MATH426 HW3

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1 1.3.3

In MatLab file

2 2.1.1

Suppose you want to interpolate the points(-1,0), (0,1), (2,0), (3,1) and (4,2) by a polynomial of as low a degree as possible. What degree should you expect this polynomial to be?

Ans: You should expect this polynomial to be of degree 4. This is because if we have n sets of data points(e.g (1,3) is one set), then a polynomial of degree n-1 can be used to interpolate the function. In our case we have 5 sets, 5-1=4

Write out a linear system of equations for the coefficients of the interpolated polynomial

Ans: Let the polynomial be $y = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4$

for(-1,0): $0 = c_0 - c_1 + c_2 - c_3 + c_4$
for(0,1): $1 = c_0$
for(2,0): $0 = c_0 + 2c_1 + 4c_2 + 8c_3 + 16c_4$
for(3,1): $1 = c_0 + 3c_1 + 9c_2 + 27c_3 + 81c_4$
for(4,2): $2 = c_0 + 4c_1 + 16c_2 + 64c_3 + 256c_4$

In matrix form:

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 4 & 8 & 16 \\ 1 & 3 & 9 & 27 & 81 \\ 1 & 4 & 16 & 64 & 256 \end{bmatrix} \begin{bmatrix} c0 \\ c1 \\ c2 \\ c3 \\ c4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Solved numerically in MatLab file

3 2.1.4

Say you want to find a cubic polynomial p such that $p(0) = 0, p'(0) = 1, p(1) = 2$ and $p'(1) = -1$, Write out a linear system of equations for the coefficients of p.

Let $p(x) = ax^3 + bx^2 + cx + d$

Let $p'(x) = 3ax^2 + 2bx + c$

$p(0) = 0 = a0^3 + b0^2 + c0 + d = 0$
$d = 0$

$p'(0) = 1 = 3a0^2 + 2b0 + c = 1$
$c = 1$

$p(1) = 2 = a1^3 + b1^2 + c1 + d = 2$
$a + b = 1$ (from above equations)

$p'(1) = -1 = 3a1^2 + 2b1 + c = -1$
$3a + 2b = -2$ (from above equations)

Thus the linear system of equations is:
d = 0
c = 1
a + b = 1
3a + 2b = -2

Solving we find that $a = -4, b = 5, c = 1, d = 0$

Thus $p(x) = -4x^3 + 5x^2 + x$

4 2.2.1

Suppose $C = \begin{bmatrix} I & A \\ -I & B \end{bmatrix}$
$C^2 = \begin{bmatrix} I & A \\ -I & B \end{bmatrix} \begin{bmatrix} I & A \\ -I & B \end{bmatrix} = \begin{bmatrix} I^2 - AI & AI + AB \\ -I^2 + I^2 & -AI + B^2 \end{bmatrix}$

$$C^3 = \begin{bmatrix} I^2 - AI & AI + AB \\ -I^2 + I^2 & -AI + B^2 \end{bmatrix} \begin{bmatrix} I & A \\ -I & B \end{bmatrix}$$

$$= \begin{bmatrix} I(I^2 - AI) - I(AI + AB) & A(I^2 - AI) + B(AI_A B) \\ I(-I^2 + I^2) - I(-I^2 + I^2) & A(-I^2 + I^2) + B(-I^2 + I^2) \end{bmatrix}$$

5 2.4.2

a)

$$T(3,-1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$

$$R(\tfrac{\pi}{5}) = \begin{bmatrix} 0.809 & 0.588 & 0 \\ -0.588 & 0.809 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(-3,1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix}$$

$$z = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0.809 & 0.588 & 0 \\ -0.588 & 0.809 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.808 & 0.588 & 0 \\ -0.588 & 0.809 & 0 \\ 0.015 & 1.955 & 1 \end{bmatrix}$$

$$\text{b} = \text{Az} = \begin{bmatrix} 0.808 & 0.588 & 0 \\ -0.588 & 0.809 & 0 \\ 0.015 & 1.955 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.794 \\ 0.442 \\ 4.94 \end{bmatrix} = b$$

b)

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = LU$$

$$\begin{bmatrix} 0.808 & 0.588 & 0 \\ -0.588 & 0.809 & 0 \\ 0.015 & 1.955 & 1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$= \begin{bmatrix} l_{11}u_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + l_{22}u_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + l_{33}u_{33} \end{bmatrix}$$

This system of equations is undetermined. To find the unique LU decomposition, some restriction needs to be placed on L and U matrices. We can set the lower triangular matrix L to be a unit triangular matrix (i.e set all the entries of the main diagonal to 1).

Thus $L = \begin{bmatrix} 1 & 0 & 0 \\ -0.727 & 1 & 0 \\ 0.018 & 1.573 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 0.809 & 0.588 & 0 \\ 0 & 1.236 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c)

$$[A|b] = \begin{bmatrix} 0.809 & 0.588 & 0 \\ -0.588 & 0.809 & 0 \\ 0.015 & 1.955 & 1 \end{bmatrix} \begin{bmatrix} 2.794 \\ 0.442 \\ 4.94 \end{bmatrix}$$

$(0.809)x_1 + (0.588)x_2 = 2.794$
$(1.236)x_2 = 2.794$
$x_3 = 0.994$
$x_2 = 2.0008$
$(0.809)x_1 + (0.588)x(2.0008) = 2.794$
$(0.809)x_1 = 1.6175$

$$x = \begin{bmatrix} 1.999 \\ 2.001 \\ 0.999 \end{bmatrix}$$

Now, $x - z = \begin{bmatrix} 1.999 \\ 2.001 \\ 0.999 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.001 \\ 0.001 \\ -0.001 \end{bmatrix}$

6 2.4.5

Done in MatLab.