SAMPLE EXAM #3A
ELEG 305 SIGNALS AND SYSTEMS
SPRING 2019

Note: Last year, the third exam included Chapter 10. So, there are some problems that will not be relevant for this year's Exam #3, which includes Chapters 5, 7, and 9. I have marked the problems that cover material that will NOT be on Exam #3 using # signs right next to the problem.

Problem #1 (25 points)

Consider a continuous-time LTI system with transfer function

$$H(s) = \frac{6s}{s^2 - 2s - 8}$$

a.) (4 pts) Determine the poles and zeros for this system, and plot them in the s-plane.
b.) (6 pts) Determine and draw the three possibilities for the regions of convergence (ROCs).
c.) (9 pts) Find the impulse responses, h(t), corresponding to each of the ROCs determined in b).
d.) (4 pts) Derive the differential equation relating the input and output of this system.
e.) (2 pts) Can this system be both causal and stable? Why or why not?

Problem #2 (25 points)

Consider the following discrete-time, LTI system with input x[n] and output y[n] given by

$$x[n] = (-3)^n u[n]$$

and

$$y[n] = 4(2)^n u[n] - \left(\frac{1}{2}\right)^n u[n]$$

a.) (8 pts) Determine the transfer function, H(z), and the region of convergence (ROC).
b.) (3 pts) Determine the poles, and plot them in the z-plane, along with the ROC.
c.) (8 pts) Determine the impulse response, h[n].
d.) (4 pts) Derive the difference equation relating the input and the output of this system.
e.) (2 pts) Is this system stable? Please justify your answer.

Problem #3 (35 points)

a.) (5 pts) Compute the inverse Laplace transform of

$$H(s) = \frac{s+2}{s+1}, \quad \mathrm{Re}\{s\} > -1$$

b.) (8 pts) Compute the Laplace transform of

$$x(t) = e^{-t}\frac{d}{dt}\left(e^{-(t+1)}u(t+1)\right)$$

c.) ##### (8 pts) Compute the z-transform of

$$x[n] = u[n-2] * \left(\frac{2}{3}\right)^n u[n]$$

d.) (4 pts) Consider the following transfer function for a causal system.

$$X(s) = e^{-5s}\frac{-2(s+3)}{s(s+2)}$$

Compute (i) x(t) at $t=0^+$ and (ii) x(t) as $t \to \infty$.

e.) ##### (2 pts) Consider the following transfer function. Is the system stable? Why or why not?

$$H(z) = \frac{2z(4z^2-1)}{8z^3-26z^2+5z+3}, \quad \frac{1}{2} < |z| < 3$$

f.) (8 pts) Determine the Laplace transform and ROC of (*This is from Sample Exam #3B.*)

$$x(t) = \int_{-\infty}^{t} e^{2\tau}\sin(\tau)u(-\tau)\,d\tau$$

Problem #4 (15 points)

In many types of transmission systems (e.g., wireless communications or broadcast television), the signal arrives at the destination through multiple paths. So, the resulting signal is the sum of the signals from the multiple paths, each having a different amplitude, phase, and time of arrival. This is called a multipath channel, and leads to some of the most difficult problems to solve. For simplicity, assume that a discrete-time signal x[n] is transmitted, and there are only two paths (the second path attenuated and delayed compared to the first); so the received signal y[n] is

$$y[n] = x[n] + ax[n-1], \quad |a| < 1$$

Determine the transfer function, ROC, and impulse response for this LTI system.

Extra Credit (10 points)

Compute the transfer function, H(s), for the following system: