MEEG 311 - Homework #3

Due: September 20, 2018

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Background: Current conducting elements and magnetic fields interact in two ways that are particularly important to understanding the operation of many electromechanical actuators and sensors.

1. Lorenz's law: If a current i (in A) flows in a conductor of length l (in m) arranged at right angles in a magnetic field B (in T), then it experiences a force at right angles to the plane defined by i and B with magnitude (in N):
$$F = B \cdot l \cdot i \tag{1}$$
Essentially, (1) implies that for a given magnetic field and geometry, the force experienced by a current conducting element is proportional to the current. Sometimes (1) is called the law of motors.
2. Faraday's law: If a conductor of length l (in m) is moving in a magnetic field B (in T) at a velocity v (in m/s) at mutually right angles, an electric voltage is established across the conductor with magnitude:
$$e = B \cdot l \cdot v \tag{2}$$
Similarly, (2) implies that for a given magnetic field and geometry, the voltage developed across the conductor is proportional to the velocity. The polarity of the voltage is specified by Lenz's rule (it is such that the induced voltage would tend to create a current—current is created only when the conductor is part of a closed loop—which in turn would create a magnetic field that tends to oppose B). Sometimes (2) is called the law of generators.

It is important to associate forces (or torques for suitable geometries) with currents and voltages with motion (i.e. velocities). As discussed in class, in a motor (or a generator), both phenomena are present. Here, we will use them to model a loudspeaker.

Problem 1: A typical geometry of a loudspeaker is shown in Figure 1. The permanent magnet establishes a radial magnetic field in the cylindrical gap between the poles of the magnet. Current i flows in the conductor wound on the bobbin, resulting in the development of a force F that causes the voice coil to move left and right (one dimensional motion), producing sound. The effects of the air can be modeled as if the cone had equivalent mass M and viscous friction coefficient b. Assume that the magnet establishes a uniform field $B = 0.5$T and that the bobbin has 20 turns at a 2cm diameter so that the total length of the wire is $l = 20 \times \frac{2\pi}{100} = 1.26$m.

1. Write the equation of motion for the system.

2. Give a state space representation of this system considering as input the current i in the conductor wound and as output the displacement x of the voice coil. What is the order of the system?
3. Give the transfer function from the input i to the output x.

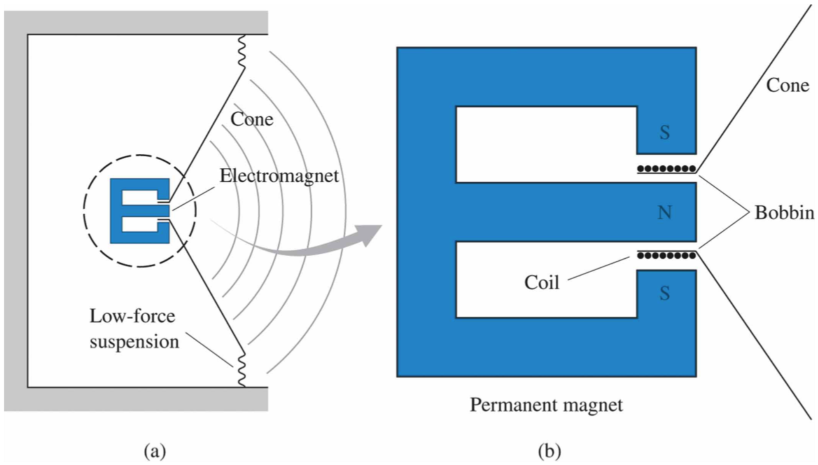

Figure 1: Geometry of a loudspeaker: (a) overall configuration; (b) the electromagnet and voice coil.

Problem 2: In this problem, we include in our model of the loudspeaker a model of the circuit that produces the current that flows in the conductor wound on the bobbin. The circuit is shown in Figure 2 and it is composed of a resistor R and inductor L driven by an input voltage source v_a. The voltage source e_{coil} represents the voltage developed across the wound due to its motion in the magnetic field.

1. Write the equations of motion of the system.
2. Give a state space representation of this system considering as input the voltage v_a and as output the displacement x of the voice coil. What is the order of the system?
3. Give the transfer function from the input v_a to the output x.

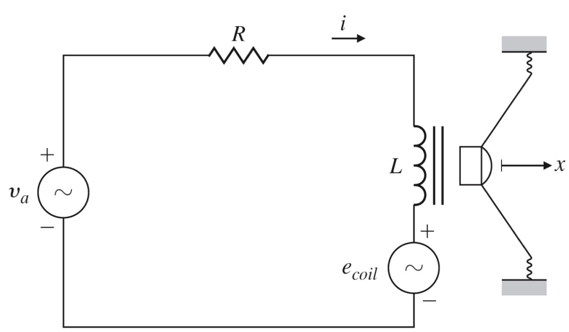

Figure 2: A loudspeaker model showing the electric circuit.