EXERCISE SHEET 20

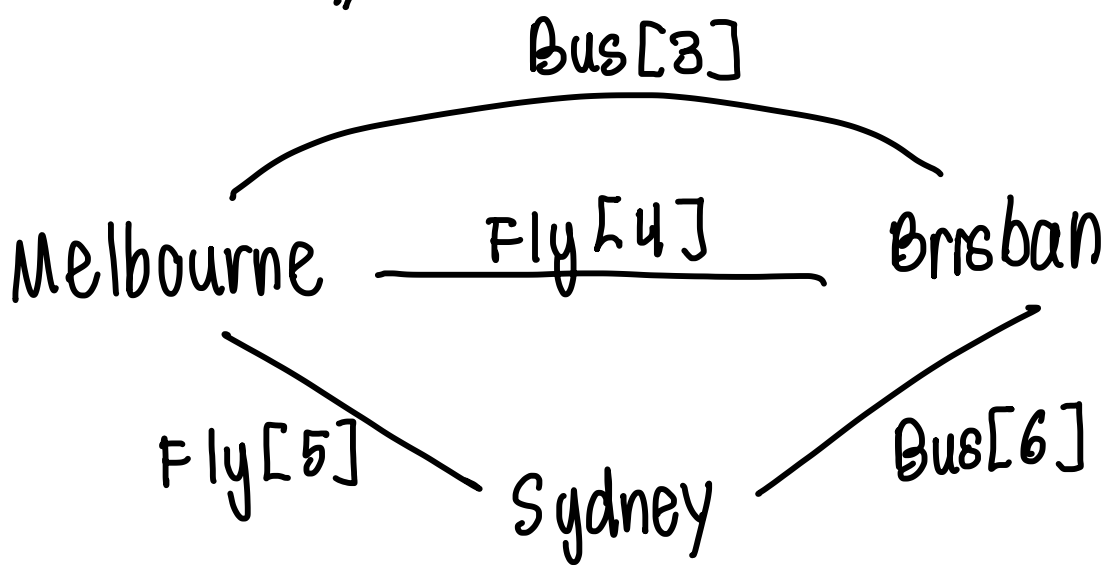
① b) $8 \times 7! = 8!$ //

c) $\frac{9!}{7!2!} = \frac{9 \times 8 \times 7!}{7!2!}$

$= \frac{72}{2}$

$= 36$ //

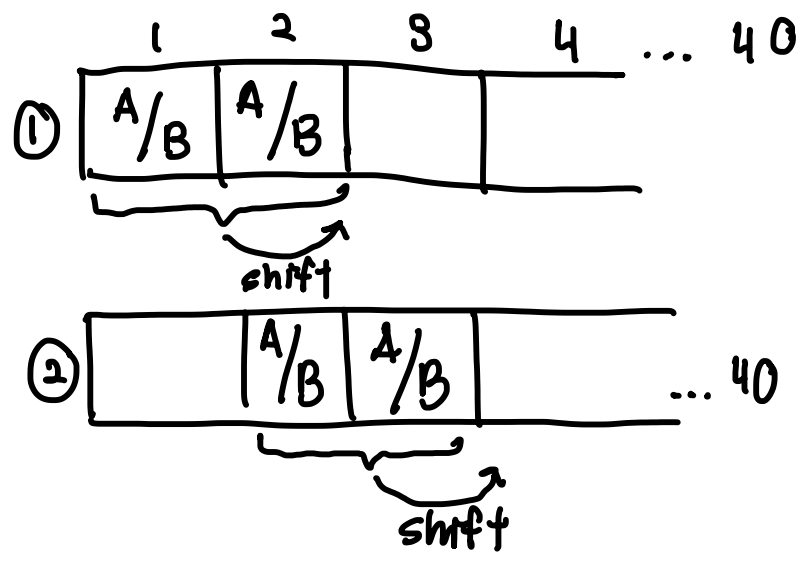
③



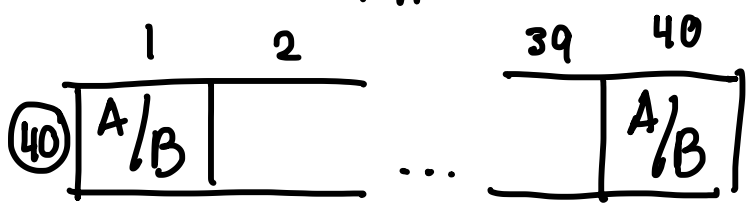
${}^3C_1 \times {}^4C_1 \times {}^5C_1 \times {}^6C_1 = 37$ ways //

④ b) ${}^{40}P_2 = 1560$ arrangements //

c)



until



(X) Not together

$39 \times 2 = 78$ ways //

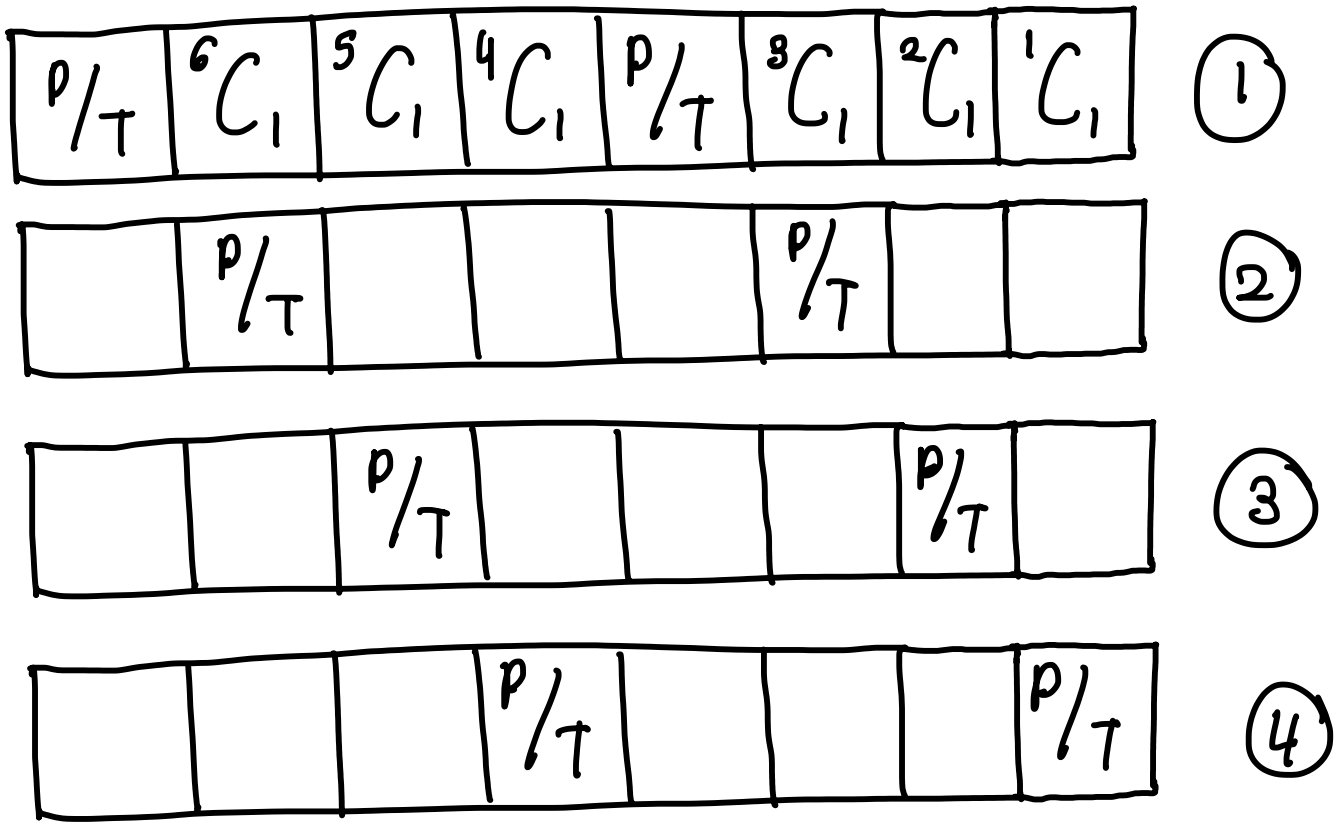
(11) COOLAGATTA

C-1 N-1
O-2 G-1
L-1 T-2
A-3

$$\frac{n!}{p!q!\ldots} = \frac{11!}{2!3!2!}$$

$= 1\ 663\ 200$ //

(13) a) APRICOTS

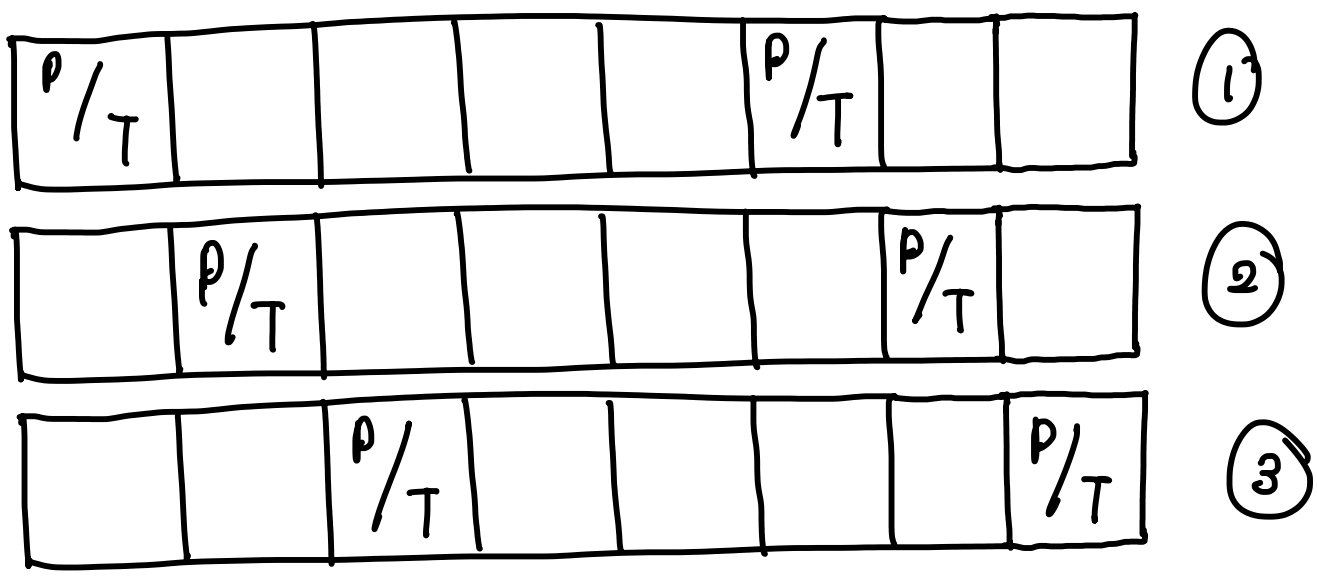


$6! \times 4 \times 2 = 5760$ //

b) APRICOTS

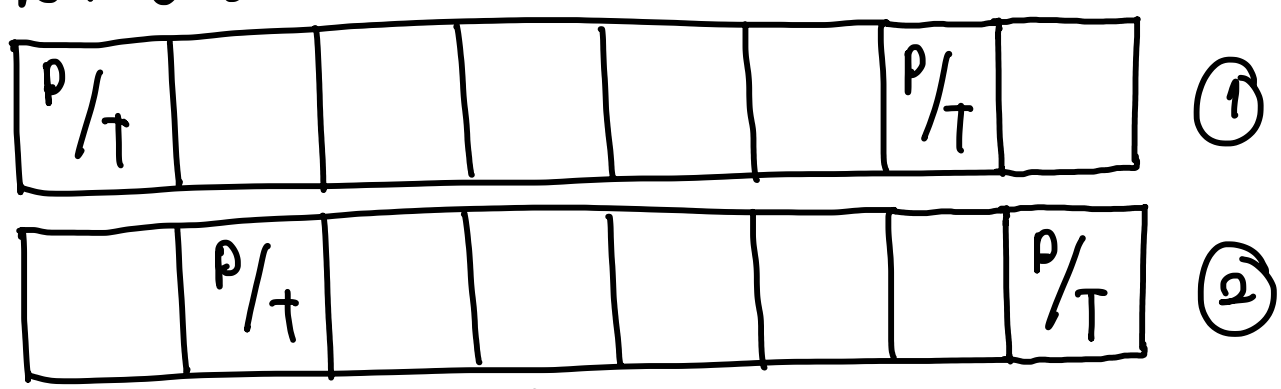
3 letters ~ 5760 <from Q13a>

4 letters



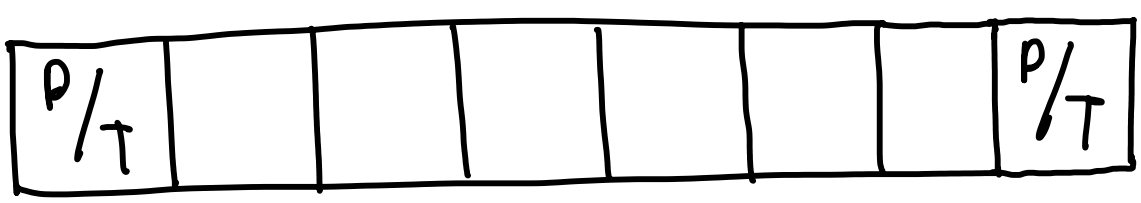
$6! \times 3 \times 2 = 4320$

5 letters



$6! \times 2 \times 2 = 2880$

6 letters



$6! \times 2 = 1440$

∴ At least 3 letters

$5760 + 4320 + 2880 + 1440 = 14\ 400$ //

EXERCISE SHEET 20

AUG 24

① a) <Selected Question>

b) $\frac{8!}{7!} = \frac{8 \times 7!}{7!}$

$= 8$ //

c) <Selected Question>

d) $\frac{450!}{449!} = \frac{450 \times 449!}{449!}$

$= 450$ //

② a) $\frac{n!}{(n-1)!}$ where $n \in \{1, 2, 3 \ldots\}$

$\frac{n!}{(n-1)!} = \frac{n(n-1)(n-2)(n-3)\ldots}{(n-1)(n-2)(n-3)\ldots}$

$= n$ //

b) $\frac{n!}{(n-2)!}$ where $n \in \{2, 3, 4 \ldots\}$

$\frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)(n-3)\ldots}{(n-2)(n-3)\ldots}$

$= n(n-1)$

③ <Selected Question>

④ a) <Selected Question>

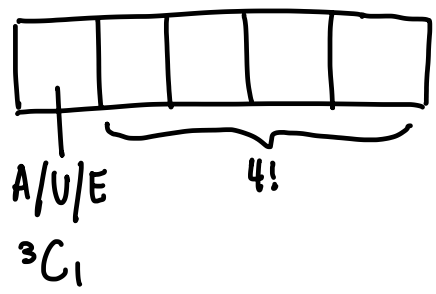
b) ${}^{40}P_2 = 1560 \text{ways}$ //

c) <Selected Question>

d) $1560 - 78 = 1482 \text{ ways}$ //

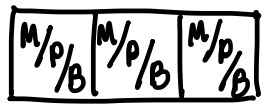
⑤ a) VALUE

$5! = 120 \text{ways}$ //

b) 

$^3C_1 \times 4! = 72$ arrangements //

⑥ 5 maths
4 physics
2 biology



$5! \times 4! \times 2! \times 3! = 34\ 560$ ways //

⑦ <Selected Questions>

⑧ a) MINIMUM

M-3 N-1
I-2 U-1

$$\frac{n!}{p!q!\ldots} = \frac{7}{3!\,2!}$$

= 420 arrangements //

b) ① M M M N _ _ _

$$\frac{3!}{}$$

② M M N M

③ M N M M

④ N M M M

$4 \times 2 \times 3! = 48$ arrangements //

⑨ a) STEEPLES

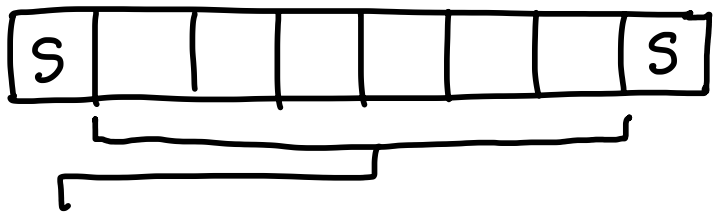
S-2 P-1
T-1 L-1
E-3

$$\frac{n!}{p!q!\ldots} = \frac{8!}{2!\,3!}$$

= 3 360 ways //

b) E E E _ _ _ _ _

$$\frac{5!}{2!} \times 6 = 360 \text{ arrangements //}$$

c) 

$\frac{6!}{3!} = 120$ arrangements //

10 a) ECONOMICS

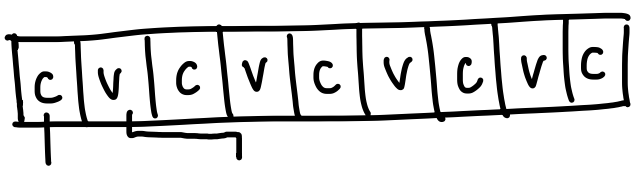
E-1 M-1
C-2 I-1
O-2 S-1
N-1

$\frac{n!}{p!q!\cdots} = \frac{9!}{2!2!}$

$= 90\ 720$ ways //

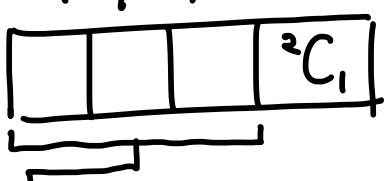
b) consonants – $\overset{2}{C}$, N, M, S

vowels – E, $\overset{2}{O}$, T



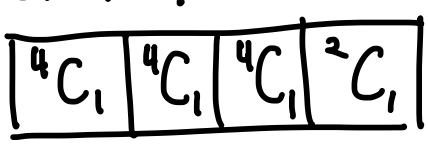
$\frac{5!}{2!} \times \frac{4!}{2!} = 720$ arrangements //

11 a) 6, 7, 8, 9



$3! \times {}^{2}C_{1} = 12$ numbers //

b) 6, 7, 8, 9



$^{4}C_{1} \times {}^{4}C_{1} \times {}^{4}C_{1} \times {}^{2}C_{1} = 128$ numbers //

12 3, 5, 7, 8, 9

4-digit numbers:

$^{4}C_{1} \times {}^{4}P_{3} = 96$

5 digit numbers:

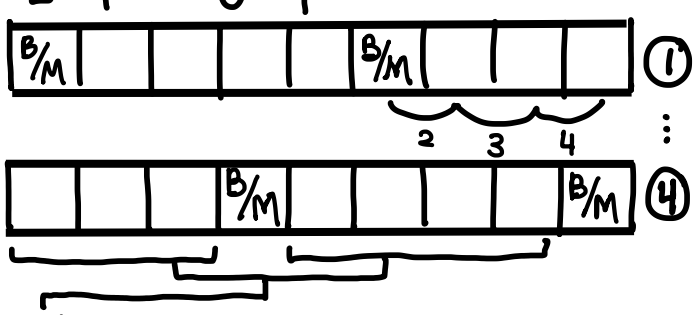
$5! = 120$

$\therefore 96 + 120 = 216$ //

⑬ <Selected Question>

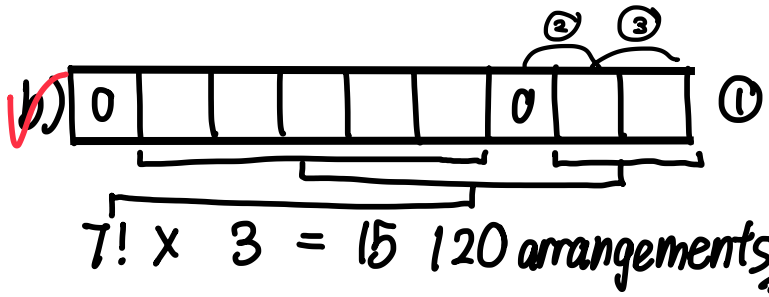
⑭ ~~a)~~ BOOMERANG

B-1 R-1
O-2 A-1
M-1 N-1
E-1 G-1



$\frac{7!}{2!} \times 4 \times 2 = 20\,160$ arrangements //

② ③

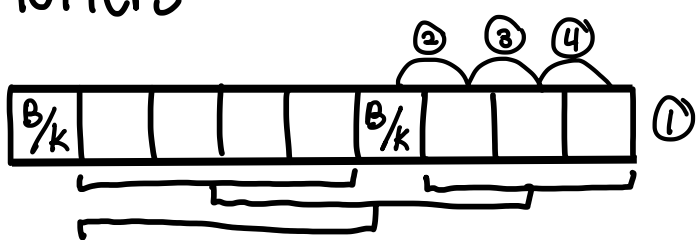


$7! \times 3 = 15\,120$ arrangements //

⑮ ~~a)~~ BALALAIKA

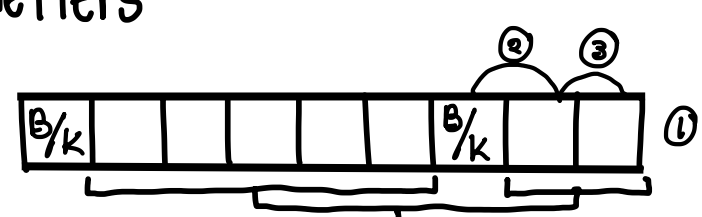
B-1 I-1
A-4 K-1
L-2

4 letters



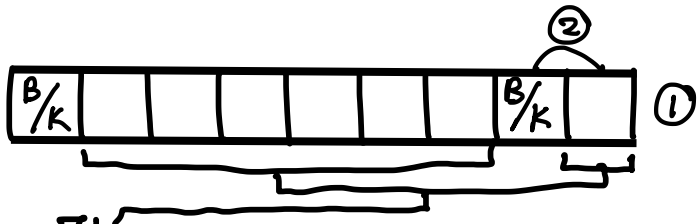
$\frac{7!}{4!2!} \times 4 \times 2 = 840$

5 letters



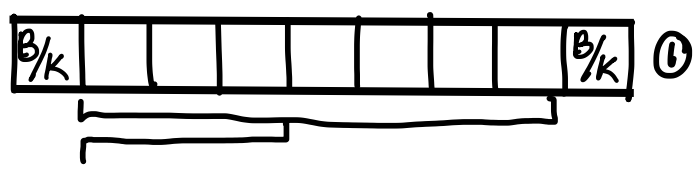
$\frac{7!}{4!2!} \times 3 \times 2 = 630$

6 letters



$\frac{7!}{4!2!} \times 2 \times 2 = 420$

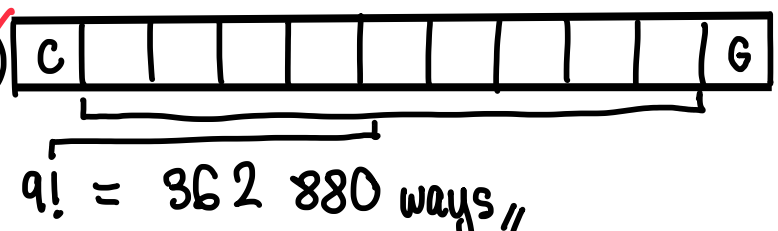
7 letters



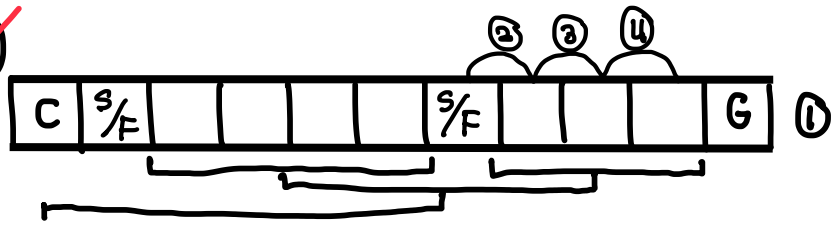
$\frac{7!}{4!2!} \times 2 = 210$

∴ At least 4 letters

840 + 630 + 420 + 210 = 2100 arrangements //

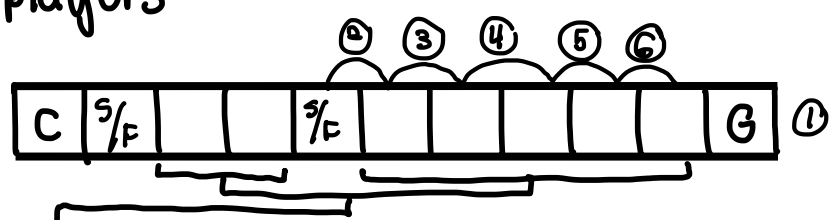
(16) a) 

9! = 362 880 ways //

b) 

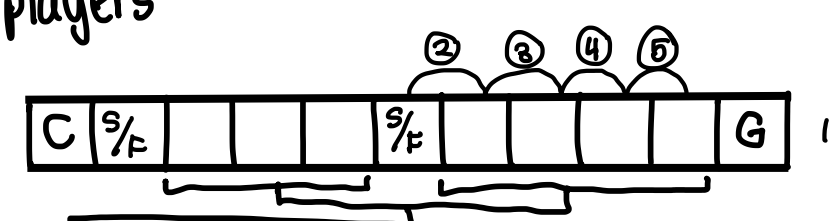
7! × 4 × 2 = 40 320 ways //

c) 2 players



7! × 6 × 2 = 60 480

3 players



7! × 5 × 2 = 50 400

4 players

$7! \times 4 \times 2 = 40\,320$

5 players

$7! \times 3 \times 2 = 30\,240$

6 players

$7! \times 2 \times 2 = 20\,160$

7 players

$7! \times 1 \times 2 = 10\,080$

$\therefore$ More than 1

$60\,480 + 50\,400 + 40\,320 + 30\,240 + 20\,160 + 10\,080 = 211\,680$ ways //

(17) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

1 digit

9

2 digits

${}^{9}C_1$	1

same

${}^{9}C_1 = 9$

3 digits

${}^{9}C_1$	${}^{10}C_1$	1

same

${}^{9}C_1 \times {}^{10}C_1 = 90$

4 digits

${}^{9}C_1$	${}^{10}C_1$	1	1

same

same

${}^{9}C_1 \times {}^{10}C_1 = 90$

5 digits

${}^{9}C_1$	${}^{10}C_1$	${}^{10}C_1$	1	1

same

same

${}^{9}C_1 \times {}^{10}C_1 \times {}^{10}C_1 = 900$

6 digits

${}^{9}C_1$	${}^{10}C_1$	${}^{10}C_1$	1	1	1

same

same

same

$^9C_1 \times {}^{10}C_1 \times {}^{10}C_1 = 900$

$\therefore$

$9+9+90+90+900+900 = 1998$ //

(18) a) OSCILLOSCOPE

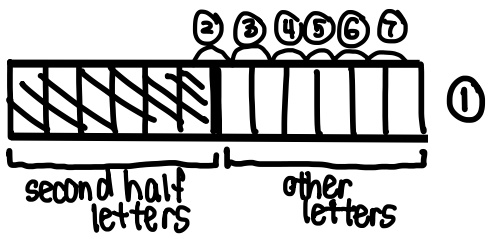
second half	first half
O-3	C-2 E-1
S-2	I-1
P-1	L-2

second half

$$\frac{n!}{p!q!\cdots} = \frac{6!}{3!2!} = 60$$

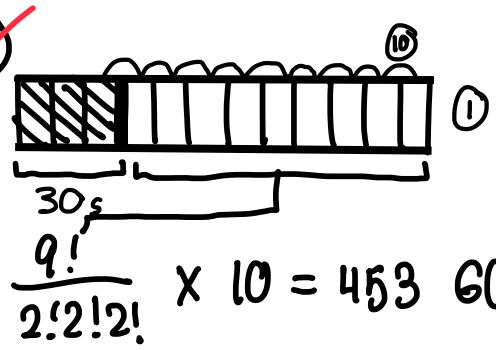
first half

$$\frac{n!}{p!q!} = \frac{6!}{2!2!} = 180$$



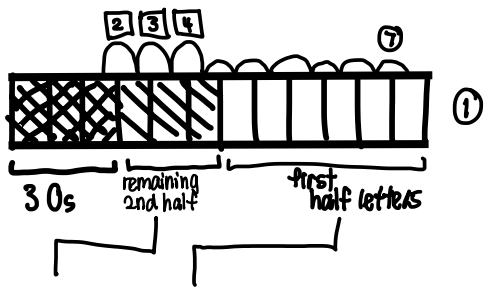
$60 \times 180 \times 7 = 75\,600$ arrangements //

b)



$$\frac{9!}{2!2!2!} \times 10 = 453\,600$$ //

c) a) 75 600
b) 453 600
Both



$$\frac{3!}{2!} \times 180 \times 7 \times 3 = 15\,120$$

$\therefore$ One or other (or both)
Exclusively a) + Exclusively b) + both

$(75\,600-15\,120)+(453\,600-15\,120)+15\,120 = 514\,080$ //

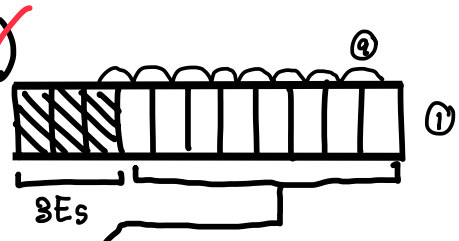
(19) a) ENGINEERING

E-3 I-2
N-3 R-1
G-2

$$\frac{n!}{p!q!\cdots} = \frac{11!}{3!3!2!2!}$$

$= 277\,200$ arrangements //

b)

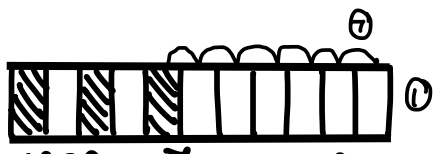


$\frac{8!}{3!2!2!} \times 9 = 15\,120$ arrangements //

c) <not examinable>

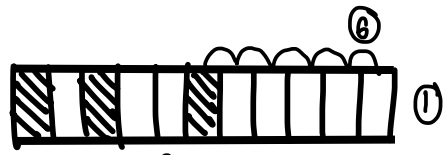
1 space between First 2 Es

Case 1



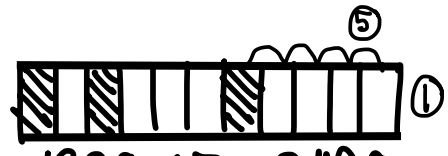
$1680 \times 7 = 11\,760$

Case 2



$1680 \times 6 = 10\,080$

Case 3



$1680 \times 5 = 8\,400$

Case 4

$1680 \times 4 = 6720$

Case 6

$1680 \times 2 = 3360$

Case 5

$1680 \times 3 = 5040$

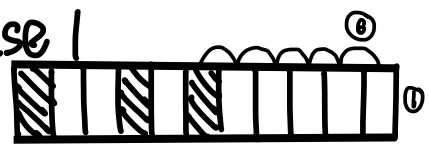
Case 7

1680

$11\,760 + 10\,080 + 8\,400 + 6720 + 5040 + 3360 + 1680 = 47\,040$

2 spaces between first 2Es

Case 1



Case 7

$47\,040 - 11\,760 = 35\,280$

3 spaces //

$35\,280 - 10\,080 = 25\,200$

4 spaces //

$25\,200 - 8\,400 = 16\,800$

5 spaces //

16 800 − 6720 = 10 080

6 spaces "

10 080 − 5040 = 5040

7 spaces "

5040 − 3360 = 1680

∴ no Es together

47 040 + 35 280 + 25 200 + 16 800 + 10 080 + 5040 + 1680 = 141 120 arrangements // ☺

(20) a) {A, B, C, D, E, F, G, H, I, J}

$^{8}C_{4} = 70$ //

b) $^{8}C_{6} = 28$ //

(21) a) Men x Women

$^{6}C_{3} \times {}^{7}C_{4} = 700$ ways //

b) $^{5}C_{2} \times {}^{5}C_{2} = 100$ ways //

(22) a) 5 prefects, 11 non-prefects

$^{5}C_{3} \times {}^{11}C_{4} = 3300$ //

b) 2 prefects

$^{5}C_{2} \times {}^{11}C_{5} = 4620$

3 prefects

$^{5}C_{3} \times {}^{11}C_{4} = 3300$

4 prefects

$^{5}C_{4} \times {}^{11}C_{3} = 825$

5 prefects

$^{5}C_{5} \times {}^{11}C_{2} = 55$

∴ At least 2 prefects

4620 + 3300 + 825 + 55 = 8800 //

(23) a) $^{7}C_{2} \times {}^{8}C_{3} = 1176$ ways //

b) 1 engineer 4 mathematicians

$^{7}C_{1} \times {}^{8}C_{4} = 490$

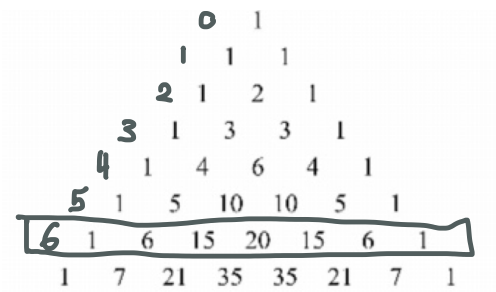
2 engineers 3 mathematicians
${}^{7}C_2 \times {}^{8}C_3 = 1176$
3 engineers 2 mathematicians
${}^{7}C_3 \times {}^{8}C_2 = 980$
∴ At least 1 engineer 2 mathematicians
$490 + 1176 + 980 = 2646$

(24) 1 Independent 3 Liberals 3 Labour
${}^{5}C_1 \times {}^{10}C_3 \times {}^{8}C_3 = 33\,600$
1 Independent 4 Liberals 2 Labour
${}^{5}C_1 \times {}^{10}C_4 \times {}^{8}C_2 = 29\,400$
1 Independent 5 Liberals 1 Labour
${}^{5}C_1 \times {}^{10}C_5 \times {}^{8}C_1 = 10\,080$
∴ 1 Independent at least 3 Liberals at least 1 Labour
$33600 + 29400 + 10\,080 = 73\,080$ //

(25) a) ${}^{5}C_2 \times {}^{5}C_2 = 100$ //

b) 1 Group A 3 Group B
${}^{5}C_1 \times {}^{5}C_3 = 50$
2 Group A 2 Group B
${}^{5}C_2 \times {}^{5}C_2 = 100$
3 Group A 1 Group B
${}^{5}C_3 \times {}^{5}C_1 = 50$
∴ At least 1 from each group
$50 + 100 + 50 = 200$ //

(26) a) $(a+x)^6 = a^6 + 6a^5y + 15a^4x^2 + 20a^3x^3 + 15a^2x^4 + 6ax^5 + x^6$
Pascal's Triangle



b)

Power	# of terms	
1	2	$(a+x)$
2	4	$(a+x)(a+x) = (a^2+ax+ax+x^2)$
3	8	$(a^2+ax+ax+x^2)(a+x) = \ldots$
4	16	
5	32	
6	64	

(x2, x2)

$\therefore (a+x)^6$ has 64 terms.

c) ${}^6C_2 = 15$

d) ${}^6C_5 = 6$

e) $(2+x)^{11} = 2^{11}x^0\,{}^{11}C_0 + 2^{10}x^1\,{}^{11}C_1 + 2^9x^2\,{}^{11}C_2 + 2^8x^3\,{}^{11}C_3 + 2^7x^4\,{}^{11}C_4 + \ldots$
$\ldots + 2^6x^5\,{}^{11}C_5 + 2^5x^6\,{}^{11}C_6 + 2^4x^7\,{}^{11}C_7 + 2^3x^8\,{}^{11}C_8 + 2^2x^9\,{}^{11}C_9 + \ldots$
$\ldots + 2^1x^{10}\,{}^{11}C_{10} + 2^0x^{11}\,{}^{11}C_{11}$
$= 2048 + 11264x + 28160x^2 + 42240x^3 + 42240x^4 + 29568x^5 + 14784x^6 + 5280x^7 + 1320x^8 + 220x^9 + 22x^{10} + x^{11}$

<not examinable>

27) ${}^xC_3 = 84$ ways

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$$84 = \frac{x!}{3!(x-3)!}$$

$$504 = \frac{x(x-1)(x-2)(x-3)!}{(x-3)!}$$

$$0 = x^3 - 3x^2 + 2x - 504$$

Let $f(x) = x^3 - 3x^2 + 2x - 504$

BTAE, $f(9) = 0$

$0 = (x-9)(Ax^2+Bx+C)$

$\therefore x = 9$

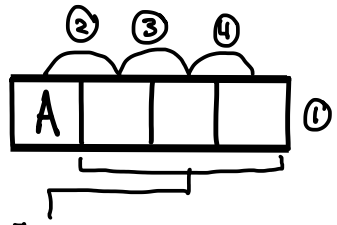
$\therefore$ 9 members //

<not examinable>

(28) K A L A O D G

repeated A

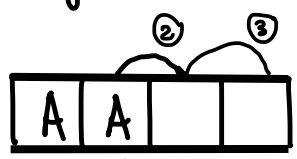
one A in:



$^5P_3 \times 4 = 240$

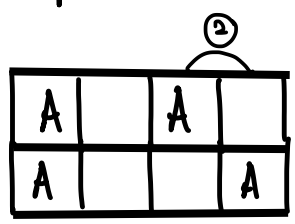
two As in:

As together



$^5P_2 \times 3 = 60$

As apart



$^5P_2 \times 2 = 40$

$^5P_2 \times 1 = 20$

No As in:

$^5P_4 = 120$

$\therefore 240 + 60 + 40 + 20 + 120 = 480$ arrangements //