EXERCISE SHEET 21 (SELECTED QUESTIONS)

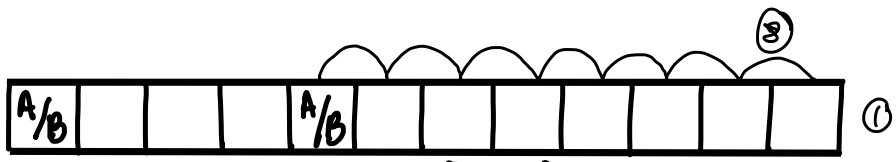
AUG
31

④ Let 1 woman be A and the other woman B

TOTAL

12! = 479 001 600

3 PEOPLE



10! x 2 x 8 = 58 060 800

4 PEOPLE

10! x 2 x 7 = 50 803 200

5 PEOPLE

10! x 2 x 6 = 43 545 600

∴ MORE THAN 2 PEOPLE =

$$\text{PROBABILITY} = \frac{261\,273\,600}{479\,001\,600}$$

$$= \frac{6}{11} //$$

6 PEOPLE

10! x 2 x 5 = 36 288 000

7 PEOPLE

10! x 2 x 4 = 29 030 400

8 PEOPLE

10! x 2 x 3 = 21 772 800

9 PEOPLE

10! x 2 x 2 = 14 515 200

10 PEOPLE

10! x 2 x 1 = 7 257 600

⑤ a) $^{12}C_7$ = 792 combinations //

b) 4 TEACHERS = $^{7}C_4 \times {}^{5}C_3$ = 350

5 TEACHERS = $^{7}C_5 \times {}^{5}C_2$ = 210

6 TEACHERS = $^{7}C_6 \times {}^{5}C_1$ = 35

7 TEACHERS = $^{7}C_7 \times {}^{5}C_0$ = 1

∴ at least 4 = 350+210+35+1 = 596 combinations //

c) $\frac{596}{792} = \frac{149}{198}$ //

⑥ $$\text{PROBABILITY} = \frac{{}^{6}C_3 \times {}^{4}C_2}{{}^{10}C_5}$$

$$= \frac{10}{21} //$$

⑦ a) $Pr(A) = 0.3$ $Pr(B') = 0.25$ $Pr(A \cap B) = 0.1$

$\cap$	A	A'	
B	0.1	0.65	0.75
B'	0.2	0.05	0.25
	0.3	0.7	1

$Pr(B) = 1 - Pr(B')$

$Pr(A') = 1 - Pr(A)$

$Pr(A \cap B) + Pr(A \cap B') = Pr(A)$

$Pr(A \cap B) + Pr(A' \cap B) = Pr(B)$

$Pr(A \cap B') + Pr(A' \cap B') = Pr(B')$

b) i) $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$= 0.3 + 0.75 - 0.1$

$= 0.95$ //

ii) $Pr(A' \cup B) = Pr(A') + Pr(B) - Pr(A' \cap B)$

$= 0.7 + 0.75 - 0.65$

$= 0.8$ //

iii) $Pr(A \cup B') = Pr(A) + Pr(B') - Pr(A \cap B')$

$= 0.3 + 0.25 - 0.2$

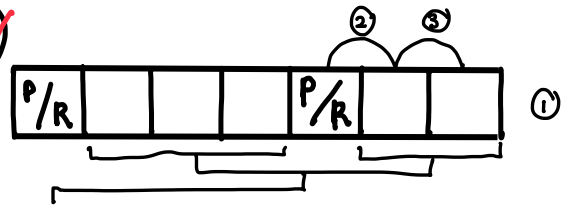
$= 0.35$ //

EXERCISE SHEET 21

AUG 31

① a) $7! = 5040$ arrangements //

b)



$5! \times 2 \times 3 = 720$ arrangements //

c) $\frac{720}{5040} = \frac{1}{7}$ //

② a) ${}^{6}P_{3} = 120$ arrangements //

b) ${}^{5}P_{2} = 20$ arrangements //

c) $\frac{20}{120} = \frac{1}{6}$ //

③ INDEPENDENCE

I - 1 E - 4

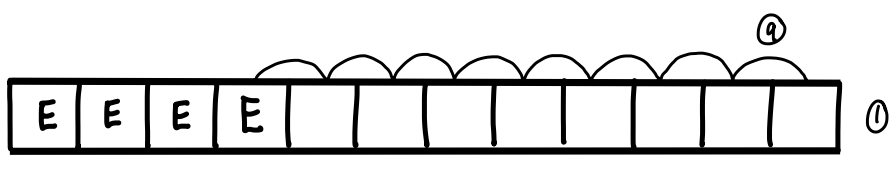
N - 3 P - 1

D - 2 C - 1

TOTAL OUTCOMES $= \frac{12!}{3!2!4!}$

$= 1\,663\,200$

Es TOGETHER



$\frac{8!}{3!2!} \times 9 = 30\,240$

PROBABILITY $= \frac{30\,240}{1\,663\,200} = \frac{1}{55}$ //

④ <selected question>

⑤ <selected question>

⑥ <selected question>

⑦ <selected question>

⑧ a) $Pr(R) = 0.4168$ $Pr(S) = 0.3275$ Mutually exclusive ~ $Pr(R \cap S) = 0$

$\cap$	R	R'	
S	0	0.3275	0.3275
S'	0.4168	0.2557	0.6725
	0.4168	0.5832	1

$Pr(S') = 1 - Pr(S)$
$Pr(R') = 1 - Pr(R)$
$Pr(R \cap S') + Pr(S' \cap R') = Pr(S')$

b) $Pr(R' \cap S') = 0.2557$
Since $Pr(R' \cap S') \neq 0$, the events R' and S' are not mutually exclusive.

⑨ Let F = the event that a guest could speak French,
J = the event that a guest could speak Japanese.

$\cap$	F	F'	
J	$x - 0.2$	$0.9 - x$	0.7
J'	0.2	0.1	0.3
	x	$1 - x$	1

$Pr(J) = 0.7$
$Pr(F) = x$
$Pr(J' \cap F') = 0.1$
$Pr(J') = 1 - Pr(J)$
$Pr(J' \cap F) = Pr(J') - Pr(J \cap F')$
$Pr(F') = 1 - Pr(F)$
$Pr(J \cap F) = Pr(F) - Pr(J' \cap F)$
$Pr(J \cap F') = Pr(F) - Pr(J \cap F')$

$$Pr(J \cap F) = \frac{x}{100} - 0.2$$ //

(10) a) January, June, July

$$\text{PROBABILITY} = \frac{{}^3C_2 \times {}^9C_3}{{}^{12}C_5}$$

$$= \frac{7}{22} //$$

b) i) <Not examinable>

$$P(m) = \frac{{}^3C_2 \times {}^9C_{m-2}}{{}^{12}C_m}$$

$$= \left(\frac{3!}{2!(3-2)!} \times \frac{9!}{(m-2)!(11-m)!}\right) \div \frac{12!}{m!(12-m)!}$$

$$= \left(3 \times \frac{9!}{(m-2)!(11-m)!}\right) \times \frac{m(m-1)(m-2)!(12-m)(11-m)!}{12 \times 11 \times 10 \times 9!}$$

$$= \frac{3m(m-1)(12-m)}{1320\ 440}$$

$$= \frac{m(m-1)(12-m)}{440} \text{ <shown>}$$

//

ii) $f(x) = \frac{x(x-1)(12-x)}{440}$

$$= \frac{x(-x^2+13x-12)}{440}$$

$$= \frac{-x^3+13x^2-12x}{440}$$

$$f'(x) = \frac{-3x^2+26x-12}{440}$$

When $f'(x) = 0$:

$$-3x^2+26x-12 = 0$$

$$3x^2-26x+12 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-26) \pm \sqrt{(-26)^2-4(3)(12)}}{2(3)}$$

$$= \frac{26 \pm \sqrt{532}}{6}$$

$= \frac{13 \pm \sqrt{133}}{3}$

$= 8.178$ or 0.489 (3dp)

<rejected ~ $f:[2,12]$>

$f(2) = 0.045$ (3dp)

$f(8.178) = 0.51$ (2dp) <maximum>

$f(12) = 0$

∴ Global maximum of f occurs at $x_{max} = 8.178$ (3dp) //

(iii) Test $m=8$

$P(8) = \frac{224}{440}$ <maximum>

Test $m=9$

$P(9) = \frac{216}{440}$

∴ $m=8$ maximises $P(m)$ //

(ii) <Not examinable>

10 e's	61 letters

Arrangements with e's together:

$$\frac{62!}{a!\,b!\,c!\,d!\,f!\ldots}$$

Total arrangements:

$$\frac{71!}{a!\,b!\,c!\,d!\,e!\,f!\ldots}$$

Probability:

$$\frac{62!}{a!\,b!\,c!\,d!\,f!\ldots} \div \frac{71!}{a!\,b!\,c!\,d!\,e!\,f!\ldots} = \frac{62!}{a!\,b!\,c!\,d!\,f!\ldots} \times \frac{a!\,b!\,c!\,d!\,e!\,f!\ldots}{71\times70\times\ldots\times63\times62!}$$

$$= \frac{10!}{71\times70\times69\times\ldots\times63}$$

$$= \frac{1}{7\ 447\ 387\ 948}$$ //

(12) <Not examinable>

(13) $V = 3 + \sin\left(\frac{t}{4}\right)$

a) when $t = 10$:

$V = 3 + \sin\left(\frac{10}{4}\right)$

$= 3.598472144104$

≈ 3.598 <3dp>

b) $\frac{dV}{dt} = \frac{1}{4}\cos\left(\frac{t}{4}\right)$

when $t = 10$:

$\frac{dV}{dt} = \frac{1}{4}\cos\left(\frac{10}{4}\right)$

$= -0.200285903886 7$

≈ -0.20 <2dp> //

(14) $\frac{dV}{dt} = 0.1 \ m^3/min \qquad V = \frac{4}{3}\pi r^3$

a) $\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$= \frac{1}{4\pi r^2} \times 0.1$

When $r = 2.5$:

$\frac{dr}{dt} = \frac{1}{4\pi(2.5)^2} \times 0.1$

$= 0.00127323954 47$

≈ 0.001 <3dp> //

b) $A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

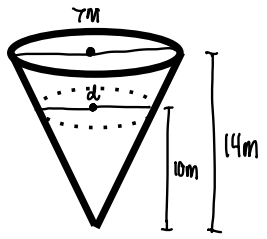
$$= 8\pi r \times 0.00127323954447$$

When $r = 2.5$:

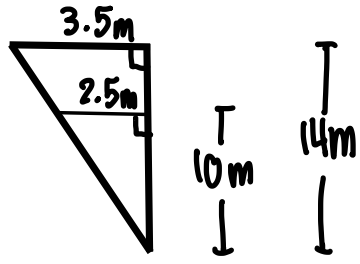
$$\frac{dA}{dt} = 8\pi(2.5) \times 0.00127323954447$$

$$= 0.08\ m^2/min_{//}$$

(15)



$\frac{dV}{dt} = 0.1\ m^3/min$ $\quad V = \frac{1}{3}\pi r^2 h$



Similar triangles

$$\frac{r}{h} = \frac{3.5}{14}$$

$$= 0.25$$

$$r = 0.25h$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(0.25h)^2 h$$

$$= \frac{1}{48}\pi h^3$$

$$\frac{dV}{dh} = \frac{1}{16}\pi h^2$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

at $h = 10$:

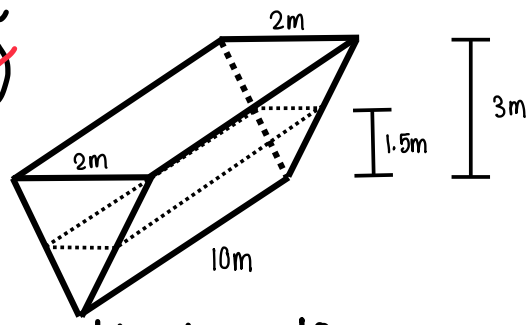
$$\frac{dh}{dt} = \frac{1}{\frac{1}{16}\pi(10)^2} \times 0.1$$

$$= 0.00509295817 89$$

$$\approx 0.0051\ m/min$$ <4dpr//

(16) $\frac{dV}{dt} = 0.1 m^3/min$

a)



Similar shapes

$\frac{w}{h} = \frac{2}{3}$

$w = \frac{2}{3}h$

$V = \frac{1}{2} wh \times 10$

$= 5wh$

$= 5 \times \frac{2}{3} h \times h$

$= \frac{10}{3} h^2$

$\frac{dV}{dh} = \frac{20}{3} h$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

when $h = 1.5$:

$\frac{dh}{dt} = \frac{1}{20/3(1.5)} \times 0.1$

$= 0.01$ m/min // The water is falling at a rate of 0.0051m/min <4 dp> //

b) Full Volume $= 5(3)(2)$

$= 30m^3$

Half Volume $= \frac{30}{2}$

$= 15m^3$

$5wh = 15$

$wh = 3$

$\left(\frac{2}{3}h\right)h = 3$

$h^2 = 2$

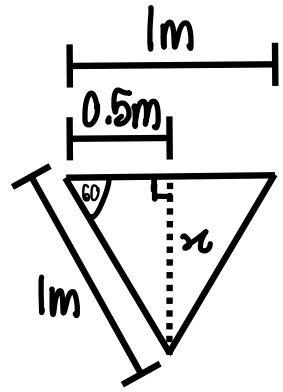
$h = \sqrt{2}$ <$-\sqrt{2}$ rejected>

$\frac{dh}{dt} = \frac{1}{20/3(\sqrt{2})} \times 0.1$

$= \frac{1}{100\sqrt{2}}$ m/min //

(17) $\frac{dV}{dr} = -0.01 m^3/min$

Similar triangles



$\sin 60° = \frac{x}{1}$

$x = \frac{\sqrt{3}}{2}$

$\frac{h}{w} = \frac{\sqrt{3}}{2} \div 1$

$= \frac{\sqrt{3}}{2}$

$w = \frac{2h}{\sqrt{3}}$

$V = \frac{1}{2} w h \times 6$

$= \frac{1}{2}\left(\frac{2h}{\sqrt{3}}\right) h \times 6^3$

$= \frac{6}{\sqrt{3}} h^2$

$\frac{dV}{dh} = \frac{12}{\sqrt{3}} h$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$= \frac{1}{12/\sqrt{3} h} \times -0.01$

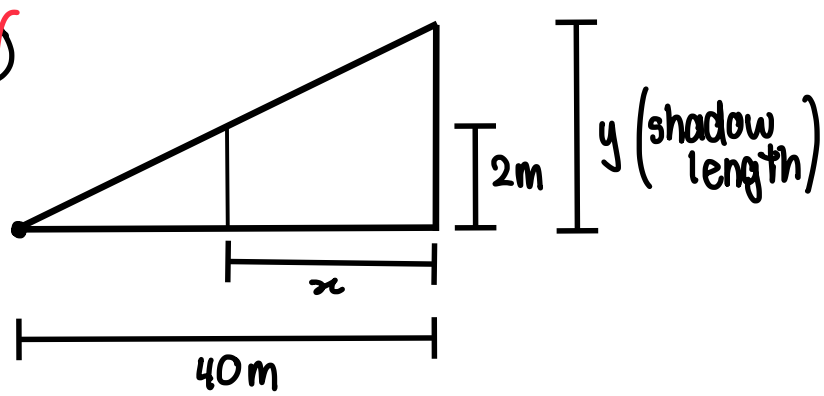
When $h = 0.2$:

$\frac{dh}{dt} = \frac{1}{12/\sqrt{3}(0.2)} \times -0.01$

$= 0.007216878364 9$

≈ 0.0072 <4dp> //

(18)



Similar shapes

$\frac{y}{2} = \frac{40}{40-x}$

$y = \frac{80}{40-x}$ ×4 ×v

$\frac{dy}{dx} = \frac{(40-x)(0) - (80)(-1)}{(40-x)^2} = \frac{80}{(40-x)^2}$

$\frac{dx}{dt} = 1.6 m/s$

$$\frac{dy}{dt} = \frac{80}{(40-10)} \times 1.6$$

$$= 0.142222\ldots$$

$$\approx 0.14 \text{ m/s } \langle 2dp \rangle //$$