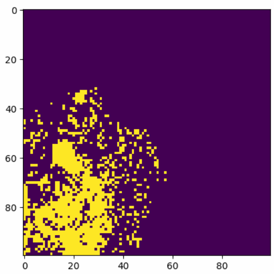
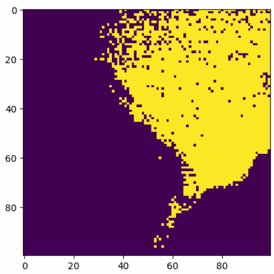
HW_06_陳芃辰

Part 1:

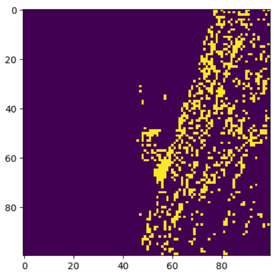
K-means clustering:



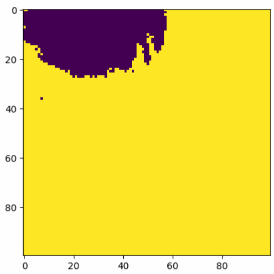
Random method



K-means++ method

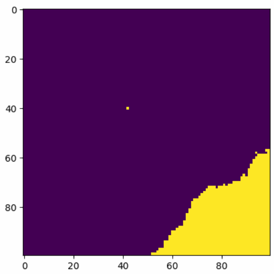


Random method

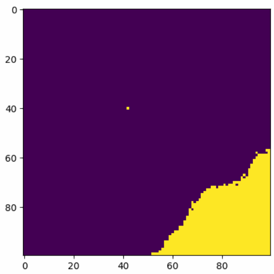


K-means++ method

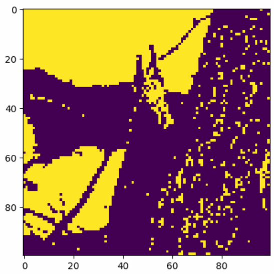
Spectral clustering:



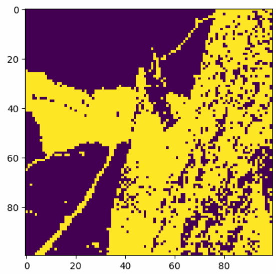
Normalize cut



Ratio cut



Normalize cut



Ratio cut

Part 2 & Part 3:

Clustering with k=2,3,4.

K-means with more clustering result:

Image1: random method

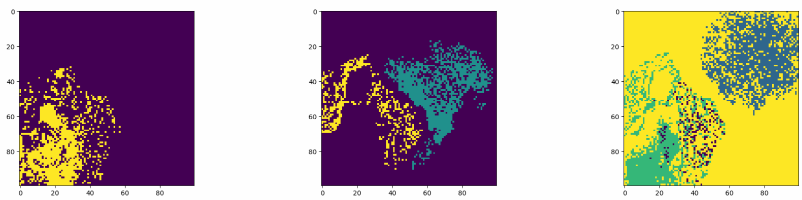


Image1: K-means++ method

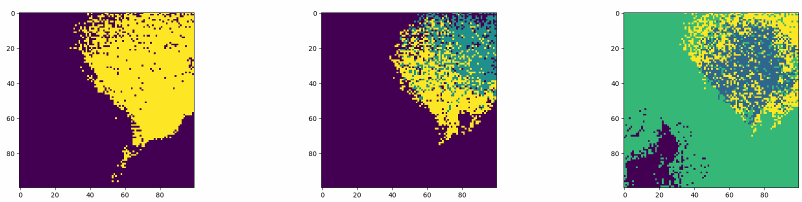


Image2: random method

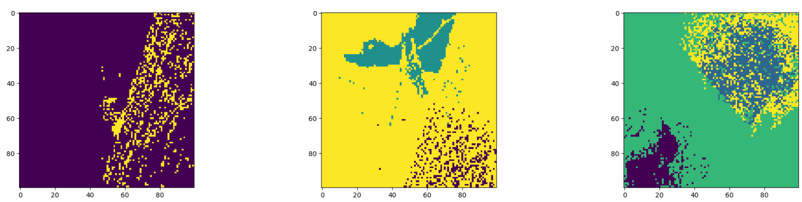
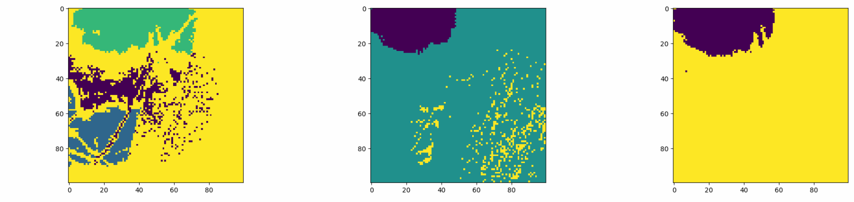


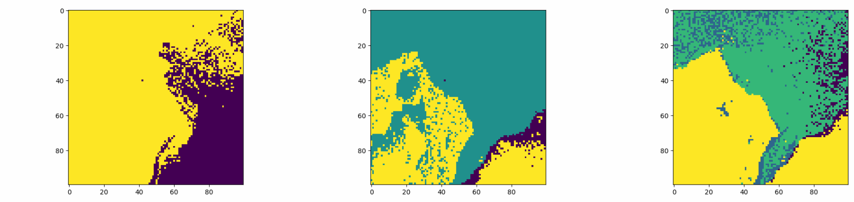
Image2: K-means++ method



Spectral Clustering:

Image1: Normalize cut

(random method)



(kmeans++ method)

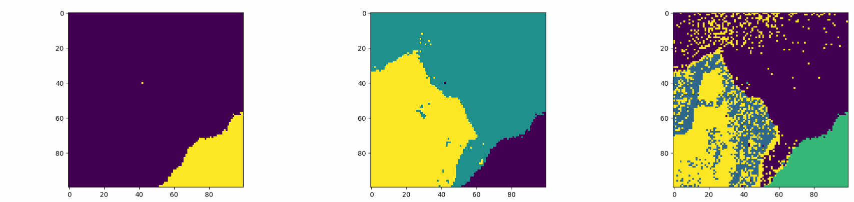
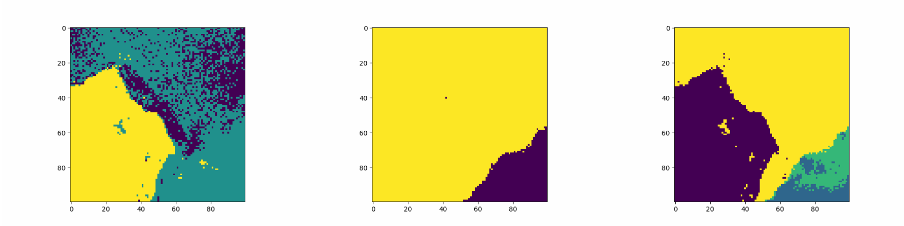


Image1:Ratio cut

(random method)



(kmeans++ method)

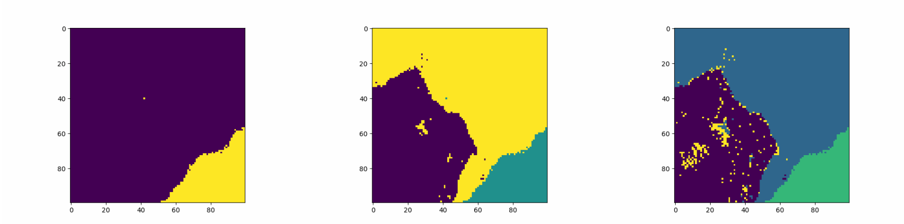
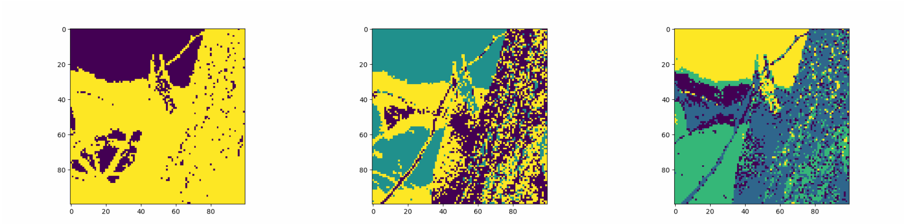


Image2: Normalize cut

(random method)



(kmeans++ method)

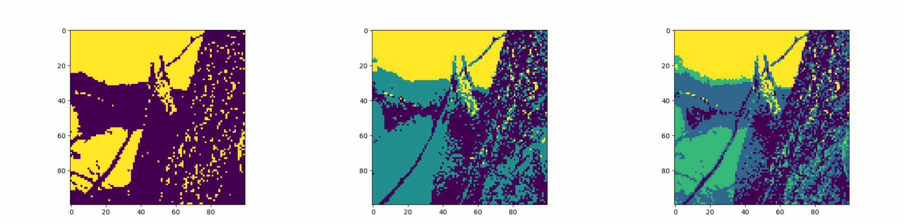
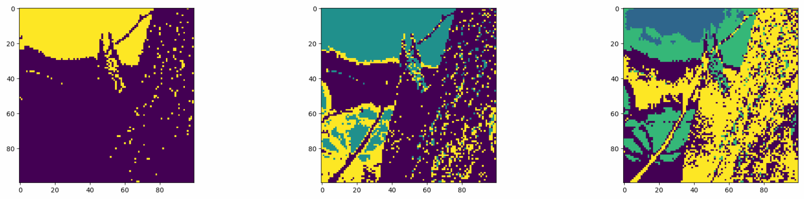
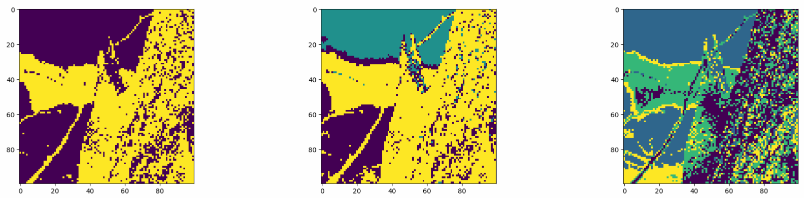


Image2:Ratio cut

(random method)



(kmeans++ method)



From the result of different clustering method, we can see that spectral clustering has the better result than K-means clustering. And with two different initialization method K-means++ has better result than random method.

ML_HW06

December 25, 2022

[1]:
```
import numpy as np
import cv2
import matplotlib.pyplot as plt
from scipy.spatial.distance import pdist, squareform
import os
from PIL import Image

from utils import load_data , kernel_function , gif_function
```

0.1 Kernel K-Means

For kernel k-means clustering, we use two different initialization method. 1. K-means++:
(1) Choose one center uniformly at random among the data points.
(2) For each data point x not chosen yet, compute D(x), the distance between x and the nearest center that has already been chosen.
(3) Choose one new data point at random as a new center, using a weighted probability distribution where a point x is chosen with probability proportional to $D(x)^2$.
(4) Repeat Steps 2 and 3 until k centers have been chosen.

2. Random:
   Randomly pick k data points as the initial centroids.

And we compute the kmeans clustering using the initialize centroids above. The alogorithms of kmeans we used is Lloyd's algorithm. For the E-step (expectation step), we keep μ_k fixed, and minimize $J = \sum_{n+1}^{N} \sum_{k=1}^{K} r_{nk} ||x_i - x_j||^2$ with respect to $r_{nk} \in \{0, 1\}$. And for the M-step (maximization step), we keep r_{nk} fixed, and minimize J with respect to μ_k. Repeat this procedure, until μ_k is converge.

[7]:
```
class Kmeans:
    def __init__(self, cluster, init):
        self.K = cluster
        self.init = init

    def initialization(self , init , data):
        init_mean = np.zeros((self.K , data.shape[1]))
        ## use kmeans++ to find the initral center
        if init == "kmeans++":
```

```
            ## randomly selected a data point as the first centroid
            init_mean[0] = data[np.random.randint(low = 0 , high = data.
↪shape[0] , size = 1) , :]

            ## find another k-1 centroid
            for k in range(1,self.K):
                k_dist = np.zeros((data.shape[0] , k))

                for i in range(k):
                    k_dist[:,i] = np.sqrt(np.sum((data- init_mean[i])**2 , axis␣
↪= 1))
                d_x = np.min(k_dist , axis = 1)
                sum_d = np.sum(d_x)*np.random.rand()
                for j in range(len(data)):
                    sum_d -= d_x[j]
                    if sum_d <= 0:
                        init_mean[k] = data[j]
                        break

        ## randomly choose k initial centroids
        elif init == "random":
            # np.random.seed(5)
            init_mean = data[np.random.randint(low = 0 , high = data.shape[0] ,␣
↪size = self.K) , :]

        return init_mean

    def train(self , png_dir , image_array , h , w):
        if not os.path.exists(png_dir):
            os.makedirs(png_dir)

        cluster = np.zeros((len(image_array)))
        ## find initial center using different method
        init_mean = self.initialization(self.init , image_array)
        mean = np.copy(init_mean)
        eps = 1e-9
        for i in range(1000):
            ## Expectation step
            dist = []
            ## classify all samples according to closet mu_k
            for j in range(self.K):
                dist.append(np.sqrt(np.sum((image_array - mean[j])**2 , axis =␣
↪1)))
            cluster = np.argmin(dist , axis = 0)

            # Maximization step
            new_mean = np.zeros(mean.shape)
```

```
            ## recompute as the mean mu_k of the points in cluter C_k
            for j in range(self.K):
                new_mean[j] = np.sum(image_array[np.argwhere(cluster == j),:] ,␣
 ↪axis = 0)
                if np.sum(cluster == j) > 0:
                    new_mean[j] /= np.sum(cluster == j)

            diff = np.sum((new_mean - mean)**2)
            mean = new_mean

            ## visualization of current clustering result
            print("iteration: {} diff: {:.6f}".format(i , diff))
            plt.imshow(cluster.reshape(h, w))
            # plt.show()
            plt.savefig(os.path.join(png_dir,'frame_'+str(i)+'.png'),dpi=100)
            plt.close()
            if diff < eps:
                break

        ## creating gif file
        gif_function(png_dir)
```

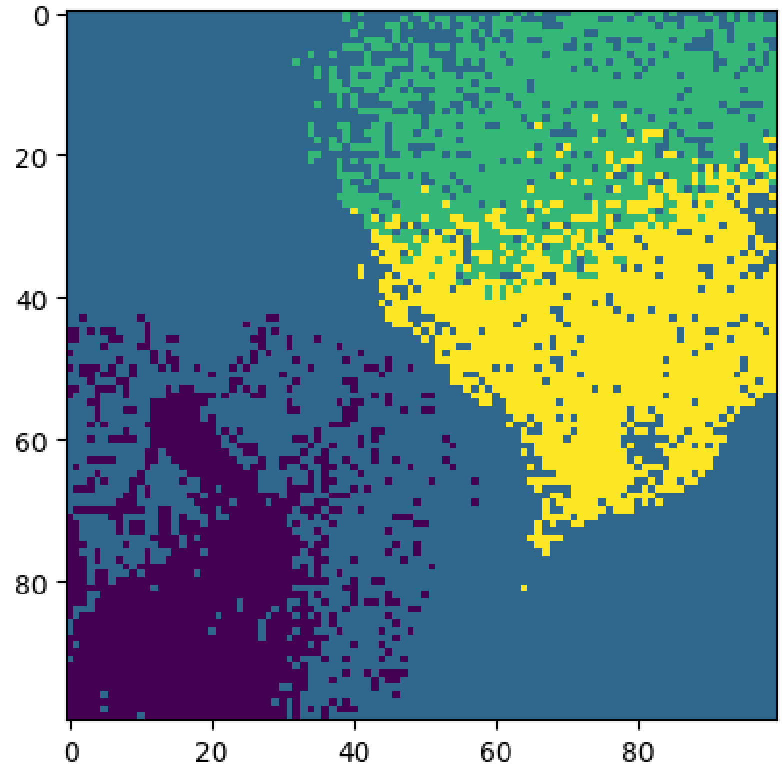
The kernel function we used is defined below:

$$k(x, x') = e^{-\gamma_s ||S(x)-S(x')||^2} \times e^{-\gamma_c ||C(x)-C(x')||^2}$$

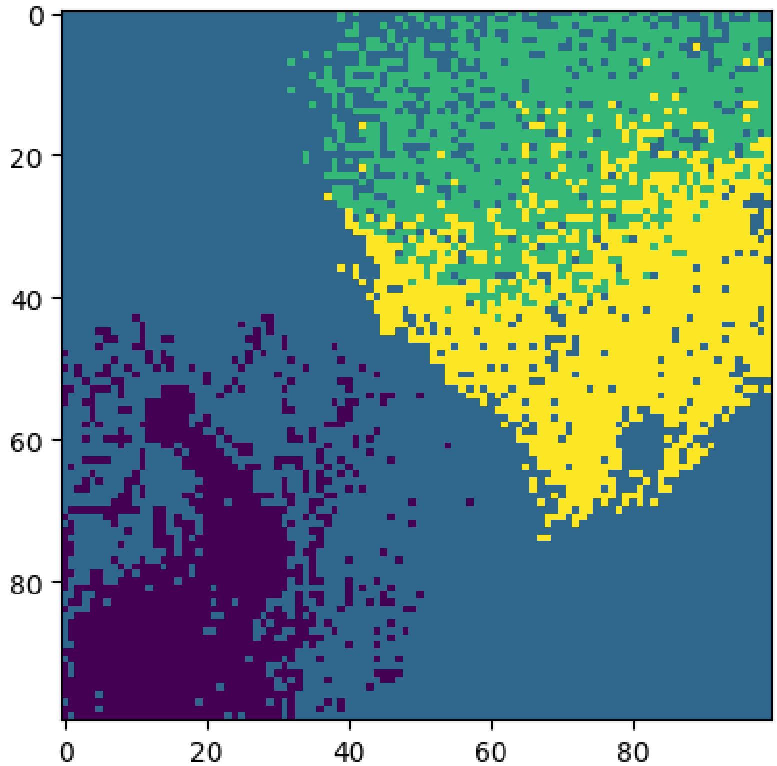
We set the γ_s to 0.001 and γ_c to 0.001.

[140]:
```
data , height , width = load_data("/Users/cindychen/Documents/ML_HW06/image1.
 ↪png")

gamma_s = 0.001
gamma_c = 0.001
## compute kernel function with gamma s equals to 0.001 and gamma c equals to 0.
 ↪001
image_array = kernel_function(data, gamma_s , gamma_c)

model = Kmeans(4, "random")
gif_dir = "/Users/cindychen/Documents/ML_HW06_output/kmeans_4"
map = model.train(gif_dir , image_array , height , width)
```

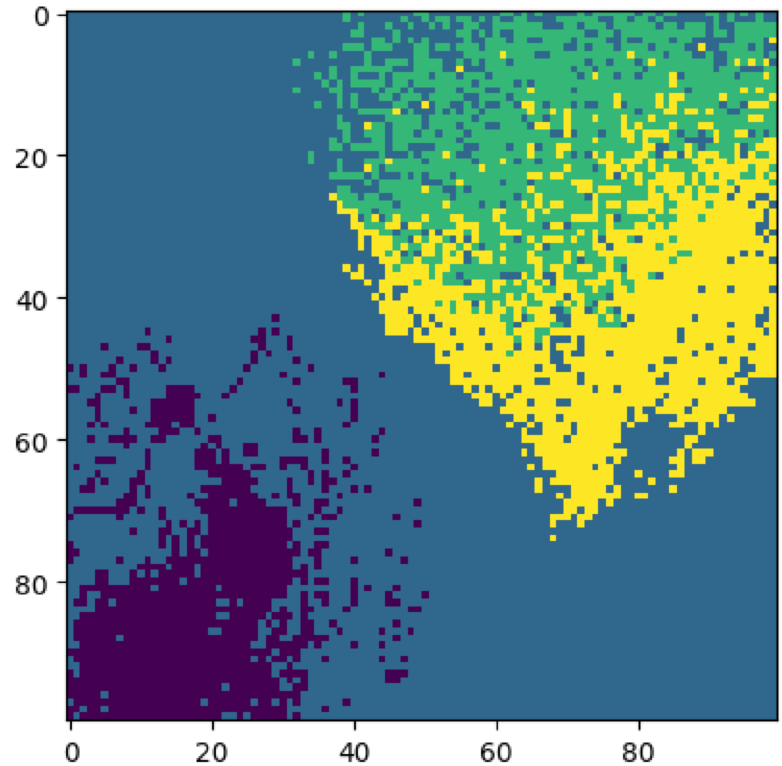
```
iteration: 0 diff: 244.525944
```



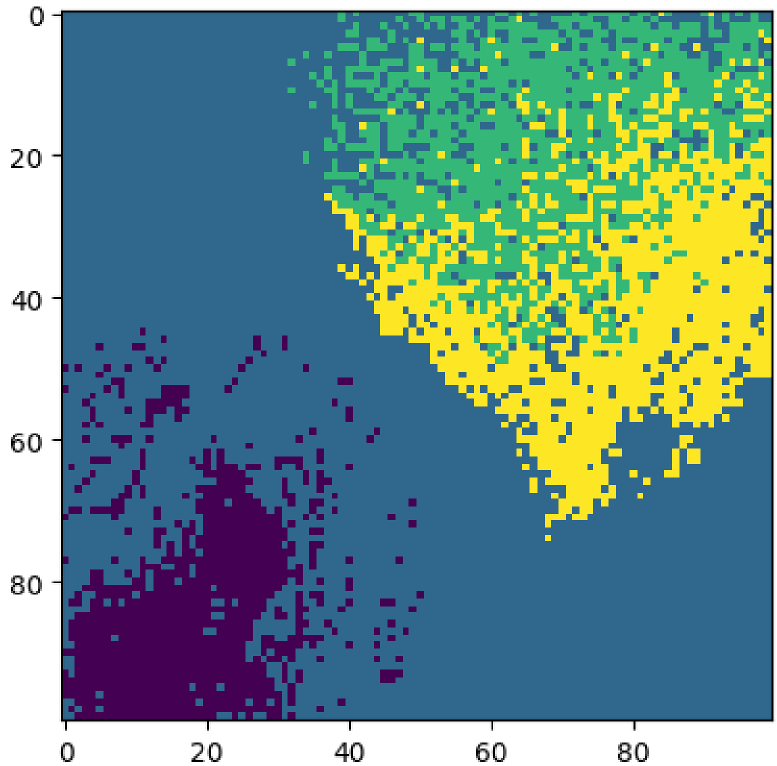
iteration: 1 diff: 3.006782



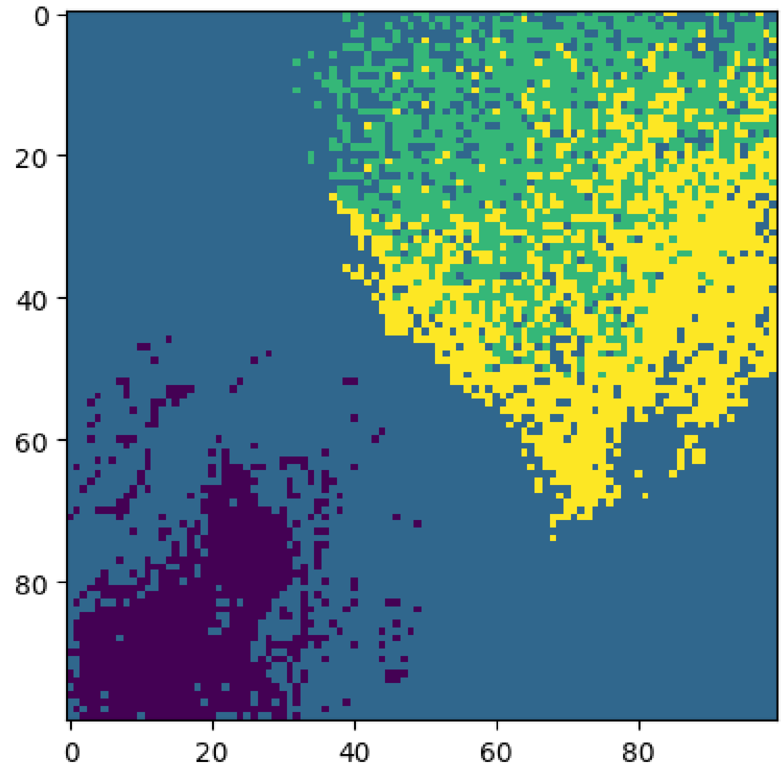
iteration: 2 diff: 2.352987



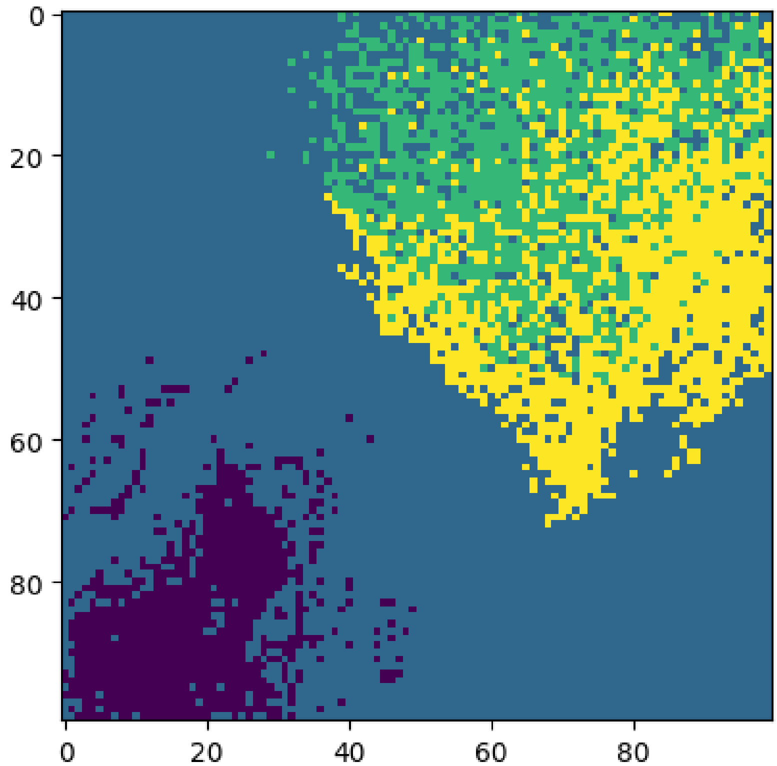

```
iteration: 3 diff: 1.759184
```



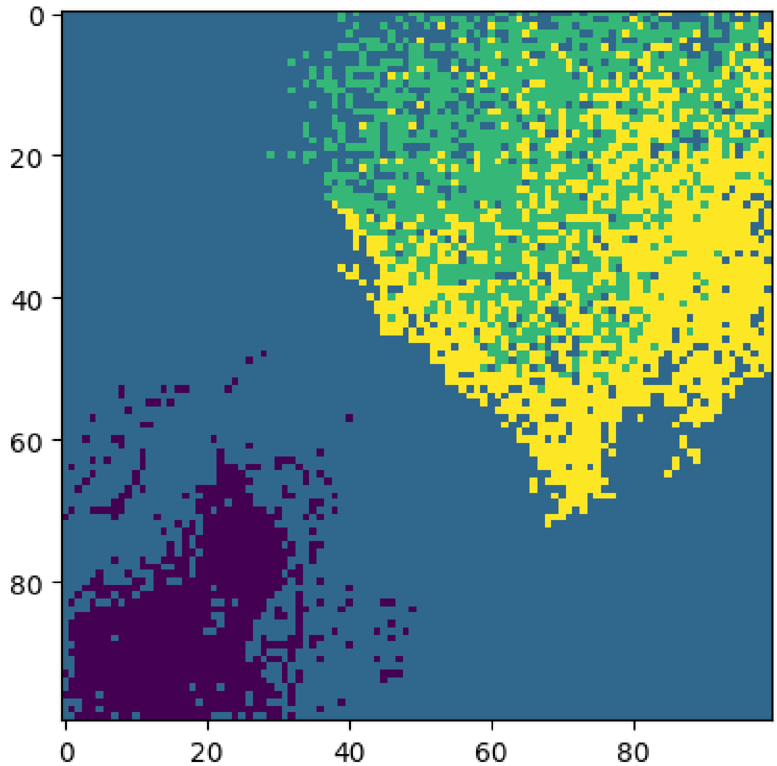

```
iteration: 4 diff: 0.936148
```



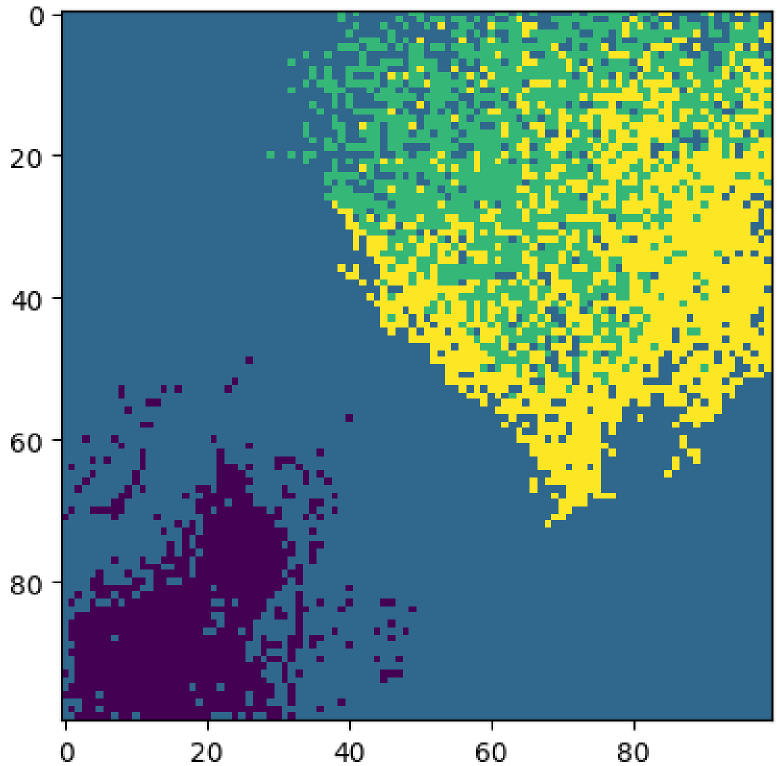

```
iteration: 5 diff: 0.559892
```



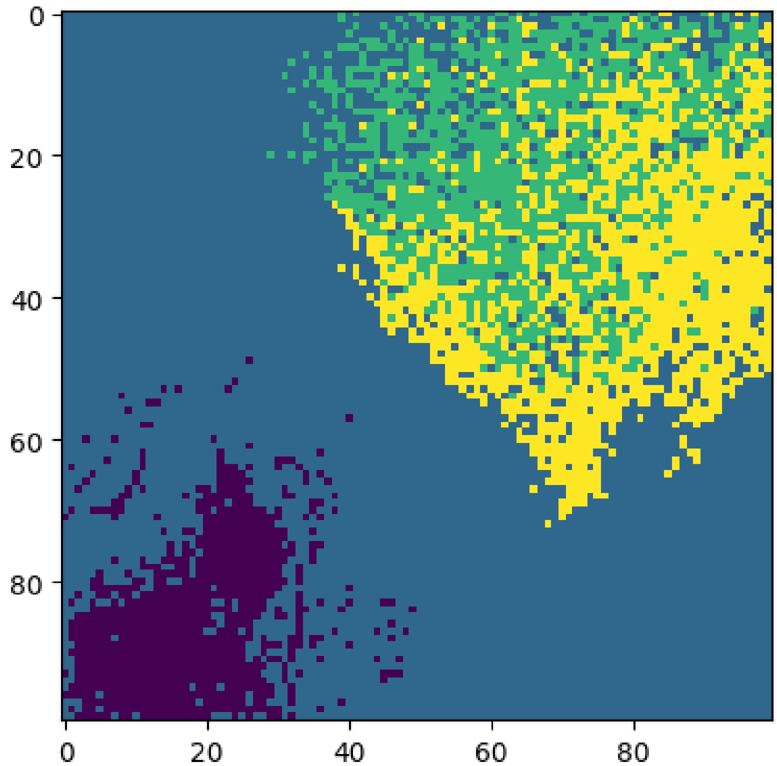
iteration: 6 diff: 0.263217



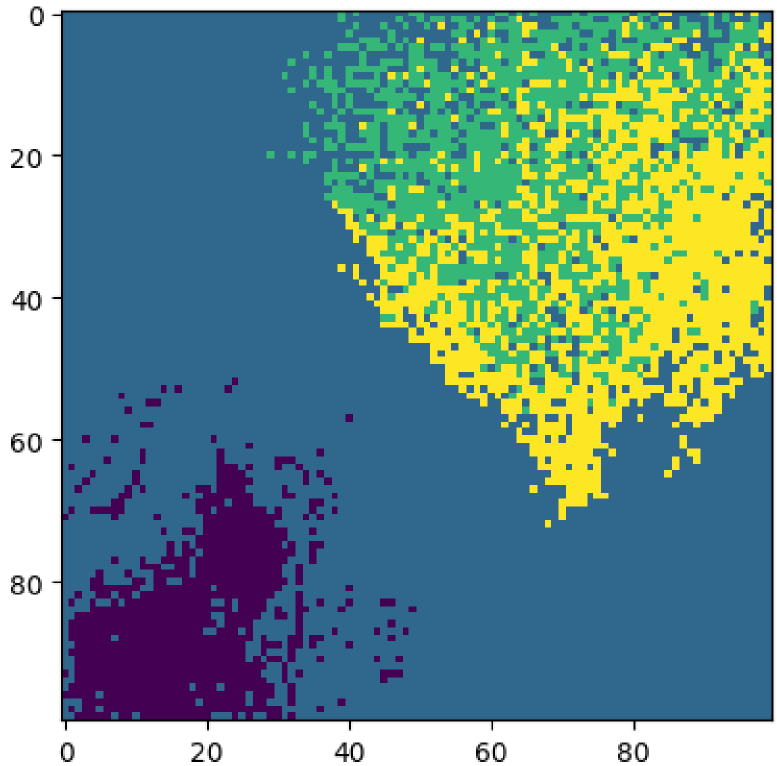

```
iteration: 7 diff: 0.118955
```



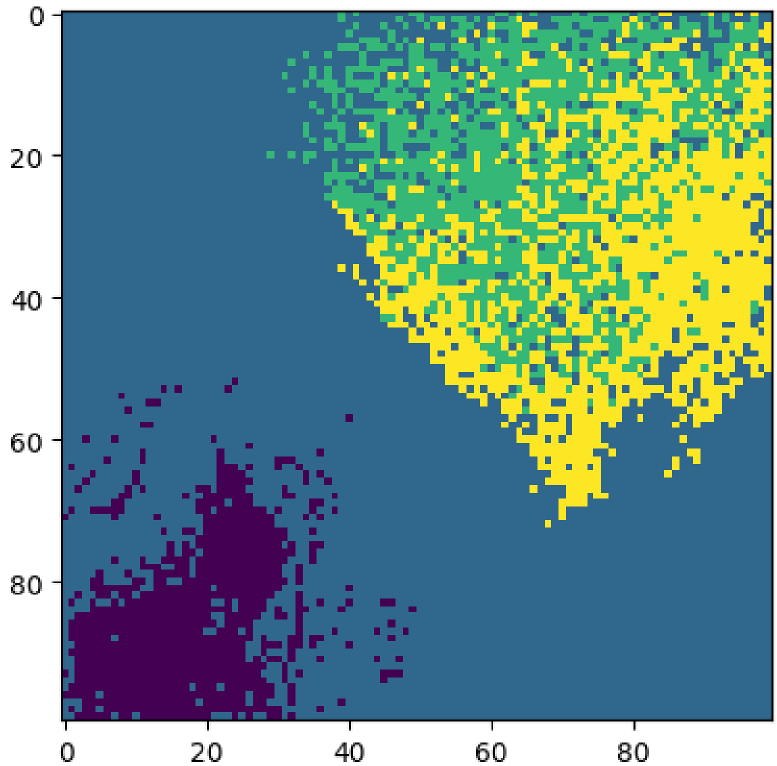

```
iteration: 8 diff: 0.054253
```



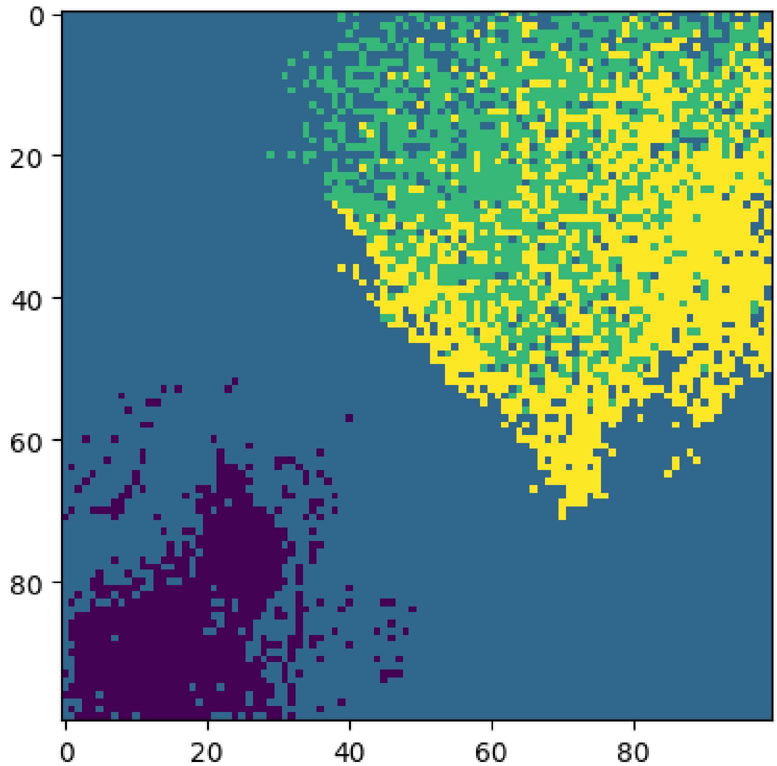
iteration: 9 diff: 0.046171



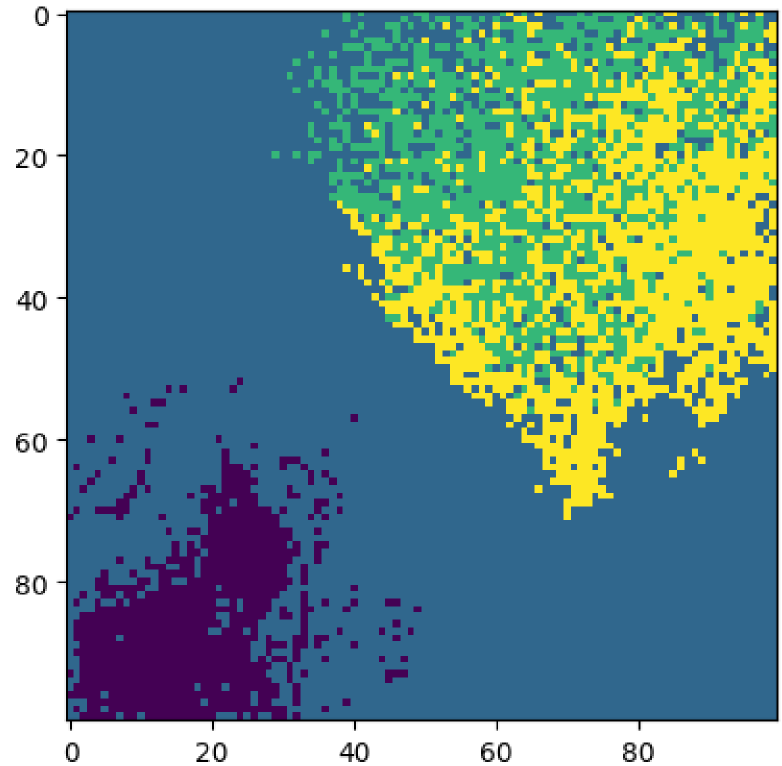

```
iteration: 10 diff: 0.013350
```



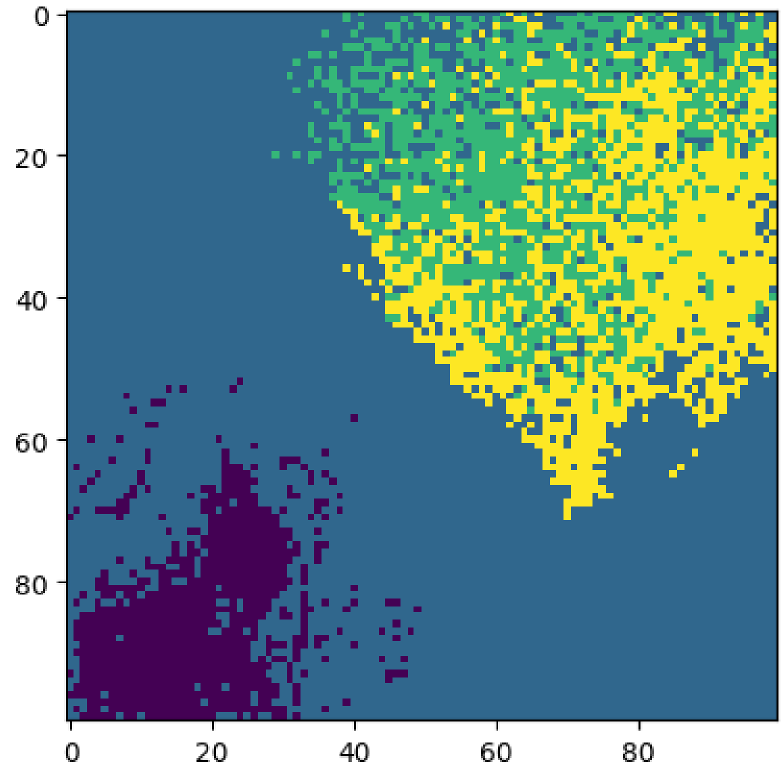

```
iteration: 11 diff: 0.018588
```



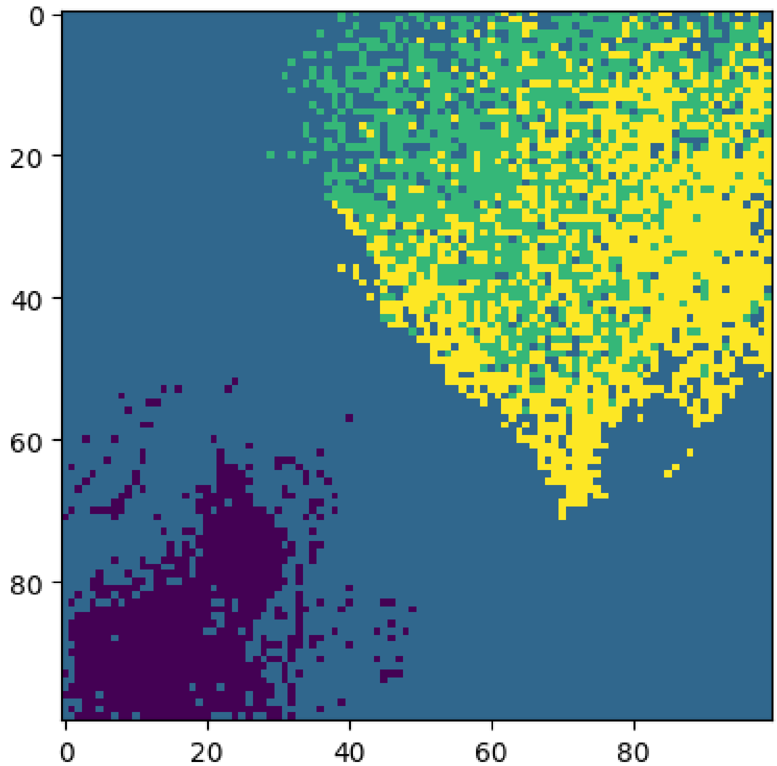

```
iteration: 12 diff: 0.018511
```



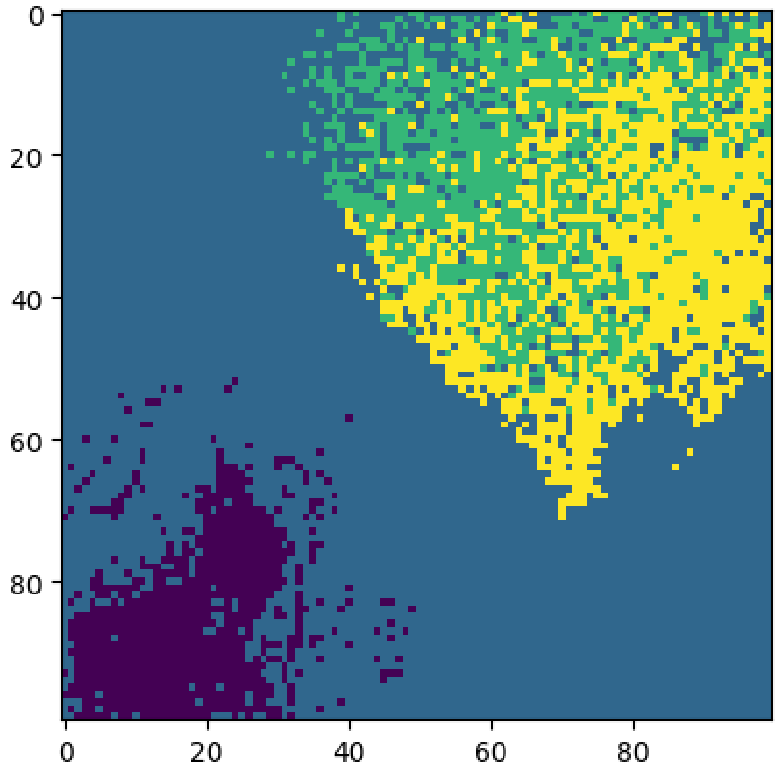

```
iteration: 13 diff: 0.027003
```



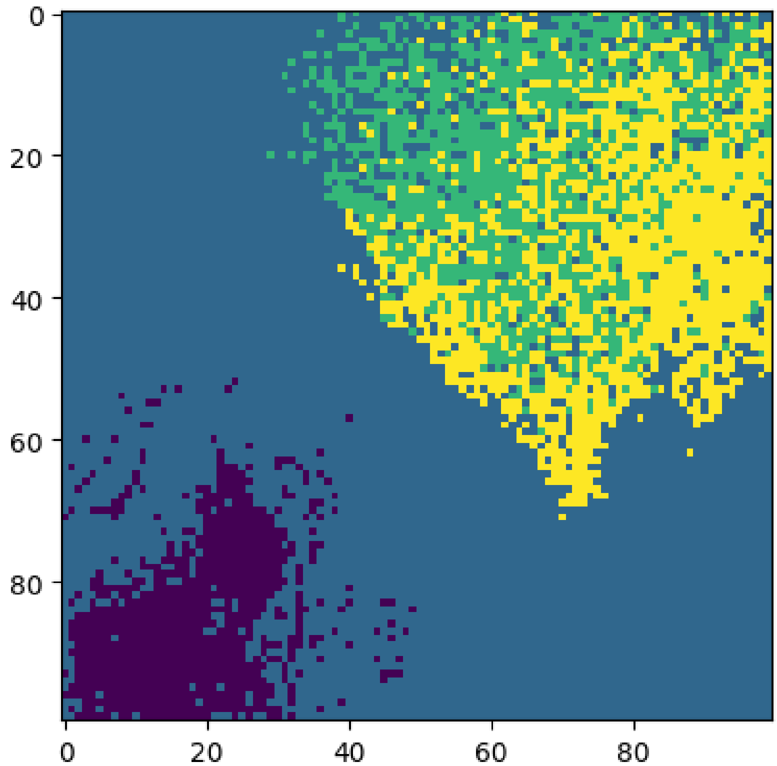

```
iteration: 14 diff: 0.031106
```



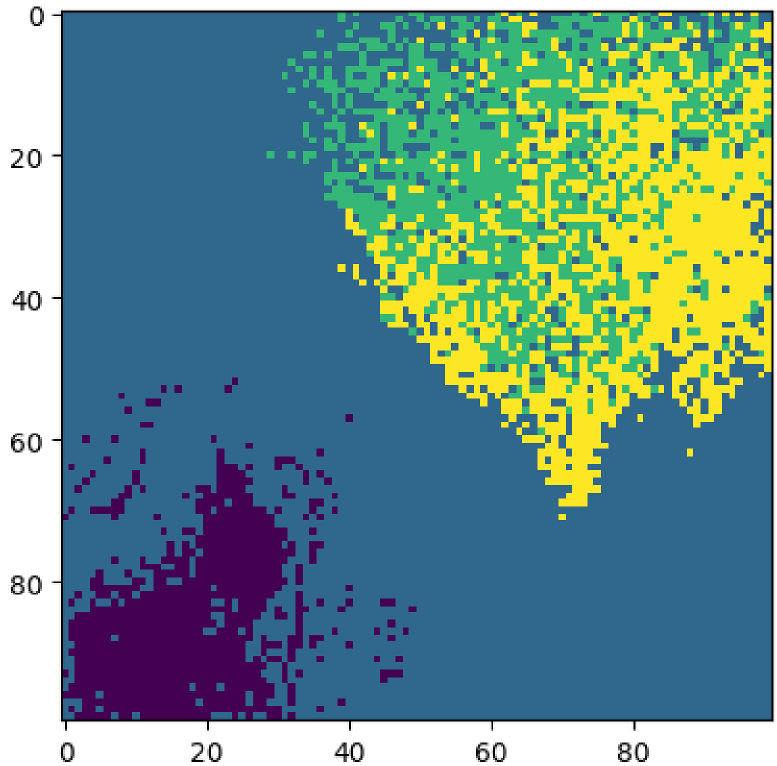
iteration: 15 diff: 0.019707



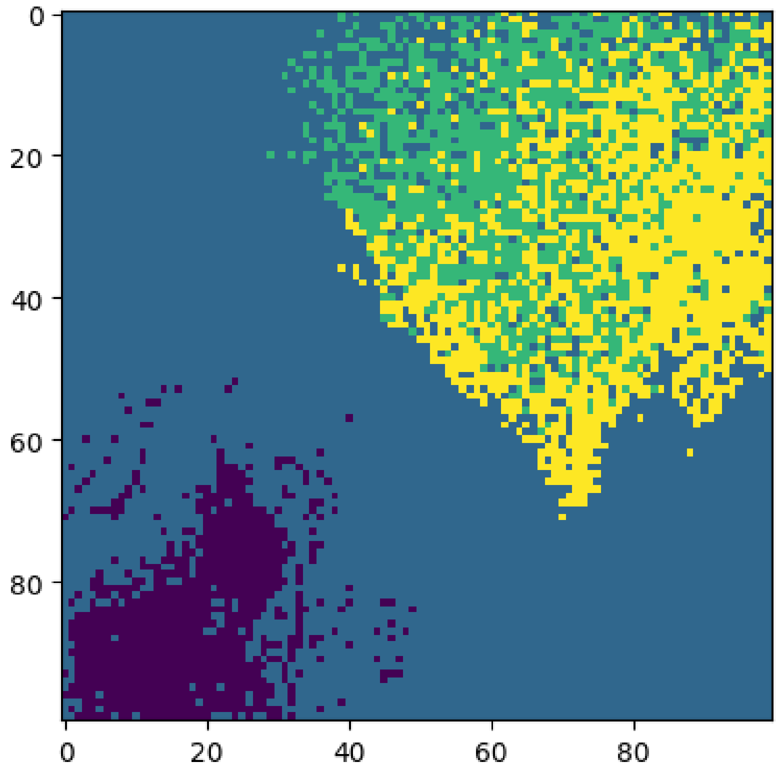
iteration: 16 diff: 0.016679



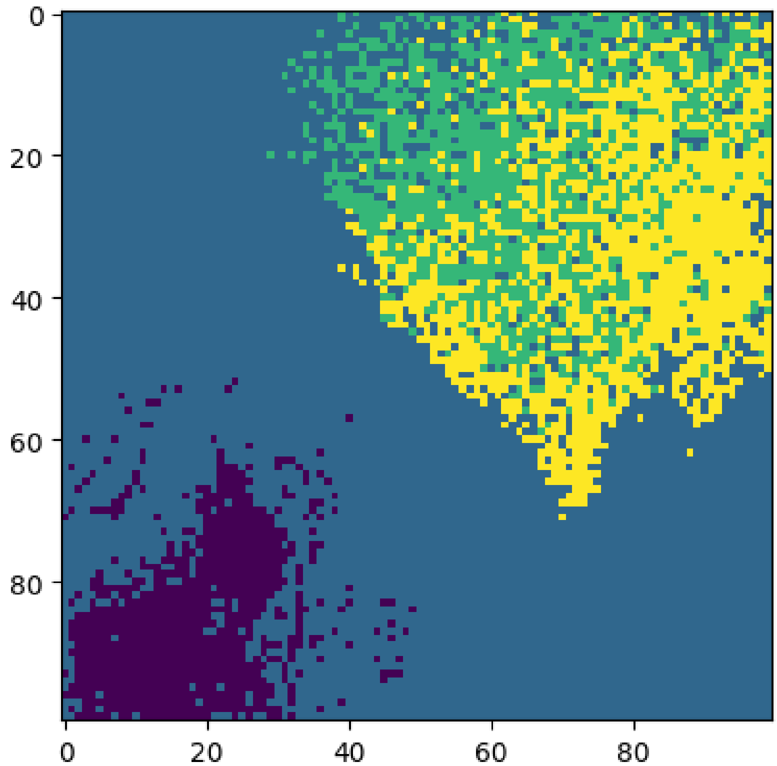

```
iteration: 17 diff: 0.009625
```



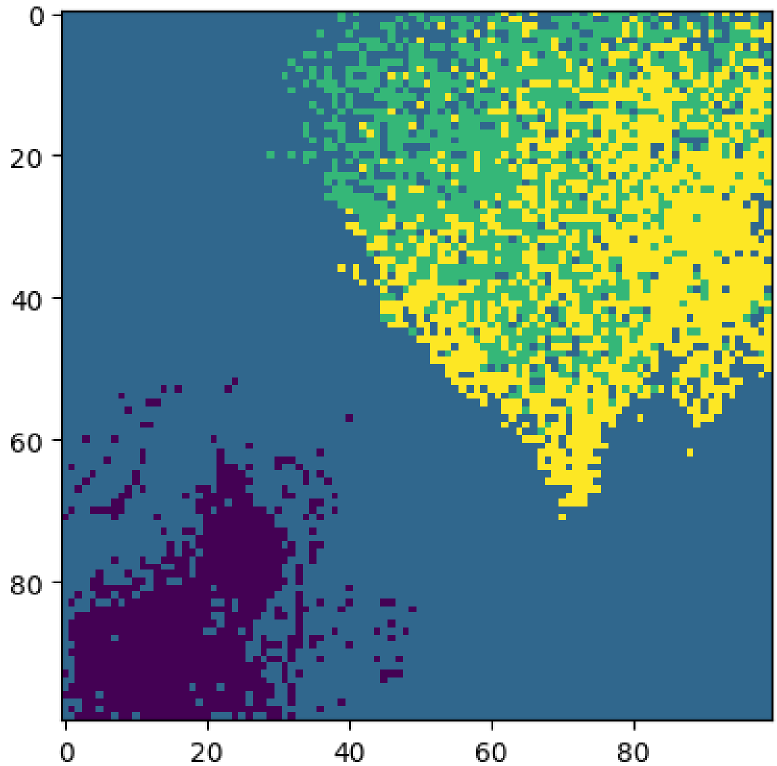
iteration: 18 diff: 0.003618



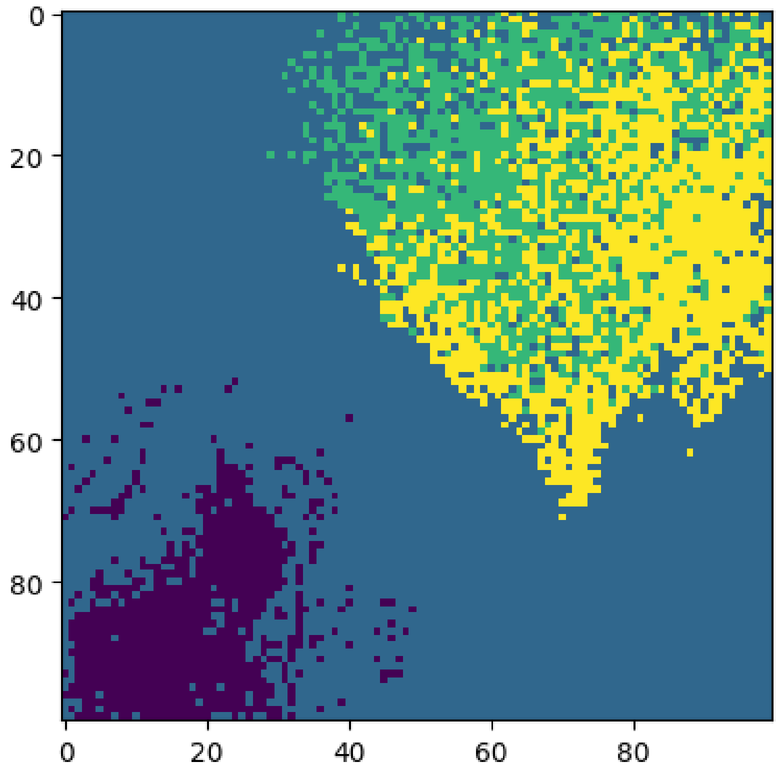

```
iteration: 19 diff: 0.000592
```



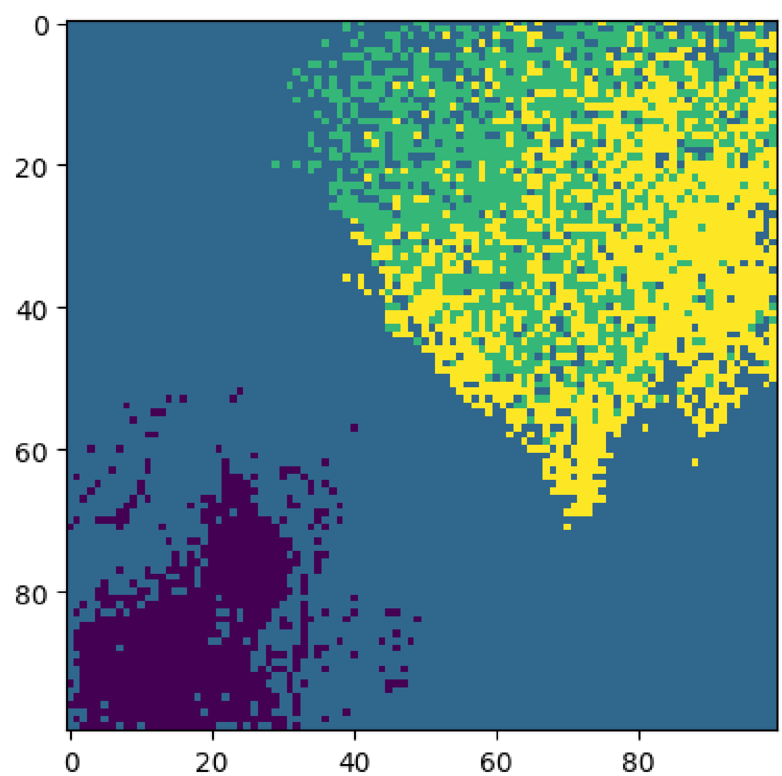
iteration: 20 diff: 0.001087



iteration: 21 diff: 0.000162




```
iteration: 22 diff: 0.000000
```



For the result of the two different initialization methods, we can see that by using kmeans++ method the clustering converges much quicker with less iterations and less time.

0.2 Spectral Clustering

For Spectral Clustering, we also used two different methods to cluster which is normalize cut and ratio cut.

[3]:
```
class Spectral_Cluster:
    def __init__(self , cluster , image_array ):
        self.K = cluster
        self.image_array = image_array

    def normalize_cut(self , eig_path , vec_path):
        if os.path.exists(eig_path) and os.path.exists(vec_path):
            eigenvalue = np.load(eig_path)
            eigenvector = np.load(vec_path)

        else:
            ## similarity matrix
            W = self.image_array
            ## degree matrix
            D = np.diag(np.sum(W , axis = 1))
```

```
            L = D - W
            ## compute normalize laplacian matrix L_sym = D^(-1/2)LD^(-1/2)
            L_sym = np.diag(1/np.diag(np.sqrt(D)))@L@np.diag(1/np.diag(np.
↪sqrt(D)))

            eigenvalue , eigenvector = np.linalg.eig(L_sym)
            np.save(eig_path , eigenvalue)
            np.save(vec_path , eigenvector)

        sort_idx = np.argsort(eigenvalue)
        U = eigenvector[:,sort_idx[:self.K]]

        T = U/np.sqrt(np.sum(U**2 , axis = 1)).reshape(-1 ,1)
        return T


    def ratio_cut(self , eig_path , vec_path):
        if os.path.exists(eig_path) and os.path.exists(vec_path):
            eigenvalue = np.load(eig_path)
            eigenvector = np.load(vec_path)

        else:
            ## similarity matrix
            W = self.image_array
            D = np.diag(np.sum(W , axis = 1))
            ## compute unormalize laplacian matrix
            L = D - W

            eigenvalue , eigenvector = np.linalg.eig(L)
            np.save(eig_path , eigenvalue)
            np.save(vec_path , eigenvector)

        sort_idx = np.argsort(eigenvalue)
        U = eigenvector[:,sort_idx[:self.K]]

        return U
```

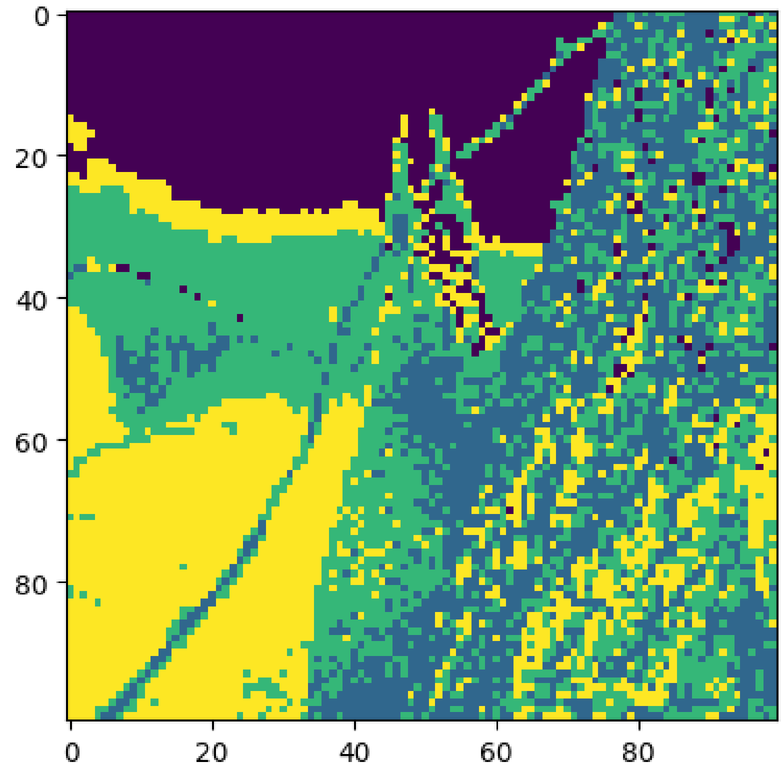
[10]:
```
data , height , width = load_data("/Users/cindychen/Documents/ML_HW06/image1.
↪png")

gamma_s = 0.001
gamma_c = 0.001
## compute kernel function with gamma s equals to 0.001 and gamma c equals to 0.
↪001
image_array = kernel_function(data, gamma_s , gamma_c)
```

```
eig_path = "/Users/cindychen/Documents/ML_HW6_310657012_ /eigenvalue_image1_0.
 ↪001_0.001_ratio.npy"
vec_path = "/Users/cindychen/Documents/ML_HW6_310657012_ /eigenvector_image1_0.
 ↪001_0.001_ratio.npy"
spectral = Spectral_Cluster(4 , image_array)
## using normalize cut and ratio cut to compute the spectral clustering
process_array = spectral.normalize_cut(eig_path , vec_path)
process_array = spectral.ratio_cut(eig_path , vec_path)

## after using spectral clustering put the pre-processed data into kmeans for␣
 ↪classification
model_2 = Kmeans(4, "random")
gif_dir = "/Users/cindychen/Documents/ML_HW06_output/spectral_4_ratio"
model_2.train(gif_dir , process_array , height , width)
```

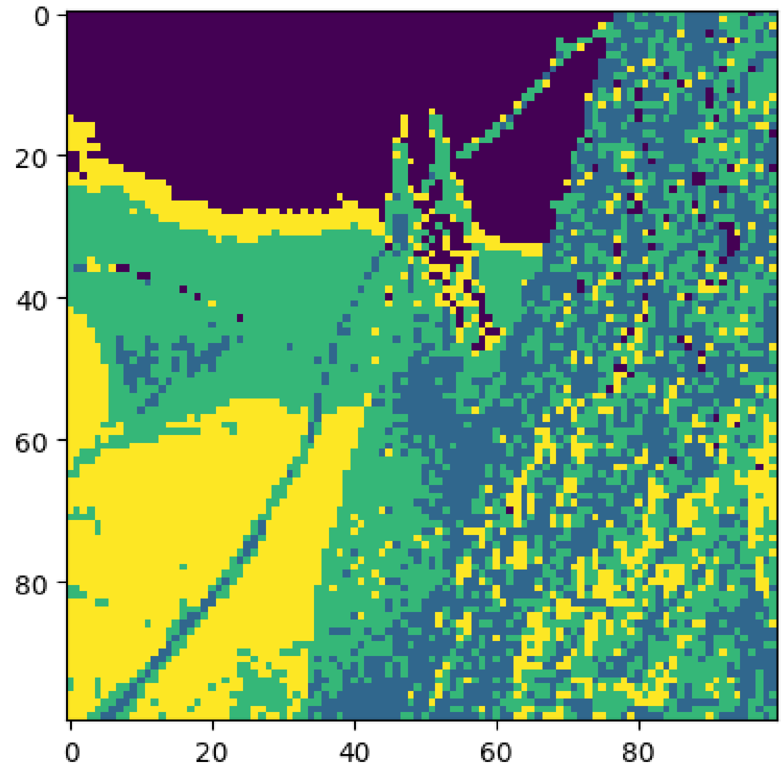
```
iteration: 0 diff: 0.000002
iteration: 1 diff: 0.000016
iteration: 2 diff: 0.000043
iteration: 3 diff: 0.000009
iteration: 4 diff: 0.000005
iteration: 5 diff: 0.000001
iteration: 6 diff: 0.000000
iteration: 7 diff: 0.000000
iteration: 8 diff: 0.000000
```

[142]:
```
data , height , width = load_data("/Users/cindychen/Documents/ML_HW06/image2.
 ↪png")

gamma_s = 0.001
gamma_c = 0.001
image_array = kernel_function(data, gamma_s , gamma_c)

eig_path = "/Users/cindychen/Documents/ML_HW6_310657012_ /eigenvalue_image2_0.
 ↪001_0.001.npy"
vec_path = "/Users/cindychen/Documents/ML_HW6_310657012_ /eigenvector_image2_0.
 ↪001_0.001.npy"
spectral = Spectral_Cluster(4 , image_array)
# process_array = spectral.ratio_cut(eig_path , vec_path)
process_array = spectral.normalize_cut(eig_path , vec_path)

model_2 = Kmeans(4, "kmeans++")
gif_dir = "/Users/cindychen/Documents/ML_HW06_output/spectral++2_4"
model_2.train(gif_dir , process_array , height , width)
```

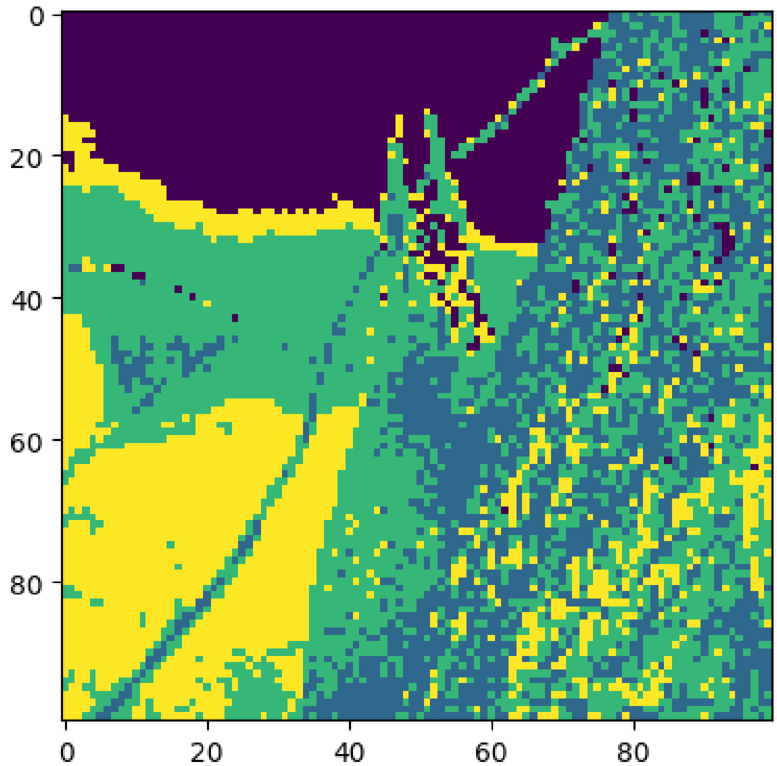
```
iteration: 0 diff: 0.184725
```



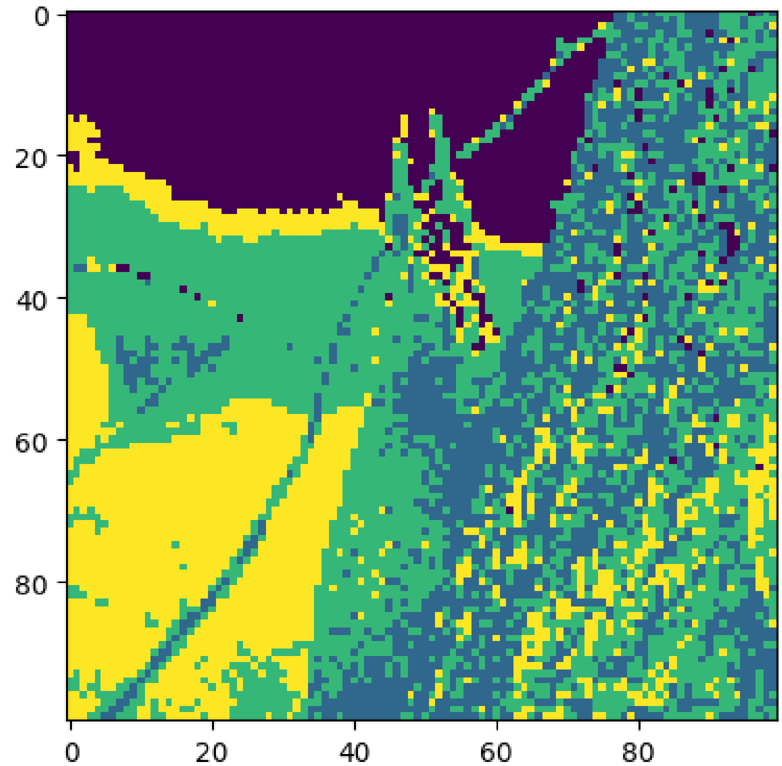

```
iteration: 1 diff: 0.011629
```



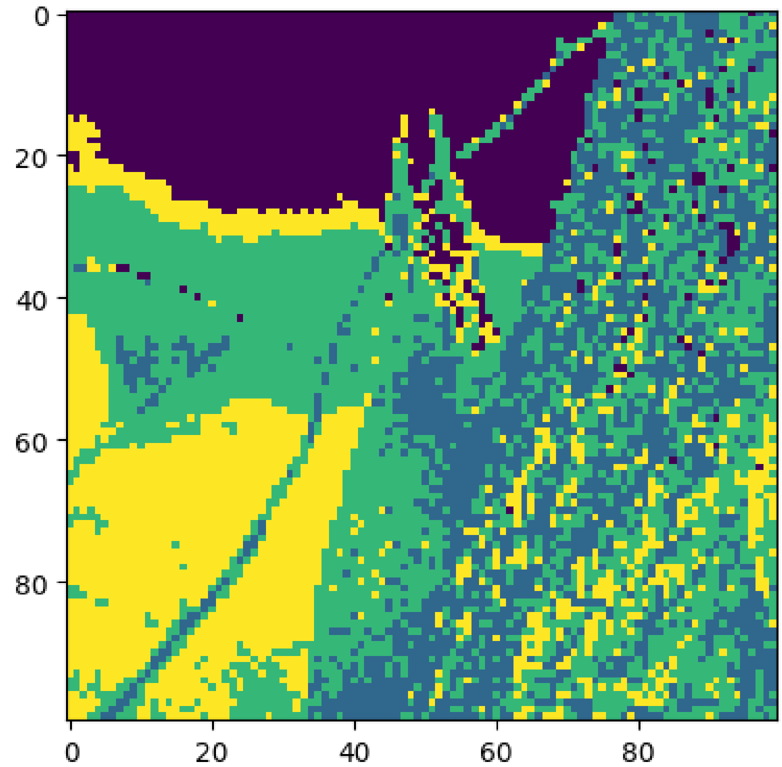

```
iteration: 2 diff: 0.001115
```



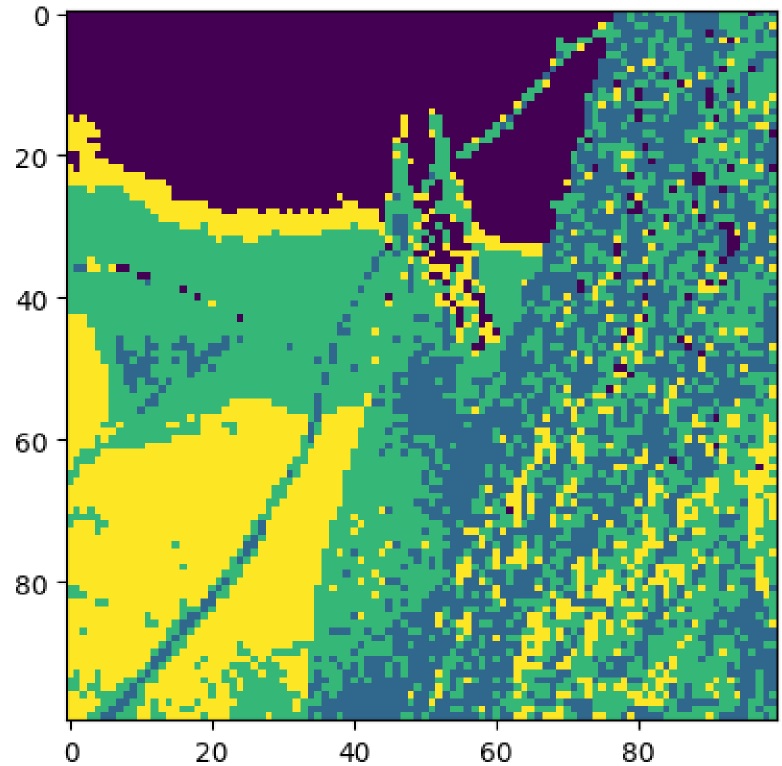

```
iteration: 3 diff: 0.000098
```



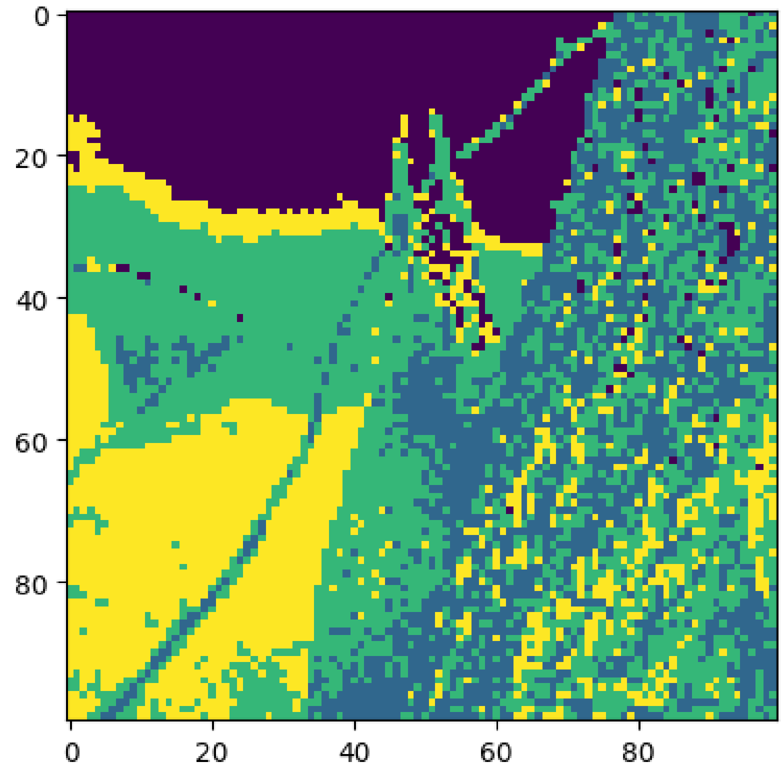
`iteration: 4 diff: 0.000007`



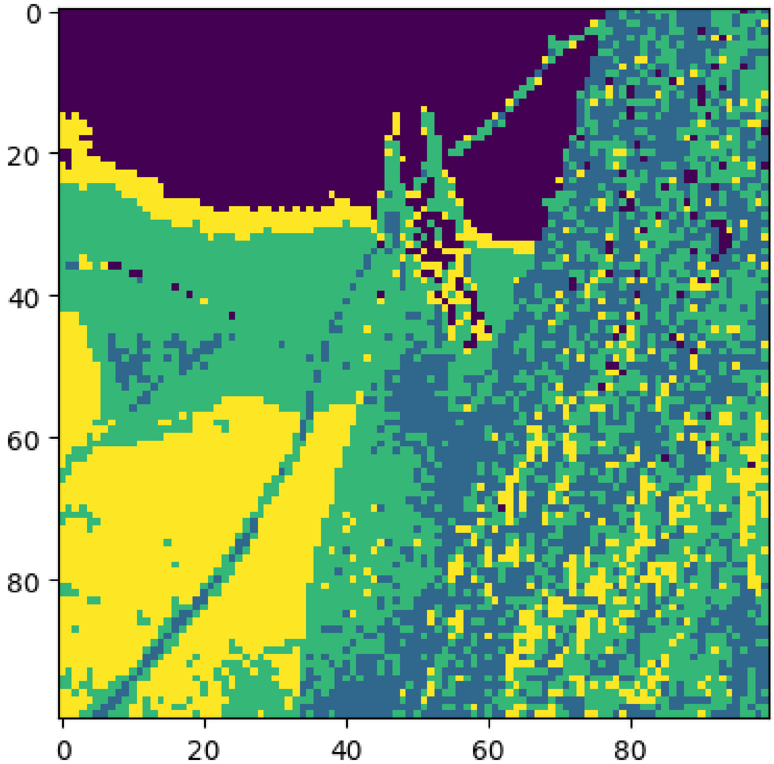
iteration: 5 diff: 0.000001



iteration: 6 diff: 0.000000



iteration: 7 diff: 0.000000


0
20
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0
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40
60
80

[ ]: