# MATH426 HW4

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## 1 2.5.4

Multiplication of two matrices C = A, B is defined by

$$C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

which can be thought of as saying the element at position (i, j) in the matrix C = A, B is found by multiplying the $i^{th}$ row of matrix A with the $j^{th}$ column of matrix B. That is: $C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = a_i b_j + a_{i2} b_{2j} + ... a_{in} b_{nj}$. This comprises n multiplications, thus the computation of $C_{ij}$ takes O(n) flops. Now since there are $n^2$ elements in the matrix $C_{ij}$, the computation of C = A, B takes $n^2 * O(n) = O(n^3)$

## 2 2.6.1

The expression on the right hand side has no precedence, the expression is simply evaluated left to right. The mathematical notation of the expression $x = U/L/b$ is equivalent to $x = (U^{-1}L)^{-1}b = (L^{-1}(U^{-1})^{-1})b = L^{-1}Ub$

## 3 2.6.3.a

$nT(q_k - q_{k-1}) + nT(q_k - q_{k+1}) = m_k g, k = 1, ..., n-1$

$q_{k+1}(-nT) + q_k(nT + nT) + q_k(-nT) = m_k g$ (Equation 1)

From equation 1, for k = 1 we have:
$q_2(-nT) + q_1(2nT) = m_1 g + 0 : [q_0 = 0]$ (Equation 2)

From equation 1, for k = 2 to n-2 we have:
$q_{k+1}(-nT) + q_k(2nT) + q_{k-1}(-nT) = m_k g$ (Equation 3)

From equation 1, for k = n-1 we have:

$q_n(-nT) + q_{n-1}(2nT) + q_{n-2}(-nT) = m_{n-1}g$
$q_{n-1}(2nT) + q_{n-2}(-nT) = m_{n-1}g$ (Equation 4)

From equations 1, 2, 3 and 4 we can form a matrix:

$$\begin{bmatrix} 2nT & -nT & 0 & 0 & 0 & ... & 0 & 0 \\ -nT & 2nT & -nT & 0 & 0 & ... & 0 & 0 \\ 0 & -nT & 2nT & -nT & 0 & ... & 0 & 0 \\ 0 & 0 & -nT & 2nT & -nT & ... & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & ... & -nT & 2nT \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_{n-1} \end{bmatrix} = \begin{bmatrix} m_1g \\ m_2g \\ m_3g \\ m_4g \\ m_{n-1}g \end{bmatrix}$$

2.6.3.b/c are in MatLab

# 4 2.7.5

The norm of the induced matrix $||A||$ is based on any vector norm $||x||$ i.e $||A|| = sub(||x|| = 1)||Ax|| = \frac{||Ax||}{||x||}$

Now we have to show that $||cA|| = |C|.||A||$ when C is a scalar. From the above,

$||CA|| = sub(||x|| = 1)||CAx|| =$

$||CA|| = sub(||x|| = 1)|C| * ||Ax|| =$

$||CA|| = |C|sub(||x|| = 1)||Ax|| =$

$|C| * ||A||$

Hence $||CA|| = |C| * ||A||$

# 5 2.7.12

a)

To show that I-A is non singular, we will assume I-A is singular.
Then det(I-A)=0. The rank of I-A is less than n, so the nullity of I-A is greater than 0, i.e the null space is non-trivial. So there exists a non-zero $x_0$ such that (I-A)$x_0 = 0$ which implies I$x_0$ = A$x_0$, which implies A$x_0 = x_0$.

Using the definition of an induced matrix and plugging in these values we find that:

$1 \leq ||A||$ which contradicts the given fact that $||A|| < 1$

Thus our assumption that I-A is singular is incorrect, thus they must be non-singular.

b)
$\lim_{m\to\infty} ||A^m - 0|| = \lim_{m\to\infty} ||A^m||$
$= \lim_{m\to\infty} ||A^{m-1} * A||$
$\leq \lim_{m\to\infty} ||A^{m-1}|| * ||A||$
$\leq \lim_{m\to\infty} ||A||^m$
$= 0$ Since $||A|| < 1$

c)

$$\sum_{k=0}^{m}(A^k)(I - A) = I - A^{m+1}$$

Taking the limit of both sides:

$$\lim_{m\to\infty}\sum_{k=0}^{m}(A^k)(I - A) = I - \lim_{m\to\infty} A^{m+1}$$

Multiply both sides by $(I - A)^{-1}$ and we get:

$$\sum_{k=0}^{\infty}(A^k) = (I - A)^{-1}$$

$||AB|| = Sup\frac{||ABx||_x}{||x||_x}$

$||ABx||_x = ||A(BX)||_x$

$\leq ||A|| * ||BX||_x$

$\leq ||A|| * ||B|| * ||X||_x$

So $\frac{||ABx||_x}{||x||_x} \leq ||A|| * ||B||$

i.e $||AB|| \leq ||A|| * ||B||$

# 6 2.8.1

Code is in MatLab file.

The growth of $k$ appears to considerably slow down at $n = 13$ because Hilbert matrices are very poorly conditioned. At n = 13, the condition number becomes so large (6.88e+17) that we are potentially losing 17 digits of data. As $k$ approaches $\frac{1}{eps}$ MatLab will notice the large condition number and warn us to not expect much from the result. In fact the error could potentially exceed 100%.