ELEG 310 Chaps 3&4 3/12/19

$\log x = \ln x$

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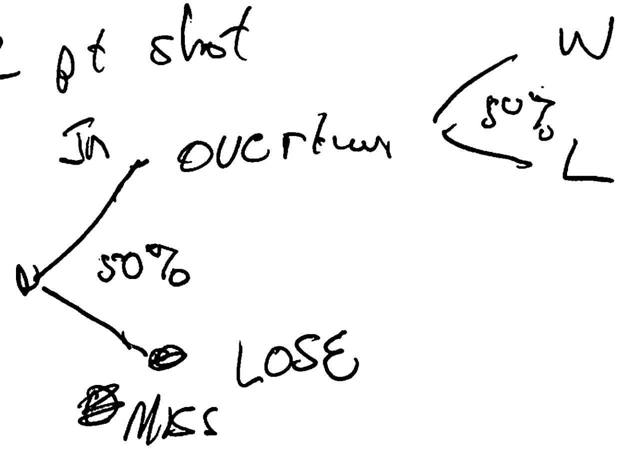
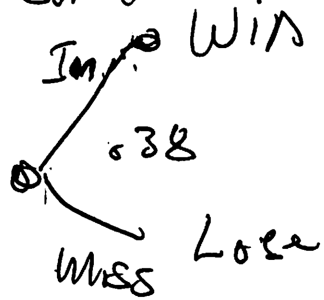
Down 2 pts, 10 secs

Shoot 2 pt shot



W – $P(W) = 0.25$

Shoot 3 pt shot



$P(Win) = .38$

Birthday Problem

$n =$ # days in year

$k =$ # people in room

P(no birthday in common)

$$= \frac{n}{n} \cdot \frac{(n-1)}{n} \cdot \frac{(n-2)}{n} \cdot \cdots \left(\frac{n-k+1}{n}\right)$$

$$= \left(1-\frac{0}{n}\right)\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{k-1}{n}\right)$$

$$\log P(\ ) = \log\left(1-\frac{0}{n}\right) + \log\left(1-\frac{1}{n}\right) + \cdots + \log\left(1-\frac{k-1}{n}\right)$$

$$\log(1+x) \approx x \quad \text{when } x \approx 0$$

$$= -\frac{0}{n} - \frac{1}{n} - \frac{2}{n} \cdots -\frac{(k-1)}{n} = -\frac{1}{n}\sum_{l=0}^{k-1} l = -\frac{1}{n}\frac{k(k-1)}{2}$$

$$P(\text{no birthday in common}) \approx e^{-k(k-1)/(2n)} \approx \frac{1}{2}$$

$$\frac{-k(k-1)}{2n} \approx +\log\frac{1}{2} = -\log 2 \Rightarrow k(k-1) \approx n\,2\log 2 \Rightarrow k \approx \sqrt{n}$$

Hypergeometric Probs

deck of cards, draw 5

$$P(3\diamondsuit, 1\heartsuit, 1\clubsuit) = \frac{\overset{\diamondsuit}{\binom{13}{3}}\overset{\heartsuit}{\binom{13}{1}}\overset{\clubsuit}{\binom{13}{1}}\overset{\spadesuit}{\binom{13}{0}}}{\binom{52}{5}}$$

What Prob (3 of one suit, 1 of another, 1 of a third)

$$= \underset{\binom{4}{1}}{4} \cdot \underset{\binom{3}{2}}{\frac{3 \cdot 2}{2}} \cdot \frac{\binom{13}{3}\binom{13}{1}\binom{13}{1}\binom{13}{0}}{\binom{52}{5}}$$

Chap 4 - Discrete RV

X = random variable

$X \; Y \; Z \quad X_1, X_2, X_3$ ← upper case

outcomes = numbers $\quad k, l, m, n$ ← lower case

$P(X = k)$ = Prob(RV X has value k) = $p(k)$

prob mass function (pmf) $= p_X(k)$

Ex. roll a die

$p(k) = \frac{1}{6} \quad k = 1, 2, 3, 4, 5, 6 \qquad p(k) = 0$ all other k

Ex. Bernoulli RV $\quad P(X=1) = p \quad P(X=0) = q = 1-p$

$$p(k) = \begin{cases} p & k = 1 \\ 1-p & k = 0 \end{cases}$$

Cumulative Distribution Function

$F_X(k) = P(X \leq k)$ ← all values of k

pmf $\equiv$ density

CDF = CDF

$F_X(u) = P(X \leq u)$

for $-\infty < u < \infty$

Continuous RVs

density $f_X(x)$

CDF $F_X(x) = \int_{-\infty}^{x} f(s)\,ds$

$F_X(x) = P(X \leq x)$

for $-\infty < x < \infty$