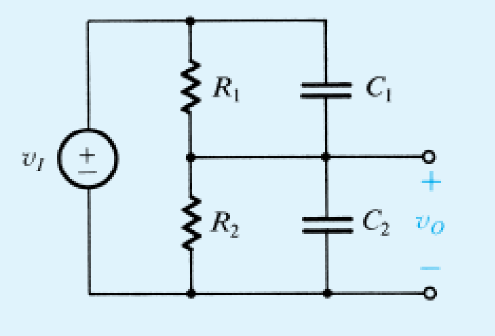
ELEG 312
Homework #5
Solutions

Problems E2, E3, E4, and E6

Problem E.2

Use the voltage divider rule to find the transfer function $V_o(s)/V_i(s)$ of the circuit in Fig. E.3(a). Show that the transfer function can be made independent of frequency if the condition $C_1R_1 = C_2R_2$ applies. Under this condition the circuit is called a *compensated attenuator*. Find the transmission of the compensated attenuator in terms of R_1 and R_2.



(a)

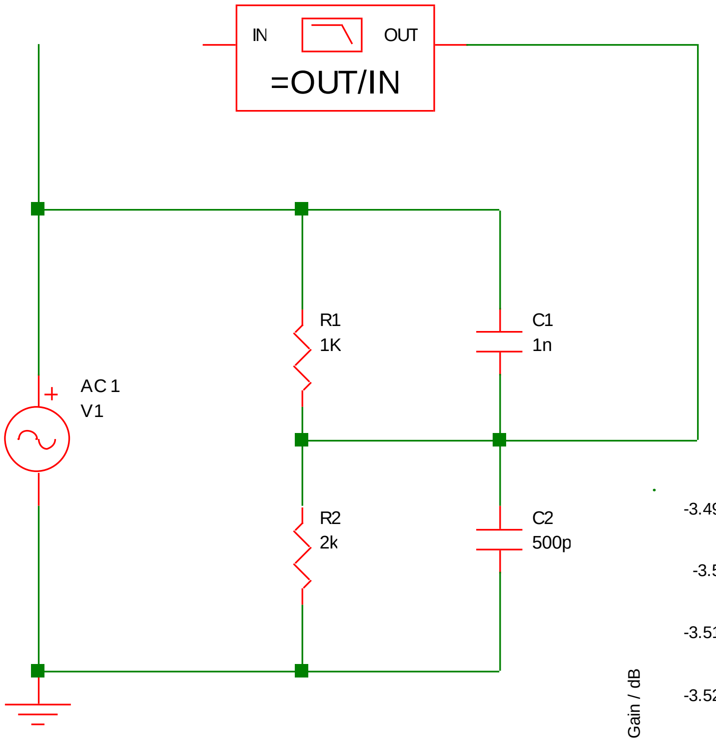
$$v_o = v_I \frac{(R_2 \parallel \chi_{C2})}{(R_1 \parallel \chi_{C1}) + (R_2 \parallel \chi_{C2})}$$

$$T(s) = \frac{v_o}{v_I} = \frac{1}{\frac{(R_1 \parallel \chi_{C1})}{(R_2 \parallel \chi_{C2})} + 1}$$

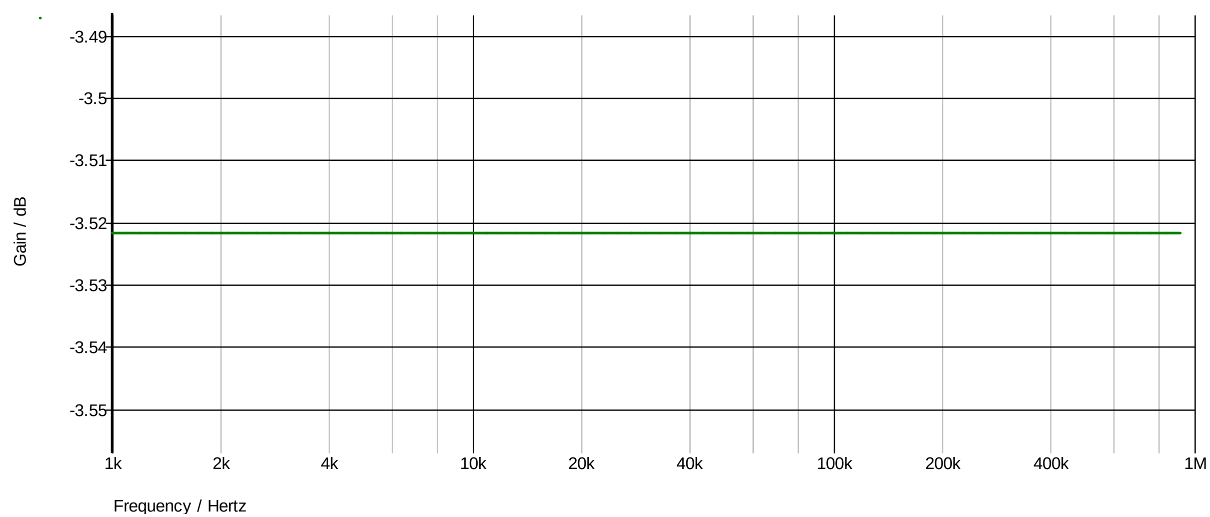
$$R_1 \parallel \chi_{C1} = \frac{R_1 \times {}^1/_{sC_1}}{R_1 + {}^1/_{sC_1}} = \frac{R_1}{sR_1C_1 + 1}$$

$$\text{if } C_1R_1 = C_2R_2 \qquad T(s) = \frac{v_o}{v_I} = \frac{1}{\frac{R_1}{R_2} + 1} = \frac{R_2}{R_1 + R_2}$$

Problem E.2

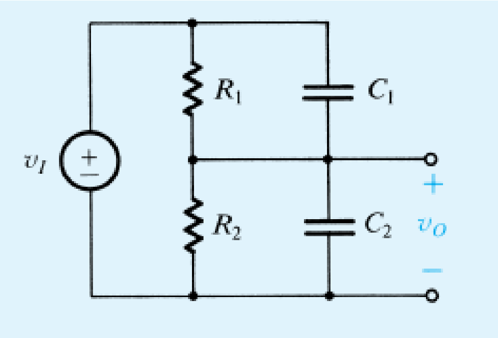


$$20\log\left(\frac{2k}{3k}\right) = -3.522\text{dB}$$



Problem E.3

The circuit of Fig. E.3(a) is used as a compensated attenuator (see Problems E.1 and E.2) for an oscilloscope probe. The objective is to reduce the signal voltage applied to the input amplifier of the oscilloscope, with the signal attenuation independent of frequency. The probe itself includes R_1 and C_1, while R_2 and C_2 model the oscilloscope input circuit. For an oscilloscope having an input resistance of 1 MΩ and an input capacitance of 30 pF, design a compensated "10-to-1 probe"—that is, a probe that attenuates the input signal by a factor of 10. Find the input impedance of the probe when connected to the oscilloscope, which is the impedance seen by v_I in Fig. E.3(a). Show that this impedance is 10 times higher than that of the oscilloscope itself. This is the great advantage of the 10:1 probe.



(a)

$$0.1 = \frac{R_2}{R_1 + R_2} = \frac{1\text{M}\Omega}{R_1 + 1\text{M}\Omega}$$

$$R_1 = 9\text{M}\Omega$$

$$C_1 = \frac{C_2}{9} = 3.33\text{pF}$$

Problem E.4

In the circuits of Figs. E.4 and E.5, let L = 10 mH, C = 0.01 µF, and R = 1 kΩ. At what frequency does a phase angle of 45° occur?

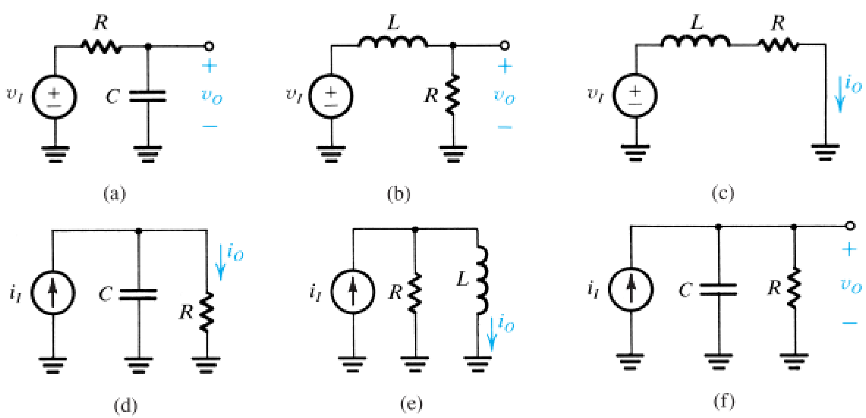


Figure E.4 STC circuits of the low-pass type.

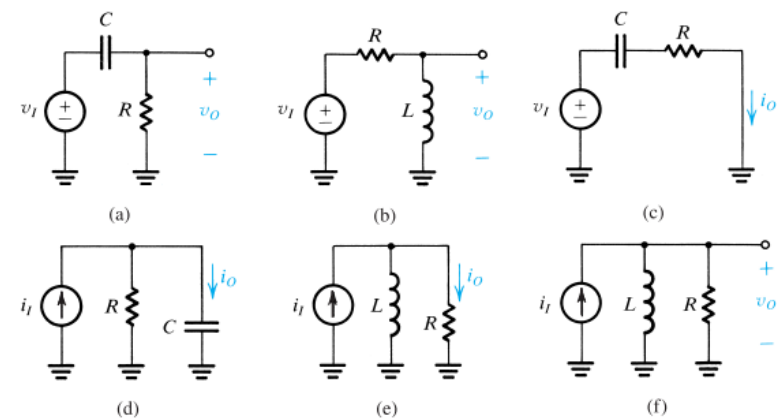


Figure E.5 STC circuits of the high-pass type.

(a) $f_0 = \dfrac{1}{2\pi RC} = 15.9\text{kHz}$

(b) $f_0 = \dfrac{R}{2\pi L} = 15.9\text{kHz}$

(a) $f_0 = \dfrac{1}{2\pi RC} = 15.9\text{kHz}$

(b) $f_0 = \dfrac{R}{2\pi L} = 15.9\text{kHz}$

Problem E.6

For the circuit in Fig. PE.6, assume the voltage amplifier to be ideal. Derive the transfer function $V_o(s)/V_i(s)$. What type of STC response is this? For C = 0.01 μF and R = 100 kΩ, find the corner frequency.

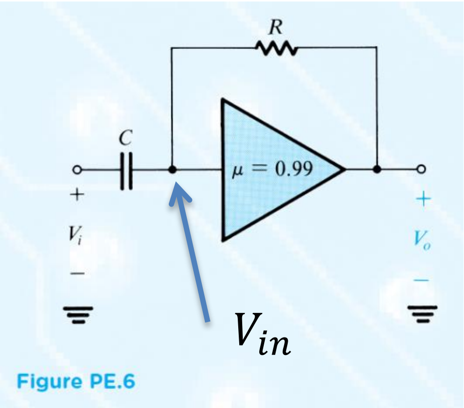


Figure PE.6

$$V_O = \mu V_{in} \Rightarrow V_{in} = \frac{V_O}{\mu} \qquad \chi_C = \frac{1}{sC} \qquad i_{in} = \frac{V_i - V_{in}}{\chi_C} = sC\left(V_i - V_{in}\right)$$

$$V_O = V_{in} - i_{in}R = V_{in} - sC\left(V_i - V_{in}\right)R = \frac{V_O}{\mu} - sC\left(V_i - \frac{V_O}{\mu}\right)R$$

$$V_O\left(1 - \frac{1}{\mu} - \frac{sCR}{\mu}\right) = -sCRV_i$$

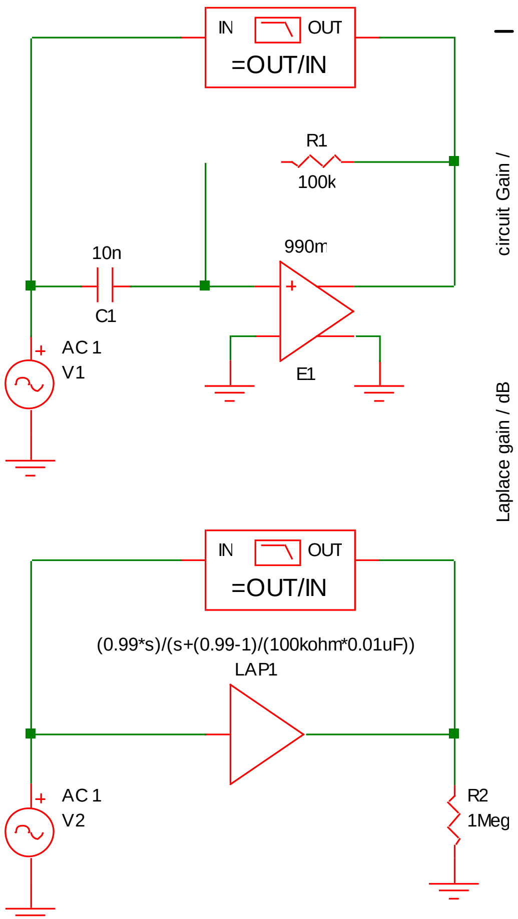
$$T(s) = \frac{V_o}{V_i} = -\frac{sCR}{\left(1 - \frac{1}{\mu} - \frac{sCR}{\mu}\right)} = \frac{\mu s}{\left(s + \frac{1-\mu}{CR}\right)}$$

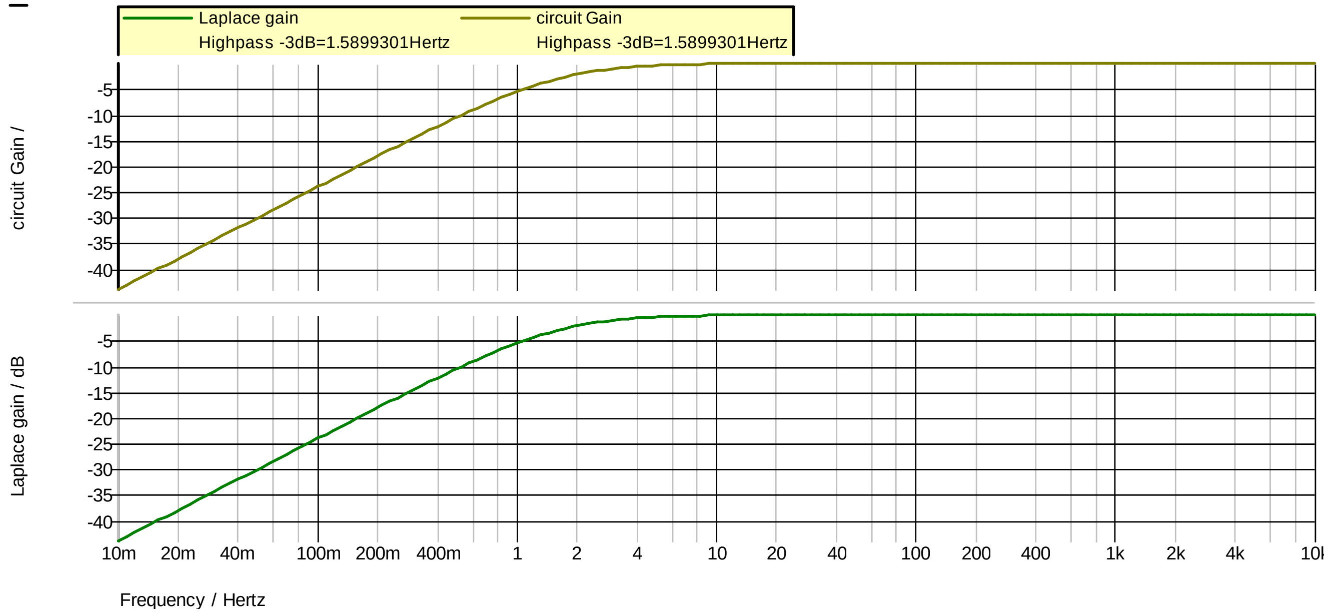
High pass response

$$T(s) = \frac{Ks}{s + \omega_0}$$

$$\omega_0 = \frac{1-\mu}{CR} = \frac{1-0.99}{0.01\text{uF} \times 100\text{k}\Omega} = 10\text{rad/s} = 1.59\text{Hz}$$

Problem E.6


IN
OUT
=OUT/IN
R1
100k
10n
C1
990m
E1
AC 1
V1
IN
OUT
=OUT/IN
(0.99*s)/(s+(0.99-1)/(100kohm*0.01uF))
LAP1
AC 1
V2
R2
1Meg


Laplace gain
Highpass -3dB=1.5899301Hertz
circuit Gain
Highpass -3dB=1.5899301Hertz
circuit Gain /
Laplace gain / dB
-5
-10
-15
-20
-25
-30
-35
-40
10m
20m
40m
100m
200m
400m
1
2
4
10
20
40
100
200
400
1k
2k
4k
10k
Frequency / Hertz