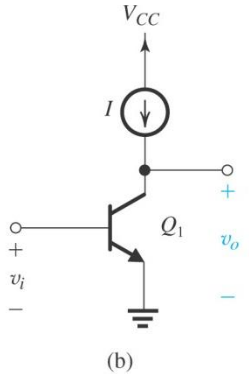
ELEG 312
Homework #2
Solutions

Problems 8.26, 8.49, 8.66, 8.71, 8.79, 8.81, and 8.83

Problem 8.26a

Consider the CE amplifiers of Fig. 8.13(b) for the case of I = 0.5 mA, β = 100, and V_A = 100 V. Find R_{in}, A_{vo}, and R_o. If it is required to raise R_{in} by a factor of 5 by changing I, what value of I is required, assuming that β remains unchanged? What are the new values of A_{vo} and R_o? If the amplifier is fed with a signal source having R_{sig} = 5 kΩ and is connected to a load of 100-kΩ resistance, find the overall voltage gain, v_o/v_{sig}.



(b)

$$g_m = \frac{I_C}{V_T} = \frac{0.5\text{mA}}{25\text{mV}} = 20\frac{\text{mA}}{\text{V}}$$

$$R_{in} = r_\pi = \frac{\beta}{g_m} = \frac{100}{20\,\text{mA}/V} = 5\text{k}\Omega$$

$$R_o = r_o = \frac{V_A}{I_C} = \frac{100\text{V}}{0.5\text{mA}} = 200\text{k}\Omega$$

$$A_{vo} = -g_m r_o = -\left(20\frac{\text{mA}}{\text{V}}\right)(200\text{k}\Omega) = -4000\frac{\text{V}}{\text{V}}$$

$$R_{in\_new} = 5R_{in} = 25\text{k}\Omega = \frac{\beta}{g_m} = \frac{\beta}{I_C/V_T}$$

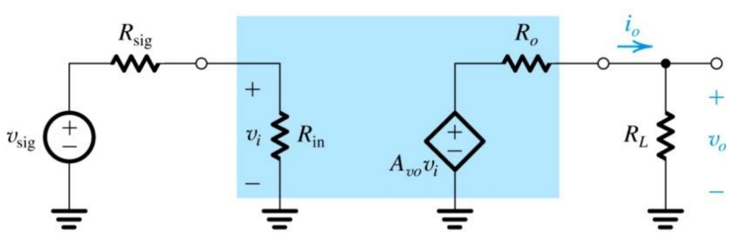
$$I_{C\_new} = \frac{\beta V_T}{R_{in\_new}} = \frac{2.5\text{V}}{25\text{k}\Omega} = 0.1\text{mA}$$

$$R_{o\_new} = r_o = \frac{V_A}{I_C} = \frac{100\text{V}}{0.1\text{mA}} = 1\text{M}\Omega$$

$$A_{vo\_new} = -g_m r_o = -\left(4\frac{\text{mA}}{\text{V}}\right)(1\text{M}\Omega) = -4000\frac{\text{V}}{\text{V}}$$

Problem 8.26b

Consider the CE amplifiers of Fig. 8.13(b) for the case of $I = 0.5$ mA, $\beta = 100$, and $V_A = 100$ V. Find R_{in}, A_{vo}, and R_o. If it is required to raise R_{in} by a factor of 5 by changing I, what value of I is required, assuming that β remains unchanged? What are the new values of A_{vo} and R_o? If the amplifier is fed with a signal source having $R_{sig} = 5$ kΩ and is connected to a load of 100-kΩ resistance, find the overall voltage gain, v_o/v_{sig}.



$$R_{in\_new} = 25\text{k}\Omega$$

$$R_{o\_new} = r_o = 1\text{M}\Omega$$

$$A_{vo\_new} = -4000\frac{\text{V}}{\text{V}}$$

$$G_v \equiv \frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{in}+R_{sig}} A_{vo} \frac{R_L}{R_L+R_o}$$

$$= \frac{25\text{k}\Omega}{25\text{k}\Omega + 5\text{k}\Omega}\left(-4000\frac{\text{V}}{\text{V}}\right)\frac{0.1\text{M}\Omega}{0.1\text{M}\Omega + 1\text{M}\Omega} = -303.03\frac{\text{V}}{\text{V}}$$

Problem 8.49a,b

Transistor Q_1 in the circuit of Fig. P8.49 is operating as a CE amplifier with an active load provided by transistor Q_2, which is the output transistor in a current mirror formed by Q_2 and Q_3. (Note that the biasing arrangement for Q_1 is *not* shown.)

(a) Neglecting the finite base currents of Q_2 and Q_3 and assuming that their V_{BE} = 0.7 V and that Q_2 has five times the area of Q_3, find the value of I.

$$I_{REF} = I_{C3} = \frac{V_{CC} - V_{BE3}}{R} = \frac{3\text{V} - 0.7\text{V}}{46\text{k}\Omega} = 0.05\text{mA}$$

$$I = I_{C2} = 5I_{C3} = 5I_{REF} = 0.25\text{mA}$$

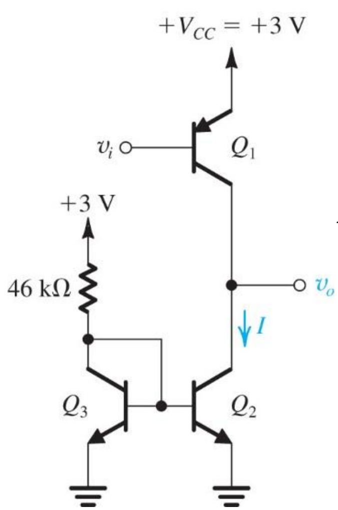


Figure P8.49

(b) If Q_1 and Q_2 are specified to have $|V_A|$ = 30 V, find r_{01} and r_{02} and hence the total resistance at the collector of Q_1.

$$r_{o1} = r_{o2} = \frac{V_A}{I_C} = \frac{30\text{V}}{0.25\text{mA}} = 120\text{k}\Omega$$

$$R_o = r_{o1} \,||\, r_{o2} = 60\text{k}\Omega$$

Problem 8.49c,d

Transistor Q_1 in the circuit of Fig. P8.49 is operating as a CE amplifier with an active load provided by transistor Q_2, which is the output transistor in a current mirror formed by Q_2 and Q_3. (Note that the biasing arrangement for Q_1 is *not* shown.)

(c) Find $r_{\pi 1}$ and g_{m1} assuming that $\beta_1 = 50$.

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{0.25\text{mA}}{25\text{mV}} = 10\frac{\text{mA}}{\text{V}}$$

$$r_{\pi 1} = \frac{\beta_1}{g_m} = \frac{50}{10\,\text{mA}/V} = 5\text{k}\Omega$$

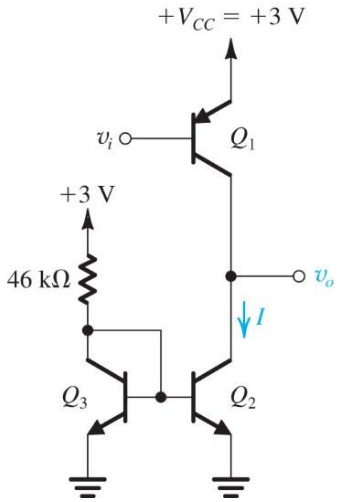


Figure P8.49

(d) Find R_{in}, A_v, and R_o.

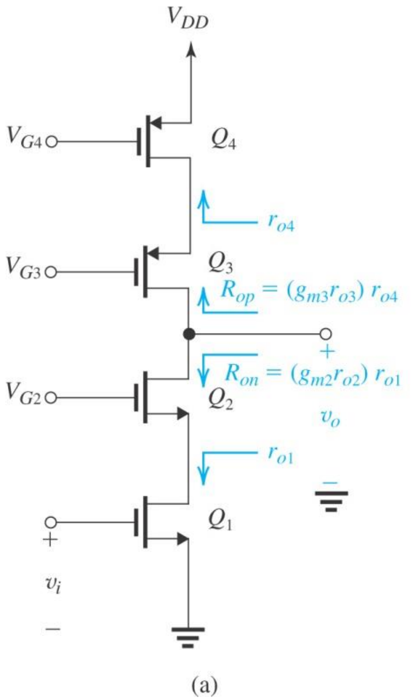
$$R_{in} = r_{\pi 1} = 5\text{k}\Omega$$

$$R_o = r_{o1} \,||\, r_{o2} = 60\text{k}\Omega$$

$$A_v = -g_m R_o = -\left(10\frac{\text{mA}}{\text{V}}\right)60\text{k}\Omega = -600\frac{\text{V}}{\text{V}}$$

Problem 8.66

The cascode amplifier of Fig. 8.33 is operated at a current of 0.2 mA with all devices operating at $|V_{OV}| = 0.20$ V. All devices have $|V_A| = 4$ V. Find g_{m1}, the output resistance of the amplifier, R_{on}, the output resistance of the current source, R_{op}, the overall output resistance, R_o, and the voltage gain, A_v.



$$g_m = \frac{2I_D}{V_{OV}} = \frac{2(0.2\text{mA})}{0.2\text{V}} = 2\frac{\text{mA}}{\text{V}}$$

Since all devices have $|V_A| = 4$ V

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{V_A}{I_D} = \frac{4\text{V}}{0.2\text{mA}} = 20\text{k}\Omega$$

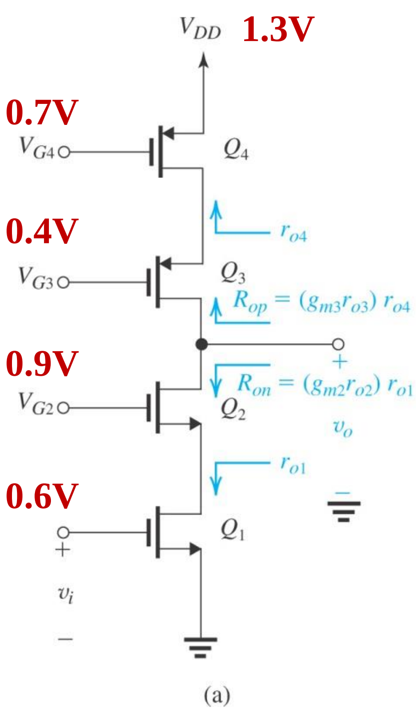
$$R_{on} = R_{op} = (g_m r_o) r_o = \left(2\frac{\text{mA}}{\text{V}} 20\text{k}\Omega\right) 20\text{k}\Omega = 800\text{k}\Omega$$

$$R_o = R_{on} \,||\, R_{op} = 400\text{k}\Omega$$

$$A_v = -g_m R_o = -\left(2\frac{\text{mA}}{\text{V}}\right)(400\text{k}\Omega) = -800\frac{\text{V}}{\text{V}}$$

Problem 8.71

Consider the cascode amplifier of Fig. 8.33 with the dc component at the input V_I = 0.6 V, V_{G2} = 0.9 V, V_{G3} = 0.4 V, V_{G4} = 0.7 V, and V_{DD} = 1.3 V. If all devices are matched, that is $k_{n1} = k_{n2} = k_{p3} = k_{p4}$, and have equal $|V_t|$ of 0.4 V, what is the overdrive voltage at which the four transistors are operating? What is the allowable voltage range at the output?



$$V_{OV} = V_{GS} - V_t \qquad \text{For } Q_1 \quad V_{OV1} = V_{GS1} - V_t = 0.6\text{V} - 0.4\text{V} = 0.2\text{V}$$

$$I_D = \frac{1}{2}k'_{n,p}\left(\frac{W}{L}\right)|V_{OV}|^2$$

Since the current is the same through all devices and they have matched $k'_{n,p}$ then $|V_{OV}|$ and $|V_{GS}|$ are the same for all devices

To be in saturation , $|V_{SD}| \geq |V_{OV}|$

$$V_{D2\min} = (V_{G2} - V_{GS2}) + V_{OV} = (0.9\text{V} - 0.6\text{V}) + 0.2\text{V} = 0.5\text{V}$$

$$V_{D3\max} = (V_{G3} + V_{GS3}) - V_{OV} = (0.4\text{V} + 0.6\text{V}) - 0.2\text{V} = 0.8\text{V}$$

The output range must be between 0.5 V and 0.8 V

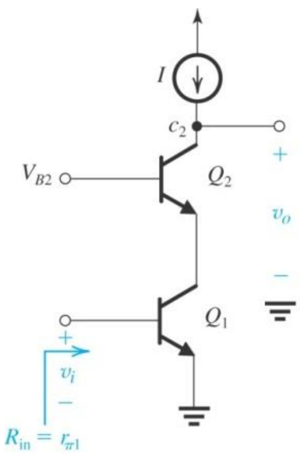
Problem 8.79

A bipolar cascode amplifier has a current-source load with an output resistance βr_o. Let $\beta = 50$, $|V_A| = 100$ V, and $I = 0.2$ mA . Find the voltage gain A_v.

$$g_{m1} = g_2 = \frac{I_C}{V_T} = \frac{0.2\text{mA}}{25\text{mV}} = 8\frac{\text{mA}}{\text{V}}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_C} = \frac{100\text{V}}{0.2\text{mA}} = 500\text{k}\Omega$$

$$r_\pi = \frac{\beta}{g_m} = \frac{50}{8\,\text{mA/V}} = 6.25\text{k}\Omega$$



(a)

$$R_{op} = \beta r_o = (50)500\text{k}\Omega = 25\text{M}\Omega$$

$$R_{on} = (g_{m2} r_{o2})(r_{o1} \,||\, r_{\pi 2}) = \left(8\frac{\text{mA}}{\text{V}}500\text{k}\Omega\right)(500\text{k}\Omega \,||\, 6.25\text{k}\Omega) = 24.69\text{M}\Omega$$

$$R_o = R_{op} \,||\, R_{on} = 25\text{M}\Omega \,||\, 24.69\text{M}\Omega = 12.422\text{M}\Omega$$

$$A_v = -g_{m1} R_o = \beta r_o = -\left(8\frac{\text{mA}}{\text{V}}\right)12.422\text{M}\Omega = 99.4\frac{\text{kV}}{\text{V}}$$

Problem 8.81a

In this problem, we will explore the difference between using a BJT as cascode device and a MOSFET as cascode device. Refer to Fig. P8.81. Given the following data, calculate G_m, R_o, and A_{vo} for the circuits (a) and (b):

I = 100 μA, β = 125, $\mu_n C_{ox}$ = 400 μA/V^2, W/L = 25, V_A = 1.8 V

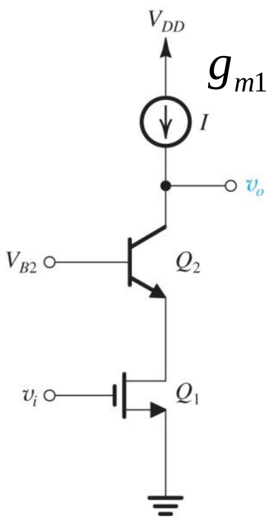


Figure P8.81

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} = \sqrt{2\left(0.4\frac{\text{mA}}{\text{V}^2}\right)25(0.1\text{mA})} = 1.414\frac{\text{mA}}{\text{V}}$$

$$r_{o1} = \frac{V_A}{I_D} = \frac{1.8\text{V}}{0.1\text{mA}} = 18\text{k}\Omega$$

$$g_{m2} = \frac{I}{V_T} = \frac{0.1\text{mA}}{25\text{mV}} = 4\frac{\text{mA}}{\text{V}} \qquad r_{o2} = \frac{V_A}{I_C} = \frac{1.8\text{V}}{0.1\text{mA}} = 18\text{k}\Omega$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{125}{4\text{mA/V}} = 31.25\text{k}\Omega$$

$$G_m = g_{m1} = 1.414\frac{\text{mA}}{\text{V}}$$

$$R_o = (g_{m2}r_{o2})(r_{o1} \,\|\, r_{\pi 2}) = \left(\left(4\frac{\text{mA}}{\text{V}}\right)18\text{k}\Omega\right)(18\text{k}\Omega \,\|\, 31.25\text{k}\Omega) = 822.34\text{k}\Omega$$

$$A_{vo} = -G_m R_o = -\left(1.414\frac{\text{mA}}{\text{V}}\right)822.34\text{k}\Omega = -1163\frac{\text{V}}{\text{V}}$$

Problem 8.81b

In this problem, we will explore the difference between using a BJT as cascode device and a MOSFET as cascode device. Refer to Fig. P8.81. Given the following data, calculate G_m, R_o, and A_{vo} for the circuits (a) and (b):

I = 100 μA, β = 125, $\mu_n C_{ox}$ = 400 μA/V^2, W/L = 25, V_A = 1.8 V

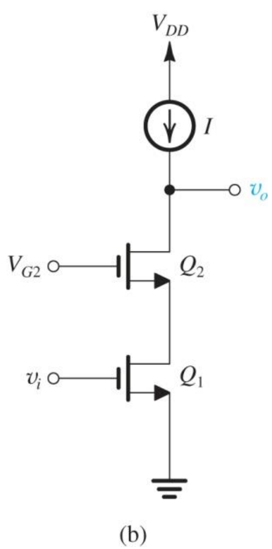


Figure P8.81

$$g_{m1} = g_{m2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} = \sqrt{2\left(0.4\frac{\text{mA}}{\text{V}^2}\right)25(0.1\text{mA})} = 1.414\frac{\text{mA}}{\text{V}}$$

$$r_{o1} = r_{o2} = \frac{V_A}{I_D} = \frac{1.8\text{V}}{0.1\text{mA}} = 18\text{k}\Omega$$

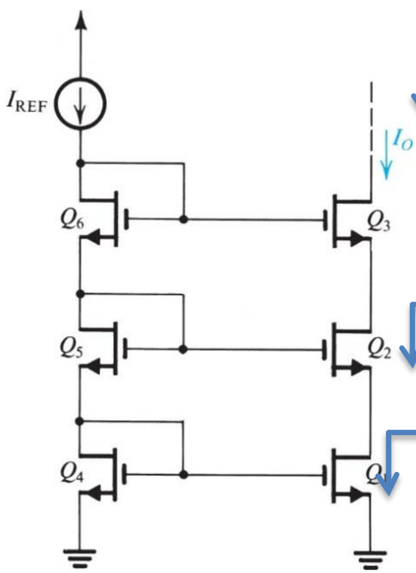
$$G_m = g_{m1} = 1.414\frac{\text{mA}}{\text{V}}$$

$$R_o = (g_{m2} r_{o2}) r_{o1} = \left(\left(4\frac{\text{mA}}{\text{V}}\right)18\text{k}\Omega\right)18\text{k}\Omega = 458\text{k}\Omega$$

$$A_{vo} = -G_m R_o = -\left(1.414\frac{\text{mA}}{\text{V}}\right)458\text{k}\Omega = -648\frac{\text{V}}{\text{V}}$$

Problem 8.83

Find the output resistance of the double-cascode current mirror of Fig. P8.83.



$$(g_{m3}r_{o3})(g_{m2}r_{o2})r_{o1}$$

$$(g_{m2}r_{o2})r_{o1}$$

$$r_{o1}$$

Figure P8.83

$$R_o = (g_{m3}r_{o3})(g_{m2}r_{o2})r_{o1}$$