ELEG 310

6 March 2018

RVs X, Y, Z, N

$$P(X=k) = p(k) = P_k$$

$$P(X=x_k) = p(k)$$

x_k = discrete sequence

PMF $= p(k)$ $k=0,1,2,\dots$

1. $p(k) \geq 0$
2. $\sum_{k=0}^{\infty} p(k) = 1$

$$E(\cos(\omega X)) = \sum_{k=0}^{\infty} \cos(\omega k)\, p(k)$$

MGF $M(u) = E(e^{uX})$

$$= \sum_{k=0}^{\infty} e^{uk}\, p(k)$$

Expected Value

$$E(X) = \text{mean} = \mu = \sum_{k=0}^{\infty} k\, p(k)$$

$$E(X^2) = \sum_{k=0}^{\infty} k^2\, p(k)$$

$$\text{Variance} = \sigma^2 \underset{\text{def}}{=} E\left((X-\mu)^2\right) \underset{\text{Theorem}}{=} E(X^2) - \mu^2$$

Ex. AM Modulation

$$X(t) = A(t)\cos(\omega_c t + \Theta) \qquad \Theta \sim RU \qquad E(\Theta) = 0$$

$$E\left(\cos(\omega_c t + \Theta)\right) = 0$$

Chap 5 Multiple RVs

PMF $P(X = k \text{ and } Y = \ell) = p(k,\ell) = P_{XY}(k,\ell)$

2nd order distribution

First Order $P(X = k) = P_X(k)$

$P(Y = \ell) = P_Y(\ell)$

may be different functions

LTP $P(A) = \sum_{i=1}^{\infty} P(A B_i)$ $B_i B_j = \phi$ $i \neq j$ $\bigcup_{i=1}^{\infty} B_i = S$

$= \sum_{i=1}^{\infty} P(A|B_i) P(B_i)$

$$P(X=k) = \sum_{\ell=0}^{\infty} P(X=k,\ Y=\ell) = \sum_{\ell=0}^{\infty} P(X=k \mid Y=\ell)\, P(Y=\ell)$$

AND

$$\begin{array}{cc|cccc} & 1 & 0.0 & 0.0 & 0.1 & 0.1 \\ Y & 0 & 0.0 & 0.1 & 0.4 & 0.2 \\ \hline & & 0 & 1 & 2 & 3 \\ & & & X & & \end{array}$$

$P(X=1 \text{ AND } Y=1) = 0.0$

$P(X=1 \text{ AND } Y=0) = 0.1$

$P(X=0) = P(X=0, Y=0) + P(X=0, Y=1) = 0 + 0.1 = 0.1$

$P(X=1) = P(X=1, Y=0) + P(X=1, Y=1) = 0.1 + 0.0 = 0.1$

$P(X=2) = 0.5$

$P(X=3) = 0.3$

$P(Y=0) = 0.7$

$P(Y=1) = 0.3$

$P(Y=0 \mid X=0) = \dfrac{P(Y=0, X=0)}{P(X=0)} = \dfrac{0.0}{0.1}$

$$P(Y=0 \mid X=2) = \frac{P(X=2, Y=0)}{P(X=2)} = \frac{0.4}{0.5} = 0.8$$

$$P(Y=1 \mid X=2) = \frac{P(X=2, Y=1)}{P(X=2)} = \frac{0.1}{0.5} = 0.2$$

$$\begin{array}{r} + \\ \hline 1.0 \end{array}$$