3、

$$P(\theta|x)=\frac{P(x|\theta)\cdot P(\theta)}{P(x)}=\frac{P(x|\theta)\cdot P(\theta)}{\int_\theta P(x|\theta)P(\theta)d\theta}$$

令 $P(x|\theta)=\binom{n}{x}\theta^x(1-\theta)^{n-x}$

$P(\theta)=\frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}=Beta(\alpha,\beta)$

则 $$P(\theta|x)=\frac{\binom{n}{x}\theta^x(1-\theta)^{n-x}\frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\int_\theta\binom{n}{x}\theta^x(1-\theta)^{n-x}\frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}d\theta}$$

$$=\frac{\frac{\binom{n}{x}}{B(\alpha,\beta)}\theta^{x+\alpha-1}(1-\theta)^{n+\beta-x-1}}{\frac{\binom{n}{x}}{B(\alpha,\beta)}\int_0^1\theta^{x+\alpha-1}(1-\theta)^{n+\beta-x-1}d\theta}$$

$$=\frac{\theta^{x+\alpha-1}(1-\theta)^{n+\beta-x-1}}{B(x+\alpha,n+\beta-x)}=Beta(x+\alpha,n+\beta-x)$$