ELEC 300

27 Feb 2018

Plugboard: 26 holes

1st cable # ways = $\binom{26}{2}$

2nd cable # ways = $\binom{24}{2}$

3rd cable # ways = $\binom{22}{2}$

k cables # ways $= \dfrac{\prod_{i=1}^{k} \binom{28-2i}{2}}{k!} = \dfrac{\binom{26}{2}\binom{24}{2}\binom{22}{2}\cdots\binom{28-2k}{2}}{k!}$ $k=10$ 1.51×10^{14}

Wheel: 26 inputs $\Rightarrow$ 26 outputs #ways = 26!

#possible wheels 4×10^{26}

3 wheels: $26!\,(26!-1)(26!-2)$ 6.56×10^{79}

Starting positions : 26^3 17576

pins : 26^2

reflector $\frac{\prod_{i=1}^{13} \binom{28-2i}{2}}{13!} = 7.91 \times 10^{12}$

Total $K=10$ 9.3×10^{113} NZ^{378}

After capture. # ways $(5)_3 \cdot 26^3 \cdot 26^2 \cdot 1.8 \times 10^{14} = 1 \times 10^{23}$

Examples: fair
roll a 6-sided die, let N = number of the side up

N = random variable

two kinds: discrete, continuous

Exam
Thurs
3/22

$P(N=2) = \frac{1}{6}$ $P(N=5) = \frac{1}{6}$ $P(N=7) = 0$

$P(N=0) = 0$

what $P(N=k)$ for $k \in 0, 1, 2, \ldots)$

k	$P(N=k)$
0	0
1	1/6
2	1/6
3	⋮
4	
5	
6	0

Uniform Distribution — equally likely

$$P(N=K) = \begin{cases} \frac{1}{m} & K = 1, 2, \ldots, m \\ 0 & K \text{ o.w.} \end{cases} \qquad K \le 0 \text{ or } K \ge m+1$$

all non-zero ~~outcome~~ probs are the same

Geometric Distribution $\quad 0 < p \le 1$

$$P(N=K) = \begin{cases} q^{K-1} p & K = \cancel{0}, 1, 2, 3, \ldots \\ 0 & K < 0 \end{cases} \qquad q = 1-p$$

$\ell = K-1$

need to show: $q^{K-1} p \overset{?}{\ge} 0$ Yes

$$\sum_{K=1}^{\infty} q^{K-1} p \overset{?}{=} 1 = p \sum_{\ell=0}^{\infty} q^{\ell} = \frac{p}{1-q} = \frac{p}{p}$$

Poisson Distribution

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$1 = e^{-x} + xe^{-x} + \frac{x^2}{2}e^{-x} + \cdots$$

1 → Poisson probs; $\frac{x^2}{2}$ → Poisson probs

$$P(N = K) = \frac{\lambda^K}{K!}e^{-\lambda}$$

λ = average number ≥ 0