ELEG 310 Midterm 1, March 24, 2011

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1. Roll a fair 6-sided die (all sides equally likely). Let $A = \{1, 2, 3\}$, $B = \{1, 3, 4, 5\}$, $C = \{2, 4, 6\}$. What are:

a) $\Pr[AB]$

b) $\Pr[ABC]$

c) $\Pr[A|B]$ and $\Pr[B|A]$

d) $\Pr[A \cup B]$

e) Are any of pair of A, B, and C independent? Is so, which ones and why?

a) $AB = \{1, 3\}$ $\quad P(AB) = \frac{2}{6} = \frac{1}{3}$

b) $ABC = \emptyset$ $\quad P(ABC) = 0$

c) $P(A|B) = \frac{P(AB)}{P(B)} = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{1}{2}$

$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$

d) $P(A \cup B) = P(A) + P(B) - P(AB) = \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \frac{5}{6}$

or $A \cup B = \{1, 2, 3, 4, 5\}$ $\quad P(A \cup B) = \frac{5}{6}$

e) $P(AB) = \frac{2}{6} = \frac{3}{6} \cdot \frac{4}{6} = P(A)P(B) \Rightarrow$ A & B <u>ind</u>

$P(AC) = \frac{1}{6} \neq \frac{3}{6} \cdot \frac{3}{6} = P(A)P(C) \Rightarrow$ A & B not ind

$P(BC) = \frac{1}{6} \neq \frac{4}{6} \cdot \frac{3}{6} = P(B)P(C) \Rightarrow$ A & B not ind

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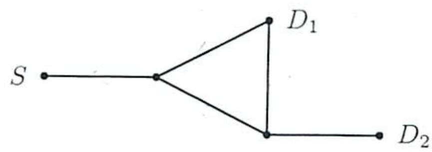
2. In the network below, assume each link (there are 5 links) operates with probability p independently of the other links. Find the following:

a) $\Pr[S \to D_1]$

b) $\Pr[S \to D_2]$

c) $\Pr[S \to D_2 | S \to D_1]$



a) let $A =$  $B =$ 

$$P(S \to D_1) = P(A \cup B) = P(A) + P(B) - P(AB) = p^2 + p^3 - p^4$$

b) let $C =$  $D =$ 

$$P(S \to D_2) = P(C \cup D) = P(C) + P(D) - P(CD) = p^4 + p^3 - p^5$$

c) $$P(S \to D_2 | S \to D_1) = \frac{P(S \to D_2 \cap S \to D_1)}{P(S \to D_1)}$$

$\{S \to D_1 \cap S \to D_2\}$ means first and last links must work and at least 2 of 3 in triangle

$$P(S \to D_1 \cap S \to D_2) = p^2(3p^2(1-p) + p^3) = p^2(3p^2 - 2p^3)$$

$$P(S \to D_2 | S \to D_1) = \frac{p^2(3p^2 - 2p^3)}{p^2 + p^3 - p^4} = \frac{3p^2 - 2p^3}{1 + p - p^2}$$

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3. Let $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ have PMF's below. What are the following:

a) $E[\boldsymbol{X}_1]$, $E[\boldsymbol{X}_2]$

b) $\mathrm{Var}[\boldsymbol{X}_1]$, $\mathrm{Var}[\boldsymbol{X}_2]$

k	0	1	2
$\Pr[\boldsymbol{X}_1 = k]$	0.5	0.3	0.2

k	0	1	2	3
$\Pr[\boldsymbol{X}_2 = k]$	0.1	0.2	0.3	0.4

a) $EX_1 = 0\times 0.5 + 1\times 0.3 + 2\times 0.2 = \boxed{0.7}$

$EX_2 = 0\times 0.1 + 1\times 0.2 + 2\times 0.3 + 3\times 0.4 = \boxed{2.0}$

b) $EX_1^2 = 0^2\times 0.5 + 1^2\times 0.3 + 2^2\times 0.2 = 1.1$

$\mathrm{Var}\,X_1 = EX_1^2 - (EX_1)^2 = 1.1 - 0.7^2 = \boxed{0.61}$

c) $EX_2^2 = 0^2\times 0.1 + 1^2\times 0.2 + 2^2\times 0.3 + 3^2\times 0.4$

$= 0.2 + 1.2 + 3.6 = 5.0$

$\mathrm{Var}\,X_2 = EX_2^2 - (EX_2)^2 = 5.0 - 2.0^2 = \boxed{1.0}$

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4. Let $\boldsymbol{S} = \boldsymbol{X}_1 + \boldsymbol{X}_2$ with $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ independent with the same PMFs as in Problem 3 (the PMFs are repeated below). What are the following:

a) $\Pr[\boldsymbol{S} = 2]$ and $\Pr[\boldsymbol{S} = 3]$?

b) What are the mean and variance of $\boldsymbol{S}$?

k	0	1	2
$\Pr[\boldsymbol{X}_1 = k]$	0.5	0.3	0.2

k	0	1	2	3
$\Pr[\boldsymbol{X}_2 = k]$	0.1	0.2	0.3	0.4

a) $P(S=2) = P(X_1=0 \cap X_2=2) + P(X_1=1 \cap X_2=1) + P(X_1=2 \cap X_2=0)$

$= P(X_1=0)P(X_2=2) + P(X_1=1)P(X_2=1) + P(X_1=2)P(X_2=0)$

$= 0.5 \times 0.3 + 0.3 \times 0.2 + 0.2 \times 0.1$

$= 0.15 + 0.06 + 0.02 = \boxed{0.23}$

$P(S=3) = 0.5 \times 0.4 + 0.3 \times 0.3 + 0.2 \times 0.2$

$= 0.20 + 0.09 + 0.04 = \boxed{0.33}$

b) $ES = EX_1 + EX_2 = 0.7 + 2.0 = \boxed{2.7}$

$\text{Var}\, S = \text{Var}\, X_1 + \text{Var}\, X_2 = 0.61 + 1.0 = \boxed{1.61}$