Homework #2

Problems 1.39, 1.43, 1.45, 1.48, and 1.68

Problem 1.39

Various amplifier and load combinations are measured as listed below using rms values. For each, find the voltage, current, and power gains (A_v, A_i, and A_p respectively) both as ratios and in dB:

(a) $v_i = 100$ mV, $i_i = 100$ μA, $v_o = 10$ V, $R_L = 100\ \Omega$

(b) $v_i = 10$ μV, $i_i = 100$ nA, $v_o = 1$ V, $R_L = 10$ kΩ

(c) $v_i = 1$ V, $i_i = 1$ mA, $v_o = 5$ V, $R_L = 10\ \Omega$

Problem 1.39a

Various amplifier and load combinations are measured as listed below using rms values. For each, find the voltage, current, and power gains (A_v, A_i, and A_p respectively) both as ratios and in dB:

(a) $v_i = 100$ mV, $i_i = 100$ μA, $v_o = 10$ V, $R_L = 100\ \Omega$

$$A_v = \frac{v_o}{v_i} = \frac{10\text{V}}{100\text{mV}} = 100\ \text{V/V} \Rightarrow 20\log(100) = 40\text{dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{v_0 / R_L}{i_i} = \frac{10\text{V}/100\Omega}{100\mu\text{A}} = \frac{100\text{mA}}{100\mu\text{A}} \Rightarrow 1000\ \text{A/A} = 20\log(1000) = 60\text{dB}$$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i}\frac{i_o}{i_i} = 100 \times 1000 = 10^5\ \text{W/W} \Rightarrow 10\log(10^5) = 50\text{dB}$$

Problem 1.39b

Various amplifier and load combinations are measured as listed below using rms values. For each, find the voltage, current, and power gains (A_v, A_i, and A_p respectively) both as ratios and in dB:

(b) $v_i = 10\ \mu\text{V}$, $i_i = 100\ \text{nA}$, $v_o = 1\ \text{V}$, $R_L = 10\ \text{k}\Omega$

$$A_v = \frac{v_o}{v_i} = \frac{1\text{V}}{10\mu\text{V}} = 1\times 10^5\ \text{V/V} \Rightarrow 20\log(1\times 10^5) = 100.0\text{dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{v_0 / R_L}{i_i} = \frac{1\text{V}/10\text{k}\Omega}{100\text{nA}} = \frac{100\mu\text{A}}{100\text{nA}} = 1000\ \text{A/A} \Rightarrow 20\log(1000) = 60.0\text{dB}$$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i}\frac{i_o}{i_i} = 1\times 10^5 \times 1000 = 1\times 10^8\ \text{W/W} \Rightarrow 10\log(1\times 10^8) = 80.0\text{dB}$$

Problem 1.39c

Various amplifier and load combinations are measured as listed below using rms values. For each, find the voltage, current, and power gains (A_v, A_i, and A_p respectively) both as ratios and in dB:

(c) $v_i = 1$ V, $i_i = 1$ mA, $v_o = 5$ V, $R_L = 10\ \Omega$

$$A_v = \frac{v_o}{v_i} = \frac{5\text{V}}{1\text{V}} = 5\ \text{V/V} \Rightarrow 20\log(5) = 14.0\text{dB}$$

$$A_i = \frac{i_o}{i_i} = \frac{v_0 / R_L}{i_i} = \frac{5\text{V}/10\Omega}{1\text{mA}} = \frac{0.5\text{A}}{1\text{mA}} = 500\ \text{A/A} \Rightarrow 20\log(500) = 54.0\text{dB}$$

$$A_p = \frac{v_o i_o}{v_i i_i} = \frac{v_o}{v_i}\frac{i_o}{i_i} = 5\times 500 = 2500\ \text{W/W} \Rightarrow 10\log(2500) = 34.0\text{dB}$$

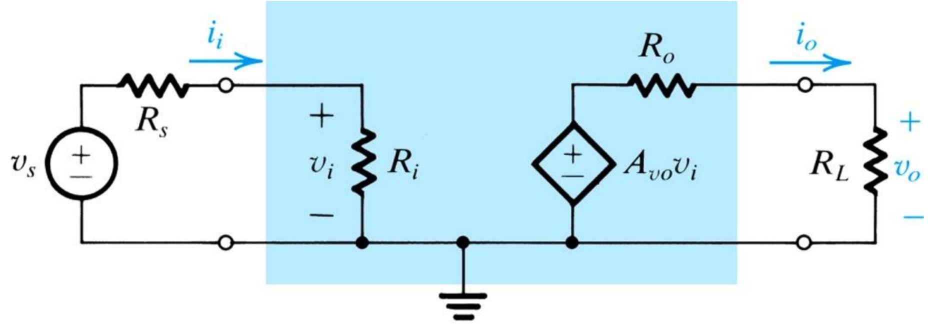
Problem 1.43

Consider the voltage-amplifier circuit model shown in Fig. 1.16(b), in which A_{vo} = 100 V/V, under the following conditions:

(a) $R_i = 10R_s$, $R_L = 10R_o$

(b) $R_i = R_s$, $R_L = R_o$

(c) $R_i = R_s/10$, $R_L = R_o/10$



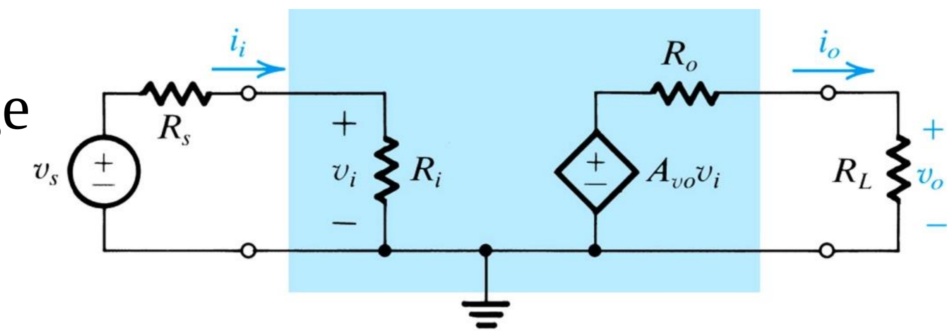
Calculate the overall voltage gain v_o/v_s, in each case, expressed both directly and in decibels.

Problem 1.43a

Consider the voltage-amplifier circuit model shown in Fig. 1.16(b), in which A_{vo} = 100 V/V, under the following conditions:

(a) $R_i = 10R_s$, $R_L = 10R_o$

Calculate the overall voltage gain v_o/v_s, in each case, expressed both directly and in decibels.



$$\frac{v_o}{v_s} = A_{vo}\,\frac{R_i}{R_i+R_s}\,\frac{R_L}{R_L+R_o} = 100\,\frac{10R_s}{10R_s+R_s}\,\frac{10R_o}{10R_0+R_o} = 100 \times \frac{10}{11} \times \frac{10}{11}$$

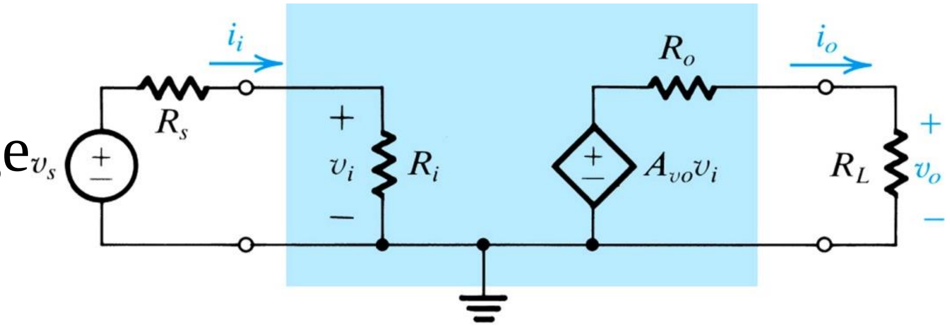
$$\frac{v_o}{v_s} = \frac{10000}{121} = 82.6\text{V/V} = 38.34\text{dB}$$

Problem 1.43b

Consider the voltage-amplifier circuit model shown in Fig. 1.16(b), in which A_{vo} = 100 V/V, under the following conditions:

(b) $R_i = R_s$, $R_L = R_o$

Calculate the overall voltage gain v_o/v_s, in each case, expressed both directly and in decibels.



$$\frac{v_o}{v_s} = A_{vo}\frac{R_i}{R_i+R_s}\frac{R_L}{R_L+R_o}=100\frac{R_s}{R_s+R_s}\frac{R_o}{R_o+R_o} = 100 \times \frac{1}{2} \times \frac{1}{2}$$

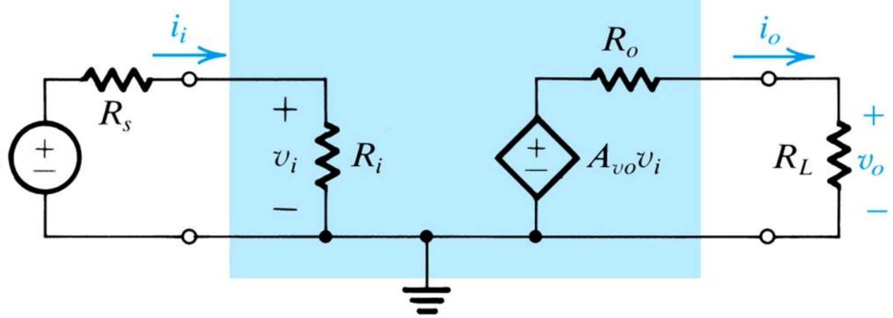
$$\frac{v_o}{v_s} = \frac{100}{4} = 25\mathrm{V/V} = 27.96\mathrm{dB}$$

Problem 1.43c

Consider the voltage-amplifier circuit model shown in Fig. 1.16(b), in which A_{vo} = 100 V/V, under the following conditions:

(c) $R_i = R_s/10$, $R_L = R_o/10$

Calculate the overall voltage gain v_o/v_s, in each case, expressed both directly and in decibels.

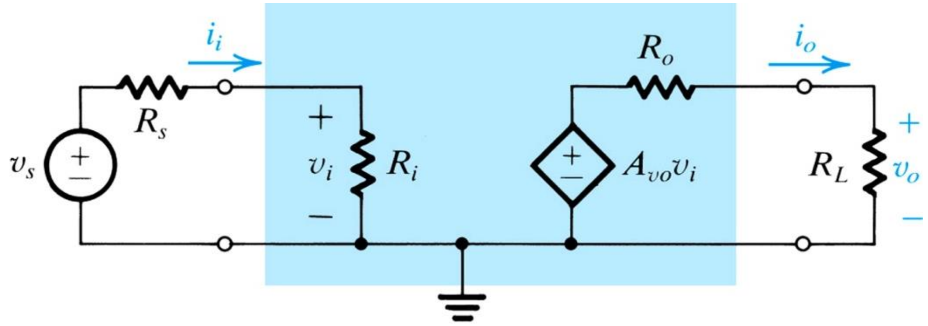


$$\frac{v_o}{v_s} = A_{vo}\frac{R_i}{R_i+R_s}\frac{R_L}{R_L+R_o}=100\frac{0.1R_s}{0.1R_s+R_s}\frac{0.1R_o}{0.1R_0+R_o} = 100\times\frac{0.1}{1.1}\times\frac{0.1}{1.1}$$

$$\frac{v_o}{v_s} = \frac{1}{1.21} = 0.83\text{V/V} = -1.66\text{dB}$$

Problem 1.45

A 10-mV signal source having an internal resistance of 100 kΩ is connected to an amplifier for which the input resistance is 10 kΩ, the open-circuit voltage gain is 1000 V/V, and the output resistance is 1 kΩ. The amplifier is connected in turn to a 100 Ω load. What overall voltage gain results as measured from the source internal voltage to the load? Where did all the gain go? What would the gain be if the source was connected directly to the load? What is the ratio of these two gains? This ratio is a useful measure of the benefit the amplifier brings.



Problem 1.45a

A 10-mV signal source having an internal resistance of 100 kΩ is connected to an amplifier for which the input resistance is 10 kΩ, the open-circuit voltage gain is 1000 V/V, and the output resistance is 1 kΩ. The amplifier is connected in turn to a 100 Ω load. What overall voltage gain results as measured from the source internal voltage to the load? Where did all the gain go?

$$\frac{v_o}{v_s} = A_{vo}\frac{R_i}{R_i+R_s}\frac{R_L}{R_L+R_o}=1000\frac{10k\Omega}{10k\Omega+100k\Omega}\frac{100\Omega}{100\Omega+1k\Omega}$$

$$= 1000 \times \frac{10k}{110k} \times \frac{100}{1100} = 8.26\mathrm{V/V} = 18.34\mathrm{dB}$$

Overall gain is 8.26 V/V (18.34 dB) as opposed to 1000 V/V (60 dB) for just the amplifier. This is due to the low input resistance wrt the source and the high output resistance wrt the load.

Problem 1.45b

A 10-mV signal source having an internal resistance of 100 kΩ is connected to an amplifier for which the input resistance is 10 kΩ, the open-circuit voltage gain is 1000 V/V, and the output resistance is 1 kΩ. The amplifier is connected in turn to a 100 Ω load. What overall voltage gain results as measured from the source internal voltage to the load? Where did all the gain go? What would the gain be if the source was connected directly to the load? What is the ratio of these two gains? This ratio is a useful measure of the benefit the amplifier brings.

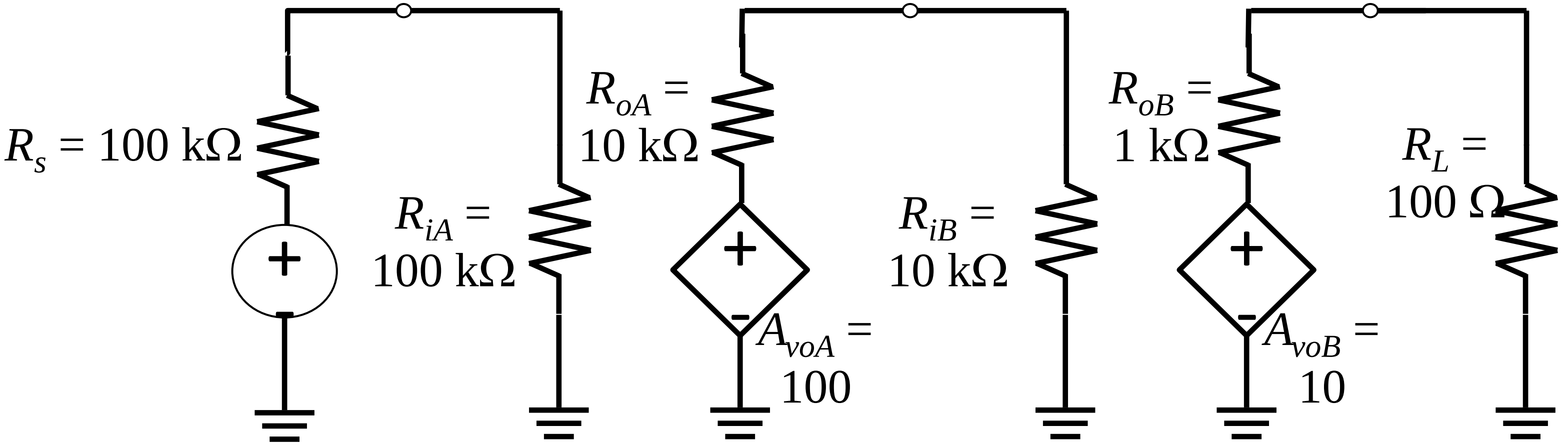
$$\frac{v_o}{v_s} = \frac{R_L}{R_L + R_s} = \frac{100\Omega}{100\Omega + 100k\Omega} = .001\text{V/V} = -60\text{dB}$$

The gain is .001 V/V (-60 dB) if the source is connected to the load which is significantly worse than when the amplifier was inserted. The ratio of the gains is 8.26/.001 = 8260. Therefore, inserting the amplifier results in an increase in gain of ~ 8260 then if it wasn't present.

Problem 1.48

You are given two amplifiers, A and B, to connect in cascade between a 10-mV, 100-kΩ source and a 100-Ω load. The amplifiers have voltage gain, input resistance, and output resistance as follows: for A, 100 V/V, 100 kΩ, 10 kΩ, respectively; for B, 10 V/V, 10 kΩ, 1 kΩ, respectively. Your problem is to decide how the amplifiers should be connected. To proceed, evaluate the two possible connections between source S and load L, namely, SABL and SBAL. Find the voltage gain for each both as a ratio and in decibels. Which amplifier arrangement is best?

Problem 1.48-SABL



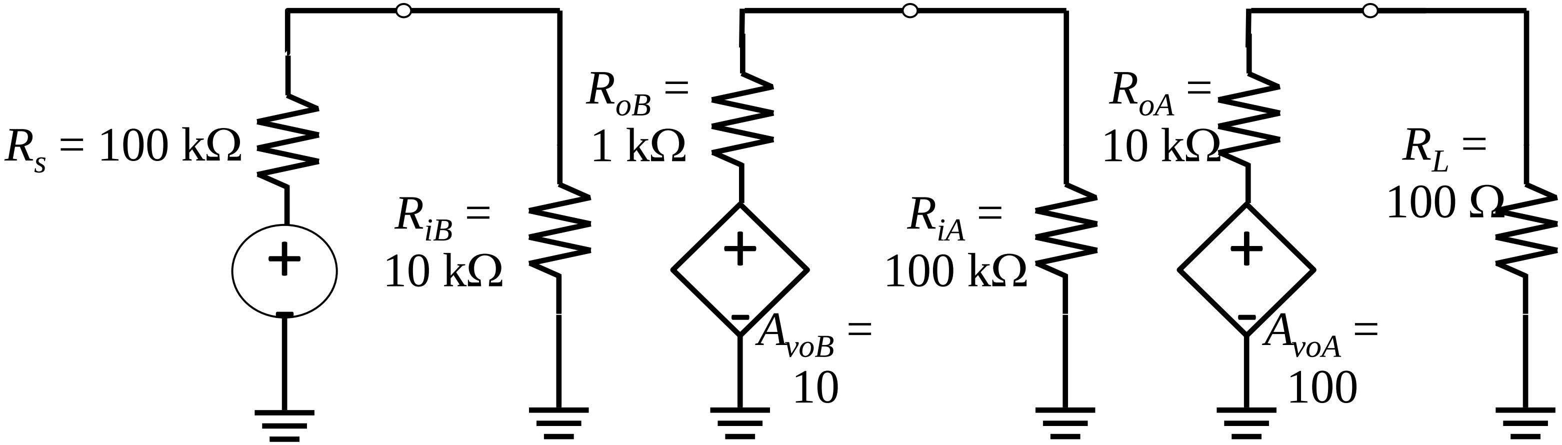
$$v_{iA} = v_s \frac{R_{iA}}{R_{iA} + R_s} = v_s \frac{100k}{100k + 100k}$$

$$v_{iB} = 100 v_{iA} \frac{R_{iB}}{R_{iB} + R_{oA}} = 100 v_{iA} \frac{10k}{10k + 10k}$$

$$v_L = 10 v_{iB} \frac{R_L}{R_L + R_{oB}} = 10 v_{iB} \frac{100}{100 + 1000}$$

$$\frac{v_L}{v_S} = 1000 \frac{100k}{200k} \frac{10k}{20k} \frac{100}{1100} = 22.73 = 27.13\text{dB}$$

Problem 1.48-SBAL



$$v_{iB} = v_s \frac{R_{iB}}{R_{iB} + R_s} = v_s \frac{10k}{10k + 100k}$$

$$v_{iA} = 10 v_{iB} \frac{R_{iA}}{R_{iA} + R_{oB}} = 10 v_{iB} \frac{100k}{100k + 1k}$$

$$v_L = 100 v_{iA} \frac{R_L}{R_L + R_{oA}} = 100 v_{iA} \frac{100}{100 + 10k}$$

$$\frac{v_L}{v_S} = 1000 \frac{10k}{110k} \frac{100k}{101k} \frac{0.1k}{10.1k} = 0.89 = -1.00\text{dB}$$

Problem 1.68

Figure P1.68 shows a signal source connected to the input of an amplifier. Here R_s is the source resistance, and R_i and C_i are the input resistance and input capacitance, respectively, of the amplifier. Derive an expression for $V_i(s)/V_s(s)$, and show that it is of the low-pass STC type. Find the 3-dB frequency for the case R_s = 10 kΩ, R_i = 40 kΩ, and C_i = 5 pF.

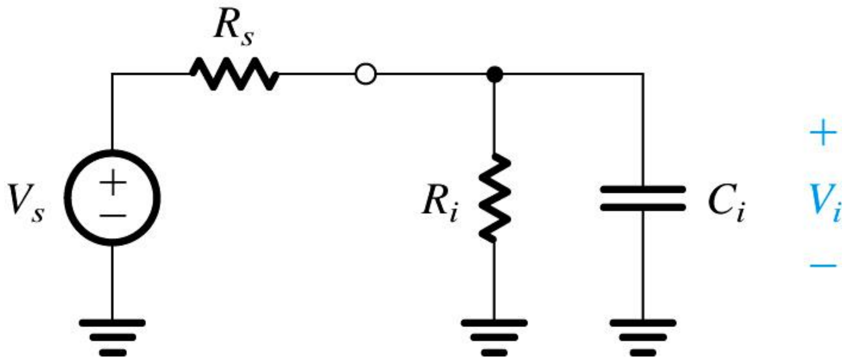
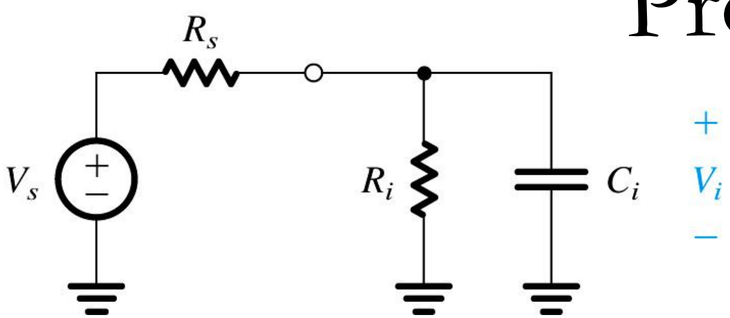


Figure P1.68

$$\chi_C = \frac{1}{sC}$$

$$T(s) = \frac{V_i(s)}{V_s(s)} = \frac{K}{1+\left(\frac{s}{\omega_0}\right)}$$

Problem 1.68a



$$\chi_{C_i} = \frac{1}{sC_i} \qquad V_i = V_s \frac{Z_i}{Z_i + R_s}$$

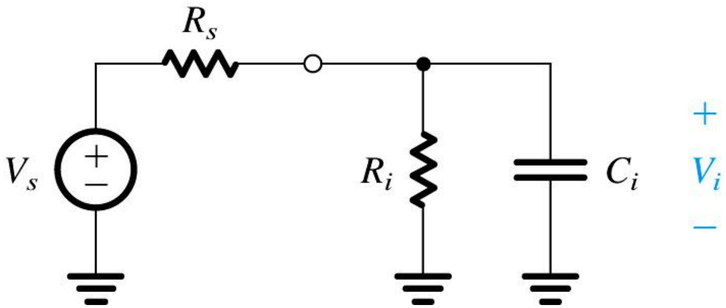
$$Z_i = R_i \parallel \chi_{C_i} = \frac{R_i \chi_{C_i}}{R_i + \chi_{C_i}} = \frac{R_i/_{sC_i}}{R_i + {}^1/_{sC_i}} = \frac{R_i}{R_i s C_i + 1}$$

$$\frac{V_i}{V_s} = \frac{Z_i}{Z_i + R_s} = \frac{\frac{R_i}{R_i s C_i + 1}}{\frac{R_i}{R_i s C_i + 1} + R_s} = \frac{R_i}{R_i + R_s R_i s C_i + R_s} = \frac{\frac{R_i}{(R_i + R_s)}}{1 + s\left(\frac{R_s R_i C_i}{(R_i + R_s)}\right)}$$

$$T_s(s)_{lowpass} = \frac{K}{1 + \left(\frac{s}{\omega_0}\right)} \Rightarrow \omega_0 = \frac{R_i + R_s}{R_i R_s C_i}$$

$$\frac{10\text{k}\Omega + 40\text{k}\Omega}{10\text{k}\Omega \cdot 40\text{k}\Omega \cdot 5\text{pF}} = 2.5000 \times 10^7 \cdot \frac{\text{rad}}{\text{s}}$$

Problem 1.68

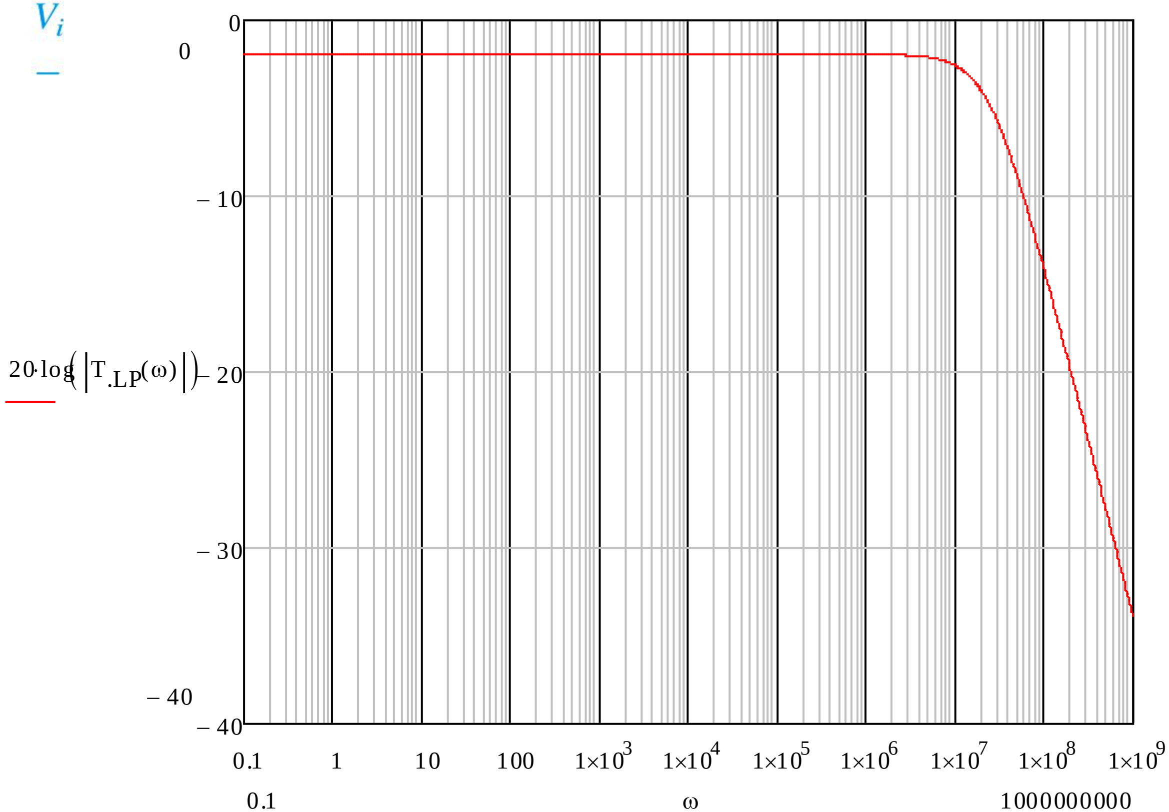


$R_i := 40k\Omega$

$R_s := 10k\Omega$

$C_i := 5pF$

$$T_{LP}(ss) := \frac{\frac{R_i}{R_i + R_s}}{1 + i \cdot ss \cdot \left(\frac{R_s \cdot R_i \cdot C_i}{R_i + R_s}\right)}$$



Problem 1.68 - SIMetrix