# MATH426 PRELAB3

Shane Cincotta

November 4 2019

## 1 4.2.4

a)

The iteration spirals towards the fixed point because within the algorithm, the x and y values are constantly being updated, this sequence of values mimics a spiral when plotted. If the iteration is to converge to the fixed point, the errors must approach zero, signifying that the new x and y values are getting closer and closer to the fixed point. Conversely, if the error is growing, we will spiral away from the fixed point.

b)

$f(x) = x^2 - 4x + 3.5$

$g(x) = x - f(x) = \frac{x^2+3.5}{4}$

$=> 20x - 5x^2 - 17.5 = 0$

x = 2.707, 1.293

$f(x) = x^2 - 4x + 3.5 = 0$

x = 2.707, 1.293.

Thus a fixed point of g is a root of f.

c)

Graph of iterations is in the Matlab PDF. Neither of my roots converged, after running the fixed point iteration method, the roots were found to be INF and DNE according to Matlab.

d)

In order for the iteration to converge, the error must converge to 0. If the errors are to converge to 0, then the error sequence $\epsilon_{k+1} \ = g'(r)\epsilon_k$. Thus if $|g'(r) > 1|$ the errors will diverge. This the case with our given function. The errors are growing instead of shrinking.

# 2 4.3.1

a)

$f(x) = x^2 - e^{-x} = 0$
$f'(x) = 2x + e^{-x}$

b)

In Matlab file HW7_431.m

c)

In Matlab file HW7_431.m

d)

In Matlab file HW7_431.m

e)

The convergence is quadratic.

Code is in Matlab file HW7_431.m

# 3 4.3.2

a)

$f(x) = 2x - \tan(x) = 0$
$f'(x) = 2 - \sec^2(x)$

b)

In Matlab file HW7_432.m

c)

In Matlab file HW7_432.m

d)

In Matlab file HW7_432.m

e)

The convergence is quadratic.

In Matlab file HW7_432.m

# 4 4.3.3

a)

$f(x) = e^{x+1} - 2 - x = 0$
$f'(x) = e^{x+1} - 1$

b)

In Matlab file HW7_433.m

c)

In Matlab file HW7_433.m

d)

In Matlab file HW7_433.m

e)

The convergence is not quadratic.

In Matlab file HW7_433.m

## 5 4.4.4

Plot and answer are in Matlab file HW7_444.m.

## 6 4.4.5

The secant method is defined as: $x_{n+1} = x_n - f(xn)\frac{x_n - x_{n-1}}{f(xn) - f(x_{n-1}})$
Where $x_0, x_1$ are the two initial guesses of the root of the function f(x) = 0.

If f(x) is a linear function then $f(x) = ax + b$, where a, b are constants.

For n = 1:
$x_2 = x_1 - f(x_1)\frac{x_1 - x_0}{f(x_1) - f(x_))}$

$= x_1 - (ax_1 + b)\frac{x_1 - x_0}{(ax_1 + b) - (ax_0 + b)}$

$= x_1 - (ax_1 + b)\frac{x_1 - x_0}{a(x_1 - x_0)}$

$= x_1 - \frac{ax_1 + b}{a}$

$= \frac{ax_1 - ax_0 - b}{a} = -\frac{b}{a}$ ,which is the root of the linear function $f(x) = 0$

Hence the secant method converges in one step for a linear function, regardless of the initialization

## 7 4.5.2

$f(x) = Ax - b$

$J = f'(x) = A$

So by the Newton Method, $X_{n+1} = x_n - J^{-1}f(x_n)$

$x_{n+1} = x_n - A^{-1}(Ax_n - b)$

$x_{n+1} = x_n - A^{-1}Ax_n + A^{-1}b$

$x_{n+1} = A^{-1}b$

So after 1 iteration, we get $x_1 = A^{-1}b$ which is the exact root.

Thus the Newton Method converges to the root after one iteration.

# 8 4.5.3

In Matlab file HW7_453.m

# 9 4.5.4

a)

In Matlab file HW7_454.m

b)

To find an intersection we have to set $x_1(t_1) = x_2(t_2)$ and $y_1(t_1) = y_2(t_2)$ for some times t. Thus we have:

$x_1(t_1) = x_2(t_2) => -5 + 10cos(t_1) = 8cos(t_2)$ and
$y_1(t_1) = y_2(t_2) => 6sin(t_1) = 1 + 12sin(t_2)$

So the nonlinear system is:

$10cos(t_1) - 8cos(t_2) - 5 = 0$ and
$6sin(t_1) - 12sin(t_2) - 1 = 0$

c)

By assigned functions $f_1$ and $f_2$ to the left hand sides of the equations above we get the following Jacobian matrix:

$$\begin{bmatrix} \frac{\partial f_1}{\partial t_1} & \frac{\partial f_1}{\partial t_2} \\ \frac{\partial f_2}{\partial t_1} & \frac{\partial f_2}{\partial t_2} \end{bmatrix} = \begin{bmatrix} -10sin(t_1) & 8sin(t_2) \\ 6cos(t_1) & -12cos(t_2) \end{bmatrix}$$

d)

In Matlab file HW7_454.m