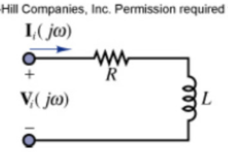
Problem 1

Solution:



Known quantities:
Figure P6.7.

Find:

a) How the input impedance, $Z(j\omega) = \dfrac{\mathbf{V}_i(j\omega)}{\mathbf{I}_i(j\omega)}$, behaves at extremely high or low frequencies.

b) An expression for the input (or driving point) impedance.

c) Show that this expression can be manipulated into the form: $Z(j\omega) = R\left[1 + j\dfrac{\omega L}{R}\right]$

d) Determine the frequency $\omega = \omega_C$ for which the imaginary part of the expression in c) is equal to 1.

e) Estimate the magnitude and angle of $Z[j\omega]$ at $\omega = 10^5, 10^6, 10^7$ rad/s.

Analysis:

a)

$$\text{As } \omega \to \infty, \quad Z_L \to \infty \Rightarrow Open \Rightarrow Z \to \infty$$

$$\text{As } \omega \to 0, \quad Z_L \to 0 \Rightarrow Short \Rightarrow Z \to R$$

b)

$$KVL: -\mathbf{V}_i + \mathbf{I}_i Z_R + \mathbf{I}_i Z_L = 0$$

$$Z(j\omega) = \frac{\mathbf{V}_i}{\mathbf{I}_i} = Z_L + Z_R = j\omega L + R$$

c) $Z(j\omega) = R + j\,\omega L = R\left(1 + j\dfrac{\omega L}{R}\right)$

d) $\dfrac{\omega_C L}{R} = 1 \quad \Rightarrow \quad \omega_C = \dfrac{R}{L} = \dfrac{2000}{2\cdot 10^{-3}} = 1000 \ \text{k}\dfrac{\text{rad}}{\text{s}}$

e)

$$Z\left(100\text{k}\frac{\text{rad}}{\text{s}}\right) = R\left(1 + j\frac{2\cdot 10^{-3}\cdot 10^{5}}{2000}\right) = 2000(1 + j\,0.1) = 2.01\,\text{k}\Omega\angle 5.71°$$

$$Z\left(1\text{M}\frac{\text{rad}}{\text{s}}\right) = R\left(1 + j\frac{2\cdot 10^{-3}\cdot 10^{6}}{2000}\right) = 2000(1 + j\,1) = 2.82\,\text{k}\Omega\angle 45.00°$$

$$Z\left(10\text{M}\frac{\text{rad}}{\text{s}}\right) = R\left(1 + j\frac{2\cdot 10^{-3}\cdot 10^{7}}{2000}\right) = 2000(1 + j\,10) = 20.10\,\text{k}\Omega\angle 84.29°$$

Note, in particular, the behavior of the impedance one decade below and one decade above the cutoff frequency.

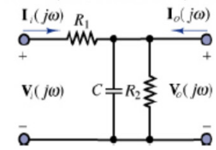
Problem 2

Solution:

Known quantities:

In the circuit of Figure P6.9:

$R_1 = 1.3\ \text{k}\Omega \quad R_2 = 1.9\ \text{k}\Omega \quad C = 0.5182\ \Omega\text{F}$



Find:

a) How the voltage transfer function:

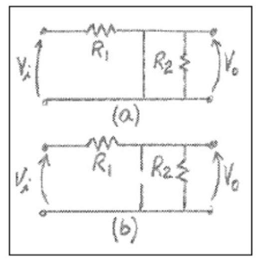
$$H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]}$$ behaves at extremes of high and low frequencies.

b) An expression for the voltage transfer function, showing that it can be manipulated into the form:

$$H_v[j\omega] = \frac{H_o}{1 + j f[\omega]} \quad \text{Where:} \quad H_o = \frac{R_2}{R_1 + R_2} \quad f[\omega] = \frac{\omega R_1 R_2 C}{R_1 + R_2}$$

c) The "cutoff" frequency at which $f[\omega] = 1$ and the value of H_o in dB.

Analysis:



a)

$$As\ \omega \to \infty: \quad Z_C \to 0\angle -90^0 \Rightarrow Short$$

$$VD: \quad H_v \to 0\angle -90^0$$

$$As\ \omega \to 0: \quad Z_C \to \infty\angle -90^0 \Rightarrow Open$$

$$VD: \quad H_v \to \frac{R_2}{R_1 + R_2}\angle 0^0$$

b)

$$Z_{eq} = \frac{Z_C\ Z_{R2}}{Z_C + Z_{R2}} = \frac{[\frac{1}{j\omega C}][R_2]}{\frac{1}{j\omega C} + R_2}\ \frac{j\omega C}{j\omega C} = \frac{R_2}{1 + j\,\omega R_2 C}$$

$$VD: \quad H_v[j\omega] = \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{Z_{eq}}{Z_{R1} + Z_{eq}} = \frac{\frac{R_2}{1 + j\,\omega R_2 C}}{R_1 + \frac{R_2}{1 + j\,\omega R_2 C}}\ \frac{1 + j\omega R_2 C}{1 + j\omega R_2 C} =$$

$$= \frac{R_2}{R_1 + R_2 + j\,\omega R_1 R_2 C} = \frac{R_2}{R_1 + R_2}\ \frac{1}{1 + j\,\frac{\omega R_1 R_2 C}{R_1 + R_2}}$$

c)

$$f[\omega_c] = \frac{\omega_c\ R_1 R_2 C}{R_1 + R_2} = 1 \qquad \omega_c = \frac{1300 + 1900}{[1300][1900][0.5182 \cdot 10^{-6}]} = 2.5\ \text{k}\frac{\text{rad}}{\text{s}}$$

$$H_o = \frac{R_2}{R_1 + R_2} = \frac{1900}{1300 + 1900} = 0.5938$$

Problem 3

Solution:

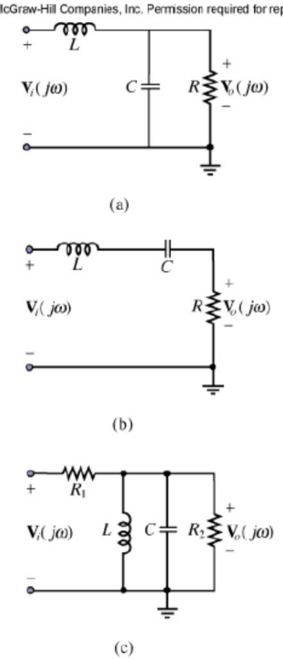
Known quantities:
Figure P6.51.

Find:
What kind of filters are the ones shown in Figure P6.51.

Analysis:
In a), as frequency increases, the impedance of the capacitor decreases and the impedance of the inductor increases. Both effects cause the magnitude of the output voltage to decrease so this is a 2[nd] order low pass filter. Note that L and C are connected neither in series nor parallel and do not form a resonant circuit.

In b), L and C are connected in series and form a series resonant circuit with an impedance which is minimum at the resonant frequency and larger above and below the resonant frequency. This series resonant circuit is in series with the output giving, because of voltage division, a maximum output voltage at the resonant frequency and less at higher and lower frequencies. Therefore, b) is a band-pass filter.

In c), L and C are connected in parallel and form a parallel resonant circuit with an impedance which is maximum at the resonant frequency and smaller above and below the resonant frequency. This parallel resonant circuit is in parallel with the output giving, because of voltage division, a maximum output at the resonant frequency and less at higher and lower frequencies. Therefore, c) is a band-pass filter.



Problem 4

Solution:

Known quantities:

Figure P6.52.

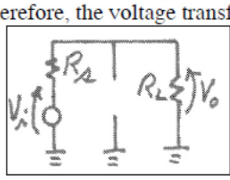
Find:

What kind of filters are the ones shown in Figure P6.52.

Analysis:

None of the inductors or capacitors is in series or parallel with any other. Therefore, there are no series or parallel resonant circuits and none of the circuits shown is band pass or band stop filters.

Circuits a) and d): As frequency approaches infinity, the inductors can be modeled as open circuits and the capacitors as short circuits. Therefore, the voltage transfer function approaches zero.



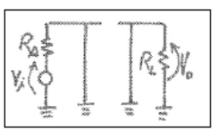
As frequency approaches zero, the inductors can be modeled as short circuits and the capacitors as open circuits.

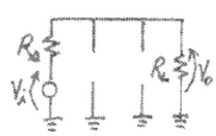
Then: $VD: \quad H_v \rightarrow \dfrac{R_L}{R_s + R_L}$

Therefore, circuits a) and d) are low pass filters.

Circuits b) and c) As frequency approaches infinity, the inductors can be modeled as open circuits and the capacitors as short circuits.

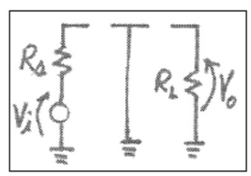
Then, $VD: \quad H_v \rightarrow \dfrac{R_L}{R_s + R_L}$

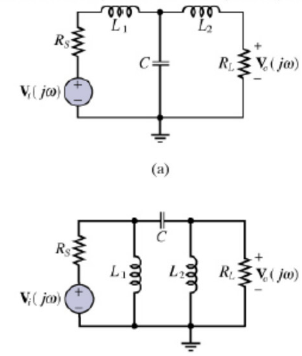




Therefore: As frequency approaches zero, inductors can be modeled as short circuits and the capacitors as open circuits. The voltage transfer function approaches zero. Therefore, circuits b) and c) are high pass filters.

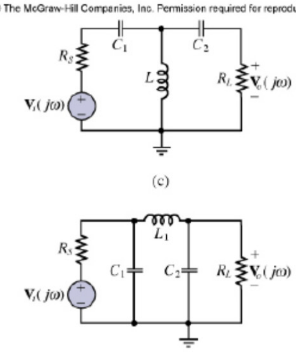
Note: Multiple capacitors and inductors give higher order low and high pass filter. Better performance is obtained outside the pass band where the response for these circuits decreases by 60 dB/decade. In first order filters, the response decreases by only 20 dB/decade.





(a)

(b)



(c)

(d)