ELEG 310

3/15/2018

$S = X_1 + X_2 + X_3$

$E(S) = E(X_1) + E(X_2) + E(X_3)$

$Var(S) = Var(X_1) + Var(X_2) + Var(X_3)$

$- 2\,Cov(X_1, X_2) - 2\,Cov(X_1, X_3) - 2\,Cov(X_2, X_3)$

if $X_1\ X_2\ X_3$ are ind $\Rightarrow Cov(X_1, X_2) = 0 \quad Cov(X_1, X_3) = 0$

$\cancel{\Leftarrow} \quad Cov(X_2, X_3) = 0$

X_1 & X_2 are ind if $P_{X_1X_2}(k, l) = P_{X_1}(k)\, P_{X_2}(l)$

$P(X_1 = k \mid X_2 = l) = P(X_1 = k)$

$S = X_1 + X_2 + X_3$ $\quad$ $X_1 \quad X_2 \quad X_3$ <u>ind</u>

$P_S(m) = P_{X_1} * P_{X_2} * P_{X_3} \ (m)$

Binomial Probabilities

Bernoulli RV $\quad P(X=0) = 1-p = q \quad P(X=1) = p$

$S = X_1 + X_2$

	q ($X_1=0$)	p ($X_1=1$)	
q ($X_2=0$)	q^2	qp	
p ($X_2=1$)		qp	p^2
	q^2	$2qp$	p^2

$S = X_1 + X_2 + X_3$

q	q^3	$2q^2p$	qp^2	
p		q^2p	$2qp^2$	p^3
	q^3	$3q^2p$	$3p^2q$	p^3

$S = X_1 + X_2 + X_3 + X_4$

	q^3	$3q^2p$	$3qp^2$	p^3	
q	q^4	$3q^3p$	$3q^2p^2$	qp^3	
p		q^3p	$3q^2p^2$	$3qp^3$	p^4

$\binom{4}{0}=1 \quad \binom{4}{1}=4 \quad \binom{4}{2}=6 \quad \binom{4}{3}=4 \quad \binom{4}{4}=1$

$n=4 \rightarrow q^4 \quad 4q^3p \quad 6q^2p^2 \quad 4qp^3 \quad p^4$

think ahead

in general, $S = X_1 + X_2 + \dots + X_n$

$$P(S=k) = \binom{n}{k} p^k q^{n-k} \qquad k=0,1,\dots,n$$

Binomial Probs

→ Prob of getting k heads in n flips

" " " k 1's in n Bernoulli trials

S_n is binomial $\qquad b(n,k,p) = P(S_n = k) = \binom{n}{k} p^k q^{n-k}$

$$ES_n = \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k} \qquad 1 = (p+q)^n = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k}$$

$$M(u) = \sum_{k=0}^{n} e^{uk} \binom{n}{k} p^k q^{n-k} = \sum_{k=0}^{n} \binom{n}{k} (pe^u)^k q^{n-k} = (pe^u + q)^n$$

$$\boxed{ES} = \frac{\partial M}{\partial u}\Big|_{u=0} = n(\;)^{n-1} pe^u \Big|_{u=0} = \boxed{np}$$

$$\boxed{\mathrm{Var}\, S = nqp}$$

$S = X_1 + X_2 + \cdots X_n$

$ES = EX_1 + EX_2 + \cdots + EX_n \qquad EX = 0 \cdot q + 1 \cdot p = p$

$= p + p + \cdots + p = np$

$E(X^2) = 0^2 q + 1^2 p = p$

$Var(X) = E(X^2) - \mu^2 = p - p^2 = p(1-p) = pq$

$\underline{Var(S) = npq}$

Negative Binomial - How many flips are required to get k heads?

N = # flips required = RV

$$P(N=n) = \binom{n-1}{k-1} p^{k-1} q^{n-k} p = \binom{n-1}{k-1} p^k q^{n-k}$$

$$n = k, k+1, k+2, \ldots$$

$$E(N) = \sum_{n=k}^{\infty} n P(N=n) = \sum_{n=k}^{\infty} n \binom{n-1}{k-1} p^k q^{n-k}$$

Ex. $k=3$ $\quad \underbrace{0001}\underbrace{01}\underbrace{1}$

runs

Geometric $P(L=l) = p q^{l-1}$

$\Rightarrow N = L_1 + L_2 + \cdots + L_K$

$EN = EL_1 + \cdots + EL_k$ $\qquad EN = \frac{k}{p}$

$EL = \frac{1}{p}$ $\qquad \mathrm{Var}\, L = \frac{\cancel{k}p}{p^2}$

$\mathrm{Var}\, L = k\left(\frac{1-p}{p^2}\right)$

Poisson Distribution

$P(N=k) = \frac{\lambda^k e^{-\lambda}}{k!}$ $\qquad k = 0, 1, 2, \ldots$

$E(N) = \lambda \qquad \mathrm{Var}(N) = \lambda$

binomial $\binom{n}{k} p^k q^{n-k} \longrightarrow \frac{\lambda^k e^{-\lambda}}{k!}$ where $\lambda = np$ and $n \to \infty$, $p \to 0$