MATH426 HW7

Shane Cincotta

November 15, 2019

1 5.2.2

a)

$$H(x) =$$

$$\begin{cases} x+1 & \text{if x } \epsilon(-1,0) \\ -x+1 & \text{if x } \epsilon(0,1) \\ 0 & \text{otherwise} \end{cases}$$

b)

Given
$$Q(x) = \int_{x-1}^{x} H(t)dt$$

We find the piecewise formula for the given Q(x)

for
$$-1 \le x \le 0$$
;

$$\int_{x-1}^{x} (t+1)dt$$

$$=x+\frac{1}{2}$$

for
$$0 \le x \le 1$$
;

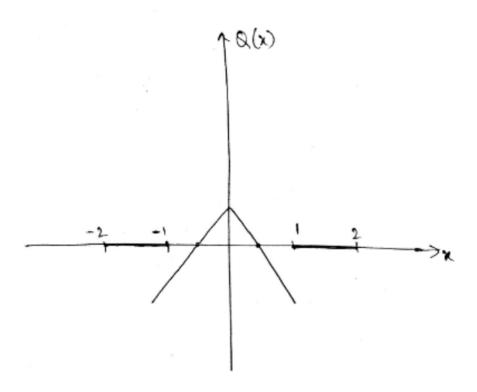
$$\int_{x-1}^{x} (-t+1)dt$$

$$= -x + \frac{1}{2}$$

Substituting both the values in H(x) we get:

$$\begin{cases} x + \frac{1}{2} & \text{if x } \epsilon(-1,0) \\ -x + \frac{1}{2} & \text{if x } \epsilon(0,1) \\ 0 & \text{otherwise} \end{cases}$$

c)



d)

Q(x) is continuous because $Q(x) = |x| + \frac{1}{2}$ for |x|

2 5.2.3

The table of values is 1.01, 1.02, 1.03...1.99, 2 is given up to four terms after the decimal

After the decimal three terms are the same and the fourth term has rounded up terms so that the upper bound of the error is 0.0005.

For example, according to the ${\rm table}(1.9)=0.6419$ and the actual result is 0.642845

Thus the error is 0.00004612.

3 5.4.3

In matlab file

I included a screenshot of the table with my submission because matlab messed up the formatting while publishing.

4 5.4.5

Code is in matlab file

The result is so inaccurate because the nodes were not chosen correctly. If the nodes were chosen more accurately the result would more closely mirror the exact answer.

$5 \quad 5.5.1$

In matlab file

$6 \quad 5.5.2$

The Taylor Series expansion is $f(x-\lambda)=f(x)-\lambda f'(x)+\frac{\lambda^2}{2!}f''(x)-\frac{\lambda^3}{3!}f'''(x)+\dots$

Now
$$\frac{f(x)-f(x-\lambda)}{\lambda} = \frac{1}{\lambda} [f(x) - f(x) + '(x) - \frac{\lambda^2}{2!} f''(x) + \frac{\lambda^3}{3!} f'''(x) - \frac{\lambda^4}{4!} f''''(x)]$$

$$= f'(x) - \frac{\lambda}{2!}f''(x) + \frac{\lambda^2}{3!}f'''(x) - \frac{\lambda^3}{4!}f''''(x)$$

 $\tau_f(\lambda) = \frac{-\lambda}{2!} f''(x) + \frac{\lambda^2}{3!} f''(x)$ -The first two non-zero terms

$$=\frac{f(x)-f(x-\lambda)}{\lambda}=f'(x)$$