SAMPLE EXAM #2B
ELEG 305 SIGNALS AND SYSTEMS
SPRING 2019

Note: Last year, the second exam included Chapter 5 and so there are some problems that will not be relevant for this year's Exam #2 which will only cover Chapters 3 and 4. I have marked the problems that cover material that will not be on Exam #2 using # signs right next to the problem.

Problem #1 (20 points)

Consider a continuous-time LTI system with a periodic input

$$x(t) = \sin\left(\frac{\pi}{4}t\right) + \cos\left(\frac{5\pi}{4}t\right)$$

a.) (2 pts) What is the period of x(t)?

b.) (6 pts) Determine the Fourier Series coefficients, a_k, of the input signal x(t).

c.) (6 pts) This signal is then passed through a *highpass filter* with frequency response

$$H(j\omega) = \begin{cases} 0, & |\omega| \le \pi \\ |\omega| - \left(\frac{\pi}{2}\right), & otherwise \end{cases}$$

Determine the Fourier Series coefficients of the output signal y(t).

d.) (6 pts) Determine the time-domain output signal y(t).

Problem #2 (20 points)

a.) (10 pts) Consider a continuous-time signal, $x(t)=e^{-at}u(t)$, $a > 0$. Compute the Fourier transform of the following expression (* denotes convolution)

$$y(t) = \beta x(t-2t_0) + 2\rho x(t) * x(t) * x(t) + \delta(t+2t_0) + d^2x(t)/dt^2$$

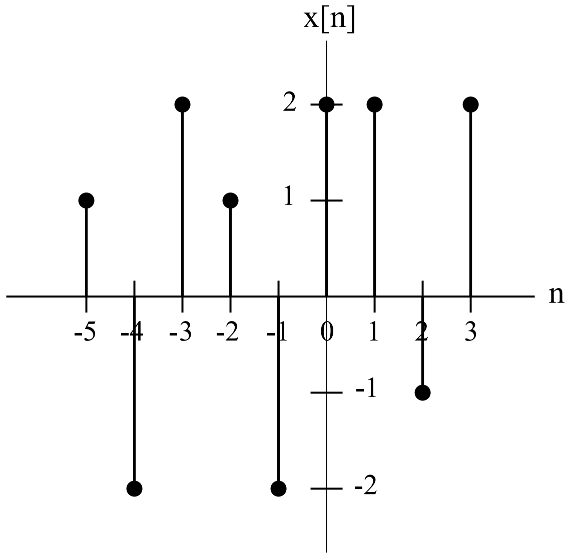
b.) ##### (10 pts) Compute the Fourier transform of

$$x[n] = \left(\left(\frac{-1}{2}\right)^n u[n]\right) * \left(\frac{1}{4}\right)^{-n} u[-n]$$

Problem #3 (20 points)

a.) ##### (8 pts) Consider the discrete-time function x[n] (on the next page). Evaluate the following without computing $X(e^{j\omega})$

(i) $X(e^{j\pi})$ (ii) $\int_{-\pi}^{+\pi} X(e^{j\omega})e^{j\omega}\, d\omega$



b.) (6 pts) Compute the value of

$$\int_{-\infty}^{+\infty} \frac{\sin^2(\pi t)}{\pi t^2}\, dt$$

c.) ##### (6 pts) Derive the difference equation for a DT LTI system with frequency response

$$H(e^{j\omega}) = \frac{-e^{-j\omega} + 3}{1 + \frac{1}{3}e^{-j\omega} - \frac{1}{3}e^{-j2\omega}}$$

Problem #4 (20 points)
Consider a continuous-time LTI system characterized by the frequency response

$$H(j\omega) = \frac{-(j\omega + 1)}{-\omega^2 + 7j\omega + 12}$$

a.) (10 pts) Derive the impulse response, h(t), for this system.
b.) (10 pts) Find the differential equation relating the input and output of this system.

Problem #5 (20 points)
A seismometer is an instrument that measures ground motion, for example, the waves generated by earthquakes. Assume we can model this system as *linear* and *time-invariant*. It is known from previous tests that *if the source of the motion is an impulse*, the measured output is given by

$$h(t) = \frac{3}{2}e^{-4t}u(t) + \frac{3}{2}e^{-2t}u(t)$$

Today, the seismometer recorded a measured output waveform

$$y(t) = 2e^{-t}u(t) - 2e^{-4t}u(t)$$

a.) (5 pts) What is the frequency response of the system?
b.) (15 pts) Determine the source, x(t), that generated today's measured output?

Extra Credit (10 points)
In Problem #5, determine the frequency response of a filter that could be used to process y(t) to recover the original signal x(t)?