ELEC 310

May 8, 2018

probability = math

statistics = science, art

Estimating the mean $X_1, X_2 \dots X_n$ IID μ, σ^2 $f(x)$

$\hat{\mu} = \overline{X}_n = \frac{1}{n}(X_1 + X_2 + \dots + X_n)$ = statistical average = sample mean

$E(\overline{X}_n) = \frac{1}{n}(EX_1 + EX_2 + \dots + EX_n) = \frac{n\mu}{n} = \mu$

$E\overline{X}_n = \mu \Leftarrow \overline{X}_n$ is an <u>unbiased</u> estimate of μ

$Var(\overline{X}_n) = Var\left(\frac{X_1 + X_2 + \dots + X_n}{n}\right) = \frac{1}{n^2}(Var X_1 + Var X_2 + \dots + Var X_n)$

$= \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n} \to 0$ as $n \to \infty$ <u>consistent</u>

$$\bar{X}_n = \frac{X_1 + X_2 + \cdots + X_n}{n} = \left(\frac{n-1}{n}\right)\underbrace{\left(\frac{X_1 + X_2 + \cdots + X_{n-1}}{n-1}\right)} + \frac{X_n}{n}$$

$$= \left(1 - \frac{1}{n}\right)\bar{X}_{n-1} + \frac{X_n}{n}$$

$$= \bar{X}_{n-1} + \frac{1}{n}\underbrace{\left(X_n - \bar{X}_{n-1}\right)}$$

= prediction + gain × innovation

Estimate Variance

1. Know μ. $\quad \widehat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$

$$E\,\widehat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}\underbrace{E(X_i - \mu)^2}_{\sigma^2} = \frac{n\sigma^2}{n} = \sigma^2$$

2. Don't know μ, must estimate it also

$$\hat{\mu} = \overline{X}_n$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \overline{X}_n)^2$$

$$E\,\hat{\sigma}^2 \neq \sigma^2$$

$$S = S_n = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X}_n)^2 \qquad E S = \sigma^2$$

Alternatives to $\overline{X}_n$, S

Order Statistics

$$X_1, \dots X_n \rightarrow \boxed{\text{Sort}} \rightarrow X_{(1)}\ X_{(2)} \cdots X_{(n)}$$

$$X_{(1)} \leq X_{(2)} \leq X_{(3)} \cdots$$

$\hat{\mu} = x_{(n/2)} =$ median

alpha trimmed mean

$\bar{X}_\alpha = \frac{1}{n(1-2\alpha)} \sum_{i=\alpha n}^{n(1-\alpha)} x_{(i)} =$ throw out αn smallest & αn largest, take average of remaining

$\hat{\sigma} = c_1 \left(X_{(3n/4)} - X_{(n/4)} \right)$ = interquartile distance

$\hat{\sigma} = \frac{c_2}{n} \sum_{i=1}^{n} |X_i - \mu| =$