Ch 24 # 26

26. (*a*) The two capacitors are in $\boxed{\text{parallel}}$. Both capacitors have their high voltage plates at the sa potential (the middle plate), and both capacitors have their low voltage plates at the same potential (the outer plates, which are connected).

(*b*) The capacitance of two capacitors in parallel is the sum of the individual capacitances.

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{d_1} + \frac{\varepsilon_0 A}{d_2} = \boxed{\varepsilon_0 A\left(\frac{1}{d_1} + \frac{1}{d_2}\right)} = \varepsilon_0 A\left(\frac{d_1 + d_2}{d_1 d_2}\right)$$

(*c*) Let $\ell = d_1 + d_2 =$ constant. Then $C = \frac{\varepsilon_0 A\ell}{d_1 d_2} = \frac{\varepsilon_0 A\ell}{d_1(\ell - d_1)}$. We see that $C \to \infty$ as $d_1 \to 0$ or $d_1 \to \ell$ (which is $d_2 \to 0$). Of course, a real capacitor would break down as the plates got too close to each other. To find the minimum capacitance, set $\frac{dC}{d(d_1)} = 0$ and solve for d_1.

$$\frac{dC}{d(d_1)} = \frac{d}{d(d_1)}\left[\frac{\varepsilon_0 A\ell}{d_1\ell - d_1^2}\right] = \varepsilon_0 A\ell\frac{(\ell - 2d_1)}{(d_1\ell - d_1^2)^2} = 0 \to d_1 = \tfrac{1}{2}\ell = d_2$$

$$C_{\min} = \varepsilon_0 A\left(\frac{d_1 + d_2}{d_1 d_2}\right)_{d_1 = \frac{1}{2}\ell} = \varepsilon_0 A\left(\frac{\ell}{(\frac{1}{2}\ell)(\frac{1}{2}\ell)}\right) = \varepsilon_0 A\left(\frac{4}{\ell}\right) = \varepsilon_0 A\left(\frac{4}{d_1 + d_2}\right)$$

$$\boxed{C_{\min} = \frac{4\varepsilon_0 A}{d_1 + d_2}\ ;\ C_{\max} = \infty}$$

Ch 24 # 29

29. (*a*) From the diagram, we see that C_1 and C_2 are in series. That combination is in parallel with C_3, and then that combination is in series with C_4. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$\frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \to C_{12} = \tfrac{1}{2}C\ ;\ C_{123} = C_{12} + C_3 = \tfrac{1}{2}C + C = \tfrac{3}{2}C\ ;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C} \to C_{1234} = \boxed{\tfrac{3}{5}C}$$

(*b*) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234}V = \frac{3}{5}CV$. This is the charge on both of the series components of C_{1234}.

$$Q_{123} = \tfrac{3}{5}CV = C_{123}V_{123} = \tfrac{3}{2}CV_{123} \to V_{123} = \tfrac{2}{5}V$$
$$Q_4 = \tfrac{3}{5}CV = C_4V_4 \to V_4 = \tfrac{3}{5}V$$

The voltage across the equivalent capacitor C_{123} is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of C_{123} is the same as the total charge on the two components, $\frac{3}{5}CV$.

$$V_{123} = \tfrac{2}{5}V = V_{12}\ ;\ Q_{12} = C_{12}V_{12} = (\tfrac{1}{2}C)(\tfrac{2}{5}V) = \tfrac{1}{5}CV$$
$$V_{123} = \tfrac{2}{5}V = V_3\ ;\ Q_3 = C_3V_3 = C(\tfrac{2}{5}V) = \tfrac{2}{5}CV$$

Finally, the charge on the equivalent capacitor C_{12} is the charge on both of the series components of C_{12}.

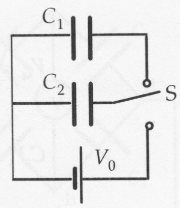
$$Q_{12} = \tfrac{1}{5}CV = Q_1 = C_1V_1 \to V_1 = \tfrac{1}{5}V\ ;\ Q_{12} = \tfrac{1}{5}CV = Q_2 = C_1V_2 \to V_2 = \tfrac{1}{5}V$$

Here are all the results, gathered together.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = \tfrac{1}{5}CV\ ;\ Q_3 = \tfrac{2}{5}CV\ ;\ Q_4 = \tfrac{3}{5}CV \\ V_1 = V_2 = \tfrac{1}{5}V\ ;\ V_3 = \tfrac{2}{5}V\ ;\ V_4 = \tfrac{3}{5}V \end{array}}$$

Ch 24 # 31

31. When the switch is down the initial charge on C_2 is calculated from Eq. 24-1.



$$Q_2 = C_2 V_0$$

When the switch is moved up, charge will flow from C_2 to C_1 until the voltage across the two capacitors is equal.

$$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1} \quad \to \quad Q_2' = Q_1'\frac{C_2}{C_1}$$

The sum of the charges on the two capacitors is equal to the initial charge on C_2.

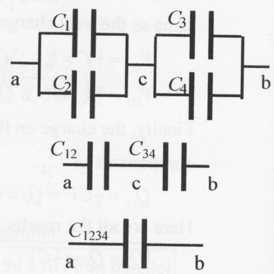
$$Q_2 = Q_2' + Q_1' = Q_1'\frac{C_2}{C_1} + Q_1' = Q_1'\left(\frac{C_2 + C_1}{C_1}\right)$$

Inserting the initial charge in terms of the initial voltage gives the final charges.

$$Q_1'\left(\frac{C_2 + C_1}{C_1}\right) = C_2 V_0 \quad \to \quad Q_1' = \boxed{\frac{C_1 C_2}{C_2 + C_1}V_0} \; ; \; Q_2' = Q_1'\frac{C_2}{C_1} = \boxed{\frac{C_2^2}{C_2 + C_1}V_0}$$

Ch 24 # 32

32. (*a*) From the diagram, we see that C_1 and C_2 are in parallel, and C_3 and C_4 are in parallel. Those two combinations are then in series with each other. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.



$$C_{12} = C_1 + C_2 \; ; \; C_{34} = C_3 + C_4 \; ;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{12}} + \frac{1}{C_{34}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3 + C_4} \quad \to$$

$$C_{1234} = \boxed{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}$$

(*b*) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234}V$. This is the charge on both of the series components of C_{1234}. Note that $V_{12} + V_{34} = V$.

$$Q_{12} = C_{1234}V = C_{12}V_{12} \to V_{12} = \frac{C_{1234}}{C_{12}}V = \frac{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}{C_1 + C_2}V = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V$$

$$Q_{34} = C_{1234}V = C_{34}V_{34} \to V_{34} = \frac{C_{1234}}{C_{34}}V = \frac{\frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}}{C_3 + C_4}V = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V$$

The voltage across the equivalent capacitor C_{12} is the voltage across both of its parallel components, and the voltage across the equivalent C_{34} is the voltage across both its parallel components.

$$V_{12} = \boxed{V_1 = V_2 = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V} \; ;$$

$$C_1V_1 = \boxed{Q_1 = \frac{C_1(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V} \; ; \; C_2V_2 = \boxed{Q_2 = \frac{C_2(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V}$$

$$V_{34} = \boxed{V_3 = V_2 = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V} \; ;$$

$$C_3V_3 = \boxed{Q_3 = \frac{C_3(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V} \; ; \; C_4V_4 = \boxed{Q_4 = \frac{C_4(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V}$$

Ch 24 # 36

36. The initial equivalent capacitance is the series combination of the two individual capacitances. Each individual capacitor will have the same charge as the equivalent capacitance. The sum of the two initial charges will be the sum of the two final charges, because charge is conserved. The final potential of both capacitors will be equal.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \ ; \ Q_{eq} = C_{eq} V_0 = \frac{C_1 C_2}{C_1 + C_2} V_0 = \frac{(3200\,\text{pF})(1800\,\text{pF})}{5000\,\text{pF}}(12.0\,\text{V}) = 13{,}824\,\text{pC}$$

$$Q_{1\,\text{final}} + Q_{2\,\text{final}} = 2Q_{eq} \ ; \ V_{1\,\text{final}} = V_{2\,\text{final}} \ \rightarrow \ \frac{Q_{1\,\text{final}}}{C_1} = \frac{Q_{2\,\text{final}}}{C_2} = \frac{2Q_{eq} - Q_{1\,\text{final}}}{C_2} \rightarrow$$

$$Q_{1\,\text{final}} = 2\frac{C_1}{C_1 + C_2} Q_{eq} = 2\frac{3200\,\text{pF}}{5000\,\text{pF}}(13{,}824\,\text{pC}) = 17{,}695\,\text{pC} \approx \boxed{1.8 \times 10^{-8}\,\text{C}}$$

$$Q_{2\,\text{final}} = 2Q_{eq} - Q_{1\,\text{final}} = 2(13{,}824\,\text{pC}) - 17{,}695\,\text{pC} = 9953\,\text{pC} \approx \boxed{1.0 \times 10^{-8}\,\text{C}}$$