ELEG 310

4/26/2018

Ex. $P(\text{76ers win}) = p$

$$E(X) = p \times 1 - (1-p)1.1 \geq 0$$

$$p + 1.1p - 1 \geq 0 \qquad 2.1p \geq 1.1 \qquad p \geq \frac{1.1}{2.1} = 0.52$$

Gaussian (Normal) Distribution

$Z \sim N(0,1)$ "standard normal"

$X \sim N(\mu, \sigma^2)$ $\qquad X = \mu + \sigma Z \qquad Z = \dfrac{X-\mu}{\sigma}$

Compute probs $\quad \Phi(z) = P(Z \le z) = F_Z(z)$

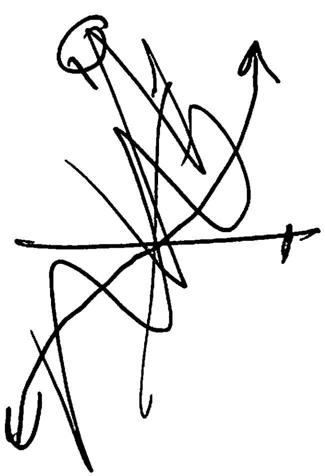
$$P(a \le Z \le b) = \Phi(b) - \Phi(a)$$

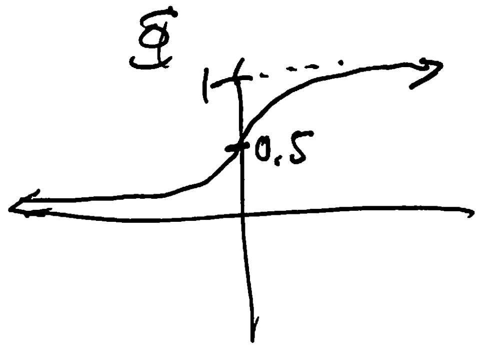
$$P(c \le X \le d) = P\left(\frac{c-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{d-\mu}{\sigma}\right)$$

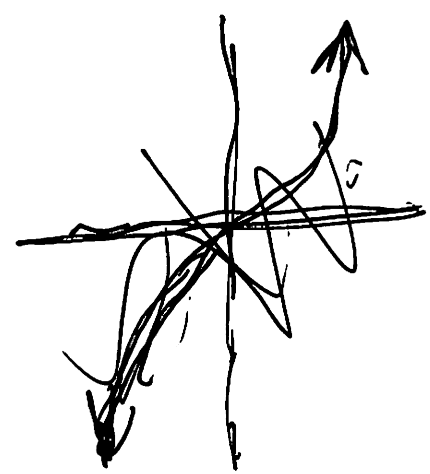
$$= P\left(\frac{c-\mu}{\sigma} \le Z \le \frac{d-\mu}{\sigma}\right) = \Phi\left(\frac{d-\mu}{\sigma}\right) - \Phi\left(\frac{c-\mu}{\sigma}\right)$$

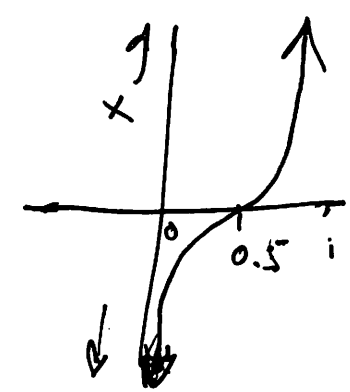
Quantile Function

$$Q(p) = x \Rightarrow p = \Phi(x) \Rightarrow Q = \Phi^{-1}$$

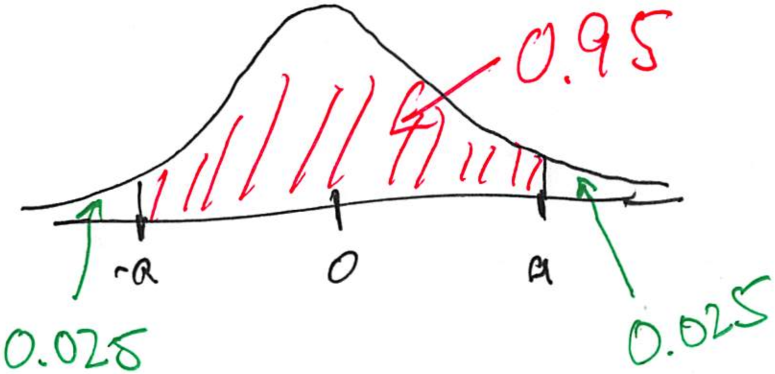








Ex. 95% prob



$$\underbrace{P(-a \le Z \le a)}_{} = 0.95$$

$$\underbrace{\Phi(a) - \Phi(-a)}_{}$$

$$2\Phi(a) - 1 = 0.95$$

$$\Phi(a) = \frac{1.95}{2} \qquad a = Q\left(\frac{1.95}{2}\right) = 1.96$$

$$P\left(-1.96 \le \frac{X-\mu}{\sigma} \le 1.96\right) = 0.95$$

95% of time within 1.96σ of μ

Central Limit Theorem $X_1\ X_2 \cdots X_n$ IID $\mu, \sigma^2 < \infty$

let $S_n = X_1 + X_2 + \cdots + X_n$

$$E(S_n) = n\mu$$

$$Var(S_n) = n\sigma^2$$

CLT $\Rightarrow S_n \approx N(n\mu, n\sigma^2)$

Ex. Cauchy Distribution

$$f(x) = \frac{c}{1+x^2} \quad -\infty < x < \infty$$

$$EX = \int_{-\infty}^{\infty} c \frac{x}{1+x^2}\,dx = \text{undefined} = \infty - \infty$$

$$EX^2 = \int_{-\infty}^{\infty} c \frac{x^2}{1+x^2}\,dx = \infty$$