ELEG 310 2/8/2018

Exp - whatever we do

outcomes: elementary results of exp

a b c x x_1 x_2 x_3

events - sets of outcomes

$A = \{x_1, x_2, x_5\}$

$B = \{x_1, x_3, x_7\}$

$C = \{x_2, x_4\}$

if x_1 is TRUE
then A is TRUE
B is TRUE
C is False

Compound Experiment

$$\mathcal{E} = \mathcal{E}_1 \times \mathcal{E}_2$$

$\mathcal{E}_1$ = flip coin

$\mathcal{E}_2$ = select person

Outcomes $(h, \text{Alice}), (h, \text{Bob}), (h, \text{Town}) \ldots$

$(t, \text{Alice}), (t, \text{Bob}), (t, \text{Town}) \ldots,$

$$A = \{\text{"tails"}\} = \{(t, \text{Alice}), (t, \text{Bob}), (t, \text{Town}) \ldots\}$$

$$B = \{\text{"bob"}\} = \{(h, \text{Bob}), (t, \text{Bob})\}$$

S = sample space = $\{$ all outcomes $\}$

ϕ = empty set = $\{\}$

Ex. Exp = flip coin 3 times

$\Rightarrow$ 8 outcomes

$\Rightarrow 2^8 = 256$ possible events

Probability — A = event,

$P(A)$ = real number

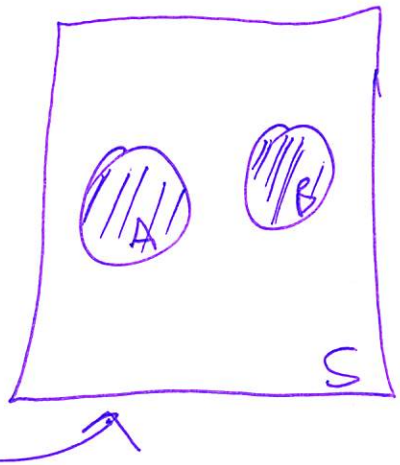
$$0 \leq P(A) \leq 1$$

$Pr(A)$

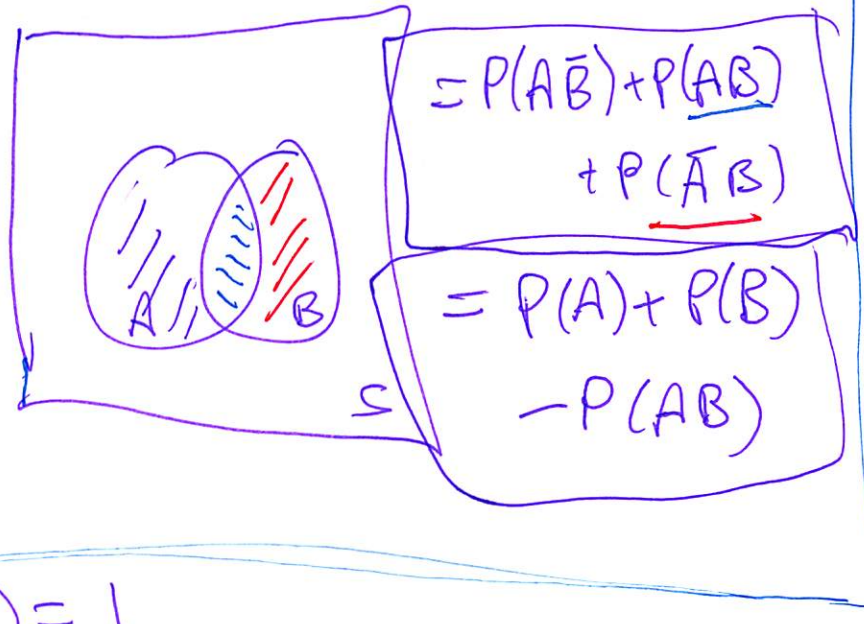
= measure of how likely event A is to occur

Three Axioms:

1. $P(A) \geq 0$ for any A
2. $P(S) = 1$
3. if $AB = \emptyset$, $P(A \cup B) = P(A) + P(B)$



$P(A \cup B) =$



$= P(A\bar{B}) + P(AB) + P(\bar{A}B)$

$= P(A) + P(B) - P(AB)$

$\leq$

$0 \leq P(\bar{A}) = 1 - P(A)$

$\uparrow$ Axiom I $\Rightarrow$ $P(A) \leq 1$

$\Rightarrow$ $\boxed{0 \leq P(A) \leq 1}$

$P(S) = 1$

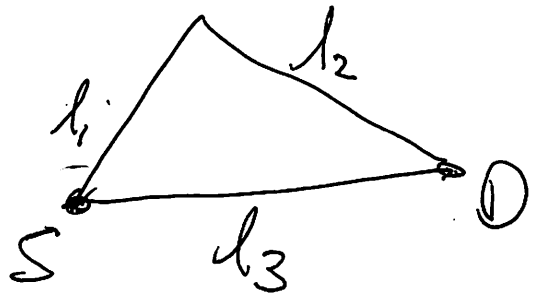
$S = A \cup \bar{A}$ $\quad \bar{A} = A$ complement

$A\bar{A} = \phi$

$1 = P(S) = P(A \cup \bar{A}) = P(A) + P(\bar{A}) \Rightarrow \boxed{P(\bar{A}) = 1 - P(A)}$

$\uparrow$ Axiom II $\quad \uparrow$ Axiom III

Ex.



each link works with prob $0 \le p \le 1$

Independence A & B are ind if $P(AB) = P(A)P(B)$

$P(S \to D) = ?$

l_1	l_2	l_3	prob	$S \to D$
0	0	0	$(1-p)^3$	0
0	0	1	$(1-p)^2 p$	1
0	1	0	$(1-p)^2 p$	0
0	1	1	$(1-p) p^2$	1
1	0	0	$(1-p)^2 p$	0
1	0	1	$(1-p) p^2$	1
1	1	0	$(1-p) p^2$	1
1	1	1	p^3	1
			1	

$P(\text{any link fails}) = 1-p$

$P(3 \text{ failed links}) =$

$P(S \to D) = \{001, 011, 101, 110, 111\}$

$= \{001\} \cup \{011\} \cup \{101\} \cup \{110\} \cup \{111\}$

$= P(001) + P(011) + P(101) + P(110) + P(111)$

$= (1-p)^3 p + (1-p) p^2 + (1-p) p^2 + (1-p) p^2 + p^3$

$$P(S \to D) = 1 - P(S \not\to D) = 1 - (1-p)^3 - (1-p)^2 p - (1-p) p$$

Ex. flu

$$P(\text{Ef}lu) = 0.01$$

$$n = 100$$

$$P(\text{nobody gets } flu) = 1 - (1-p)^{100}$$