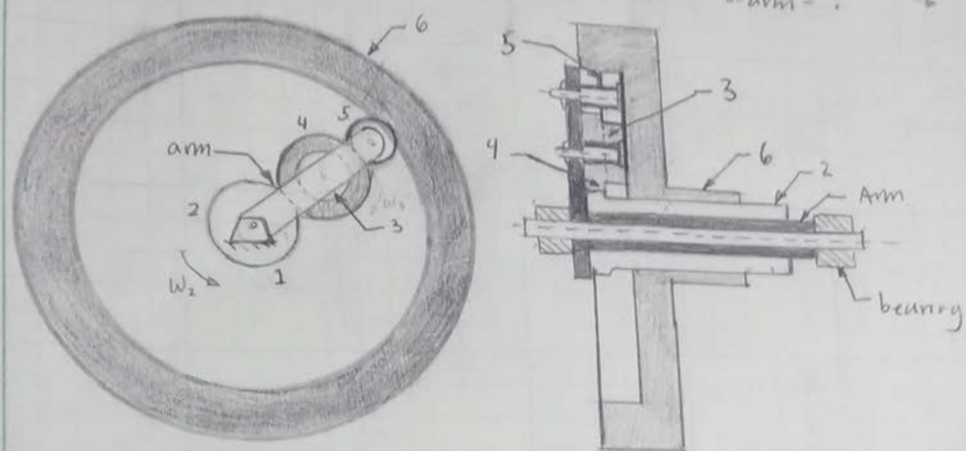
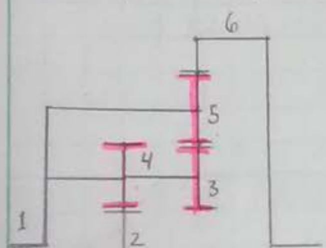
9-25 (row A) $N_2 = 30$ $N_3 = 25$ $N_4 = 45$ $N_5 = 30$ $N_6 = 160$ $W_6 = 0$ $W_2 = 50$ $W_{arm} = ?$ (+



Compound Planetary gear train



$$R = \pm \frac{\text{Product \# of teeth on driver gears}}{\text{Product \# of teeth on driven gears}} = \frac{W_L - W_{arm}}{W_F - W_{arm}}$$

$$\left(\frac{-N_2}{N_4}\right)\left(\frac{-N_3}{N_5}\right)\left(\frac{N_5}{N_6}\right) = \frac{W_L - W_{arm}}{W_F - W_{arm}}$$

$$\boxed{\left(\frac{N_2}{N_4}\right)\left(\frac{N_3}{N_6}\right) = \frac{W_6 - W_{arm}}{W_2 - W_{arm}}}$$

Plug in values:

$$\left(\frac{30}{45}\right)\left(\frac{25}{160}\right) = \frac{0 - W_{arm}}{50 - W_{arm}}$$

$$0.1042 = \frac{-W_{arm}}{50 - W_{arm}}$$

$$0.1042(50 - W_{arm}) = -W_{arm}$$

$$5.21 = (0.1042 - 1)W_{arm}$$

$$5.25 = -0.8458\,W_{arm}$$

$$\boxed{-5.414 = W_{arm}}$$

20/20

9.26

$N_2 = 30$ $\omega_2 = 40$

$N_3 = 30$ $\omega_6 = 0$

$N_4 = 45$

$N_5 = 40$

$N_6 = 35$

$$R = \frac{\omega_6 - \omega_{arm}}{\omega_3 - \omega_{arm}}$$

$$R = \left(\frac{N_3}{N_4}\right)\left(\frac{N_5}{N_6}\right)$$

$$R = 0.7619$$

$$\frac{\omega_2}{\omega_3} = \frac{-N_3}{N_2}$$

$$\omega_3 = \frac{-\omega_2}{N_3} N_2$$

$$\omega_3 = -40 \text{ rad/s}$$

$$0.7619 = \frac{0 - \omega_{arm}}{-40 - \omega_{arm}}$$

$$-30.476 - \omega_{arm}(0.7619) = -\omega_{arm}$$

$$-30.476 = -0.2381\,\omega_{arm}$$

$$\omega_{arm} = 127.997$$

$$\boxed{\omega_{arm} \approx 128 \ \text{rad/s}}$$

PROBLEM 9-9

Statement: Design a simple, spur gear train for a ratio of +6.5:1 and a diametral pitch of 5. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 6.5$ Diametral pitch $p_d := 5 \cdot in^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth for a 20-deg pressure angle (see Table 9-4b).

Design Choice: Pressure angle $\phi := 20 \cdot deg$

Solution: See Mathcad file P0909.

1. From inspection of Table 9-5a, we see that 17 teeth is the least number that the pinion can have for a gear ratio of 6.5. therefore, let the number of teeth on the pinion be (an even number so the gear tooth number will be an integer).

   $N_p := 18$ and $N_g := m_G \cdot N_p$ $N_g = 117$

2. Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

   $d_p := \dfrac{N_p}{p_d}$ $d_p = 3.6000\,in$ $d_g := \dfrac{N_g}{p_d}$ $d_g = 23.4000\,in$

3. Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

   $r_p := 0.5 \cdot d_p$ $r_p = 1.8000\,in$ $r_g := 0.5 \cdot d_g$ $r_g = 11.7000\,in$

   $a_p := \dfrac{1}{p_d}$ $a_p = 0.2000\,in$ $a_g := \dfrac{1}{p_d}$ $a_g = 0.2000\,in$

   Center distance $C := r_p + r_g$ $C = 13.5000\,in$

   $$Z := \sqrt{\left(r_p + a_p\right)^2 - \left(r_p \cdot cos(\phi)\right)^2} + \sqrt{\left(r_g + a_g\right)^2 - \left(r_g \cdot cos(\phi)\right)^2} - C \cdot sin(\phi) \qquad Z = 1.0033\,in$$

   Contact ratio $m_p := \dfrac{p_d \cdot Z}{\pi \cdot cos(\phi)}$ $m_p = 1.699$

4. An idler gear of any diameter is needed to get the positive ratio. If the idler is does not have the same number of teeth as the gear, the calculation of contact ratio (step 3) will not be correct.

PROBLEM 9-17

Statement: Design a compound, reverted, spur gear train for a ratio of 7:1 and diametral pitch of 4. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 7$ Diametral pitch $p_d := 4 \cdot in^{-1}$

Solution: See Mathcad file P0917.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be 2.
2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 3.5$ Stage 2 ratio $r_2 := 2$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \dfrac{N_3}{N_2}$ $r_2 := \dfrac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 13.500$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 6 \cdot K_{min}$ $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   $K = 81.000$ $N_2 = 18$ $N_4 = 27$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 63$ $N_5 := r_2 \cdot N_4$ $N_5 = 54$

The pitch diameters are: $d_i := \dfrac{N_i}{p_d}$

$i =$	$\dfrac{d_i}{in} =$	$N_i =$
2	4.5000	18
3	15.7500	63
4	6.7500	27
5	13.5000	54

Checking the overall gear ratio: $\left(-\dfrac{N_3}{N_2}\right) \cdot \left(-\dfrac{N_5}{N_4}\right) = 7.000$

PROBLEM 9-42

Statement: Figure P9-9*a* shows a compound epicyclic train. Shaft 1 is driven at 300 rpm CCW and gear A is fixed to ground. The tooth numbers are indicated in the figure. Determine the speed and direction of shaft 2.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Tooth numbers:

$N_A := 56 \quad N_B := 18 \quad N_C := 48 \quad N_D := 26 \quad N_E := 60 \quad N_F := 18 \quad N_G := 68$

Shaft 1 speed: $\omega_1 := 300\cdot rpm$ CCW

Solution: See Figure P9-9*a* and Mathcad file P0942.

1. Shaft 1 drives arm-1, the first stage arm, and arm-2, the second stage arm. The first stage is composed of gears A, B, C, and D, with gear A fixed. The second stage is composed of gears D, E, F, and G. Second stage inputs are from gear D and arm-2.

$$\omega_{arm} := \omega_1$$

2. Determine the speed of gear D using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R \qquad \omega_A := 0\cdot rpm$$

Calculate R using equation 9.14 and inspection of Figure P9-9*a*.

$$R := \left(-\frac{N_A}{N_B}\right)\cdot\left(-\frac{N_C}{N_D}\right) \qquad R = 5.74359$$

Solve the right-hand equation above for ω_D with $\omega_A = 0$.

$$\omega_D := (1 - R)\cdot\omega_{arm} \qquad \omega_D = -1423.08\,rpm$$

3. Determine the speed of gear G using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be D and last be G. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_G - \omega_{arm}}{\omega_D - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-9*a*.

$$R := \left(-\frac{N_D}{N_E}\right)\cdot\left(-\frac{N_F}{N_G}\right) \qquad R = 0.11471$$

Solve the right-hand equation above for ω_G.

$$\omega_G := R\cdot\left(\omega_D - \omega_{arm}\right) + \omega_{arm} \qquad \omega_G = 102.4\,rpm$$

Gear G drives shaft 2, so

$$\omega_2 := \omega_G \qquad \omega_2 = 102.4\,rpm$$