ELEG 312
Homework #7
Solutions

Problems 10.1, 10.3, 10.5, 10.6, 10.13, 10.17, and 10.21

Problem 10.1

For the amplifier in Fig. 10.3(a), if R_{G1} = 2 MΩ, R_{G2} = 1 MΩ, and R_{sig} = 200 kΩ, find the value of the coupling capacitor C_{C1} (specified to one significant digit) that places the associated pole at 10 Hz or lower.

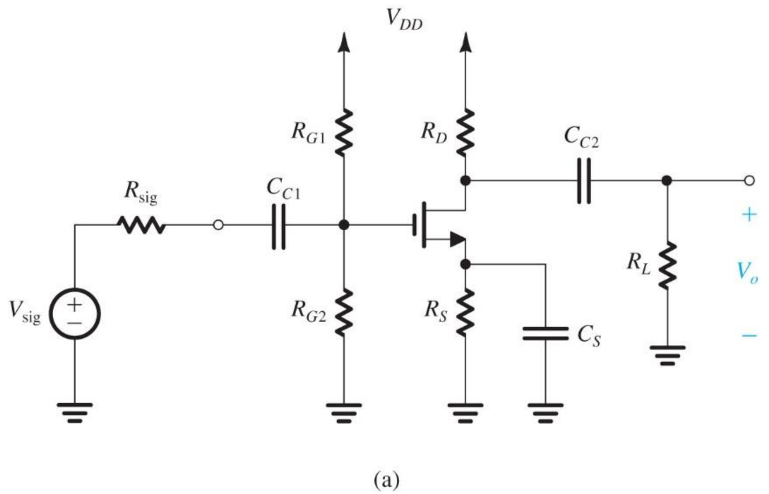


Figure 10.3 (a) Capacitively coupled common-source amplifier.

$$f_{P1} = \frac{1}{(2\pi)C_{C1}\left(R_G + R_{sig}\right)} = 10\text{Hz}$$

$$R_G = R_{G1} \,||\, R_{G2} = 2\text{M}\Omega \,||\, 1\text{M}\Omega = 0.6666\text{M}\Omega$$

$$C_{C1} = \frac{1}{(2\pi f_{P1})\left(R_G + R_{sig}\right)}$$

$$= \frac{1}{(2\pi 10\text{Hz})(666.66\text{k}\Omega + 200\text{k}\Omega)}$$

$$= 18.36\text{nF} = 20\text{nF}$$

Problem 10.3

The amplifier in Fig. 10.3(a) is biased to operate at g_m = 5 mA/V, and R_S = 1.8 kΩ. Find the value of C_S (specified to one significant digit) that places its associated pole at 100 Hz or lower. What are the actual frequencies of the pole and zero realized?

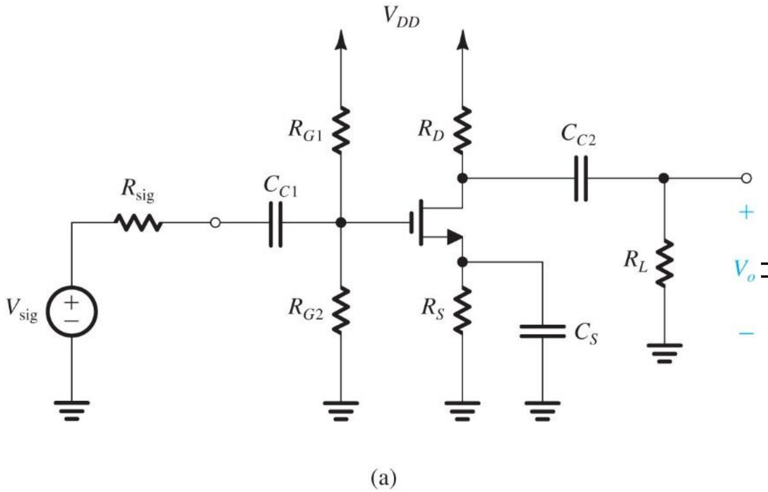


Figure 10.3 (a) Capacitively coupled common-source amplifier.

$$f_{P2} = \frac{g_m + 1/R_S}{2\pi C_S}$$

$$\Rightarrow C_S = \frac{g_m + 1/R_S}{2\pi f_{P2}} = \frac{5\frac{\text{mA}}{\text{V}} + \frac{1}{1.8\text{k}\Omega}}{2\pi 100\text{Hz}} = 8.84\mu\text{F} = 9\mu\text{F}$$

$$f_{P2} = \frac{5\frac{\text{mA}}{\text{V}} + \frac{1}{1.8\text{k}\Omega}}{(2\pi)9\mu\text{F}} = 98.244\text{Hz}$$

$$f_Z = \frac{1}{2\pi C_S R_S} = 9.824\text{Hz}$$

Problem 10.5 a,b

The amplifier in Fig. P10.5 is biased to operate at g_m = 2 mA/V. Neglect r_o.

(a) Determine the value of R_D that results in a midband gain of −20 V/V.

(b) Determine the value of C_S that results in a pole frequency of 100 Hz.

(c) What is the frequency of the transmission zero introduced by C_S?

(d) Give an approximate value for the 3-dB frequency f_L.

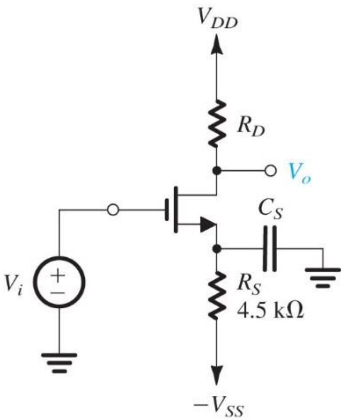


Figure P10.5

$$A_M = -g_m R_D \qquad \Rightarrow R_D = -\frac{A_M}{g_m} = -\frac{-20\frac{\text{V}}{\text{V}}}{2\frac{\text{mA}}{\text{V}}} = 10\text{k}\Omega$$

$$R_{eff} = 1/g_m \,||\, R_S = \frac{\left(\frac{1}{2\text{mA/V}}\right)4.5\text{k}\Omega}{\left(\frac{1}{2\text{mA/V}}\right)+4.5\text{k}\Omega} = 450\Omega$$

$$C_S = \frac{1}{2\pi f_L R_{eff}} = \frac{1}{(2\pi 100\text{Hz})450\Omega} = 3.537\,\mu\text{F}$$

Problem 10.5 c,d

The amplifier in Fig. P10.5 is biased to operate at g_m = 2 mA/V. Neglect r_o.

(a) Determine the value of R_D that results in a midband gain of −20 V/V.

(b) Determine the value of C_S that results in a pole frequency of 100 Hz.

(c) What is the frequency of the transmission zero introduced by C_S?

(d) Give an approximate value for the 3-dB frequency f_L.

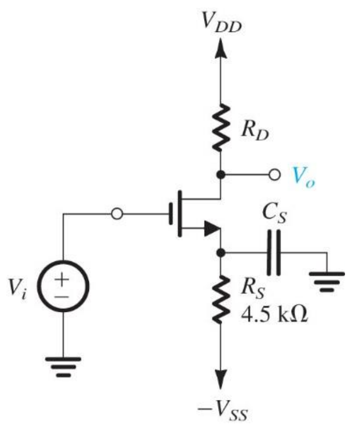


Figure P10.5

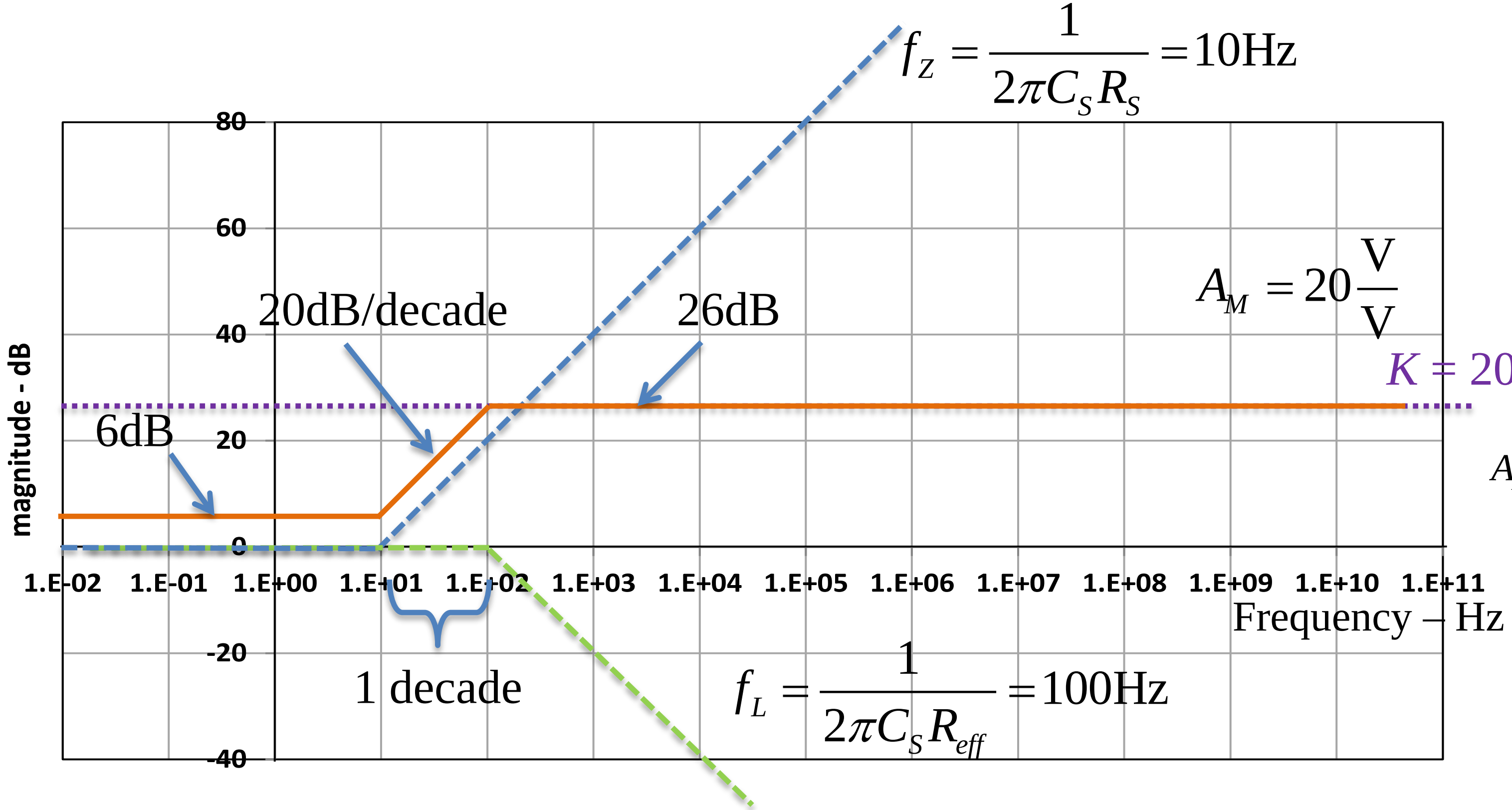
$$f_Z = \frac{1}{2\pi C_S R_S} = 10\text{Hz}$$

$$f_L = \frac{1}{2\pi C_S R_{eff}} = 100\text{Hz}$$

Problem 10.5 e

The amplifier in Fig. P10.5 is biased to operate at g_m = 2 mA/V. Neglect r_o.

(e) Sketch a Bode plot for the gain of this amplifier. What does the plot tell you about the gain at DC? Does this make sense? Why or why not?



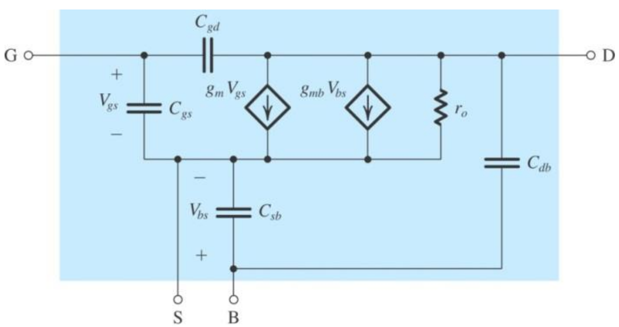
$$A_{DC} = -\frac{g_m R_D}{1 + g_m R_L}$$

$$= -\frac{20}{1 + \left(2\frac{\text{mA}}{\text{V}}\right)4.5\text{k}\Omega}$$

$$= \frac{20}{1+9} = 2 = 6\text{dB}$$

Problem 10.13a

Refer to the MOSFET high-frequency model in Fig. 10.12(a). Evaluate the model parameters for an NMOS transistor operating at I_D = 200 μA, V_{SB} = 1 V, and V_{DS} = 1.5 V. The MOSFET has W = 20 μm, L = 1 μm, t_{ox} = 8 nm, μ_n = 450 cm²/V·s, $\gamma = 0.5\ \mathrm{V}^{1/2}$, $2\varphi_f = 0.65$ V, $\lambda = 0.05\ \mathrm{V}^{-1}$, $V_0 = 0.7$ V, $C_{sb0} = C_{db0}$ = 20 fF, and L_{ov} = 0.05 μm. [Recall that $g_{mb} = \chi g_m$, where $\chi = \gamma \big/ \left(2\sqrt{2\phi_f + V_{SB}}\right)$ and that $\varepsilon_{ox} = 3.45\times10^{-11}$ F/m.]



(a)

Figure 10.12 (**a**) High-frequency, equivalent-circuit model for the MOSFET.

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{3.45\times10^{-11}\ \mathrm{F/m}}{8\mathrm{nm}} = 4.3\frac{\mathrm{fF}}{\mu\mathrm{m}^2}$$

$$k_n = \mu_n C_{ox}\frac{W}{L} = 450\frac{cm^2}{V\cdot s}\left(4.3\frac{\mathrm{fF}}{\mu\mathrm{m}^2}\right)\left(\frac{20\mu\mathrm{m}}{1\mu\mathrm{m}}\right) = 3.88\frac{\mathrm{mA}}{\mathrm{V}^2}$$

$$V_{OV} = \sqrt{\frac{2I_D}{k_n}} = 0.316\mathrm{V}$$

$$\chi := \frac{\gamma}{2\cdot\sqrt{2\phi_f + V_{SB}}} = 0.195$$

$$g_m = k_n V_{OV} = 1.25\frac{\mathrm{mA}}{\mathrm{V}} \qquad g_m = \frac{2I_D}{V_{OV}} = \frac{2(0.2\mathrm{mA})}{0.316\mathrm{V}} = 1.25\frac{\mathrm{mA}}{\mathrm{V}} \qquad g_{mb} = \chi g_m = 0.243\frac{\mathrm{mA}}{\mathrm{V}}$$

Problem 10.13b

Refer to the MOSFET high-frequency model in Fig. 10.12(a). Evaluate the model parameters for an NMOS transistor operating at I_D = 200 μA, V_{SB} = 1 V, and V_{DS} = 1.5 V. The MOSFET has W = 20 μm, L = 1 μm, t_{ox} = 8 nm, μ_n = 450 cm^2/V·s, $\gamma = 0.5\ V^{1/2}$, $2\varphi_f = 0.65$ V, $\lambda = 0.05\ V^{-1}$, $V_0 = 0.7$ V, $C_{sb0} = C_{db0}$ = 20 fF, and L_{ov} = 0.05 μm. [Recall that $g_{mb} = \chi g_m$, where $-\chi = \gamma \Big/ \left(2\sqrt{2\phi_f + V_{SB}}\right)$ and that $\varepsilon_{ox} = 3.45\times10^{-11}$ F/m.]

$$V_A = \frac{1}{\lambda} = \frac{1}{0.05\mathrm{V}^{-1}} = 20\mathrm{V} \qquad r_o = \frac{V_A}{I_D} = \frac{20\mathrm{V}}{0.2\mathrm{mA}} = 100\mathrm{k\Omega}$$

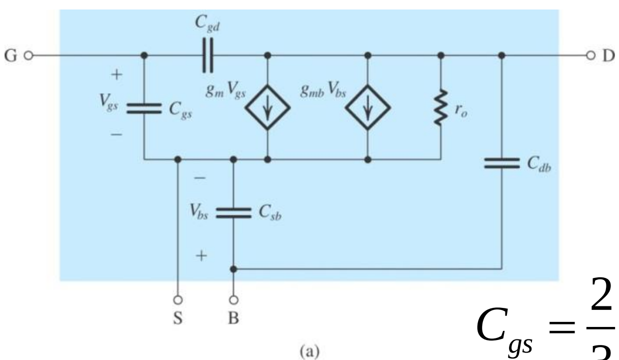


Figure 10.12 (**a**) High-frequency, equivalent-circuit model for the MOSFET.

$$C_{ov} = WL_{ov}C_{ox} = (20\mu\mathrm{m})(0.05\mu\mathrm{m})\left(4.312\frac{\mathrm{fF}}{\mu\mathrm{m}^2}\right) = 4.312\mathrm{fF}$$

$$C_{gd} = C_{ov} = 4.312\mathrm{fF}$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + C_{ov} = \frac{2}{3}(20\mu\mathrm{m})(1\mu\mathrm{m})\left(4.312\frac{\mathrm{fF}}{\mu\mathrm{m}^2}\right) + 4.312\mathrm{fF} = 61.812\mathrm{fF}$$

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1+\frac{V_{SB}}{V_0}}} = \frac{20\mathrm{fF}}{\sqrt{1+\frac{1\mathrm{V}}{0.7\mathrm{V}}}} = 12.83\mathrm{fF} \qquad C_{db} = \frac{C_{db0}}{\sqrt{1+\frac{V_{DB}}{V_0}}} = \frac{20\mathrm{fF}}{\sqrt{1+\frac{2.5\mathrm{V}}{0.7\mathrm{V}}}} = 9.35\mathrm{fF}$$

Problem 10.13c

Refer to the MOSFET high-frequency model in Fig. 10.12(a). Evaluate the model parameters for an NMOS transistor operating at I_D = 200 µA, V_{SB} = 1 V, and V_{DS} = 1.5 V. The MOSFET has W = 20 µm, L = 1 µm, t_{ox} = 8 nm, μ_n = 450 cm²/V·s, γ = 0.5 $V^{1/2}$, $2\varphi_f$ = 0.65 V, λ = 0.05 V^{-1}, V_0 = 0.7 V, $C_{sb0} = C_{db0}$ = 20 fF, and L_{ov} = 0.05 µm. [Recall that $g_{mb} = \chi g_m$, where $-\chi = \gamma / \left(2\sqrt{2\phi_f + V_{SB}}\right)$ and that $\varepsilon_{ox} = 3.45\times10^{-11}$ F/m.]

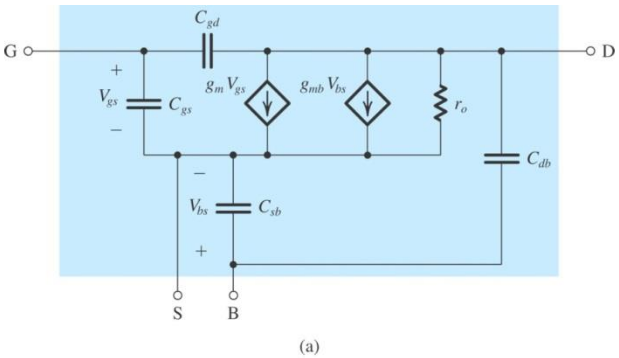


Figure 10.12 (**a**) High-frequency, equivalent-circuit model for the MOSFET.

$$g_m = \frac{2I_D}{V_{OV}} = 1.25\frac{\text{mA}}{\text{V}}$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + C_{ov} = 61.812\text{fF}$$

$$C_{gd} = C_{ov} = 4.312\text{fF}$$

$$f_T = \frac{g_m}{2\pi\left(C_{gs}+C_{gd}\right)} = \frac{1.25\frac{\text{mA}}{\text{V}}}{2\pi\left(61.81\text{fF}+4.31\text{fF}\right)} = 3.0\text{GHz}$$

Problem 10.17

It is required to calculate the intrinsic gain A_0 and the unity-gain frequency f_T of an n-channel transistor fabricated in a 0.13-μm CMOS process for which L_{ov} = 0.1 L, μ_n =400 cm²/V·s, and V'_A = 5 V/μm. The device is operated at V_{OV} = 0.2 V. Find A_0 and f_T for devices with $L = L_{min}$, $2L_{min}$, $3L_{min}$, $4L_{min}$, and $5L_{min}$. Present your results in a table. (Hint: For f_T, use the approximate expression $f_T \simeq 3\mu_n V_{OV}/(4\pi L^2)$.)

$$A_0 = \frac{2V_A}{V_{OV}} = \frac{2V'_A L}{V_{OV}} = \frac{2(5\text{V}/\mu\text{m})L}{0.2\text{V}} = \frac{50L}{\mu\text{m}}$$

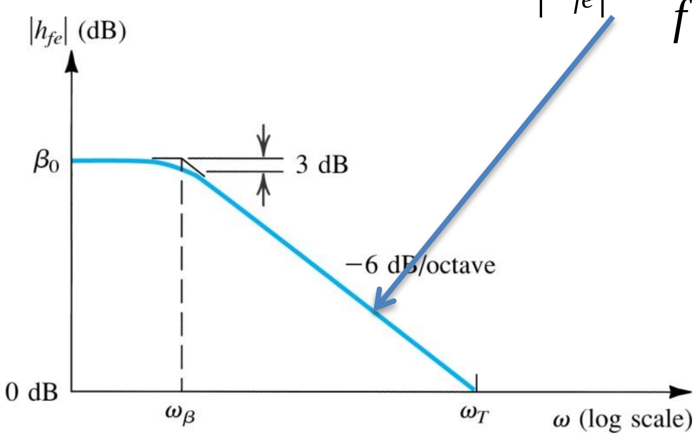
$$f_T \simeq \frac{3\mu_n V_{OV}}{4\pi L^2} = \frac{3\left(400\dfrac{\text{cm}^2}{\text{V}\cdot\text{s}}\right)0.2\text{V}}{4\pi L^2} = \frac{1.91\text{GHz}\cdot\mu\text{m}^2}{L^2}$$

	Lmin	2Lmin	3Lmin	4Lmin	5Lmin
L,um	0.13	0.26	0.39	0.52	0.65
A0, V/V	6.5	13	19.5	26	32.5
fT, GHz	113.0178	28.25444	12.55753	7.063609	4.52071

Problem 10.21a

Measurement of h_{fe} of an *npn* transistor at 50 MHz shows that $|h_{fe}| = 10$ at $I_C = 0.2$ mA and 12 at $I_C = 1.0$ mA. Furthermore, C_μ was measured and found to be 0.1 pF. Find f_T at each of the two collector currents used. What must τ_F and C_{je} be?

$$\left|h_{fe}\right| = \frac{f_T}{f} \quad \Rightarrow f_T = f\left|h_{fe}\right|$$



at $I_C = 0.2$ mA

$$f_T = f\left|h_{fe}\right| = 50\text{MHz}(10) = 500\text{MHz}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.2\text{mA}}{0.025\text{V}} = 8\frac{\text{mA}}{\text{V}}$$

at $I_C = 1.0$ mA

$$f_T = f\left|h_{fe}\right| = 50\text{MHz}(12) = 600\text{MHz}$$

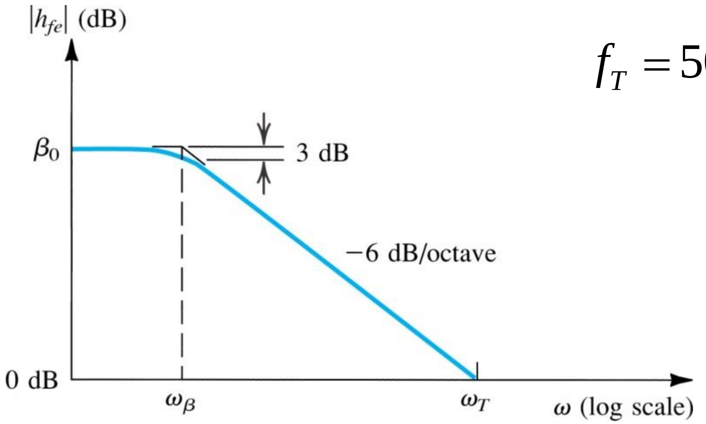
$$g_m = \frac{I_C}{V_T} = \frac{1\text{mA}}{0.025\text{V}} = 40\frac{\text{mA}}{\text{V}}$$

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$$

Problem 10.21b

Measurement of h_{fe} of an *npn* transistor at 50 MHz shows that $|h_{fe}| = 10$ at $I_C = 0.2$ mA and 12 at $I_C = 1.0$ mA. Furthermore, C_μ was measured and found to be 0.1 pF. Find f_T at each of the two collector currents used. What must τ_F and C_{je} be?

at $I_C = 0.2$ mA $\quad f_T = f\left|h_{fe}\right| = 50\text{MHz}(10) = 500\text{MHz}$



$$f_T = 500\text{MHz} = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$$

$$C_\pi + C_\mu = \frac{8\dfrac{\text{mA}}{\text{V}}}{2\pi(500\text{MHz})} = 2.546\text{pF}$$

$$C_\mu = 0.1\text{pF} \Rightarrow C_\pi = 2.446\text{pF}$$

Note at $I_C = 1.0$ mA $\quad f_T = f\left|h_{fe}\right| = 50\text{MHz}(12) = 600\text{MHz}$

$$C_\pi + C_\mu = \frac{40\dfrac{\text{mA}}{\text{V}}}{2\pi(600\text{MHz})} = 10.61\text{pF} \qquad C_\mu = 0.1\text{pF} \Rightarrow C_\pi = 10.51\text{pF}$$

Problem 10.21c

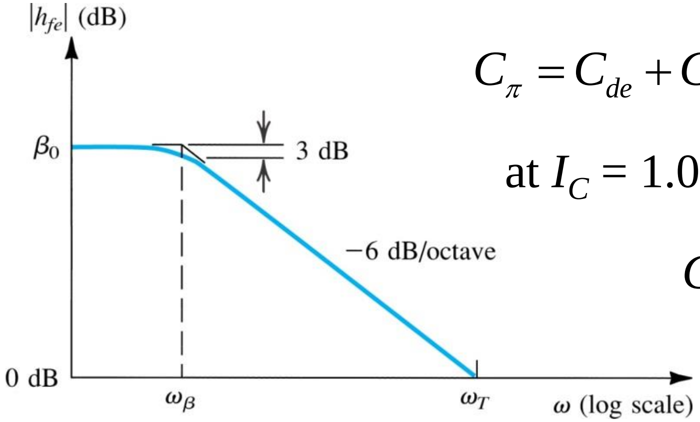
Measurement of h_{fe} of an *npn* transistor at 50 MHz shows that $|h_{fe}|$ = 10 at I_C = 0.2 mA and 12 at I_C = 1.0 mA. Furthermore, C_μ was measured and found to be 0.1 pF. Find f_T at each of the two collector currents used. What must τ_F and C_{je} be?

at I_C = 0.2 mA $\quad f_T = f\left|h_{fe}\right| = 50\text{MHz}(10) = 500\text{MHz} \quad C_\pi = 2.446\text{pF}$

$$C_\pi = C_{de} + C_{je} = \tau_F g_m + C_{je} = \tau_F\left(8\frac{\text{mA}}{\text{V}}\right) + C_{je} = 2.446\text{pF}$$

at I_C = 1.0 mA $\quad f_T = f\left|h_{fe}\right| = 50\text{MHz}(12) = 600\text{MHz} \quad C_\pi = 10.51\text{pF}$

$$C_\pi = \tau_F g_m + C_{je} = \tau_F\left(40\frac{\text{mA}}{\text{V}}\right) + C_{je} = 10.51\text{pF}$$



$$C_{de} = \tau_F g_m$$

$$C_{je} = 2.446\text{pF} - \tau_F\left(8\frac{\text{mA}}{\text{V}}\right) \quad C_{je} = 10.51\text{pF} - \tau_F\left(40\frac{\text{mA}}{\text{V}}\right)$$

$$\Rightarrow \tau_F = 252\text{ps} \qquad C_{je} = 0.43\text{pF}$$

$$\tau_F \simeq \frac{1}{2\pi f_t} = \frac{1}{2\pi(600\text{MHz})} = 265\text{ps}$$

$$\tau_F \simeq \frac{1}{2\pi f_t} = \frac{1}{2\pi(500\text{MHz})} = 318\text{ps}$$