ELEG 3~~1~~0

3/20/2018

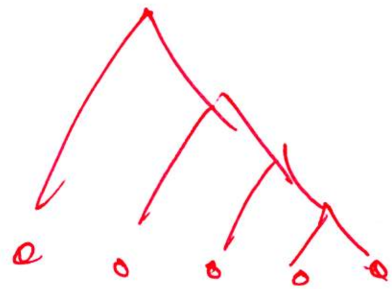
Entropy $H(X) = -\sum_{k=1}^{m} p(k) \log_2 p(k) \geq 0$

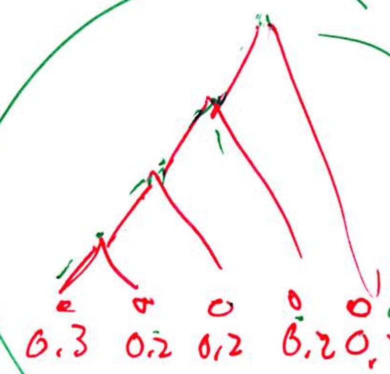
= measure of randomness (unpredictability)

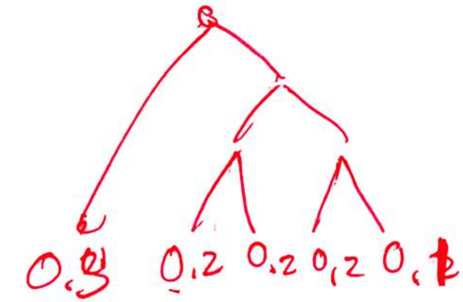
Theorem: $0 \leq H(X) \leq \log_2 M$

with $H(X) = \log_2 m$ when $p(k) = \frac{1}{m}$

Uniform distribution







Theorem $H(x) + 1 > EL_{Huffman} \geq H(x)$

$EL(X) = 1 \times 0.3 + 3 \times 0.7 = 2.4 \ \frac{\text{bits}}{\text{letter}}$

$\underbrace{00}_{a}\underbrace{00}_{a}\underbrace{11}_{c}\underbrace{011}_{e}\underbrace{00}_{a}\underbrace{10}_{b}\underbrace{11}_{c}0 \ldots$

$EL = 4 \times 0.3 + 4 \times 0.2 + 3 \times 0.2 + 2 \times 0.2 + 1 \times 0.1$

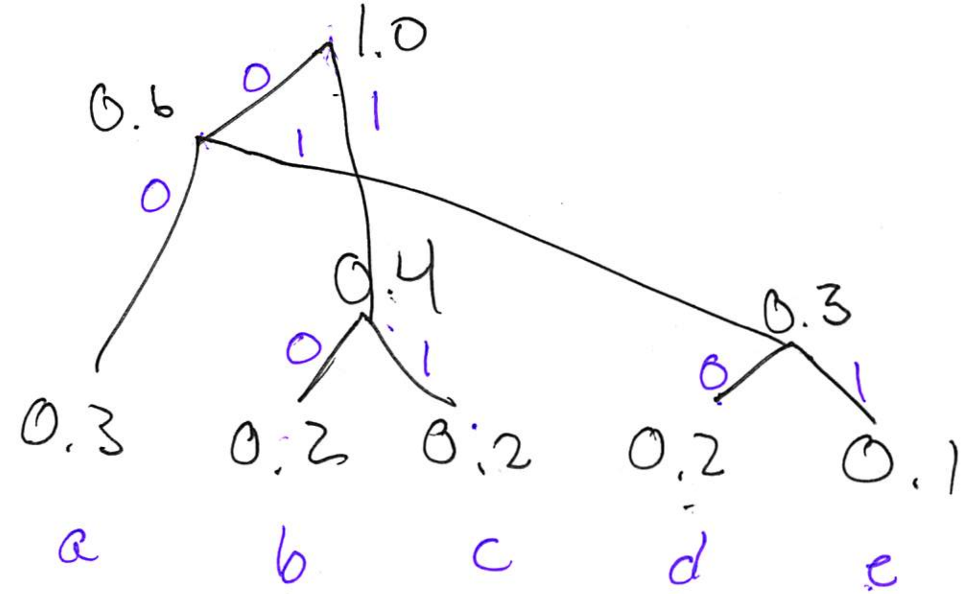
$= 3.1 \ \frac{\text{bits}}{\text{letter}}$

$2.3 \geq 2.24$

Theorem: (Source Coding) $\boxed{EL(x) \geq H(x)}$

where $EL(X) =$ expected # of bits needed to encode X

Huffman Code

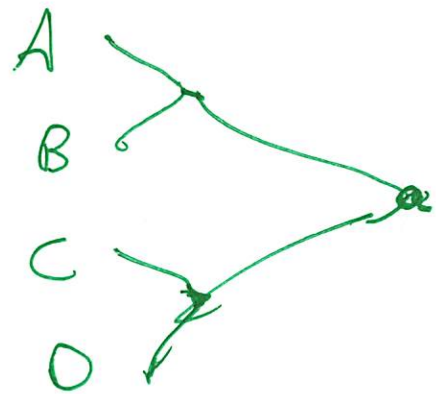


letter	P	Code	L(x)
a	0.3	00	2
b	0.2	10	2
c	0.2	11	2
d	0.2	010	3
e	0.1	011	3

$$EL(x) = 2\times0.3 + 2\times0.2 + 2\times0.2 + 3\times0.2 + 3\times0.1 = 2.3 \ \frac{\text{bits}}{\text{letter}}$$

≈ 2.24 bits/letter

$$H(x) = -0.3\log 0.3 - 0.2\log 0.2 - 0.2\log 0.2 - 0.2\log 0.2 - 0.1\log 0.1$$



$P(A \text{ wins tournament})$

$= P(A \text{ wins } 1^{st} \text{ game} \cap A \text{ win } 2^{nd} \text{ game})$

$= P(A \text{ wins } 1^{st} \text{ game}) P(A \text{ wins } 2^{nd} \text{ game})$

$= 0.7 \times 0.7 = 0.49$

$P(A \text{ beats } X) = 0.7$

$P(B \text{ wins tournament}) = P(B \text{ wins } 1^{st} \text{ game}) \times P(B \text{ wins } 2^{nd} \text{ game}$

$= 0.3 \times 0.5 = 0.15$

$P(C \text{ wins tour}) = 0.5 \times$ ____

$P(C \text{ wins Tour}) = P(C \text{ wins Tour} \mid A \text{ wins } 1^{st} \text{ game}) P(A \text{ win } 1^{st} \text{ game}) + P(C \text{ wins Tour} \mid B \text{ wins } 1^{st} \text{ game}) P(B \text{ wins } 1^{st} \text{ game})$

$= 0.5 \times 0.3 \times 0.7 + 0.5 \times 0.5 \times 0.3 = 0.18$

$X_1, X_2 \sim \text{Geometric}(p)$ $\underline{\text{ind}}$

$S = X_1 + X_2$ pmf of S ?

$P(X=k) = (1-p)^{k-1} p \quad k = 1, 2, \ldots, \infty$

	p	$(1-p)p$	$(1-p)^2 p$	$(1-p)^3 p$	$\ldots$
p	p^2	$(1-p)p^2$	$q^2 p^2$	$q^3 p^2$	$\ldots$
$(1-p)p$		$q^2 p^2$	$q^3 p^2$	$q^3 p^2$	
$q^2 p = (1-p)^2 p$			$q^3 p^3$	$q^3 p^3$	
$(1-p)^3 p$				$q^3 p^3$	
$\vdots$					
	p^2	$2qp^2$	$3q^2p^2$	$4q^3p^2$	