MATH426 HW6

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1 3.3.1

We begin with the 2-norm of matrix Q, $||Q||_2$, then by using the definition of an induced matrix norm we find that,

$||Q||_2 = \max_{||x||=1} \frac{||Qx||_2}{||x||_2}$ for a vector norm $||x||_2$. From part 2 of the theorem, we know that $||Qx||_2 = ||x||_2$.

Therefore $\max_{||x||=1} \frac{||Qx||_2}{||x||_2} = \max_{||x||=1} \frac{||x||_2}{||x||_2} = 1$.

2 3.3.2

1.

By definition of an orthogonal matrix X, $XX^T = X^T X = I$

Since $(X^T)^T = X$, it follows that

$(X^T)^T X^T = X^T (X^T)^T = I$

$\implies X^T$ is an orthogonal matrix

2.

By the definition of 2-norm of X, $||X||_2 = \sqrt{P(X^T X)}$

Where $P(X^T X)$ denotes the maximum of the eigenvalues of $X^T X$

$= \sqrt{P(I)}$ $(\because X^T X = I)$

$= 1$

Also for an orthogonal matrix X, $X^T = X^{-1}$

$\therefore ||X^{-1}|| = ||X^T||_2$

$\sqrt{P(X^T)^T X^T)}$

$= \sqrt{P(I)} \therefore X^T$ is orthogonal by(1)

3.

Suppose A is any nxn matrix.

Now, $||AX||_2 = \sqrt{P((AX)^T AX)} = \sqrt{P(AX(AX)^T)}$

$= \sqrt{P(AXX^T A^T)}$

$= \sqrt{P(AIA^T)} = ||A||_2$

4.

Suppose U is an nxn orthogonal matrix

$\therefore UU^T = U^TU = I$

Then, $(XU)(XU)^T = XUU^TX^T$

$= XIX^T$ ($\therefore UU^T = I$)

$= XX^T = I$

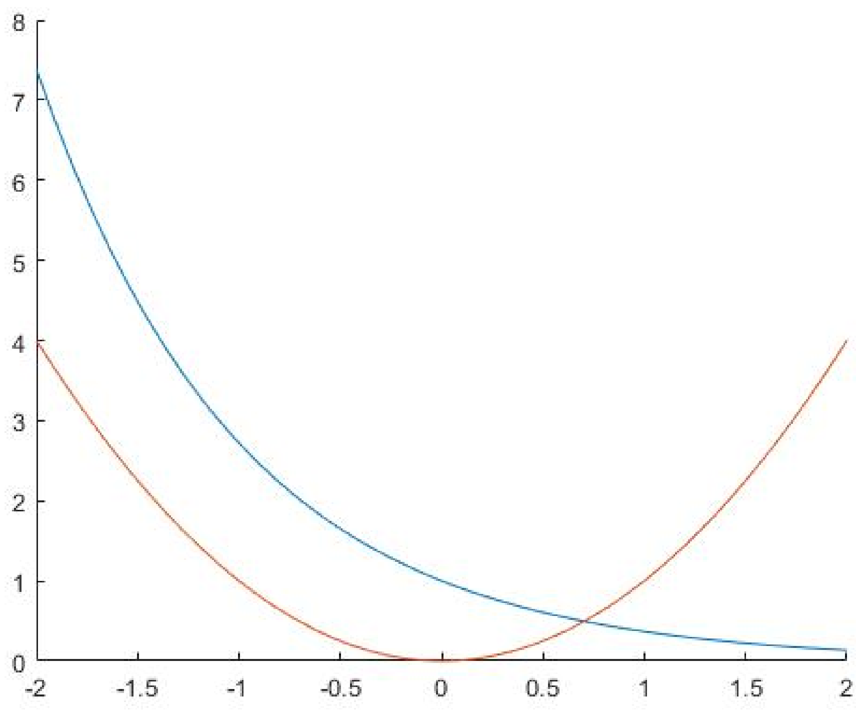
$\implies$ XU is orthogonal

3 4.1.1

a.

$f(x) = x^2 - e^{-x} = 0$

To find the roots of $f(x) = 0$ between [-2,2] we need to sketch the graph of x^2, e^{-x} in [-2,2]:



From this graph one can see that there one a root between 0 and 1, it is around 0.7

b.

After running fzero in MatLab with an initial guess of 0.7, the root was calculated to be 0.7035.

c.

$k_r = |f'(r)|^{-1}$

$f'(r) = -e^{-r} - 2r,\ f'(0.7035) = -1.902$

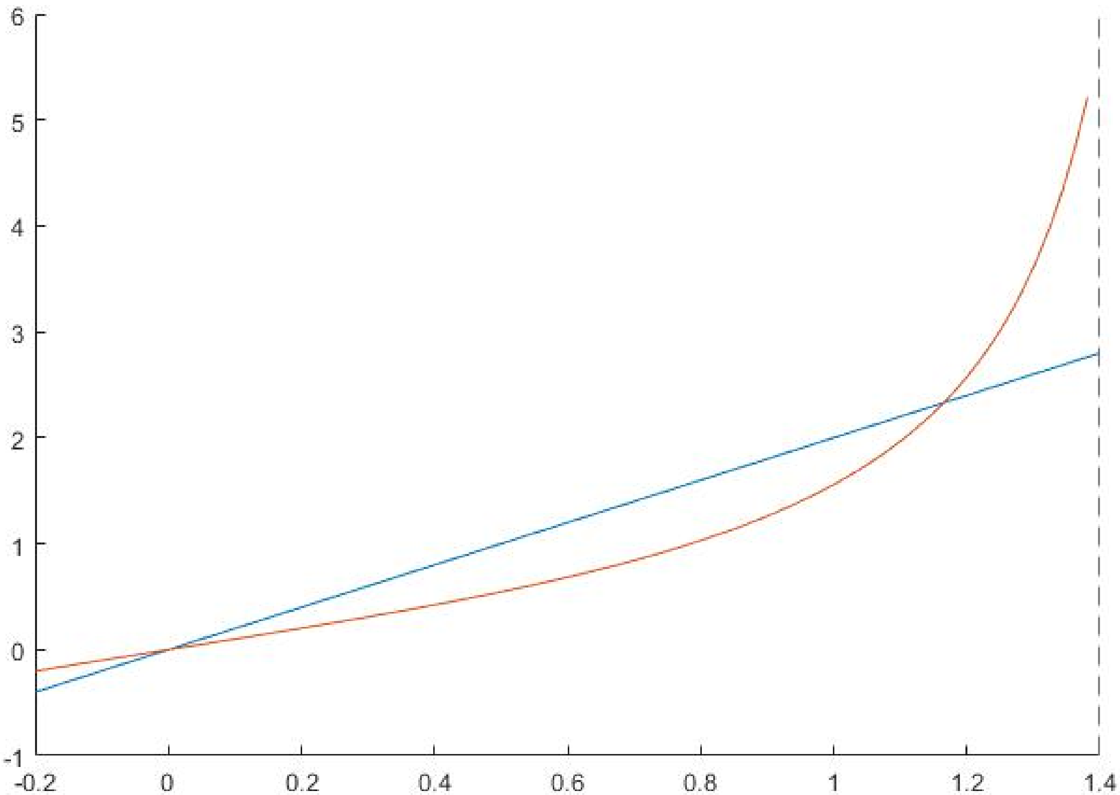
$k_{r=.7035} = |-1.902|^{-1} = 0.526$

4 4.1.2

a.

$f(x) = 2x - \tan(x) = 0$

To find the roots of $f(x) = 0$ between [-0.2,1.4] we need to sketch the graph of $2x, \tan(x)$ in [-0.2,1.4]:



From this graph one can see that there are 2 roots between -0.2 and 1.4, they are around 0 and 1.2

b.

After running fzero in MatLab with initial guesses of 0 and 1.15, the roots were calculated to be 0 and 1.1656.

c.

$k_r = |f'(r)|^{-1}$

$f'(r) = 2 - sec^2(r)$

$k_{r=0} = |1|^{-1} = 1$

$k_{r=1.1656} = |-4.435|^{-1} = 0.2255$

5 4.1.4

Table in MatLab file

6 4.1.5

Code in MatLab file