ELEG 310 Chap 7 4/09

X RV Cts

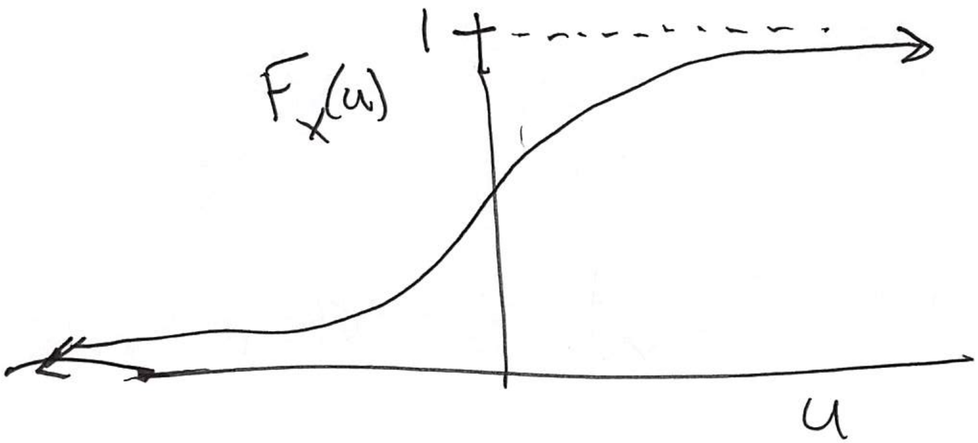
CDF $\boxed{F_X(u) = P(X \le u)}$ $-\infty < u < \infty$

$\boxed{P(X = u) = 0 \quad \text{for all } u}$

$P(X \le b) = F_X(b)$

$P(X \le a) = F_X(a)$

$$\boxed{P(a < X \le b) = F_X(b) - F_X(a)}$$



$P(a < X \le b) = F_X(b) - F_X(a) \ge 0$

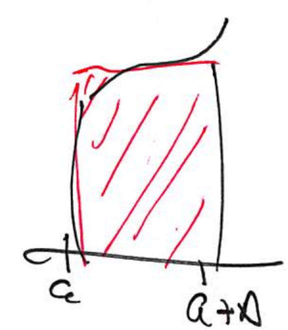
$$\boxed{\Rightarrow F_X(b) \ge F_X(a) \quad b \ge a}$$

$\Rightarrow$ slope ≥ 0 all u.

(like the PMF)

Define density $f_X(u) = \dfrac{dF_X(u)}{du} \ge 0$

$$P(a < X \le b) = F_X(b) - F_X(a) = \boxed{\int_a^b f_X(u)\,du}$$



$$P(a < X \le a+\Delta) = \int_a^{a+\Delta} f_X(u)\,du \approx \Delta f_X(a)$$

$$\approx P(X \approx a)$$

Δ small

$f_X(a) \approx f_X(a+\Delta)$

$$EX = \int_{-\infty}^{\infty} g(u) f_X(u)\,du = \int_a^b u \frac{1}{b-a}\,du = \frac{1}{b-a}\left(\frac{b^2}{2} - \frac{a^2}{2}\right) = \frac{b+a}{2}$$

$g(X) = X$

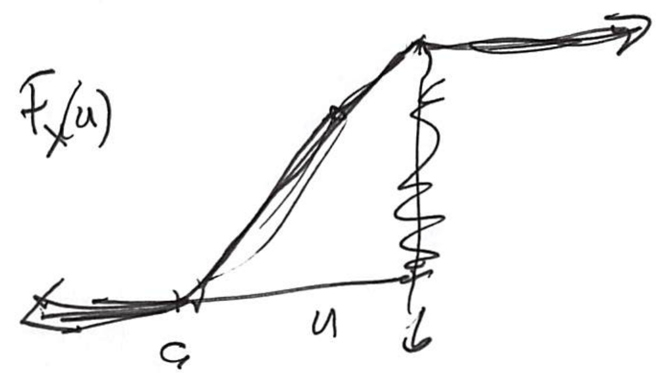
$$EX^2 = \frac{1}{b-a}\int_a^b u^2\,du = \frac{b^3 - a^3}{3(b-a)}$$

$g(X) = X^2$

$$\sigma^2 = E(X^2) - (EX)^2 = \frac{b^3 - a^3}{3(b-a)} - \left(\frac{b+a}{2}\right)^2 = \frac{(b-a)^2}{12}$$

$$= E(X-\mu)^2$$

$g(x) = (X-\mu)^2$



Expected Values

$$E\,g(X) = \int_{-\infty}^{\infty} g(u)\, f_X(u)\,du$$

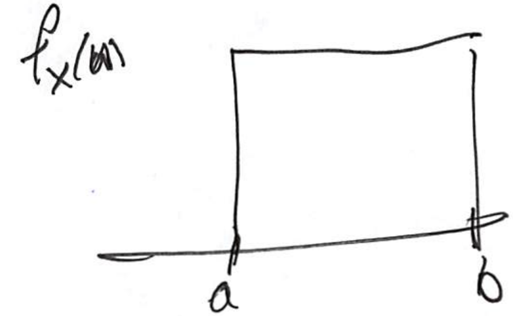
mean $\mu = E(X) = \int_{-\infty}^{\infty} u\, f_X(u)\,du$

Variance $\sigma^2 = E(X-\mu)^2 = \int_{-\infty}^{\infty} (u-\mu)^2 f_X(u)\,du$

CDF $F_X(u) = \int_{-\infty}^{u} f_X(v)\,dv$

Ex. $X \sim$ Uniform (a, b) $\quad X \sim U(a,b)$

"is distributed as"



$$f_X(u) = \begin{cases} c & a<u<b \\ 0 & \text{o.w.} \end{cases}$$

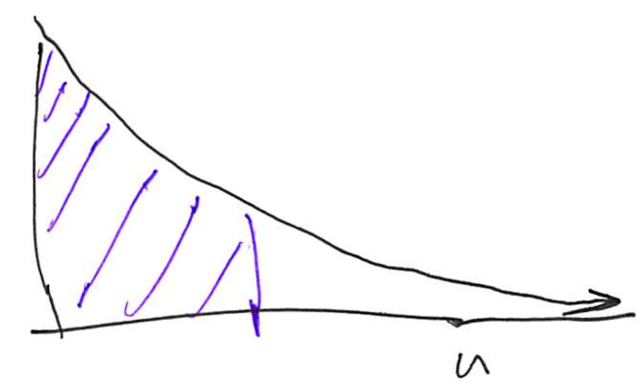
$$1 = \int_{-\infty}^{\infty} f_X(u)\,du = \int_a^b c\,du = c(b-a)$$

$$\Rightarrow c = \frac{1}{b-a}$$

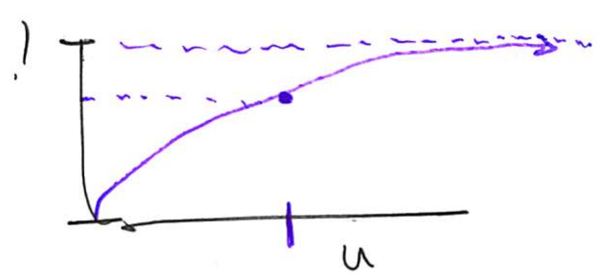
Exponential Distribution — often used to model waiting time experiments

cts version of geometric distribution

$$f_X(u) = \begin{cases} \lambda e^{-\lambda u} & u \geq 0 \\ 0 & u < 0 \end{cases}$$



CDF $F_X(u) = P(X \leq u)$



$$EX = \int_0^\infty u \lambda e^{-\lambda u}\, du \qquad s = \lambda u \quad ds = \lambda du$$

$$= \int_0^\infty \frac{s}{\lambda} e^{-s}\, ds = \frac{1}{\lambda} \qquad \text{Var } X = \frac{1}{\lambda^2}$$

Conditional Probs.

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$P(X \le u \mid X \ge b) = \frac{P(X \le u \cap X \ge b)}{P(X \ge b)}$$

$$F_X(u \mid X \ge b) = \frac{P(b \le X \le u)}{\underbrace{P(X \ge b)}_{P(b \le X < \infty)}} = \frac{F_X(u) - F_X(b)}{1 - F_X(b)}$$

$F_X(\infty)$

Ex. $P(X \le u \mid X \le b) = \dfrac{P(X \le u \cap X \le b)}{P(X \le b)} = \dfrac{P(X \le \min(u,b))}{F_X(b)}$

$$= \frac{F_X(\min(u,b))}{F_X(b)}$$