



# Applied Cryptography

## CPEG 472/672

### Lecture 7A

Instructor: Nektarios Tsoutsos

# Keyed Hashing

- ◉ Anyone can compute a hash value
  - ◉ There is no secret involved in the process
  - ◉ Attackers can modify a msg and create hash
- ◉ Is there a way to prevent that?
  - ◉ We want a type of hash for authorized users
  - ◉ Only those who know a secret can generate the hash, or can verify the hash
- ◉ Keyed Hashing
  - ◉ Message Authentication Codes (MACs)
  - ◉ Pseudo Random Functions (PRFs)

# MACs

- ◉ Validate if a message has been modified
  - ◉ Protect msg integrity and authenticity
  - ◉ Create a tag  $T = \text{MAC}(\text{key}, \text{msg})$
  - ◉ Return pair  $(M, T)$
- ◉ Can generate/verify tag iff you know  $K$ 
  - ◉ Attackers do not know  $K$
- ◉ Uses of MACs
  - ◉ Network packets IPSec, SSH, TLS
  - ◉ 3G/4G mobile telephony standards

# Secure MACs

Define security goals/attack models

- ◉ Forgery

- ◉ Create valid (msg, tag) pair without  $K$
- ◉ In secure MACs this should be impossible
- ◉ Security goal: Unforgeability

- ◉ Attack Model

- ◉ Basic model: known-msg attack (KMA)
  - ◉ Attacker collects valid (M,T) pairs
- ◉ Standard model: chosen-msg attack (CMA)
  - ◉ Attacker asks oracle for tags of chosen msgs

- ◉ Replay attack

- ◉ Attacker sends duplicate valid (M,T) pair
- ◉ We want replay resistance (unique msg IDs)

# PRFs & PRF security

- ◉ Inputs: message  $M$ , secret key  $K$
- ◉ Output: indistinguishable from random
  - ◉ Attack cannot tell if a value is the output of a PRF or truly random without knowing  $K$
  - ◉ Cannot find patterns for distinguishing PRF output from truly random values
- ◉ Notion: indistinguishability from random
- ◉ Used in other crypto constructions
  - ◉ Feistel networks, challenge/response ID
    - ◉ Server sends chal msg  $M$ , client sends  $\text{PRF}(K, M)$
  - ◉ PRP/PRF Switching lemma (<https://eprint.iacr.org/2004/331.pdf>)

# MACs vs PRFs

- ◉ Both keyed hashes but PRFs are stronger
    - ◉ MACs: weaker security requirements
      - ◉ Are secure if cannot be forged
      - ◉ MAC outputs cannot be guessed
    - ◉ PRFs: outputs IND-able from random strings
      - ◉ Stronger requirements compared to MACs
      - ◉ If PRF output IND-able => can't be guessed
  - ◉ Any secure PRF is also a secure MAC
    - ◉ But: secure MAC not necessarily secure PRF
- Can distinguish PRF2 from random
- ◉  $\text{PRF2}(K, M) = \text{sec\_PRF}(K, M) \parallel 0$
- ◉ PRF2 is not a secure PRF but a secure MAC
- If you can forge a tag for PRF2 you can forge a tag for sec\_PRF, which should be impossible

# Constructions from hashes

- ◉ Secret-prefix construction:  $\text{Hash}(K||M)$ 
  - ◉ Vulnerable to length extension attack
    - ◉ Compute  $\text{Hash}(K||M_1||M_2)$  given  $\text{Hash}(K||M_1)$ 
      - ◉ Forgery: you don't know  $K$  or  $M_1$
    - ◉ SHA-3 finalists not vulnerable to LEA
      - ◉ Mandatory NIST requirement for SHA-3
  - ◉ Insecurity with variable key lengths
    - ◉ Consider:  $\text{Hash}(K||M)$  with  $K=123abc$ ,  $M=def000$ 
      - ◉ Same as  $K=123a$ ,  $M=bcdef000$
    - ◉ Solution: Encode key length as well
      - ◉  $\text{Hash}(L||K||M)$  with  $L=\text{encode\_length}(K)$

# Constructions from hashes

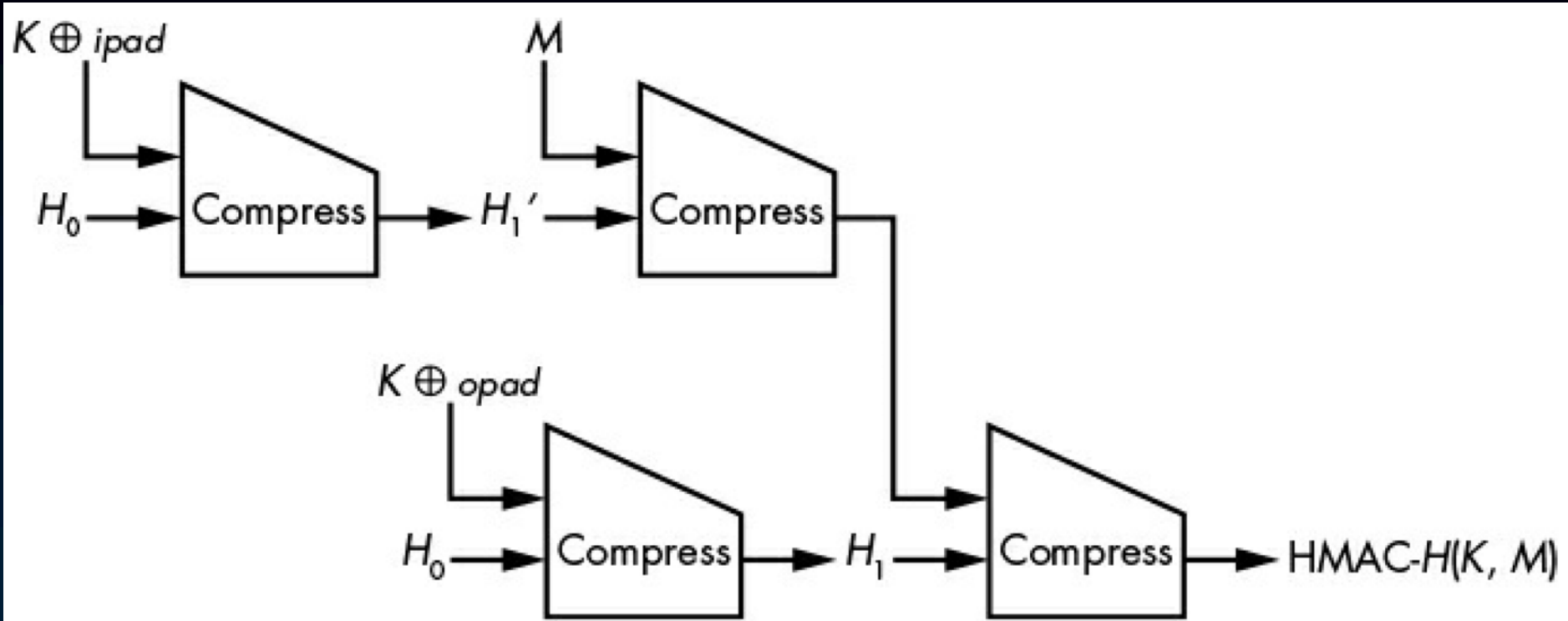
- ◉ Secret-suffix construction:  $\text{Hash}(M||K)$ 
  - ◉ Solves the LEA problem
    - ◉  $\text{Hash}(M_1||K||M_2)$  is not extension of  $\text{Hash}(M_1||K)$ 
      - ◉ The extension would be  $\text{Hash}(M_1||M_2||K)$  instead
  - ◉ Vulnerable if Hash has collisions
    - ◉ Assume  $\text{Hash}(M_1) = \text{Hash}(M_2)$
    - ◉ Do CMA attack: Request  $\text{Hash}(M_1||K)$
    - ◉ Return pair  $M_2$ ,  $\text{tag} = \text{Hash}(M_2||K)$
- ◉ Envelope method:  $\text{Hash}(K||M||K)$ 
  - ◉ More secure than prefix/suffix constructions



# Constructions from hashes: HMAC

- ◉ Secure PRF construction from a hash  $H$ 
  - ◉ Needs collision resistant hash
    - ◉ At least PRF as compression function
  - ◉ Defines two paddings: opad, ipad
    - ◉ opad = 0x5c5c...5c5c (as long as blk size of  $H$ )
    - ◉ ipad = 0x3636...3636 (as long as blk size of  $H$ )
  - ◉ Pad  $K$  with 0x00's to match blk size of  $H$
- ◉ Definition of HMAC PRF using  $H$ 
  - ◉  $\text{PRF} = H( (K \text{ xor opad}) || H( (K \text{ xor ipad}) || M) )$
  - ◉ More secure than envelope construction

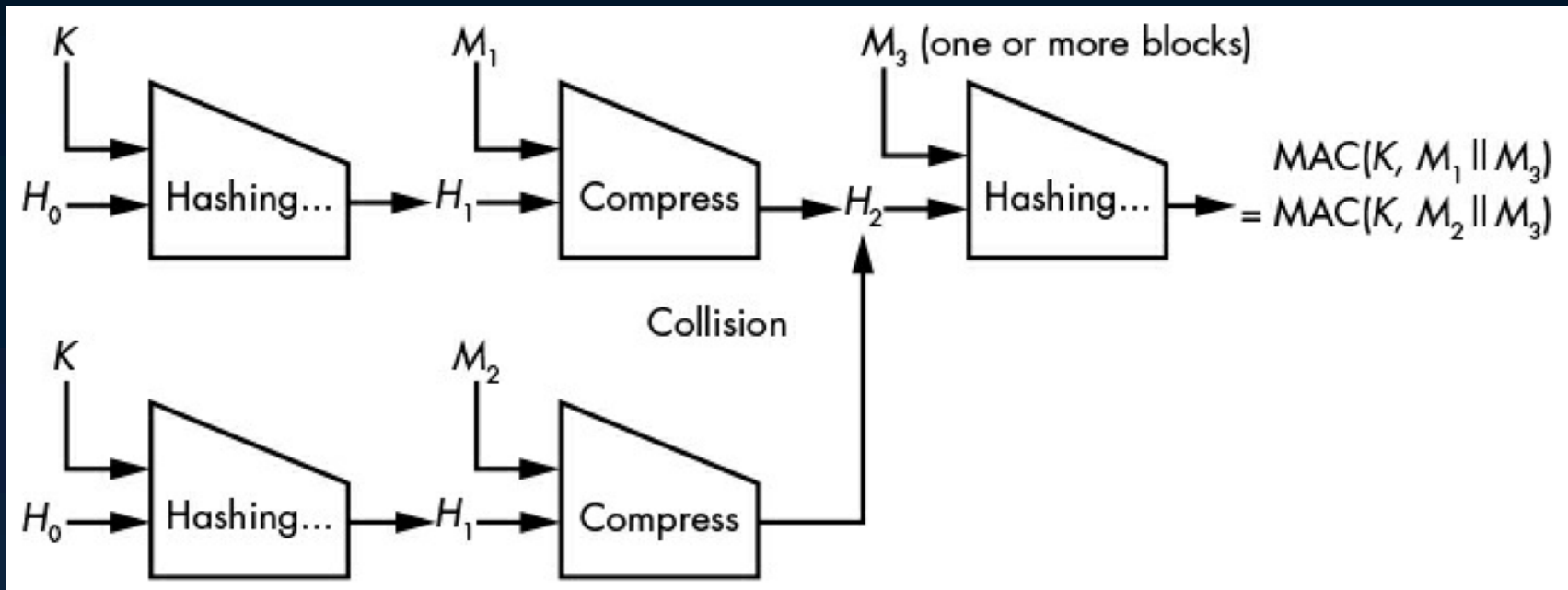
# HMAC



# Generic attack on hash-based MACs

Applicable to MACs based on iterated hash functions

- Collision on hash  $\Rightarrow$  MAC collision
  - Assume  $\text{Hash}(K \parallel M_1) = \text{Hash}(K \parallel M_2)$
  - Assume a LEA possible on Hash (e.g., SHA2)
  - Request  $2^{(n/2)}$  tags (i.e., CMA attack)



# Keyed hashes from block ciphers

- ◉ Block ciphers already used before
  - ◉ E.g., HMAC-SHA-256 uses block cipher => D-M compression function => M-D hash
  - ◉ Can we use block cipher directly?
- ◉ CBC-MAC (broken)
  - ◉ Tag=Encrypt M with CBC, keep last ctxt blk
    - ◉ Keep  $C_i = E(K, M_i \text{ xor } C_{i-1})$  when i is last blk
    - ◉ Use 0x00..000 as IV
  - ◉ Forgery: given  $T1 = E(K, M1)$ ,  $T2 = E(K, M2)$  create new pair  $T2, (M1 || (M2 \text{ xor } T1))$

# Keyed hashes from block ciphers

- ◉ Solution: CMAC

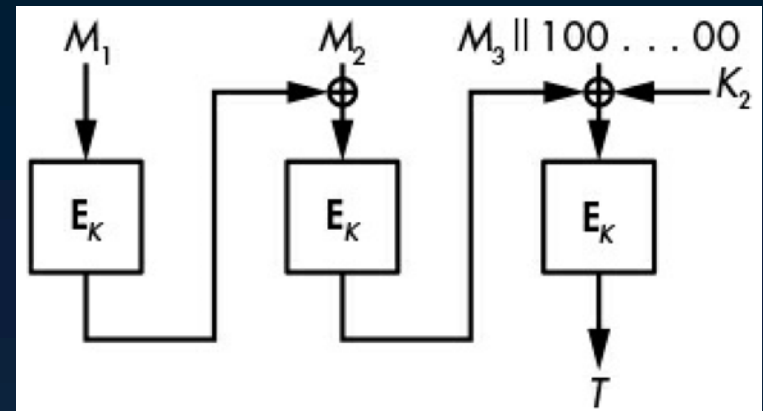
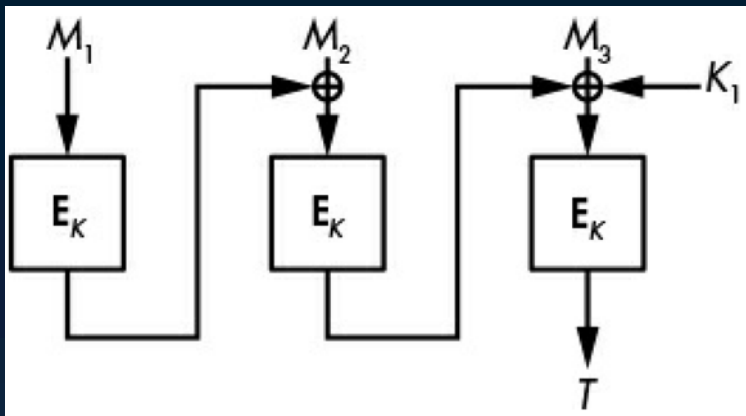
- ◉ Fixes CBC-MAC using two extra keys  $K_1, K_2$

- ◉  $L = E(0x00, K)$  encrypt  $K$  with zero as key

- ◉  $K_1 = L \ll 1$  if  $\text{MSB}(L) = 0$  else  $(L \ll 1) \oplus 87$

- ◉  $K_2 = K_1 \ll 1$  if  $\text{MSB}(K_1) = 0$  else  $(K_1 \ll 1) \oplus 87$

- ◉ Use  $K_1$  if  $M$  does not need padding, else  $K_2$



# Dedicated MAC designs

Algorithms created to specifically serve as PRFs and MACs

- ◉ Universal hash functions (UHs)
  - ◉ Weaker notion vs cryptographic hashes
  - ◉ UHs don't need to be collision resistant
    - ◉ Only requirement:  $\text{UH}(K, M1) = \text{UH}(K, M2)$  with negligible probability
    - ◉ Use a secret key
  - ◉ No need to be pseudorandom like PRFs
- ◉ UHs can be used as ONE TIME MACs
  - ◉ Security collapses if used twice

# Example UH using polynomials

- ◉ Select prime number  $p$
- ◉ Select secret values  $R, K$  in range  $[1, p]$
- ◉ Define  $UH(R, K, M)$  of message  $M$  as
$$R + M_1 * K + \dots + M_n * K^n \bmod p$$
- ◉ Limitations:
  - ◉ Can be used only for one msg  $M$
  - ◉ Attacker can break after requesting 2 tags
    - ◉ Recover  $R$  by requesting  $UH(R, K, 0) = R$
    - ◉ Recover  $K$  by requesting  $UH(R, K, 1) = R + K$

# Carter Wegman (C-W) construction

- ◉ Convert 1-time MAC to many-time MAC
  - ◉ "Encrypt" the UH using a PRF
  - ◉  $\text{MAC}(K1, K2, N, M) = \text{UH}(K1, M) + \text{PRF}(K2, N)$ 
    - ◉ Use nonce N (use only once per key K2)
    - ◉ PRF acts like a stream cipher
- ◉ Poly1305-AES: very fast C-W MAC
  - ◉ Faster than HMAC- or CMAC-based MACs
  - ◉  $\text{Poly1305}(K1, M) + \text{AES}(K2, N) \% 2^{128}$
  - ◉ Define UH
    - ◉  $\text{Poly1305}(K, M) = M_1 * K^n + \dots + M_n * K \% 2^{130-5}$
    - ◉ Integer at most 129 bits



# Timing attacks on MAC verification

- ◉ Problem: Variable time in equality check
  - ◉ Servers check the provided tag byte by byte
  - ◉ Use early termination if bytes are different

```
def compare_mac(x, y, n):  
    for i in range(n):  
        if x[i] != y[i]:  
            return False  
    return True
```

- ◉ Information leakage: divide & conquer
  - ◉ Attacker can tell if tag byte was correct
  - ◉ Solution: constant-time implementation

# Hands-on exercises

- ◉ Poly1305-AES
- ◉ HMAC
- ◉ CMAC
- ◉ SipHash

# Reading for next week

- ◉ Aumasson: Chapter 8 (up to AES-GCM)
  - ◉ We will have a short quiz
- ◉ Midterm: Thu April 9th, 2:00-3:15pm
  - ◉ All material during first 6 weeks
  - ◉ Chapters 1-6
  - ◉ Lectures 1A-6B