Homework #4

Problems 2.60, 2.62, 2.68, 2.104, and 2.111

Problem 2.60a

Find the voltage gain for the difference amplifier of Fig. 2.16 for the case R_1 = R_3 = 5 kΩ and R_2 = R_4 = 100 kΩ. What is the differential input resistance R_{id}? If the two key resistance ratios (R_2/R_1) and (R_4/R_3) are different from each other by 1%, what do you expect the common-mode gain A_{cm} to be? Also, find the CMRR in this case. Neglect the effect of the ratio mismatch on the value of A_d.

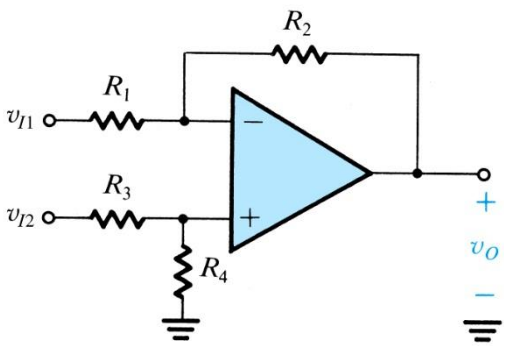


Figure 2.16 A difference amplifier.

$$v_O = \left(\frac{R_4}{R_3 + R_4}\right)\left(1 + \frac{R_2}{R_1}\right)v_{I2} - \frac{R_2}{R_1}v_{I1}$$

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} \Rightarrow \qquad v_O = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{R_1 + R_2}{R_1}\right)v_{I2} - \frac{R_2}{R_1}v_{I1}$$

$$= \frac{R_2}{R_1}(v_{I2} - v_{I1}) = \frac{R_2}{R_1}v_{Id}$$

$$R_{id} = 5\text{ k}\Omega + 5\text{ k}\Omega = 10\text{ k}\Omega$$

$$A_d \equiv \frac{v_O}{v_{Id}} = \frac{R_2}{R_1} = \frac{100\text{ k}\Omega}{5\text{ k}\Omega} = 20\text{V/V}$$

Problem 2.60b

Find the voltage gain for the difference amplifier of Fig. 2.16 for the case $R_1 = R_3 = 5$ kΩ and $R_2 = R_4 = 100$ kΩ. What is the differential input resistance R_{id}? If the two key resistance ratios (R_2/R_1) and (R_4/R_3) are different from each other by 1%, what do you expect the common-mode gain A_{cm} to be? Also, find the CMRR in this case. Neglect the effect of the ratio mismatch on the value of A_d.

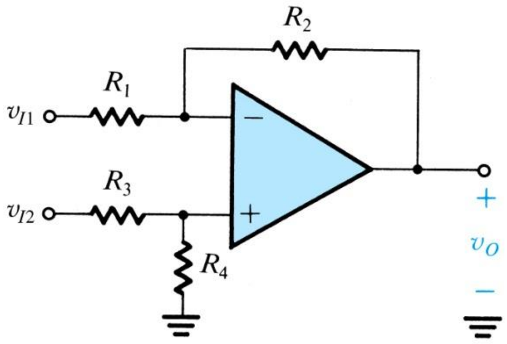


Figure 2.16 A difference amplifier.

$$v_O = \left(\frac{R_4}{R_3+R_4}\right)\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)v_{Icm}$$

$$R_1 = R_3, R_2 = R_4 \Rightarrow \quad v_O = \left(\frac{R_2}{R_2+R_1}\right)\left(1-\frac{R_2}{R_1}\frac{R_1}{R_2}\right)v_{Icm}$$

$$= \frac{R_2}{R_2+R_1}(1-1)v_{Icm} = 0\text{V}$$

$$A_{cm} \equiv \frac{v_O}{v_{Ism}} = 0\text{V/V}$$

Problem 2.60c

Find the voltage gain for the difference amplifier of Fig. 2.16 for the case $R_1 = R_3 = 5$ kΩ and $R_2 = R_4 = 100$ kΩ. What is the differential input resistance R_{id}? If the two key resistance ratios (R_2/R_1) and (R_4/R_3) are different from each other by 1%, what do you expect the common-mode gain A_{cm} to be? Also, find the CMRR in this case. Neglect the effect of the ratio mismatch on the value of A_d.

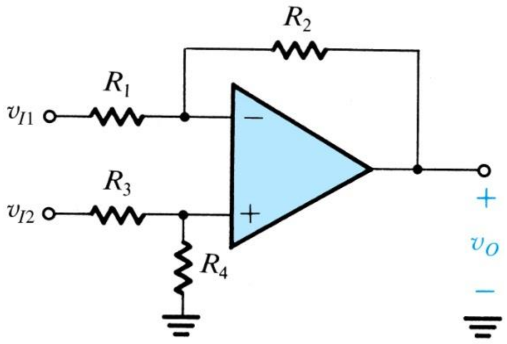


Figure 2.16 A difference amplifier.

$$\frac{R_2}{R_1} = 0.99\frac{R_4}{R_3} = 0.99\frac{100\text{ k}\Omega}{5\text{ k}\Omega}$$

$$A_d \equiv \frac{v_O}{v_{Id}} = \frac{R_2}{R_1} = \frac{100\text{ k}\Omega}{5\text{ k}\Omega} = 20\text{V/V}$$

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = \left(\frac{R_4}{R_4+R_3}\right)\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

$$= \frac{100\text{ k}\Omega}{100\text{ k}\Omega+5\text{ k}\Omega}\left(1-\left(0.99\frac{R_4}{R_3}\right)\frac{R_3}{R_4}\right)$$

$$= \frac{100\text{ k}\Omega}{105\text{ k}\Omega}(.01) = 0.0095\text{V/V}$$

Problem 2.60d

Find the voltage gain for the difference amplifier of Fig. 2.16 for the case $R_1 = R_3 = 10\ \text{k}\Omega$ and $R_2 = R_4 = 100\ \text{k}\Omega$. What is the differential input resistance R_{id}? If the two key resistance ratios (R_2/R_1) and (R_4/R_3) are different from each other by 1%, what do you expect the common-mode gain A_{cm} to be? Also, find the CMRR in this case. Neglect the effect of the ratio mismatch on the value of A_d.

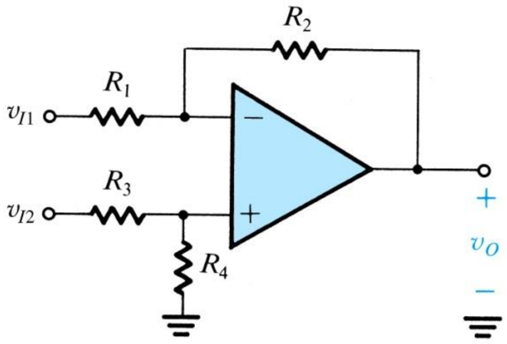


Figure 2.16 A difference amplifier.

$$\frac{R_2}{R_1} = 0.99\frac{R_4}{R_3} = 0.99\frac{100\ \text{k}\Omega}{5\ \text{k}\Omega}$$

$$A_d \equiv \frac{v_O}{v_{Id}} = \frac{R_2}{R_1} = \frac{100\ \text{k}\Omega}{5\ \text{k}\Omega} = 20\text{V/V}$$

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = 0.0095\text{V/V}$$

$$CMRR \equiv 20\log\left(\frac{|A_d|}{|A_{cm}|}\right) = 20\log\left(\frac{|20\text{V/V}|}{|0.0095\text{V/V}|}\right)$$

$$= 66.47\text{dB}$$

Problem 2.62a

For the circuit shown in Fig P2.62, express v_O as a function of v_1 and v_2. What is the input resistance seen by v_1 alone? By v_2 alone? By a source connected between the two input terminals? By a source connected to both input terminals simultaneously?

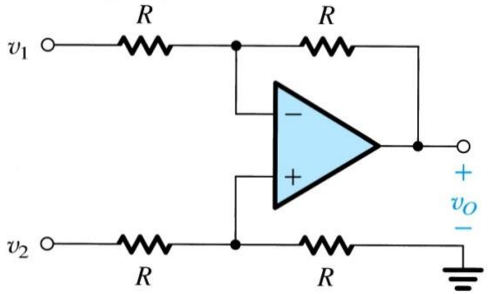


Figure P2.62

Method 1 – using nodal analysis noting $v_+ = v_-$

$$v_+ = v_2 \frac{R}{R+R} = v_- = v_1 + (v_O - v_1)\frac{R}{R+R}$$

$$\frac{v_2}{2} = v_1 + \frac{(v_O - v_1)}{2} \Rightarrow v_O = v_2 - v_1 = v_{id}$$

Method 2 – Differential Gain Equation

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} \Rightarrow \qquad A_d \equiv \frac{v_O}{v_{Id}} = \frac{R_2}{R_1} = \frac{R}{R} = 1\text{V/V} \qquad \Rightarrow v_O = A_d v_{Id} = v_{Id} = v_2 - v_1$$

Problem 2.62b

For the circuit shown in Fig P2.62, express v_O as a function of v_1 and v_2. What is the input resistance seen by v_1 alone? By v_2 alone? By a source connected between the two input terminals? By a source connected to both input terminals simultaneously?

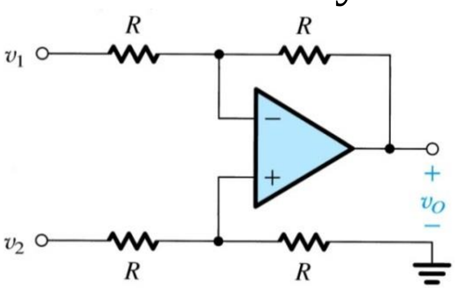


Figure P2.62

What is the input resistance seen by v_1 alone?

Ground v_2 so v_+ = gnd and v_- = gnd

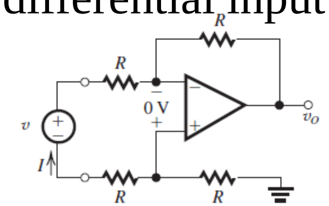
$$\Rightarrow R_{in1} = \frac{v_1}{i_1} = R$$

What is the input resistance seen by v_2 alone?

Ground v_1 so $v_+ = v_2/2$

$$\Rightarrow R_{in2} = \frac{v_2}{i_2} = R + (R \,||\, \infty) = 2R$$

What is the differential input resistance seen if v_s is applied between the two terminals?

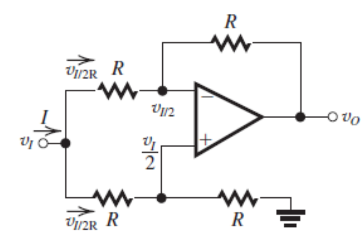


$$\Rightarrow R_{ind} = \frac{v_s}{i_s} = R + R = 2R$$

By a source connected to both input terminals simultaneously?

since $v_+ = v_-$

$$\Rightarrow R_{icm} = \frac{v_{icm}}{i_{cm}} = \frac{v_{icm}}{i_+ + i_-} = \frac{v_{icm}}{\frac{v_{icm}}{R}} = R$$



Problem 2.68 (a)

(a) Find A_d and A_{cm} for the difference amplifier circuit shown in Fig. P2.68.

(b) If the op amp is specified to operate properly as long as the common-mode voltage at its positive and negative inputs falls in the range ± 2.5 V, what is the corresponding limitation on the range of the input common-mode signal v_{icm}? (This is known as the common-mode range of the differential amplifier.)

(c) The circuit is modified by connecting a 10-kΩ resistor between node A and ground, and another 10-kΩ resistor between node B and ground. What will now be the values of A_d, A_{cm}, and the input common-mode range?

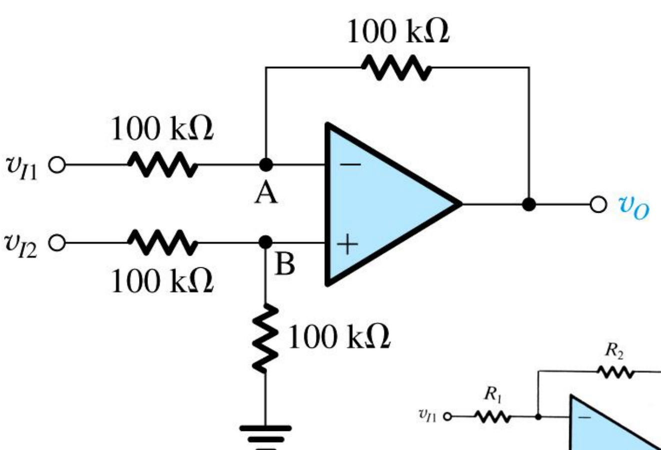
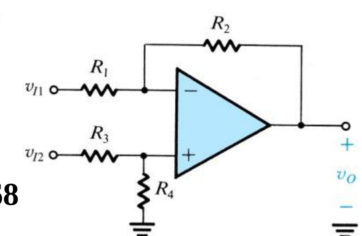


Figure P2.68



$$\frac{R_2}{R_1} = \frac{R_4}{R_3} \Rightarrow \quad A_d \equiv \frac{v_O}{v_{Id}} = \frac{R_2}{R_1} = \frac{100\text{ k}\Omega}{100\text{ k}\Omega} = 1\text{V/V}$$

$$A_{cm} \equiv \frac{v_O}{v_{Icm}} = \left(\frac{R_4}{R_4+R_3}\right)\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right) = 0\text{V/V}$$

Problem 2.68 (b)

(b) If the op amp is specified to operate properly as long as the common-mode voltage at its positive and negative inputs falls in the range ± 2.5 V, what is the corresponding limitation on the range of the input common-mode signal v_{icm}? (This is known as the common-mode range of the differential amplifier.)

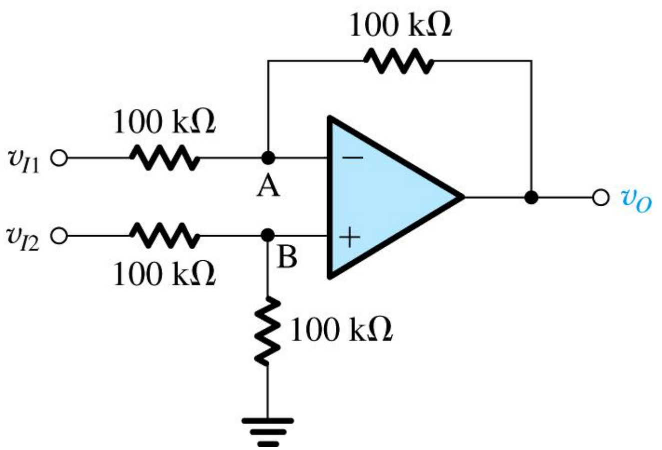
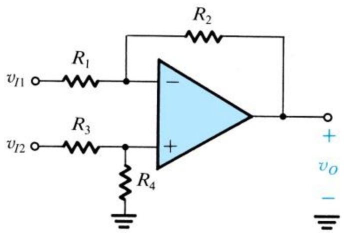


Figure P2.68

$$-2.5V \leq V_A, V_B \leq 2.5V$$

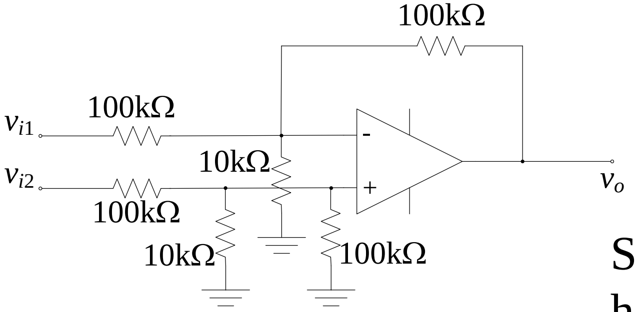
$$V_A = V_B = v_{Icm}\frac{100k}{200k} = \frac{v_{Icm}}{2}$$

$$-5V \leq V_{Icm} \leq 5V$$



Problem 2.68 (c)

(c) The circuit is modified by connecting a 10-kΩ resistor between node A and ground, and another 10-kΩ resistor between node B and ground. What will now be the values of A_d, A_{cm}, and the input common-mode range?



Using the superposition principal, ground v_{i2} and then v_{i1}

$$v_{O1} = -\frac{R_2}{R_1} v_{I1} = -\frac{100\text{k}\Omega}{100\text{k}\Omega} v_{I1} = -v_{I1}$$

Since v_{i2} is gnd, v_A (v_-) = virtual ground so 10kΩ resistor has no effect on gain

$$v_{O2} = v_{I2}\left(\frac{R_4}{R_4 + R_3}\right)\left(1 + \frac{R_2}{R_1}\right) = v_{I2}\frac{100\text{k}\Omega \,||\, 10\text{k}\Omega}{(100\text{k}\Omega \,||\, 10\text{k}\Omega) + 100\text{k}\Omega}\left(1 + \frac{100\text{k}\Omega}{100\text{k}\Omega \,||\, 10\text{k}\Omega}\right)$$

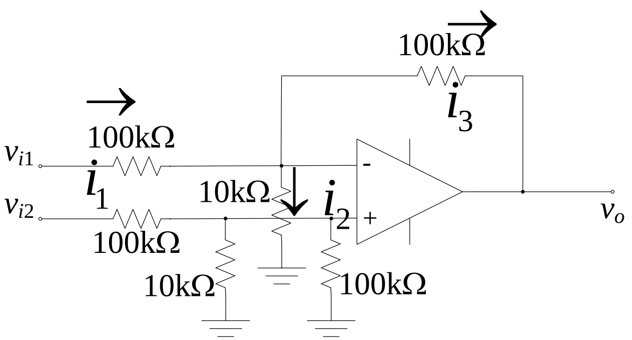
$$= v_{I2}\frac{1}{12}(1+11) = v_{I2}$$

$$v_O = v_{O1} + v_{O2} = -v_{I1} + v_{I2} = (v_{I2} - v_{I1}) = v_{Id}$$

$$A_d \equiv \frac{v_O}{v_{Id}} = 1\text{V/V}$$

Problem 2.68 (c)

(c) The circuit is modified by connecting a 10-kΩ resistor between node A and ground, and another 10-kΩ resistor between node B and ground. What will now be the values of A_d, A_{cm}, and the input common-mode range?



$$v_B = v_+ = v_{icm}\frac{100\text{k}\Omega \,||\, 10\text{k}\Omega}{(100\text{k}\Omega \,||\, 10\text{k}\Omega)+100\text{k}\Omega} = \frac{v_{icm}}{12}$$

$$v_A = v_- = v_+ = \frac{v_{icm}}{12} \qquad i_1 = \frac{v_{icm}-v_-}{100\text{k}\Omega} = \frac{v_{icm}-v_{icm}/12}{100\text{k}\Omega}$$

$$i_2 = \frac{v_-}{10\text{k}\Omega} = \frac{v_{icm}/12}{10\text{k}\Omega}$$

$$i_3 = i_1 - i_2 = \frac{v_{icm}-v_{icm}/12}{100\text{k}\Omega} - \frac{v_{icm}/12}{10\text{k}\Omega} = \frac{11v_{icm}}{1200\text{k}\Omega} - \frac{10v_{icm}}{1200\text{k}\Omega} = \frac{v_{icm}}{1200\text{k}\Omega}$$

$$v_O = v_A - i_3 100\text{k}\Omega = \frac{v_{icm}}{12} - \frac{v_{icm}}{1200\text{k}\Omega}100\text{k}\Omega = v_{icm}\left(\frac{1}{12} - \frac{100\text{k}\Omega}{1200\text{k}\Omega}\right)$$

$$A_{CM} \equiv \frac{v_O}{v_{icm}} = \frac{1}{12} - \frac{100\text{k}\Omega}{1200\text{k}\Omega} = 0\text{V/V}$$

$$-30V \le V_{Icm} \le 30V$$

Problem 2.104

An op amp is connected in a closed loop with gain of +100 utilizing a feedback resistor of 1 MΩ.

(a) If the input bias current is 200 nA, what output voltage results with the input grounded?

(b) If the input offset voltage is ±2mV and the input bias current as in (a), what is the largest possible output that can be observed with the input grounded?

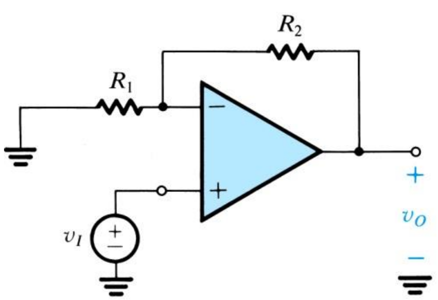
(c) If bias-current compensation is used, what is the value of the required resistor? If the offset current is no more than one-tenth the bias current, what is the resulting output offset voltage (due to offset current alone)?

(d) With bias-current compensation as in (c) in place what is the largest dc voltage at the output due to the combined effect of offset voltage and offset current?

Problem 2.104 (a)

An op amp is connected in a closed loop with gain of +100 utilizing a feedback resistor of 1 MΩ.

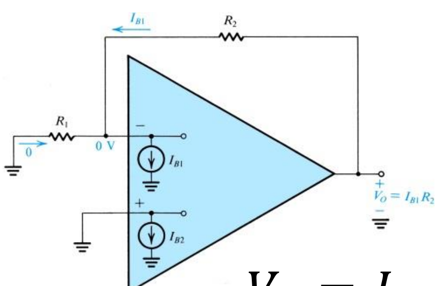
(a) If the input bias current is 200 nA, what output voltage results with the input grounded?



$$R_2 = 1\text{M}\Omega \qquad 100 = 1 + \frac{R_2}{R_1}$$

$$R_1 = \frac{R_2}{99} = \frac{1\text{M}\Omega}{99} = 10.1\text{k}\Omega$$

$$V_o = 200nA \times 1\text{M}\Omega = 0.2\text{V}$$



$$V_O = I_{B1}R_2 \approx I_B R_2$$

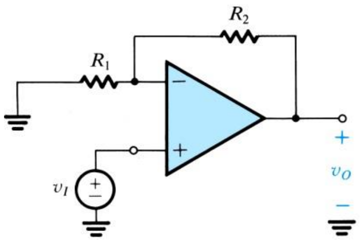
Problem 2.104 (b,c)

An op amp is connected in a closed loop with gain of +100 utilizing a feedback resistor of 1 MΩ.

(b) If the input offset voltage is ±2mV and the input bias current as in (a), what is the largest possible output that can be observed with the input grounded?

$$V_o = (2\text{mV} \times 100) + (200\text{nA} \times 1\text{M}\Omega) = 0.4\text{V}$$

(c) If bias-current compensation is used, what is the value of the required resistor? If the offset current is no more than one-tenth the bias current, what is the resulting output offset voltage (due to offset current alone)?



for bias-current compensation we connect a resistor in series with the +input terminal that is equal to:

$$R_{bc} = R_1 \parallel R_2 = 10k\Omega$$

$$V_o = I_{os} \times R_2 = 20\text{nA} \times 1\text{M}\Omega = 20\text{mV}$$

Problem 2.104 (d)

An op amp is connected in a closed loop with gain of +100 utilizing a feedback resistor of 1 MΩ.

(d) With bias-current compensation as in (c) in place what is the largest dc voltage at the output due to the combined effect of offset voltage and offset current?

$$V_o = 200\text{mV} + 20\text{mV} = 220\text{mV}$$

Problem 2.111

An inverting amplifier with nominal gain of - 50 V/V employs an op amp having a dc gain of 10^4 and a unity-gain frequency of 10^6 Hz. What is the 3-dB frequency f_{3dB} of the closed-loop amplifier? What is its gain at 0.1 f_{3dB} and at 10 f_{3dB}?

$$A_0 = 10^4 = 80\text{dB}$$

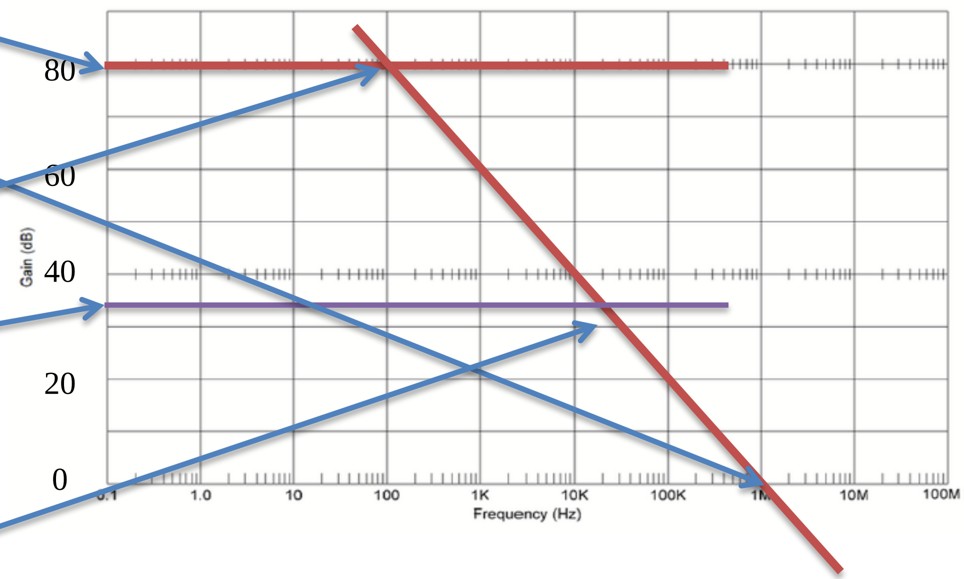
$$f_t = 10^6\,\text{Hz} = 1\text{MHz}$$

At 20 dB/decade (4 decades)

$$f_b = \frac{10^6\,\text{Hz}}{10^4} = 100\text{Hz}$$

$$A_v = |-50| = 34\text{dB}$$

$$f_{3dB} = \frac{f_T}{1+\frac{R_2}{R_1}} = \frac{10^6\,\text{Hz}}{1+50} = 19.6\text{kHz}$$



gain at 0.1 f_{3dB} = 34 dB (50V/V)

gain at 10 f_{3dB} = 14 dB (5V/V)