Assignment 6, FIN 560

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1 1

1.1 a

To write $D1_i$ in terms of the rest of the variables $D2_i, D3_i, X_{0,i,t}$, it can be seen that this individual-level heterogeneous effect/intercept is fully determined given values of other variables and residual term.

$$D_{i1} = \frac{y_{it} + \beta_0 X_{0,i,t} + \beta_1 X_{it} + \gamma_2 D2_i + \gamma_3 D3_i + u_{it}}{\gamma_1}$$

$$D_{i1} = \frac{y_{it} + \beta_0(1) + \beta_1 X_{it} + \gamma_2 D2_i + \gamma_3 D3_i + u_{it}}{\gamma_1}$$

If the estimated model is put into matrix form $\hat{y_{it}} = \hat{\beta}_0 + \hat{\beta}_1 X_{it} + \hat{\gamma_1} D1_i + ... + \hat{\gamma_3} D3_i$, the first column will be made entirely of 1's, and if one of the n dummies is not excluded, then the design matrix X will be perfectly collinear as the square sub-matrix with columns 2 to 4 (first column corresponds to $X_{0,i,t}$) will form an identity matrix.

1.2 b

Generalizing above:

$$D_{i1} = \frac{y_{it} + \beta_0 X_{0,i,t} + \beta_1 X_{it} + \gamma_2 D2_i + ... + \gamma_n Dn_i + u_{it}}{\gamma_1}$$

$$D_{i1} = \frac{y_{it} + \beta_0(1) + \beta_1 X_{it} + \gamma_2 D2_i + ... + \gamma_n Dn_i + u_{it}}{\gamma_1}$$

Again, if the estimated model is put into matrix form $\hat{y_{it}} = \hat{\beta}_0 + \hat{\beta}_1 X_{it} + \hat{\gamma_1} D1_i + ... + \hat{\gamma_n} Dn_i$, the first column will be made entirely of 1's, and if one of the n dummies is not excluded, then the design matrix X will be perfectly collinear as the square sub-matrix with columns 2 to now $n+1$ (first column corresponds to $X_{0,i,t}$) will form an identity matrix.

1.3 c

Because the design matrix is perfectly collinear, the model will be undefined because the matrix will be singular and have a non-existent inverse. Estimates that are the unique solution to minimizing the sum of square errors cannot be found.

2 2

2.1 a

$$\mathbf{Pr}(deny = 1|PI, Black) = \Phi(-2.26 + 2.74 \times 0.35 + 0.71) = 0.2772602$$

2.2 b

$$\mathbf{Pr}(deny = 1|PI, Black) = \Phi(-2.26 + 2.74 \times 0.3 + 0.71) = 0.2333068$$

The lower PI ratio lowered the probability of denial. The difference in probabilities is 4.39534 percentage points lower.

2.3 c

$$\mathbf{Pr}(deny = 1|PI, White\Phi(-2.26 + 2.74 \times 0.35) = 0.09662923$$

$$\mathbf{Pr}(deny = 1|PI, White) = \Phi(-2.26 + 2.74 \times 0.3) = 0.07521703$$

Again the lower PI ratio lowered the probability of denial, and it was a difference of 2.14122 percentage points.

2.4 d

Take the t-statistic of the race estimated coefficient to compare magnitudes to critical values at standard levels of significance:

$$\frac{0.71}{0.083} = 8.554217 > 2.58$$

This is greater than the 1% level critical value, so the null that race has no effect on the marginal effect of the P/I ratio on the probability of mortgage denial can be rejected.

3 3

The logit results show that the magnitudes are slightly larger. However, the magnitudes across probit and logit models are not comparable directly. But t-statistics and predicted probabilities can be examined.

Repeating the same four computations as question 2:

$$\mathbf{Pr}(deny = 1|PI, Black) = \frac{1}{1 + \exp(-4.13 + 5.37(0.35) + 1.27)} = 0.2728$$

$$\mathbf{Pr}(deny = 1|PI, Black) = \frac{1}{1 + \exp(-4.13 + 5.37(0.3) + 1.27)} = 0.2229$$

$$\mathbf{Pr}(deny = 1|PI, White) = \frac{1}{1 + \exp(-4.13 + 5.37(0.35))} = 0.0953$$

$$\mathbf{Pr}(deny = 1|PI, White) = \frac{1}{1 + \exp(-4.13 + 5.37(0.3))} = 0.0745$$

Examining the t-statistics for the coefficients, it can be seen that

$$\frac{5.37}{0.96} = 5.59375 > 2.58$$

$$\frac{1.27}{0.15} = 8.466667 > 2.58$$

Probit model t-statistic for the estimated coefficient on PI ratio:

$$\frac{2.74}{0.44} = 6.227273 > 2.58$$

So the race dummy variable has a significant impact on the marginal effect of the P/I ratio on the probability of mortgage denial across both logit and probit models.