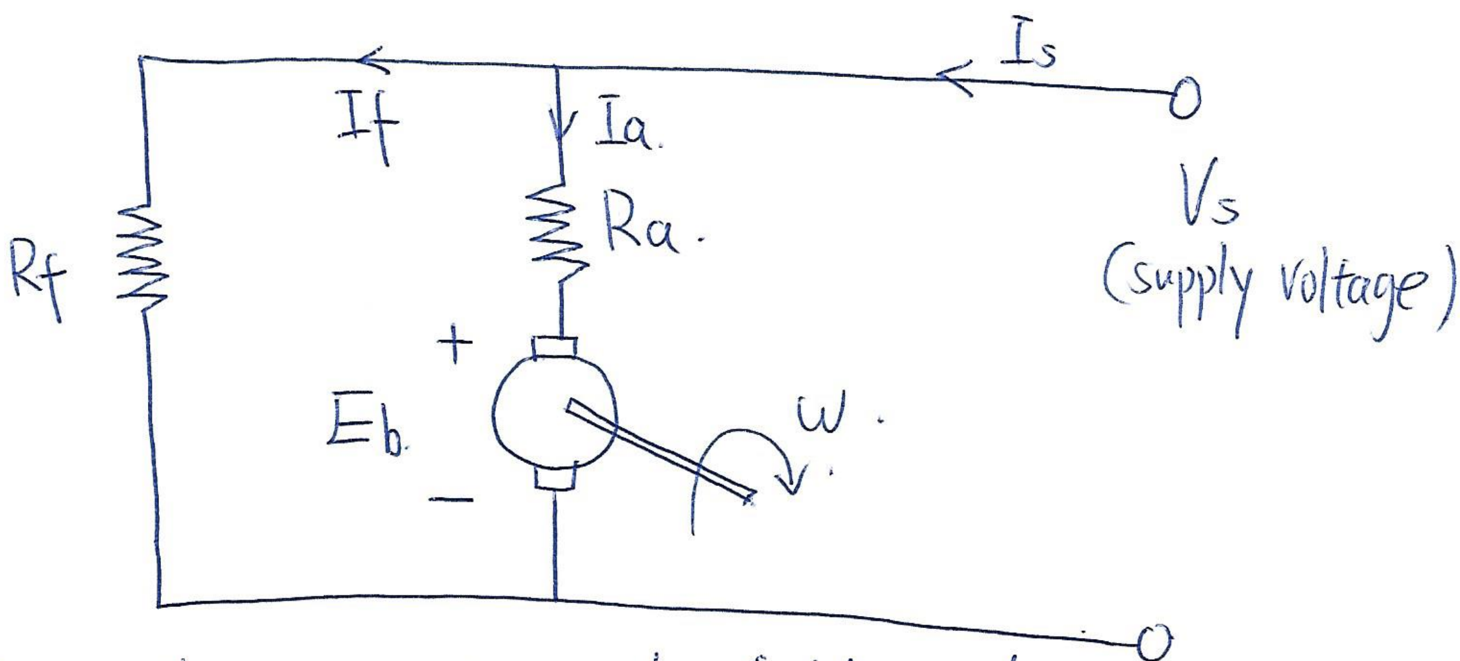
D.C. motors.

More specifically D.C. shunt motors



I_f is the current into the field windings on stator.
R_f is the field winding resistance.
I_a is the current into the armature windings.
R_a is the armature winding resistance.
V_s is the supply voltag; I_s is supply current.
E_b is the back emf. caused by armature ~~rotation~~ rotation. inside magnetic flux.

Basic equations

$$E_b = k_a \phi \omega. \quad (1)$$

torque $T = k_a \phi I_a. \quad (2)$. developed torque.

$$V_s = E_b + I_a R_a. \quad (3).$$

$$I_s = I_f + I_a. \quad (4).$$

k_a is armature constant. (determined by the geometry and configuration of armature windings)

ϕ is the magnetic flux $\phi \propto I_f$

Eq. (1) comes from $\varepsilon = B l v$. motional emf.

Eq. (2) comes from. $F = B i l$. Ampere force.

Additional formulas

$P = T\omega$ (5) mechanical power delivered.

in analogy to $P = F \cdot v$ for linear motion.

$P = E_b I_a.$ (6). electrical power into the ~~moto~~ armature.

$T\omega = E_b I_a.$ energy conserved.

Units for variables

T: $N \cdot m$

ω: standard unit: rad/s

conventional unit: rpm

$1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s}$

ϕ: weber (Wb).

k_a: dimensionless

P: standard unit: W.

conventional unit: hp

$1 \text{ hp} = 746 \text{ W}$.

From Eq (3), $I_a = \frac{V_s - E_b}{R_a}$

Using ~~From~~ Eq (1), $I_a = \frac{V_s - k_a \phi \omega}{R_a}$

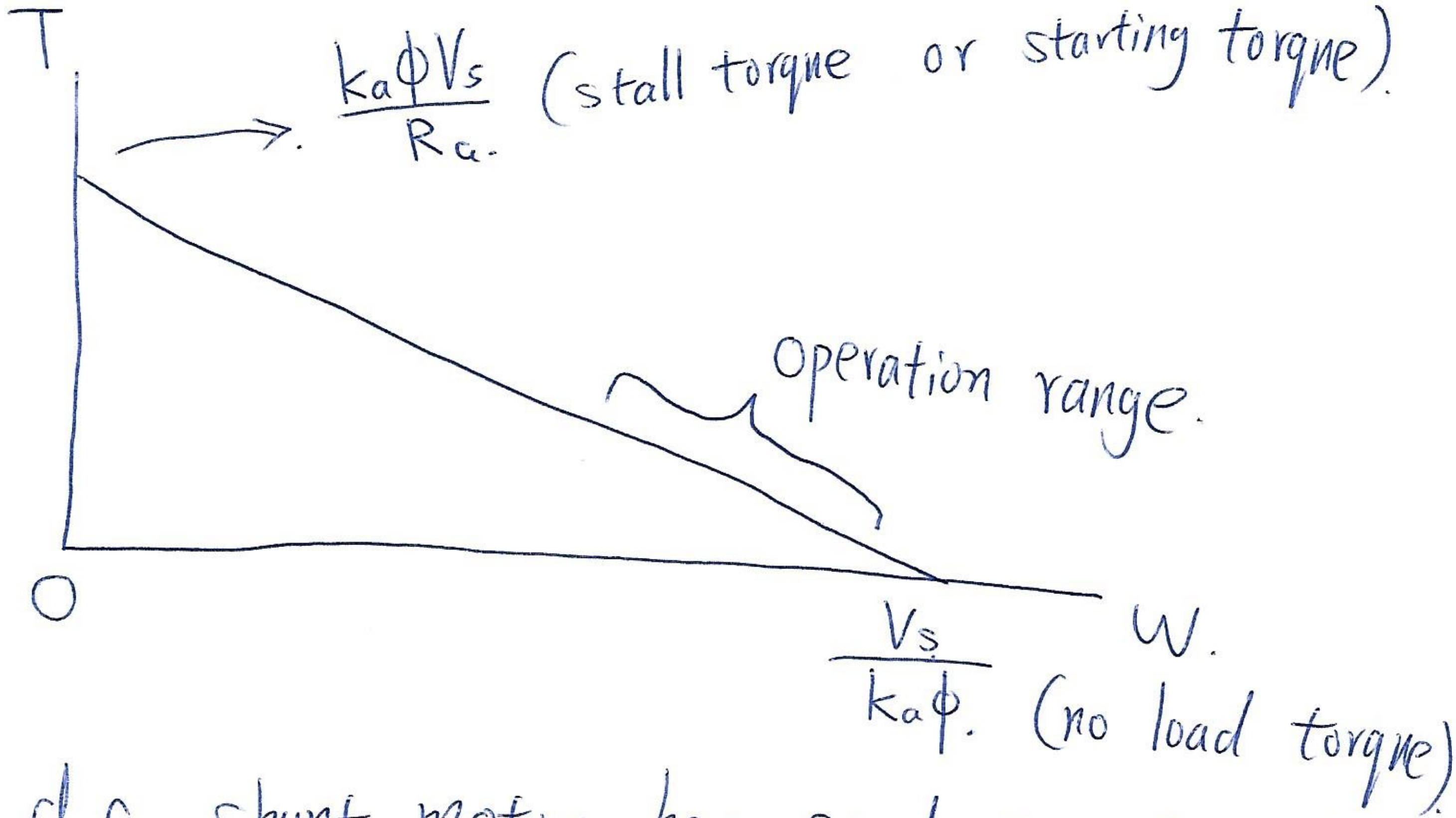
From Eq. (2), $I_a = \frac{T}{k_a \phi}$

$$\therefore \frac{T}{k_a \phi} = \frac{V_s - k_a \phi \omega}{R_a}$$

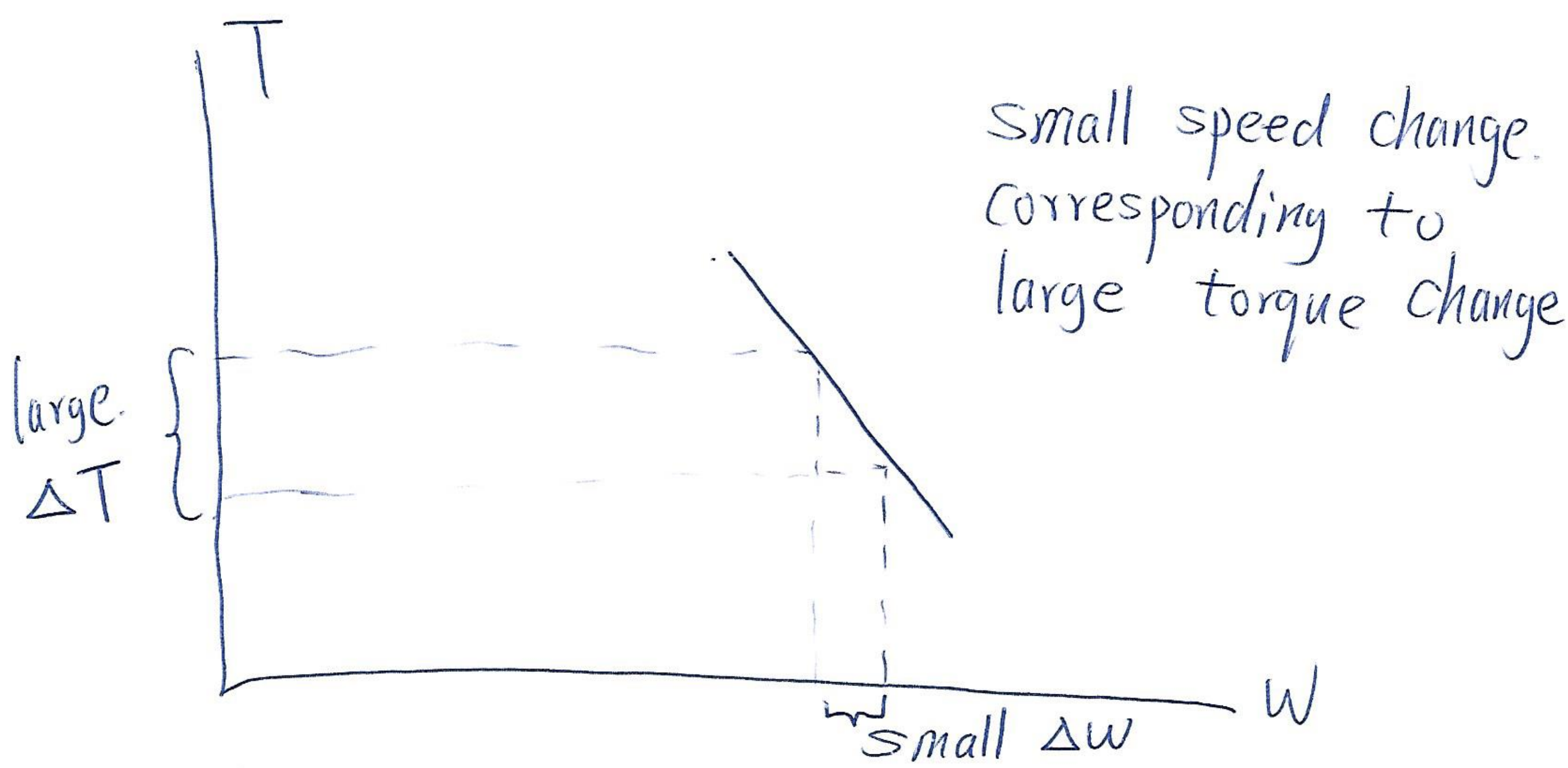
$$T = \frac{V_s k_a \phi}{R_a} - \frac{k_a^2 \phi^2}{R_a} \omega$$

$$T = \frac{k_a \phi}{R_a} (V_s - k_a \phi \omega).$$

torque — speed relation.



d.c. shunt motor has good speed regulation.
Offers nearly constant speed.
R_a is usually very small. Therefore the slope of T-w curve is large.

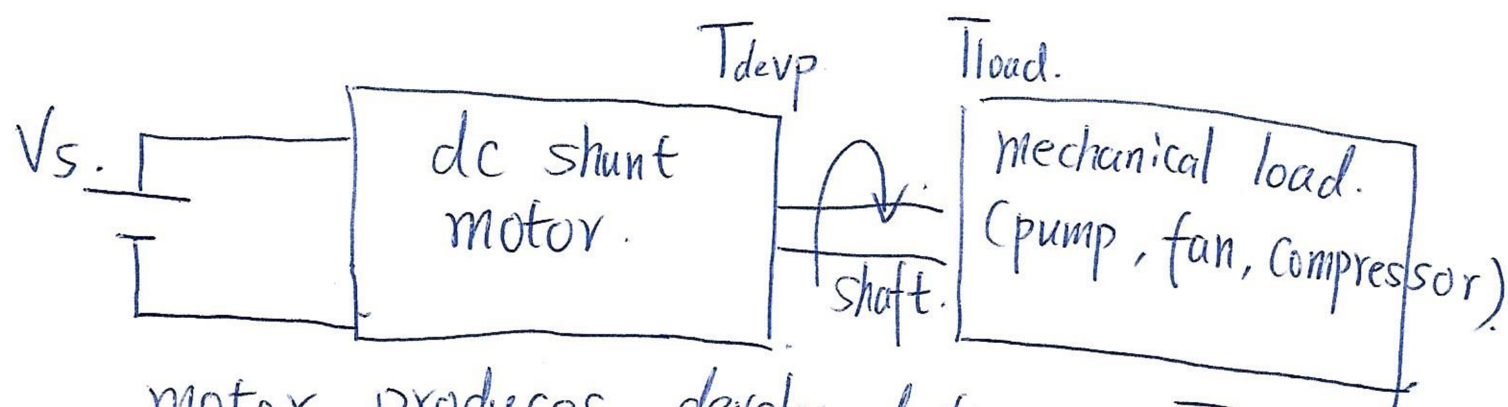


- dc shunt motor is good for applications that require a constant speed.

— Increase of load torque → decrease of ω → decrease of $E_b (= k_a \phi \omega)$ → increase of I_a, → increase of torque.
It is self-regulating

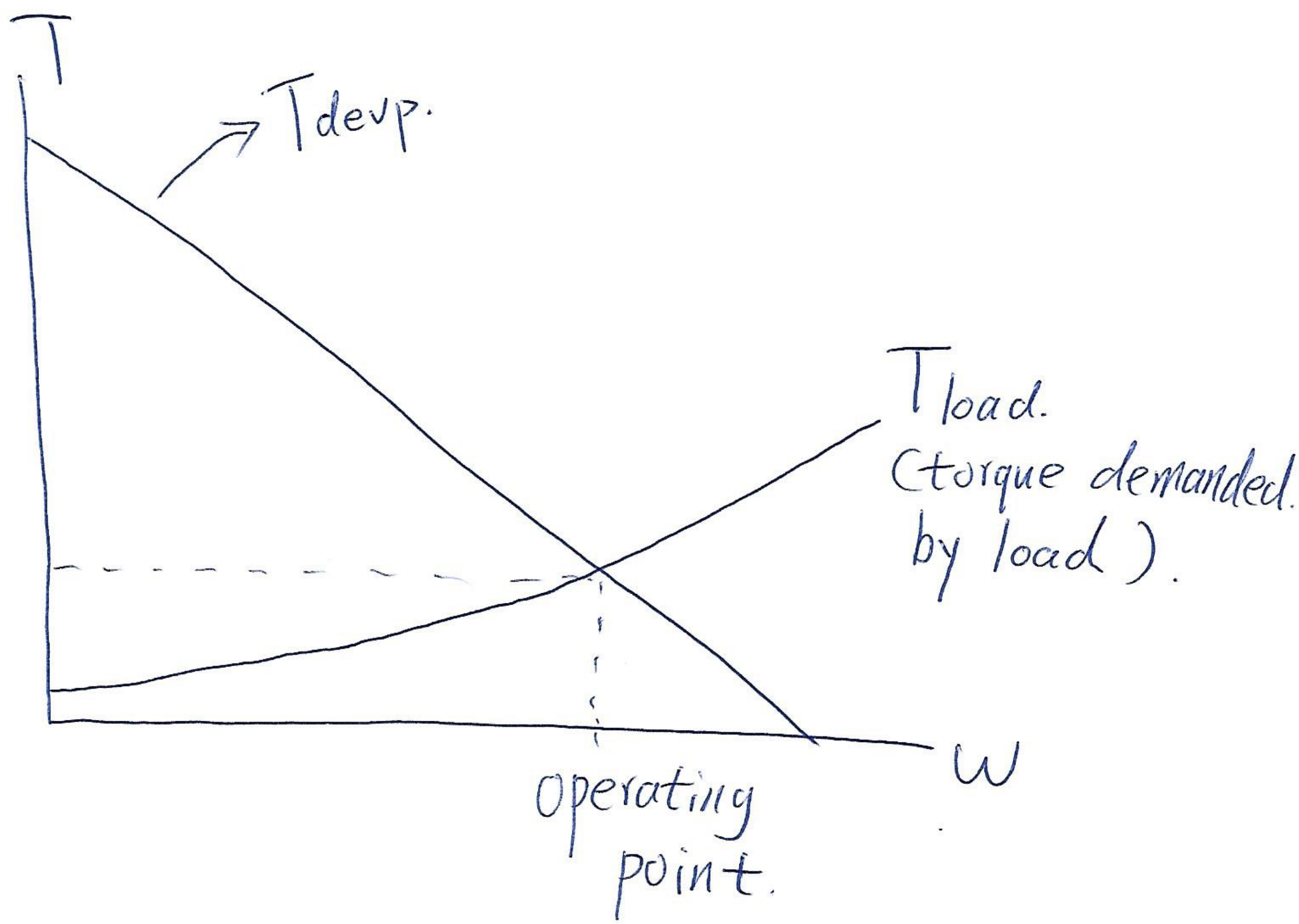
— small R_a. So $E_b = V_s - I_a R_a$ is nearly constant unless I_a is very large.

*Match d.c. shunt motor to a load.



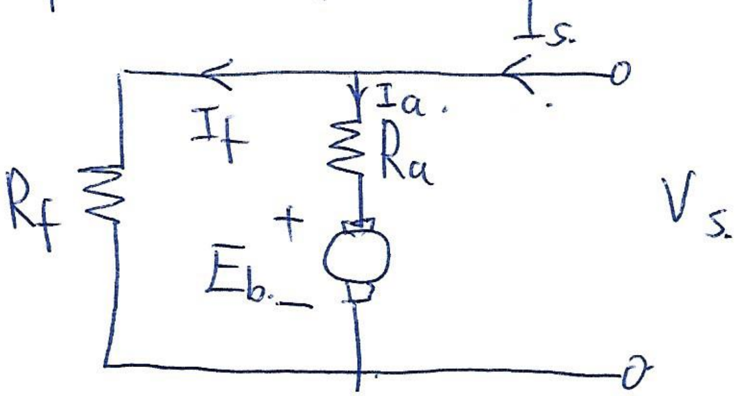
motor produces developed torque : T_{devp}.
load requires speed-dependent torque : T_{load}.

$T_{devp} - \omega$ curve
$T_{load} - \omega$ curve
are different.


T
Tdevp.
Tload.
(torque demanded
by load).
w
operating
point.

Example.

D.C. Shunt motor is driven by $V_s = 240V$. The source current is $I_s = 30A$, and the field current $I_f = 1.4A$. Armature resistance is $R_a = 0.6\,\Omega$. Flux is $\phi = 0.020\,Wb$. Machine Constant is $k_a = 159.15$. Find the speed (ω) and torque (T) of motor.



$$I_a = I_s - I_f = 30 - 1.4$$
$$= 28.6\,A$$

$$E_b = V_s - I_a R_a$$
$$= 240 - 28.6 \times 0.6$$
$$= 222.8\,(V)$$

$$E_b = k_a \phi \omega$$

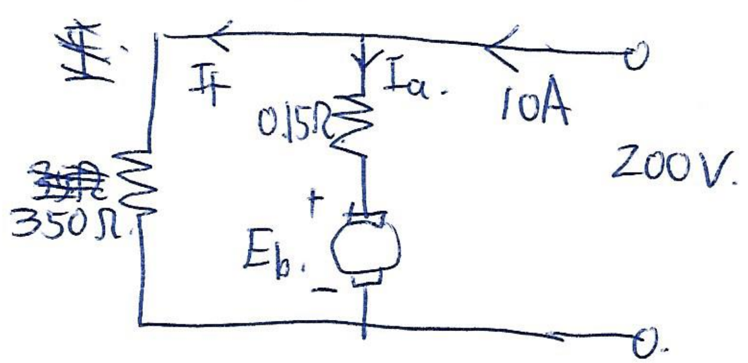
$$\omega = \frac{E_b}{k_a \phi} = \frac{222.8}{159.15 \times 0.02}$$
$$= 70\ rad/s$$
$$= 70 \times \frac{60}{2\pi}\ rpm = 669\ rpm$$

$$T = k_a \phi I_a = 159.15 \times 0.02 \times 28.6 = 91\ N \cdot m$$

Example. A 200-V dc shunt motor draws 10A at 1800 rpm. The armature resistance is $R_a = 0.15\Omega$. Field winding resistance is $R_f = 350\Omega$. What is the torque and power developed by the motor?

$V_s = 200\,V \quad I_s = 10\,A \quad R_a = 0.15\,\Omega$

$R_f = 350\,\Omega$



$$I_f = \frac{200V}{350\Omega} = 0.571\,A$$

$$I_a = 10 - 0.571$$
$$= 9.43\,A$$

$$E_b = 200 - 9.43 \times 0.15 = 198.58\,V$$

$$P = I_a E_b = T\omega \qquad \omega = 1800 \times \frac{2\pi}{60} = 188.5\,\frac{rad}{s}$$

$$T = \frac{I_a \cdot E_b}{\omega} = \frac{9.43 \times 198.58}{188.5} = 9.93\,(N \cdot m)$$

$$P = I_a E_b = T\omega = 1872\,(W) = 2.51\,hp$$

You could also use $E_b = k_a\phi\omega$ to calculate $k_a\phi$ first. Then use $T = k_a\phi I_a$ to calculate T.

Example. dc shunt motor

$V_s = 7.2V \quad R_f = 12\,\Omega \quad R_a = 0.2\,\Omega$

Motor draws a total current of 8.6 A when speed is 120rpm.

(1) Calculate torque and power at 120 rpm.

(2) Calculate no-load speed.

$$I_f = \frac{7.2}{12} = 0.6\ A$$

$$I_a = 8.6 - 0.6 = 8.0 A$$

$$E_b = 7.2 - 8.0 \times 0.2 = 5.6\ (V)$$

$$E_b = k_a\phi\,\omega \qquad \omega = 120 \times \frac{2\pi}{60} = 12.57\left(\frac{rad}{s}\right)$$

$$k_a\phi = \frac{E_b}{\omega} = \frac{5.6}{12.57} = 0.445\ (Wb)$$

$$T = k_a\phi I_a = 0.445 \times 8.0 = 3.56\ (N.m)$$

Here you can also calculate power at 120rpm

$$P = I_a E_b = T\omega = 44.7(W) = 0.06\,hp$$

"No-load" means $T' = 0$

So $I_a' = 0$ $\quad \therefore E_b' = V_s = 7.2V$

$$E_b' = k_a\phi\,\omega'$$

$$\omega' = \frac{E_b'}{k_a\phi} = \frac{7.2}{0.445} = 16.18\ \frac{rad}{s} = 154\ rpm$$