Assignment 8, FIN 560

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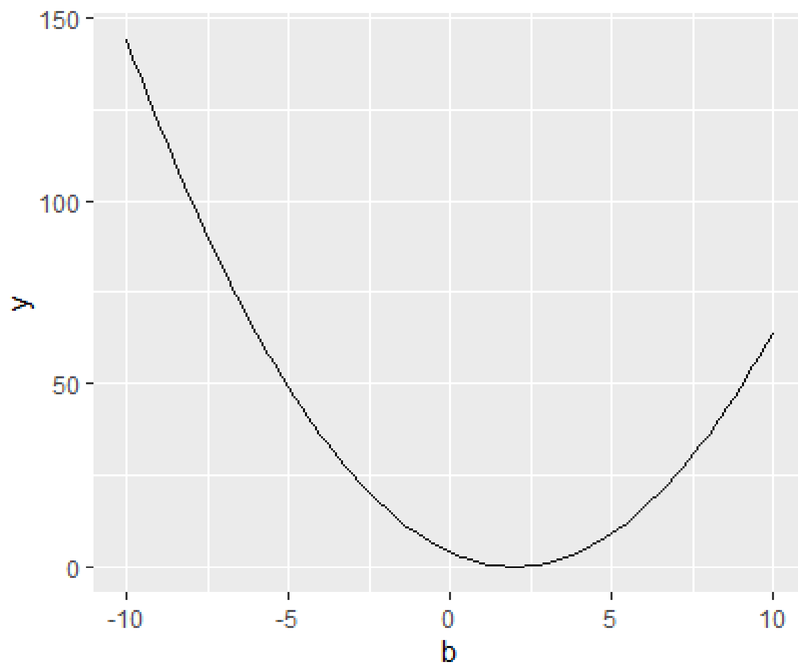
April 21, 2021

1 1

1.1 a



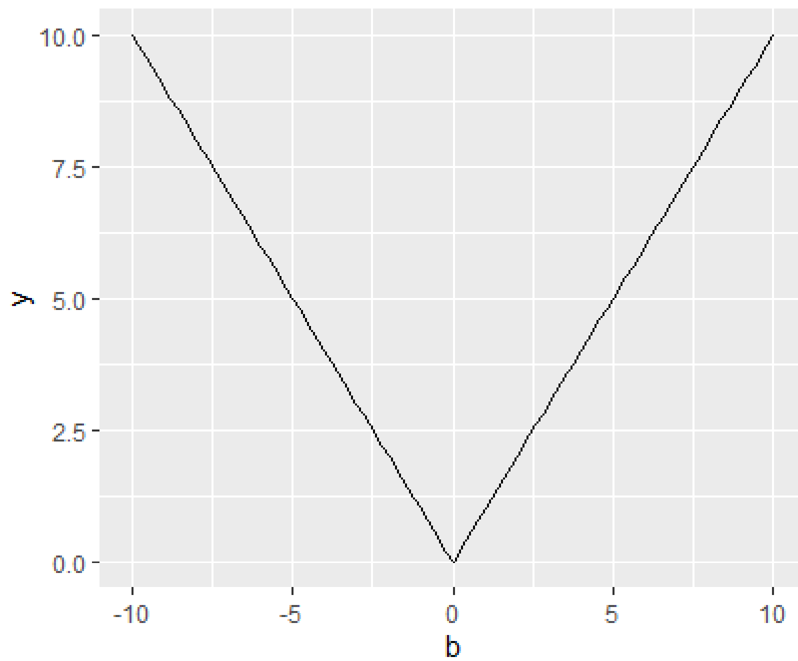
1.2 b

Using the familiar formula (with no intercept) with a trivial matrix X:

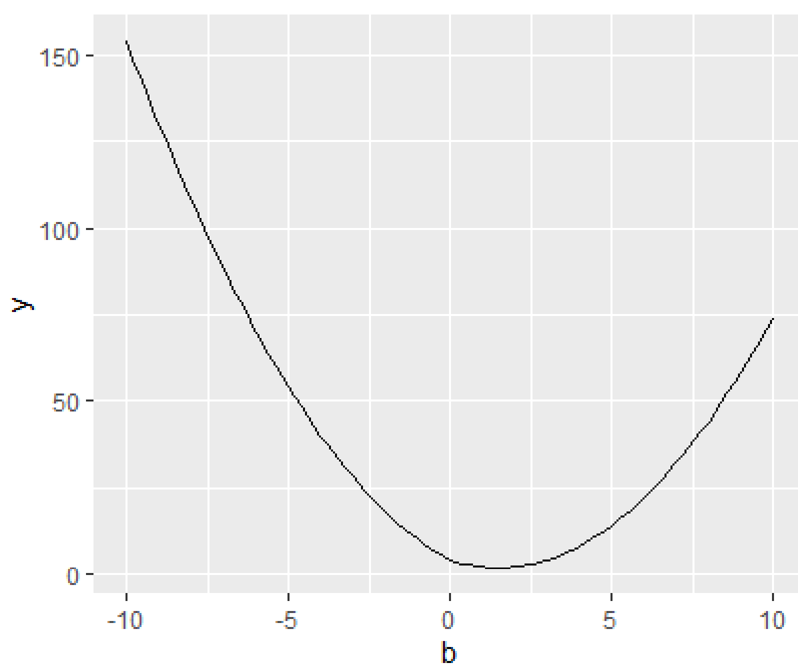
$$\hat{\beta}^{OLS} = (X^\top X)^{-1} X^\top y$$

$$= 1^{-1} 1^\top (2) = 2$$

1.3 c



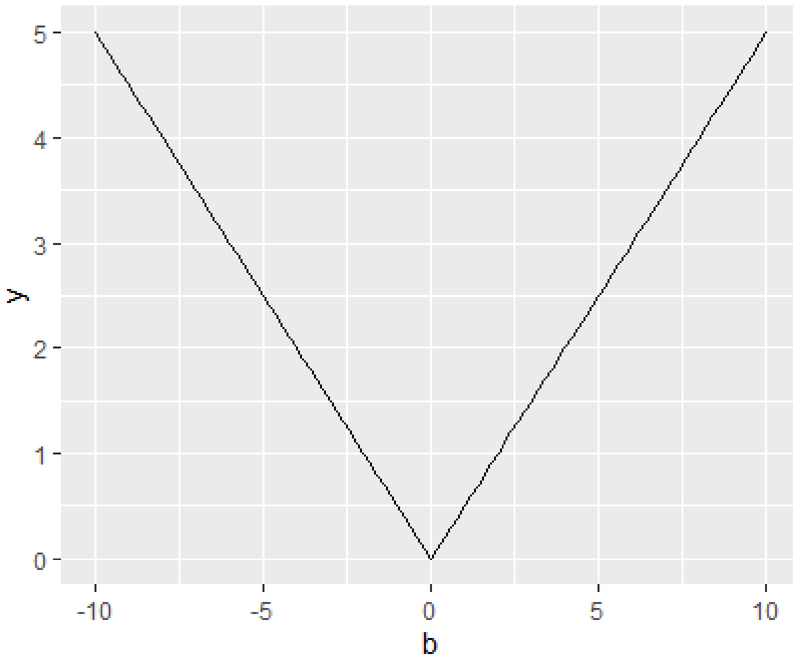
1.4 d

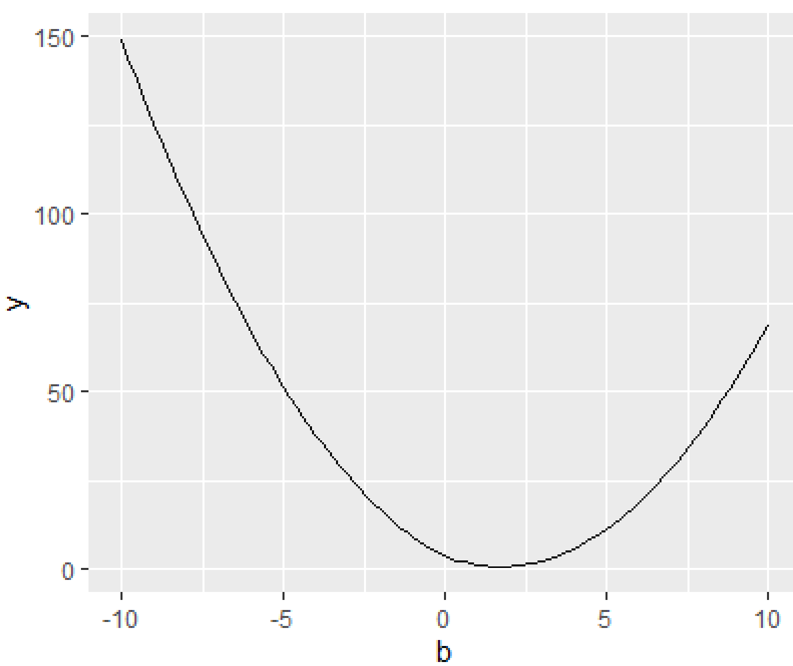


1.5 e

It is evident from the plots that the minimizing value of b for this loss function of the LASSO model is $\hat{\beta}^{LASSO} = 1.5$.

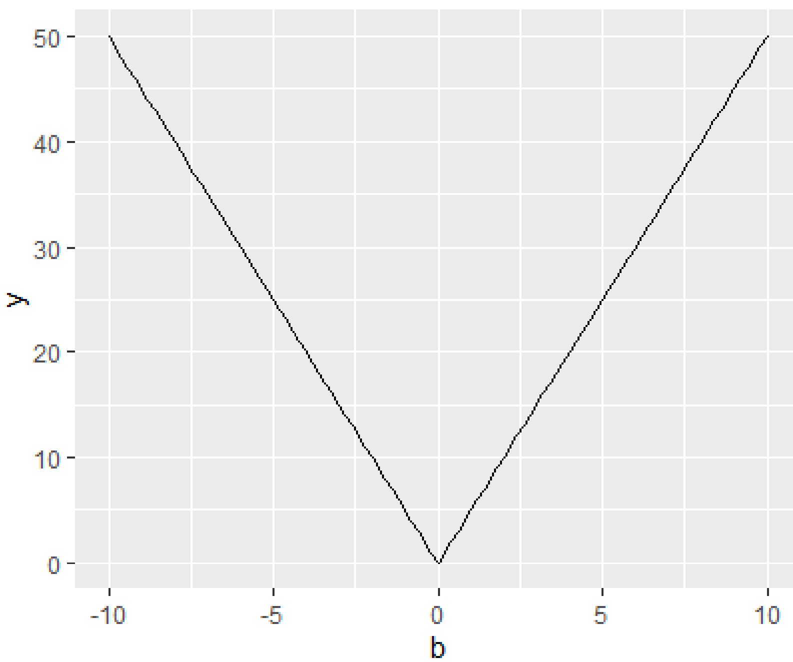
1.6 f

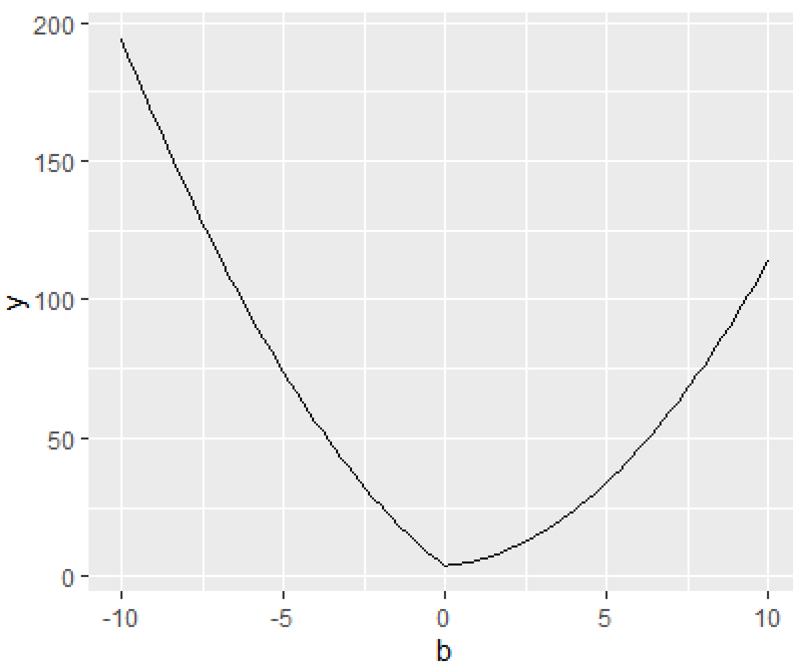




Here $\hat{\beta}^{LASSO} = 1.75$.

1.7 g





Here $\hat{\beta}^{LASSO} = 0$.

1.8 h

A larger λ^{LASSO} shrinkage parameter will strengthen the regularization of the coefficients. In the loss function that is being optimized, if there is a larger weight or if the tuning parameter is larger, then choosing more extreme (higher or more negative) values for β, even if it were beneficial to minimize the squared residual term, it will incur a large cost/penalty due to the ℓ_1 norm term being added on. In the special case of LASSO regularization, in contrast to Ridge regression, most coefficients will be set equal to zero if the tuning parameter is high enough. This is due to the level sets of the squared residuals being elliptical, and one of these function level sets will intersect the feasible region (convex polytope) formed by the constraint at a vertex.

That explains why at the end, with $\lambda = 5$ in the previous part, it is strong enough to put the optimal solution to be $\hat{\beta}^{LASSO} = 0$.

2 Code used

```
######################################################################
# Assignment 8 Textbook
#
# Cindy Lu
# FIN 560
#
# Last modified: 21-Apr-2021
######################################################################

###############
# Load packages
###############

library(ggplot2)

#############
# Question 1a
#############

# For x-axis values (b)
b <- -10:10

# Plot as function of b
ggplot(data.frame(b), aes(b))+
  stat_function(fun = function(x) (2-x)^2)

#############
# Question 1c
#############

# Plot as a function of b
ggplot(data.frame(b), aes(b))+
  stat_function(fun = function(x) abs(x))

#############
# Question 1d
#############

# Plot lasso penalty sum
ggplot(data.frame(b), aes(b))+
  stat_function(fun = function(x) (2-x)^2+abs(x))

#############
# Question 1f
#############

# Plot as a function of b
ggplot(data.frame(b), aes(b))+
  stat_function(fun = function(x) 0.5*abs(x))

# Plot lasso penalty sum
ggplot(data.frame(b), aes(b))+
```

```
  stat_function(fun = function(x) (2-x)^2+0.5*abs(x))

#############
# Question 1g
#############

# Plot as a function of b
ggplot(data.frame(b), aes(b))+
  stat_function(fun = function(x) 5*abs(x))

# Plot lasso penalty sum
ggplot(data.frame(b), aes(b))+
  stat_function(fun = function(x) (2-x)^2+5*abs(x))
```