Problem 1

KCL: $-i_1(t)+i_2(t)+i_3(t)=0 \Rightarrow +i_3(t)=i_1(t)-i_2(t)$

$$i_3(t)=141.4\cos\left(\omega t+135^o\right)\mathbf{mA}-50\sin\left(\omega t-53.13^o\right)\mathbf{mA}=$$

$$=141.4\cos\left(\omega t+135^o\right)\mathbf{mA}-50\cos\left(\omega t-53.13^o-90^o\right)\mathbf{mA}$$

$$\mathbf{I}_3=141.4\ \mathbf{mA}\ \angle 135^o-50\ \mathbf{mA}\ \angle -143.13^o=$$

$$=(-99.98+j\cdot 99.98)\ \mathbf{mA}-(-40.00-j\cdot 30.00)\ \mathbf{mA}=$$

$$=(-59.98+j\cdot 129.98)\ \mathbf{mA}=143.2\ \mathbf{mA}\ \angle 114.8^o$$

$$i_3(t)=143.2\cos\left(\omega t+114.8^o\right)\mathbf{mA}$$

If sine functions were used, the result in phasor notation would differ in phase by 90 degrees.

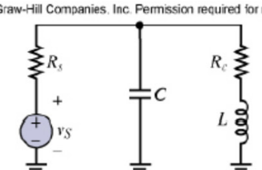
Problem 2

Solution:

Known quantities:

The values of the impedance, $R_s=50\Omega$, $R_c=40\Omega$, $L=20\ \mu\mathbf{H}$, $C=1.25\ \mathbf{nF}$, and the voltage applied to the circuit shown in Figure P4.53,

$v_s(t)=V_0\cos\left(\omega t+0^o\right)$ $V_0=10\ \mathbf{V}$, $\omega=6\ \mathbf{M}\dfrac{\mathbf{rad}}{\mathbf{s}}$.



Find:

The current supplied by the source.

Analysis:

Assume clockwise currents:

$$X_L=\omega L=\left(6\ \mathbf{M}\frac{\mathbf{rad}}{\mathbf{s}}\right)(20\ \mu\mathbf{H})=1203\Omega \Rightarrow Z_L=0+j120\ \Omega=120\angle 90^o\ \Omega$$

$$X_C=\frac{1}{\omega C}=\frac{1}{\left(6\ \mathbf{M}\dfrac{\mathbf{rad}}{\mathbf{s}}\right)(1.25\ \mathbf{nF})}=133.3\ \Omega \Rightarrow Z_C=0-j133.3\ \Omega=133.3\angle -90^o\ \Omega$$

$$Z_{R_c}=40-j\Omega=40\angle 0^o\ \Omega,\ Z_{R_s}=50-j\ \Omega=50\angle 0^o\ \Omega$$

Equivalent impedances:

$$Z_{eq1}=Z_{R_c}+Z_L=40+j120\ \Omega=126.5\angle 71.56^o\ \Omega$$

$$Z_{eq}=Z_{R_s}+\frac{Z_C\cdot Z_{eq1}}{Z_C+Z_{eq1}}=50+j0\ \Omega+\frac{\left(133.3\angle -90^o\ \Omega\right)\cdot\left(126.5\angle 71.56^o\ \Omega\right)}{133.3\angle -90^o\ \Omega+126.5\angle 71.56^o\ \Omega}=$$

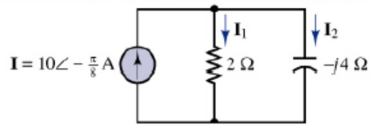
$$=50+j0\ \Omega+\frac{16.87\angle -18.44^o\ \mathbf{k}\Omega^2}{42.161\angle -18.44^o\ \Omega}=50\angle 0^o\ \Omega+400\angle 0^o\ \Omega=450\angle 0^o\ \Omega$$

OL: $$\mathbf{I}_s=\frac{\mathbf{V}_s}{Z_{eq}}=\frac{10\angle 0^o\ \mathbf{V}}{450\angle 0^o\ \Omega}=22.22\angle 0^o\ \mathbf{mA} \Rightarrow i_s(t)=22.22\cos\left(\omega t+0^o\right)\mathbf{mA}$$

Note:

The equivalent impedance of the parallel combination is purely resistive; therefore, the frequency given is the resonant frequency of this network.

Problem 3



Solution:

Known quantities:

The values of the impedance and the current source for the circuit shown in Figure P4.56.

Find:

The current $\mathbf{I}_1$.

Analysis:

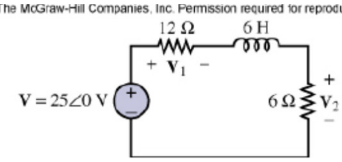
Specifying the positive directions of the currents as in figure P4.45:

$$Z_{eq} = \frac{1}{\frac{1}{2} + \left(\frac{1}{-j4}\right)} = 1.79\angle 26.56^o \ \Omega$$

$$V_S = I_S Z_{eq} = \left(10\angle -22.5^o\right)\mathbf{A} \cdot \left(1.79\angle 26.56^o\right)\Omega = 17.9\angle 4.06^o \ \ \mathbf{V}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_S}{R} = 8.95\angle 4.06^o \ \ \mathbf{A}$$

Problem 4



Solution:

Known quantities:

The values of the impedance and the voltage source for circuit shown in Figure P4.57.

Find:

The voltage $\mathbf{V}_2$.

Analysis:

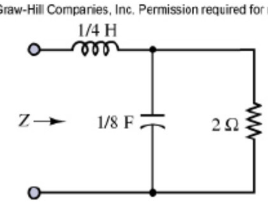
Specifying the positive directions as in figure P4.57:

$Z_L = j\omega L = j12 \ \Omega$

$$V_2 = \frac{R_{6\Omega}}{R_{12\Omega} + Z_L + R_{6\Omega}} V = \frac{6 \ \Omega}{(12 + j12 + 6)\ \Omega} 25\angle 0^o \ \mathbf{V} = \frac{150\angle 0^o \ \Omega}{18 + j12 \ \Omega}\mathbf{V} = 6.93\angle -33.7^o \ \ \mathbf{V}$$

Problem 5

Solution:



Known quantities:

The circuit shown in Figure P4.61, the values of the resistance, $R = 2 \ \Omega$, capacitance, $C = 1/8 \ \mathbf{F}$, inductance, $L = 1/4 \ \mathbf{H}$, and the frequency $\omega = 4 \ \frac{\mathbf{rad}}{\mathbf{s}}$.

Find:

The impedance Z.

Analysis:

$$Z_L = j\omega L = j4\frac{1}{4} \ \Omega = j \ \Omega, \ Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C} = -j\frac{1}{4\cdot(1/8)} = -j2 \ \Omega$$

$$Z = Z_L + Z_C \| R = Z_L + \frac{1}{\frac{1}{Z_C} + \frac{1}{R}} = j + \frac{1}{\frac{1}{-j2} + \frac{1}{2}} = j + \frac{j2}{-1+j} = j + \frac{(j2)}{(-1+j)}\frac{(-1-j)}{(-1-j)}$$

$$= j + \frac{j2(-1-j)}{1+1} = j - j + 1 = 1 \ \Omega$$

Problem 6

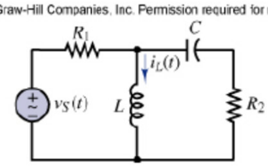
Solution:

Known quantities:

Circuit shown in Figure P4.72 the values of the resistance, $R_1 = 4\ \Omega$, $R_2 = 4\Omega$, capacitance, $C = 1/4$ F, inductance, $L = 2$ **H**, and the voltage source $v_s(t) = 2\cos(2t)$ **V**.

Find:

The current in the circuit $i_L(t)$ using phasor techniques.



Analysis:

$$\mathbf{V}_S(t) = 2\angle 0^o\ \mathbf{V}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2\,\frac{1}{4}} = -j2\ \Omega$$

$$Z_L = j\omega L = j2 \cdot 2 = j4\ \Omega$$

Applying the voltage divider rule:

$$V_L = \frac{(Z_L \,\|\, (Z_C + Z_2))}{Z_1 + (Z_L \,\|\, (Z_C + Z_2))} V_S = \frac{4\angle 36.8°}{4\angle 0° + 4\angle 36.8°} 2\angle 0^o = 1.05\angle 18.4°\,\text{V}$$

Therefore, the current is:

$$I_L = \frac{V_L}{Z_L} = \frac{1.05\angle 18.4°}{4\angle 90°} = 0.2635\angle -71.6^o\ \text{A}$$

$$i_L(t) = 0.2635\cos\left(2t - 71.6^o\right)\ \text{A}$$