The first 8 questions are multiple choice. Circle the correct answer. Circle only one answer.

1. (5 points) The binomial coefficient, $\binom{n}{k}$ is
   (a) $\frac{n!}{k!}$
   (b) $\frac{n!}{(n-k)!}$
   (c) $\frac{(n+k)!}{n!k!}$
   (d) $\frac{n!}{(n-k)!k!}$
   (e) $\frac{n!k!}{(n+k)!}$

2. (5 points) Let X be uniform on $k = 1, 2, \ldots, m$. What is $E(X)$?
   (a) $\frac{m}{2}$
   (b) $\frac{m-1}{2}$
   (c) $\frac{m+1}{2}$
   (d) m
   (e) $\frac{m(m-1)}{2}$

3. (5 points) Let $p(k) = ck$ for $k = 1, 2, 3, 4$ be a PMF. What is c?
   (a) $\frac{1}{15}$
   (b) $\frac{1}{10}$
   (c) $\frac{1}{5}$
   (d) $\frac{1}{4}$
   (e) $\frac{1}{2}$

4. (5 points) Let X be uniform on $k = 1, 2, \ldots, m$. What is the entropy $H(X)$?
   (a) $\frac{m}{2}$
   (b) $\frac{(m+1)(m-1)}{2}$
   (c) $\log_2(m/2)$
   (d) $\log_2(m)$
   (e) $\log_2(m-1)$

5. (5 points) What are the mean and variance of a binomial random variable with parameters n and p (with $q = 1 - p$)?

   (a) np and pq

   (b) nq and npq

   (c) p and n

   (d) pq and npq

   (e) np and npq

6. (5 points) If X and Y are independent with PMFs $[0.4, 0.4, 0.2]$ and $[0.5, 0.5]$, respectively, what is the PMF of $S = X + Y$?

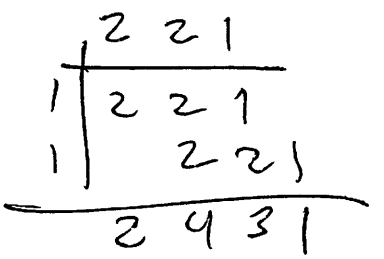
   (a) $[0.2, 0.4, 0.3, 0.1]$

   (b) $[0.4, 0.4, 0.2]$

   (c) $[0.2, 0.2, 0.1, 0.5]$

   (d) $[0.1, 0.2, 0.4, 0.2, 0.1]$

   (e) $[0.1, 0.3, 0.4, 0.2]$



7. (5 points) Roll an ordinary 6-sided die. Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 5\}$. What is $\Pr(A|B)$?

   (a) 1/3

   (b) 1/2

   (c) 1

   (d) 2/3

   (e) 0

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(\{2,3\})}{P(\{2,3,5\})} = \frac{2/6}{3/6} = \frac{2}{3}$$

8. (5 points) Let X have PMF $p(0) = 1/3$, $p(1) = 1/3$, $p(2) = 0$, and $p(3) = 1/3$. What is the variance of X?

   (a) 1/3

   (b) 10/3

   (c) 4/3

   (d) 14/9

   (e) 2

$$EX^2 = 0^2 \cdot \frac{1}{3} + 1^2 \cdot \frac{1}{3} + 2^2 \cdot 0 + 3^2 \cdot \frac{1}{3} = \frac{10}{3}$$

$$EX = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 2 \cdot 0 + 3 \cdot \frac{1}{3} = \frac{4}{3}$$

$$Var(X) = \frac{10}{3} - \left(\frac{4}{3}\right)^2 = \frac{30 - 16}{9} = \frac{14}{9}$$

9. (15 points) For parts (a) through (e) below, write one of Bernoulli, Binomial, Geometric, Negative Binomial, Poisson, or Uniform next to each phrase.
   (a) count of rare events Poisson
   (b) number of flips to get a heads Geometric
   (c) a coin flip Bernoulli
   (d) count of heads in fixed number of flips Binomial
   (e) number of flips to get k heads Negative Binomial

10. (15 points) For parts (a) through (e) below, write one of Bernoulli, Binomial, Geometric, Negative Binomial, Poisson, or Uniform next to each expression for the PMF (using the usual form for each one).

(a) $\binom{n}{k}p^k(1-p)^{n-k}$ Binomial

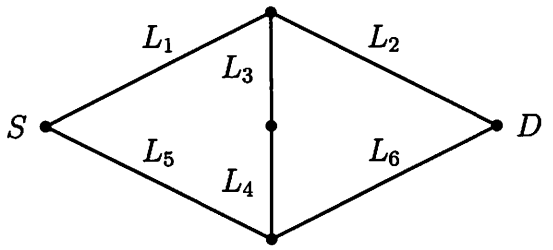
(b) $p(1-p)^{k-1}$ Geometric

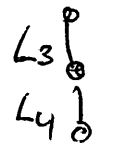
(c) $\frac{1}{m}$ Uniform

(d) $\binom{n-1}{k-1}pq^{k-1}$ Negative Binomial

(e) $\frac{\lambda^k e^{-\lambda}}{k!}$ Poisson

11. (15 points) In the network below, each link works independently from the others with probability p. What is the probability S can send a message to D?



let $E =$ 

$$P(S\to D) = P(S\to D \mid E)P(E) + P(S\to D \mid \bar{E})P(\bar{E})$$

$$= (2p-p^2)^2 p^2 + (2p^2-p^4)(1-p^2)$$

$$= 4p^4 - 4p^5 + p^6 + 2p^2 - p^4 - 2p^4 + p^6$$

$$= 2p^2 + p^4 - 4p^5 + 2p^6$$

Check $p=1 \Rightarrow P(S\to D) = 2+1-4+2 = 1$

12. (15 points) Let X and Y have joint PMF below.

y \ x	0	1	2	3
1	0.2	0.1	0.1	0.2
0	0.1	0.1	0.1	0.1

What are the following?

(a) EX

(b) EY

(c) $\mathrm{Cov}(X,Y)$

(d) $Pr(Y=1|X=3)$

(e) Are X and Y independent? Why or why not?

a) $EX = 0\times 0.3 + 1\times 0.2 + 2\times 0.2 + 3\times 0.3 = 1.5$

b) $EY = 0\times 0.4 + 1\times 0.6 = 0.6$

c) $\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y)$

$E(XY) = 1\times 1\times 0.1 + 2\times 1\times 0.1 + 3\times 1\times 0.2 = 0.9$

$\mathrm{Cov}(X,Y) = 0.9 - 0.6\times 1.5 = 0.9 - 0.9 = 0$

d) $P(Y=1|X=3) = \frac{0.2}{0.2+0.1} = \frac{2}{3}$

e) No. Eg. $P(X=0 \cap Y=0) = 0.1 \neq P(X=0)P(Y=0)$

$= 0.3\times 0.4 = 0.12$