ML_HW07

January 4, 2023

0.1 Kernel Eigenfaces

[214]:
```python
import numpy as np
from scipy.spatial import distance
import matplotlib.pyplot as plt
import os
from PIL import Image
```

0.1.1 PCA and LDA Eigenfaces and Fisherfaces

0.1.2 Part I

0.1.3 PCA eigenfaces

First, try to use PCA to find the first 25 eigenfaces. To find the eigenvalues and eigenvectors of PCA analysis, we should compute the covariance matrix of the data first. And after obtaining the eigenvalues we can sort the value from the largest to the smallest.

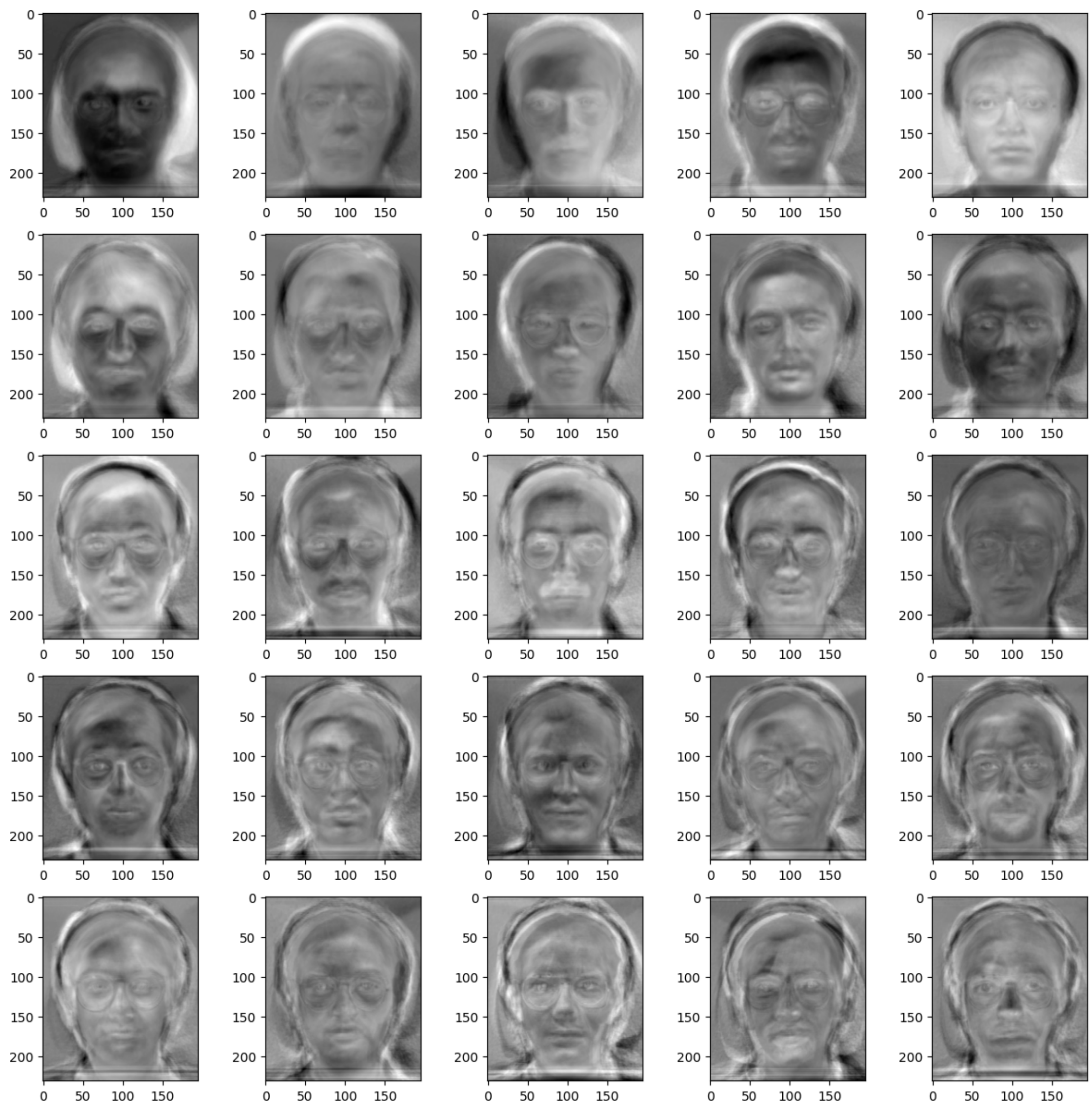
[4]:
```python
## read file and also the label of the file to classify different person's face.
def read_file(dir , H , W):
    file_path = os.listdir(dir)
    images = np.zeros((H*W, len(file_path)))
    labels = np.zeros(len(file_path) , dtype = int)
    for i,name in enumerate(file_path):
        image_dir = os.path.join(dir , name)
        images[:,i] = np.array(Image.open(image_dir).resize((W , H) , Image.
↪Resampling.LANCZOS)).flatten()
        labels[i] = int(name.split('.')[0][7:9]) - 1

    return images , labels
```

[5]:
```python
## compute PCA
def PCA(data):
    x_mean = data.mean(axis = 1)
    data_center = data - x_mean.reshape(-1,1)

    eigenvalue , eigenvector = np.linalg.eig(data_center.T@data_center)

    ## sort the eigenvalue from largest to smallest
```

```
    sort_idx = np.argsort(eigenvalue)[::-1]
    eigenvalue = eigenvalue[sort_idx]

    ## transform the data by projecting it onto the space of N eigenfaces
    eigenvector = data_center@eigenvector[:,sort_idx]
    norm = np.linalg.norm(eigenvector , axis = 0)
    eigenvector = eigenvector / norm

    return eigenvalue , eigenvector , x_mean.reshape(-1,1)
```

[6]:
```
## show eigenfaces from top N eigenvectors
def show_eigenface(vec , num , H , W):
    vec = vec[:,:num].reshape(H , W , -1)
    row , col = num//5 , 5

    figure , axis = plt.subplots(row , col , figsize = (15,15))

    for i in range(row):
        for j in range(col):
            axis[i ,j].imshow(vec[:,:,j+i*5] , cmap = "gray")

## show reconstructed faces
def reconstructed_faces(x_train , vec , num , H , W):
    pick = np.random.randint(low = 0 , high = vec.shape[1] , size = num)
    recon_image = vec[:,pick].reshape(H , W , -1)
    origin_image = x_train[:,pick].reshape(H , W , -1)

    row , col = num//5 , 5
    figure , axis = plt.subplots(row , col , figsize = (15,5))
    figure_2 , axis_2 = plt.subplots(row , col , figsize = (15,5))

    for i in range(row):
        for j in range(col):
            axis[i , j].imshow(recon_image[:,:,j+i*5] , cmap = "gray")
            axis_2[i , j].imshow(origin_image[:,:,j+i*5] , cmap = "gray")
```

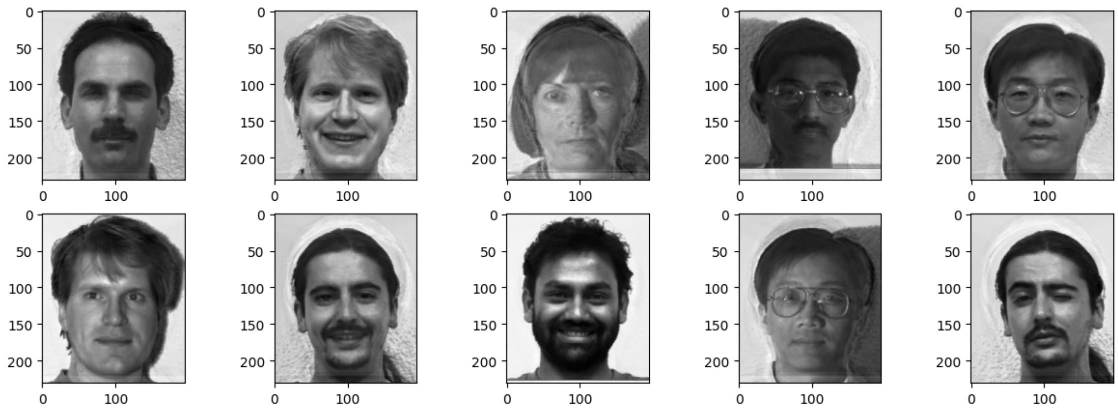
[60]:
```
dir = "/Users/cindychen/Documents/ML_HW07/Yale_Face_Database/Training"
H , W = 231 , 195
x_train , y_train = read_file(dir , H , W)

value , vector , x_mean = PCA(x_train)
show_eigenface(vector , 25 , H , W)
```

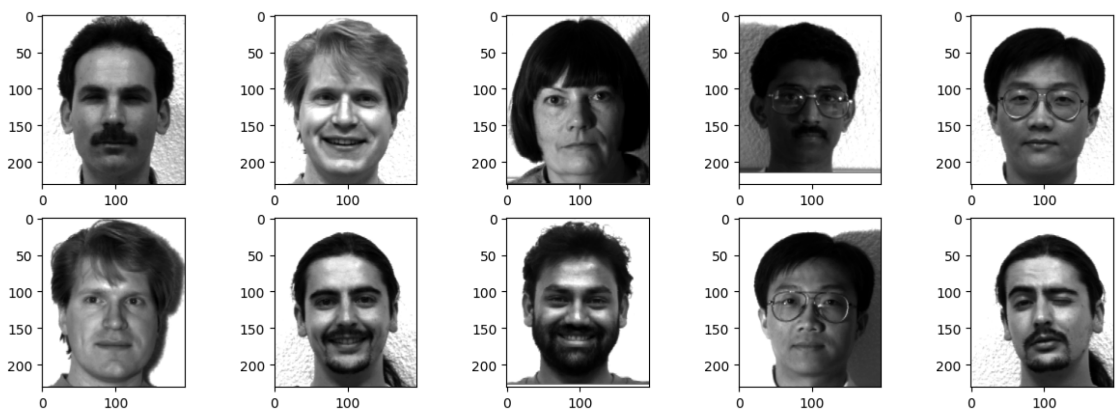


From the result of the eigenfaces, we can see that it almost capture the facial features of the images and the contour of different faces. We know that lighter regions correspond to higher variation, where darker regions correspond to little to no variation. Since that PCA focuses on the maximum variance, the output of the eigenfaces is not so bad that we can still recognize these are human faces.

0.1.4 PCA reconstructed faces

[8]:
```
## projecting the vector into lower dimension
vec = vector.T@(x_train - x_mean)
## then projecting back to real space
recon = vector@vec + x_mean
reconstructed_faces(x_train , recon , 10 , H , W)
```





For the reconstructed faces, the result also turns out good.

0.1.5 LDA fisherfaces

Different from PCA method, for LDA analysis we have to compute the within-class scatter and between-class scatter for computing eigenvalues and eigenvectors.

$$S_{within} = S_w = \sum_{i=1}^{c} \sum_{x_j \in X_c} (x_j - \mu_i)(x_j - \mu_i)^T$$

$$S_{between} = S_b = \sum_{i=1}^{c} N_i(\mu_i - \mu)(\mu_i - \mu)^T$$

where μ is the total mean of all the image, and μ_i is the mean of different class. There are 15 different people so c equals to 15, and in the training dataset each person has 9 different pictures $N_i = 9$.

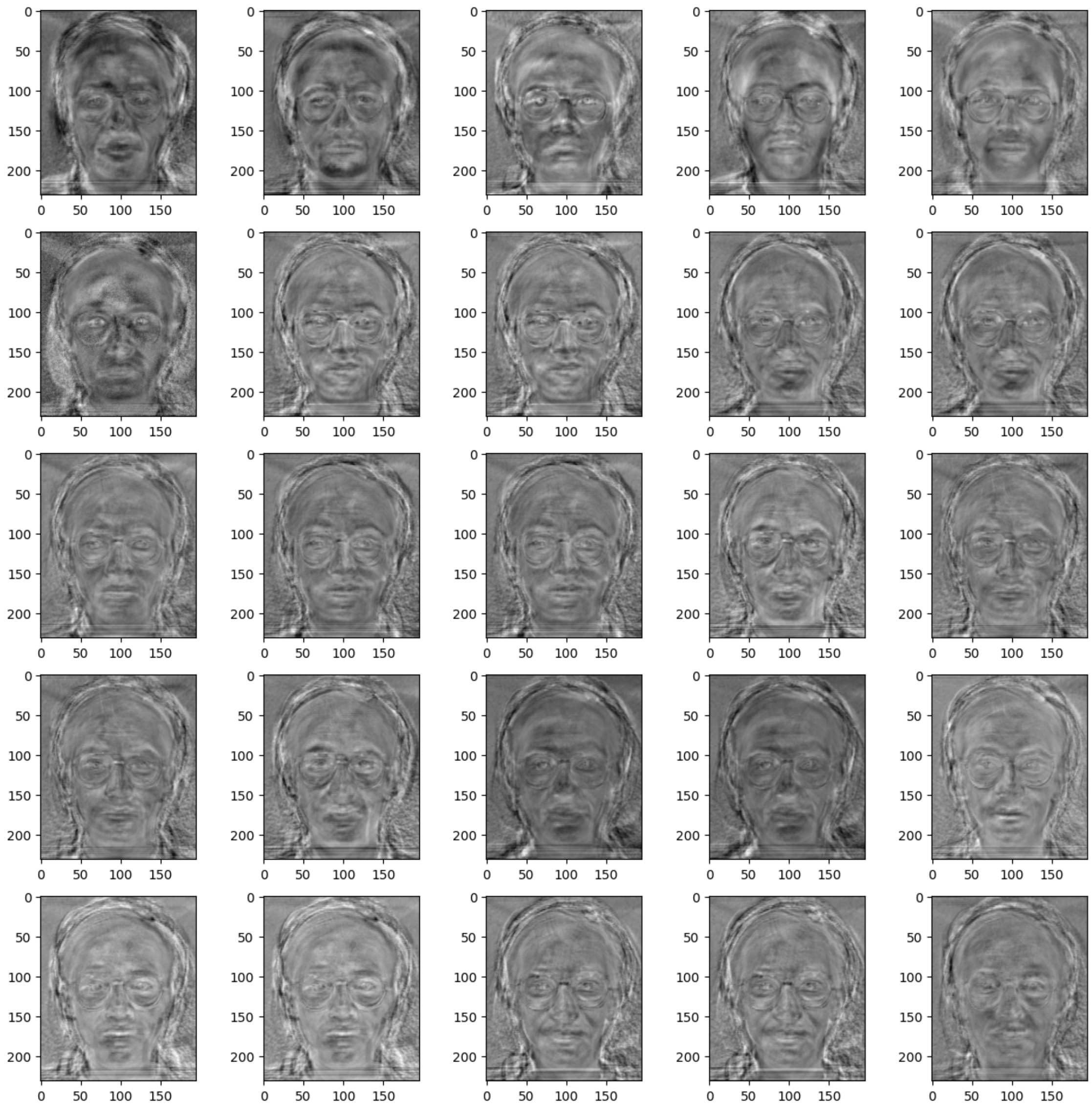
We can obtain the eigenvalues and eigenvectors by solving the equation of $S_w^{-1} S_b v_i = \lambda_i v_i$.

[57]:

```
def LDA(data , label):
    x_mean = data.mean(axis = 1).reshape(-1,1)

    ## compute within-class classes means
    class_mean = np.zeros((data.shape[0] , 15) , dtype = np.float32)
    for i in range(15):
        idx = np.argwhere(label == i).reshape(-1)
        class_mean[:,i] = np.mean(data[:,idx] , axis = 1)

    S_w = np.zeros((data.shape[0] , data.shape[0]) , dtype = np.float32)
    for i in range(data.shape[1]):
        diff = data[:,i].reshape(-1,1) - class_mean[:,label[i]].reshape(-1,1)
        S_w += diff@diff.T

    ## compute between-class means
    S_b = np.zeros((data.shape[0], data.shape[0]) , dtype = np.float32)
    for i in range(15):
        diff = class_mean[:,i].reshape(-1,1) - x_mean
        S_b += 9*diff@diff.T

    ## compute eigenvalue and eigenvector from within-class scatter and␣
 ↪between-class scatter
    eigenvalue , eigenvector = np.linalg.eig(np.linalg.inv(S_w)@S_b)

    ## sort the eigenvalue from largest to smallest
    sort_idx = np.argsort(eigenvalue)[::-1]
    eigenvalue = eigenvalue[sort_idx].real
    eigenvector = eigenvector[:,sort_idx].real
    norm = np.linalg.norm(eigenvector , axis = 0)
    eigenvector = eigenvector / norm

    return eigenvalue , eigenvector
```

[58]:

```
value , vector , X_mean = PCA(x_train)
vec = vector.T@(x_train - X_mean)

value_lda , vector_lda = LDA(vec, y_train)
vec_lda = vector@vector_lda

show_eigenface(vec_lda , 25 , H , W)
```

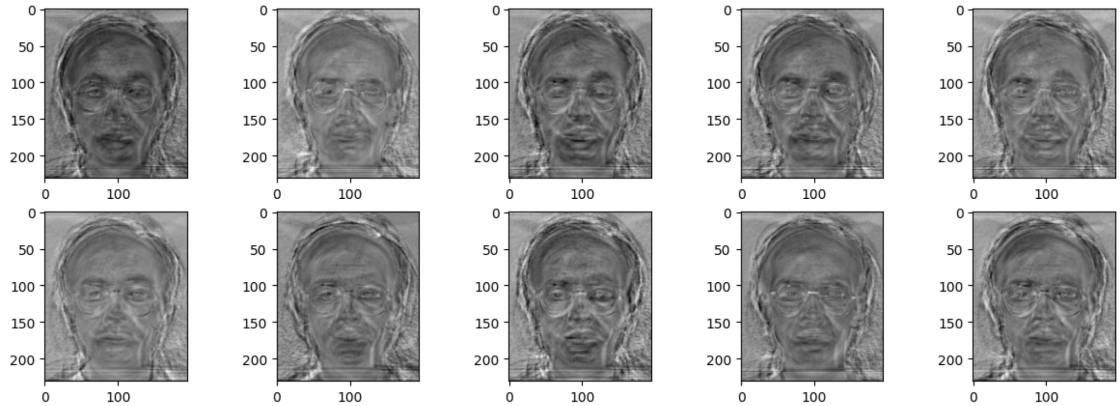


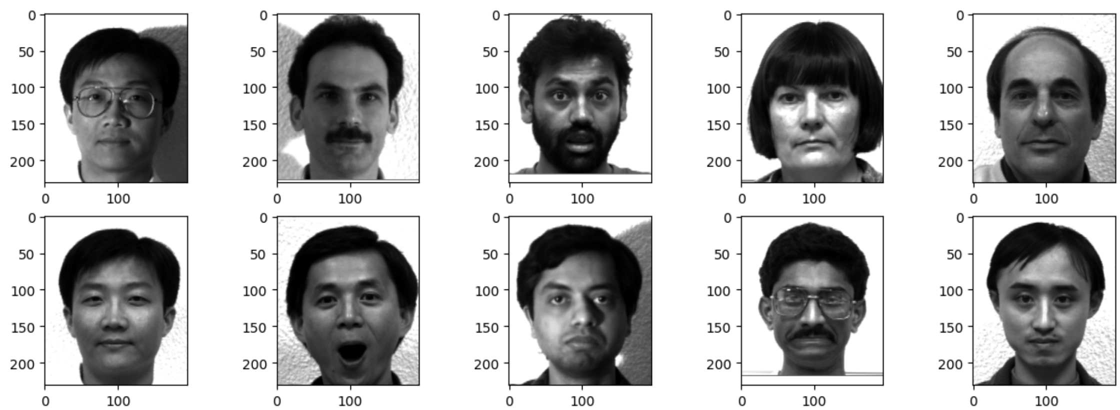
Using PCA method to reduce the dimension of images into 135 dimensions first for easy computation. And then use LDA to show the first 25 fisherfaces.

Since LDA analysis is more focus on the between classes and within classes different, the top 25 fisherfaces can only capture the contour of a human face. The facial features of the top 25 fisherfaces are not so clearly to be classify from each others.

0.1.6 LDA reconstructed faces

[59]:
```
vec_trans = vec_lda.T@x_train
recon_lda = vec_lda@vec_trans + X_mean
reconstructed_faces(x_train , recon_lda , 10 , H , W)
```





For the reconstructed result, the LDA analysis also give a bad result different from the PCA method.

0.1.7 Part II

0.1.8 Face recognition without kernel

[176]:
```python
def classification(x_test , y_test , x_train , y_train , k):
    predict = np.zeros(len(y_test))
    for i in range(y_test.shape[0]):
        dist = np.zeros(x_train.shape[1])
        for j in range(x_train.shape[1]):
            dist[j] = np.sum(np.square(x_test[:,i] - x_train[:,j]))
        sort_idx = np.argsort(dist)
        n_neighbor = y_train[sort_idx[:k]]
        neighbor , count = np.unique(n_neighbor , return_counts = True)
        predict[i] = neighbor[np.argsort(-count)[0]]

    acc = np.sum(predict == y_test) / len(y_test)
```

```
    print("K:{} , accuracy = {:3f}".format(k , acc))

    return predict
```

0.1.9 Face recognition for pca method

[183]:
```
dir_test = "/Users/cindychen/Documents/ML_HW07/Yale_Face_Database/Testing"
H , W = 231 , 195
x_test , y_test = read_file(dir_test , H , W)

value , vector , x_mean = PCA(x_train)

## turn the test dataset into low dimension
z_test = vector.T@(x_test - x_mean)
x_proj = vector.T@(x_train - x_mean)

for i in range(1,16,2):
    predict = classification(z_test , y_test , x_proj , y_train , i)
```

```
K:1 , accuracy = 0.866667
K:3 , accuracy = 0.833333
K:5 , accuracy = 0.866667
K:7 , accuracy = 0.866667
K:9 , accuracy = 0.766667
K:11 , accuracy = 0.866667
K:13 , accuracy = 0.933333
K:15 , accuracy = 0.833333
```

From the result, we can see that when k = 13 we have the highest accuracy.

0.1.10 Face recognition for lda method

[181]:
```
value , vector , X_mean = PCA(x_train)
vec = vector.T@(x_train - X_mean)

value_lda , vector_lda = LDA(vec, y_train)
vec_lda = vector@vector_lda
vec_trans = vec_lda.T@x_train

## turn the test dataset into low dimension
z_test = vec_lda.T@(x_test - X_mean)

for i in range(1,16,2):
    predict = classification(z_test , y_test , vec_trans , y_train , i)
```

```
K:1 , accuracy = 0.266667
K:3 , accuracy = 0.133333
K:5 , accuracy = 0.600000
```

```
K:7 , accuracy = 0.800000
K:9 , accuracy = 0.766667
K:11 , accuracy = 0.733333
K:13 , accuracy = 0.766667
K:15 , accuracy = 0.833333
```

From the result, we can see that when k = 15 we have the highest accuracy.

0.1.11 Part III

0.1.12 Face recognition with kernel

[215]:
```
def linearkernel(x):
    return x.T@x

def rbfkernel(x , gamma):
    return np.exp(-gamma * distance.cdist(x , x , "sqeuclidean"))
```

[245]:
```
def kernelPCA(data , dims , type):
    if type == "linear":
        kernel = linearkernel(data)
    else:
        kernel = rbfkernel(data , 1e-4)

    n = kernel.shape[0]
    one = np.ones((n, n), dtype=np.float64) / n
    kernel = kernel - one @ kernel - kernel @ one + one @ kernel @ one
    eigen_val, eigen_vec = np.linalg.eigh(kernel)

    for i in range(eigen_vec.shape[1]):
        eigen_vec[:, i] = eigen_vec[:, i] / np.linalg.norm(eigen_vec[:, i])

    idx = np.argsort(eigen_val)[::-1]
    W = eigen_vec[:, idx][: , :].real

    return kernel @ W.T
```

[246]:
```
dir = "/Users/cindychen/Documents/ML_HW07/Yale_Face_Database/Training"
dir_test = "/Users/cindychen/Documents/ML_HW07/Yale_Face_Database/Testing"
H , W = 50 , 50
x_train , y_train = read_file(dir , H , W)
x_test , y_test = read_file(dir_test , H , W)

data = np.hstack((x_train, x_test))

new_coor = kernelPCA(data , 25 , "linear")
new_train = new_coor[:, :len(y_train)]
new_test = new_coor[:,len(y_train):]
```

[253]:
```
preidct = classification(new_test, y_test, new_train, y_train, 5)
```

```
K:5 , accuracy = 0.066667
```

0.2 t-SNE

[254]:
```
import sys, os
import numpy as np
import seaborn as sns
import scipy.spatial.distance
import matplotlib.pyplot as plt
```

0.2.1 Part I

0.2.2 t-SNE and symmetric SNE

0.2.3 SNE (Symmetric-SNE)

SNE works as dimensionality reduction by converting the high-dimensional euclidean distances into conditional probability that represents similarities. So, we project high-dimensional data into low-dimensional data which look like the cost function of KL divergence.

$$C = \sum_i KL(P_i||Q_i) = \sum_i \sum_j p_{j|i} log \frac{p_{j|i}}{q_{j|i}}$$

We want the value of KL divergence to be as small as possible, therefore we can get high similarities in low dimensionality space. The formula of p_{ij} and q_{ij} is shown below.

$$p_{ij} = \frac{exp(-||x_i - x_j||^2/(x\sigma^2))}{\sum_{k \neq l} exp(-||x_l - x_k||^2/(x\sigma^2))}$$

$$q_{ij} = \frac{exp(-||y_i - y_j||^2)}{\sum_{k \neq l} exp(-||y_l - y_k||^2)}$$

0.2.4 t-SNE (t-Distributed SNE)

Different from symmetric SNE, t-SNE using student's t distribution probability function in low dimensionality space.

$$p_{ij} = \frac{exp(-||x_i - x_j||^2/(x\sigma^2))}{\sum_{k \neq l} exp(-||x_l - x_k||^2/(x\sigma^2))}$$

$$q_{ij} = \frac{(1 + ||y_i - y_j||^2)^{-1}}{\sum_{k \neq l} (1 + ||y_i - y_j||^2)^{-1}}$$

[ ]:
```
for iter in range(max_iter):
    # Compute pairwise affinities
    sum_Y = np.sum(np.square(Y), 1)
    num = -2. * np.dot(Y, Y.T)
    num = 1. / (1. + np.add(np.add(num, sum_Y).T, sum_Y))
```

```
    num[range(n), range(n)] = 0.
    Q = num / np.sum(num)
    Q = np.maximum(Q, 1e-12)

    # Compute gradient
    PQ = P - Q
    for i in range(n):
        dY[i, :] = np.sum(np.tile(PQ[:, i] * num[:, i], (no_dims, 1)).T * (Y[i,␣
↪:] - Y), 0)
```

From the reference code, we can see that it uses conditional probability on low dimensional space q. The formula using t-distribution formula to achieve the conditional probability function.

$$q_{ij} = \frac{(1 + ||y_i - y_j||^2)^{-1}}{\sum_{k \neq l}(1 + ||y_i - y_j||^2)^{-1}}$$

And for computing the gradient in t-SNE we use the formula down below.

$$\frac{\delta C}{\delta y_i} = 4 \sum_j (p_{ij} - q_{ij})(y_i - y_j)(1 + ||y_i - y_j||^2)^{-1}$$

Now we can change the code to symmetric SNE.

```
[ ]: for iter in range(max_iter):
         # Compute pairwise affinities
         sum_Y = np.sum(np.square(Y), 1)
         num = -2. * np.dot(Y, Y.T)
         num = np.exp(-1.*np.add(np.add(num , sum_Y).T , sum_Y))

         num[range(n), range(n)] = 0.
         Q = num / np.sum(num)
         Q = np.maximum(Q, 1e-12)

         # Compute gradient
         PQ = P - Q
         for i in range(n):
             dY[i, :] = np.dot(PQ[i , :] , Y[i,:] - Y)
```

That formula using an exponent component with refers to the Gaussian or normal distribution formula to achieve the conditional probability function.

$$q_{ij} = \frac{exp(-||y_i - y_j||^2)}{\sum_{k \neq l} exp(-||y_l - y_k||^2)}$$

And for computing the gradient in SNE we use the formula down below.

$$\frac{\delta C}{\delta y_i} = 2 \sum_j (p_{ij} - q_{ij})(y_i - y_j)$$

0.2.5 Part II

[309]:
```
def plotResult(Y, labels, idx, interval, method, perplexity):
    plt.clf()
    scatter = plt.scatter(Y[:, 0], Y[:, 1], 20, labels)
    plt.legend(*scatter.legend_elements(), loc='lower left', title='Digit')
    plt.title(f'{method}, perplexity: {perplexity}, iteration: {idx}')
    plt.tight_layout()
    if interval:
        plt.savefig(f'{method}_{perplexity}_{idx // interval}.png')
    else:
        plt.savefig(f'{method}_{perplexity}_{idx}.png')
```

[310]:
```
def Hbeta(D=np.array([]), beta=1.0):
    # Compute P-row and corresponding perplexity
    P = np.exp(-D.copy() * beta)
    sumP = sum(P)
    H = np.log(sumP) + beta * np.sum(D * P) / sumP
    P = P / sumP
    return H, P

def x2p(X, tol, perplexity):
    print('Computing pairwise distances...')
    (n, d) = X.shape
    D = distance.cdist(X, X, 'sqeuclidean')
    P = np.zeros((n, n))
    beta = np.ones((n, 1))
    logU = np.log(perplexity)

    for i in range(n):
        if i % 500 == 0:
            print(f'Computing P-values for point {i} of {n}...')
        # Compute the Gaussian kernel and entropy for the current precision
        betamin = -np.inf
        betamax = np.inf
        Di = D[i, np.concatenate((np.r_[0:i], np.r_[i+1:n]))]
        (H, thisP) = Hbeta(Di, beta[i])

        # Evaluate whether the perplexity is within tolerance
        Hdiff = H - logU
        tries = 0
        while np.abs(Hdiff) > tol and tries < 50:
            # If not, increase or decrease precision
            if Hdiff > 0:
                betamin = beta[i].copy()
                if betamax == np.inf or betamax == -np.inf:
                    beta[i] = beta[i] * 2
                else:
                    beta[i] = (beta[i] + betamax) / 2
```

```
            else:
                betamax = beta[i].copy()
                if betamin == np.inf or betamin == -np.inf:
                    beta[i] = beta[i] / 2
                else:
                    beta[i] = (beta[i] + betamin) / 2
            (H, thisP) = Hbeta(Di, beta[i])
            Hdiff = H - logU
            tries += 1
        P[i, np.concatenate((np.r_[0:i], np.r_[i+1:n]))] = thisP
    print(f'Mean value of sigma: {np.mean(np.sqrt(1 / beta))}')
    return P
```

[294]:
```
def pca(X, dims):
    print('Preprocessing the data using PCA...')
    (n, d) = X.shape
    X = X - np.tile(np.mean(X, axis=0), (n, 1))
    (eigen_val, eigen_vec) = np.linalg.eig(np.dot(X.T, X))
    Y = np.dot(X, eigen_vec[:, 0:dims])
    return Y
```

[311]:
```
def sne(X, dims, init_dims, perplexity, labels, method, interval):
    X = pca(X, init_dims).real
    (n, d) = X.shape
    Y = np.random.randn(n, dims)
    dY = np.zeros((n, dims))
    iY = np.zeros((n, dims))
    gains = np.ones((n, dims))
    max_iter = 1000
    initial_momentum = 0.5
    final_momentum = 0.8
    eta = 500
    min_gain = 0.01

    # Compute P-values
    P = x2p(X, 1e-5, perplexity)
    P = P + np.transpose(P)
    P = P / np.sum(P)
    P = P * 4                           # early exaggeration
    P = np.maximum(P, 1e-12)

    for itr in range(max_iter):
        # Compute pairwise affinities
        if method == 'tsne':
            num = 1 / (1 + scipy.spatial.distance.cdist(Y, Y, 'sqeuclidean'))
        else:
            num = np.exp(-1 * scipy.spatial.distance.cdist(Y, Y, 'sqeuclidean'))
```

```python
        num[range(n), range(n)] = 0
        Q = num / np.sum(num)
        Q = np.maximum(Q, 1e-12)

        # Compute gradient
        PQ = P - Q
        for i in range(n):
            if method == 'tsne':
                dY[i, :] = np.sum(np.tile(PQ[:, i] * num[:, i], (dims, 1)).T *␣
↪(Y[i, :] - Y), axis=0)
            else:
                dY[i, :] = np.sum(np.tile(PQ[:, i], (dims, 1)).T * (Y[i, :] -␣
↪Y), axis=0)

        # Perform the update
        if itr < 20:
            momentum = initial_momentum
        else:
            momentum = final_momentum
        gains = (gains + 0.2) * ((dY > 0) != (iY > 0)) + (gains * 0.8) * ((dY >␣
↪0) == (iY > 0))
        gains[gains < min_gain] = min_gain
        iY = momentum * iY - eta * (gains * dY)
        Y = Y + iY
        Y = Y - np.tile(np.mean(Y, 0), (n, 1))

        if itr % interval == 0:
            plotResult(Y, labels, itr, interval, method, perplexity)

        # Compute current value of cost function
        if (itr + 1) % 10 == 0:
            C = np.sum(P * np.log(P / Q))
            print(f'Iteration {itr + 1}: error is {C}')

        # Stop lying about P-values
        if itr == 100:
            P = P / 4

    return Y, P, Q
```

[305]:

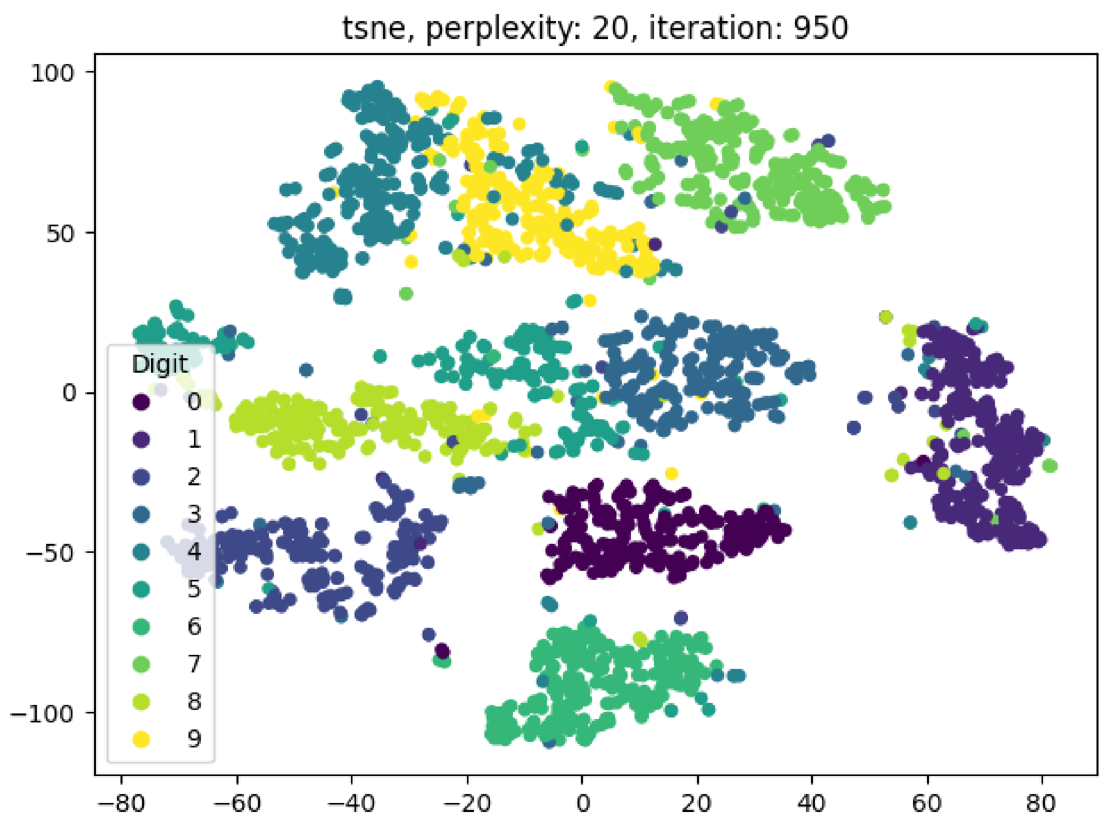
```python
X = np.loadtxt("/Users/cindychen/Documents/tsne_python/mnist2500_X.txt")
labels = np.loadtxt("/Users/cindychen/Documents/tsne_python/mnist2500_labels.
↪txt")
Y, P, Q = sne(X, 2, 50, 20, labels, "tsne", 50)
```

Preprocessing the data using PCA…

```
Computing pairwise distances…
Computing P-values for point 0 of 2500…
Computing P-values for point 500 of 2500…
Computing P-values for point 1000 of 2500…
Computing P-values for point 1500 of 2500…
Computing P-values for point 2000 of 2500…
Mean value of sigma: 2.386596621341598
Iteration 10: error is 23.67132573898476
Iteration 20: error is 21.006746482086506
Iteration 30: error is 17.928447707923528
Iteration 40: error is 16.849853702899942
Iteration 50: error is 16.517391978695585
Iteration 60: error is 16.346203264091578
Iteration 70: error is 16.201771292663675
Iteration 80: error is 16.114435558665967
Iteration 90: error is 16.119297818135333
Iteration 100: error is 16.07352492632593
Iteration 110: error is 2.359894221382359
Iteration 120: error is 2.1024957559192043
Iteration 130: error is 1.911612786346212
Iteration 140: error is 1.76798679230404
Iteration 150: error is 1.6581414949190356
Iteration 160: error is 1.5725964471737663
Iteration 170: error is 1.5046738350489601
Iteration 180: error is 1.4492885590545208
Iteration 190: error is 1.4033220088339353
Iteration 200: error is 1.3648582211649107
Iteration 210: error is 1.3321654663462494
Iteration 220: error is 1.3041149491252066
Iteration 230: error is 1.2801081830358978
Iteration 240: error is 1.2593594631174372
Iteration 250: error is 1.2411530933586046
Iteration 260: error is 1.2249465510358981
Iteration 270: error is 1.2104190365991254
Iteration 280: error is 1.1973626142745963
Iteration 290: error is 1.1855571437792065
Iteration 300: error is 1.1749052741309876
Iteration 310: error is 1.1652659013560198
Iteration 320: error is 1.1564741420664433
Iteration 330: error is 1.1483868387448166
Iteration 340: error is 1.1408372847607524
Iteration 350: error is 1.133947706552321
Iteration 360: error is 1.1276127368933444
Iteration 370: error is 1.1217653938910277
Iteration 380: error is 1.1163470854282953
Iteration 390: error is 1.1112827634206746
Iteration 400: error is 1.1065386527416605
Iteration 410: error is 1.1020903174400203
```

```
Iteration 420: error is 1.0979096383011826
Iteration 430: error is 1.093957971394924
Iteration 440: error is 1.0902187120263402
Iteration 450: error is 1.0867305458458765
Iteration 460: error is 1.0834656316758111
Iteration 470: error is 1.0803838780399984
Iteration 480: error is 1.0774527031218135
Iteration 490: error is 1.0746667973791841
Iteration 500: error is 1.0720214803783032
Iteration 510: error is 1.0695190650449897
Iteration 520: error is 1.0671523379937624
Iteration 530: error is 1.0649078394269276
Iteration 540: error is 1.0627718724935946
Iteration 550: error is 1.060741843053611
Iteration 560: error is 1.0588043750352023
Iteration 570: error is 1.0569461339092765
Iteration 580: error is 1.0551644607162307
Iteration 590: error is 1.053470939819303
Iteration 600: error is 1.0518638454865366
Iteration 610: error is 1.05032154726007
Iteration 620: error is 1.048704877114935
Iteration 630: error is 1.0472654806197728
Iteration 640: error is 1.045902626951627
Iteration 650: error is 1.044601098721346
Iteration 660: error is 1.0433833165851296
Iteration 670: error is 1.0422206440106763
Iteration 680: error is 1.0411034909838126
Iteration 690: error is 1.0400294120847173
Iteration 700: error is 1.0389910022420588
Iteration 710: error is 1.0379809794241661
Iteration 720: error is 1.0369994353972192
Iteration 730: error is 1.0360487799898235
Iteration 740: error is 1.0351271841760947
Iteration 750: error is 1.0342285550108419
Iteration 760: error is 1.0333536615293368
Iteration 770: error is 1.0325092477497635
Iteration 780: error is 1.0316915630643821
Iteration 790: error is 1.0309001757078102
Iteration 800: error is 1.0301297465438908
Iteration 810: error is 1.0293810679661177
Iteration 820: error is 1.028656486931813
Iteration 830: error is 1.0279537717610412
Iteration 840: error is 1.0272700398005716
Iteration 850: error is 1.0266051614501368
Iteration 860: error is 1.0259564797829306
Iteration 870: error is 1.0253225658040832
Iteration 880: error is 1.0247046672459952
Iteration 890: error is 1.0241040925916305
```

```
Iteration 900: error is 1.0235150697773256
Iteration 910: error is 1.0229345794892042
Iteration 920: error is 1.0223814464085543
Iteration 930: error is 1.0218453761364648
Iteration 940: error is 1.0213239217310741
Iteration 950: error is 1.0208165849046702
Iteration 960: error is 1.0203208157564765
Iteration 970: error is 1.0198361634534885
Iteration 980: error is 1.0193613740816974
Iteration 990: error is 1.0188999862596042
Iteration 1000: error is 1.0184541850233346
```



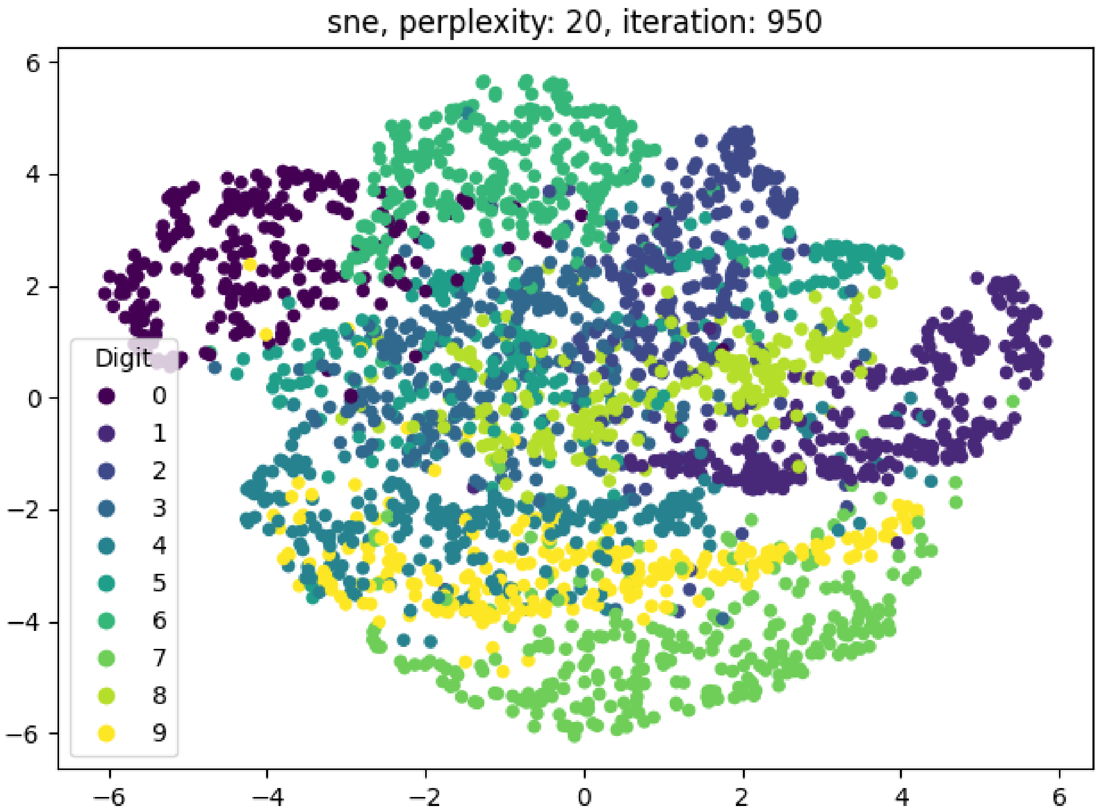
[306]:
```
Y_sne, P_sne, Q_sne = sne(X, 2, 50, 20, labels, "sne", 50)
```

```
Preprocessing the data using PCA…
Computing pairwise distances…
Computing P-values for point 0 of 2500…
Computing P-values for point 500 of 2500…
Computing P-values for point 1000 of 2500…
Computing P-values for point 1500 of 2500…
Computing P-values for point 2000 of 2500…
```

```
Mean value of sigma: 2.386596621341598
Iteration 10: error is 23.542497163274312
Iteration 20: error is 19.228628997145915
Iteration 30: error is 17.850320450208855
Iteration 40: error is 17.292568389572416
Iteration 50: error is 17.077153909929155
Iteration 60: error is 16.882856075529595
Iteration 70: error is 16.835364265801743
Iteration 80: error is 16.830541683416236
Iteration 90: error is 16.819417711893397
Iteration 100: error is 16.812856968083434
Iteration 110: error is 2.2370328917818503
Iteration 120: error is 2.0844610699864483
Iteration 130: error is 2.084956799958217
Iteration 140: error is 2.098392461892874
Iteration 150: error is 2.103495697042625
Iteration 160: error is 2.100586288223392
Iteration 170: error is 2.098271604863654
Iteration 180: error is 2.0977054689439067
Iteration 190: error is 2.0977031164580326
Iteration 200: error is 2.097446930096143
Iteration 210: error is 2.0971959374386264
Iteration 220: error is 2.09711771823751
Iteration 230: error is 2.0971023776515247
Iteration 240: error is 2.097096273989041
Iteration 250: error is 2.0970959400646567
Iteration 260: error is 2.097096397752981
Iteration 270: error is 2.097094853437034
Iteration 280: error is 2.097093567327207
Iteration 290: error is 2.0970927984661594
Iteration 300: error is 2.09709257178408
Iteration 310: error is 2.0970924595195424
Iteration 320: error is 2.0970923762264886
Iteration 330: error is 2.097092346989505
Iteration 340: error is 2.0970923515778197
Iteration 350: error is 2.09709235715197
Iteration 360: error is 2.09709235463852
Iteration 370: error is 2.097092353761787
Iteration 380: error is 2.0970923572369133
Iteration 390: error is 2.097092360460746
Iteration 400: error is 2.097092362846917
Iteration 410: error is 2.0970923635559964
Iteration 420: error is 2.097092363534745
Iteration 430: error is 2.0970923634855394
Iteration 440: error is 2.0970923634656047
Iteration 450: error is 2.0970923634600407
Iteration 460: error is 2.0970923634614667
Iteration 470: error is 2.097092363459426
```

```
Iteration 480: error is 2.0970923634540544
Iteration 490: error is 2.0970923634492378
Iteration 500: error is 2.0970923634469574
Iteration 510: error is 2.09709236344655
Iteration 520: error is 2.097092363446687
Iteration 530: error is 2.097092363446714
Iteration 540: error is 2.0970923634466856
Iteration 550: error is 2.097092363446667
Iteration 560: error is 2.0970923634466585
Iteration 570: error is 2.0970923634466563
Iteration 580: error is 2.097092363446652
Iteration 590: error is 2.0970923634466514
Iteration 600: error is 2.09709236344665
Iteration 610: error is 2.09709236344665
Iteration 620: error is 2.097092363446652
Iteration 630: error is 2.097092363446653
Iteration 640: error is 2.0970923634466536
Iteration 650: error is 2.0970923634466545
Iteration 660: error is 2.097092363446654
Iteration 670: error is 2.097092363446654
Iteration 680: error is 2.097092363446654
Iteration 690: error is 2.097092363446654
Iteration 700: error is 2.097092363446654
Iteration 710: error is 2.097092363446654
Iteration 720: error is 2.097092363446654
Iteration 730: error is 2.097092363446654
Iteration 740: error is 2.097092363446654
Iteration 750: error is 2.097092363446654
Iteration 760: error is 2.097092363446654
Iteration 770: error is 2.097092363446654
Iteration 780: error is 2.097092363446654
Iteration 790: error is 2.097092363446654
Iteration 800: error is 2.097092363446654
Iteration 810: error is 2.0970923634466536
Iteration 820: error is 2.097092363446654
Iteration 830: error is 2.097092363446654
Iteration 840: error is 2.097092363446654
Iteration 850: error is 2.097092363446654
Iteration 860: error is 2.097092363446654
Iteration 870: error is 2.097092363446654
Iteration 880: error is 2.097092363446654
Iteration 890: error is 2.097092363446654
Iteration 900: error is 2.097092363446654
Iteration 910: error is 2.097092363446654
Iteration 920: error is 2.097092363446654
Iteration 930: error is 2.097092363446654
Iteration 940: error is 2.097092363446654
Iteration 950: error is 2.097092363446654
```

```
Iteration 960: error is 2.097092363446654
Iteration 970: error is 2.097092363446654
Iteration 980: error is 2.097092363446654
Iteration 990: error is 2.097092363446654
Iteration 1000: error is 2.097092363446654
```



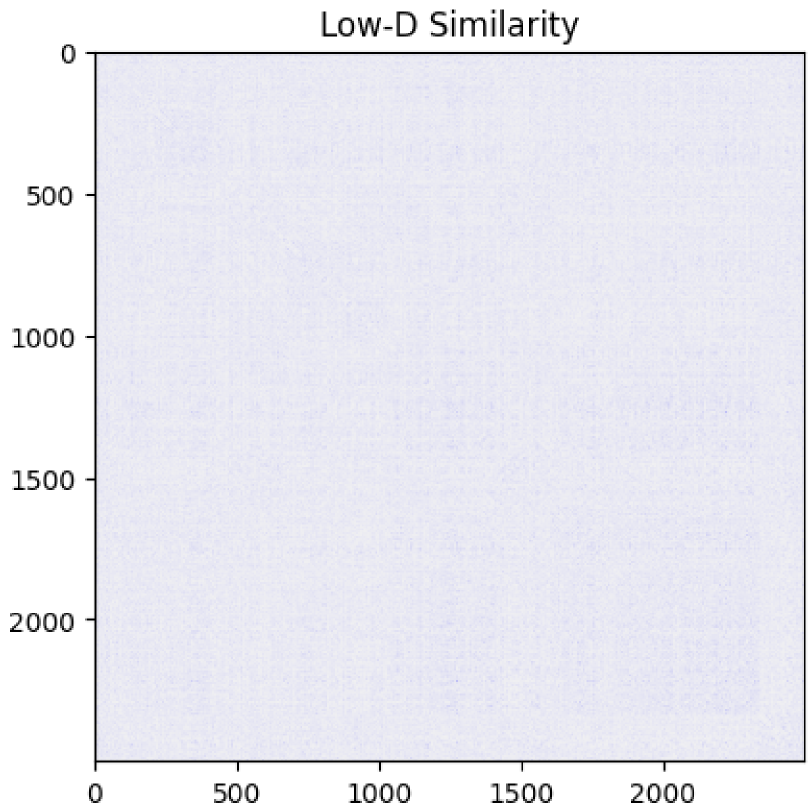
From the result, we can see that the scatter plot of t-SNE is not so crowded, every group is separated from each other.

0.2.6 Part III

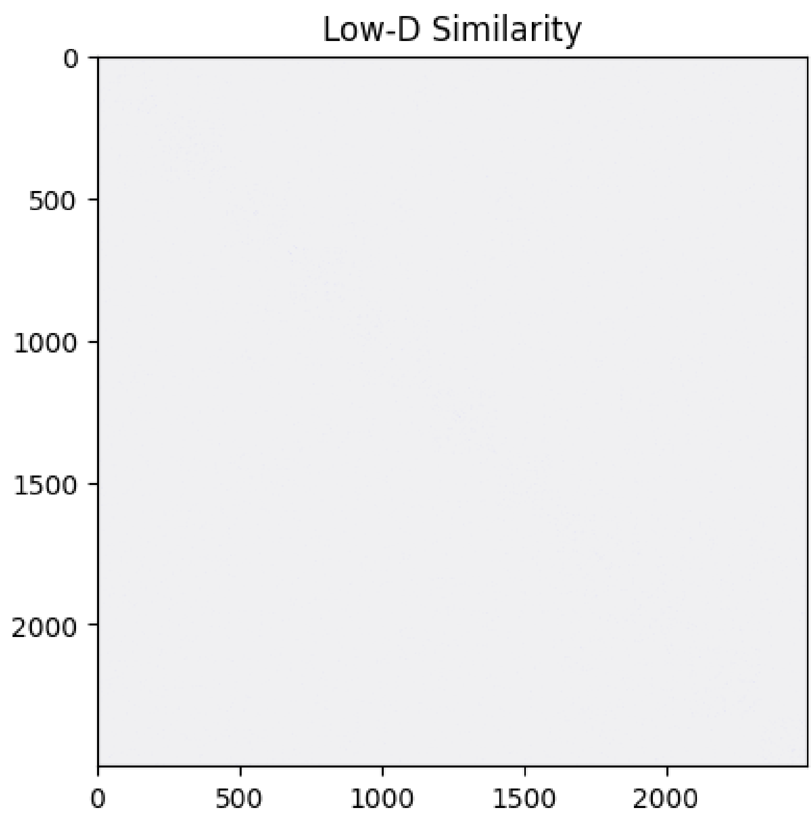
[298]:
```
def plotHighDLowD(P, Q, method, perplexity):
    pal = sns.light_palette('blue', as_cmap=True)
    plt.clf()
    plt.title('High-D Similarity')
    plt.imshow(P, cmap=pal)
    plt.savefig(f'./{method}_{perplexity}/High_D.png')

    plt.clf()
    plt.title('Low-D Similarity')
    plt.imshow(Q, cmap=pal)
```

```
    plt.savefig(f'./{method}_{perplexity}/Low_D.png')
```

[302]:
```
plotHighDLowD(P_sne, Q_sne, 'sne', 20)
```



[301]:
```
plotHighDLowD(P, Q, 'tsne', 20)
```



0.2.7 Part IIII

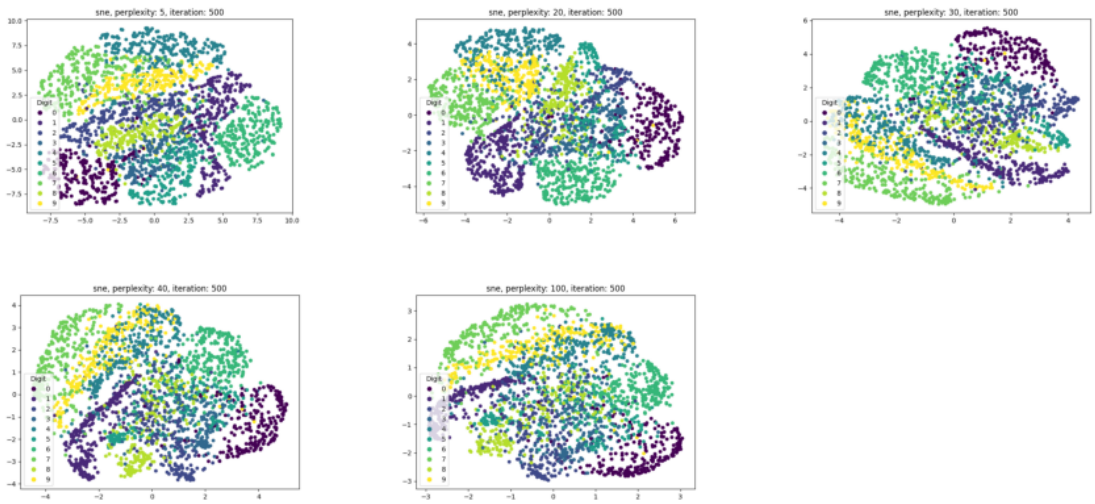
0.2.8 Play with different perplexity

[415]:
```python
image1 = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_ /sne_5_1.
 ↪png")
image2 = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_ /sne_20_1.
 ↪png")
image3 = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_ /sne_30_1.
 ↪png")
image4 = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_ /sne_40_1.
 ↪png")
image5 = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_ /sne_100_1.
 ↪png")
file = [image1 , image2 , image3 , image4 , image5]
```

0.2.9 Symmetric SNE

From the result of different perplexity, it shows that there is no different between each plots.

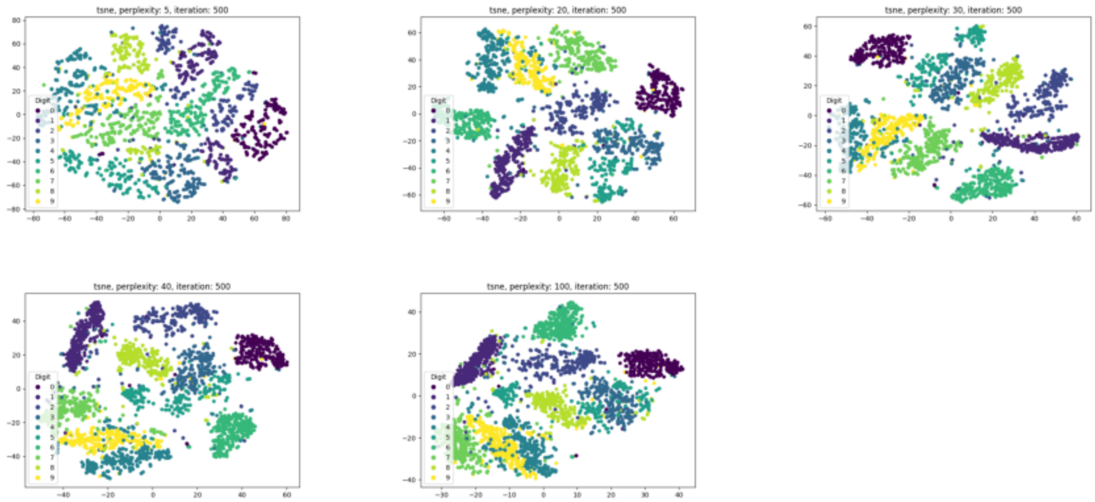
[416]:
```
fig = plt.figure(figsize=(11, 5))
for i in range(len(file)):
    fig.add_subplot(2, 3, i+1)
    plt.imshow(file[i])
    plt.axis('off')
```



[413]:
```
image1_t = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_  /tsne_5_1.
 ↪png")
image2_t = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_  /tsne_20_1.
 ↪png")
image3_t = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_  /tsne_30_1.
 ↪png")
image4_t = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_  /tsne_40_1.
 ↪png")
image5_t = Image.open("/Users/cindychen/Documents/ML_HW7_310657012_  /
 ↪tsne_100_1.png")
file_t = [image1_t , image2_t , image3_t , image4_t , image5_t]
```

0.2.10 t-SNE

[414]:
```
fig = plt.figure(figsize=(11, 5))
for i in range(len(file_t)):
    fig.add_subplot(2, 3, i+1)
    plt.imshow(file_t[i])
    plt.axis('off')
```


tsne, perplexity: 5, iteration: 500
tsne, perplexity: 20, iteration: 500
tsne, perplexity: 30, iteration: 500
tsne, perplexity: 40, iteration: 500
tsne, perplexity: 100, iteration: 500

[ ]: