ELEG 310

3/8/2018

$P(X=k) = p_X(k)$ $\qquad EX = \sum_{k=0}^{\infty} p(k)\,k$

$P(X=k \text{ and } Y=l) = p_{XY}(k,l)$

$$P(X=k) = \sum_{l=0}^{\infty} P(X=k \text{ and } Y=l) = \sum_{l=0}^{\infty} P(X=k \mid Y=l)\,P(Y=l)$$

$$E(X) = \sum_{k=0}^{\infty} p_X(k)\,k = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} k\, p_{XY}(k,l)$$

$$= \sum_{l=0}^{\infty} \sum_{k=0}^{\infty} k\, P(X=k \mid Y=l)\, P(Y=l) = \sum_{l=0}^{\infty} E(X \mid Y=l)\, p_Y(l)$$

$E(X \mid Y=l)$

$$E(XY) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} kl \, P_{XY}(k,l) = \text{correlation between } X \text{ and } Y$$

$$= \mu_{XY}$$

if $E(XY) = E(X)E(Y)$ say X and Y are uncorrelated

$$E[(X-\mu_X)(Y-\mu_Y)] \underset{\text{theorem}}{=} E[XY] - E[X]E[Y] = \sigma_{XY}$$

$= \text{covariance between } X \text{ and } Y$

if X & Y are uncorrelated $\Rightarrow \sigma_{XY} = 0$

X and Y are <u>independent</u> if

$$P_{XY}(k,l) = P_X(k)\,P_Y(l) \quad \text{for all } k, l$$

Ex.

Y \ X	0	1	2	3
1	0.1	0.0	0.1	0.1
0	0.0	0.1	0.4	0.2

$P_Y(1) = 0.3$

$P_Y(0) = 0.7$

$P_X(0) = 0.1$

$P_X(1) = 0.1$

$P_X(2) = 0.5$

$P_X(3) = 0.3$

$$0.1 = P_{XY}(0,1) \cancel{\overset{?}{=}} P_X(0)\,P_Y(1) = 0.1 \times 0.3$$

NO!

$$E[Y] = 0 \times 0.7 + 1 \times 0.3 = 0.3$$

$$E[X] = 0 \times 0.1 + 1 \times 0.1 + 2 \times 0.5 + 3 \times 0.3 = 2.0$$

$$E[XY] = 0\times 0\times 0.0 + 0\times 1\times 0.1 + 1\times 0\times 0.1 + 1\times 1\times 0.0$$

$$+ 2\times 0\times 0.4 + \underbrace{2\times 1\times 0.1} + 3\times 0\times 0.2 + \underbrace{3\times 1\times 0.1}$$

$$= 0.5$$

Is $\cancel{0.5} \stackrel{?}{=} 0.3 \times 2.0$ NO!

$X, Y, \& Z$ are independent if both are true

a). $X \& Y$, $X \& Z$, $Y \& Z$ are ind.

b) $P_{XYZ}(k, l, m) = P_X(k)\, P_Y(l)\, P_Z(m)$ for all k, l, m

Sums of RVs

$Z = X + Y$ $\qquad$ X, Y ind

$P(X = k) = p_X(k) \quad k = 0, \ldots L-1$

$P(Y = l) = p_Y(l) \quad l = 0, \ldots N-1$

$P(Z = m)$

$=$

$0 \le m \le L + N - 2$

$P(Z=0) = P(X+Y=0) = P(X=0 \text{ and } Y=0) = P(X=0)P(Y=0)$

Ind

$= p_X(0)\, p_Y(0)$

$P(Z=1) = P(X+Y=1) = P(X=0)P(Y=1) + P(X=1)P(Y=0)$

$= p_X(0)\, p_Y(1) + p_X(1)\, p_Y(0)$

Axiom III

$P(X=0 \wedge Y=1 \cup X=1 \wedge Y=0)$

$$P(Z=m) = \sum_{l=0}^{N-1} P(X+Y=m \mid Y=l)\, P(Y=l)$$

$$P(A) = \sum_{l} P(A \mid B_l)\, P(B_l)$$

AND

$$\underbrace{P(X=m-l \mid Y=l)} = \frac{P(X=m-l,\ Y=l)}{P(Y=l)}$$

$$P(A \mid B) = \frac{P(AB)}{P(B)} = P(A) \text{ if } A \,\&\, B \text{ ind}$$

$$= \frac{P(X=m-l)\, P(Y=l)}{P(Y=l)} = \underbrace{P(X=m-l)}$$

$$P(Z=m) = \sum_{l=0}^{N-1} P(X=m-l)\, P(Y=l) = \sum_{l=0}^{N-1} P_X(m-l)\, P_Y(l)$$

$$P(Z=m) = \sum_{l=0}^{N-1} P(X=m-l \text{ and } Y=l)$$

if X & Y are ind, $\quad P_Z = P_X * P_Y$

Ex. $P_X = \left[\frac{1}{6} \;\; \frac{2}{6} \;\; \frac{3}{6}\right]$ $\qquad P_Y = \left[\frac{1}{5} \;\; \frac{1}{5} \;\; \frac{1}{5} \;\; \frac{2}{5}\right]$

$= \frac{1}{6}[1, 2, 3]$ $\qquad = \frac{1}{5}[1, 1, 1, 2]$

	1	1	1	2		
1	1	1	1	2		
2		2	2	2	4	
3			3	3	3	6
	1	3	6	7	7	6

/ (6×5)