UNIVERSITY *of* DELAWARE

Chapter 10

Frequency Response

IN THIS CHAPTER YOU WILL LEARN

1. How coupling and bypass capacitors cause the gain of discrete-circuit amplifiers to fall off at low frequencies, and how to obtain an estimate of the frequency f_L at which the gain decreases by 3 dB below its value at midband.

2. The internal capacitive effects present in the MOSFET and the BJT and how to model these effects by adding capacitances to the hybrid-π model of each of the two transistor types.

3. The high-frequency limitation on the gain of the CS and CE amplifiers and how the gain falloff and the upper 3-dB frequency f_H are mostly determined by the small capacitance between the drain and gate (collector and base).

IN THIS CHAPTER YOU WILL LEARN

4. Powerful methods for the analysis of the high-frequency response of amplifier circuits of varying complexity.

5. How the cascode amplifier studied in Chapter 7 can be designed to obtain wider bandwidth than is possible with the CS and CE amplifiers.

6. The high-frequency performance of the source and emitter followers.

7. The high-frequency performance of differential amplifiers.

8. Circuit configurations for obtaining wideband amplification.

Frequency Response of a Discrete Circuit Amplifier

Low-frequency band:

- Gain drops at frequencies lower than f_L
- Large capacitors can no longer be treated as short circuit
- The gain roll-off is mainly due to coupling and by-pass capacitors

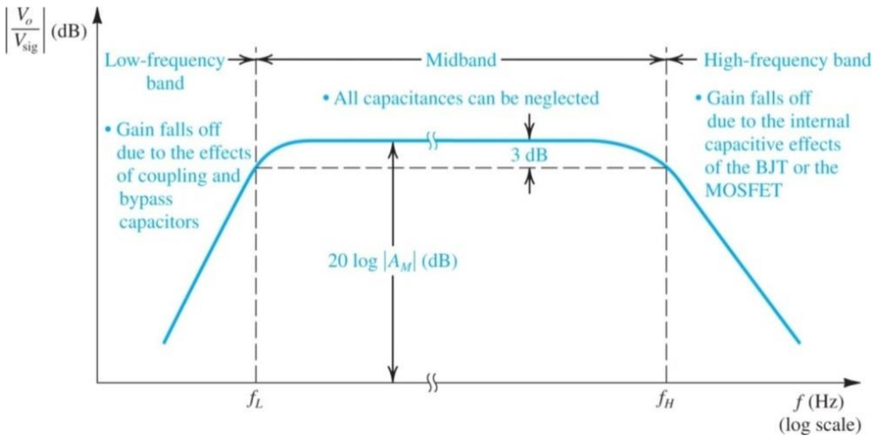


Figure 10.1 Sketch of the magnitude of the gain of a discrete-circuit BJT or MOS amplifier versus frequency. The graph delineates the three frequency bands relevant to frequency-response determination.

High-frequency band:

- Gain drops at frequencies higher than f_H
- Small capacitors can no longer treated as open circuit
- The gain roll-off is mainly due to parasitic capacitances of the MOSFETs and BJTs

Midband:

- The frequency range of interest for amplifiers
- Large capacitors can be treated as short circuit and small capacitors can be treated as open circuit
- Gain is constant and can be obtained by small-signal analysis

Frequency Response of an Integrated Circuit Amplifier

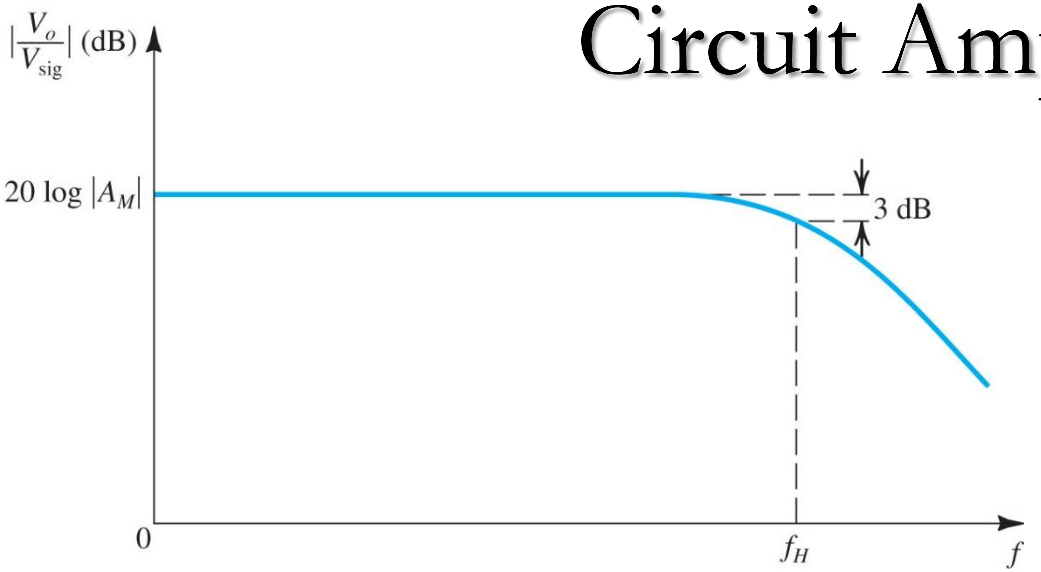


Figure 10.2 Frequency response of a direct-coupled (dc) amplifier. Observe that the gain does *not* fall off at low frequencies, and the midband gain A_M extends down to zero frequency.

The analysis presented here applies equally well to discrete-circuit, capacitively coupled amplifiers and to integrated circuit (IC) amplifiers. At the frequencies of interest to us here (the high-frequency band), all coupling and bypass capacitors behave as perfect short circuits and amplifiers of both types have identical high-frequency equivalent circuits.

$BW = f_H - f_L$ (discrete-circuit amplifiers)

$BW = f_H$ (integrated-circuit amplifiers)

A figure of merit for the amplifier is its **gain–bandwidth product**, defined as $GB = |A_M|\ BW$

Objective is to identify the mechanism that limits the bandwidth as well as find A_M.

10.1 LOW-FREQUENCY RESPONSE OF DISCRETE CIRCUIT CS AND CE AMPLIFIERS

Determining the Low-Frequency Cutoff of a CS Amplifier

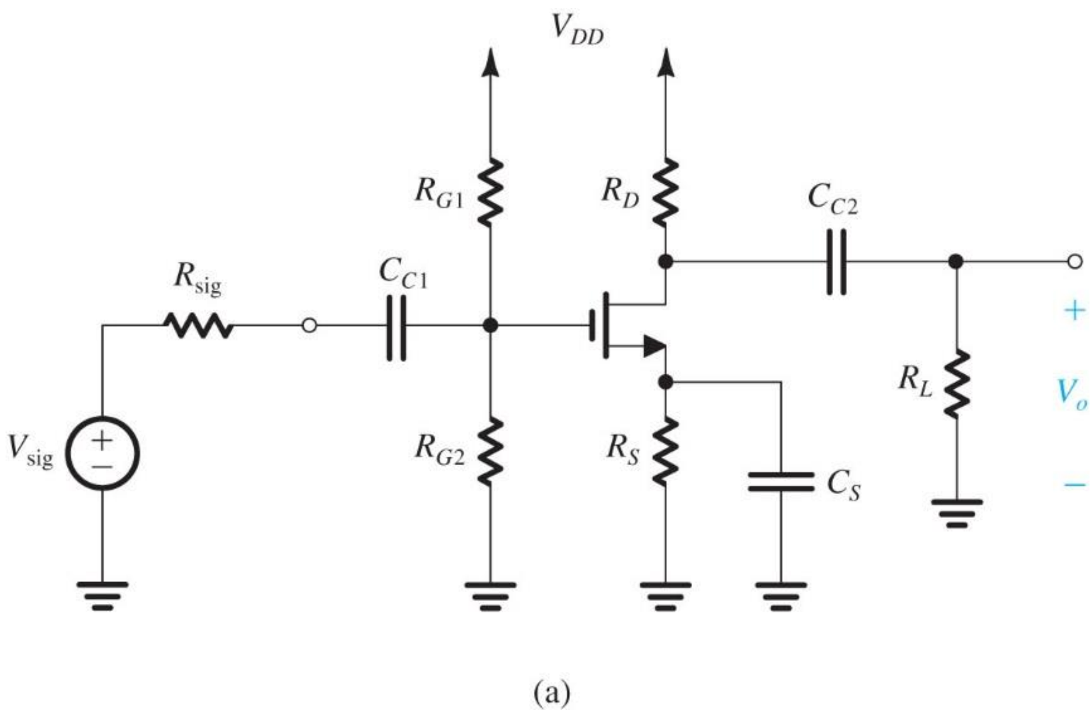
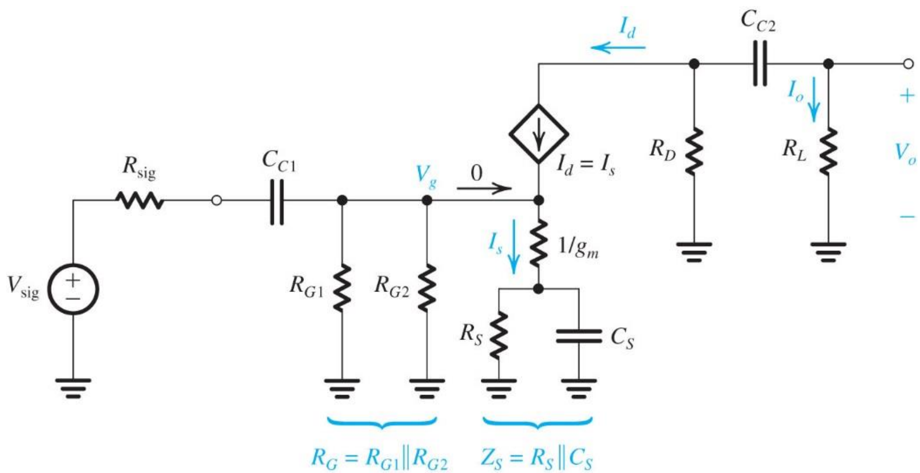


Figure 10.3 (a) Capacitively coupled common-source amplifier.

- Figure 10.3(a) shows a discrete-circuit, common-source amplifier.
  - coupling capacitors C_{C1} and C_{C2}
  - bypass capacitor C_S
- Objective is to determine the effect of these capacitances on gain (V_o/V_{sig}).
- At low frequencies, their reactance ($1/j\omega C$) is high and gain is reduced.

Discrete Common Source Equivalent Circuit



(b)

Figure 10.3 (**b**) The amplifier equivalent circuit at low frequencies. Note that the T model is used for the MOSFET and r_o is neglected.

$$\frac{V_o}{V_{sig}} = \frac{V_g}{V_{sig}} \times \frac{I_d}{V_g} \times \frac{V_o}{I_d}$$

Input resistance at V_g is: $R_G = R_{G1} \| R_{G2}$

$$\Rightarrow V_g = V_{sig} \frac{R_G}{(R_G + \chi_{C1}) + R_{sig}}$$

$$\Rightarrow V_g = V_{sig} \frac{R_G}{R_G + \frac{1}{sC_{C1}} + R_{sig}}$$

$$\frac{V_g}{V_{sig}} = \frac{R_G}{R_G + R_{sig}} \frac{s}{s + \frac{1}{C_{C1}\left(R_G + R_{sig}\right)}}$$

C_{C1} Frequency Response

$$\frac{V_g}{V_{sig}} = \frac{\left(\frac{R_G}{R_G + R_{sig}}\right)s}{s + \frac{1}{C_{C1}\left(R_G + R_{sig}\right)}}$$

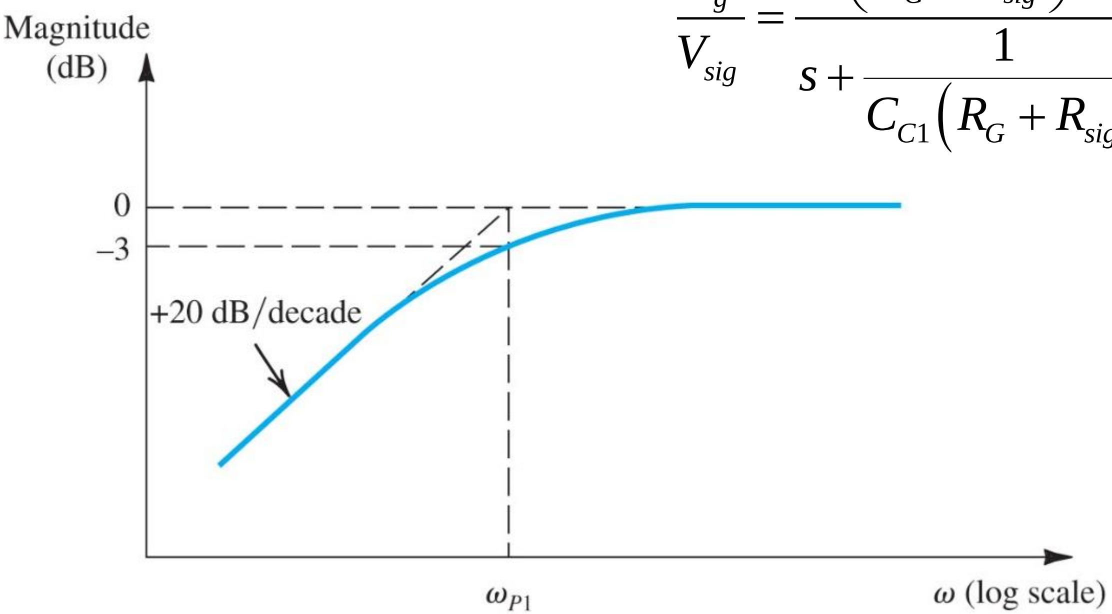


Figure 10.4 Sketch of the magnitude of the high pass function $\frac{s}{s+\omega_{P1}}$, that is, $\frac{\omega}{\sqrt{\omega^2+\omega_{P1}^2}}$ versus frequency ω.

$$T(s)_{\text{highpass}} = \frac{Ks}{s+\omega_0}$$

$$\omega_{P1} = \omega_0 = \frac{1}{C_{C1}\left(R_G + R_{sig}\right)}$$

Break frequency #1

Discrete Common Source Equivalent Circuit

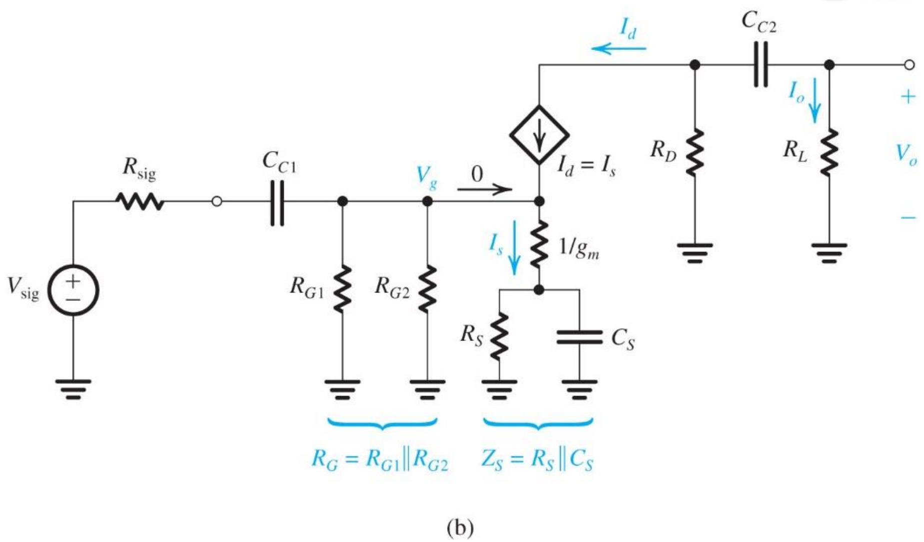


Figure 10.3 (**b**) The amplifier equivalent circuit at low frequencies. Note that the T model is used for the MOSFET and r_o is neglected.

$$\frac{V_o}{V_{sig}} = \frac{V_g}{V_{sig}} \times \frac{I_d}{V_g} \times \frac{V_o}{I_d}$$

$$Z_S = R_S \,||\, C_S = \frac{R_S \times 1/sC_S}{R_S + 1/sC_S} = \frac{R_S}{1 + sR_S C_S}$$

$$Y_S = \frac{1}{Z_S} = \frac{1 + sR_S C_S}{R_S} = \frac{1}{R_S} + sC_S$$

$$I_d = I_s = \frac{V_g}{\dfrac{1}{g_m} + Z_S} = g_m V_g \frac{Y_S}{g_m + Y_S}$$

$$\Rightarrow \frac{I_d}{V_g} = g_m \frac{s + \dfrac{1}{C_S R_S}}{s + \dfrac{g_m + 1/R_S}{C_S}}$$

C_S Frequency Response

$$\frac{I_d}{V_g} = g_m \frac{s + \dfrac{1}{C_S R_S}}{s + \dfrac{g_m + 1/R_S}{C_S}}$$

$$\omega_{P2} = \frac{g_m + 1/R_S}{C_S}$$

Break frequency #2

$$s_Z = -\frac{1}{C_S R_S}$$ Transmission zero

$$\Rightarrow \omega_Z = \frac{1}{C_S R_S}$$

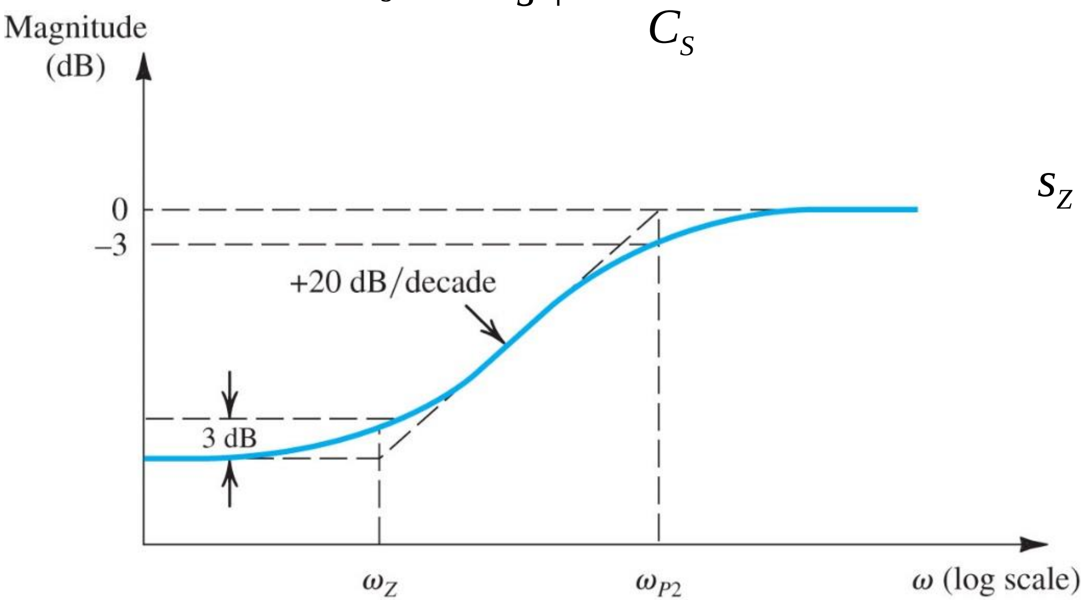


Figure 10.5 Sketch of the magnitude of the function $(s + \omega_Z)/(s + \omega_{P2})$ versus frequency ω.

Since g_m is usually large, $\omega_{P2} \gg \omega_Z$. That is, ω_{P2} will be closer to the midband, and thus it plays a more significant role in determining ω_L than does ω_Z.

Discrete Common Source Equivalent Circuit

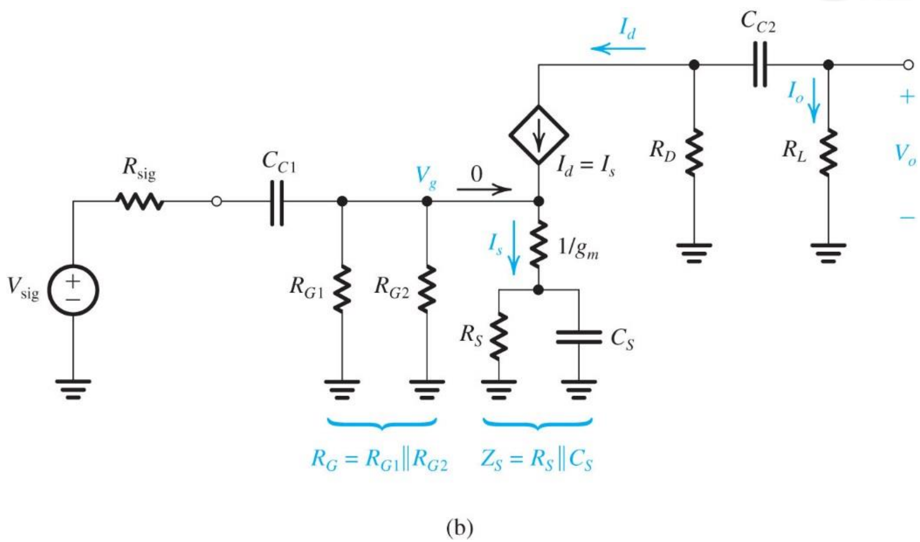


Figure 10.3 (**b**) The amplifier equivalent circuit at low frequencies. Note that the T model is used for the MOSFET and r_o is neglected.

$$\frac{V_o}{V_{sig}} = \frac{V_g}{V_{sig}} \times \frac{I_d}{V_g} \times \frac{V_o}{I_d}$$

The fraction of I_d that flows through R_L is:

$$I_o = -I_d \frac{R_D}{R_D + \frac{1}{sC_{C2}} + R_L}$$

$$\Rightarrow V_o = I_o R_L = -I_d R_L \frac{R_D}{R_D + \frac{1}{sC_{C2}} + R_L}$$

$$\frac{V_o}{I_d} = -\left(\frac{R_D R_L}{R_D + R_L}\right) \frac{s}{s + \frac{1}{C_{C2}\left(R_D + R_L\right)}}$$

C_{C2} Frequency Response

$$\frac{V_o}{I_d} = -\frac{\left(\frac{R_D R_L}{R_D + R_L}\right)s}{s + \frac{1}{C_{C2}\left(R_D + R_L\right)}}$$

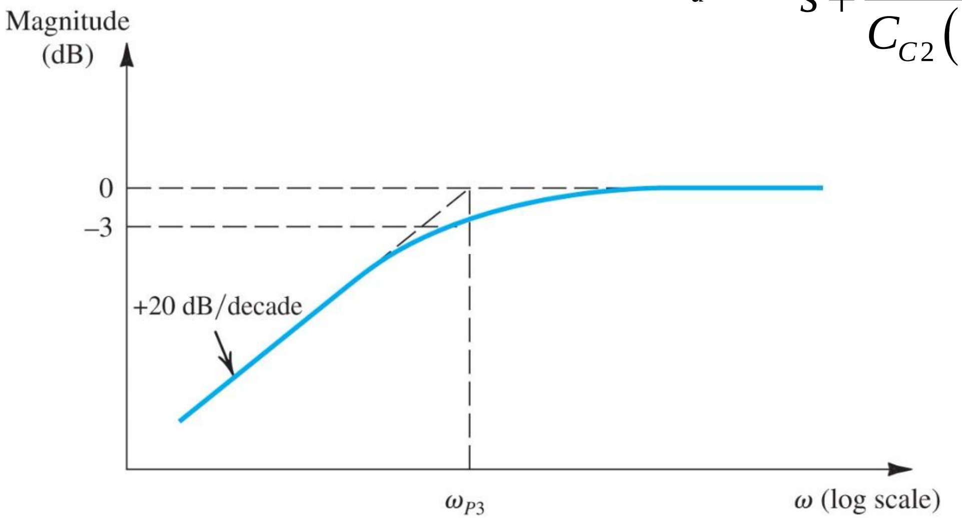


Figure 10.6 Sketch of the magnitude of the high pass function $\frac{s}{s+\omega_{P3}}$ versus frequency ω.

$$T(s)_{\text{highpass}} = \frac{Ks}{s + \omega_0}$$

$$\omega_{P3} = \frac{1}{C_{C2}\left(R_D + R_L\right)}$$

Break frequency #3

Determining the Low-Frequency Cutoff

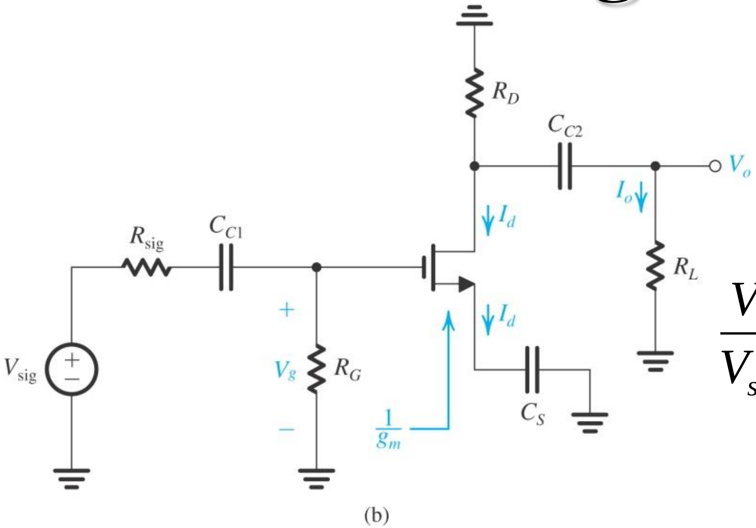


Figure 10.3 (b) Analysis of the CS amplifier to determine its low-frequency transfer function. For simplicity, r_o is neglected.

$$\frac{V_o}{V_{sig}} = \frac{V_g}{V_{sig}} \times \frac{I_d}{V_g} \times \frac{V_o}{I_d}$$

$$\frac{V_o}{V_{sig}} = -\frac{R_G}{R_G + R_{sig}} g_m \left(R_D \| R_L\right)\left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s+\omega_Z}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

$$= A_M \left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s+\omega_Z}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m \left(R_D \| R_L\right)$$

$$\omega_{P1} = \frac{1}{C_{C1}\left(R_G + R_{sig}\right)}$$

Pole frequency #1

$$\omega_{P2} = \frac{g_m + 1/R_S}{C_S}$$

Pole frequency #2

$$\omega_{P3} = \frac{1}{C_{C2}\left(R_D + R_L\right)}$$

Pole frequency #3

$$\omega_Z = \frac{1}{C_S R_S}$$

Zero frequency

Low-Frequency Magnitude Response

$$f_{P1} = \frac{1}{2\pi C_{C1}\left(R_G + R_{sig}\right)} \qquad f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S} \qquad f_{P3} = \frac{1}{2\pi C_{C2}\left(R_D + R_L\right)} \qquad f_Z = \frac{1}{2\pi C_S R_S}$$

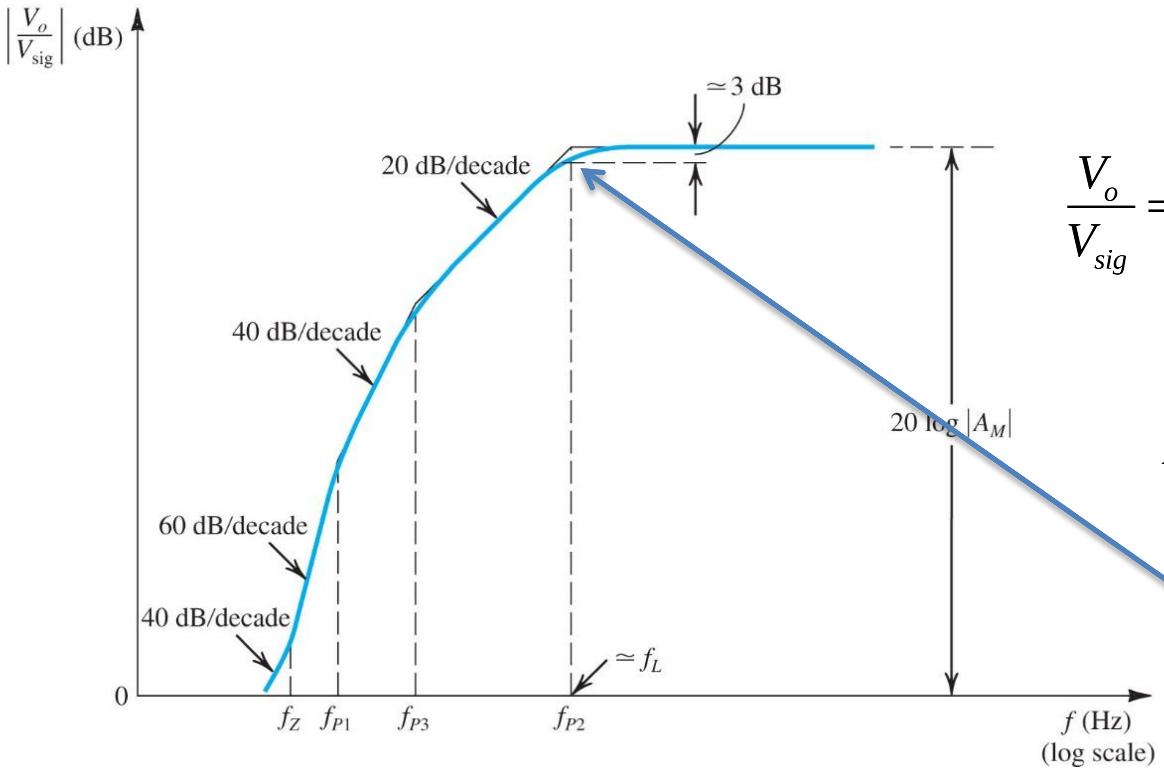


Figure 10.7 Sketch of the low-frequency magnitude response of a CS amplifier for which the three pole frequencies are sufficiently separated for their effects to appear distinct.

Transfer function

$$\frac{V_o}{V_{sig}} = A_M\left(\frac{s}{s+\omega_{P1}}\right)\left(\frac{s+\omega_Z}{s+\omega_{P2}}\right)\left(\frac{s}{s+\omega_{P3}}\right)$$

Midband gain

$$A_M = -\left(\frac{R_G}{R_G + R_{sig}}\right)\left[g_m\left(R_D \| R_L\right)\right]$$

Response dominated by highest frequency pole – usually associated with C_S

Determining Poles by Inspection

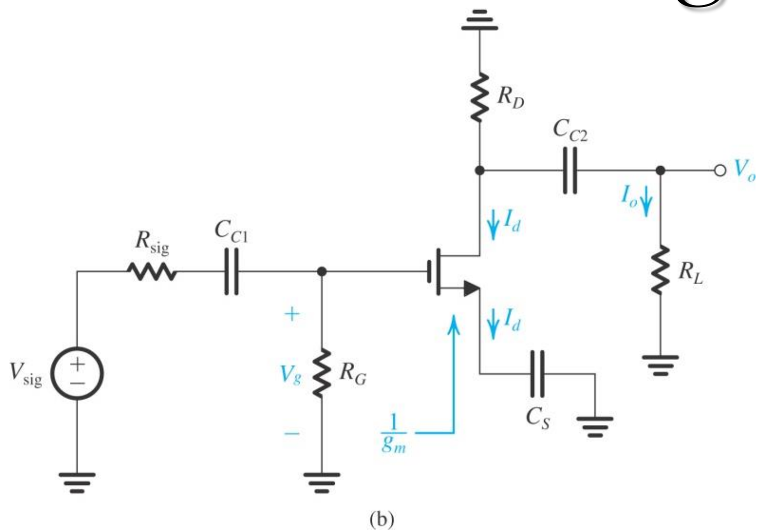


Figure 10.3 (b) Analysis of the CS amplifier to determine its low-frequency transfer function. For simplicity, r_o is neglected.

- Reduce V_{sig} to zero.
- Consider each capacitor separately.
- Assume that the other capacitors are acting as perfect short circuits.
- Find the total resistance seen between terminals of each capacitor.

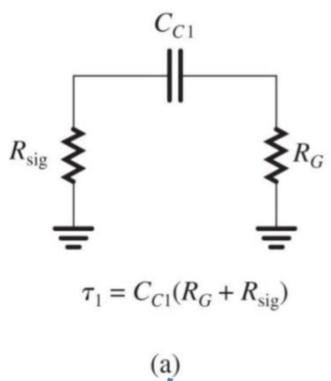
$$\tau_{P1} = \frac{1}{\omega_{01}} = C_{C1}R_{eff\,1} = C_{C1}\left(R_G + R_{sig}\right)$$

$$\tau_{P2} = \frac{1}{\omega_{02}} = C_S R_{eff\,2} = C_S\left(\frac{1}{g_m}\right)$$

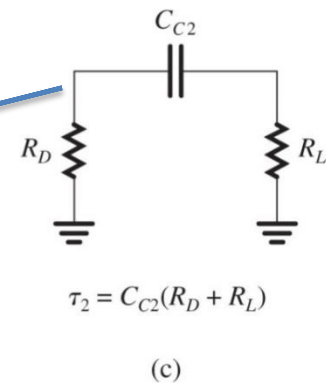
$$\tau_{P3} = \frac{1}{\omega_{03}} = C_{C2}R_{eff\,3} = C_{C2}\left(R_D + R_L\right)$$

selecting appropriate values for C_{C1}, C_S, and C_{C2} is done by placing the lower 3-dB frequency f_L at a specified value while minimizing the capacitor values. Since C_S results in the highest of the three pole frequencies, the total capacitance is minimized by selecting C_S so that its pole frequency $f_{P2} = f_L$. We then decide on the location of the other two pole frequencies, say 5 to 10 times lower than f_L.

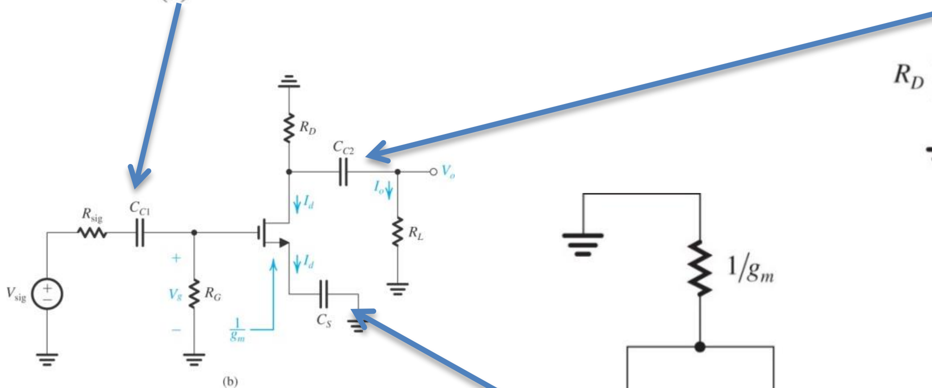
Using Time Constants

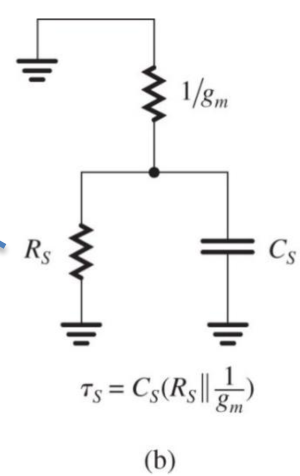


$$f_{P1} = \frac{1}{2\pi C_{C1}\left(R_G + R_{sig}\right)}$$



$$f_{P3} = \frac{1}{2\pi C_{C2}\left(R_D + R_L\right)}$$





$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

Figure 10.8 Circuits for determining the time constant of each of the three capacitors, and hence the pole associated with each one. Note that this determination is possible because in the circuit of Fig. 10.3, the capacitors do not interact.

Example 10.1

We wish to select appropriate values for the coupling capacitors C_{C1} and C_{C2} and the bypass capacitor C_S for a CS amplifier for which R_G = 4.7 MΩ, R_D = R_L = 15 kΩ, R_{sig} = 100 kΩ, R_S = 10 kΩ, and g_m = 1 mA/V. It is required to have f_L at 100 Hz and that the nearest break frequency be at least a decade lower.

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S} = f_L \qquad \Rightarrow C_S = \frac{g_m + 1/R_S}{2\pi f_{P2}} = \frac{1\text{mA/V} + 0.1\text{mA/V}}{2\pi \times 100\text{Hz}} = 1.75\mu\text{F} = f_L$$

$$f_{P1} = f_{P3} = 10\text{Hz}$$

$$f_{P1} = \frac{1}{2\pi C_{C1}\left(R_G + R_{sig}\right)} \qquad C_{C1} = \frac{1}{2\pi f_{P1}\left(R_G + R_{sig}\right)} = \frac{1}{2\pi \times 10\text{Hz}\left(4.7\text{M}\Omega + 0.1\text{M}\Omega\right)} = 3.3\text{nF}$$

$$f_{P3} = \frac{1}{2\pi C_{C2}\left(R_D + R_L\right)} \qquad C_{C2} = \frac{1}{2\pi f_{P1}\left(R_D + R_L\right)} = \frac{1}{2\pi \times 10\text{Hz}\left(15\text{k}\Omega + 15\text{k}\Omega\right)} = 0.53\mu\text{F}$$

The Method of Short-Circuit Time Constants

In some circuits, such as that of the common-emitter amplifier discussed shortly, the capacitors interact, making it difficult to determine the pole frequencies. Fortunately, however, there is a simple method for obtaining an estimate for f_L without the need to determine the frequencies of the poles. Although the method is predicated on the assumption that one of the poles is dominant, the resulting estimate for f_L is usually very good even if this assumption is not strictly valid. The method is as follows:

1. Set the input signal $V_{sig} = 0$.
2. Consider the capacitors one at a time. That is, while considering capacitor C_i, set all the other capacitors to infinite values (i.e., replace them with short circuits—hence the name of the method).
3. For each capacitor C_i, find the total resistance R_i seen by C_i. This can be determined either by inspection or by replacing C_i with a voltage source V_x and finding the current I_x drawn from V_x; $R_i \equiv V_x/I_x$.
4. Calculate the 3-dB frequency f_L using $f_L \simeq \dfrac{1}{2\pi}\sum_{i=1}^{n}\dfrac{1}{C_i R_i}$ where n is the total number of capacitors.

Low-Frequency Cutoff of a CE Amplifier

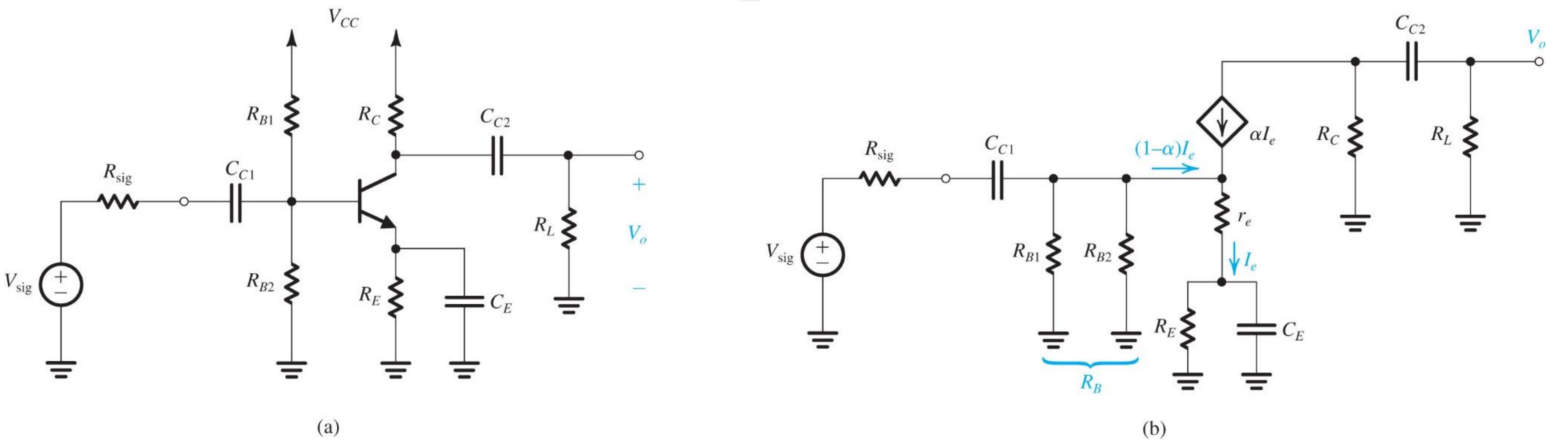


Figure 10.9 (**a**) A discrete-circuit common-emitter amplifier. (**b**) Equivalent circuit of the amplifier in (**a**).

Figure 10.4. shows common-emitter amplifier.

- coupling capacitors C_{C1} and C_{C2}
- emitter bypass capacitor C_E
- ignore r_o

- Effect of these capacitors felt at low frequencies.
- Objective is to determine amplifier gain and transfer function.
- This analysis is somewhat more complicated than CS case.

Using SCTC Method

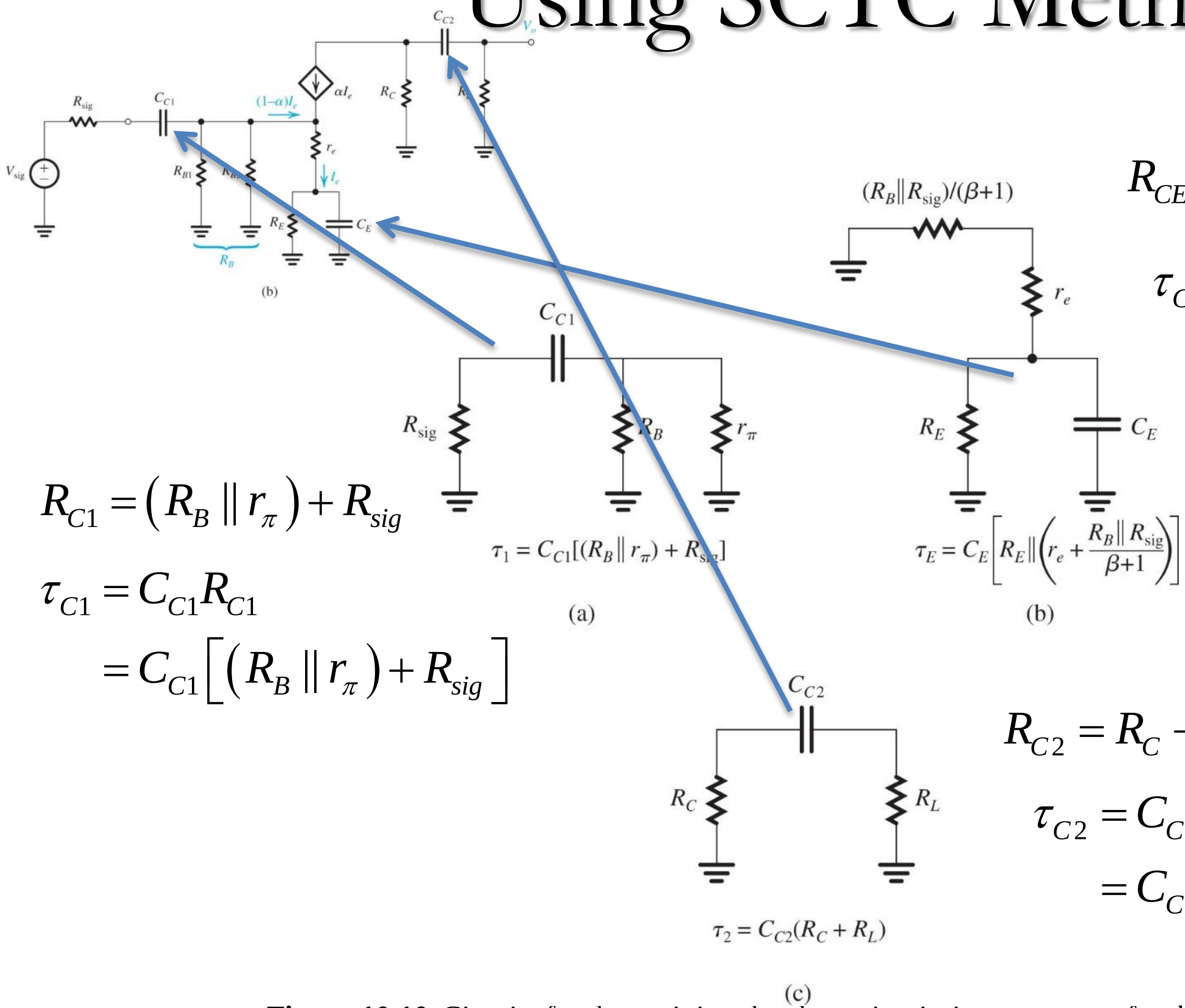


Figure 10.10 Circuits for determining the short-circuit time constants for the amplifier in Fig. 10.9.

$$R_{CE} = R_E \| \left[r_e + \frac{(R_B \| R_{sig})}{\beta + 1} \right]$$

$$\tau_{CE} = C_E R_{CE}$$

$$= C_E \left[R_E \| \left[r_e + \frac{(R_B \| R_{sig})}{\beta + 1} \right] \right]$$

$$R_{C1} = (R_B \| r_\pi) + R_{sig}$$

$$\tau_{C1} = C_{C1} R_{C1}$$

$$= C_{C1}\left[(R_B \| r_\pi) + R_{sig} \right]$$

$$R_{C2} = R_C + R_L$$

$$\tau_{C2} = C_{C2} R_{C2}$$

$$= C_{C2}\left[R_C + R_L \right]$$

Determining the Low-Frequency Cutoff

$$\tau_{C1} = C_{C1}R_{C1}$$
$$= C_{C1}\left[\left(R_B \parallel r_\pi\right) + R_{sig}\right]$$

$$\tau_{CE} = C_E R_{CE}$$
$$= C_E\left[R_E \parallel \left[r_e + \frac{\left(R_B \parallel R_{sig}\right)}{\beta + 1}\right]\right]$$

$$\tau_{C2} = C_{C2}R_{C2}$$
$$= C_{C2}\left[R_C + R_L\right]$$

$$f_L \simeq \sum_{i=1}^{n} \frac{1}{\tau_i} = \sum_{i=1}^{n} \frac{1}{C_i R_i}$$

Effect of all three capacitors

$$f_L = \frac{\omega_L}{2\pi} = \frac{1}{2\pi}\left[\frac{1}{C_{C1}R_{C1}} + \frac{1}{C_E R_E} + \frac{1}{C_{C2}R_{C2}}\right]$$

it is obvious that the capacitor that has the smallest time constant has contributes the most to f_L. In the CE amplifier, this is usually CE because the associated resistance R_{CE} is typically small.

The design objective is to place the lower 3-dB frequency f_L at a specified location while minimizing the capacitor values. Since C_E usually sees the lowest of the three resistances, the total capacitance is minimized by selecting C_E so that its contribution to f_L is dominant.

Example 10.2

We wish to select appropriate values for C_{C1}, C_{C2}, and C_E for the common-emitter amplifier, which has R_B =100 kΩ, R_C = 8 kΩ, R_L = 5 kΩ, R_{sig} =5 kΩ, R_E = 5 kΩ, β = 100, g_m = 40 mA/V, and r_π = 2.5 kΩ. It is required to have f_L = 100 Hz.

$$R_{C1} = \left(R_B \,||\, r_\pi\right) + R_{sig} = (100\text{k}\Omega \,||\, 2.5\text{k}\Omega) + 5\text{k}\Omega = 7.44\text{k}\Omega$$

$$R_{CE} = R_E \,||\left[r_e + \frac{\left(R_B \,||\, R_{sig}\right)}{\beta + 1}\right] = 5\text{k}\Omega \,||\left[\frac{2.5\text{k}\Omega}{100} + \frac{100\text{k}\Omega \,||\, 5\text{k}\Omega}{101}\right] = 71\Omega$$

$$R_{C2} = R_C + R_L = 8\text{k}\Omega + 5\text{k}\Omega = 13\text{k}\Omega$$

Now, selecting C_E so that it contributes 80% of the value of ω_L gives

$$\frac{1}{C_E R_{CE}} = (80\%)2\pi f_L = 502.65\frac{\text{rad}}{\text{sec}} \qquad \Rightarrow C_E = 28\mu\text{F}$$

Example 10.2

We wish to select appropriate values for C_{C1}, C_{C2}, and C_E for the common-emitter amplifier, which has R_B =100 kΩ, R_C = 8 kΩ, R_L = 5 kΩ, R_{sig} =5 kΩ, R_E = 5 kΩ, β = 100, g_m = 40 mA/V, and r_π = 2.5 kΩ. It is required to have f_L = 100 Hz.

Now, select C_{C1} and C_{C2} so that they each contribute 10% of the value of ω_L gives

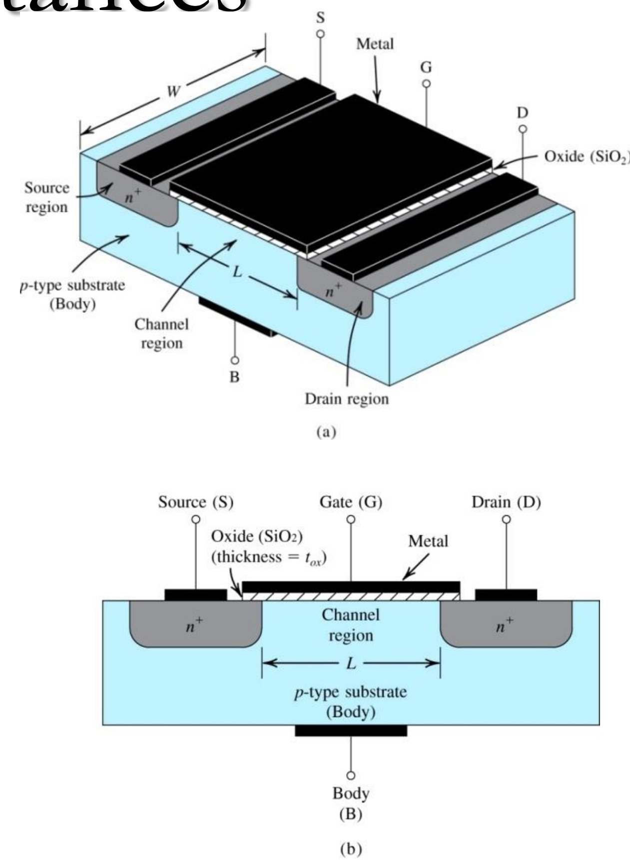
$$\frac{1}{C_{C1}R_{C1}} = (10\%)2\pi f_L = 62.83\frac{\text{rad}}{\text{sec}} \qquad \Rightarrow C_{C1} = 2.1\mu\text{F}$$

$$\frac{1}{C_{C2}R_{C2}} = (10\%)2\pi f_L = 62.83\frac{\text{rad}}{\text{sec}} \qquad \Rightarrow C_{C2} = 1.2\mu\text{F}$$

10.2 INTERNAL CAPACITIVE EFFECTS AND THE HIGH-FREQUENCY MODEL OF THE MOSFET AND THE BJT

MOSFET Capacitances

- MOSFET has internal capacitance.
  - The gate capacitive effect: The gate electrode forms a parallel plate capacitor with the channel.
  - The source-body and drain-body depletion layer capacitances: These are the capacitances of the reverse-biased *pn*-junctions.
- Previously, it was assumed that charges are acquired instantaneously - resulting in steady-state model.
  - This assumption poses problem for frequency analysis.



MOSFET Gate Capacitances

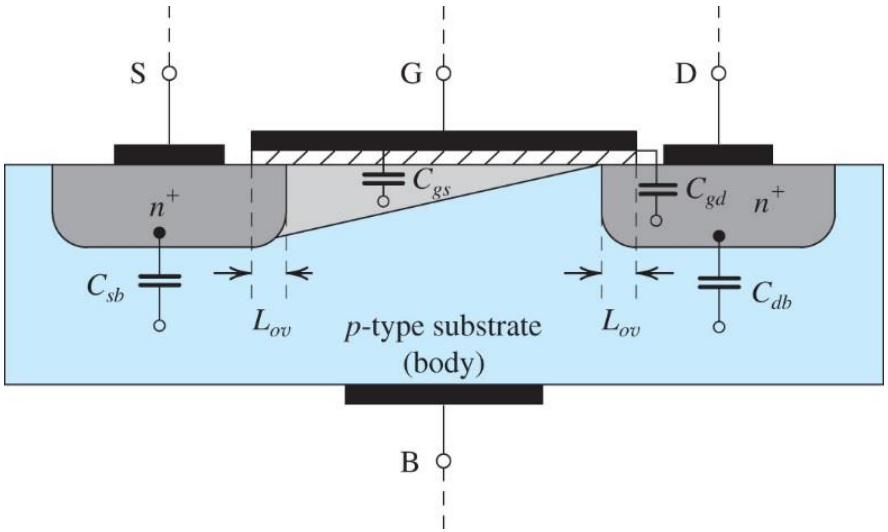


Figure 10.11 A cross section of the n-channel MOSFET operating in the saturation region. The four internal capacitances, C_{gs}, C_{gd}, C_{sb}, and C_{db}, are indicated. Note that the bias voltages are not shown. Also not shown, to keep the diagram simple, is the depletion region.

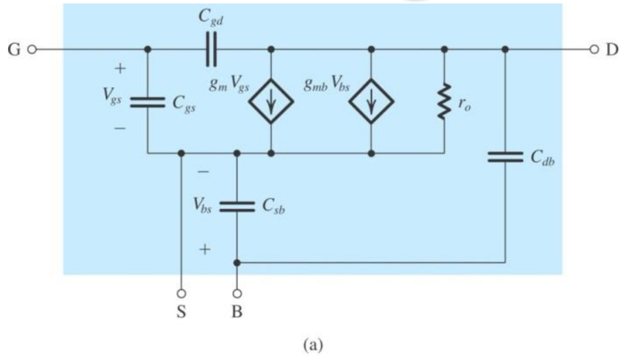
Triode region: $$C_{gs} = C_{gd} = \frac{1}{2} WLC_{ox}$$

Saturation region: $$\begin{cases} C_{gs} = \frac{2}{3} WLC_{ox} + C_{ov} \\ C_{gd} = C_{ov} \end{cases}$$

Cutoff region: $$\begin{cases} C_{gs} = C_{gd} = C_{ov} \\ C_{gb} = WLC_{ox} + C_{ov} \end{cases}$$

Overlap capacitance that should be added to C_{gs} and C_{gd} in all cases: $$C_{ov} = WL_{ov}C_{ox}$$

High-Frequency Model of the MOSFET



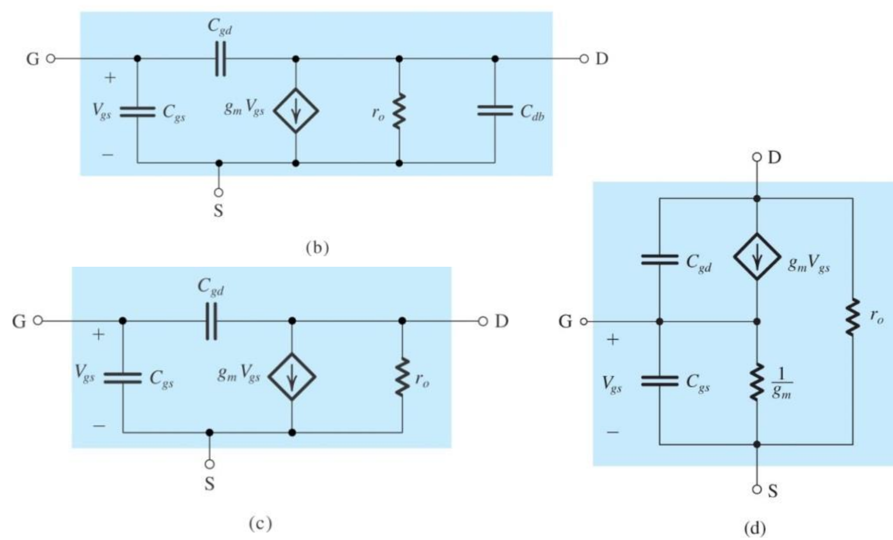


Figure 10.12 (**a**) High-frequency, equivalent-circuit model for the MOSFET. (**b**) The equivalent circuit for the case in which the source is connected to the substrate (body). (**c**) The equivalent-circuit model of (**b**) with C_{db} neglected (to simplify analysis). (**d**) The simplified high-frequency T model.

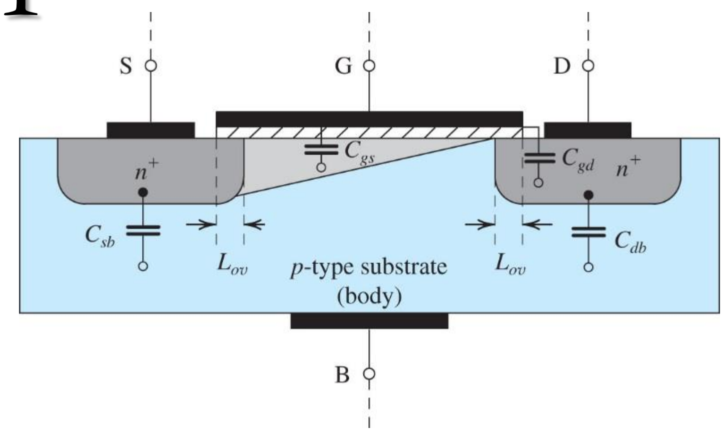


Figure 10.11 A cross section of the *n*-channel MOSFET operating in the saturation region. The four internal capacitances, C_{gs}, C_{gd}, C_{sb}, and C_{db}, are indicated. Note that the bias voltages are not shown. Also not shown, to keep the diagram simple, is the depletion region.

The body capacitances

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1+\frac{V_{SB}}{V_0}}} \qquad C_{db} = \frac{C_{db0}}{\sqrt{1+\frac{V_{DB}}{V_0}}}$$

MOSFET Unity-Gain Frequency (f_T)

A figure of merit for the high-frequency operation of the MOSFET as an amplifier is the unity-gain frequency, f_T, also known as the transition frequency. This is defined as the frequency at which the short-circuit current-gain of the common-source configuration becomes unity.

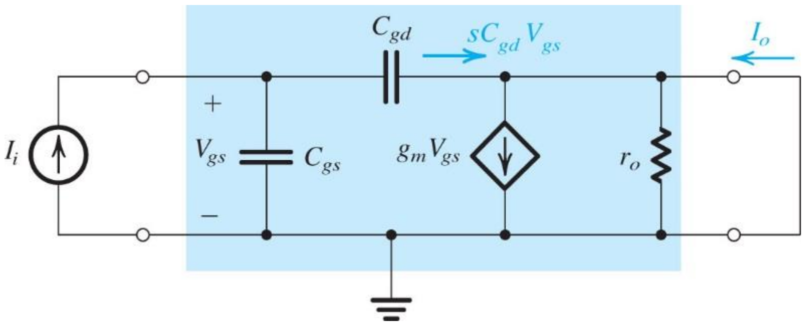


Figure 10.13 Determining the short-circuit current gain I_o/I_i.

$$I_o = g_m V_{gs} - sC_{gd}V_{gs} \simeq g_m V_{gs}$$

$$V_{gs} = \frac{I_i}{s\left(C_{gs} + C_{gd}\right)}$$

$$I_i = sV_{gs}\left(C_{gs} + C_{gd}\right)$$

$$\frac{I_o}{I_i} = \frac{g_m V_{gs}}{sV_{gs}\left(C_{gs} + C_{gd}\right)} = \frac{g_m}{s\left(C_{gs} + C_{gd}\right)}$$

For physical frequencies $s = j\omega$, it can be seen that the magnitude of the current gain becomes unity at the frequency

$$\omega_T = \frac{g_m}{\left(C_{gs} + C_{gd}\right)} \text{rad/s} \qquad f_T = \frac{g_m}{2\pi\left(C_{gs} + C_{gd}\right)} \text{Hz}$$

MOSFET High-Frequency Model

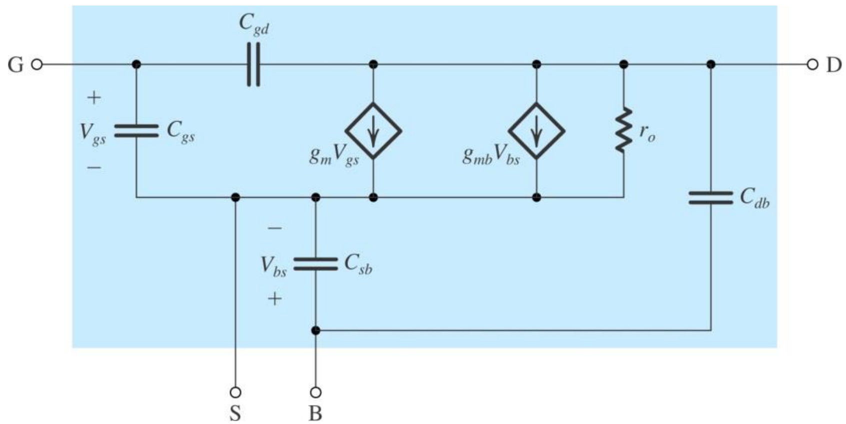


Table 10.1 The MOSFET High-Frequency Model

$$g_m = \mu_n C_{ox} \frac{W}{L} |V_{OV}| = \sqrt{\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{|V_{OV}|}$$

$$g_{mb} = \chi g_m, \chi = 0.1 \text{ to } 0.2$$

$$r_o = \frac{|V_A|}{I_D}$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov}C_{ox}$$

$$C_{gd} = WL_{ov}C_{ox}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{|V_{DB}|}{V_0}}} \qquad C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{|V_{SB}|}{V_0}}}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

High Frequency Model of the BJT

Like the MOSFET, previously it was assumed that transistor action was instantaneous.

steady-state model

neglects frequency-dependence

Actual transistors exhibit charge-storage.

An augmented BJT model is required to examine this dependence.

High Frequency Model of the BJT

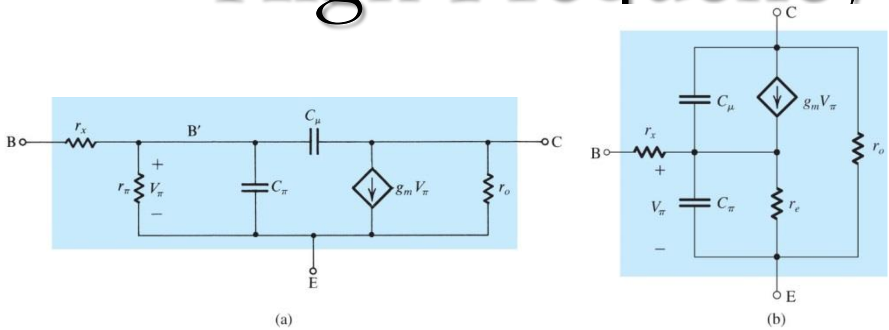


Figure 10.14 The high-frequency models of the BJT: (**a**) hybrid-π model and (**b**) T model.

the collector-base junction capacitance C_μ is:

$$C_\mu = \frac{C_{\mu 0}}{\left(1 + \frac{V_{CB}}{V_{0c}}\right)^m}$$

(a fraction of a pF to a few pF)

where $C_{\mu 0}$ is the value of C_μ at zero voltage; V_{CB} is the magnitude of the CBJ reverse-bias voltage, V_{0c} is the CBJ built-in voltage (typically, 0.75 V), and m is its grading coefficient (typically, 0.2–0.5)

the small-signal diffusion capacitance C_{de} is

$$C_{de} = \tau_F g_m = \tau_F \frac{I_C}{dv_{BE}}$$

where τ_F is a device constant with the dimension of time known as the forward base-transit time

the base–emitter junction capacitance C_{je} is:

$$C_{je} = 2C_{je0}$$

the emitter-base capacitance C_π is:

$$C_\pi = C_{de} + C_{je}$$

(a few pF to a few tens of pF)

The Cutoff Frequency

The transistor data sheets do not usually specify the value of C_π. Rather, the behavior of β (or h_{fe}) versus frequency is normally given. In order to determine C_π and C_μ, we shall derive an expression for h_{fe}, the CE short-circuit current gain, as a function of frequency in terms of the hybrid-π components

$$I_c = (g_m - sC_\mu)V_\pi$$

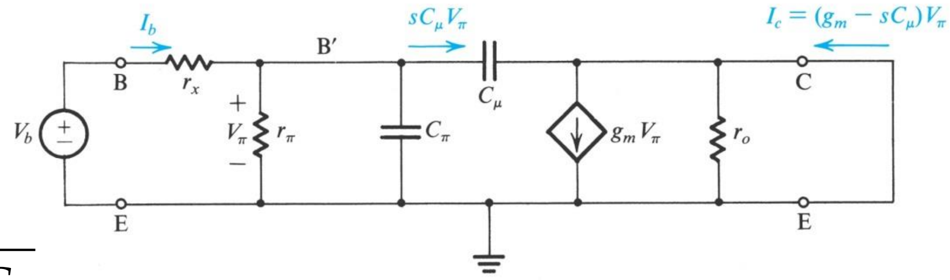


Figure 10.15 Circuit for deriving an expression for $h_{fe}(s) \equiv I_c/I_b$.

$$V_\pi = I_b (r_\pi \parallel C_\pi \parallel C_\mu) = \frac{I_b}{1/r_\pi + sC_\pi + sC_\mu}$$

$$h_{fe} \equiv \frac{I_c}{I_b} = \frac{g_m - sC_\mu}{1/r_\pi + sC_\pi + sC_\mu}$$

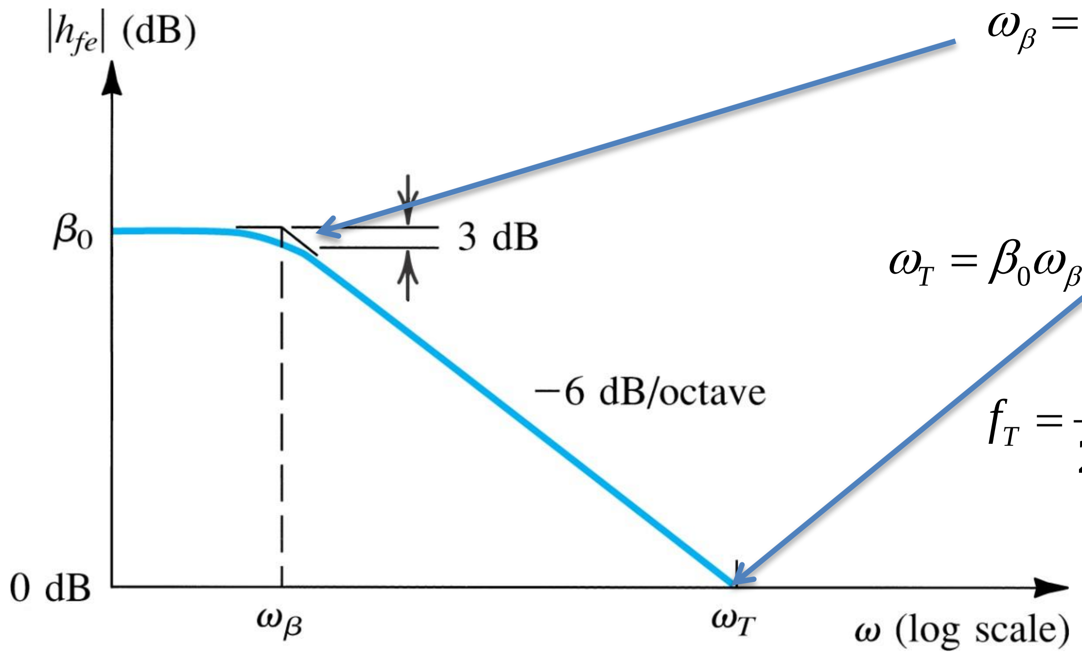
$$\omega C_\mu \ll g_m$$

$$h_{fe} \simeq \frac{g_m r_\pi}{1 + s(C_\pi + C_\mu)r_\pi} = \frac{\beta_0}{1 + s(C_\pi + C_\mu)r_\pi}$$

The Cutoff and Unity-Gain Bandwidth Frequencies

$$h_{fe} = \frac{\beta_0}{1 + s\left(C_\pi + C_\mu\right) r_\pi}$$

$$\omega_\beta = \frac{1}{\left(C_\pi + C_\mu\right) r_\pi}$$



$$\omega_T = \beta_0 \omega_\beta = \frac{g_m}{C_\pi + C_\mu}$$

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$$

Figure 10.16 Bode plot for $|h_{fe}|$.

The Transition Frequency with I_C

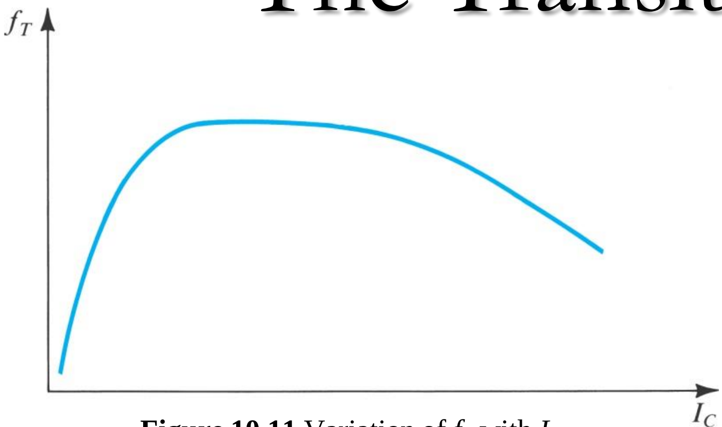
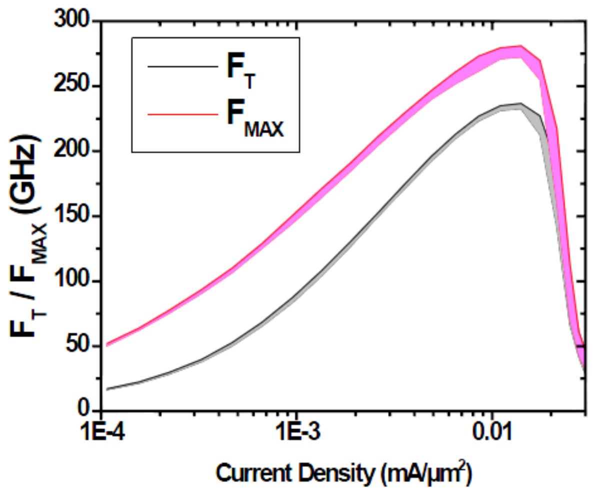


Figure 10.11 Variation of f_T with I_C.



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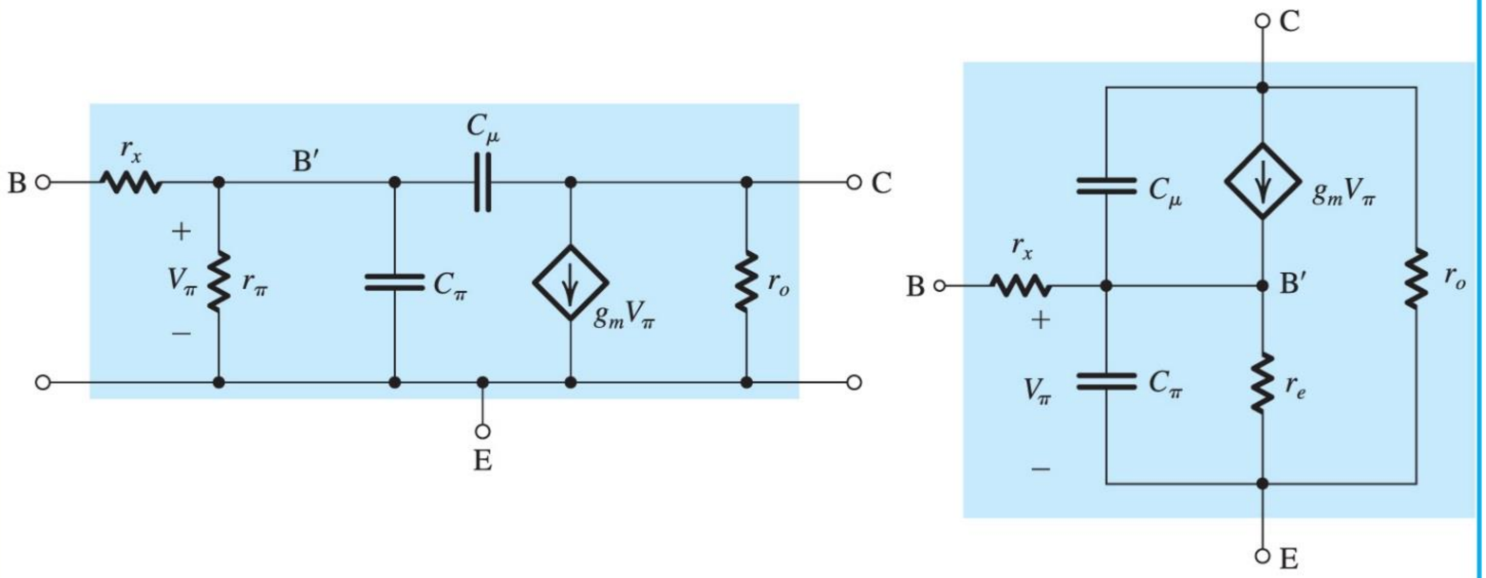
In some cases f_T is given as a function of I_C and V_{CE}. To see how f_T changes with I_C, recall that g_m is directly proportional to I_C, but only the diffusion capacitance part of C_π (C_{de}) is directly proportional to I_C. It follows that f_T decreases at low currents, as shown in Fig. 10.11. However, the decrease in f_T at high currents, also shown in Fig. 10.11, cannot be explained by this argument; rather, it is due to the same phenomenon that causes β_0 to decrease at high currents. In the region where f_T is almost constant, C_π is dominated by the diffusion part.

Typically, f_T is in the range of 100 MHz to tens of GHz. The value of f_T can be used to determine $C_\pi + C_\mu$. The capacitance C_μ is usually determined separately by measuring the capacitance between base and collector at the desired reverse-bias voltage V_{CB}.

The BJT High Frequency Model

Table 10.2 The BJT High-Frequency Model



$g_m = I_C/V_T$ $\qquad r_o = |V_A|/I_C$ $\qquad r_\pi = \beta_0/g_m$ $\qquad r_e = r_\pi/(\beta+1)$

$C_\pi + C_\mu = \dfrac{g_m}{2\pi f_T}$ $\qquad C_\pi = C_{de} + C_{je}$ $\qquad C_{de} = \tau_F g_m$ $\qquad C_{je} \simeq 2C_{je0}$

$C_\mu = C_{jc0}\Big/\left(1 + \dfrac{|V_{CB}|}{V_{0c}}\right)^m$ $\qquad m = 0.3\text{–}0.5$

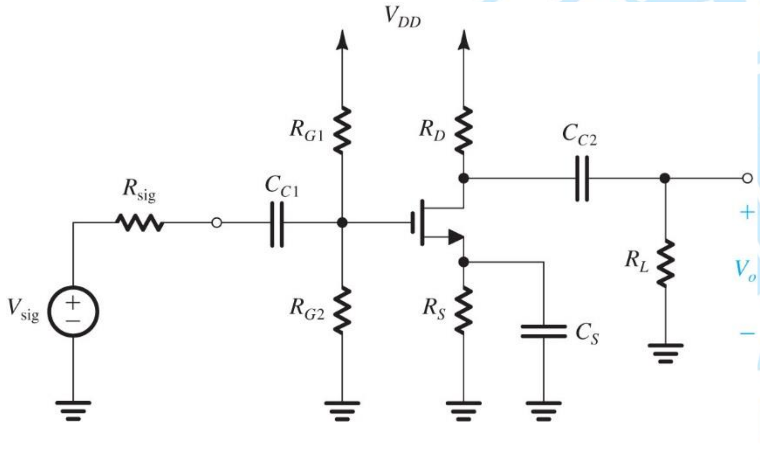
Homework #7

Read Chapter 10

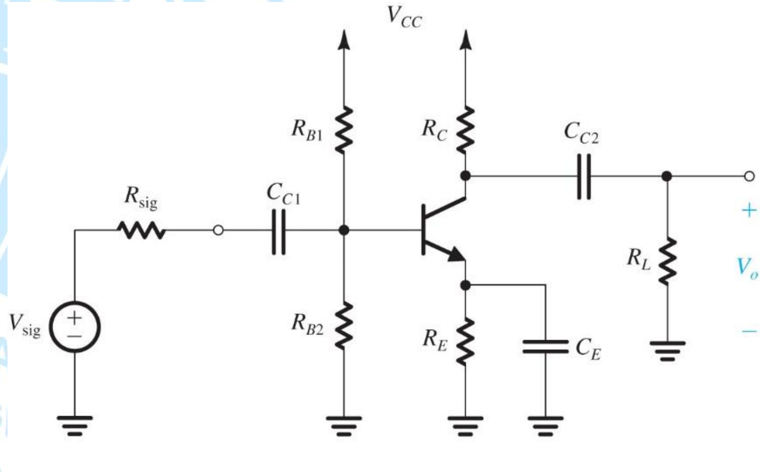
Chapter 10 Problems:

1) 10.1*
2) 10.3*
3) 10.5*
4) 10.13*
5) 10.17*
6) 10.21*

* Answers in Appendix L



(a)



(a)

10.3 HIGH-FREQUENCY RESPONSE OF THE CS AND CE AMPLIFIERS

The Common-Source Amplifier

high-frequency equivalent-circuit model of a CS amplifier.

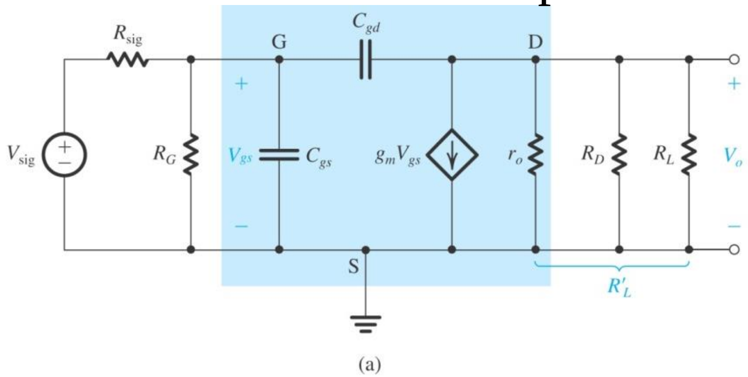


Figure 10.18 Determining the high-frequency response of the CS amplifier: **(a)** equivalent circuit;

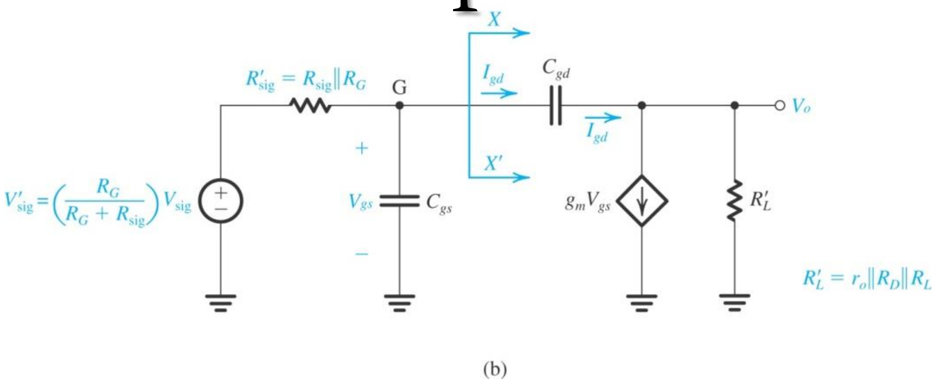


Figure 10.18; **(b)** the circuit of **(a)** simplified at the input and the output;

It may be simplified using Thevenin's theorem.

$$R'_{sig} = R_{sig} \,||\, R_G \qquad V'_{sig} = \frac{R_G}{R_G + R_{sig}} V_{sig}$$

$$R'_L = r_o \,||\, R_D \,||\, R_L$$

$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_G}{R_G + R_{sig}} \left(g_m R'_L\right)$$

Also, bridging capacitor (C_{gd}) may be redefined.

$$I_L = g_m V_{gs} - I_{gd} \simeq g_m V_{gs}$$

$$V_o \simeq -\left(g_m V_{gs}\right) R'_L = -g_m R'_L V_{gs} = V_{ds}$$

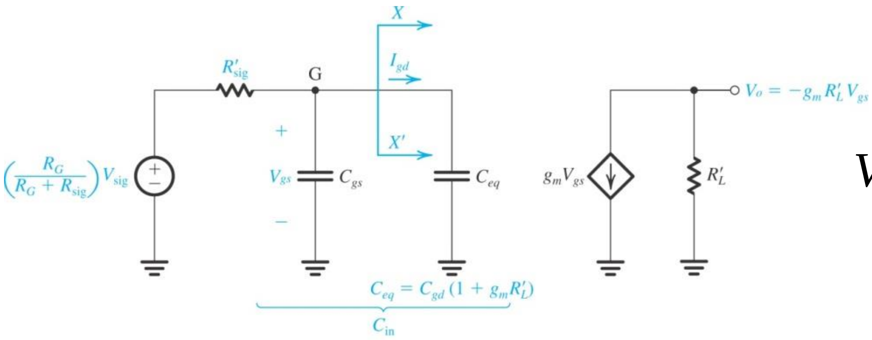
$$I_{gd} = sC_{gd}\left(V_{gs} - V_o\right) = sC_{gd}\left(1 + g_m R'_L\right) V_{gs}$$

replace C_{gd} by an equivalent capacitance C_{eq} between the gate and ground as long as C_{eq} draws a current equal to I_{gd}

$$sC_{eq} V_{gs} = sC_{gd}\left(1 + g_m R'_L\right) V_{gs}$$

$$C_{eq} = C_{gd}\left(1 + g_m R'_L\right)$$

The Common-Source Amplifier



(c)

Figure 10.18 (*Continued*) **(c)** the equivalent circuit with C_{gd} replaced at the input side with the equivalent capacitance C_{eq};

$$C_{eq} = C_{gd}\left(1 + g_m R'_L\right)$$

C_{gd} gives rise to much larger capacitance C_{eq}. The multiplication effect that it undergoes is known as the **Miller Effect**.

$$C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}\left(1 + g_m R'_L\right)$$

$$V_{gs} = \frac{R_G}{R_G + R_{sig}} V_{sig} \frac{1}{1 + sC_{in}R'_{sig}} = \frac{R_G}{R_G + R_{sig}} V_{sig} \frac{1}{1 + \frac{s}{\omega_0}}$$

$$\omega_0 = \frac{1}{C_{in}R'_{sig}}, \text{ where } R'_{sig} = R_{sig} \,||\, R_G$$

$$V_o = -g_m R'_L V_{gs}$$

$$\frac{V_o}{V_{sig}} = -\left(\frac{R_G}{R_G + R_{sig}}\right)\left(g_m R'_L\right)\frac{1}{1 + \frac{s}{\omega_0}}$$

$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + \frac{s}{\omega_H}} \qquad \omega_H = \omega_0 = \frac{1}{C_{in}R'_{sig}}$$

$$f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi C_{in}R'_{sig}}$$

The Common-Source Amplifier

$$T(s)=\frac{V_o}{V_{sig}}=\frac{A_M}{1+\frac{s}{\omega_H}}$$

$$\omega_H=\omega_0=\frac{1}{C_{in}R'_{sig}}$$

$$A_M=-\left(\frac{R_G}{R_G+R_{sig}}\right)(g_m R'_L)$$

$$f_H=\frac{\omega_H}{2\pi}=\frac{1}{2\pi C_{in}R'_{sig}}$$

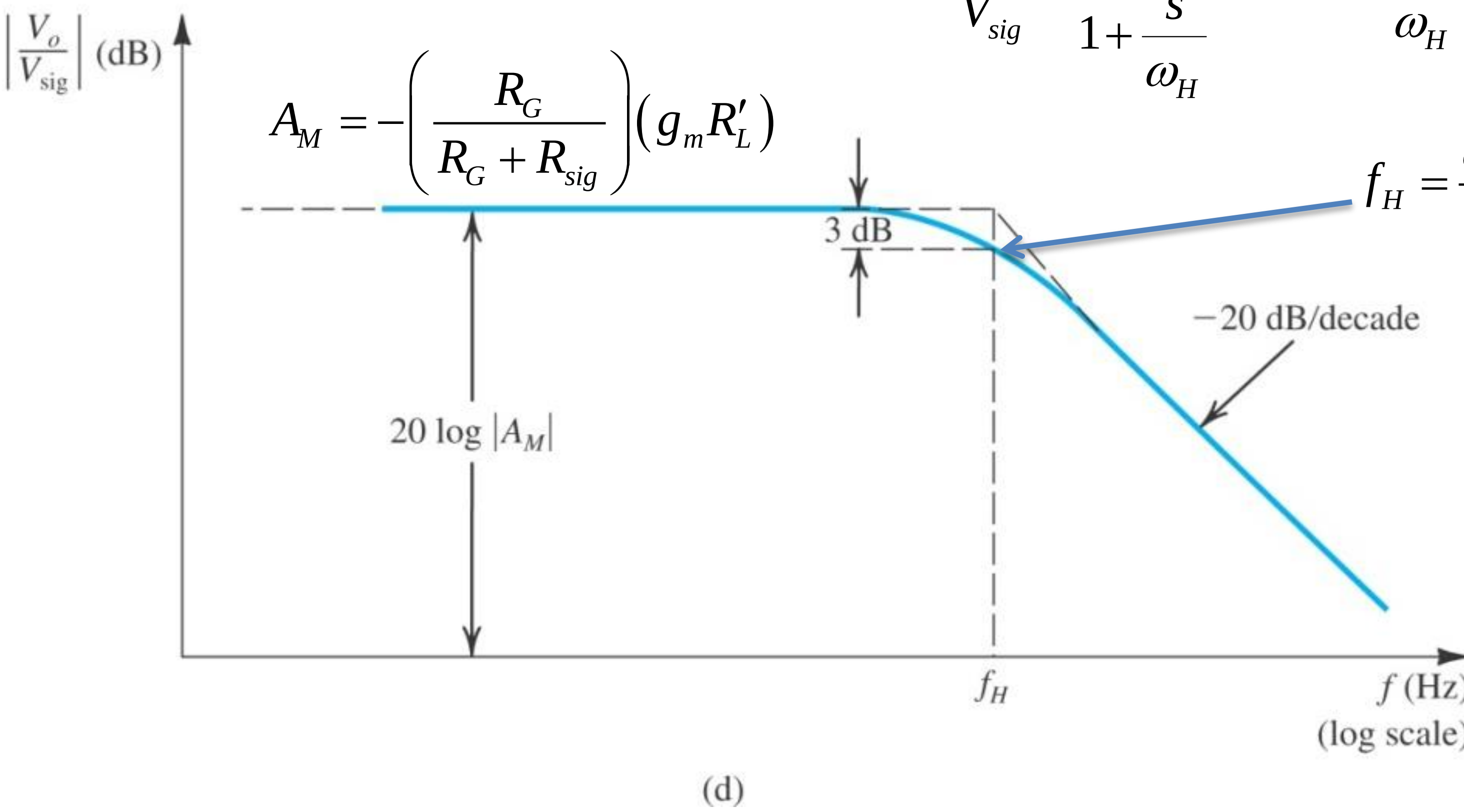


Figure 10.18 (*Continued*) **(d)** the frequency response plot, which is that of a low-pass, single-time-constant circuit.

The Common-Source Amplifier

1. The upper 3-dB frequency is determined by the interaction of R'_{sig} and C_{in}. Since the bias resistance R_G is usually very large, it can be neglected, resulting in the resistance of the signal source. It follows that a large value of R_{sig} will cause f_H to be lowered.
2. The total input capacitance C_{in} is usually dominated by C_{eq}, which in turn is made large by the multiplication effect that C_{gd} undergoes. Thus, although C_{gd} is usually a very small capacitance, its effect on the amplifier frequency response can be very significant as a result of its multiplication by the Miller Effect.
3. To extend the high-frequency response of a MOSFET amplifier, we have to find configurations in which the Miller effect is absent or at least reduced.
4. The above analysis, resulting in an STC or a single-pole response, is approximate. Specifically, it is based on neglecting I_{gd} relative to $g_m V_{gs}$, an assumption that applies well at frequencies not too much higher than f_H. An exact analysis of the circuit will be carried out in Section 10.5. The results above, however, are more than sufficient for a quick estimate of f_H. As well, the approximate approach helps to reveal the primary limitation on the high-frequency response: the Miller effect.

Example 10.3

Find the midband gain A_M and the upper 3-dB frequency f_H of a CS amplifier fed with a signal source having an internal resistance R_{sig} = 100 kΩ. The amplifier has R_G = 4.7 MΩ, R_D = R_L = 15 kΩ, g_m = 1 mA/V, r_o = 150 kΩ, C_{gs} = 1 pF, and C_{gd} = 0.4 pF.

$$R_L' = r_o \,||\, R_D \,||\, R_L = 150\text{k}\Omega \,||\, 15\text{k}\Omega \,||\, 15\text{k}\Omega = 7.14\text{k}\Omega$$

$$R_{sig}' = R_{sig} \,||\, R_G = 100\text{k}\Omega \,||\, 4.7\text{M}\Omega = 97.9\text{k}\Omega$$

$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_G}{R_G + R_{sig}}\left(g_m R_L'\right)$$
$$= -\frac{4700\text{k}\Omega}{4700\text{k}\Omega + 100\text{k}\Omega}\left(1\text{mA/V} \times 7.14\text{k}\Omega\right)$$
$$= -7\text{V/V}$$

$$C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd}\left(1 + g_m R_L'\right)$$
$$= 1\text{pF} + 0.4\text{pF}\left(1 + 1\text{mA/V} \times 7.14\text{k}\Omega\right)$$
$$= 4.26\text{pF}$$

$$f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi C_{in} R_{sig}'}$$
$$= \frac{1}{2\pi\left(4.26\text{pF}\right)97.9\text{k}\Omega}$$
$$= 381.6\text{kHz}$$

The Common-Emitter Amplifier

high-frequency equivalent circuit of a CE amplifier.

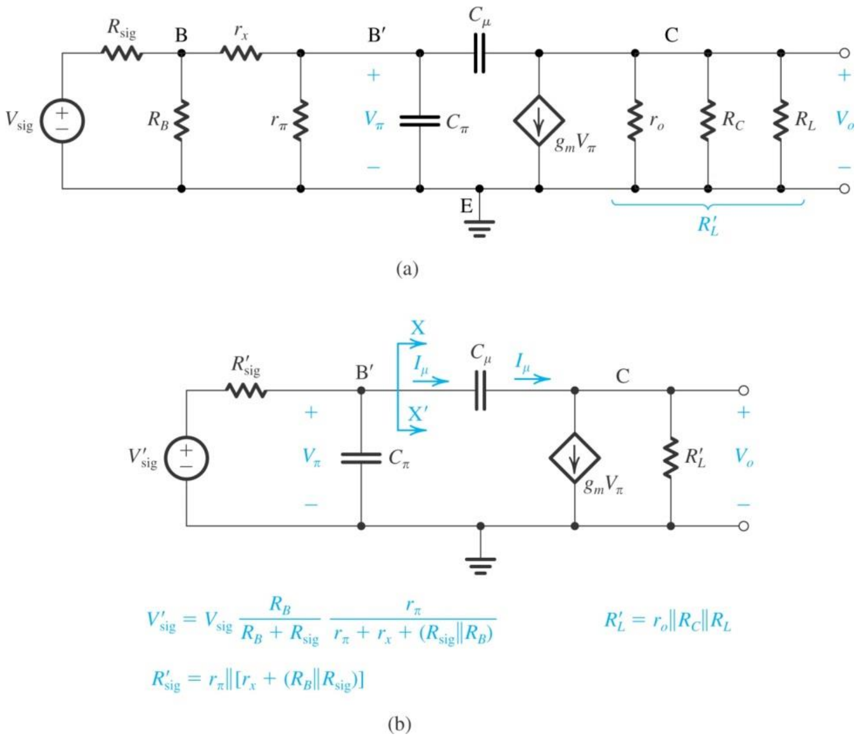


Figure 10.19 Determining the high-frequency response of the CE amplifier: **(a)** equivalent circuit; **(b)** the circuit of **(a)** simplified at both the input side and the output side; (*continued*)

It may be simplified using Thevenin's theorem.

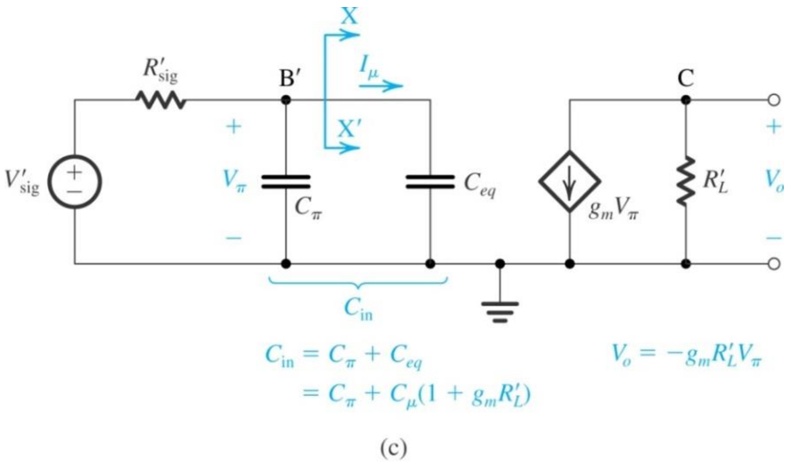
$$R'_{sig} = r_\pi \,\|\, \left[r_x + \left(R_B \,\|\, R_{sig} \right) \right]$$

$$V'_{sig} = V_{sig} \frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + \left(R_{sig} \,\|\, R_B \right)}$$

$$R'_L = r_o \,\|\, R_C \,\|\, R_L$$

$$A_M = \frac{V_o}{V_{sig}} = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + \left(R_{sig} \,\|\, R_B \right)} \left(g_m R'_L \right)$$

The Common-Emitter Amplifier

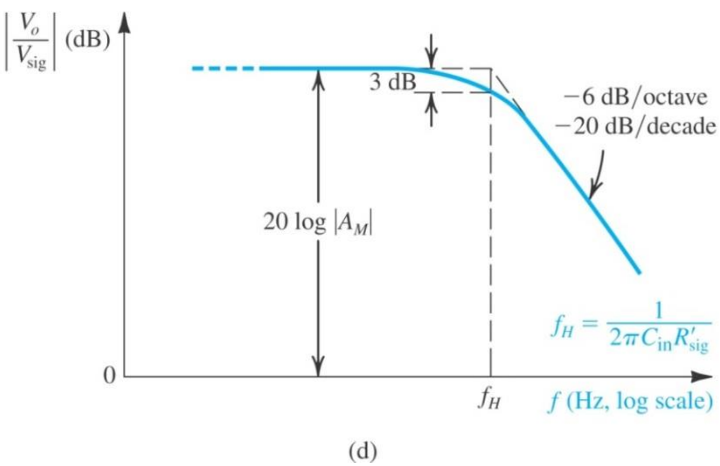


$$C_{eq} = C_\mu \left(1 + g_m R'_L\right)$$

C_μ gives rise to much larger capacitance C_{eq}. The multiplication effect that it undergoes is known as the **Miller Effect**.

$$C_{in} = C_\pi + C_{eq} = C_\pi + C_\mu \left(1 + g_m R'_L\right)$$

$$R'_{sig} = r_\pi \,\|\, \left[r_x + \left(R_B \,\|\, R_{sig} \right) \right]$$



$$\frac{V_o}{V_{sig}} = \frac{A_M}{1 + \dfrac{s}{\omega_H}}$$

$$\omega_H = \omega_0 = \frac{1}{C_{in} R'_{sig}}$$

$$f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi C_{in} R'_{sig}}$$

Figure 10.19 (**c**) equivalent circuit with C_μ replaced at the input side with the equivalent capacitance C_{eq}; (**d**) sketch of the frequency-response plot, which is that of a low-pass STC circuit.

Example 10.4a

It is required to find the midband gain and the upper 3-dB frequency of the common-emitter amplifier of Fig. 10.9(a) for the following case: $V_{CC} = V_{EE} = 10$ V, $I = 1$ mA, $R_B = 100$ kΩ, $R_C = 8$ kΩ, $R_{sig} = 5$ kΩ, $R_L = 5$ kΩ, $\beta_0 = 100$, $V_A = 100$ V, $C_\mu = 1$ pF, $f_T = 800$ MHz, and $r_x = 50$ Ω. Also, find the frequency of the transmission zero.

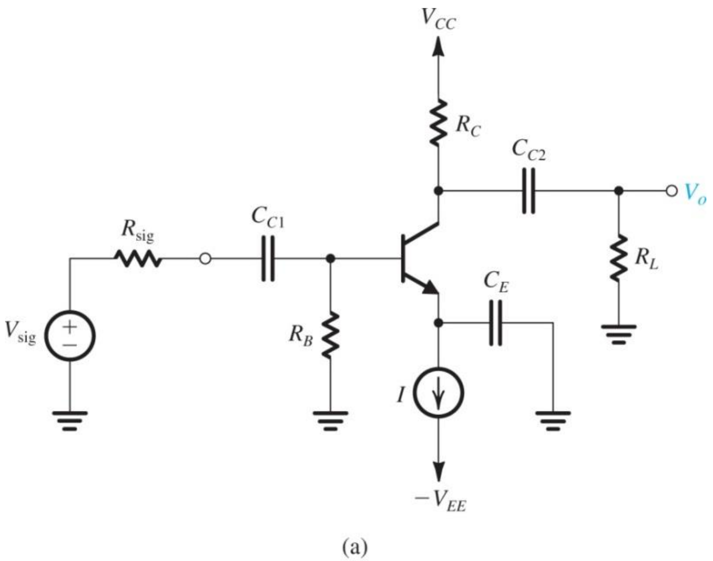


Figure 10.9 (a) A capacitively coupled common-emitter amplifier..

$$g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{25\text{mV}} = 40\text{mA/V}$$

$$r_o = \frac{|V_A|}{I_C} = \frac{100\text{V}}{1\text{mA}} = 100\text{k}\Omega$$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{40\text{mA/V}} = 2.5\text{k}\Omega$$

$$R_L' = r_o \,||\, R_C \,||\, R_L = 100\text{k}\Omega \,||\, 8\text{k}\Omega \,||\, 5\text{k}\Omega = 3\text{k}\Omega$$

$$R_{sig}' = r_\pi \,||\left[r_x + \left(R_B \,||\, R_{sig}\right)\right]$$

$$= 2.5\text{k}\Omega \,||\left[0.050\text{k}\Omega + \left(100\text{k}\Omega \,||\, 5\text{k}\Omega\right)\right] = 1.65\text{k}\Omega$$

Example 10.4b

It is required to find the midband gain and the upper 3-dB frequency of the common-emitter amplifier of Fig. 10.9(a) for the following case: $V_{CC} = V_{EE} = 10$ V, $I = 1$ mA, $R_B = 100$ kΩ, $R_C = 8$ kΩ, $R_{sig} = 5$ kΩ, $R_L = 5$ kΩ, $\beta_0 = 100$, $V_A = 100$ V, $C_\mu = 1$ pF, $f_T = 800$ MHz, and $r_x = 50$ Ω.

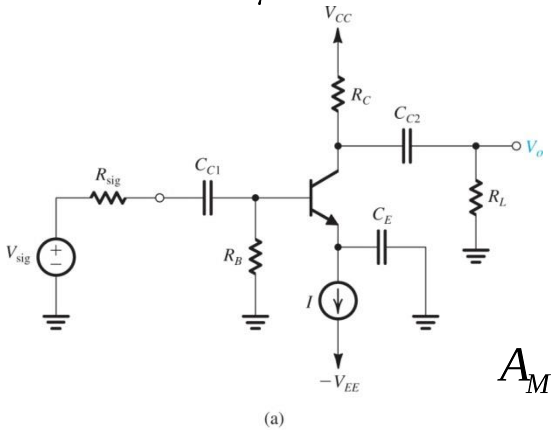


Figure 10.9 (a) A capacitively coupled common-emitter amplifier..

$$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu = \frac{40\text{mA/V}}{2\pi \times 800\text{MHz}} - 1\text{pF} = 7\text{pF}$$

$$C_{in} = C_\pi + C_\mu\left(1 + g_m R'_L\right) = 7\text{pF} + 1\text{pF}\left(1 + 40\text{mA/V} \times 3\text{k}\Omega\right)$$
$$= 128\text{pF}$$

$$A_M = \frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + \left(R_{sig} \,||\, R_B\right)}\left(g_m R'_L\right)$$
$$= \frac{100\text{k}\Omega}{100\text{k}\Omega + 5\text{k}\Omega} \frac{2.5\text{k}\Omega}{2.5\text{k}\Omega + 0.05\text{k}\Omega + \left(5\text{k}\Omega \,||\, 100\text{k}\Omega\right)}\left(40\text{mA/V} \times 3\text{k}\Omega\right)$$
$$= -39\text{V/V}$$

$$f_H = \frac{\omega_H}{2\pi} = \frac{1}{2\pi C_{in} R'_{sig}} = \frac{1}{2\pi \times 128\text{pF} \times 1.65\text{k}\Omega} = 754\text{kHz}$$

Miller's Theorem

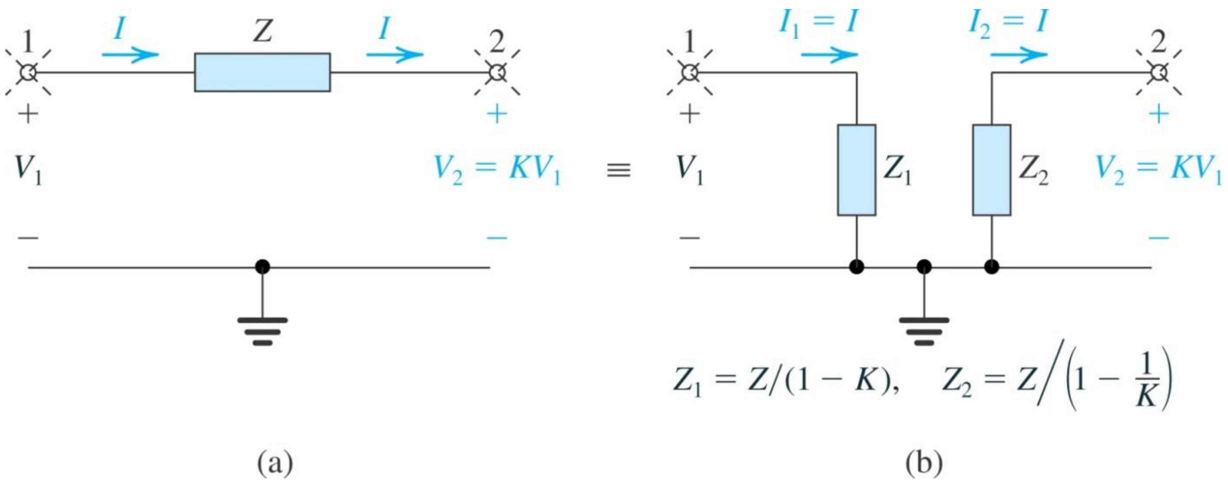


Figure 10.20 The Miller equivalent circuit.

- Two circuit nodes
- Impedance Z between them
- $V_2 = KV_1$

Miller's Theorem states that impedance Z can be replaced with two impedances:

Z_1 connected between node 1 and ground

$$Z_1 = Z/(1-K)$$

Z_2 connected between node 2 and ground

$$Z_2 = Z/(1-1/K)$$

Miller's Theorem Derivation

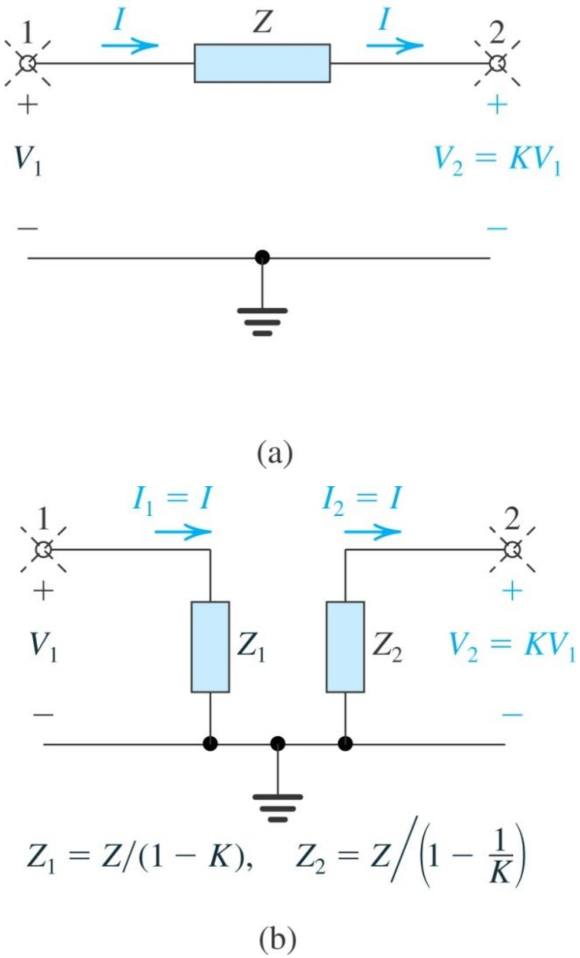


Figure 10.20 The Miller equivalent circuit.

In our analysis of the high- frequency response of the common-source and common-emitter amplifiers, we employed a technique for replacing the bridging capacitance (C_{gs} or C_{μ}) by an equivalent input capacitance. This very useful and effective technique is based on a general theorem known as Miller's theorem.

$$I_1 = \frac{V_1}{Z_1} = I = \frac{V_1 - KV_1}{Z}$$

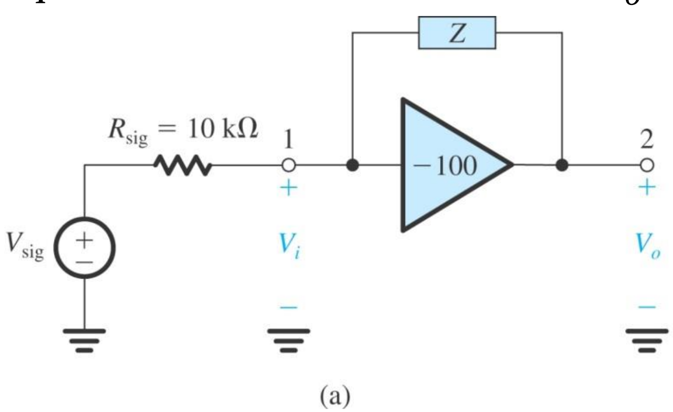
$$Z_1 = \frac{V_1 Z}{V_1 - KV_1} = \frac{Z}{1-K}$$

$$I_2 = \frac{0 - V_2}{Z_2} = \frac{0 - KV_1}{Z_2} = I = \frac{V_1 - KV_1}{Z}$$

$$Z_2 = \frac{-ZKV_1}{V_1 - KV_1} = \frac{-ZK}{1-K} = \frac{ZK}{K-1} = \frac{Z}{1 - \dfrac{1}{K}}$$

Example 10.5

Figure 10.21(a) shows an ideal voltage amplifier having a gain of with an impedance Z connected between its output and input terminals. Find the Miller equivalent circuit when Z is (a) a 1-MΩ resistance and (b) a 1-pF capacitance. In each case, use the equivalent circuit to determine V_o/V_{sig}.



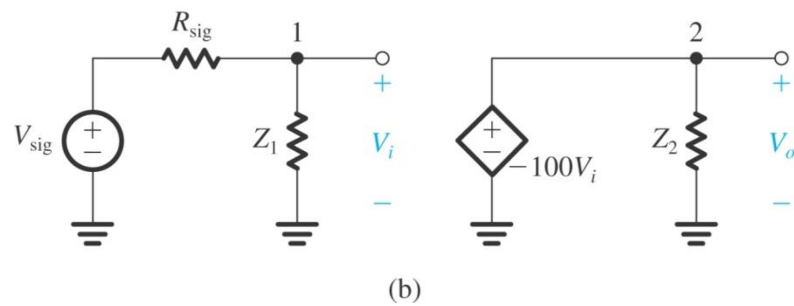


Figure 10.21 Circuits for Example 10.5.

(a) a 1-MΩ resistance

$$Z_1 = \frac{Z}{1-K} = \frac{1\text{M}\Omega}{1-(-100)} = 9.9\text{k}\Omega$$

$$Z_2 = \frac{1\text{M}\Omega}{1+\dfrac{1}{100}} = 0.99\text{M}\Omega$$

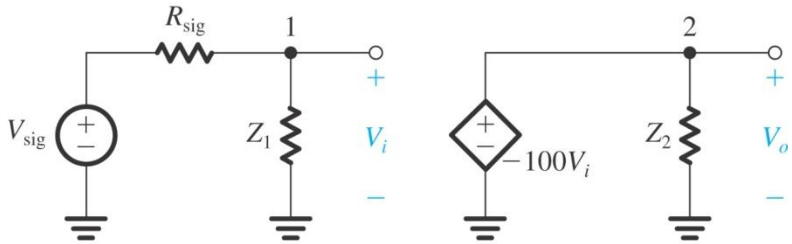
$$\frac{V_o}{V_{sig}} = \frac{V_o}{V_i}\frac{V_i}{V_{sig}} = -100\frac{Z_1}{Z_1+R_{sig}}$$

Example 10.5

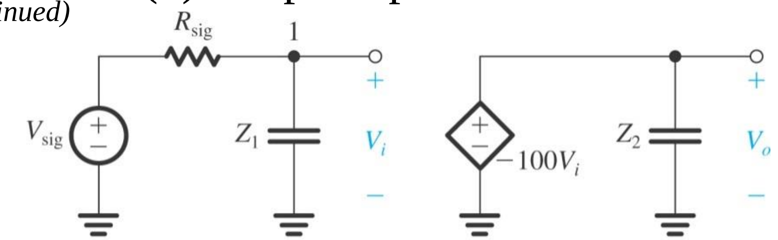
(a) a 1-MΩ resistance

(b) a 1-pF capacitance

Figure 10.21 *(continued)*



(b)



(c)

$$Z_1 = \frac{Z}{1-K} = \frac{1\text{M}\Omega}{1-(-100)} = 9.9\text{k}\Omega$$

$$Z_2 = \frac{1\text{M}\Omega}{1+\dfrac{1}{100}} = 0.99\text{M}\Omega$$

$$\frac{V_o}{V_{sig}} = \frac{V_o}{V_i}\frac{V_i}{V_{sig}} = -100\frac{Z_1}{Z_1+R_{sig}}$$

$$= -100\frac{9.9\text{k}\Omega}{9.9\text{k}\Omega+10\text{k}\Omega}$$

$$= -49.7\text{V/V}$$

$$Z_1 = \frac{Z}{1-K} = \frac{1/sC}{1-(-100)} = \frac{1}{(101)sC}$$

$$Z_2 = \frac{1/sC}{1+\dfrac{1}{100}} = \frac{1}{(1.01)sC}$$

$$\frac{V_o}{V_{sig}} = \frac{V_o}{V_i}\frac{V_i}{V_{sig}} = -100\frac{Z_1}{Z_1+R_{sig}}$$

$$= -100\frac{1/\left[(101)sC\right]}{1/\left[(101)sC\right]+10\text{k}\Omega}$$

$$= \frac{-100}{1+s(101)1\text{pF}\times 10\text{k}\Omega}\text{V/V}$$

CS Amp with Low R_{sig}

$$\frac{V_o}{V'_{sig}} = \frac{-(g_m R'_L)\left[1 - s(C_{gd}/g_m)\right]}{1 + s\left\{\left[C_{gs} + C_{gd}(1 + g_m R'_L)\right]R'_{sig} + (C_L + C_{gd})R'_L\right\} + s^2\left[(C_L + C_{gd})C_{gs} + C_L C_{gd}\right]R'_{sig}R'_L}$$

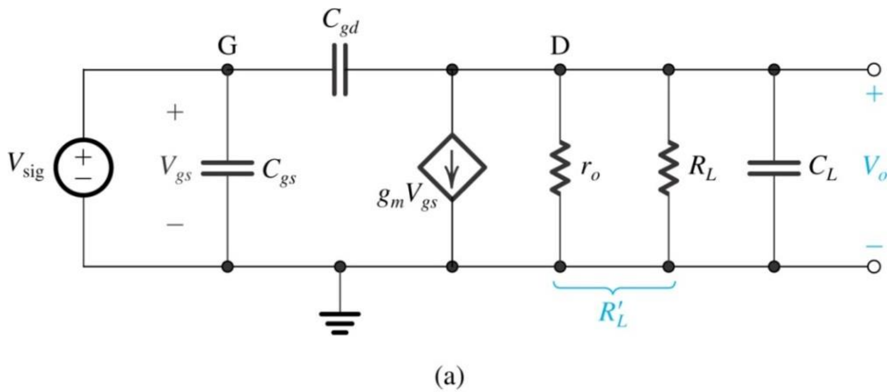


Figure 10.22 (a) High-frequency equivalent circuit of a CS amplifier fed with a signal source having a very low (effectively zero) resistance.

There are applications in which the CS amplifier is fed with a low-resistance signal source. In this case, the high-frequency gain will no longer be limited by the interaction of the source resistance and the input capacitance. Rather, the high-frequency limitation happens at the amplifier output. If we set R_{sig} = 0 in the transfer function above we get:

$$\frac{V_o}{V'_{sig}} = \frac{-(g_m R'_L)\left[1 - s(C_{gd}/g_m)\right]}{1 + s(C_L + C_{gd})R'_L}$$

Note that the dc gain and the frequency of the zero do not change but the high-frequency response is now determined by a pole formed by (C_L + C_{gd}) together with R'_L.

CS Amp with Low R_{sig}

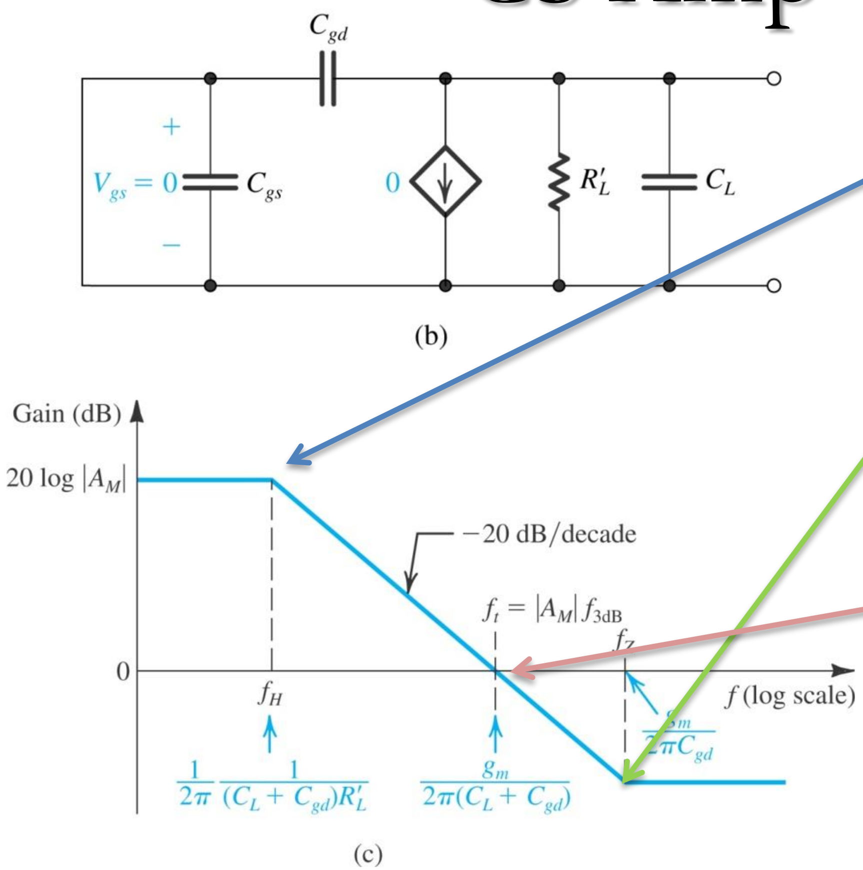


Figure 10.22 **(b)** The circuit with V_{sig} reduced to zero. **(c)** Bode plot for the gain of the circuit in **(a)**.

$$f_H = \frac{1}{2\pi\left(C_L + C_{gd}\right)R'_L}$$

$$f_Z = \frac{g_m}{2\pi C_{gd}}$$

still very high in frequency and doesn't play a significant role

The high frequency response equals unity (0 dB) at the frequency f_t given by:

$$f_t = |A_M|\, f_H$$

$$= \left|-g_m R'_L\right| \frac{1}{2\pi\left(C_L + C_{gd}\right)R'_L}$$

$$= \frac{g_m}{2\pi\left(C_L + C_{gd}\right)}$$

This is known as the **gain-bandwidth product (GBW) product** of the amplifier

Example 10.6a

Consider an IC CS amplifier fed with a source having R_{sig} = 0 and having an effective load resistance R'_L composed of r_o of the amplifier transistor in parallel with an equal resistance r_o of the current-source load. Let g_m = 1.25 mA/V, r_o = 20 kΩ, C_{gs} = 20 fF, C_{gd} = 5 fF, and C_L = 25 fF. Find A_M, f_H, f_t, and f_Z. If the amplifying transistor is to be operated at twice the original overdrive voltage while W and L remain unchanged, by what factor must the bias current be changed? What are the new values of A_M, f_H, f_t, and f_Z?

$$R'_L = r_o \,||\, r_o = 20\text{k}\Omega \,||\, 20\text{k}\Omega = 10\text{k}\Omega$$

$$A_M = -g_m R'_L = -1.25\frac{\text{mA}}{\text{V}}\left(10\text{k}\Omega\right) = -12.5\frac{\text{V}}{\text{V}}$$

$$f_H = \frac{1}{2\pi\left(C_L + C_{gd}\right)R'_L} = \frac{1}{2\pi\left(25\text{fF} + 5\text{fF}\right)10\text{k}\Omega} = 530.5\text{MHz}$$

$$f_t = \left|A_M\right| f_H = \left|-12.5\frac{\text{V}}{\text{V}}\right|\left(530\text{MHz}\right) = 6.63\text{GHz}$$

$$f_Z = \frac{g_m}{2\pi C_{gd}} = \frac{1.25\,\text{mA/V}}{2\pi\left(5\text{fF}\right)} = 39.78\text{GHz}$$

Example 10.6b

Consider an IC CS amplifier fed with a source having R_{sig} = 0 and having an effective load resistance R'_L composed of r_o of the amplifier transistor in parallel with an equal resistance r_o of the current-source load. Let g_m = 1.25 mA/V, r_o = 20 kΩ, C_{gs} = 20 fF, C_{gd} = 5 fF, and C_L = 25 fF. Find A_M, f_H, f_t, and f_Z. If the amplifying transistor is to be operated at twice the original overdrive voltage while *W* and *L* remain unchanged, by what factor must the bias current be changed? What are the new values of A_M, f_H, f_t, and f_Z?

$$V_{OV}=\sqrt{\frac{2I_D}{k_n'(W/L)}}\Rightarrow I_{D\_new}=4I_D \text{ to double } V_{OV}\quad\Rightarrow g_{m\_new}=\frac{4}{2}\frac{(2I_D)}{V_{OV}}=2g_m=2.5\frac{\text{mA}}{\text{V}}$$

$$r_{o\_new}=\frac{1}{4}\frac{V_A}{I_D}=\frac{r_o}{4}=5\text{k}\Omega \qquad R'_{L\_new}=r_{o\_new}\,||\,r_{o\_new}=5\text{k}\Omega\,||\,5\text{k}\Omega=2.5\text{k}\Omega$$

$$A_M=-g_mR'_L=-2.5\frac{\text{mA}}{\text{V}}(2.5\text{k}\Omega)=-6.25\frac{\text{V}}{\text{V}}$$

$$f_H=\frac{1}{2\pi\left(C_L+C_{gd}\right)R'_L}=\frac{1}{2\pi(25\text{fF}+5\text{fF})2.5\text{k}\Omega}=2.12\text{GHz}$$

$$f_t=|A_M|\,f_H=\left|-6.25\frac{\text{V}}{\text{V}}\right|(2.12\text{GHz})=13.3\text{GHz} \qquad f_Z=\frac{g_m}{2\pi C_{gd}}=\frac{2.5\text{mA/V}}{2\pi(5\text{fF})}=79.56\text{GHz}$$

Homework #8

Read Chapter 10

Chapter 10 Problems:

1. 10.29
2. 10.32
3. 10.34
4. 10.39*

* Answers in Appendix L

10.4 USEFUL TOOLS FOR THE ANALYSIS OF THE HIGH-FREQUENCY RESPONSE OF AMPLIFIERS

The High Frequency Gain Function

- The approximate method used in previous sections to analyze the high-frequency response of amps provides an “ok” estimate.
- However, it does not apply to more complex circuits.
- Amp gain can be expressed as a function of *s* as: $A(s) = A_M F_H(s)$
- The value of A_M may be determined by assuming transistor internal capacitances are open circuited.

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})\cdots(1+s/\omega_{Zn})}{(1+s/\omega_{P1})(1+s/\omega_{P2})\cdots(1+s/\omega_{Pn})}$$

- where ω_{P1}, ω_{P2} ..., ω_{Pn} are positive numbers representing the frequencies of the *n* real poles and ω_{Z1}, ω_{Z2} ..., ω_{Zn} are positive, negative, or infinite numbers representing the frequencies of the *n* real transmission zeros. Note that as *s* approaches 0, F_H(s) approaches unity and the gain approaches A_M.

Determining the 3-dB Frequency f_H

High-frequency band closest to midband is generally of greatest concern as the designer needs to estimate upper 3*dB* frequency. If one pole (predominantly) dictates the high-frequency response of an amplifier, this pole is called the dominant-pole response.

$$F_H(s) \simeq \frac{1}{1+s/\omega_{P1}} \qquad \omega_H \simeq \omega_{P1}$$

As rule of thumb, a dominant pole exists if the lowest-frequency pole is at least two octaves (a factor of 4) away from the nearest pole or zero.

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})}{(1+s/\omega_{P1})(1+s/\omega_{P2})} \qquad |F_H(j\omega)|^2 = \frac{(1+\omega^2/\omega_{Z1}^2)(1+\omega^2/\omega_{Z2}^2)}{(1+\omega^2/\omega_{P1}^2)(1+\omega^2/\omega_{P2}^2)}$$

By definition, at $\omega = \omega_H$, $|F_H|^2 = ½$;

$$\omega_H \simeq \frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}+\frac{1}{\omega_{P2}^2}-\frac{2}{\omega_{Z1}^2}-\frac{2}{\omega_{Z2}^2}}}$$

if one of the poles, say P_1 is dominant, then $\omega_{P1} << \omega_{P2}, \omega_{Z1}, \omega_{Z2}$ and can be approximated as:

$$\omega_H \simeq \frac{1}{\sqrt{\frac{1}{\omega_{P1}^2}}} \simeq \omega_{P1}$$

Example 10.7

The high-frequency response of an amplifier is characterized by the transfer function:

$$F_H(s) = \frac{1 - s/10^5}{\left(1 + s/10^4\right)\left(1 + s/4\times10^4\right)}$$

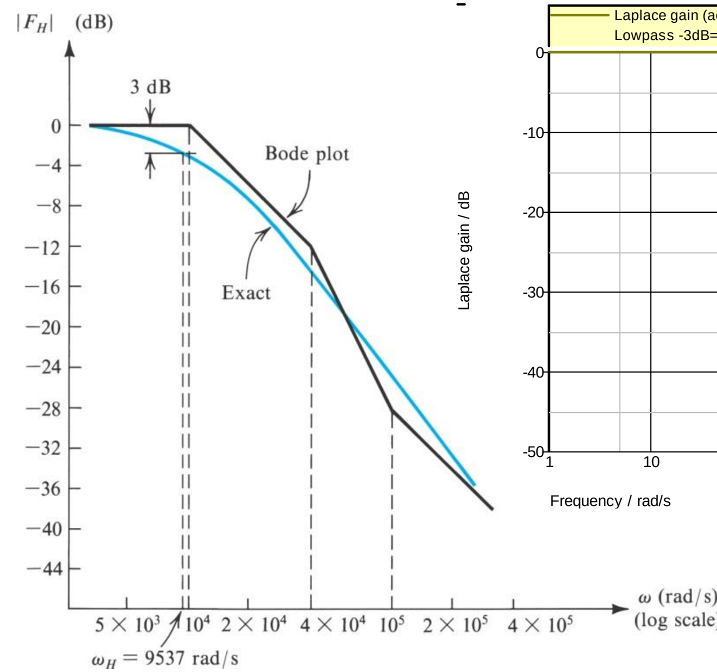
Determine the 3-dB frequency approximately and exactly.

$$\omega_H \simeq \omega_{P1} = 10000\text{rad/s}$$

$$\omega_H \simeq \frac{1}{\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} - \frac{2}{\omega_{Z1}^2} - \frac{2}{\omega_{Z2}^2}}}$$

$$= \frac{1}{\sqrt{\frac{1}{10^8} + \frac{1}{16\times10^8} - \frac{2}{10^{10}}}}$$

$$= 9794\text{rad/s}$$



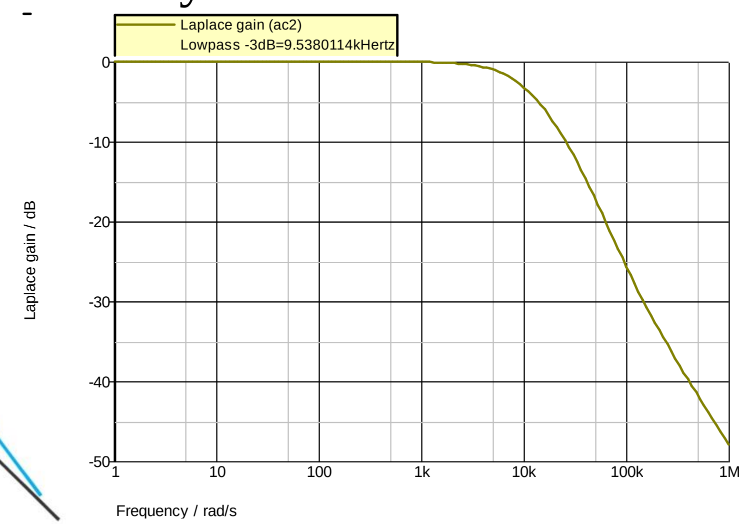


Figure 10.23 Normalized high-frequency response of the amplifier in Example 10.7.

Using Open-Circuit Time Constants

If the poles and zeros of the amplifier transfer function can be determined easily, then we can determine f_H using the techniques above. In many cases, however, it is not a simple matter to determine the poles and zeros by quick hand analysis. In such cases an approximate value for f_H can be obtained using the following method. Consider the function $F_H(s)$, which determines the high-frequency response of the amplifier.

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})\cdots(1+s/\omega_{Zn})}{(1+s/\omega_{P1})(1+s/\omega_{P2})\cdots(1+s/\omega_{Pn})}$$

The numerator and denominator factors can be multiplied out and expressed in the alternative form:

$$F_H(s) = \frac{1+a_1 s+a_2 s^2+\cdots+a_n s^n}{1+b_1 s+b_2 s^2+\cdots+b_n s^n}$$

where the coefficients a and b are related to the frequencies of the zeros and poles, respectively.

Using Open-Circuit Time Constants

Specifically, the coefficient b_1 is given by

$$b_1 = \frac{1}{\omega_{P1}} + \frac{1}{\omega_{P2}} + \cdots + \frac{1}{\omega_{Pn}}$$

It can be shown that the value of b_1 can be obtained by considering the various capacitances in the high-frequency equivalent circuit one at a time while reducing all other capacitors to zero (or, equivalently, replacing them with open circuits). That is, to obtain the contribution of capacitance C_i we reduce all other capacitances to zero, reduce the input signal source to zero, and determine the resistance R_i seen by C_i. This process is then repeated for all other capacitors in the circuit. The value of b_1 is computed by summing the individual time constants, called **open-circuit time constants**,

$$b_1 = \sum_{i=1}^{n} C_i R_i$$

where we have assumed that there are n capacitors in the high-frequency equivalent circuit.

Using Open-Circuit Time Constants

This method for determining b_1 is exact; the approximation comes about in using the value of b_1 to determine ω_H. Specifically, if the zeros are not dominant and if one of the poles, say P_1, is dominant, then

$$b_1 \simeq \frac{1}{\omega_{P1}}$$

But, also, the upper 3-dB frequency will be approximately equal to ω_{P1}, leading to the approximation

$$\omega_H \simeq \omega_{P1} = \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{n} C_i R_i}$$

Here it should be pointed out that in complex circuits we usually do not know whether a dominant pole exists. Nevertheless, using this equation to determine ω_H normally yields remarkably good results even if a dominant pole does not exist.

Application of the Method of OCTCs

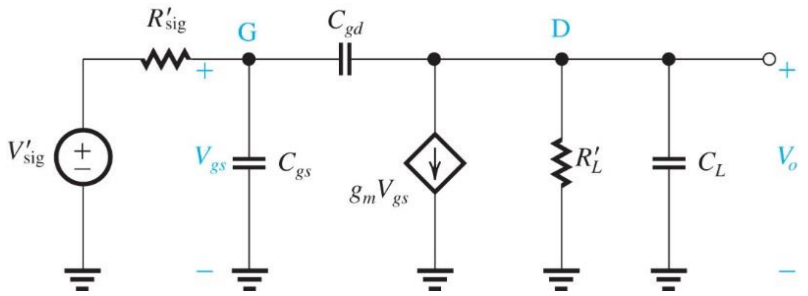


Figure 10.24 Generalized high-frequency equivalent circuit for the CS amplifier.

Figure 10.24 shows a generalized high-frequency equivalent circuit for the common-source amplifier. Here, V'_{sig} and R'_{sig} are the Thevenin equivalent of the signal generator together with whatever bias circuit may be present at the amplifier input. Resistance R'_L represents the total resistance between the output (drain) node and ground and includes R_D, r_o, and R_L. Similarly, C_L represents the total capacitance between the drain node and ground and includes the MOSFET's drain-to-body capacitance (C_{db}), the capacitance introduced by a current-source load, the input capacitance of a succeeding amplifier stage, and in some cases, as we will see in later chapters, a deliberately introduced capacitance. In IC MOS amplifiers, C_L can be substantial.

Application of the Method of OCTCs: CS Amplifier

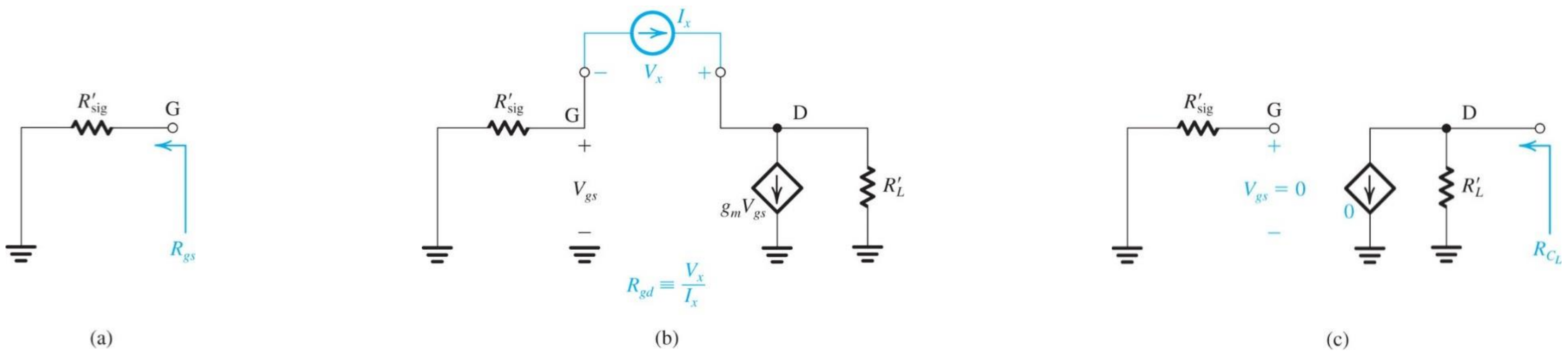


Figure 10.25 Application of the open-circuit time-constants method to the CS equivalent circuit of Fig. 10.24.

$$R_{gs} = R'_{\text{sig}} \qquad V_{gs} = -I_x R'_{\text{sig}} \qquad V_d = V_x + V_{gs} \qquad R_{C_L} = R'_L$$

$$I_x = g_m V_{gs} + \frac{V_d}{R'_L} = g_m V_{gs} + \frac{V_x + V_{gs}}{R'_L}$$

$$R_{gd} \equiv \frac{V_x}{I_x} = R'_{\text{sig}}\left(1 + g_m R'_L\right) + R'_L$$

$$\tau_H = b_1 = C_{gs}R_{gs} + C_{gd}R_{gd} + C_L R_{C_L} = C_{gs}R'_{\text{sig}} + C_{gd}\left[R'_{\text{sig}}\left(1 + g_m R'_L\right) + R'_L\right] + C_L R'_L$$

Example 10.8a

An integrated circuit CS amplifier has g_m = 1.25 mA/V, C_{gs} = 20 fF, C_{gd} = 5 fF, C_L = 25 fF, R'_{sig} = 10 kΩ , and R'_L = 10 kΩ. Determine f_H and the transmission zero f_z caused by C_{gd}.

$$R_{gs} = R'_{sig} = 10\text{k}\Omega$$

$$R_{gd} = R'_{\text{sig}}\left(1+g_m R'_L\right)+R'_L = 10\text{k}\Omega\left(1+1.25\frac{\text{mA}}{\text{V}}10\text{k}\Omega\right)+10\text{k}\Omega = 145\text{k}\Omega$$

$$R_{C_L} = R'_L = 10\text{k}\Omega$$

$$\tau_{gs} = C_{gs}R_{gs} = (20\text{fF})(10\text{k}\Omega) = 200\text{ps}$$

$$\tau_{gd} = C_{gd}R_{gd} = (5\text{fF})(145\text{k}\Omega) = 725\text{ps}$$

$$\tau_{C_L} = C_{C_L}R_{C_L} = (25\text{fF})(10\text{k}\Omega) = 250\text{ps}$$

$$\tau_H = \tau_{gs}+\tau_{gd}+\tau_{C_L} = 1175\text{ps}$$

$$f_H = \frac{1}{(2\pi)\tau_H} = \frac{1}{(2\pi)1175\text{ps}} = 135.5\text{MHz}$$

$$f_Z = \frac{g_m}{(2\pi)C_{gd}} = \frac{1.25\frac{\text{mA}}{\text{V}}}{(2\pi)5\text{fF}} = 39.8\text{GHz}$$

$$f_Z \gg f_H$$

Application of the Method of OCTCs: CE Amplifier

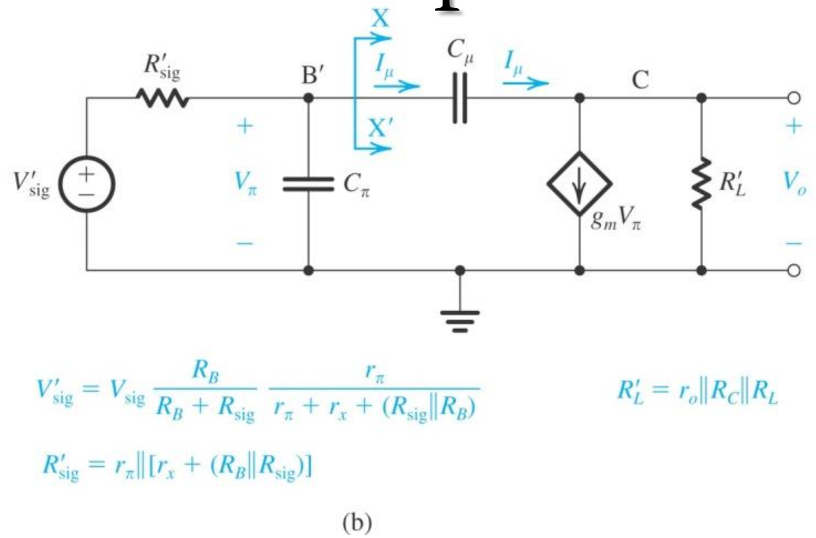


Figure 10.19 Determining the high-frequency response of the CE amplifier: **(a)** equivalent circuit; **(b)** the circuit of **(a)** simplified at both the input side and the output side.

$$R_\pi = R'_{sig} \qquad R_\mu = R'_{sig}\left(1 + g_m R'_L\right) + R'_L \qquad R_{C_L} = R'_L$$

$$\tau_H = b_1 = C_\pi R_\pi + C_\mu R_\mu + C_L R_{C_L} = C_\pi R'_{sig} + C_\mu\left[R'_{sig}\left(1 + g_m R'_L\right) + R'_L\right] + C_L R'_L$$

$$f_H = \frac{1}{(2\pi)\tau_H}$$

10.5 HIGH-FREQUENCY RESPONSE OF THE COMMON-GATE AND CASCODE AMPLIFIERS

High-Frequency Amplifier Response

Although common-source and common-emitter amplifiers provide substantial gain at midband frequencies, their gain falls off in the high-frequency band at a relatively low frequency. This is primarily due to the large input capacitance C_{in}, whose value is significantly increased by the Miller multiplication effect, which the bridging capacitance C_{gd} (or C_{μ}) experiences. It follows that the key to obtaining wideband operation, that is, high f_H, is to use circuit configurations that do not suffer from the Miller effect. One such configuration is the common-gate circuit (Figure 10.26). Note that C_L and C_{gd} are in parallel. Also all three capacitors are shunt (i.e. to ground) so there is no Miller multiplication.

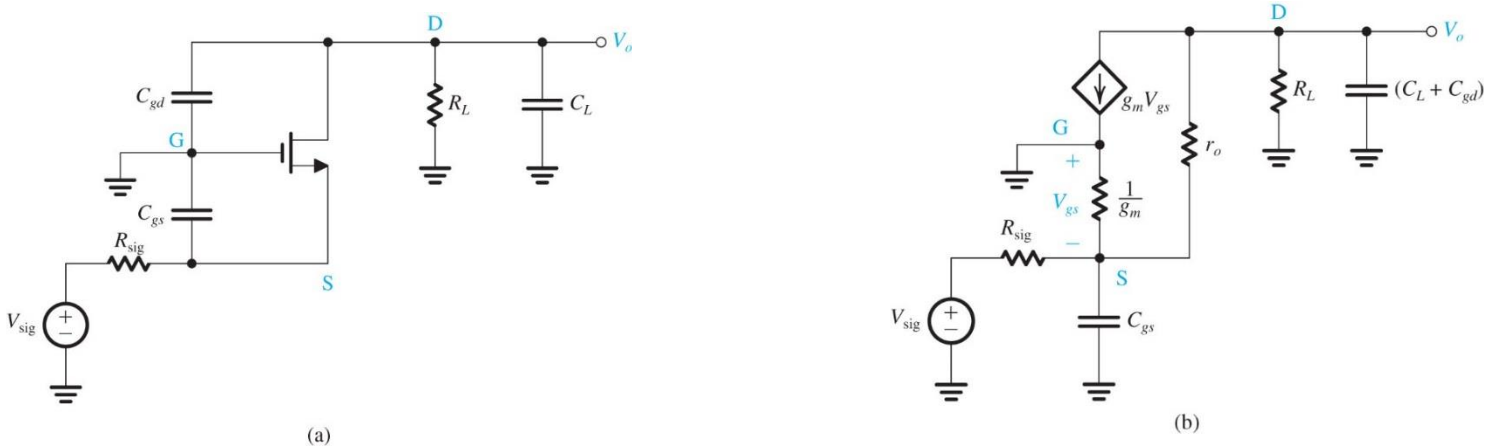


Figure 10.26 (a) The common-gate amplifier with the transistor internal capacitances shown. A load capacitance C_L is also included. **(b)** Equivalent circuit of the CG amplifier with the MOSFET replaced with its T model

High-Frequency Response of the Common Gate (CG) Amplifier

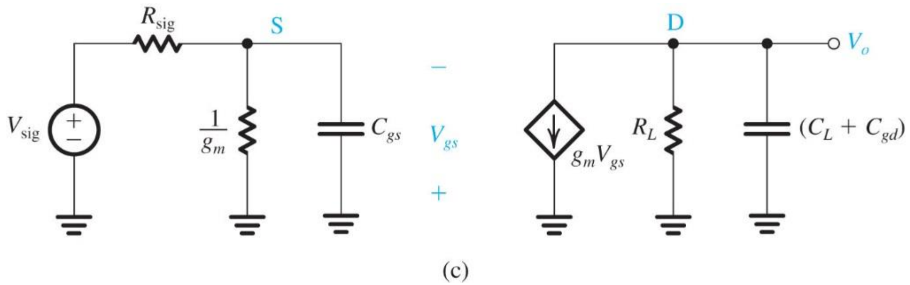


Figure 10.26 (c) Equivalent circuit for the case in which r_o is neglected.

Note that C_L and C_{gd} are in parallel. Also all three capacitors are shunt (i.e. to ground) so there is no Miller multiplication. Finally, to simplify the analysis we will neglect r_o.

There are obviously 2 poles – 1 at the input and 1 at the output:

$$f_{P1} = \frac{1}{2\pi C_{gs}\left(R_{sig} \,||\, \frac{1}{g_m}\right)} \qquad f_{P2} = \frac{1}{2\pi\left(C_{gd} + C_L\right)R_L}$$

The relative locations of the two poles will depend on the specific situation. However, f_{P2} is usually lower than f_{P1}; thus f_{P2} can be dominant. The important point to note is that both f_{P1} and f_{P2} are usually much higher than the frequency of the dominant input pole in the CS stage.

$$f_H = \frac{1}{(2\pi)\tau_H} = \frac{1}{\frac{1}{f_{P1}} + \frac{1}{f_{P2}}}$$

Using OCTC Method (including r_o)

In an IC amplifier we will have to take into account the effect of r_o. We can use the tool of open circuit time constants (OCTCs) to find an approximate high frequency 3-dB frequency, f_H.

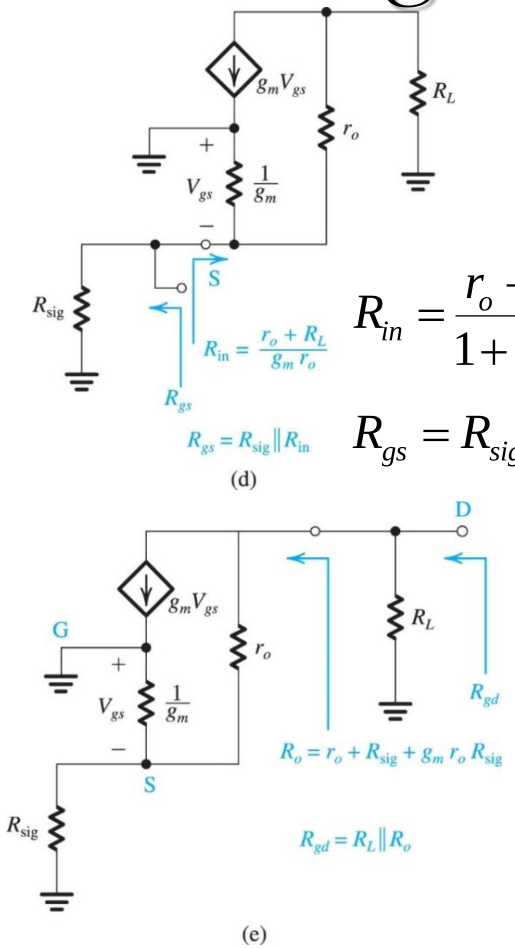


Figure 10.26 (d) Circuit for determining the resistance R_{gs} seen by C_{gs}. **(e)** Circuit for determining the resistance R_{gd} seen by $(C_L + C_{gd})$.

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} \simeq \frac{1}{g_m}$$

$$R_{gs} = R_{sig} \,||\, R_{in} \simeq R_{sig} \,||\, \frac{1}{g_m}$$

$$f_H = \frac{1}{2\pi\left[C_{gs} R_{gs} + \left(C_{gd} + C_L\right) R_{gd}\right]}$$

$$R_o = r_o + R_{sig} + \left(g_m r_o\right) R_{sig}$$

$$R_{gd} = R_L \,||\, R_o \simeq R_L$$

a properly designed CG circuit can have a wide bandwidth. However, the input resistance will be low and the overall midband gain can be very low.

Example 10.9

Consider a common-gate amplifier with g_m = 1.25 mA/V, r_o = 20 kΩ, C_{gs} = 20 fF, C_{gd} = 5 fF, C_L = 25 fF, R_{sig} = 10 kΩ, and R_L = 20 kΩ. Assume that C_L includes C_{db}. Determine the input resistance, the midband gain, and the upper 3-dB frequency f_H.

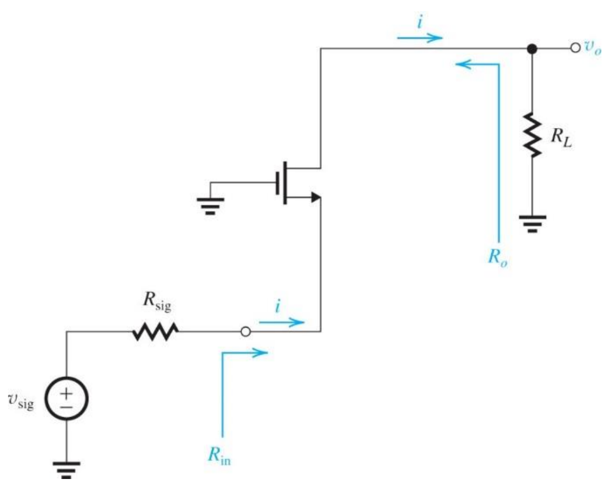


Figure 10.27 The CG amplifier circuit at midband.

$$R_{in} = \frac{r_o + R_L}{1 + g_m r_o} = \frac{20\text{k}\Omega + 20\text{k}\Omega}{1 + 1.25\text{mA/V} \times 20\text{k}\Omega} = 1.54\text{k}\Omega$$

$$v_o = iR_L$$

$$v_{sig} = i\left(R_{sig} + R_{in}\right)$$

$$G_v = \frac{v_o}{v_{sig}} = \frac{R_L}{R_{sig} + R_{in}} = 1.73 \text{ V/V}$$

$$R_{gs} = R_{sig} \,||\, R_{in} = 10\text{k}\Omega \,||\, 1.54\text{k}\Omega = 1.33\text{k}\Omega$$

$$R_o = r_o + R_{sig} + \left(g_m r_o\right) R_{sig}$$

$$= 20\text{k}\Omega + 10\text{k}\Omega + \left(1.25\text{mA/V} \times 20\text{k}\Omega\right) \times 10\text{k}\Omega$$

$$= 280\text{k}\Omega$$

$$R_{gd} = R_L \,||\, R_o = 20\text{k}\Omega \,||\, 280\text{k}\Omega = 18.7\text{k}\Omega$$

$$f_H = \frac{1}{2\pi\left[C_{gs}R_{gs} + \left(C_{gd} + C_L\right)R_{gd}\right]} = 270.9\text{MHz}$$

High-Frequency Response of the MOS Cascode Amplifier

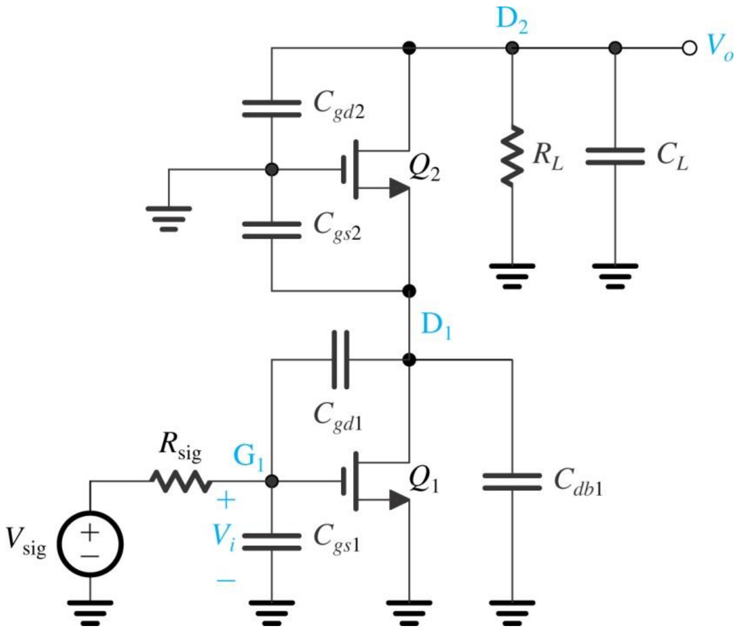


Figure 10.28 The cascode circuit with the various transistor capacitances indicated.

by combining the CS and CG configurations, the cascode amplifier exhibits a very high input resistance and a voltage gain that can be as high as A_0^2, where A_0 is the intrinsic gain of the MOSFET. We shall show that the versatility of the cascode circuit allows us to trade off some of this high midband gain in return for a wider bandwidth.

Note that C_{db1} and C_{gs2} are in parallel as are C_L and C_{gd2} and they will be combined in performing an OCTC analysis.

High-Frequency Response of the MOS Cascode Amplifier

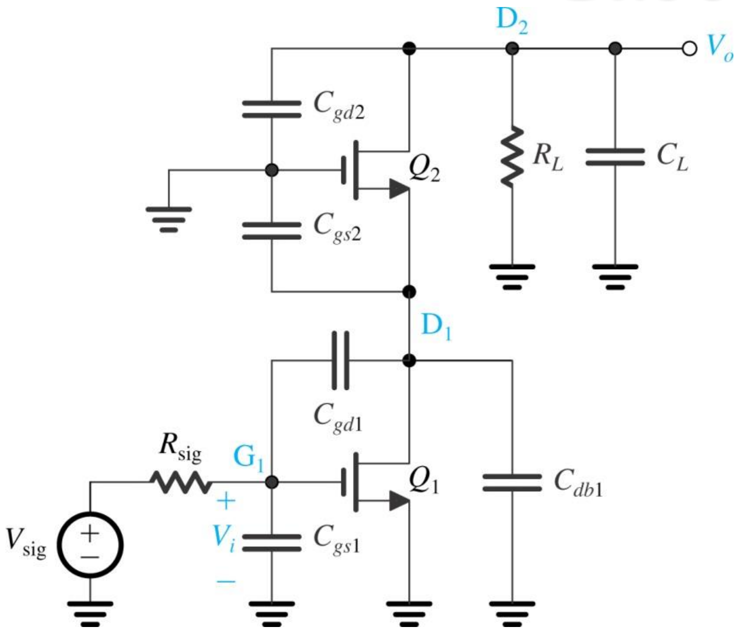


Figure 10.28 The cascode circuit with the various transistor capacitances indicated.

1.) C_{gs1} "sees" resistance R_{sig}.

2.) C_{gd1} "sees" resistance R_{gd1}.

$$R_{gd1} = (1 + g_{m1}R_{d1})R_{sig} + R_{d1}$$

$$R_{d1} = r_{o1} \,||\, R_{in2} = r_{o1} \,||\, \frac{r_{o2} + R_L}{g_{m2}r_{o2}}$$

3.) $(C_{db1} + C_{gs2})$ "sees" resistance R_{d1}.

4.) $(C_L + C_{gd2})$ "sees" resistance $(R_L \| R_o)$

$$R_o = r_{o2} + r_{o1} + (g_{m2}r_{o2})r_{o1}$$

$$\tau_H = C_{gs1}R_{sig} + C_{gd1}\left[(1 + g_{m1}R_{d1})R_{sig} + R_{d1}\right] + (C_{db1} + C_{gs2})R_{d1} + (C_L + C_{gd2})(R_L \,||\, R_o)$$

$$f_H \simeq \frac{1}{2\pi\tau_H}$$

High-Frequency Response of the MOS Cascode Amplifier

$$\tau_H = C_{gs1}R_{sig} + C_{gd1}\left[\left(1+g_{m1}R_{d1}\right)R_{sig} + R_{d1}\right] + \left(C_{db1} + C_{gs2}\right)R_{d1} + \left(C_L + C_{gd2}\right)\left(R_L \parallel R_o\right)$$

Can be rewritten as:

$$\tau_H = \boxed{R_{sig}\left[C_{gs1} + C_{gd1}\left(1+g_{m1}R_{d1}\right)\right]} + R_{d1}\left(C_{gd1} + C_{db1} + C_{gs2}\right) + \left(R_L \parallel R_o\right)\left(C_L + C_{gd2}\right)$$

If R_{sig} is large, the first term can dominate, especially if the Miller multiplier is large. This in turn happens when the load resistance R_L is large (on the order of $A_0 r_o$), causing R_{in2} to be large and requiring the first stage, Q_1, to provide a large proportion of the gain. It follows that when R_{sig} is large, to extend the bandwidth we have to lower R_L to the order of r_o. This in turn lowers R_{in2} and hence R_{d1} and renders the Miller effect in Q_1 insignificant. Note, however, that the dc gain of the cascode will then be A_0. Thus, while the dc gain will be the same as (or a little higher than) that achieved in a CS amplifier, the bandwidth will be greater.

High-Frequency Response of the MOS Cascode Amplifier

$$\tau_H = R_{sig}\left[C_{gs1} + C_{gd1}\left(1+g_{m1}R_{d1}\right)\right] + R_{d1}\left(C_{gd1} + C_{db1} + C_{gs2}\right) + \boxed{\left(R_L \parallel R_o\right)\left(C_L + C_{gd2}\right)}$$

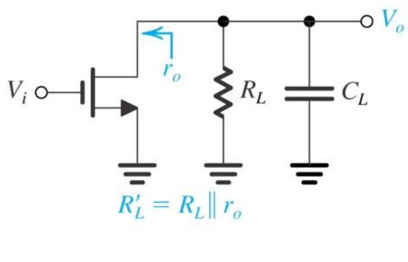
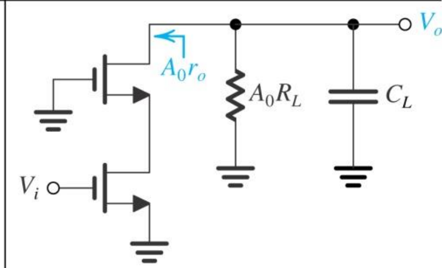
If R_{sig} is small, the Miller effect in Q_1 will not be a concern. A large value of R_L can be used to achieve high dc gain. In this case the third term above will dominate and the time constant will be roughly:

$$\tau_H = \left(C_L + C_{gd2}\right)\left(R_L \parallel R_o\right)$$

$$f_H = \frac{1}{2\pi\left(C_L + C_{gd2}\right)\left(R_L \parallel R_o\right)}$$

which is of the same form as the formula for the CS amplifier with R_{sig} = 0. Here, however, $(R_L \parallel R_o)$ is larger than R'_L by a factor of about A_0. Thus the f_H of the cascode will be lower than that of the CS amplifier by the same factor A_0.

Benefits of Cascoding

	Common Source	Cascode
Circuit		
DC Gain	$-g_m R'_L$	$-A_0 g_m R'_L$
f_{3dB}	$\frac{1}{2\pi(C_L + C_{gd})R'_L}$	$\frac{1}{2\pi(C_L + C_{gd})A_0 R'_L}$
f_t	$\frac{g_m}{2\pi(C_L + C_{gd})}$	$\frac{g_m}{2\pi(C_L + C_{gd})}$

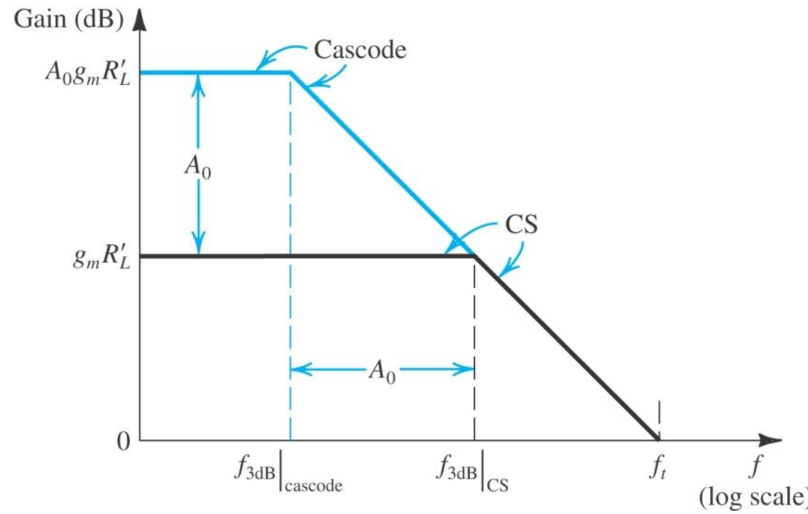


Figure 10.29 Effect of cascoding on gain and bandwidth in the case $R_{sig} = 0$. Cascoding can increase the dc gain by the factor A_0 while keeping the unity-gain frequency constant. Note that to achieve the high gain, the load resistance must be increased by the factor A_0.

1) Increases the dc gain by the factor A_0 over a CS amplifier by using a large load resistance
2) Reduces the 3-dB frequency by the factor A_0
3) Keeps the same unity-gain frequency as the CS amplifier
4) Therefore, increases the gain-bandwidth product by A_0

Example 10.10a

This example illustrates the advantages of cascoding by comparing the performance of a cascode amplifier with that of a common-source amplifier in two cases:

(a) The resistance of the signal source is significant, R_{sig} = 10 kΩ.

(b) R_{sig} is negligibly small.

Assume all MOSFETs have g_m = 1.25 mA/V, r_o = 20 kΩ, C_{gs} = 20 fF, C_{gd} = 5 fF, C_{db} = 5 fF, and C_L (excluding C_{db}) = 10 fF. For case (a), let $R_L = r_o$ = 20 kΩ for both amplifiers. For case (b), let $R_L = r_o$ = 20 kΩ for the CS amplifier and $R_L = R_o$ for the cascode amplifier. For all cases, determine A_v, f_H, and f_t.

Example 10.10b

Assume all MOSFETs have g_m = 1.25 mA/V, r_o = 20 kΩ, C_{gs} = 20 fF, C_{gd} = 5 fF, C_{db} = 5 fF, and C_L (excluding C_{db}) = 10 fF. For case (a), let R_L = r_o = 20 kΩ for both amplifiers. For case (b), let R_L = r_o = 20 kΩ for the CS amplifier and R_L = R_o for the cascode amplifier. For all cases, determine A_v, f_H, and f_t.

(a) The resistance of the signal source is significant, R_{sig} = 10 kΩ.

CS Amp

$$R'_L = (r_o \,||\, R_L) = (20\text{k}\Omega \,||\, 20\text{k}\Omega) = 10\text{k}\Omega$$

$$A_0 = g_m r_o = 1.25\text{mA/V} \times 20\text{k}\Omega = 25\text{V/V}$$

$$A_v = -g_m R'_L = -1.25\text{mA/V} \times 10\text{k}\Omega = -12.5\text{V/V}$$

$$\tau_H = \left[C_{gs} + C_{gd}\left(1 + g_m R'_L\right)\right] R'_{sig} + \left(C_L + C_{gd}\right) R'_L = 1.075\text{ns}$$

$$f_H = \frac{1}{2\pi\tau_H} = 148.05\text{MHz}$$

$$f_t = |A_v|\, f_H = 1.85\text{GHz}$$

Example 10.10c

Assume all MOSFETs have g_m = 1.25 mA/V, r_o = 20 kΩ, C_{gs} = 20 fF, C_{gd} = 5 fF, C_{db} = 5 fF, and C_L (excluding C_{db}) = 10 fF. For case (a), let $R_L = r_o$ = 20 kΩ for both amplifiers. For case (b), let $R_L = r_o$ = 20 kΩ for the CS amplifier and $R_L = R_o$ for the cascode amplifier. For all cases, determine A_v, f_H, and f_t.

(a) The resistance of the signal source is significant, R_{sig} = 10 kΩ.

Cascode Amp

$$R_o = 2r_0 + \left(g_m r_o\right) r_o = 540\text{k}\Omega \qquad R_o \,||\, R_L = 540\text{k}\Omega \,||\, 20\text{k}\Omega = 19.3\text{k}\Omega$$

$$A_v = -g_m\left(R_o \,||\, R_L\right) = -1.25\text{mA/V} \times 19.3\text{k}\Omega = -24.1\text{V/V}$$

$$R_{in2} = \frac{r_o + R_L}{g_m r_o} = 1.6\text{k}\Omega \qquad R_{d1} = r_{o1} \,||\, R_{in2} = 1.48\text{k}\Omega$$

$$\tau_H = R_{sig}\left[C_{gs1} + C_{gd1}\left(1 + g_{m1}R_{d1}\right)\right] + R_{d1}\left(C_{gd1} + C_{db1} + C_{gs2}\right) + \left(R_L \,||\, R_o\right)\left(C_L + C_{gd2}\right) = 772.6\text{ps}$$

$$f_H = \frac{1}{2\pi\tau_H} = 206\text{MHz} \qquad f_t = \left|A_v\right| f_H = 4.96\text{GHz}$$

Example 10.10d

Assume all MOSFETs have g_m = 1.25 mA/V, r_o = 20 kΩ, C_{gs} = 20 fF, C_{gd} = 5 fF, C_{db} = 5 fF, and C_L (excluding C_{db}) = 10 fF. For case (a), let $R_L = r_o$ = 20 kΩ for both amplifiers. For case (b), let $R_L = r_o$ = 20 kΩ for the CS amplifier and $R_L = R_o$ for the cascode amplifier. For all cases, determine A_v, f_H, and f_t.

(b) R_{sig} is negligibly small.

CS Amp

$$A_v = -g_m R'_L = -12.5\text{V/V}$$

$$\tau_H = \left(C_L + C_{db} + C_{gd}\right)R'_L = 200\text{ps}$$

$$f_H = \frac{1}{2\pi\tau_H} = 796\text{MHz}$$

$$f_t = |A_v|\, f_H = 9.95\text{GHz}$$

Cascode Amp

$$R_L = R_o = 540\text{k}\Omega$$

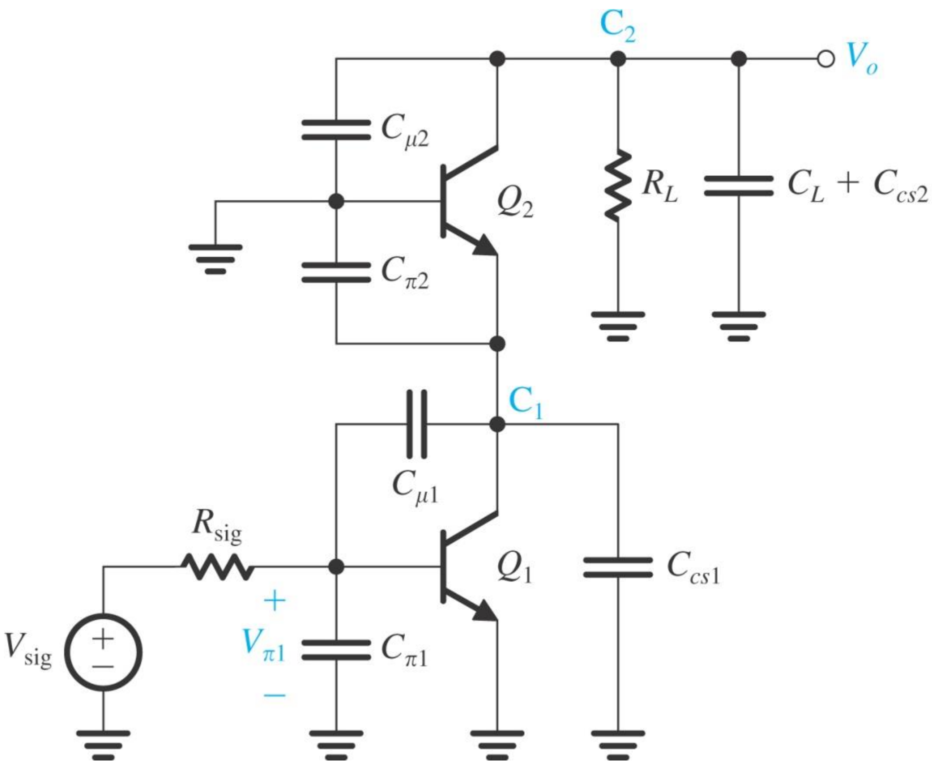
$$A_v = -g_m\left(R_o \,||\, R_L\right) = -1.25\text{mA/V} \times 270\text{k}\Omega = -337.5\text{V/V}$$

$$R_{in2} = \frac{r_o + R_L}{g_m r_o} = 22.4\text{k}\Omega \qquad R_{d1} = r_{o1} \,||\, R_{in2} = 10.6\text{k}\Omega$$

$$\tau_H = R_{d1}\left(C_{gd1} + C_{db1} + C_{gs2}\right) + \left(R_L \,||\, R_o\right)\left(C_L + C_{gd2}\right) = 5718\text{ps}$$

$$f_H = \frac{1}{2\pi\tau_H} = 27.8\text{MHz} \qquad f_t = |A_v|\, f_H = 9.39\text{GHz}$$

High-Frequency Response of a BJT Cascode Amplifier



$$R'_{sig} = r_{\pi 1} \| (r_{x1} + R_{sig})$$

$$R_{\pi 1} = R'_{sig}$$

$$R_{\mu 1} = R'_{sig}(1 + g_{m1}R_{c1}) + R_{c1}$$

$$R_{c1} = r_{o1} \left\| \left[r_{e2}\left(\frac{r_{o2} + R_L}{r_{o2} + R_L/(\beta_2 + 1)} \right) \right] \right.$$

$$\tau_H = C_{\pi 1}R_{\pi 1} + C_{\mu 1}R_{\mu 1} + (C_{cs1} + C_{\pi 2})R_{c1} + (C_L + C_{cs2} + C_{\mu 2})(R_L \| R_o)$$

$$R_o \simeq \beta_2 r_{o2}$$

$$f_H \simeq \frac{1}{2\pi\tau_H}$$

$$A_M = -\frac{r_\pi}{r_\pi + r_x + R_{sig}} g_m(\beta r_o \| R_L)$$

Figure 10.30 Determining the frequency response of the BJT cascode amplifier. Note that in addition to the BJT capacitances C_π and C_μ, the capacitance between the collector and the substrate C_{cs} for each transistor are included.

10.6 HIGH-FREQUENCY RESPONSE OF THE SOURCE AND EMITTER FOLLOWERS

High Frequency Response of the Common Drain/Source Follower Amp

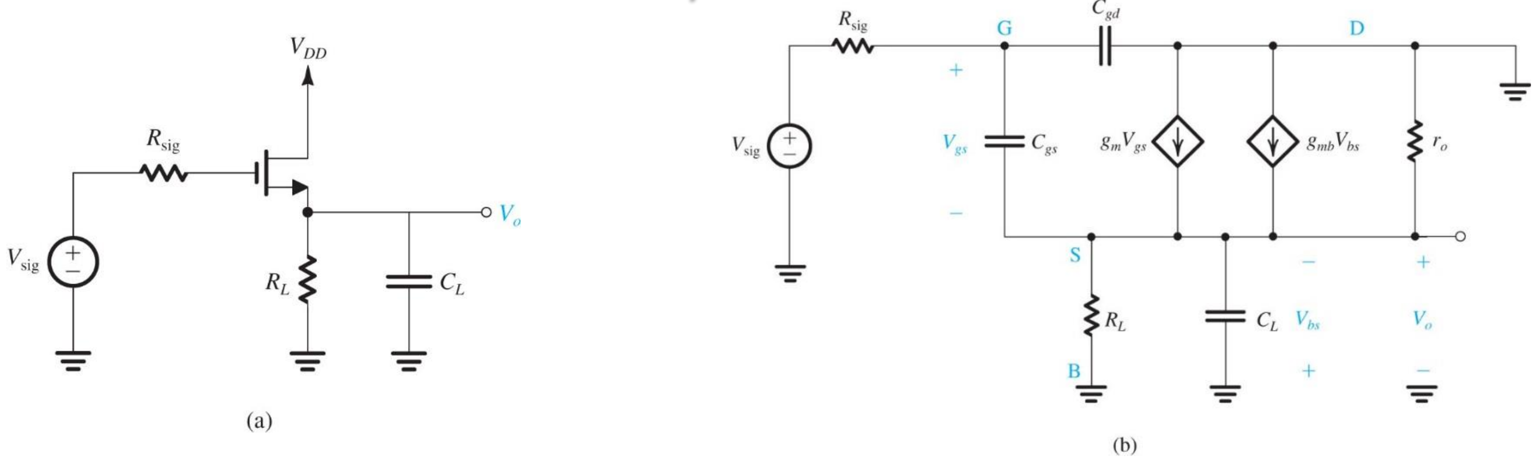


Figure 10.31 (**a**) A directly coupled source follower without the bias detail; (**b**) high-frequency equivalent circuit of the source follower;

Figure 10.31(a) shows a source follower without the biasing arrangement. The follower is driven by a signal source (V_{sig}, R_{sig}) and is loaded with a resistance R_L and, for generality, a capacitance C_L. Replacing the MOSFET with its hybrid-π equivalent-circuit model results in The equivalent circuit shown in Fig. 10.31(b). Here, we have included the body-effect generator $g_{mb}V_{bs}$ because it plays an important role in determining the source-follower gain. Also, we are assuming that whatever capacitances exist between the MOSFET source and ground, such as C_{sb} of Fig. 10.12(a), have been lumped into C_L.

High Frequency Response of the Common Drain/Source Follower Amp

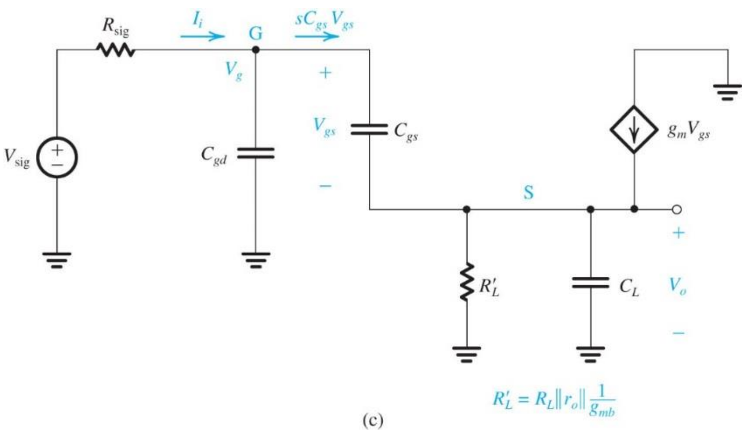


Figure 10.31 (**c**) a simplified version of the equivalent circuit.

Noting that the drain terminal is grounded, we see that C_{gd} in fact appears across the input terminals of the source follower. Also, r_o is in parallel with R_L and can be combined with it. Finally, we observe that since the body terminal B is connected to ground, the voltage V_{bs} appears across the controlled source $g_{mb}V_{bs}$. Thus we can utilize the source-absorption theorem to replace the controlled source with a resistance $1/g_{mb}$. Since the latter appears between source and ground, it is in parallel with R_L and can be combined with it.

Derived in Sedra (7th edition) on pages 762 - 763

$$R'_L = R_L \,||\, r_o \,||\, \frac{1}{g_{mb}}$$

$$\frac{V_o(s)}{V_{sig}(s)} = A_M \frac{1+\left(\frac{s}{\omega_Z}\right)}{1+b_1 s+b_2 s^2}$$

$$A_M = \frac{R'_L}{R'_L+\frac{1}{g_m}} = \frac{g_m R'_L}{g_m R'_L+1} \qquad \omega_Z = \frac{g_m}{C_{gs}}$$

$$b_1 = \left(C_{gd}+\frac{C_{gs}}{g_m R'_L+1}\right)R_{sig}+\left(\frac{C_{gs}+C_L}{g_m R'_L+1}\right)R'_L \qquad b_2 = \frac{\left(C_{gs}+C_{gd}\right)C_L+C_{gs}C_{gd}}{g_m R'_L+1}R_{sig}R'_L$$

High Frequency Response of the Common Drain/Source Follower Amp

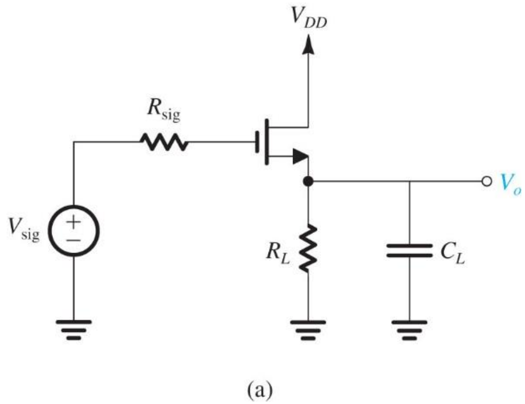


Figure 10.31 (**a**) A directly coupled source follower without the bias detail;

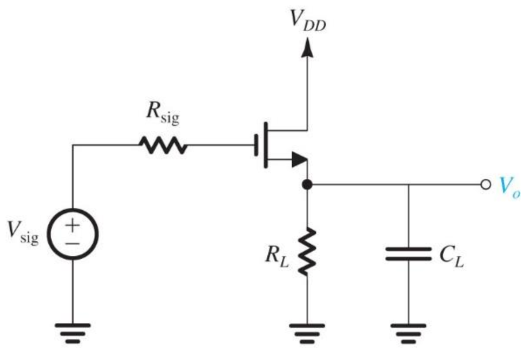
$$\frac{V_o(s)}{V_{sig}(s)} = A_M \frac{1+\left(\frac{s}{\omega_Z}\right)}{1+b_1 s+b_2 s^2}$$

1. Since the source follower in Fig. 10.31(a) is directly coupled, the gain at dc is equal to A_M.
2. Although the equivalent circuit of Fig. 10.31(c) has three capacitors, the transfer function is of second order. This is because the three capacitors form a continuous loop.
3. The two transmission zeros can be found as the values of s for which $V_o/V_{sig} = 0$. We see that V_o/V_{sig} approaches 0 as s approaches ∞. Thus one transmission zero is at s = ∞. Physically, this zero is a result of C_{gd}, which appear s across the input terminals, becoming a short circuit at infinite frequency and thus making $V_o = 0$. From the numerator we see that the other transmission zero is at s = $-\omega_Z$. We note that ω_Z is slightly higher than the unity-gain frequency ω_T of the MOSFET,

$$\omega_Z = \frac{g_m}{C_{gs}} \qquad \omega_T = \frac{g_m}{C_{gs}+C_{gd}}$$

Thus the finite transmission zero is at such a high frequency that its effect on the frequency response of the follower is negligibly small.

High Frequency Response of the Common Drain/Source Follower Amp



(a)

Figure 10.31 (**a**) A directly coupled source follower without the bias detail;

$$\frac{V_o(s)}{V_{sig}(s)} = A_M \frac{1+\left(\frac{s}{\omega_Z}\right)}{1+b_1 s+b_2 s^2}$$

4. The two poles of the source follower can be found as the roots of the denominator polynomial ($1 + b_1 s + b_2 s^2$). If the poles are real, their frequencies, say ω_{P1} and ω_{P2}, can be found from

$$1+b_1 s+b_2 s^2 = \left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)$$

Now if $\omega_{P2} \gg \omega_{P1}$ (at least four times larger), a dominant pole exists with frequency ω_{P1} and the 3-dB frequency f_H is given by

$$f_H \simeq f_{P1} \simeq \frac{1}{2\pi b_1}$$

Where b_1 is also τ_H, the effective high-frequency time constant evaluated in the method of open-circuit time constants.

$$b_1 = \left(C_{gd} + \frac{C_{gs}}{g_m R_L' + 1}\right) R_{sig} + \left(\frac{C_{gs} + C_L}{g_m R_L' + 1}\right) R_L'$$

High Frequency Response of the Common Drain/Source Follower Amp

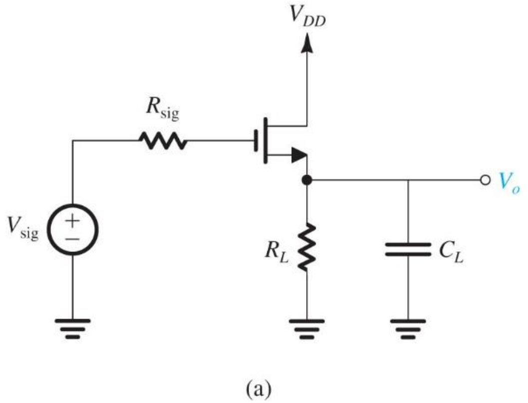


Figure 10.31 (**a**) A directly coupled source follower without the bias detail;

$$\frac{V_o(s)}{V_{sig}(s)} = A_M \frac{1+\left(\dfrac{s}{\omega_Z}\right)}{1+b_1 s+b_2 s^2}$$

5. If the poles are real but none is dominant, the 3-dB frequency can be determined analytically from the transfer function as the frequency at which $|V_o/V_{sig}| = A_M/\sqrt{2}$. An approximate value can be obtained using the formula

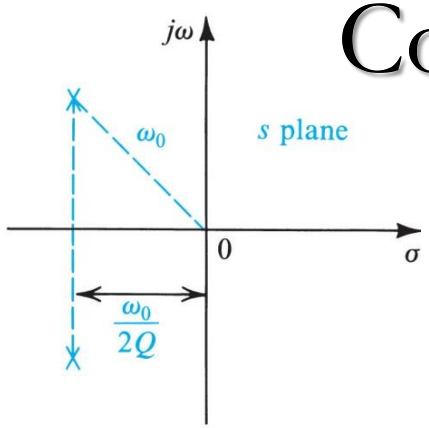
$$f_H \simeq \frac{1}{\sqrt{\dfrac{1}{f_{P1}^2}+\dfrac{1}{f_{P2}^2}-\dfrac{1}{f_Z^2}}}$$

6. If the poles are complex, they are best described in terms of their frequency ω_0 and Q-factor, where

$$1+b_1 s+b_2 s^2 = 1+\frac{1}{Q}\frac{s}{\omega_0}+\frac{s^2}{\omega_0^2}$$

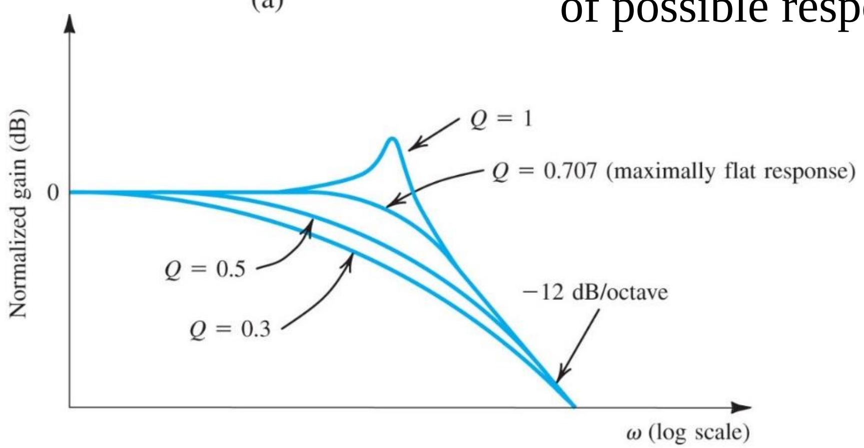
and for complex poles, $Q > 0.5$.

Complex Conjugate Poles



(a)

Figure 10.32(a) provides a geometrical interpretation of ω_0 and Q. From the study of second-order network responses in Chapter 17, it will be seen that the response of the source follower shows no peaking for $Q \le 0.707$. The boundary case corresponding to Q = 0.707 (poles at 45◦ angles) results in what is known as a maximally flat response for which $f_{3dB} = f_0$. Figure 10.32(b) shows a number of possible responses obtained for various values of Q.



(b)

In terms of the component values of the source follower,

$$\omega_0 = \frac{1}{\sqrt{b_2}} = \sqrt{\frac{g_m R_L' + 1}{R_{sig} R_L' \left[\left(C_{gs} + C_{gd} \right) C_L + C_{gs} C_{gd} \right]}}$$

$$Q = \frac{\sqrt{b_2}}{b_1}$$

$$= \frac{\sqrt{g_m R_L' + 1} \sqrt{\left[\left(C_{gs} + C_{gd} \right) C_L + C_{gs} C_{gd} \right] R_{sig} R_L'}}{\left[C_{gs} + C_{gd} \left(g_m R_L' + 1 \right) \right] R_{sig} + \left(C_{gs} + C_L \right) R_L'}$$

Figure 10.32 (**a**) A pair of complex-conjugate poles with the definition of ω_o and Q indicated. (**b**) Magnitude response of a source (or emitter) follower for different values of the parameter Q. Note that the response is normalized relative to A_M.

Example 10.11a

A source follower operated at g_m = 2 mA/V and r_o = 20 kΩ is fed with a signal source for which R_{sig} = 10 kΩ and is loaded in a resistance R_L = 20 kΩ. The MOSFET has C_{gs} = 20 fF, C_{gd} = 5 fF, and $g_{mb} = \chi g_m$ where χ = 0.2, and the total capacitance at the output C_L = 15 fF. Determine A_M, f_T, f_Z, Q, f_{P1}, f_{P2}, and f_{3dB}.

$$R_L' = R_L \,||\, r_o \,||\, \frac{1}{g_{mb}} = 20\text{k}\Omega \,||\, 20\text{k}\Omega \,||\, \frac{1}{(0.2\times 2\,\text{mA/V})} = 2\text{k}\Omega$$

$$A_M = \frac{g_m R_L'}{g_m R_L' + 1} = \frac{(2\,\text{mA/V})2\text{k}\Omega}{(2\,\text{mA/V})2\text{k}\Omega + 1} = \frac{4}{5} = 0.8\frac{\text{V}}{\text{V}}$$

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})} = \frac{2\,\text{mA/V}}{(2\pi)25\text{fF}} = 12.7\text{GHz}$$

$$f_Z = \frac{g_m}{2\pi C_{gs}} = \frac{2\,\text{mA/V}}{(2\pi)20\text{fF}} = 15.9\text{GHz}$$

$$Q = \frac{\sqrt{g_m R_L' + 1}\sqrt{\left[(C_{gs}+C_{gd})C_L + C_{gs}C_{gd}\right]R_{sig}R_L'}}{\left[C_{gs} + C_{gd}(g_m R_L' + 1)\right]R_{sig} + (C_{gs}+C_L)R_L'} = 0.42$$

Example 10.11b

A source follower operated at g_m = 2 mA/V and r_o = 20 kΩ is fed with a signal source for which R_{sig} = 10 kΩ and is loaded in a resistance R_L = 20 kΩ. The MOSFET has C_{gs} = 20 fF, C_{gd} = 5 fF, and $g_{mb} = \chi g_m$ where χ = 0.2, and the total capacitance at the output C_L = 15 fF. Determine A_M, f_T, f_Z, Q, f_{P1}, f_{P2}, and $f_{3\text{dB}}$.

$$\tau_H = b_1 = \left(C_{gd} + \frac{C_{gs}}{g_m R_L' + 1}\right) R_{sig} + \left(\frac{C_{gs} + C_L}{g_m R_L' + 1}\right) R_L' = 104\text{ps}$$

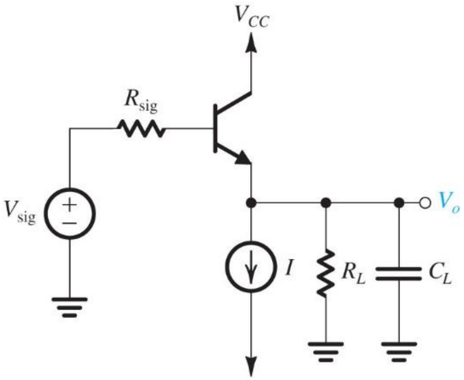
$$b_2 = \frac{\left(C_{gs} + C_{gd}\right) C_L + C_{gs} C_{gd}}{g_m R_L' + 1} R_{sig} R_L' = 1.9 \times 10^{-21}\text{s}$$

$$f_{P1} = \left(\frac{1}{2\pi}\right) \frac{-b_1 + \sqrt{b_1^2 - 4b_2}}{2b_2} = 1.98\text{GHz} \qquad f_{P2} = \left(\frac{1}{2\pi}\right) \frac{-b_1 - \sqrt{b_1^2 - 4b_2}}{2b_2} = 6.73\text{GHz}$$

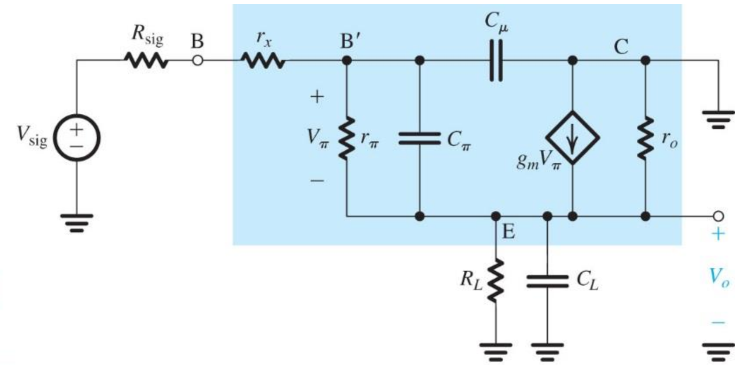
Since f_{P2}/f_{P1} = 3.4 < 4, no dominant pole exists. An approximate value for f_H can be obtained as

$$f_{3dB} = f_H = \frac{1}{\sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2} - \frac{2}{f_Z^2}}} = 1.93\text{GHz}$$

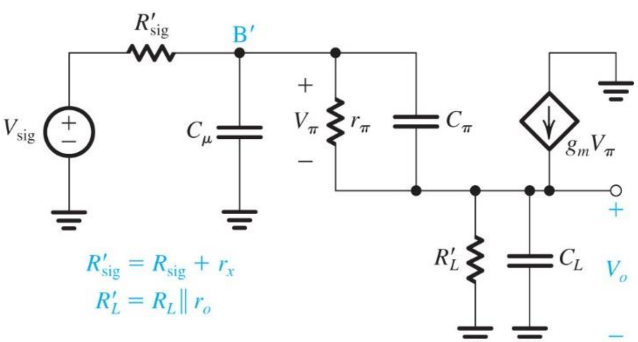
High-Frequency Response of the Emitter Follower



(a)



(b)



(c)

$$\frac{V_o(s)}{V_{sig}(s)} = A_M \frac{1 + (s/\omega_Z)}{1 + b_1 s + b_2 s^2}$$

$$A_M = \frac{R'_L}{R'_L + r_e + R'_{sig}/(\beta +}$$

$$f_Z = 1/2\pi C_\pi r_e$$

$$b_1 = \frac{\left[C_\pi + C_\mu\left(1 + \frac{R'_L}{r_e}\right)\right]R'_{sig} + \left[C_\pi + C_L\left(1 + \frac{R'_{sig}}{r_\pi}\right)\right]R'_L}{1 + \frac{R'_L}{r_e} + \frac{R'_{sig}}{r_\pi}}$$

$$b_2 = \frac{[(C_\pi + C_\mu)C_L + C_\pi C_\mu]R'_L R'_{sig}}{1 + \frac{R'_L}{r_e} + \frac{R'_{sig}}{r_\pi}}$$

(d)

Figure 10.33 **(a)** Emitter follower. **(b)** High-frequency equivalent circuit. **(c)** Simplified equivalent circuit. **(d)** Transfer function.

Homework #9

Read Chapter 10

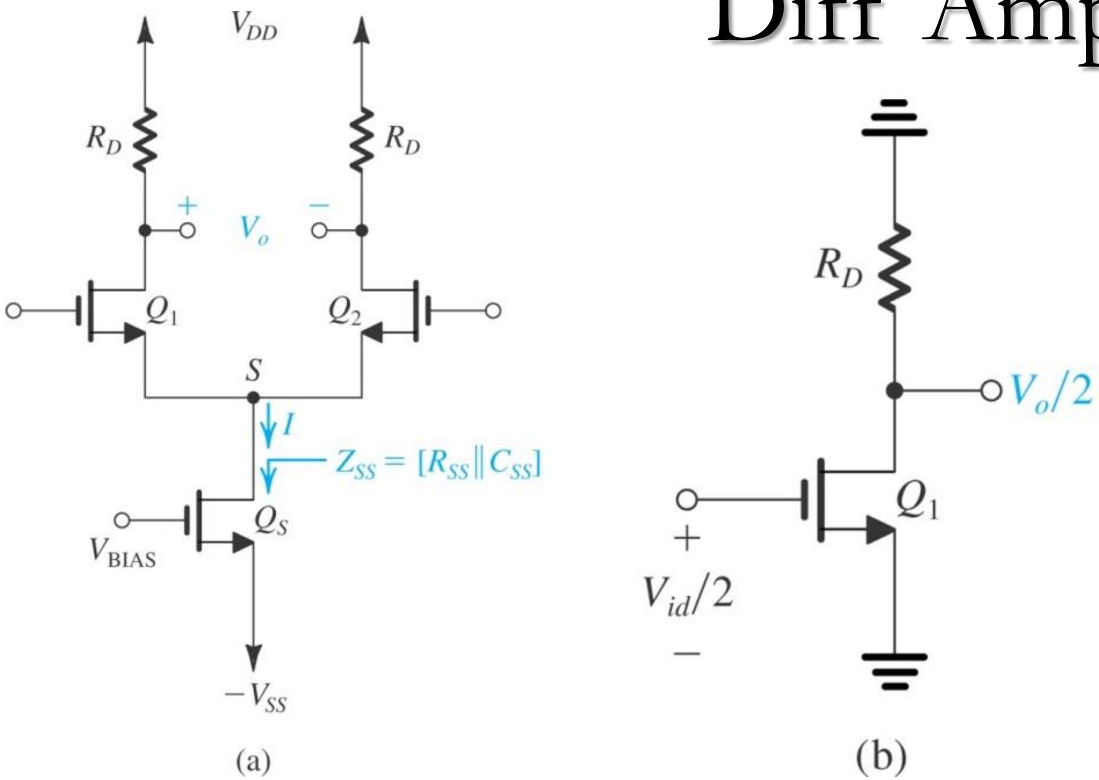
Chapter 10 Problems:

1) 10.52
2) 10.55*
3) 10.59*
4) 10.63*
5) 10.66*
6) 10.79*
7) 10.84*

* Answers in Appendix L

10.7 HIGH-FREQUENCY RESPONSE OF DIFFERENTIAL AMPLIFIERS

Analysis of Resistively Loaded MOS Diff Amp



The differential half-circuit shown in Fig. 10.34(b) can be used to determine the frequency dependence of the differential gain V_o/V_{id}. Indeed the gain function $A_d(s)$ of the differential amplifier will be identical to the transfer function of this CS amplifier.

Figure 10.34 (a) A resistively loaded MOS differential pair; the transistor supplying the bias current is explicitly shown. It is assumed that the total impedance between node *S* and ground, *ZSS*, consists of a resistance R_{SS} in parallel with a capacitance C_{SS}. **(b)** Differential half-circuit.

Analysis of Resistively Load MOS Diff Amp

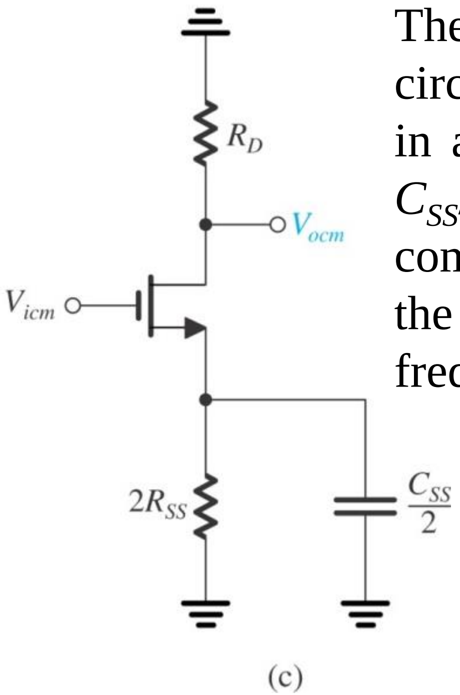


Figure 10.34 (c) Common-mode half-circuit.

The common-mode half-circuit is shown in Fig. 10.34(c). Although this circuit has other capacitances, namely C_{gs}, C_{gd}, and C_{db} of the transistor in addition to other stray capacitances, we have chosen to show only $C_{SS}/2$. This is because together with ($2R_{SS}$) forms a real-axis zero in the common-mode gain function at a frequency much lower than those of the other poles and zeros of the circuit. This zero then dominates the frequency dependence of A_{cm} and CMRR.

$$A_{CM} = -\left(\frac{R_D}{2R_{SS}}\right)\frac{\Delta R_D}{R_D}$$

$$A_{CM} = -\frac{R_D}{2Z_{SS}}\left(\frac{\Delta R_D}{R_D}\right) = -\frac{1}{2}R_D\left(\frac{\Delta R_D}{R_D}\right)Y_{SS}$$

$$= -\frac{1}{2}R_D\left(\frac{\Delta R_D}{R_D}\right)\left(\frac{1}{R_{SS}} + sC_{SS}\right) = -\frac{R_D}{2R_{SS}}\left(\frac{\Delta R_D}{R_D}\right)(1 + sC_{SS}R_{SS})$$

Analysis of Resistively Load MOS Diff Amp

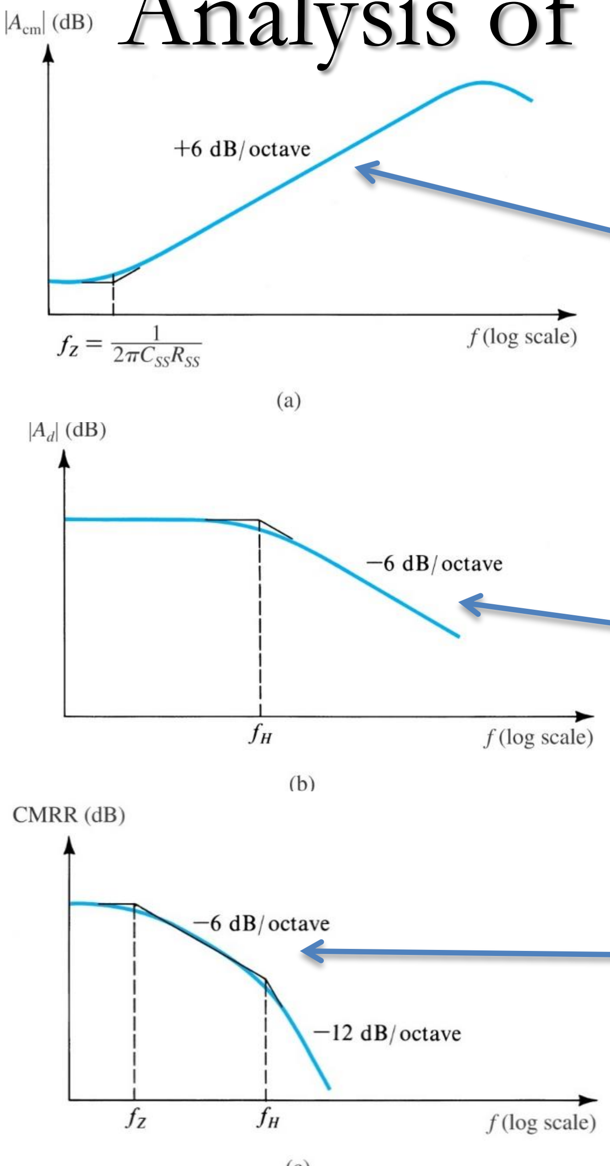


Figure 10.35 Variation of **(b)** differential gain, and **(c)** common-mode rejection ratio with frequency.

usually f_Z is much lower than the frequencies of the other poles and zeros. As a result, the common-mode gain increases at the rate of +6 dB/octave (20 dB/ decade) starting at a relatively low frequency.

$$A_{CM} = -\frac{R_D}{2R_{SS}}\left(\frac{\Delta R_D}{R_D}\right)(1 + sC_{SS}R_{SS}) \qquad f_Z = \frac{\omega_Z}{2\pi} = \frac{1}{2\pi C_{SS}R_{SS}}$$

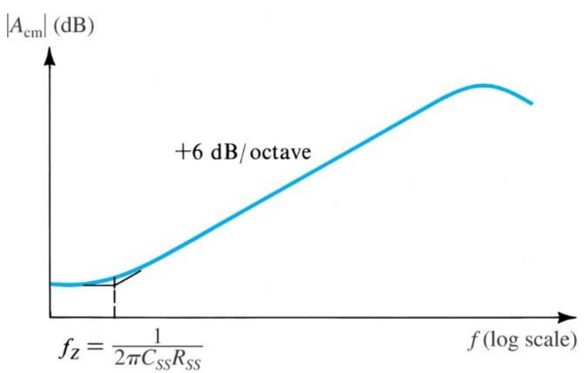
A_{cm} drops off at high frequencies because of the other poles of the common- mode half-circuit. It is, however, f_Z that is significant, for it is the frequency at which the CMRR of the differential amplifier begins to decrease.

Note that if both A_d and A_{cm} are expressed and plotted in dB, then CMRR in dB is simply the difference between A_d and A_{cm}.

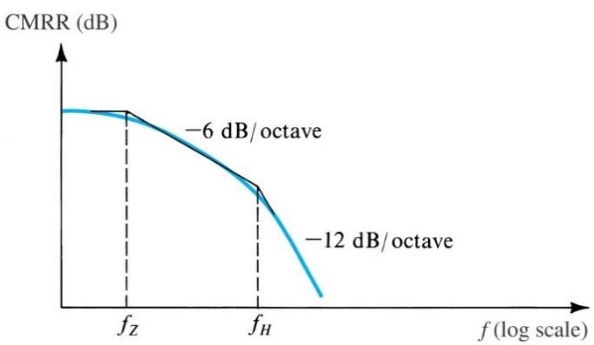
Low V_{OV}/CMRR Tradeoff

an important trade-off exists in the design of the current-source transistor Q_S: In order to operate this current source with a small V_{DS} (to conserve the already low V_{DD}), we desire to operate the transistor at a low over-drive voltage V_{OV}. For a given value of the current I, however, this means using a large W/L ratio (i.e., a wide transistor). This in turn increases C_{SS} and hence lowers f_Z with the result that the CMRR deteriorates (i.e., decreases) at a relatively low frequency.

$$V_{OV} = \sqrt{\frac{2I_D}{k_n'\left(\frac{W}{L}\right)}}$$



(a)



(c)

High Frequency Noise Suppression

To appreciate the need for high CMRR at higher frequencies, consider two stages of a differential amplifier whose power- supply voltage V_{DD} is corrupted with high- frequency noise. The quiescent voltage at each of the drains of Q_1 and Q_2 (v_{D1} and v_{D2}) will have the same high-frequency noise as V_{DD}. This high- frequency noise then constitutes a common-mode input signal to the second differential stage, formed by Q_3 and Q_4.

If the second differential stage is perfectly matched, its differential output voltage V_o should be free of high-frequency noise. However, in practice there is no such thing as perfect matching, and the second stage will have a finite common-mode gain. Furthermore, because of the zero formed by R_{SS} and C_{SS} of the second stage, the common-mode gain will increase with frequency, causing some of the noise to make its way to V_o.

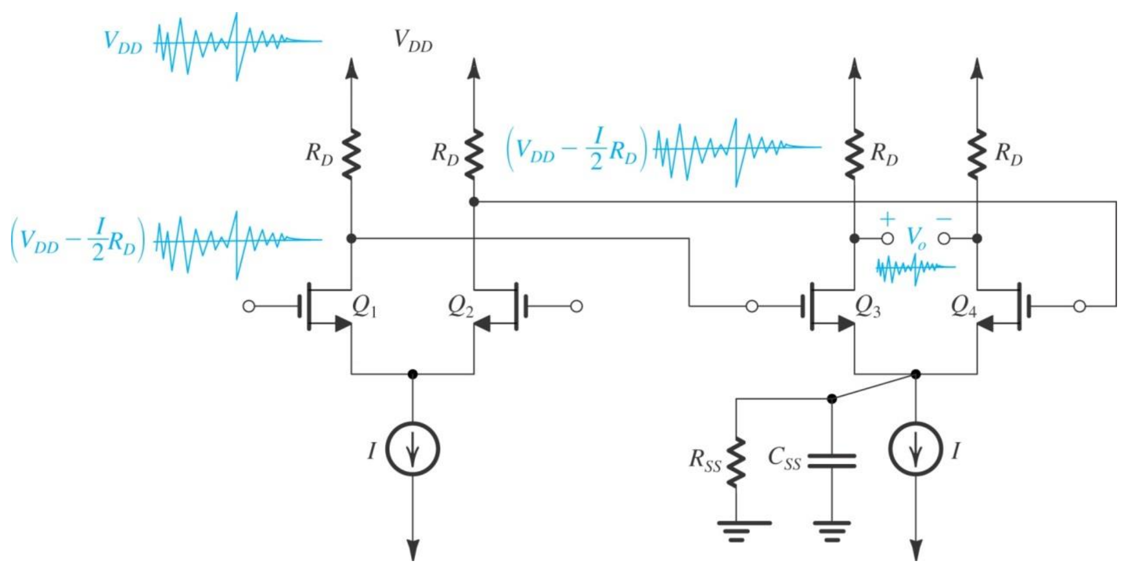


Figure 10.36 The second stage in a differential amplifier, which is relied on to suppress high-frequency noise injected by the power supply of the first stage, and therefore must maintain a high CMRR at higher frequencies.

Active-Loaded MOS Amplifier

We next consider the frequency response of the current-mirror-loaded MOS differential-pair circuit. The circuit is shown in Fig. 10.37(a) with two capacitances indicated: C_m, which is the total capacitance at the input node of the current mirror, and C_L, which is the total capacitance at the output node. Capacitance C_m is mainly formed by C_{gs3} and C_{gs4} but also includes C_{gd1}, C_{db1}, and C_{db3}:

$$C_m = C_{gd1} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4}$$

$$C_L = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x$$

Where capacitance C_x is the load capacitance or the input capacitance of the next stage.

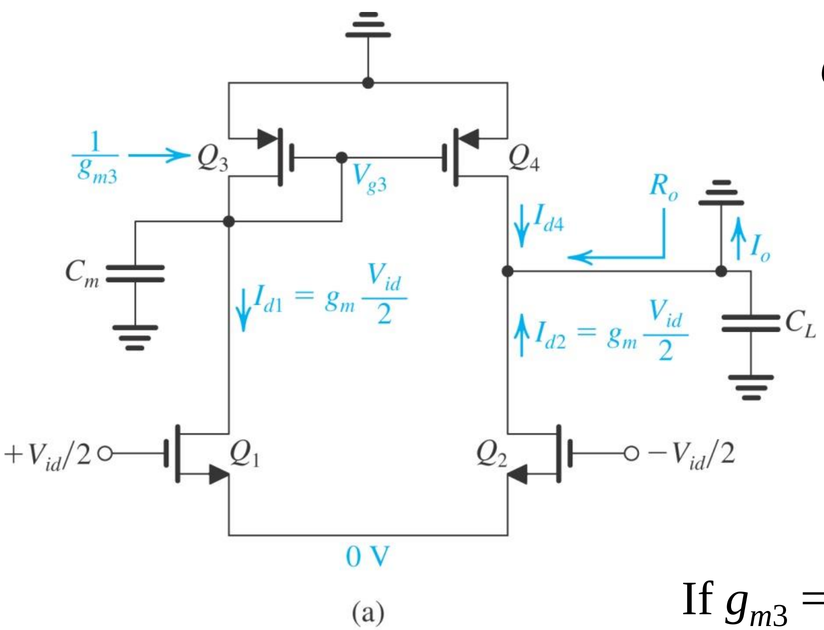


Figure 10.37 (a) Frequency–response analysis of the active-loaded MOS differential amplifier.

$$V_{g3} = -\frac{g_m V_{id}/2}{g_{m3} + sC_m}$$

$$I_{d4} = -g_{m4} V_{g3} = \frac{g_{m4} g_m V_{id}/2}{g_{m3} + sC_m}$$

If $g_{m3} = g_{m4}$ then:

$$I_{d4} = \frac{g_m V_{id}/2}{1 + s\dfrac{C_m}{g_{m3}}}$$

Active-Loaded MOS Amplifier

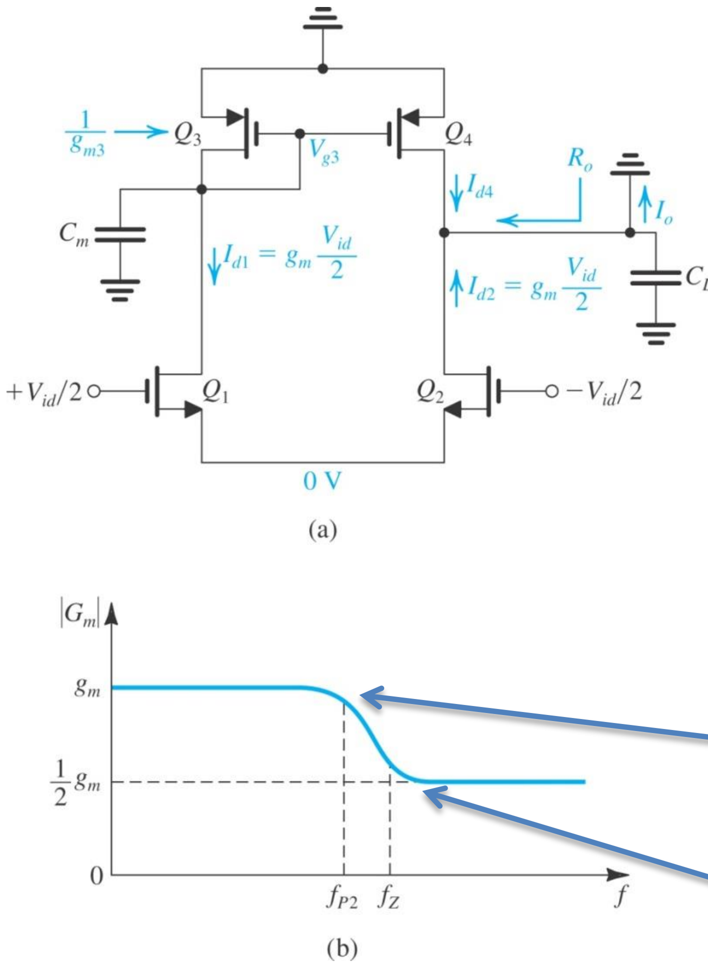


Figure 10.37 (a) Frequency–response analysis of the active-loaded MOS differential amplifier. **(b)** The overall transconductance G_m as a function of frequency.

$$I_o = I_{d4} + I_{d2} = g_{m4} V_{g3} = \frac{g_m V_{id}/2}{1 + s\dfrac{C_m}{g_{m3}}} + g_m V_{id}/2$$

$$G_m \equiv \frac{I_o}{V_{id}} = g_m \frac{1 + s\dfrac{C_m}{2g_{m3}}}{1 + s\dfrac{C_m}{g_{m3}}}$$

Thus, as expected, the low-frequency value of G_m is equal to g_m of Q_1 and Q_2. At high frequencies, G_m acquires a pole and a zero, the frequencies of which are

$$f_{P2} = \frac{g_{m3}}{2\pi C_m} \simeq \frac{g_{m3}}{2\pi\left(2C_{gs}\right)} \simeq \frac{f_T}{2}$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} \simeq f_T$$

Active-Loaded MOS Amplifier

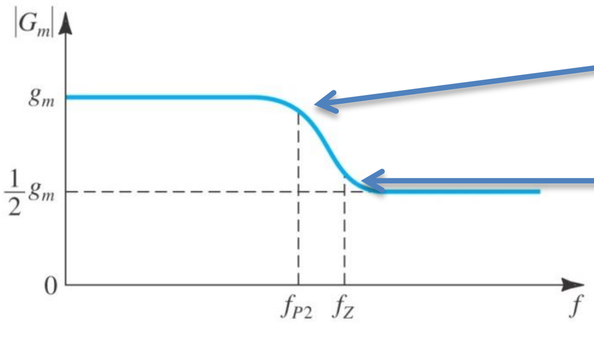


Figure 10.37 (b) The overall transconductance G_m as a function of frequency.

$$f_{P2} = \frac{g_{m3}}{2\pi C_m} \simeq \frac{g_{m3}}{2\pi\left(2C_{gs}\right)} \simeq \frac{f_T}{2}$$

$$f_Z = \frac{2g_{m3}}{2\pi C_m} \simeq f_T$$

The mirror pole and zero occur at very high frequencies. Nevertheless, their effect can be significant.

$$R_o = r_{o2} \,||\, r_{o4}$$

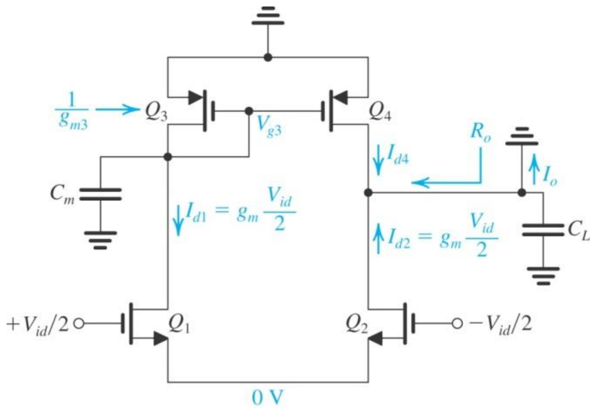
$$V_o = I_o \frac{1}{\frac{1}{R_o} + sC_L} = G_m V_{id} \frac{R_o}{1 + sC_L R_o}$$

$$\frac{V_o}{V_{id}} = G_m \frac{R_o}{1 + sC_L R_o} = \left(g_m R_o\right) \left[\frac{1 + s\frac{C_m}{2g_{m3}}}{1 + s\frac{C_m}{g_{m3}}}\right] \left(\frac{1}{1 + sC_L R_o}\right)$$

$$f_{P1} = \frac{1}{2\pi C_L R_o}$$

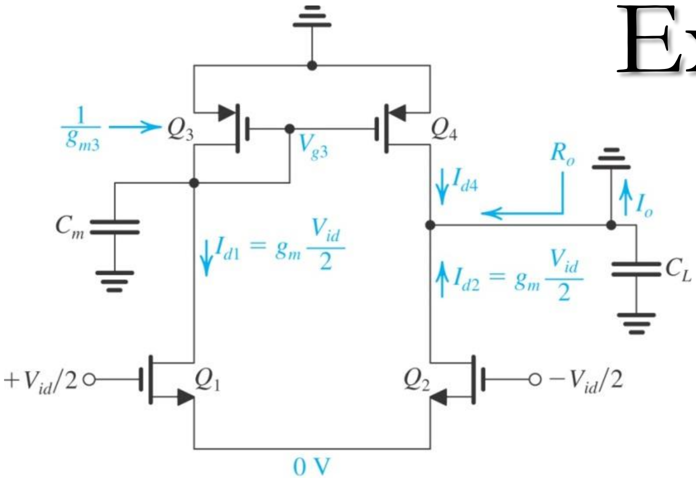
Example 10.12a

Consider an active-loaded MOS differential amplifier of the type shown in Fig. 10.37(a). Assume that for all transistors, W/L = 7.2 µm/ 0.36 µm, C_{gs} = 20 fF, C_{gd} = 5 fF, and C_{db} = 5 fF. Also, let $\mu_n C_{ox}$ = 387 µA/V^2, $\mu_p C_{ox}$ = 86 µA/V^2, V'_{An} = 5 V/µm, $|V'_{Ap}|$ =6 V/µm. The bias current I = 0.2 mA, and the bias current source has an output resistance R_{SS} = 25 kΩ and an output capacitance C_{SS} = 0.2 pF. In addition to the capacitances introduced by the transistors at the output node, there is a capacitance C_x of 25 fF. It is required to determine the low-frequency values of A_d, A_{cm}, and CMRR. It is also required to find the poles and zero of A_d and the dominant pole of CMRR.



(a)

Example 10.12b



(a)

$W = 7.2$ µm

$L = 0.36$ µm

$C_{gs} = 20$ fF

$C_{gd} = 5$ fF

$C_{db} = 5$ fF

$\mu_n C_{ox} = 387$ µA/V²

$\mu_p C_{ox} = 86$ µA/V²

$V'_{An} = 5$ V/µm

$|V'_{Ap}| = 6$ V/µm

$I = 0.2$ mA

$R_{SS} = 25$ kΩ

$C_{SS} = 0.2$ pF

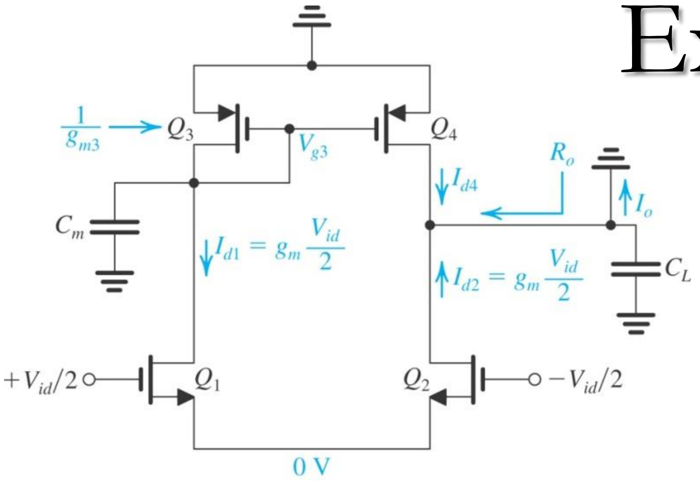
C_x of 25 fF

$$V_{OV1,2} = \sqrt{2I_D / \left[\mu_n C_{ox}\left(W/L\right)\right]} = \sqrt{\frac{0.2\text{mA}}{0.387\frac{\text{mA}}{\text{V}^2}\left(\frac{7.2\mu\text{m}}{0.36\mu\text{m}}\right)}} = 0.161\text{V}$$

$$g_m = g_{m1} = g_{m2} = \frac{2I_D}{V_{OV1}} = \frac{200\mu\text{A}}{0.161\text{V}} = 1.244\text{mA/V}$$

$$r_{o1} = r_{o2} = \frac{V'_{An}L}{I_D} = \frac{5\text{V}/\mu\text{m} \times 0.36\mu\text{m}}{0.1\text{mA}} = 18\text{k}\Omega$$

Example 10.12c



(a)

$W = 7.2\ \mu\text{m}$

$L = 0.36\ \mu\text{m}$

$C_{gs} = 20$ fF

$C_{gd} = 5$ fF

$C_{db} = 5$ fF

$\mu_n C_{ox} = 387\ \mu\text{A/V}^2$

$\mu_p C_{ox} = 86\ \mu\text{A/V}^2$

$V'_{An} = 5$ V/µm

$|V'_{Ap}| = 6$ V/µm

$I = 0.2$ mA

$R_{SS} = 25$ kΩ

$C_{SS} = 0.2$ pF

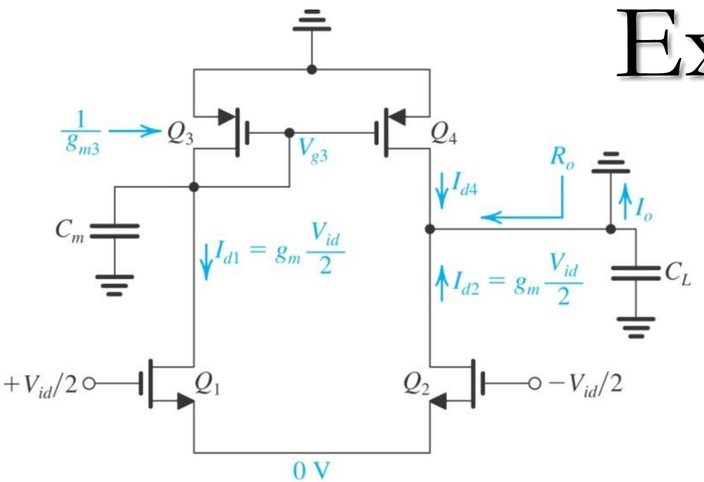
C_x of 25 fF

$$V_{OV3,4} = \sqrt{2I_D / \left[\mu_p C_{ox}(W/L)\right]} = \sqrt{\frac{0.2\text{mA}}{0.086\frac{\text{mA}}{\text{V}^2}\left(\frac{7.2\mu\text{m}}{0.36\mu\text{m}}\right)}} = 0.341\text{V}$$

$$g_{m3} = g_{m4} = \frac{2I_D}{V_{OV3}} = \frac{200\mu\text{A}}{0.341\text{V}} = 0.587\text{mA/V}$$

$$r_{o3} = r_{o4} = \frac{V'_{Ap} L}{I_D} = \frac{6\text{V}/\mu\text{m} \times 0.36\mu\text{m}}{0.1\text{mA}} = 21.6\text{k}\Omega$$

Example 10.12d



(a)

$W = 7.2\ \mu\text{m}$

$L = 0.36\ \mu\text{m}$

$C_{gs} = 20\text{ fF}$

$C_{gd} = 5\text{ fF}$

$C_{db} = 5\text{ fF}$

$\mu_n C_{ox} = 387\ \mu\text{A/V}^2$

$\mu_p C_{ox} = 86\ \mu\text{A/V}^2$

$V'_{An} = 5\text{ V}/\mu\text{m}$

$|V'_{Ap}| = 6\text{ V}/\mu\text{m}$

$I = 0.2\text{ mA}$

$R_{SS} = 25\text{ k}\Omega$

$C_{SS} = 0.2\text{ pF}$

C_x of 25 fF

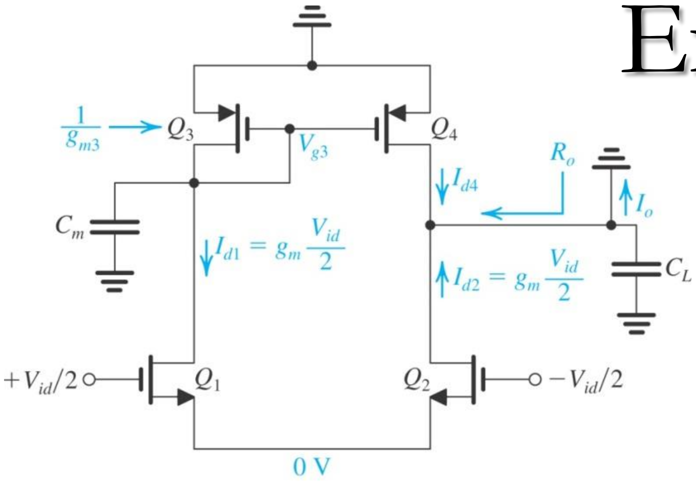
$$R_o = r_{o2} \,||\, r_{o4} = 18\text{k}\Omega \,||\, 21.6\text{k}\Omega = 9.818\text{k}\Omega$$

$$A_d = g_m R_o = 1.244\text{mA/V} \times 9.818\text{k}\Omega = 12.216\text{V/V}$$

$$A_{CM} = -\frac{1}{2g_{m3}R_{SS}} = -\frac{1}{2 \times 0.5\text{mA/V} \times 10\text{k}\Omega} = -0.034\text{V/V}$$

$$\text{CMMR} = 20\log\left(\frac{|A_d|}{|A_{cm}|}\right) = 20\log\left(\frac{|12.216|}{|-0.034|}\right) = 51.08\text{dB}$$

Example 10.12e



(a)

$$C_m = C_{gd1} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4} = 55\text{fF}$$

$$C_L = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x = 45\text{fF}$$

$W = 7.2\ \mu\text{m}$
$L = 0.36\ \mu\text{m}$
$C_{gs} = 20$ fF
$C_{gd} = 5$ fF
$C_{db} = 5$ fF
$\mu_n C_{ox} = 387\ \mu\text{A/V}^2$
$\mu_p C_{ox} = 86\ \mu\text{A/V}^2$
$V'_{An} = 5$ V/µm
$|V'_{Ap}| = 6$ V/µm
$I = 0.2$ mA
$R_{SS} = 25$ kΩ
$C_{SS} = 0.2$ pF
C_x of 25 fF

Differential mode poles and zero

$$f_{P1} = \frac{1}{2\pi C_L R_o} = \frac{1}{2\pi(45\text{fF})(9.818\text{k}\Omega)} = 360.2\text{MHz}$$

$$f_{P2} = \frac{g_{m3}}{2\pi C_m} = \frac{0.587\text{mA/V}}{2\pi(55\text{fF})} = 1.697\text{GHz}$$

$$f_Z = 2 f_{P2} = 3.39\text{GHz}$$

Common mode zero

$$f_Z = \frac{\omega_Z}{2\pi} = \frac{1}{2\pi C_{SS} R_{SS}} = \frac{1}{2\pi(0.2\text{pF})25\text{k}\Omega} = 31.83\text{MHz}$$

10.8 OTHER WIDEBAND AMPLIFIER CONFIGURATIONS

Obtaining Wideband Amplification by Source and Emitter Degeneration

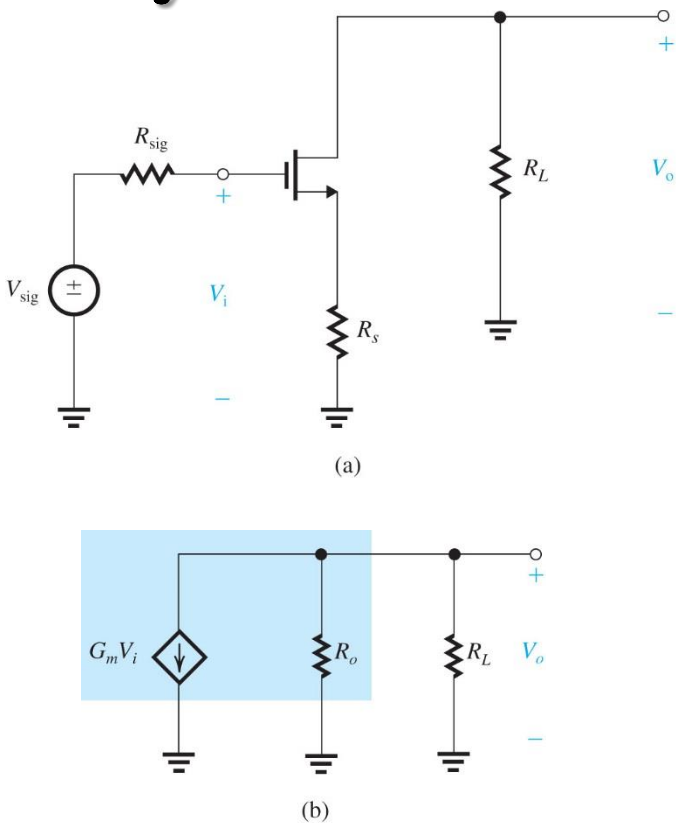


Figure 10.38 (a) The CS amplifier circuit, with a source resistance R_s. **(b)** Equivalent-circuit representation of the amplifier output.

Figure 10.38(a) shows a common-source amplifier with a source-degeneration resistance R_s. As indicated in Fig. 10.38(b), the output of the amplifier can be modeled at low frequencies by a controlled current-source $G_m V_i$ and an output resistance R_o, where the transconductance G_m is given by

$$G_m \simeq \frac{g_m}{1+g_m R_S}$$

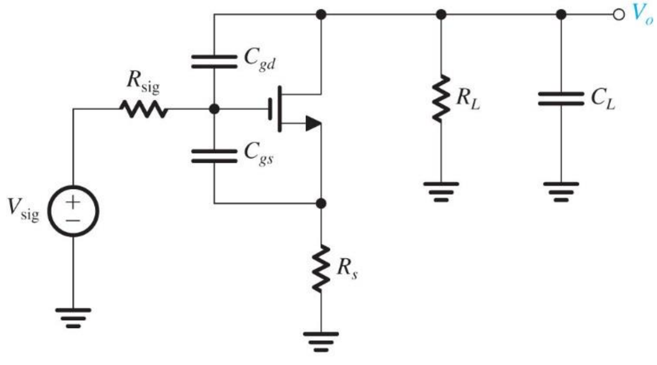
and the output resistance is given by

$$R_o \simeq r_o\left(1+g_m R_S\right)$$

$$A_M = \frac{V_o}{V_{sig}} = -G_m\left(R_o \,||\, R_L\right) = -G_m R_L'$$

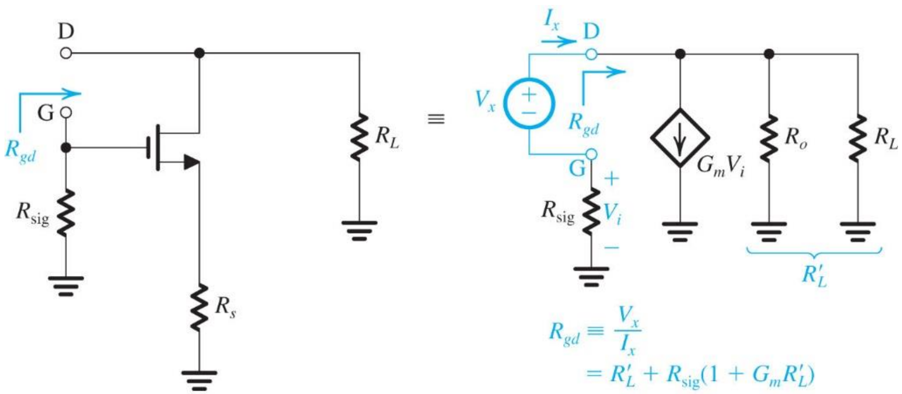
$$R_L' = R_L \,||\, R_o$$

Obtaining Wideband Amplification by Source and Emitter Degeneration



(c)

Let's now consider the high-frequency response of the source-degenerated amplifier. Figure 10.38(c) shows the amplifier, indicating the capacitances C_{gs} and C_{gd}. A capacitance C_L that *includes* the MOSFET capacitance C_{db} is also shown at the output. The method of open circuit time constants can be employed to obtain an estimate of the 3-dB frequency f_H.



(d)

Figure 10.38 (c) The circuit prepared for frequency-response analysis. **(d)** Determining the resistance R_{gd} seen by the capacitance C_{gd}

Toward that end, we show in Fig. 10.38(d) the circuit for determining R_{gd} , which is the resistance seen by C_{gd}. We observe that R_{gd} can be determined to be:

$$R_{gd} \simeq R_{sig}\left(1 + G_m R_L'\right) + R_L'$$

$$R_L' = R_L \,||\, R_o$$

Obtaining Wideband Amplification by Source and Emitter Degeneration

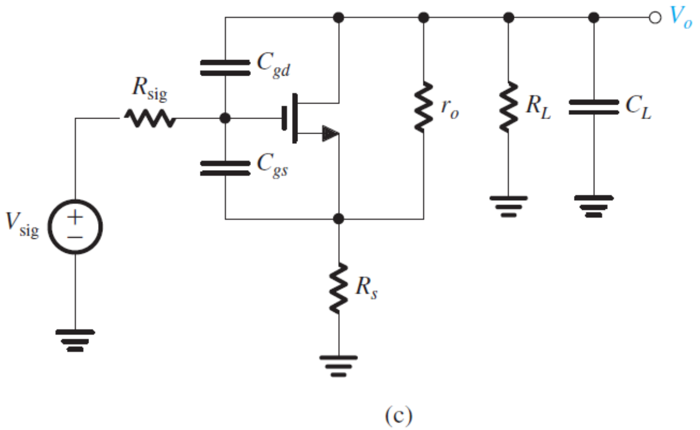


Figure 10.38 (**c**) The circuit prepared for frequency-response analysis.

$$R_{gd} \simeq R_{sig}\left(1+G_m R_L'\right)+R_L'$$

$$R_L' = R_L \,||\, R_o$$

The formula for R_{CL} can be seen to be simply

$$R_{C_L} = R_L \,||\, R_o = R_L'$$

The formula for R_{gs} is the most difficult to derive, and the derivation should be performed with the hybrid-π model explicitly utilized. The result is

$$R_{gs} \simeq \frac{R_{sig}+R_S}{1+g_m R_S\left(\frac{r_0}{r_0+R_L}\right)}$$

$$\tau_H = C_{gs}R_{gs} + C_{gd}R_{dg} + C_L R_{C_L}$$

Obtaining Wideband Amplification by Source and Emitter Degeneration

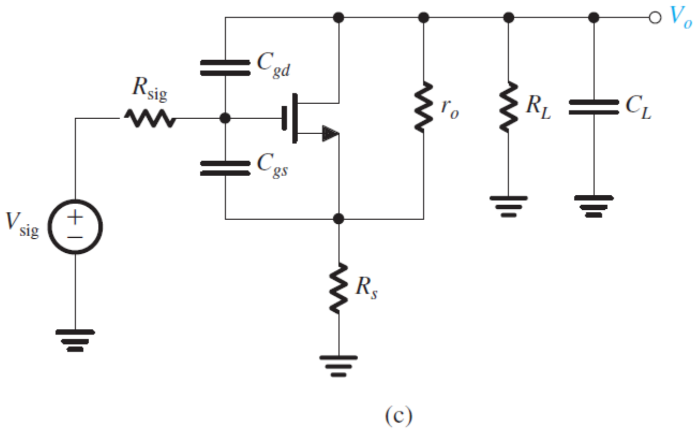


Figure 10.38 (**c**) The circuit prepared for frequency-response analysis.

$$R_{gd} \simeq R_{sig}\left(1+G_m R_L'\right)+R_L'$$

$$R_{C_L} = R_L \,||\, R_o = R_L'$$

$$R_{gs} \simeq \frac{R_{sig}+R_S}{1+g_m R_S\left(\dfrac{r_0}{r_0+R_L}\right)}$$

When R_{sig} is relatively large, the frequency response will be dominated by the Miller multiplication of C_{gd}. Another way for saying this is that $C_{gd}R_{gd}$ will be the largest of the three open-circuit time constants that make up τ_H,

$$\tau_H \simeq C_{gd}R_{gd} \Rightarrow f_H \simeq \frac{1}{2\pi C_{gd}R_{gd}}$$

Now, as R_S is increased, the gain magnitude, $|A_M|$, will decrease, causing R_{gd} to decrease, which in turn causes f_H to increase. If we simplify the expression for R_{gd} by assuming that $G_m R_L' \gg 1$ and $G_m R_{sig} \gg 1$,

$$R_{gd} \simeq G_m R_L' R_{sig} = |A_M| R_{sig}$$

$$f_H \simeq \frac{1}{2\pi C_{gd}|A_M| R_{sig}}$$

And the Gain-BW product is: $$|A_M| f_H = \frac{1}{2\pi C_{gd}R_{sig}}$$

The CD–CS, CC–CE and CD–CE Configurations

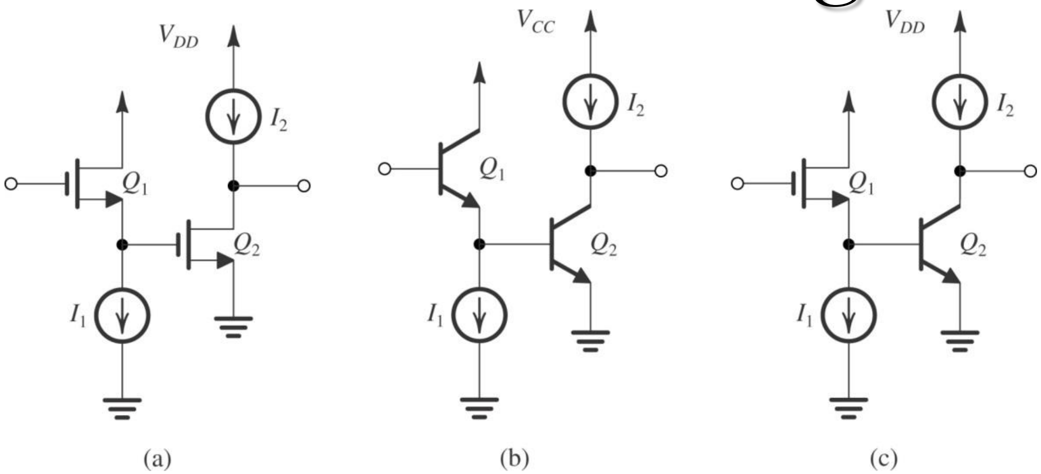


Figure 10.39 (a) CD–CS amplifier. **(b)** CC–CE amplifier. **(c)** CD–CE amplifier.

In Section 8.7.1 we discussed the performance improvements obtained by preceding the CS and CE amplifiers by a buffer implemented by a CD or a CC amplifier, as in the circuits shown in Fig. 10.39. A major advantage of each of these circuits is wider bandwidth than that obtained in the CS or CE stage alone.

To see how this comes about, consider as an example the CD–CS amplifier in Fig 10.39(a) and note that the CS transistor Q_2 will still exhibit a Miller effect that results in a large input capacitance, C_{in2}, between its gate and ground. However, the resistance that this capacitance interacts with will be much lower than R_{sig}; the buffering action of the source follower causes a relatively low resistance, approximately equal to a $1\,/\,g_{m1}$, to appear between the source of Q_1 and ground across C_{in2}.

Example 10.13 CC-CE circuit (a)

Consider a CC–CE amplifier such as that in Fig. 10.39(b) with the following specifications: $I_1 = I_2 = 1$ mA and identical transistors with $\beta = 100$, $f_T = 400$ MHz, and $C_\mu = 2$ pF. Let the amplifier be fed with a source V_{sig} having a resistance $R_{sig} = 4$ kΩ, and assume a load resistance of 4 kΩ. Find the voltage gain A_M, and estimate the 3-dB frequency, f_H. Compare the results with those obtained with a CE amplifier operating under the same conditions. For simplicity, neglect r_o and r_x.

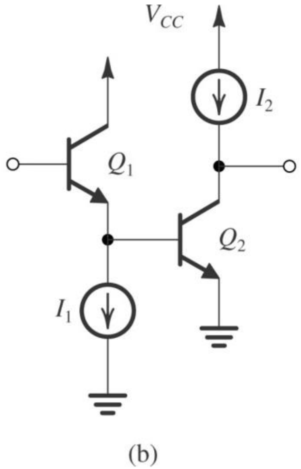


Figure 10.39 (b) CC–CE amplifier.

$$g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{0.025\text{V}} = 40\frac{\text{mA}}{\text{V}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\,\text{mA/V}} = 2.5\text{k}\Omega \qquad r_e \simeq \frac{r_\pi}{\beta} = 25\Omega$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{40\,\text{mA/V}}{(2\pi)400\text{MHz}} = 15.9\text{pF}$$

$$C_\mu = 2\text{pF} \Rightarrow C_\pi = 13.9\text{pF}$$

Example 10.13 CC-CE circuit (b)

Consider a CC–CE amplifier such as that in Fig. 10.39(b) with the following specifications: $I_1 = I_2 = 1$ mA and identical transistors with $\beta = 100$, $f_T = 400$ MHz, and $C_\mu = 2$ pF. Let the amplifier be fed with a source V_{sig} having a resistance $R_{sig} = 4$ kΩ, and assume a load resistance of 4 kΩ. Find the voltage gain A_M, and estimate the 3-dB frequency, f_H. Compare the results with those obtained with a CE amplifier operating under the same conditions. For simplicity, neglect r_o and r_x.

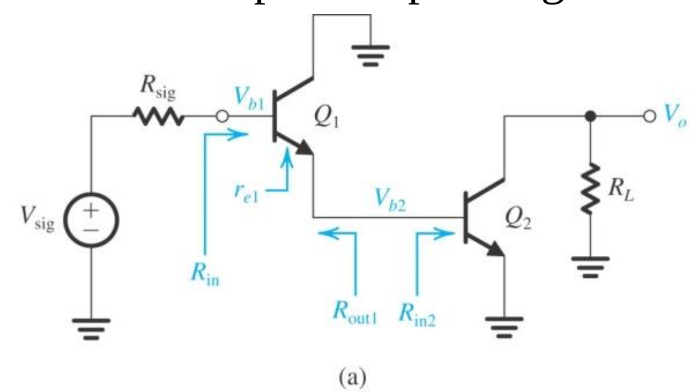


Figure 10.40 Circuits for Example 10.13: **(a)** the CC–CE circuit prepared for low-frequency, small-signal analysis;

$$R_{in2} = r_{\pi 2} = 2.5\text{k}\Omega$$

$$R_{in} = (\beta+1)(r_{e1} + R_{in2}) = 101(2.525\text{k}\Omega) = 255\text{k}\Omega$$

$$\frac{V_{b1}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_{sig}} = \frac{255\text{k}\Omega}{255\text{k}\Omega + 4\text{k}\Omega} = 0.98\frac{\text{V}}{\text{V}}$$

$$\frac{V_{b2}}{V_{b1}} = \frac{R_{in2}}{R_{in2} + r_{\pi 1}} = \frac{2.5\text{k}\Omega}{2.5\text{k}\Omega + 0.025\text{k}\Omega} = 0.99\frac{\text{V}}{\text{V}}$$

$$\frac{V_o}{V_{b2}} = -g_{m2}R_L = -160\frac{\text{V}}{\text{V}}$$

$$\frac{V_o}{V_{sig}} = \left(\frac{V_{b1}}{V_{sig}}\right)\left(\frac{V_{b2}}{V_{b1}}\right)\left(\frac{V_o}{V_{b2}}\right) = -155\frac{\text{V}}{\text{V}}$$

Example 10.13 CC-CE circuit (c)

Consider a CC–CE amplifier such as that in Fig. 10.39(b) with the following specifications: $I_1 = I_2 = 1$ mA and identical transistors with $\beta = 100$, $f_T = 400$ MHz, and $C_\mu = 2$ pF. Let the amplifier be fed with a source V_{sig} having a resistance $R_{sig} = 4$ kΩ, and assume a load resistance of 4 kΩ. Find the voltage gain A_M, and estimate the 3-dB frequency, f_H. Compare the results with those obtained with a CE amplifier operating under the same conditions. For simplicity, neglect r_o and r_x.

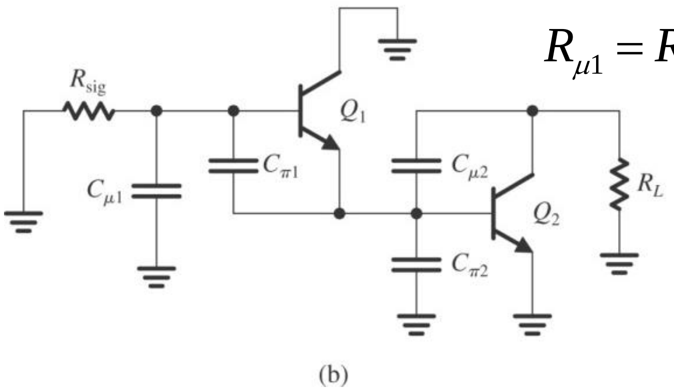


Figure 10.40 Circuits for Example 10.13: (**(b)** the circuit at high frequencies, with V_{sig} set to zero to enable determination of the open-circuit time constants;

$$R_{\mu 1} = R_{sig} \| R_{in} = 4\text{k}\Omega \| 255\text{k}\Omega = 3.94\text{k}\Omega$$

$$R_{\pi 1} \equiv \frac{V_x}{I_x} = \frac{R_{sig} + R_{in2}}{1 + \dfrac{R_{sig}}{r_{\pi 1}} + \dfrac{R_{in2}}{r_{e1}}} = 63.4\Omega$$

$$R_{\pi 2} = R_{in2} \| R_{out1} = r_{\pi 2} \| \left[r_{e1} + \frac{R_{sig}}{\beta_1 + 1} \right]$$

$$= 2500\Omega \| \left[25\Omega + \frac{4000\Omega}{101} \right] = 63\Omega$$

$$R_{\mu 2} = (1 + g_{m2} R_L)(R_{in2} \| R_{out1}) + R_L = 14.143\text{k}\Omega$$

Example 10.13 CC-CE circuit (d)

Consider a CC–CE amplifier such as that in Fig. 10.39(b) with the following specifications: $I_1 = I_2 = 1$ mA and identical transistors with $\beta = 100$, $f_T = 400$ MHz, and $C_\mu = 2$ pF. Let the amplifier be fed with a source V_{sig} having a resistance $R_{sig} = 4$ kΩ, and assume a load resistance of 4 kΩ. Find the voltage gain A_M, and estimate the 3-dB frequency, f_H. Compare the results with those obtained with a CE amplifier operating under the same conditions. For simplicity, neglect r_o and r_x.

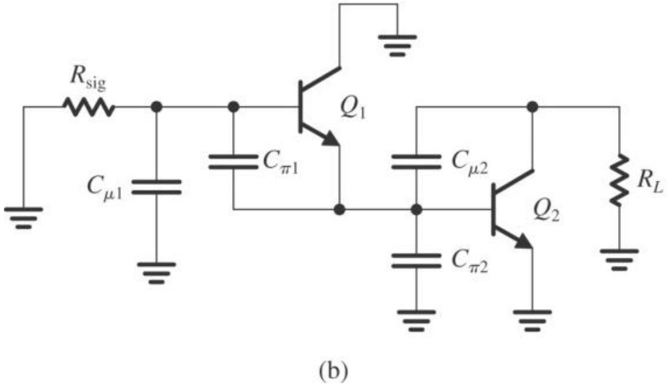


Figure 10.40 Circuits for Example 10.13: (**(b)** the circuit at high frequencies, with V_{sig} set to zero to enable determination of the open-circuit time constants;

$$\tau_H = C_{\mu 1}R_{\mu 1} + C_{\pi 1}R_{\pi 1} + C_{\mu 2}R_{\mu 2} + C_{\pi 2}R_{\pi 2} = 37.8\text{ns}$$

$$f_H = \frac{1}{2\pi\tau_H} = 4.2\text{MHz}$$

Example 10.13 CC-CE circuit (e)

Consider a CC–CE amplifier such as that in Fig. 10.39(b) with the following specifications: $I_1 = I_2 = 1$ mA and identical transistors with $\beta = 100$, $f_T = 400$ MHz, and $C_\mu = 2$ pF. Let the amplifier be fed with a source V_{sig} having a resistance $R_{sig} = 4$ kΩ, and assume a load resistance of 4 kΩ. Find the voltage gain A_M, and estimate the 3-dB frequency, f_H. Compare the results with those obtained with a CE amplifier operating under the same conditions. For simplicity, neglect r_o and r_x.

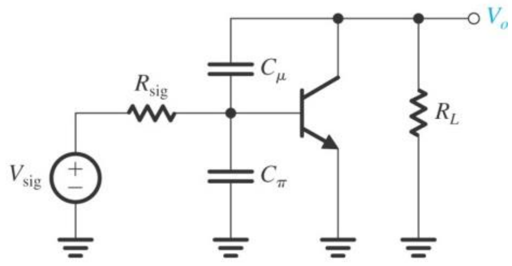


Figure 10.40 Circuits for Example 10.13: **(c)** a CE amplifier for comparison.

$$A_M = \frac{R_{in}}{R_{in}+R_{sig}}\left(-g_m R_L\right) = \frac{r_\pi}{r_\pi + R_{sig}}\left(-g_m R_L\right)$$

$$= \frac{2.5\text{k}\Omega}{2.5\text{k}\Omega + 4\text{k}\Omega}\left(-40\frac{mA}{V}4\text{k}\Omega\right) = -61.5\frac{\text{V}}{\text{V}}$$

$$R_\pi = r_\pi \,||\, R_{sig} = 1.54\text{k}\Omega$$

$$R_\mu = \left(1 + g_m R_L\right)\left(R_{sig} \,||\, r_\pi\right) + R_L = 251.7\text{k}\Omega$$

$$\tau_H = C_\mu R_\mu + C_\pi R_\pi = 524.8\text{ns} \qquad f_H = \frac{1}{2\pi\tau_H} = 303\text{kHz}$$

The CC–CB and CD–CG Configurations

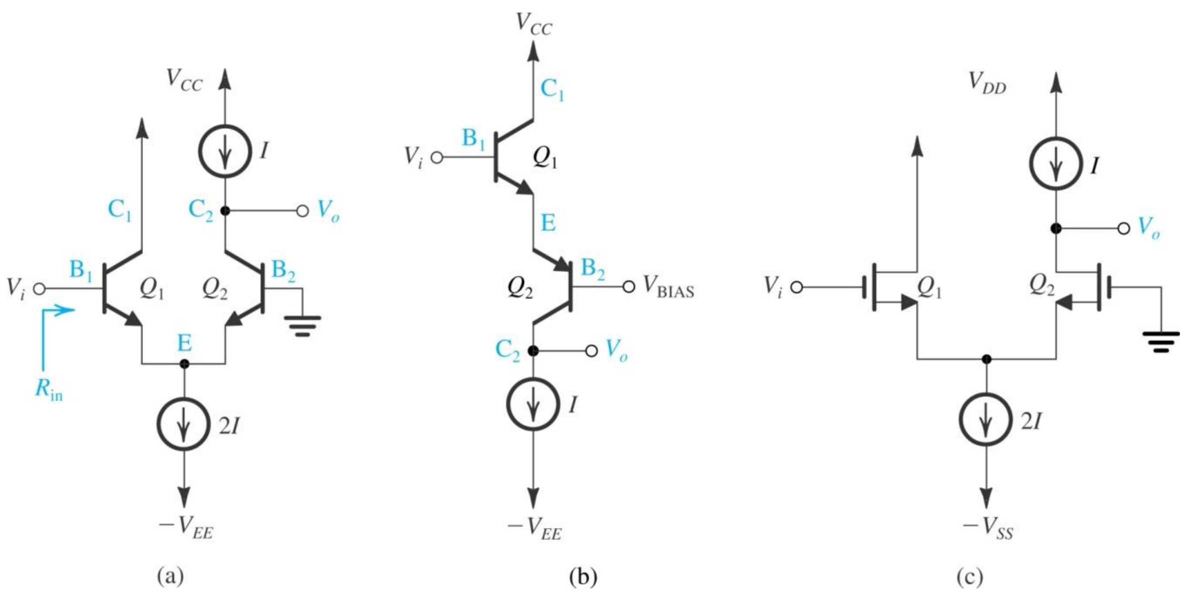


Figure 10.41 (a) A CC–CB amplifier. **(b)** Another version of the CC–CB circuit with Q_2 implemented using a *pnp* transistor. **(c)** The MOSFET version of the circuit in **(a)**.

In Section 8.7.3 we showed that preceding a CB or CG transistor with a buffer implemented with a CC or a CD transistor solves the low-input-resistance problem of the CB and CG amplifiers. Examples of the resulting compound-transistor amplifiers are shown in Fig. 10.41. Since in each of these circuits, neither of the two transistors suffers from the Miller effect, the resulting amplifiers have even wider bandwidths than those achieved in the compound amplifier stages of the last section.

The CC–CB Configuration

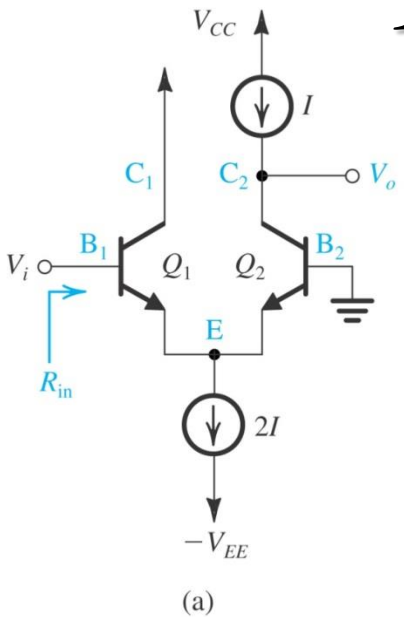


Figure 10.41 (a) A CC–CB amplifier.

To illustrate, consider as an example the circuit in Fig. 10.41(a).

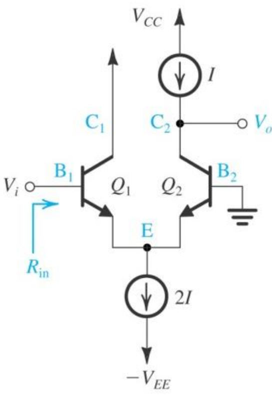
$$R_{in} = (\beta + 1)(r_{e1} + r_{e2}) = 2r_\pi \qquad (\text{if } r_{e1} = r_{e2} = r_e \text{ and } \beta_1 = \beta_2 = \beta)$$

If a load resistance R_L is connected at the output, the voltage gain will be

$$\frac{V_o}{V_i} = \frac{\alpha_2 R_L}{r_{e1} + r_{e2}} = \frac{1}{2} g_m R_L$$

Now, if the amplifier is fed with a voltage signal from a source with a resistance R_{sig}, the overall voltage gain will be

$$\frac{V_o}{V_{sig}} = \frac{1}{2}\left(\frac{R_{in}}{R_{in} + R_{sig}}\right)(g_m R_L)$$



The CC–CB Configuration

The high-frequency analysis is illustrated in Fig. 10.42(a). Here we have drawn the hybrid-π equivalent circuit for each of Q_1 and Q_2. Recalling that the two transistors are operating at equal bias currents, their corresponding model components will be equal. With this in mind the reader should be able to see that $V_{\pi 1} = -V_{\pi 2}$ and the horizontal line through the node labeled E in Fig. 10.42(a) can be deleted. Thus the circuit reduces to that in Fig. 10.42(b).

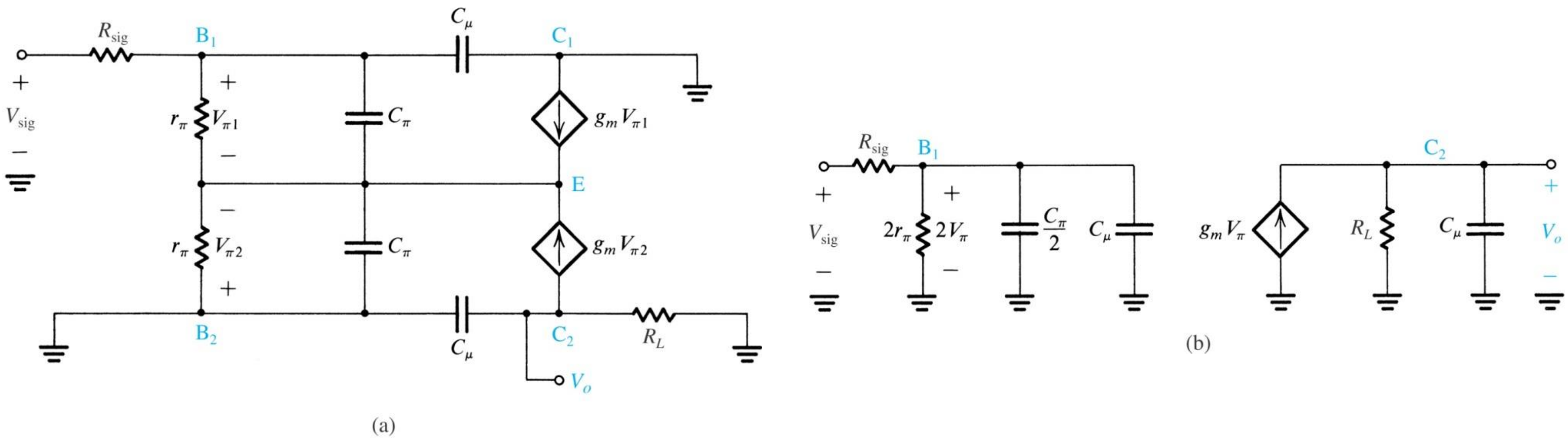
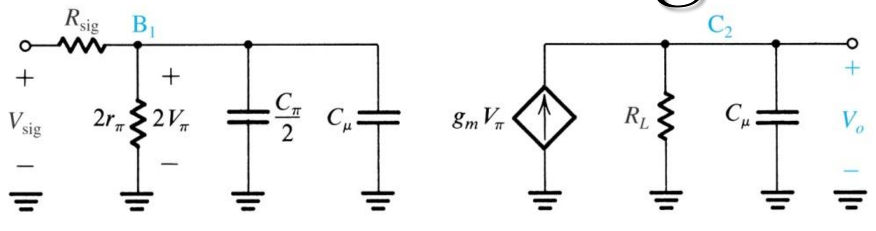


Figure 10.42 **(a)** Equivalent circuit for the amplifier in Fig. 10.41(a). **(b)** Simplified equivalent circuit. Note that the equivalent circuits in **(a)** and **(b)** also apply to the circuit shown in Fig. 10.41(b). In addition, they can be easily adapted for the MOSFET circuit in Fig. 10.41(c), with $2r_\pi$ eliminated, C_π replaced with C_{gs}, C_μ replaced with C_{gd}, and V_π replaced with V_{gs}

The CC–CB Configuration



(b)

This circuit shows clearly the two poles that determine the high-frequency response: The pole at the input, with a frequency f_{P1}, is

$$f_{P1} = \frac{1}{2\pi\left(\frac{C_\pi}{2} + C_\mu\right)\left(R_{sig} \parallel 2r_\pi\right)}$$

and the pole at the output, with a frequency f_{P2}, is

$$f_{P2} = \frac{1}{2\pi C_\mu R_L}$$

Whether one of the two poles is dominant will depend on the relative values of R_{sig} and R_L. If the two poles are close to each other, then the 3-dB frequency can be determined either by exact analysis - that is, finding the frequency at which the gain is down by 3 dB - or by using the approximate

$$f_H \simeq 1 \Big/ \sqrt{\frac{1}{f_{P1}^2} + \frac{1}{f_{P2}^2}}$$

Summary

- The coupling and bypass capacitors utilized in discrete-circuit amplifiers cause the amplifier gain to fall off at low frequencies. The frequencies of the low-frequency poles can be estimated by considering each of these capacitors separately and determining the resistance seen by the capacitor. The highest-frequency pole is that which determines the lower 3-dB frequency (f_L).
- Both MOSFET and the BJT have internal capacitive effects that can be modeled by augmenting the device hybrid-pi model with capacitances.

MOSFET: $f_T = g_m/2\pi(C_{gs}+C_{gd})$

BJT: $f_T = g_m/2\pi(C_\pi+C_\mu)$

Summary

- The internal capacitances of the MOSFET and the BJT cause the amplifier gain to fall off at high frequencies. An estimate of the amplifier bandwidth is provided by the frequency f_H at which the gain drops 3dB below its value at midband (A_M). A figure-of-merit for the amplifier is the gain-bandwidth product ($GBW = A_M f_H$). Usually, it is possible to trade gain for increased bandwidth, with GBW remaining nearly constant. For amplifiers with a dominant pole with frequency f_H, the gain falls off at a uniform 6dB/octave rate, reaching 0 dB at $f_T = GB$.
- The high-frequency response of the CS and CE amplifiers is severely limited by the Miller effect.

Summary

- The method of open-circuit time constants provides a simple and powerful way to obtain a reasonably good estimate of the upper 3-dB frequency f_H. The capacitors that limit the high-frequency response are considered one at a time with $V_{sig} = 0$ and all other capacitances are set to zero (open circuited). The resistance seen by each capacitance is determined, and the overall time constant (τ_H) is obtained by summing the individual time constants. Then f_H is found as $1/2\pi\tau_H$.
- The CG and CB amplifiers do not suffer from the Miller effect.
- The source and emitter followers do not suffer from Miller effect.

Summary

- The high-frequency response of the differential amplifier can be obtained by considering the differential and common-mode half-circuits. The CMRR falls off at a relatively low frequency determined by the output impedance of the bias current source.
- The high-frequency response of the current-mirror-loaded differential amplifier is complicated by the fact that there are two signal paths between input and output: a direct path and one through the current mirror.
- Combining two transistors in a way that eliminated or minimizes the Miller effect can result in much wider bandwidth.