ELEG 310 2/28/19

Conditional Probabilities

sometimes have partial information

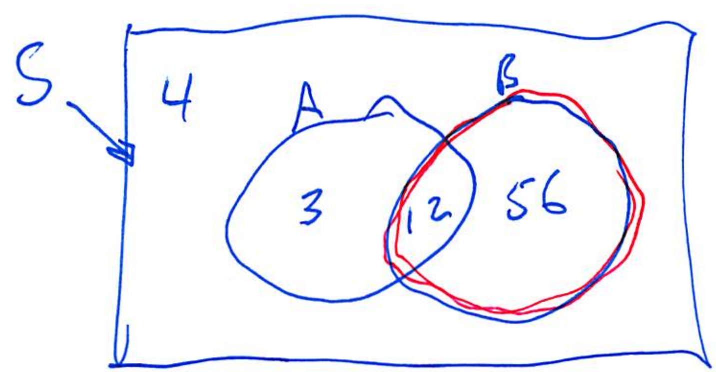
$P(A|B) =$ " prob of A given B "

$= \frac{P(AB)}{P(B)}$ def'n

Ex. roll a die $A = \{1, 2, 3\}$ $B = \{1, 2, 5, 6\}$

$P(A) = \frac{3}{6} = \frac{1}{2}$ $P(B) = \frac{4}{6} = \frac{2}{3}$

$P(A|B) = \frac{P(AB)}{P(B)}$



$AB = \{1, 2\}$

$= \frac{2/6}{4/6} = \frac{1}{2}$

Ex roll a die

$A = \{1, 2\}$ $\quad B = \{1, 2, 5, 6\}$

$P(A) = \frac{1}{3}$ $\quad P(B) = \frac{2}{3}$ $\quad AB = \{1, 2\}$ $\quad P(AB) = \frac{2}{6} = \frac{1}{3}$

$$P(A \mid B) = \frac{P(AB)}{P(B)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

Ex roll a die

$A = \{1, 2, 3, 4\}$ $\quad P(A) = \frac{4}{6} = \frac{2}{3}$ $\quad AB = \{1, 2\}$

$$P(A \mid B) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$P(A|B) = \frac{P(AB)}{P(B)} \Rightarrow P(AB) = P(A|B)\,P(B)$$

if A & B ind, $P(AB) = P(A)P(B)$

$$P(A|B) = \frac{P(A)\cancel{P(B)}}{\cancel{P(B)}} = P(A)$$

$$P(AB) = P(A|B)P(B) = P(A)\,P(B)$$

defini $P(A|B) = \frac{P(AB)}{P(B)}$ when $P(B) > 0$

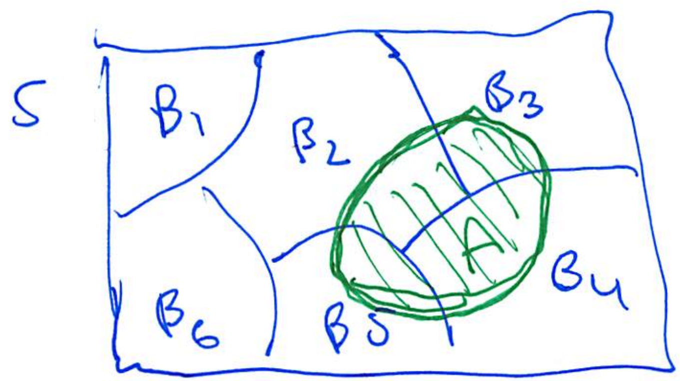
Chain rule $P(ABC) = \frac{P(ABC)}{P(BC)} \frac{P(BC)}{P(C)} P(C)$

$$= P(A|BC)\,P(B|C)\,P(C)$$

Law of Total Prob (LTP)

let B_i $i=1,2,\ldots$ be a <u>partition</u> of S,

meaning $B_iB_j=\phi$ $i\neq j$ and $\bigcup_{i=1}^{\infty} B_i = S$



$$\boxed{P(A)} = P(A\cap S) = P\left(A\cap \bigcup_{i=1}^{\infty} B_i\right) = P\left(\bigcup_{i=1}^{\infty} AB_i\right)$$

$$= \boxed{\sum_{i=1}^{\infty} P(AB_i)} = \boxed{\sum_{i=1}^{\infty} P(A|B_i)P(B_i)}$$

Axiom 3

Bayes Theorem

$$P(B|A) = \frac{P(AB)}{P(A)} \frac{P(B)}{P(B)} = \frac{P(A|B)P(B)}{P(A)}$$

In general, $P(A|B) \neq P(B|A)$

Urn Models

U	R	B
1	5	5
2	2	4

$P(U_1) = \frac{2}{3}$

$P(U_2) = \frac{1}{3}$

$$P(R) = P(R|U_1)P(U_1) + P(R|U_2)P(U_2)$$

$$= \frac{5}{5+5} \cdot \frac{2}{3} + \frac{2}{2+4} \cdot \frac{1}{3} = \frac{4}{9}$$

Exp. ~~pull~~ pick one marble, pick another

let R_1 = red first pick

R_2 = red second pick

$$P(R_2 | R_1) = \frac{P(R_1 R_2)}{P(R_1)}$$

$$P(R_1 R_2) = P(R_1 R_2 | U_1) P(U_1) + P(R_1 R_2 | U_2) P(U_2)$$

$$= \frac{5}{5+5} \cdot \frac{4}{4+5} \cdot \frac{2}{3} + \frac{2}{2+4} \cdot \frac{1}{1+4} \cdot \frac{1}{3}$$

$$P(R_2 | R_1) = \frac{P(R_1 R_2)}{P(R_1)} = 0.383$$

assuming we do not replace first marble