SAMPLE EXAM #1A
ELEG 305 SIGNALS AND SYSTEMS
SPRING 2019

Problem #1 (15 points)
Why is Fourier analysis an important tool for the study of linear, time-invariant systems?

Problem #2 (25 points)
a.) (2 pts) Consider a discrete-time signal

$$x[n] = e^{-(1+j\pi)n}$$

Is this signal periodic? Explain why or why not.

b.) (4 pts) Evaluate the following integral

$$\int_0^{\infty} e^{-2t}\delta(t-2)dt$$

c.) (4 pts) Evaluate the following sum

$$\sum_{k=0}^{\infty}(1+k)^3\delta[k-2]$$

d.) (15 pts) Consider a continuous-time, linear, time-invariant system with input, x(t), and output, y(t), related by

$$y(t) = \int_{t-1}^{t} x(\tau)\,d\tau$$

i.) Determine the system impulse response. (Do NOT leave your result in integral form.)

ii.) Is this system causal? Please **explain** why or why not (a simple "yes" or "no" will get you no points even if it is the correct answer).

iii.) Is this system stable? Again, please **explain** why or why not.

Problem #3 (15 points)
Consider a continuous-time, linear, time-invariant system with input

$$x(t) = u(t) - u(t-1)$$

Assume the output for this input is

$$y(t) = 2x(t-1) + x(t-2)$$

Derive and sketch the output when the input is g(t) = x(t-1) + x(t-3).

Problem #4 (25 points)
Compute the convolution y(t) = x(t)* h(t) for

$$x(t) = u(t+1) - u(t-1)$$

$$h(t) = u(4-t) - u(1-t)$$

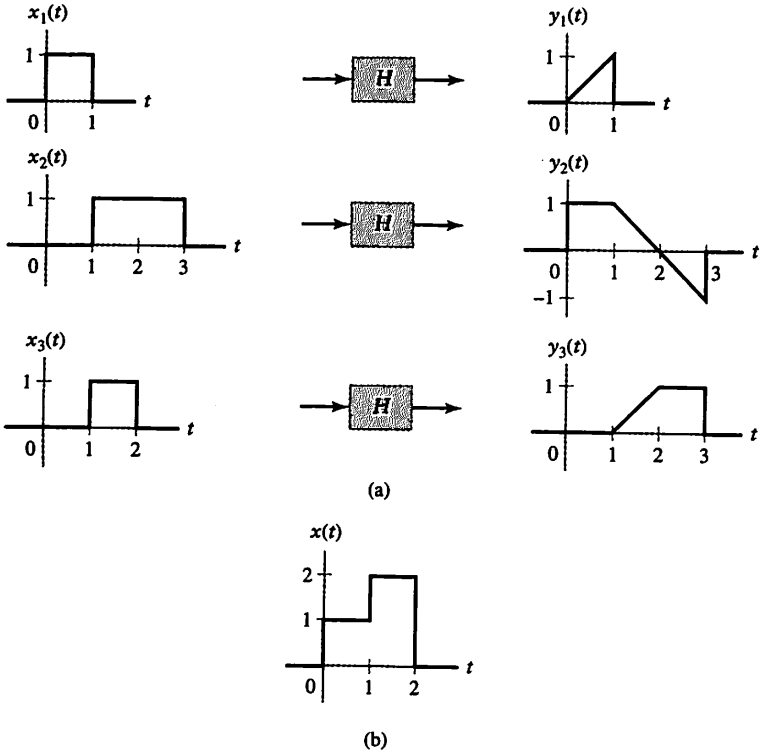
Problem #5 (20 points)

A discrete-time signal (for example, a sampled audio signal or EKG signal) is processed by a linear, time-invariant system to remove some of the noise (we call this *filtering*). The filter has impulse response $h[n] = \beta^n u[n]$, where $u[n]$ is the unit step function and where β is positive. What is the discrete-time signal at the output of the filter if the input is $x[n] = \alpha^n u[n-1]$, where α is positive and not equal to β?

Extra Credit (10 points)

This is one of the problems from Sample Exam #1A.

A *linear* system has the following input-output pairs:



Answer the following questions, and please **explain** your answers (as before, a simple "yes" or "no" will get you no points even if it is the correct answer).

a.) (5 pts) Could this system be causal?

b.) (5 pts) Could this system be time-invariant?