ELEG 305 SIGNALS AND SYSTEMS
SPRING 2019

- **All Homeworks and Homework Quizzes are worth 25 points.**
- **Homeworks and Solutions from Spring 2018 have been posted on Canvas.**

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HOMEWORK #4 → *Hand-in on Thursday March 21 (Collected in Lecture)*

Read Chapter 3 in Oppenheim, Willsky, and Nawab (**O&W**)

Problem #1

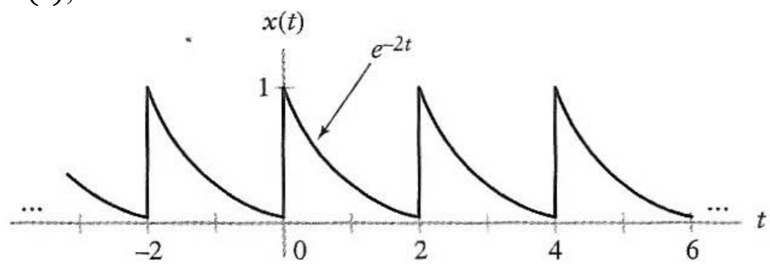
Determine the Fourier Series coefficients for the following *continuous-time* periodic signals.

a.) $x(t) = 3\cos(\frac{\pi t}{2} + \frac{\pi}{4})$ Please use the "inspection method" to find the coefficients. Also, plot the magnitude and phase of the coefficients as a function of k (the frequency index).

b.) For the next two signals, use the Fourier Series analysis equation (3.39) to compute the coefficients. In each case, first determine the fundamental period of the signal.

(i) $x(t) = \sum_{m=-\infty}^{m=\infty} \delta(t - 2m)$

(ii) The signal, $x(t)$, shown below:



Problem #2

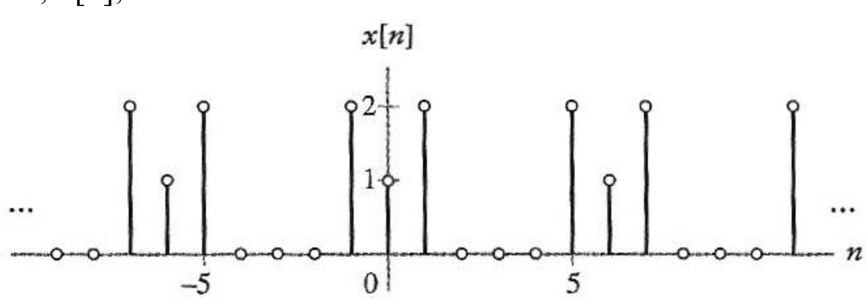
Determine the Fourier Series coefficients for the following *discrete-time* periodic signals.

a.) $x[n] = 1 + \sin(\frac{n\pi}{12} + \frac{3\pi}{8})$ Please use the "inspection method" to find the coefficients. Also, plot the magnitude and phase of the coefficients as a function of k (the frequency index).

b.) For the next two signals, use the Fourier Series analysis equation (3.95) to compute the coefficients. In each case, first determine the fundamental period of the signal.

(i) $x[n] = \sum_{m=-\infty}^{m=\infty} (-1)^m \delta(n - m)$

(ii) The signal, $x[n]$, shown below:



Problem #3

a.) Determine the continuous-time periodic signal, $x(t)$ (with fundamental radian frequency $\omega_0 = \pi$), if its Fourier Series coefficients are given by

$$a_2 = -j,\ a_{-2} = j, a_3 = a_{-3} = 2$$
$$a_k = 0, \text{ for all other values of } k$$

b.) Determine the discrete-time periodic signal, $x[n]$ (with period $N = 10$), if one period of its Fourier Series coefficients is given by

$$a_k = (\tfrac{1}{2})^k,\ 0 \le k \le 9$$

Problem #4

Consider a continuous-time LTI system with a periodic input

$$x(t) = \sin(\tfrac{\pi}{4}t) + \cos(\tfrac{5\pi}{4}t)$$

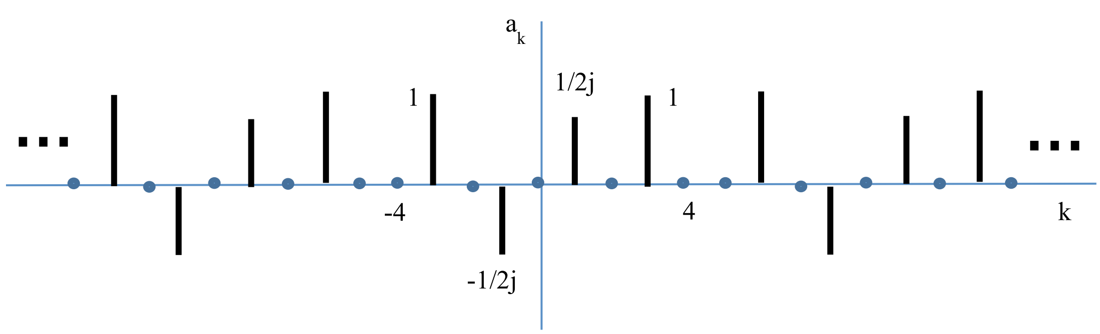
a.) Determine the Fourier Series coefficients, a_k, of the input signal $x(t)$.
b.) This signal is then passed through a *highpass filter* with frequency response

$$H(j\omega) = \begin{cases} 0, & |\omega| \le \pi \\ |\omega| - \left(\frac{\pi}{2}\right), & otherwise \end{cases}$$

Determine the Fourier Series coefficients of the output signal $y(t)$.
c.) Determine the time-domain output signal $y(t)$.

Problem #5

Consider a *periodic* time-domain signal (with period 9), with Fourier Series coefficients



a.) Are these the Fourier Series coefficients for a continuous- or discrete-time signal?
b.) Derive the periodic, time-domain, signal from its Fourier Series coefficients.
c.) Assume that the signal is passed through an ideal *lowpass* filter with cutoff frequency $\omega_c = 2\omega_0$. What is the resulting filtered time-domain signal?

Conceptual: We can make an analogy between a prism and the Fourier Series representation of a periodic signal. A prism breaks up light into different colors (i.e, electromagnetic radiation at different frequencies). Provide another example of an analogy with the Fourier Series representation, and explain.

Math Review: Please use partial fraction expansion to rewrite these algebraic expressions. If you don't remember how to do this, there is a tutorial in O&W, with examples. There are also examples in Ch. 4 (see 4.19 and 4.26).

$$\frac{4x-19}{x^2-9x+20}$$

$$\frac{x^2+x+3}{x+1}$$

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EXAM # 2 Tuesday April 16

- Closed everything: no calculators, cellphones, laptops, …
- Chapters 3 and 4
- A formula sheet will be provided with trigonometric identities, and the defining equations and properties for Fourier Series/Transforms.
- Review on Monday April 15