + I_3 = 0$

Define mesh currents i_1 and i_2



$$I_1 = i_1$$

$$I_3 = -i_2$$

$$I_2 = i_2 - i_1$$

$$I_1 + I_2 + I_3 = \cancel{i_1} + (-\cancel{i_2}) + (\cancel{i_2} - \cancel{i_1}) = 0$$

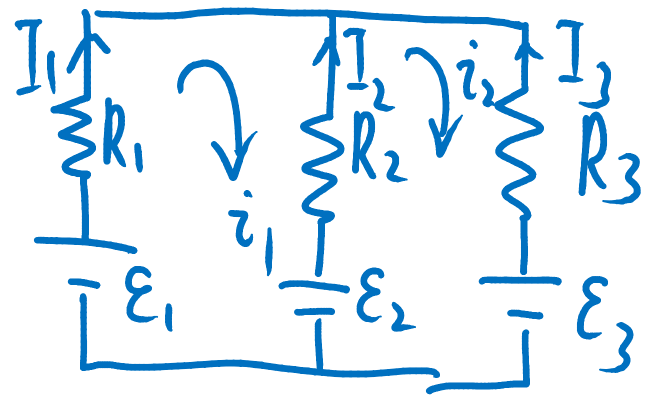
By defining mesh current i_1 and i_2, KCL is automatically satisfied.

loop equations (KVL)

$$I_1 = i_1$$

$$I_2 = i_2 - i_1$$

$$I_3 = -i_2$$



left mesh $\quad \varepsilon_1 - I_1 R_1 + I_2 R_2 - \varepsilon_2 = 0$

$$\varepsilon_1 - i_1 R_1 + (i_2 - i_1) R_2 - \varepsilon_2 = 0$$

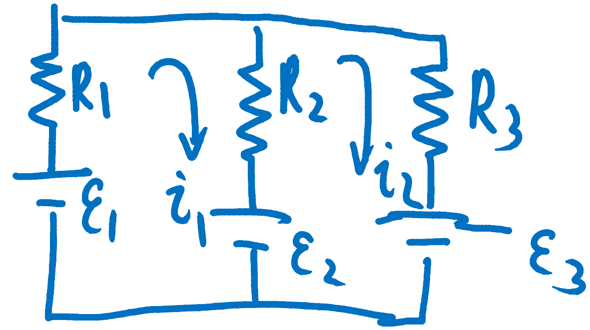
$$i_1 (R_1 + R_2) - i_2 R_2 = \varepsilon_1 - \varepsilon_2 \quad (1)$$

similarly right mesh $\quad -R_2 i_1 + (R_2 + R_3) i_2 = \varepsilon_2 - \varepsilon_3 \quad (2)$

only two equations.

Mesh analysis

Step 1: assignment of mesh currents (CCW)



mesh is a loop that does not contain other loops.

Step 2. Apply KVL to each mesh

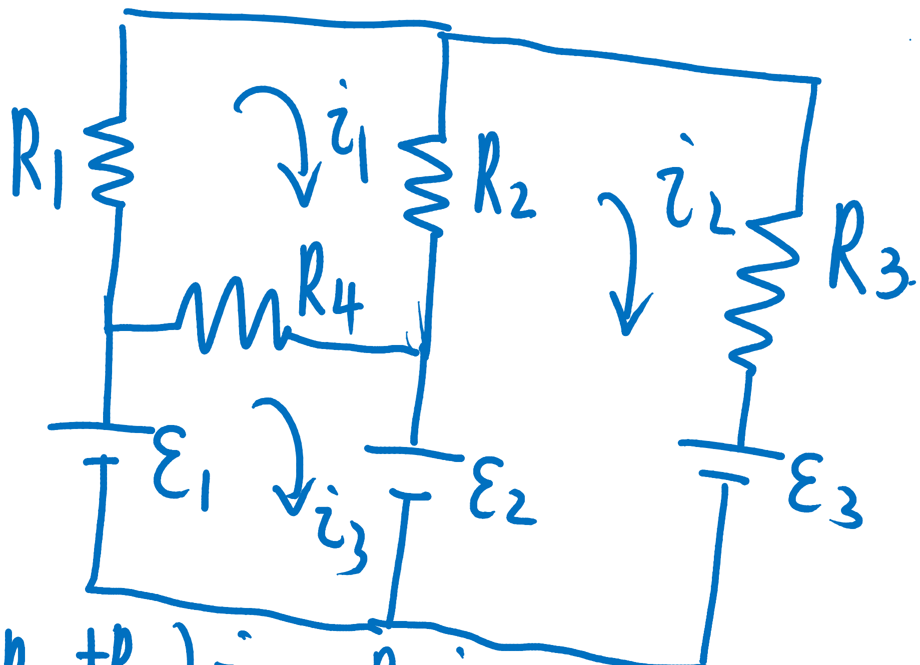
- The so-called self-resistance is the effective resistance of the resistors in series within a mesh. The mutual resistance is the resistance that the mesh has in common with a neighboring mesh.
- To write down the mesh equation, evaluate the self-resistance, then multiply by the mesh current. This have units of voltage
- From that, subtract the product of mutual resistance and current from the neighboring mesh for each such neighbor.
- Equate the result above to the driving voltage, taken to be positive if it's polarity tends to push current in the same direction as the

assigned mesh current.

mesh 1. $(R_1+R_2)i_1 - R_2 i_2 = \mathcal{E}_1 - \mathcal{E}_2$

mesh 2. $-R_2 i_1 + (R_2+R_3) i_2 = \mathcal{E}_2 - \mathcal{E}_3$

Example



mesh. 1 $(R_1+R_2+R_4)i_1 - R_2 i_2 - R_4 i_3 = 0$

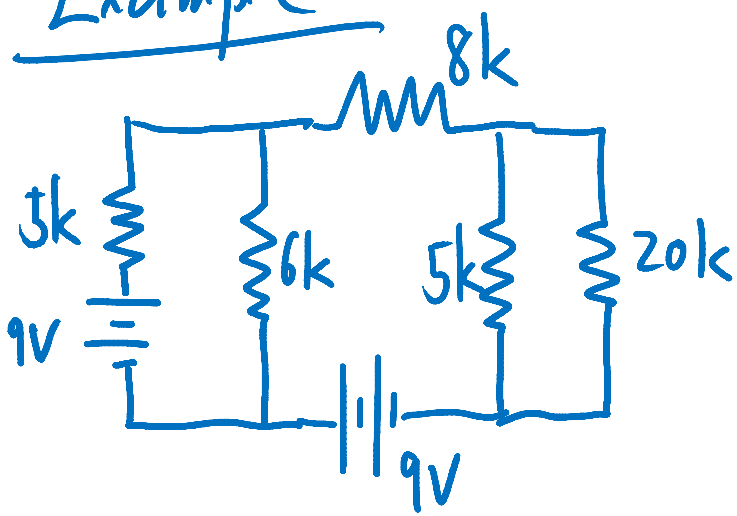
mesh 2. $-R_2 i_1 + (R_2+R_3) i_2 + 0 = \mathcal{E}_2 - \mathcal{E}_3$

mesh 3.

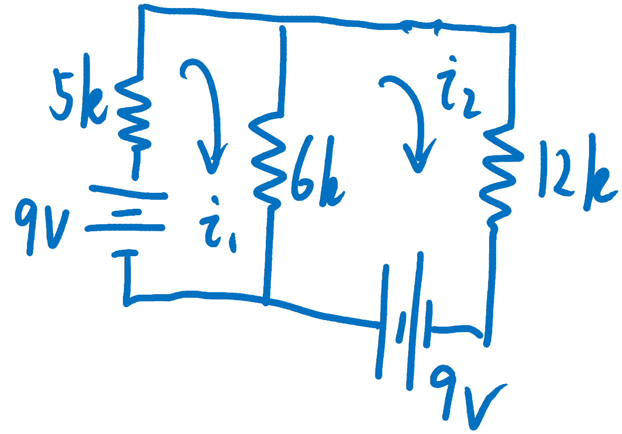
mesh 3. $-R_4 i_1 \qquad + R_4 i_3 = \varepsilon_1 - \varepsilon_2$

Example

$20 // 5 = \frac{20 \times 5}{20+5} = 4k \qquad 4k + 8k = 12k$



$\Rightarrow$



left mesh: $11 i_1 - 6 i_2 = 9$

right mesh: $18 i_2 - 6 i_1 = 9$

$i_1 = 1.33$ (mA)

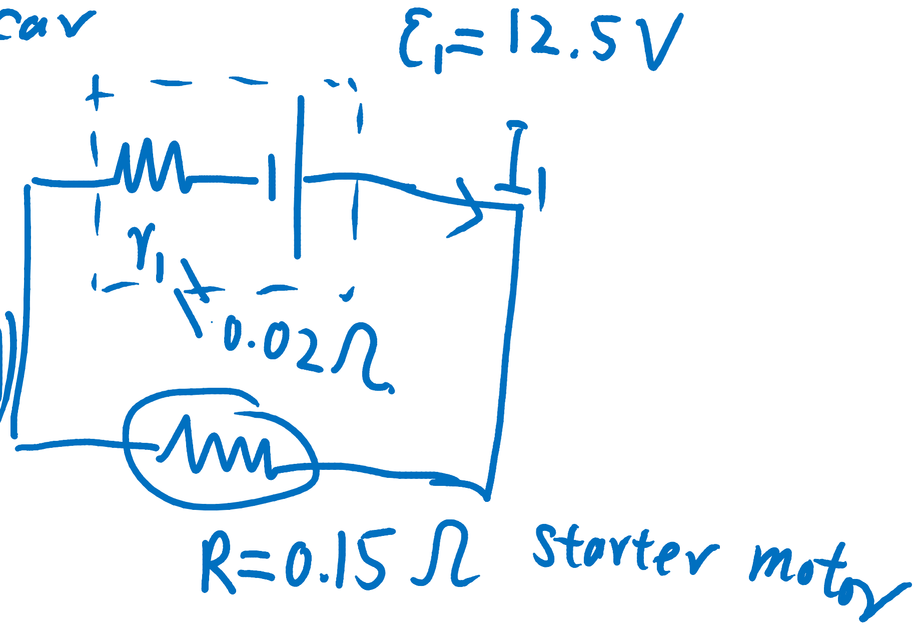
$i_2 = 0.94$ (mA)

A comparison between KVL/KCL method and mesh analysis

	KVL/KCL	mesh analysis
independent variables	branch currents	mesh currents
number of equations	number of branches	number of meshes
underlying principles	KCL and KVL	KVL explicit KCL implicit
sign conventions	evaluate voltage change around loop	self resistance, mutual resistance source direction
How to deal with current sources	current in branch is the preset value of current source	

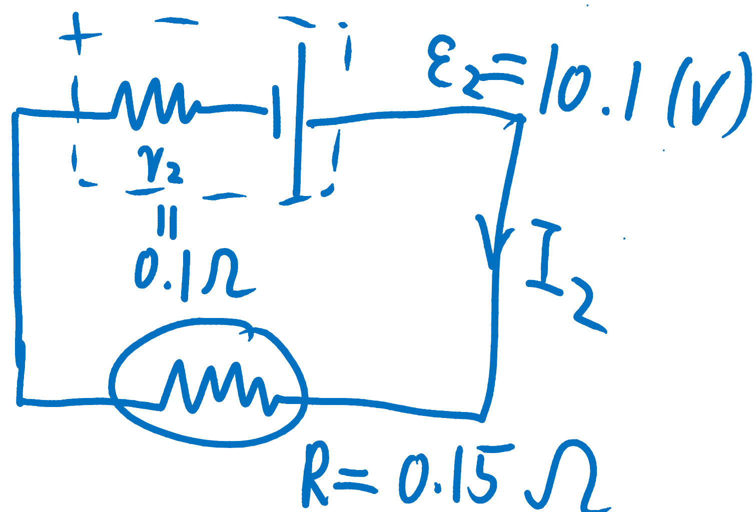
Application   jump start a car

good car battery:



$$I_1 = \frac{\varepsilon_1}{r_1 + R} = \frac{12.5}{0.02 + 0.15} = 73.5(A)$$

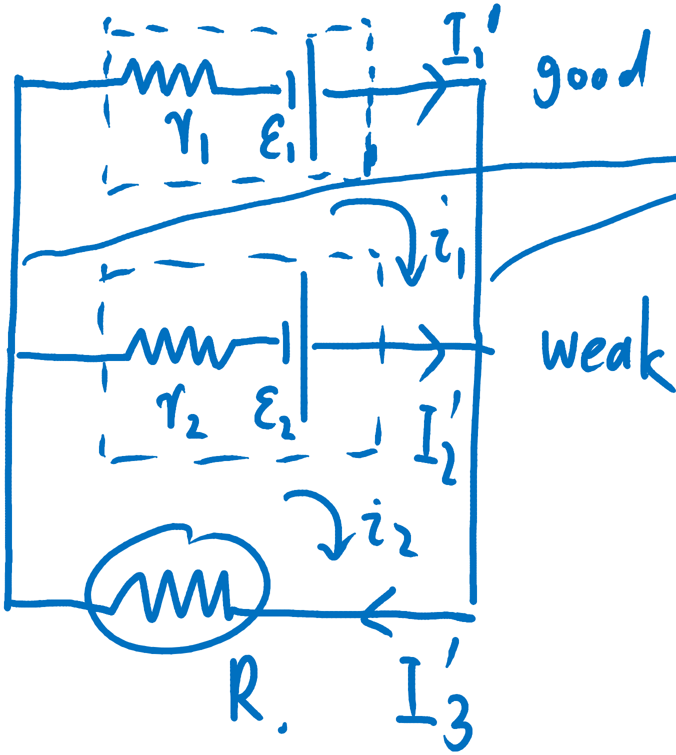
weak car battery



$$I_2 = \frac{\varepsilon_2}{r_2 + R} = \frac{10.1V}{(0.1 + 0.15)\Omega}$$

$$= 40.4\,(A) < I_1$$

Jump start: good battery starts motor and recharge weak battery.



jump start cables

I_1', I_2', I_3'

$I_3' = i_2 = 72.6\,(A) \approx I_1$

$I_1' = i_1 = 80.5\,(A)$

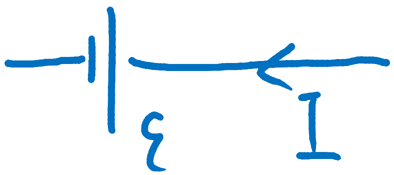
$I_2' = i_2 - i_1 = -7.9\,(A)$

opposite to defined direction

$$(r_1 + r_2)\, i_1 - r_2 i_2 = \varepsilon_1 - \varepsilon_2$$

$$(r_2 + R)\, i_2 - r_2 i_1 = \varepsilon_2$$

$$\begin{cases} 0.12\, i_1 - 0.1\, i_2 = 2.4 & (1) \\ -0.1\, i_1 + 0.25\, i_2 = 10.1 & (2) \end{cases} \Rightarrow \begin{cases} i_1 = 80.5\,(A) \\ i_2 = 72.6\,(A) \end{cases}$$

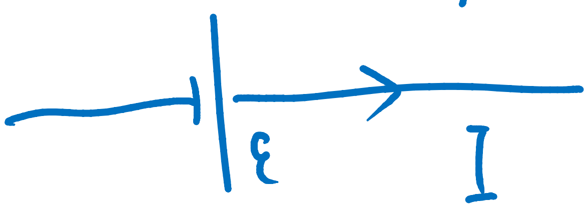


recharge a battery

electric energy converts to chemical energy inside battery

rate of energy conversion (power): $P = I\varepsilon$

Normal operation of a battery (discharging)



chemical energy inside battery converts to electric energy

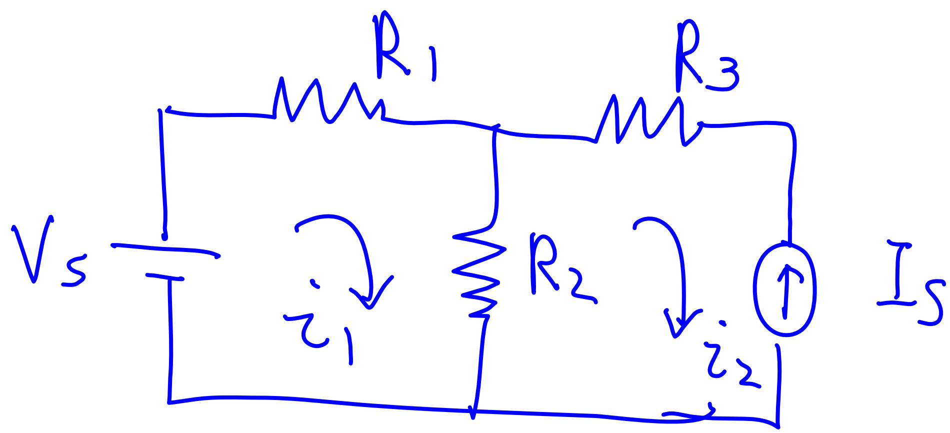
rate of energy conversion (power): $P = I\varepsilon$

Discharging power of ε_1

$=$ charging power of ε_1 + heating on resistors (r_1, r_2, R)

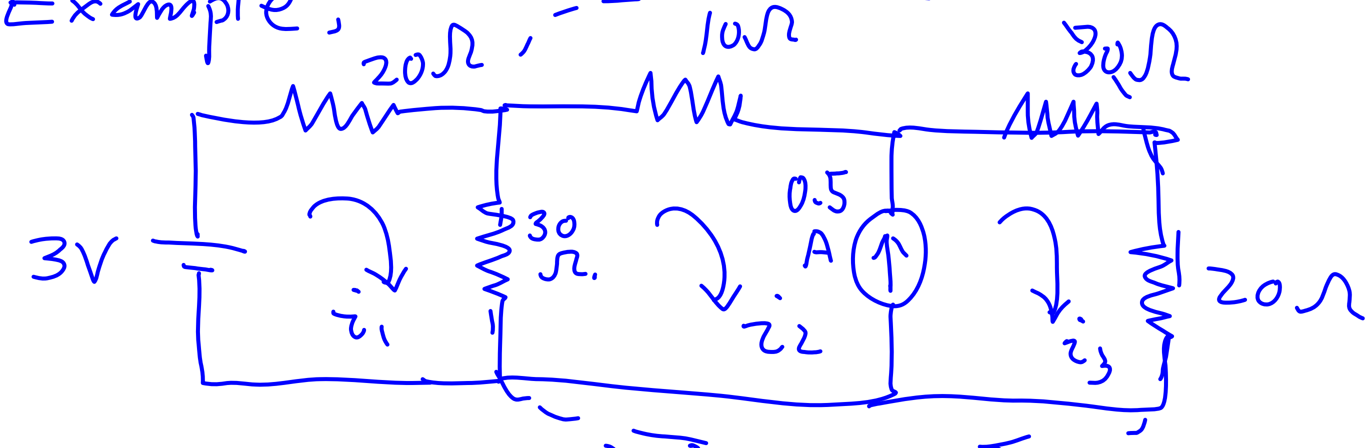
Mesh analysis with current sources

Example



mesh
$$\begin{cases} (R_1+R_2)i_1 - R_2 i_2 = V_s \\ i_2 = -I_s \end{cases}$$

Example,



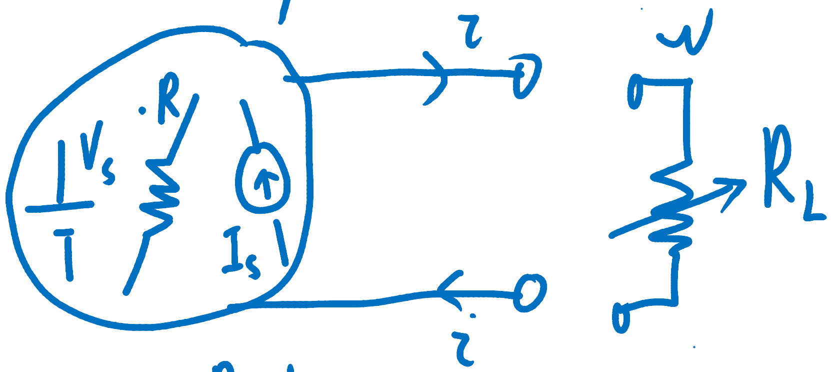
$$(20+30)i_1 - 30\,i_2 = 3 \qquad (1)$$

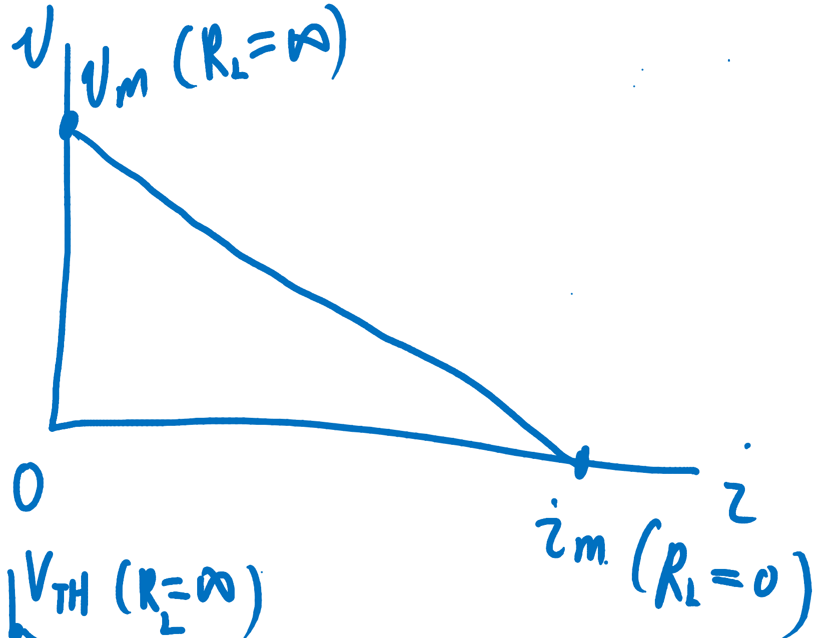
$$i_3 - i_2 = 0.5 \qquad (2)$$

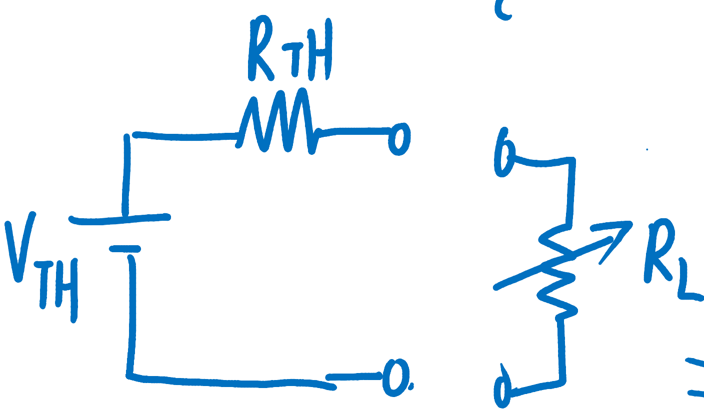
$$30(i_1 - i_2) - 10\,i_2 - (30+20)i_3 = 0 \qquad (3)$$

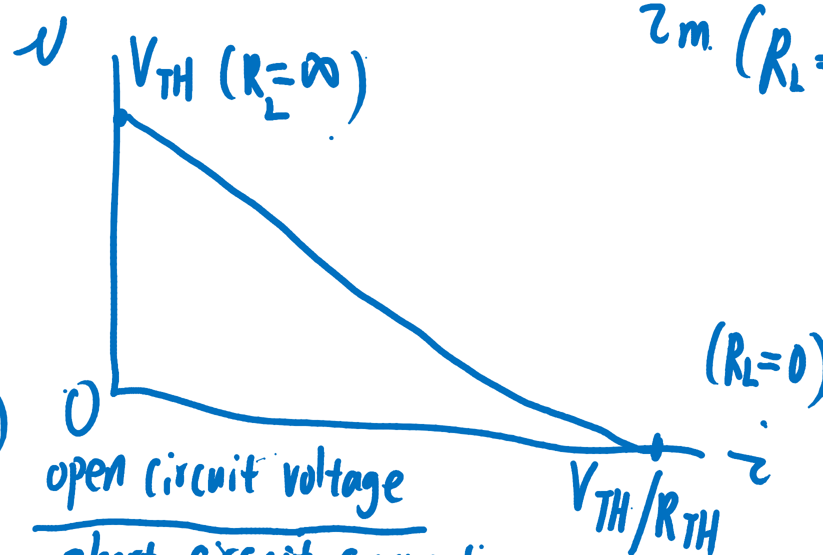
* Thevenin and Norton equivalent circuits

Arbitrary Network Consisting of sources and resistors









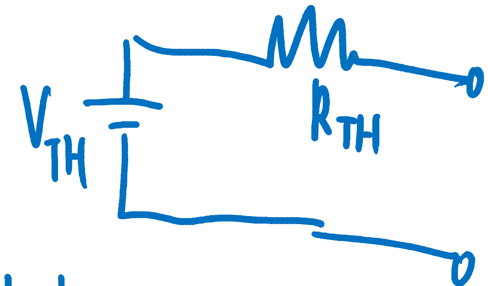
$V_{TH} = v_m$ (open circuit voltage)

$\frac{V_{TH}}{R_{TH}} = i_m$ (short circuit current)

$$R_{TH} = \frac{V_{TH}}{i_m} = \frac{v_m}{i_m} = \frac{\text{open circuit voltage}}{\text{short circuit current}}$$

R_{TH} is also the resistance of network seen from the two terminals.

Thevenin's Theorem

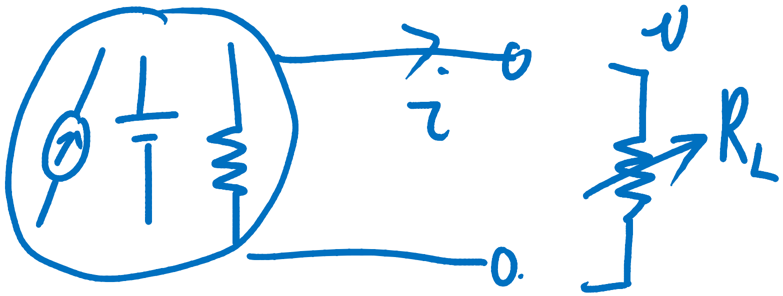


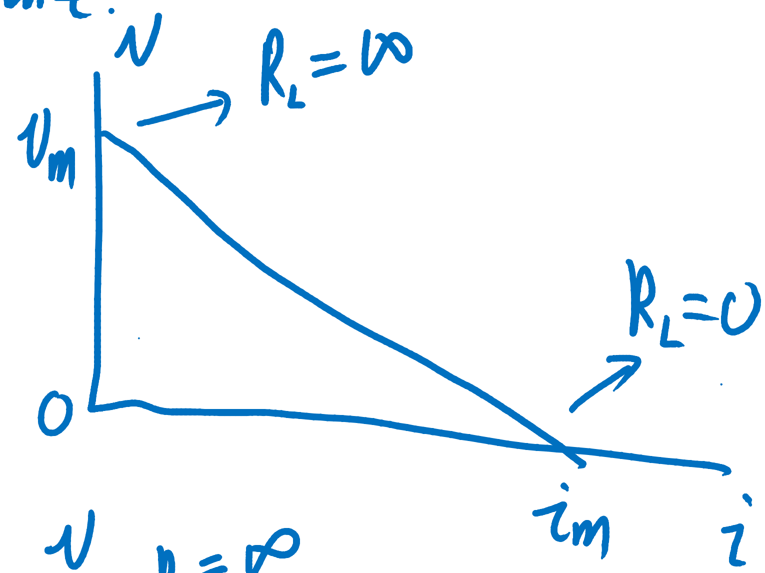
— Any two terminal linear circuit can be replaced with a voltage source in series with a resistance which will produce the same effects at the terminals.

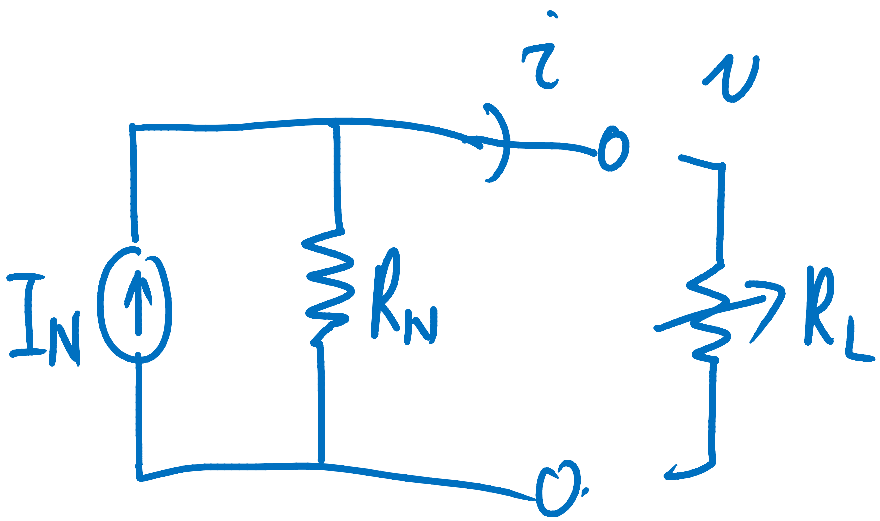
— V_{TH} is the open circuit voltage V_{oc} between the two terminals of the circuit that the Thevenin equivalent circuit is replacing.

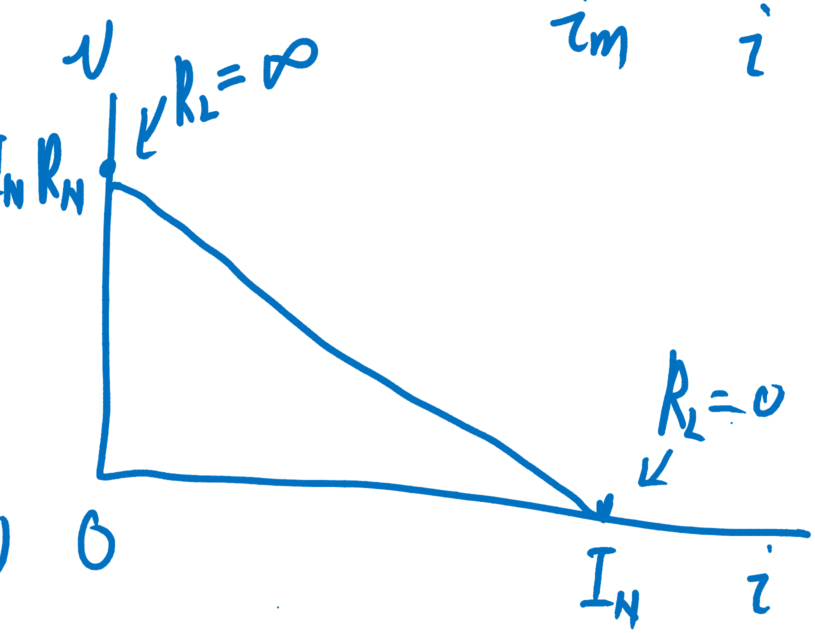
— R_{TH} is the ratio of V_{oc} to the short circuit current I_{sc}.
In linear circuits, this is equivalent to "killing" the sources and evaluating the resistance between the terminals. Voltage sources are killed by replacing it with a wire; Current sources are killed by disconnecting them.

Norton Equivalent Circuit.









$I_N R_N = V_m$ (open circuit voltage)

$I_N = i_m$ (short circuit current)

$$R_N = \frac{V_m}{I_N} = \frac{V_m}{i_m} = \frac{\text{open circuit voltage}}{\text{short circuit current}} = R_{TH}$$

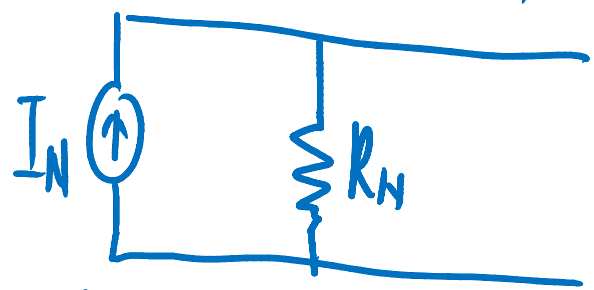
$$R_N = R_{TH}$$

$$V_{TH} = I_N R_N = V_{OC}$$

$$I_N = \frac{V_{TH}}{R_N} = \frac{V_{TH}}{R_{TH}}$$

$$R_N = R_{TH} = \frac{V_{OC}}{I_{SC}}$$

Norton Theorem



- Any two terminal linear circuit can be replaced with a current source in parallel with a resistance which will produce the same effects at the terminals
- I_N is short circuit current I_{sc} of the circuit that the Norton equivalent circuit is replacing.
- R_N is the ratio of V_{oc} to the short circuit current I_{sc}. Or calculating the equivalent resistance after killing the sources.

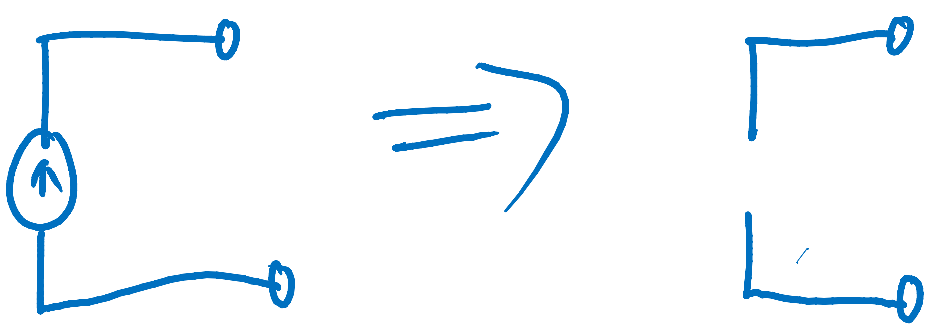
Calculation of R_{TH} and R_N

$R_N = R_{TH}$

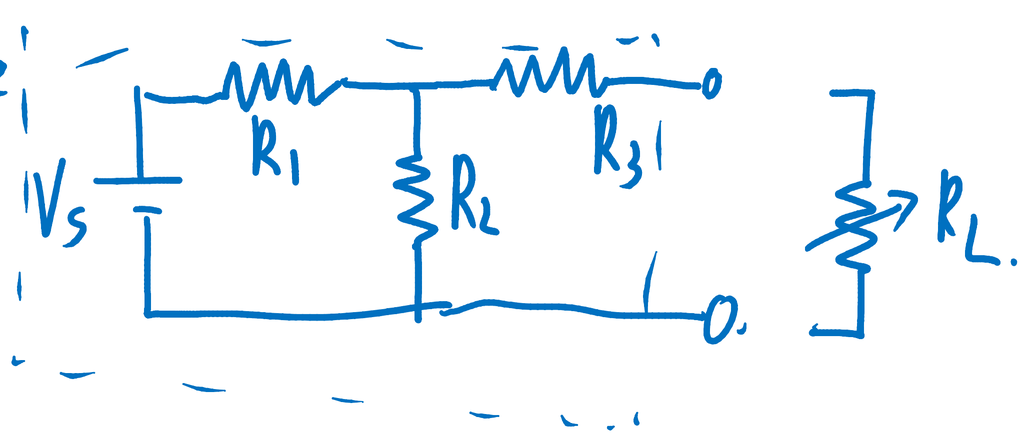
~~see~~ Set all sources to zero ("killing" the source)

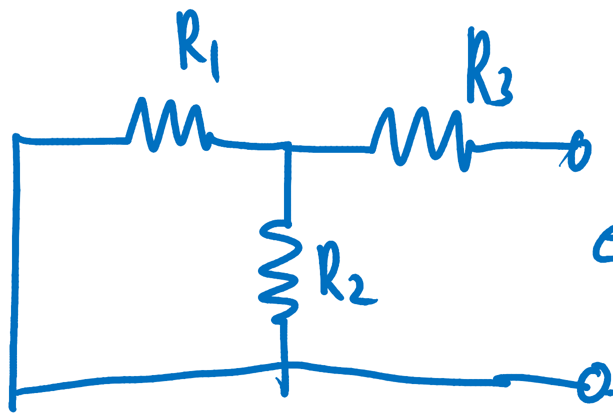
voltage source : replace with a wire

Current Source : replace with open circuit



Example

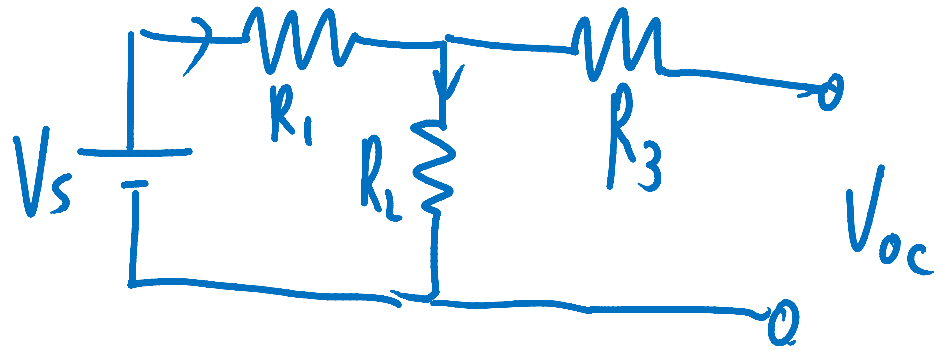




equivalent resistance

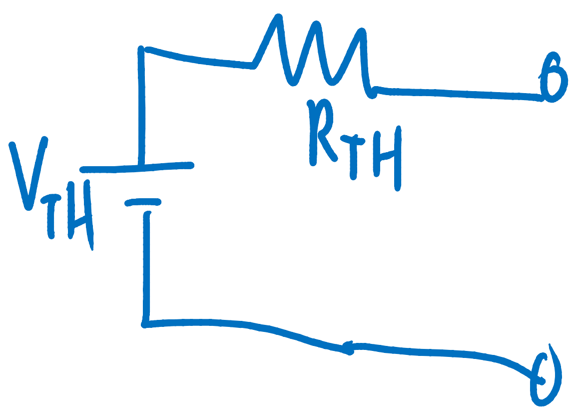
$$R_N = R_{TH} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Calculate $V_{TH} = V_{oc}$ open circuit voltage.



$$V_{oc} = V_{R_2} = V_S \cdot \frac{R_2}{R_1 + R_2} = V_{TH}$$

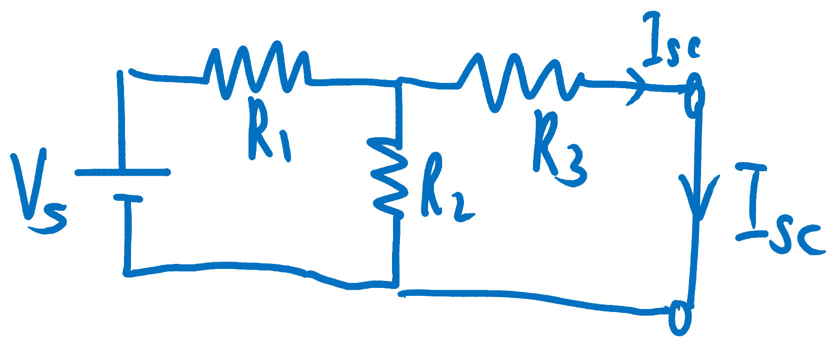
Thevenin equivalent:

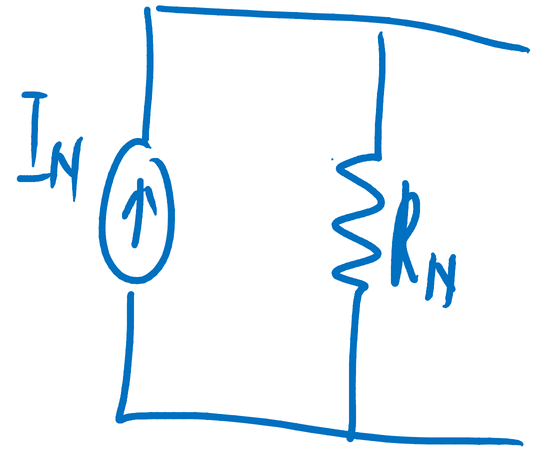


$$V_{TH} = V_S \cdot \frac{R_2}{R_1 + R_2}$$

$$R_{TH} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Same network Do Norton equivalent. Calculate I_N (I_{sc})





$$I_{sc} = \left(\frac{V_s}{R_1 + R_2 /\!/ R_3}\right) \cdot \left(\frac{1/R_3}{1/R_2 + 1/R_3}\right)$$

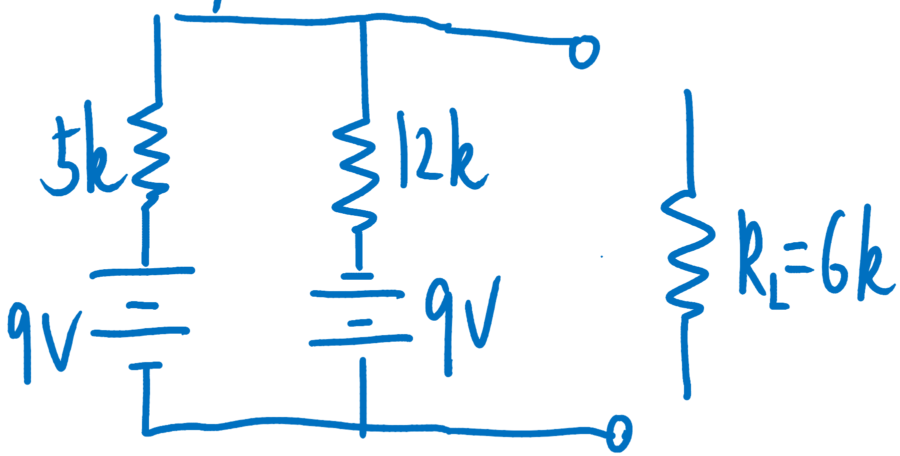
$$= \frac{V_s}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \cdot \frac{R_2}{R_2 + R_3}$$

$$= \frac{V_s R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} = I_N$$

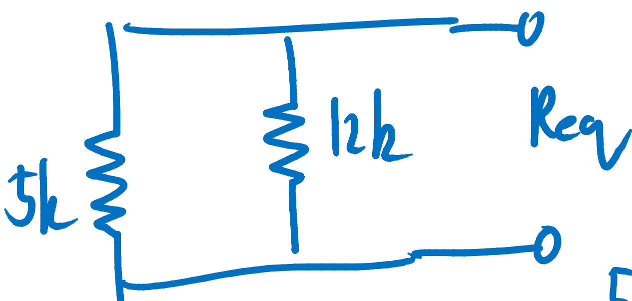
$$R_N = R_{TH} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Verify $V_{TH} = I_N R_N$

Example.



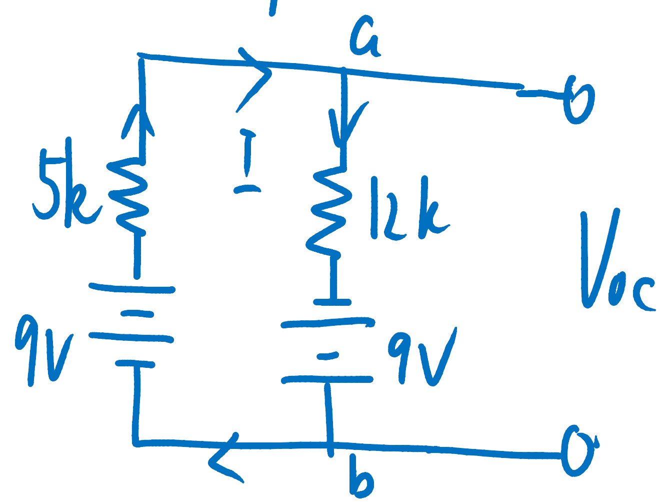
$R_{TH} = R_N = R_{eq}$



$R_N = R_{TH} = 5k // 12k = \frac{5 \times 12}{5+12} = 3.53\,k\Omega$

$\Rightarrow \quad I_N = \frac{V_{TH}}{R_{TH}} = \frac{3.71}{3.53} = 1.05\,(mA)$

$V_{oc} = ?$ open circuit voltage



$I = \frac{9+9}{5+12} = 1.06\,(mA)$

$V_{oc} = V_a - V_b$

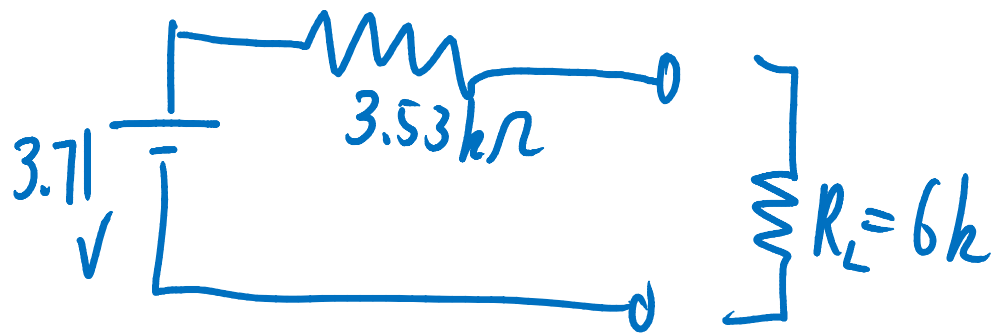
$= \Delta V_{b-a}$

$= -9 + 12 \times (1.06)$

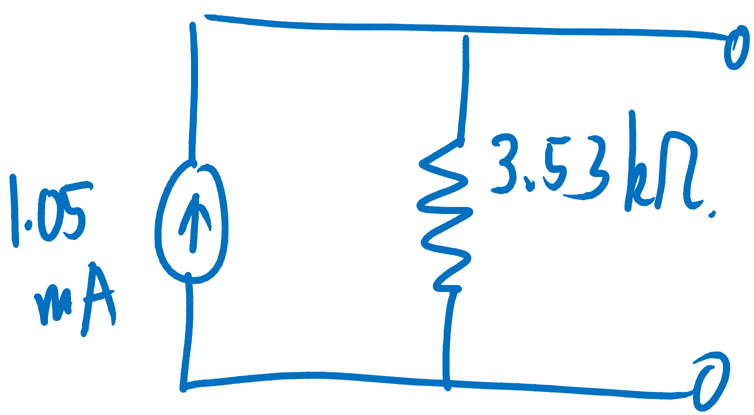
$= 3.71\,(V)$

$V_{TH} = 3.71\,(V)$

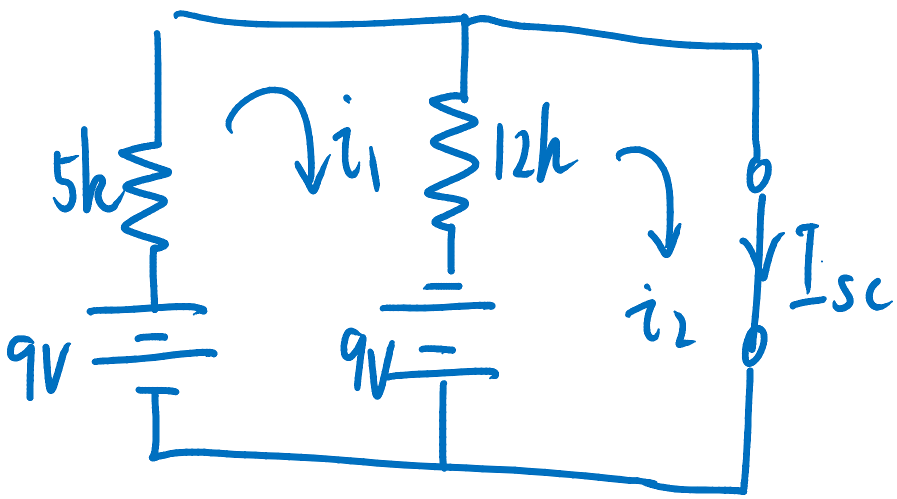
Thevenin equivalent



Norton equivalent



$I_{sc} = ?$ short circuit current



$I_{sc} = i_2$

$$17 i_1 - 12 i_2 = 18$$

$$12 i_2 - 12 i_1 = -9$$

$$I_N = I_{sc} = i_2 = 1.05\ (mA)$$