SPRING 2023

Information Technology Research

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Lecture slides are based on the supplemental materials of the textbook: https://algs4.cs.princeton.edu

http://algs4.cs.princeton.edu

2.2 MERGESORT

- *mergesort*
- *bottom-up mergesort*
- *comparators*
- *stability*

Two classic sorting algorithms: mergesort and quicksort

Critical components in the world's computational infrastructure.

- Full scientific understanding of their properties has enabled us to develop them into practical system sorts.
- Quicksort honored as one of top 10 algorithms of 20th century in science and engineering.

Mergesort.









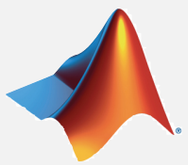
...

Quicksort.









...

2.2 Mergesort

- *mergesort*
- *bottom-up mergesort*
- *comparators*
- *stability*

Mergesort

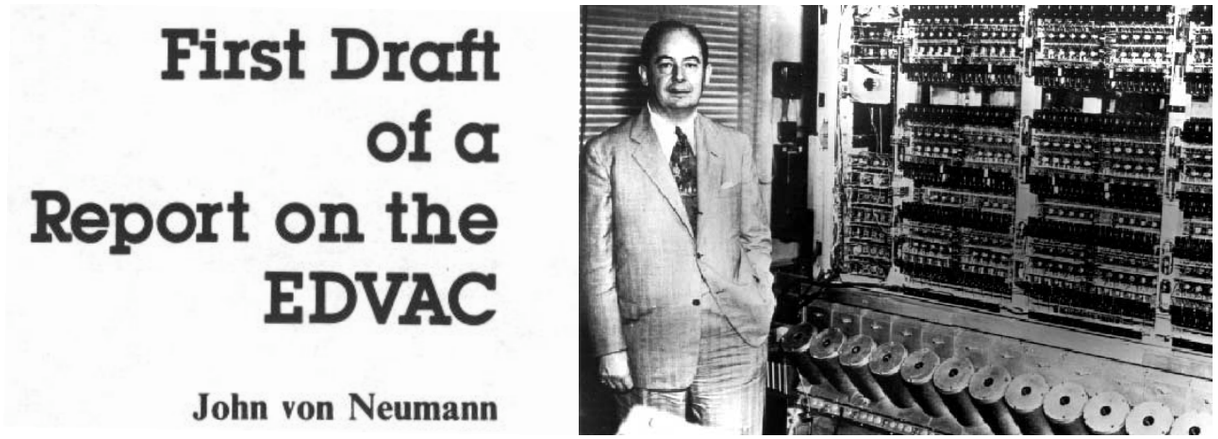
Basic plan.

- Divide array into two halves.
- Recursively sort each half.
- Merge two halves.

input	M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
sort left half	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E
sort right half	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge results	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Mergesort overview

Electronic Discrete Variable Automatic Computer (EDVAC): first program on it was mergesort.



Von Neumann architecture

Processing unit
Control unit
Memory
External mass storage
I/O mechanisms

https://en.wikipedia.org/wiki/Von_Neumann_architecture

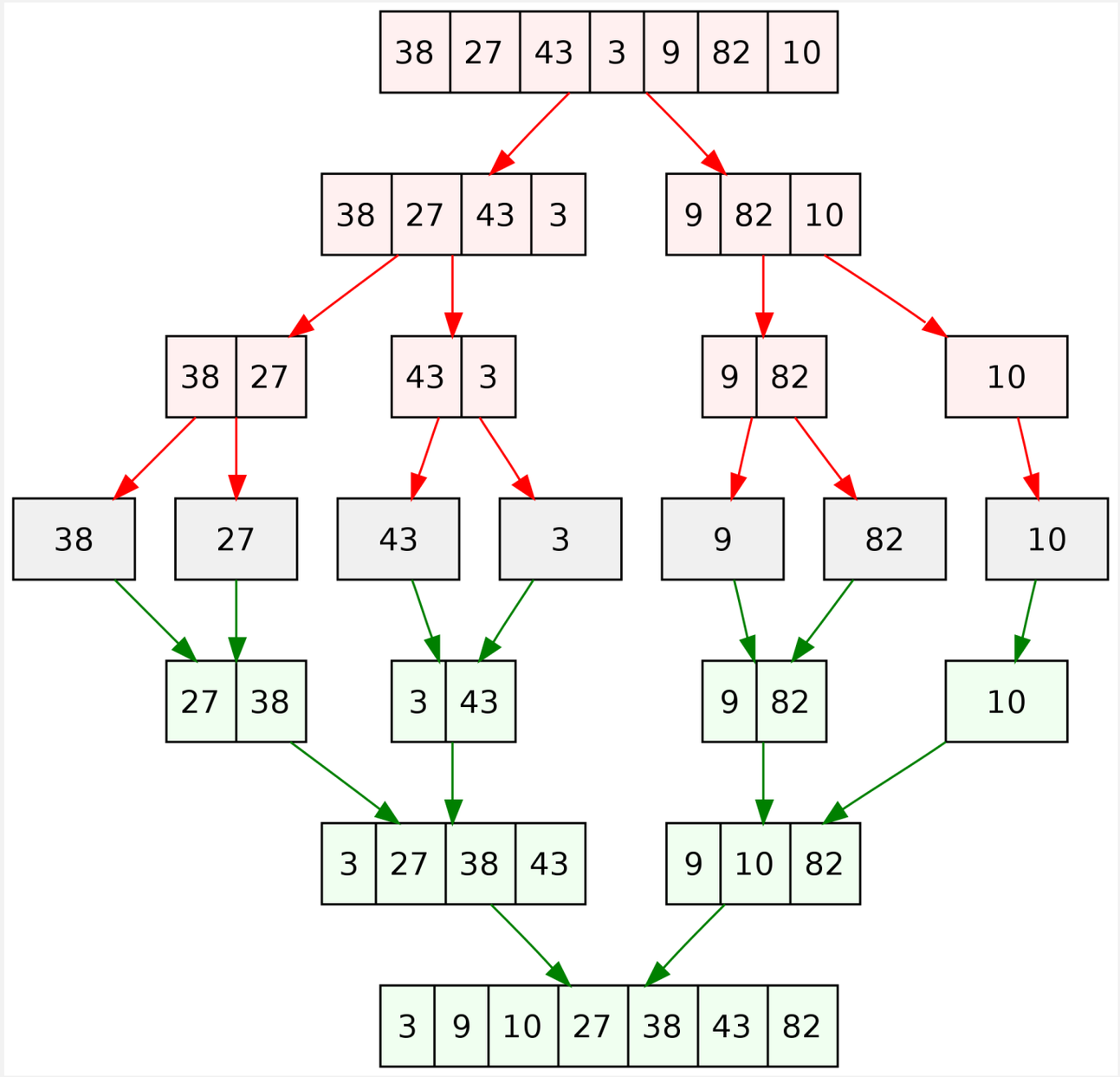
Mergesort: Divide & merge

Basic plan.

- Divide array into two halves.
- Recursively sort each half.
- Merge two halves.

Base case of single element.

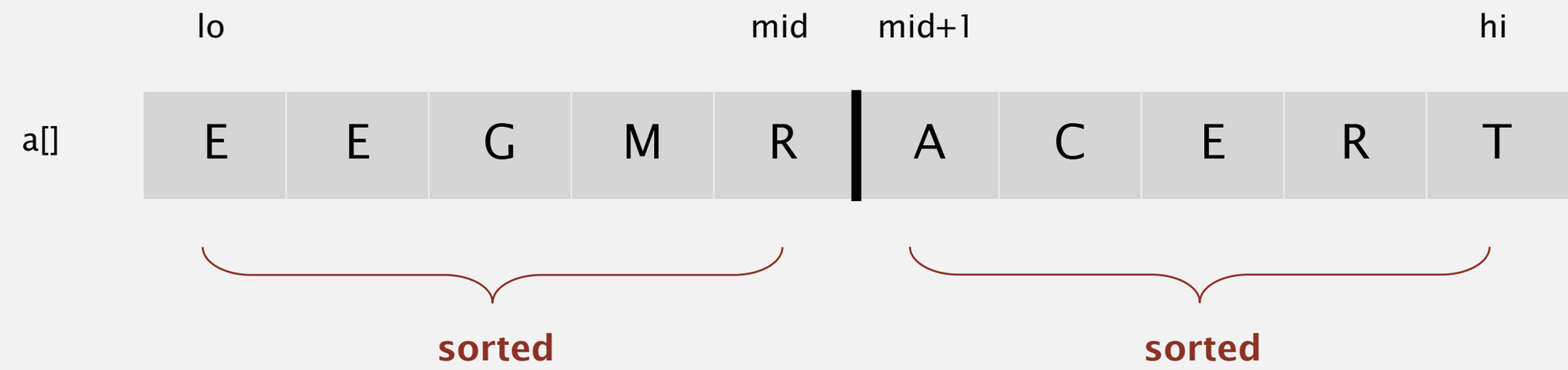
- Is well... sorted.



https://en.wikipedia.org/wiki/Merge_sort

Merging demo

Goal. Given two sorted subarrays `a[lo]` to `a[mid]` and `a[mid+1]` to `a[hi]`, replace with sorted subarray `a[lo]` to `a[hi]`.



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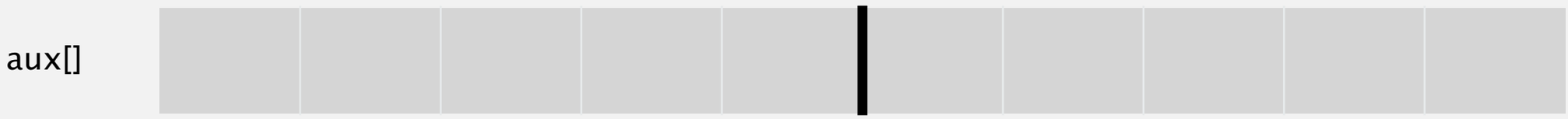
copy to auxiliary array



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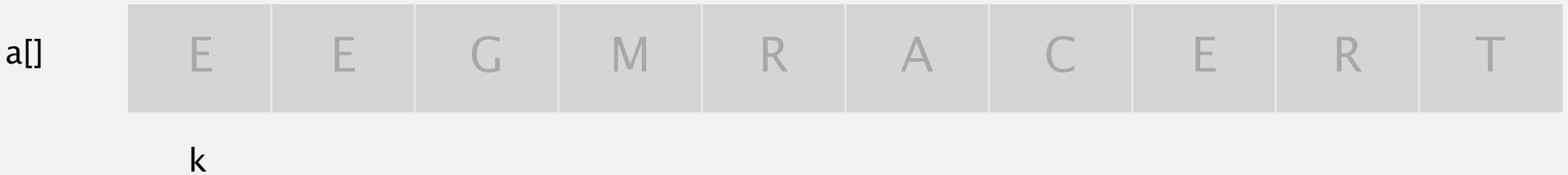
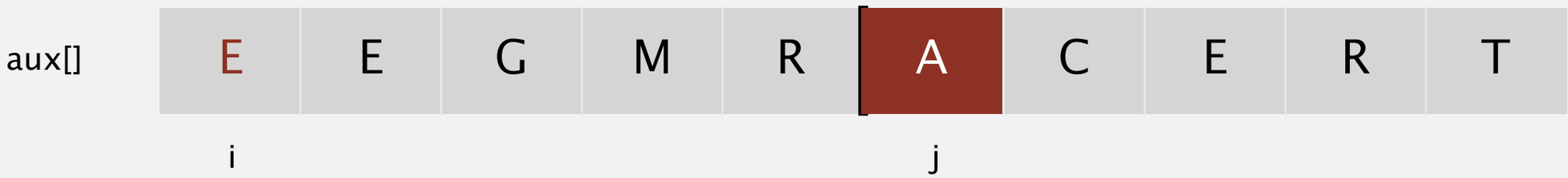


compare minimum in each subarray



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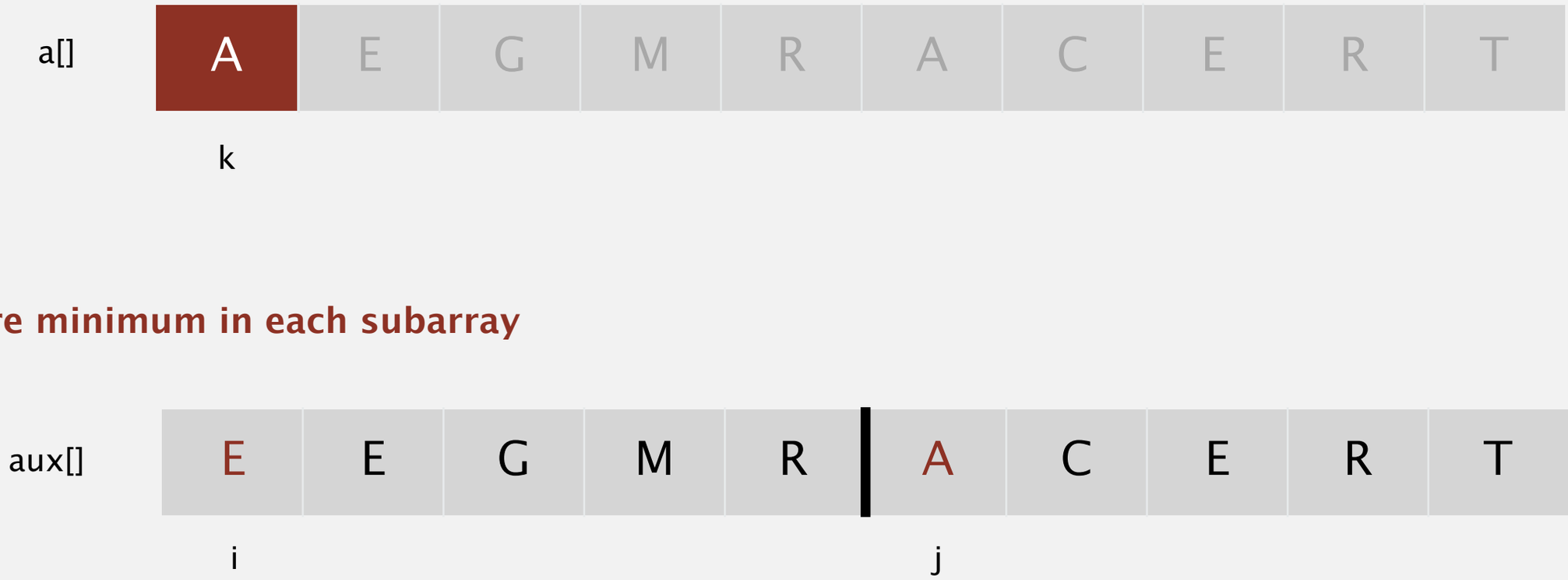

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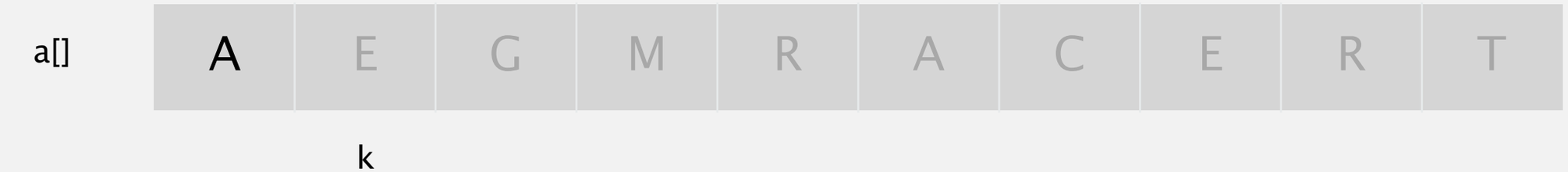

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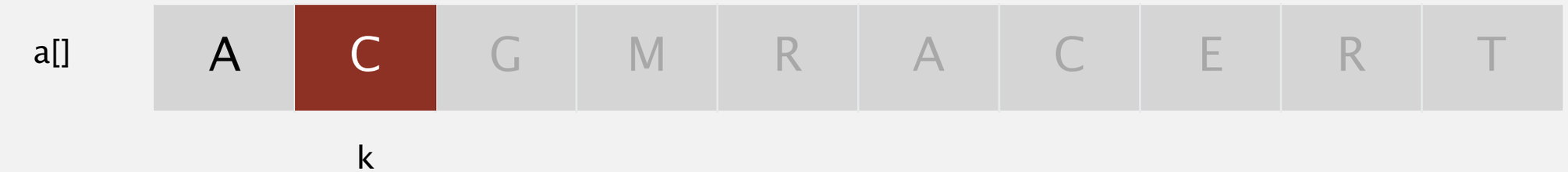

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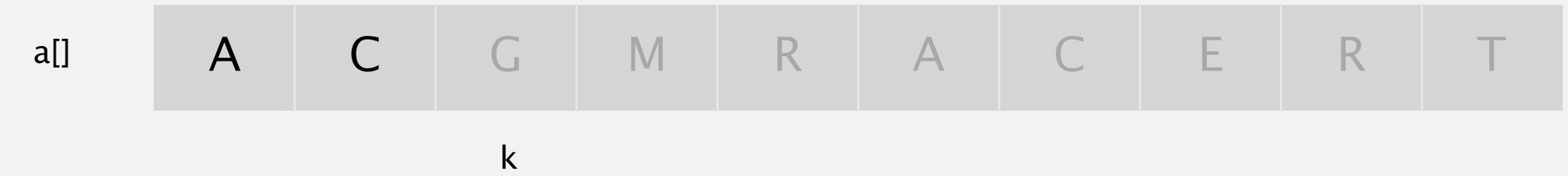

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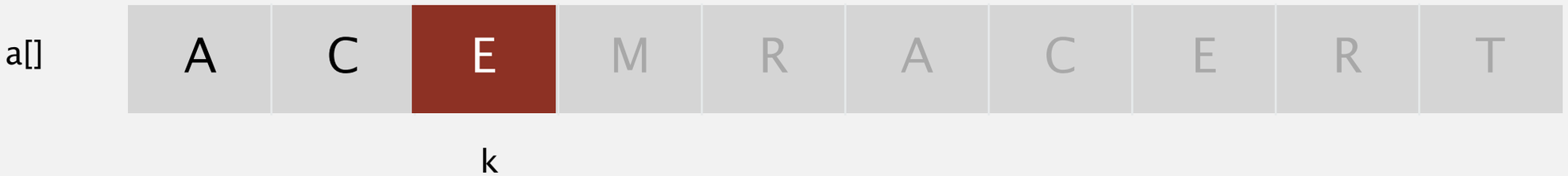

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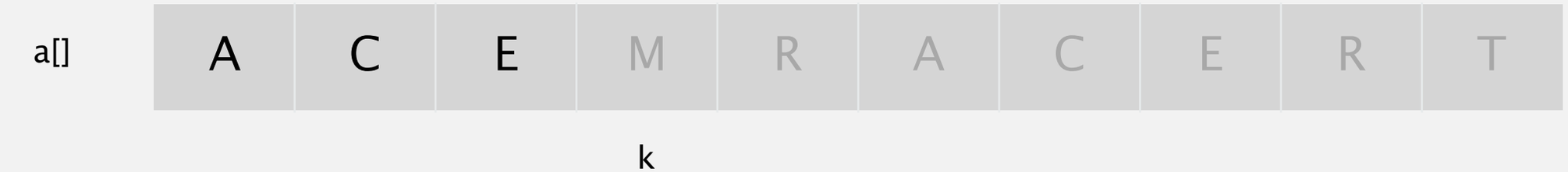

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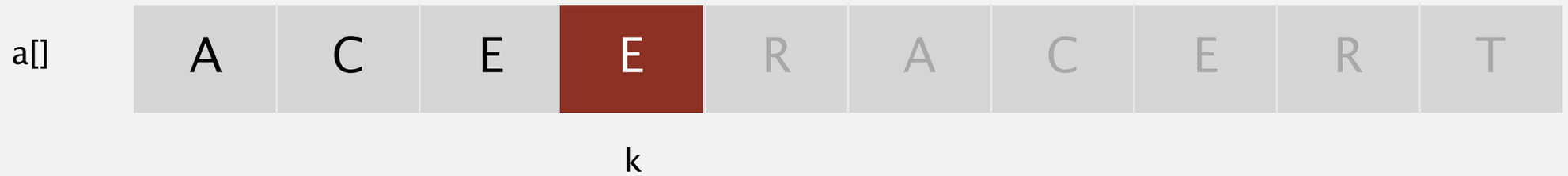

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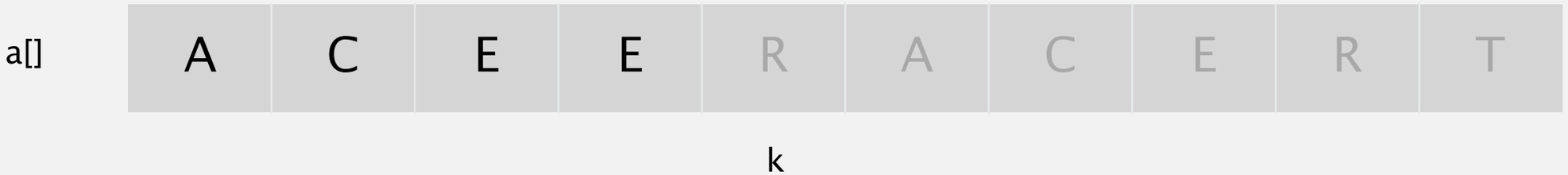

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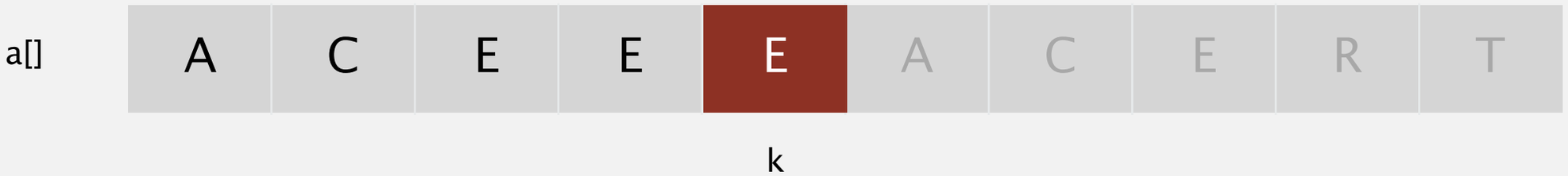

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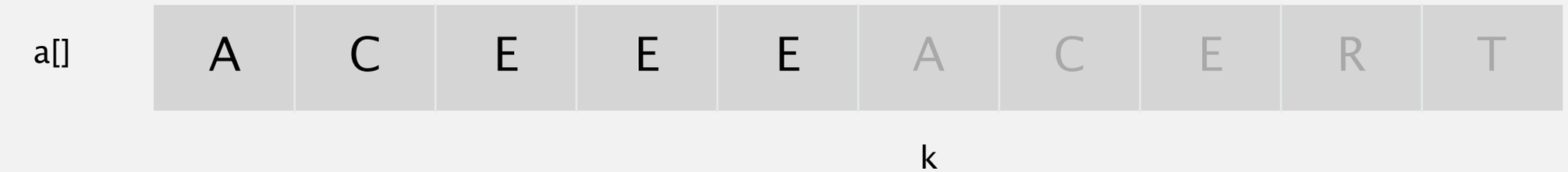

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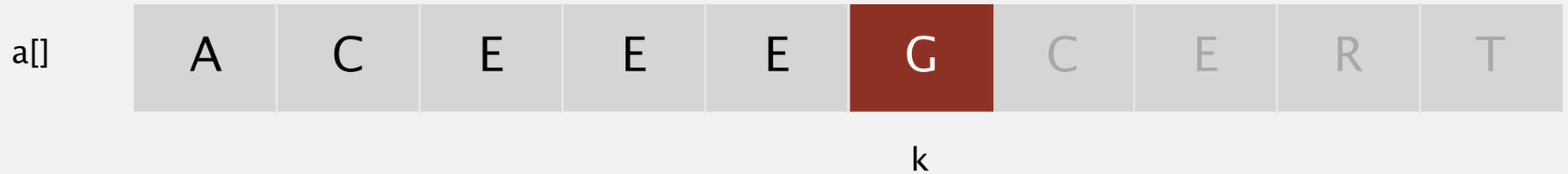

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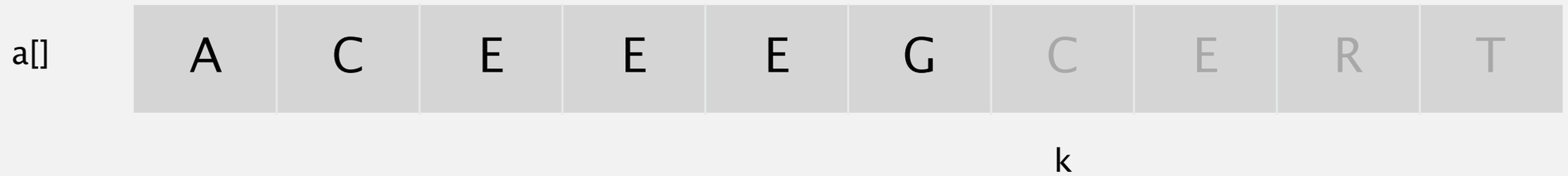

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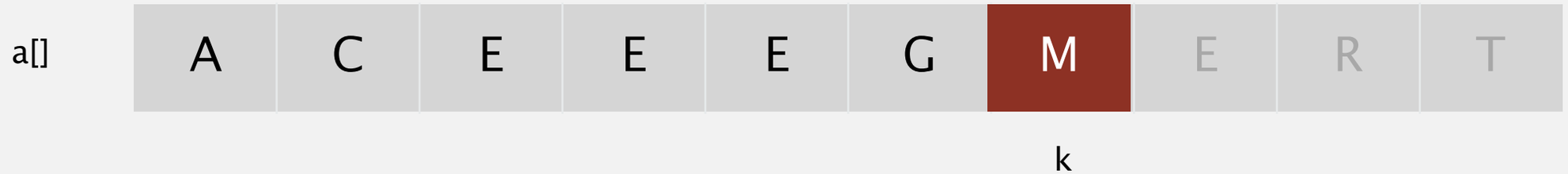

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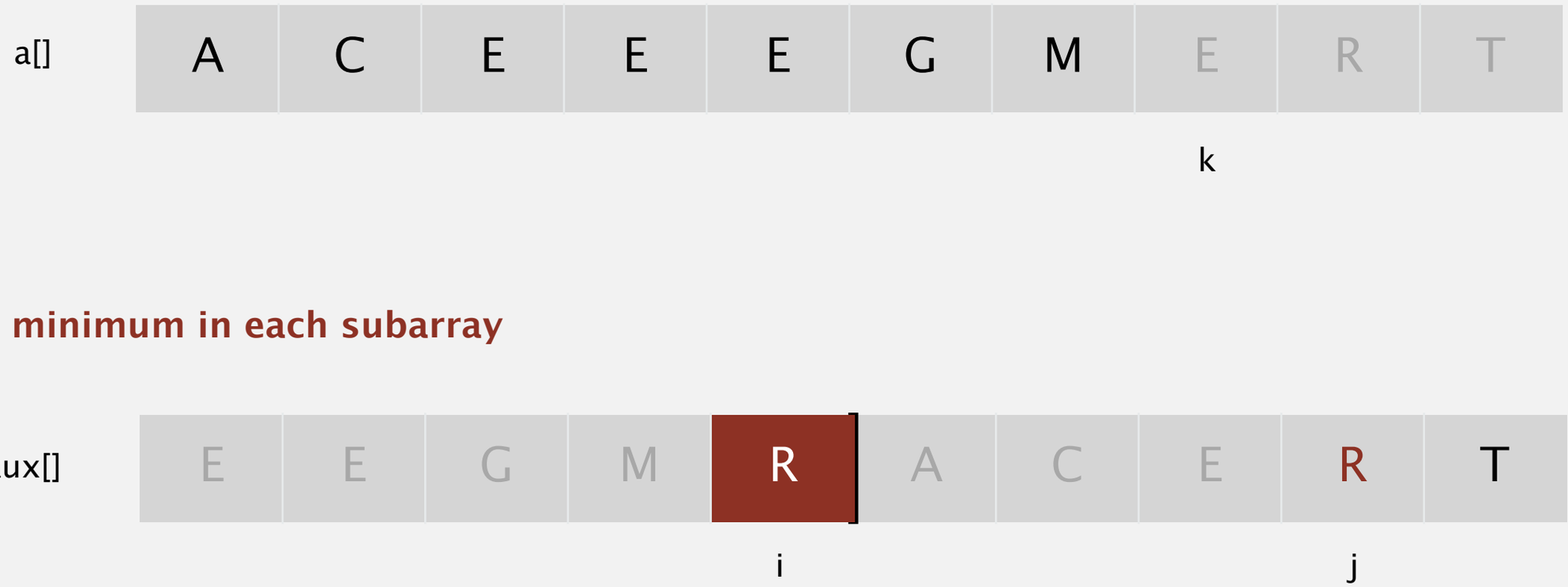

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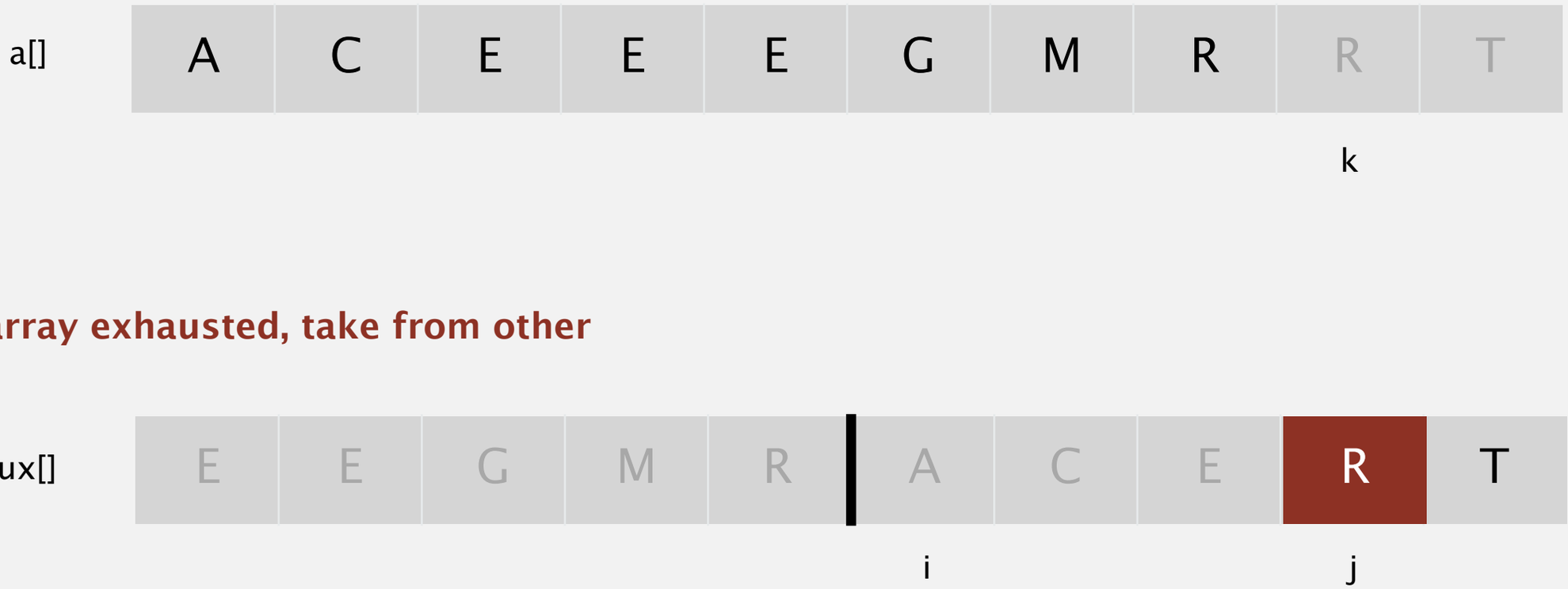

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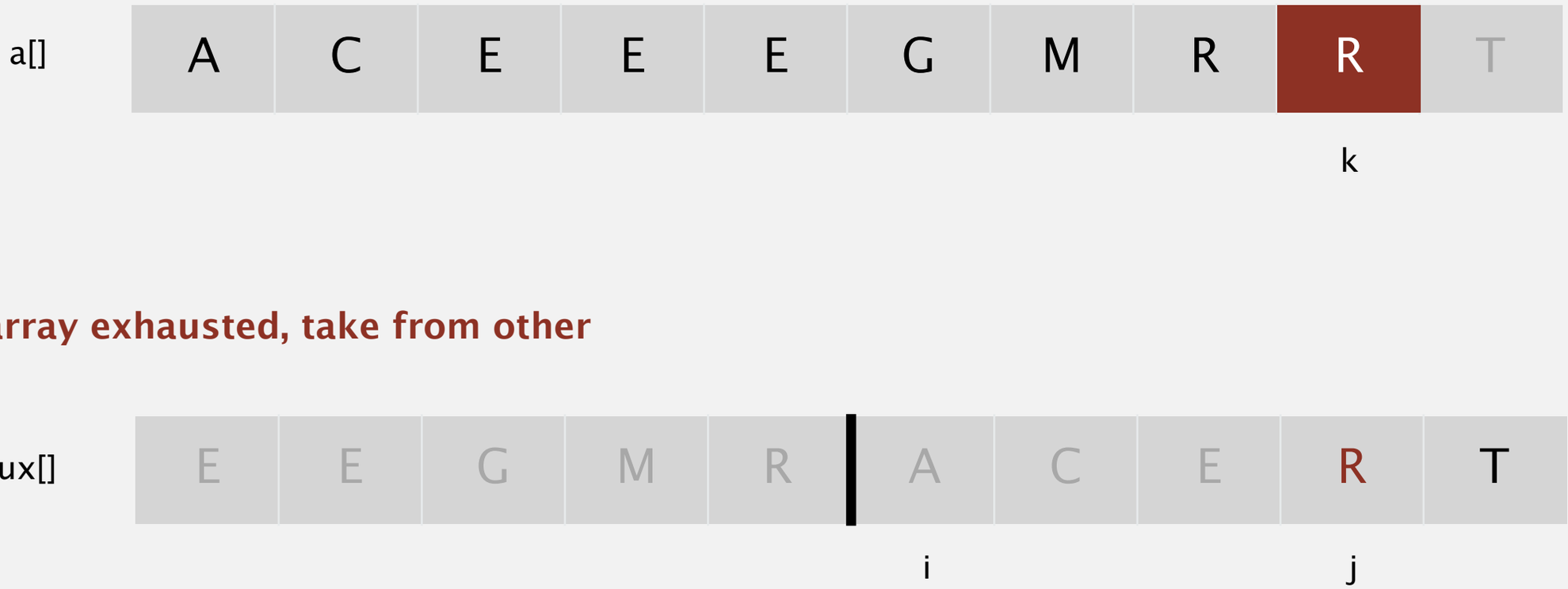

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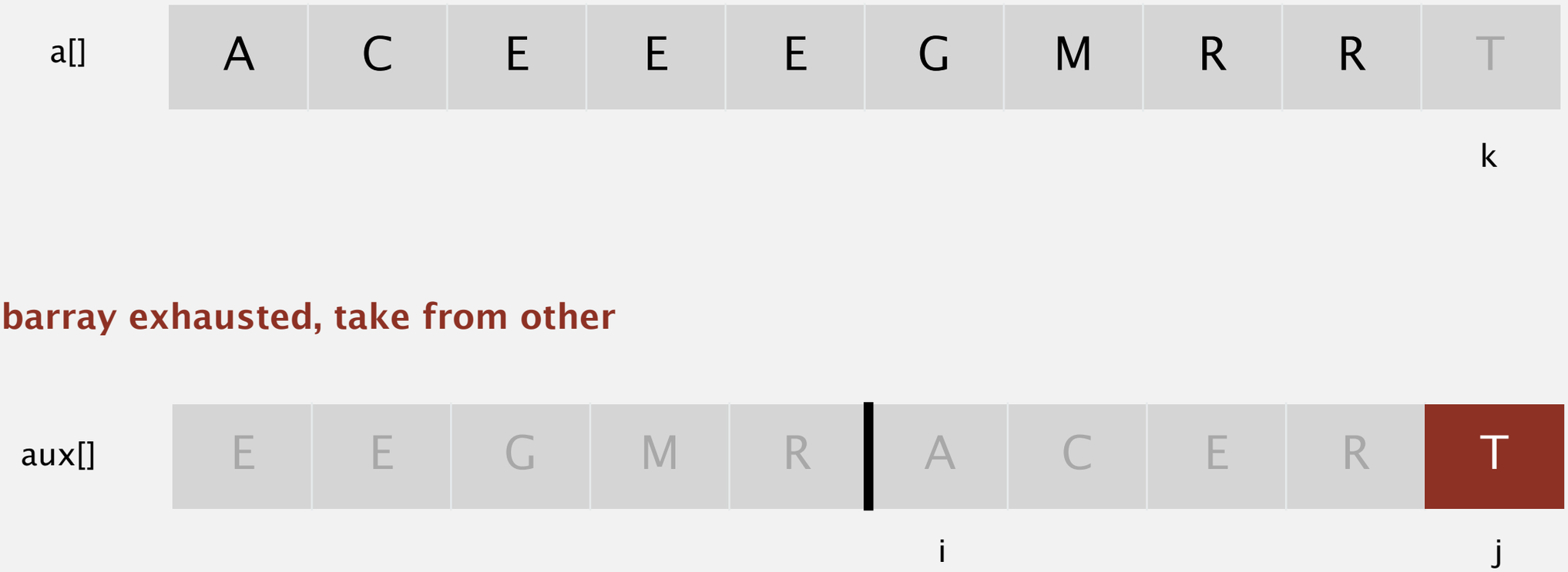

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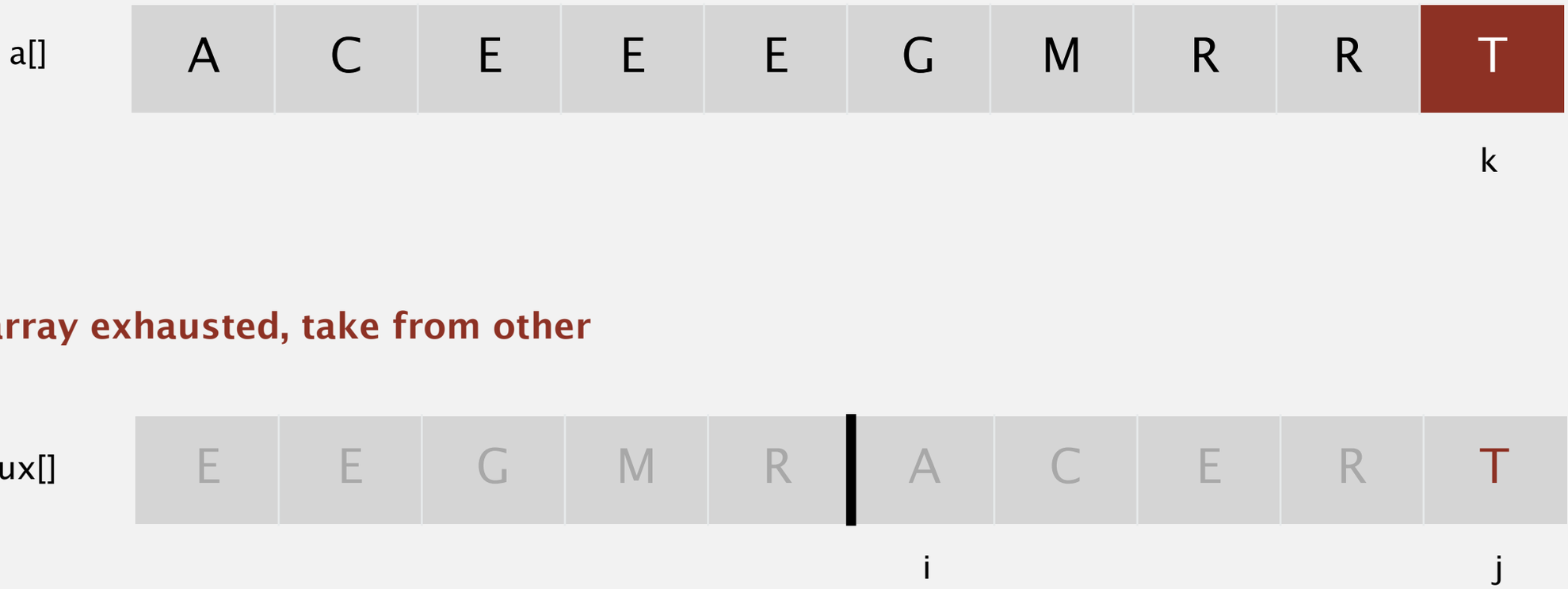

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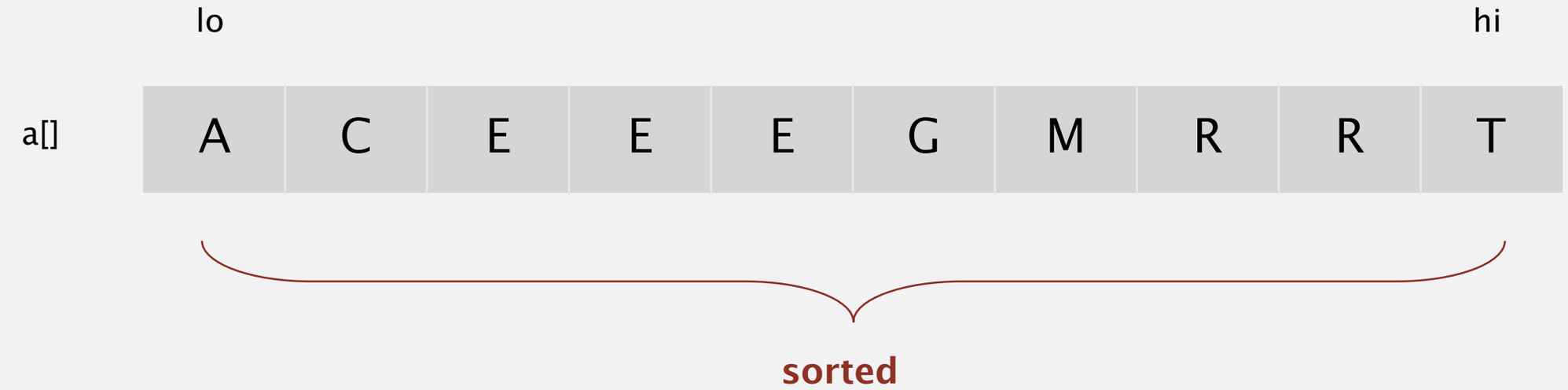


Merging: Java implementation

```
private static void merge(Comparable[] a, Comparable[] aux, int lo, int mid, int hi)
{

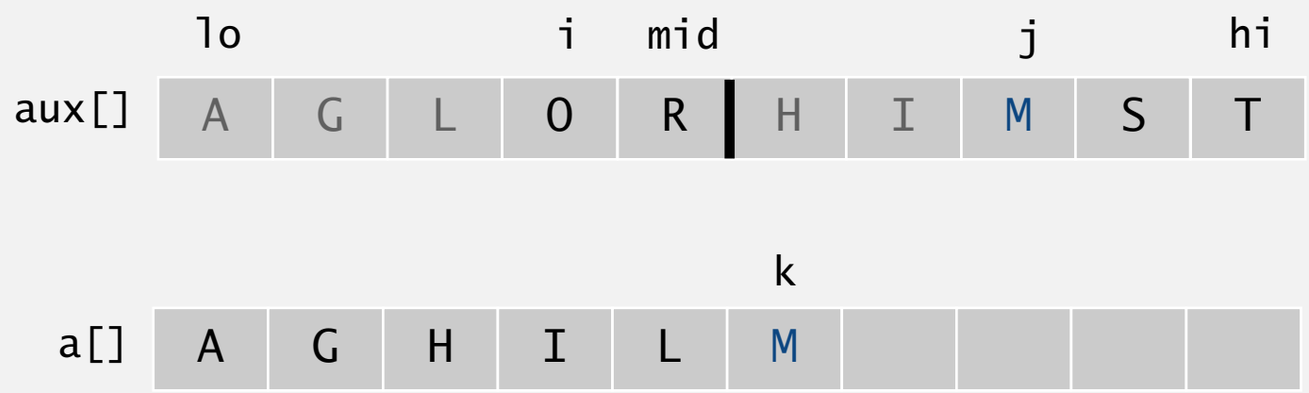
   for (int k = lo; k <= hi; k++)
      aux[k] = a[k];

   int i = lo, j = mid+1;
   for (int k = lo; k <= hi; k++)
   {
      if      (i > mid)                a[k] = aux[j++];
      else if (j > hi)                 a[k] = aux[i++];
      else if (less(aux[j], aux[i]))   a[k] = aux[j++];
      else                             a[k] = aux[i++];
   }
}
```

copy

merge

What to write here? Pen and Paper, 4 minutes. Use less for comparison.



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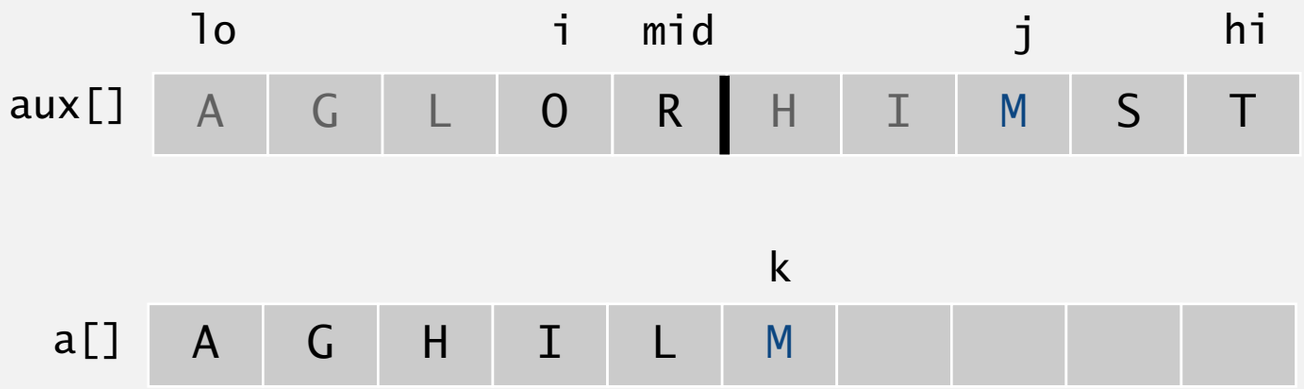
      else                               a[k] = aux[i++];
   }
}
```

copy

merge



Assertions

Assertion. Statement to test assumptions about your program.

- Helps detect logic bugs.
- Documents code.

Java assert statement. Throws exception unless boolean condition is true.

```
assert isSorted(a, lo, hi);
```

Can enable or disable at runtime. ⇒ No cost in production code.

```
% java -ea MyProgram   // enable assertions
% java -da MyProgram   // disable assertions (default)
```

Best practices. Use assertions to check internal invariants; assume assertions will be disabled in production code.

Merging: Java implementation

```
private static void merge(Comparable[] a, Comparable[] aux, int lo, int mid, int hi)
{
   assert isSorted(a, lo, mid);    // precondition: a[lo..mid]   sorted
   assert isSorted(a, mid+1, hi);  // precondition: a[mid+1..hi] sorted

   for (int k = lo; k <= hi; k++)                                          copy
      aux[k] = a[k];

   int i = lo, j = mid+1;
   for (int k = lo; k <= hi; k++)                                          merge
   {
      if       (i > mid)                  a[k] = aux[j++];
      else if (j > hi)                    a[k] = aux[i++];
      else if (less(aux[j], aux[i])) a[k] = aux[j++];
      else                                a[k] = aux[i++];
   }

   assert isSorted(a, lo, hi);     // postcondition: a[lo..hi] sorted
}
```

Mergesort: Java implementation

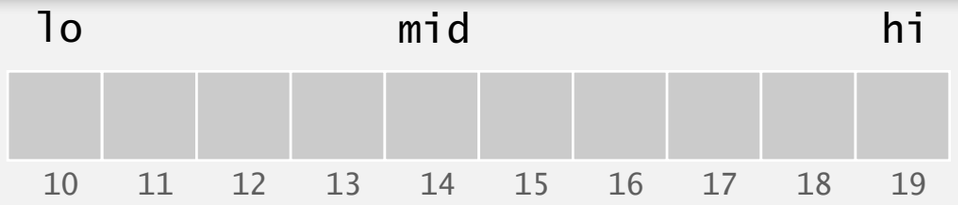
```
public class Merge
{
   private static void merge(Comparable[] a, Comparable[] aux, int lo, int mid,
int hi)
   {  /* as before */  }

   private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
   {
      if (hi <= lo) return;
      int mid = lo + (hi - lo) / 2;

      What to write here?
      Pen and Paper, 3 minutes. Hint: recursive.

   }

   public static void sort(Comparable[] a)
   {
      Comparable[] aux = new Comparable[a.length];
      sort(a, aux, 0, a.length - 1);
   }
}
```



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   private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
   {
      if (hi <= lo) return;
      int mid = lo + (hi - lo) / 2;
      sort(a, aux, lo, mid);
      sort(a, aux, mid+1, hi);
      merge(a, aux, lo, mid, hi);
   }

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   {
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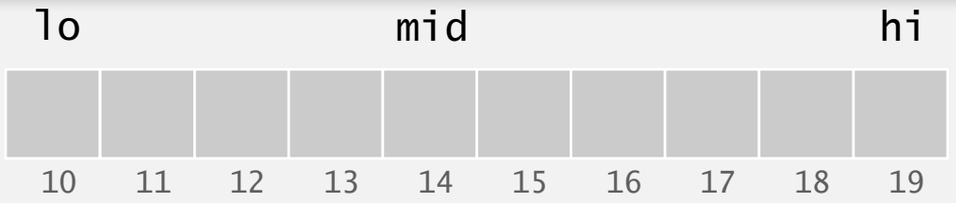


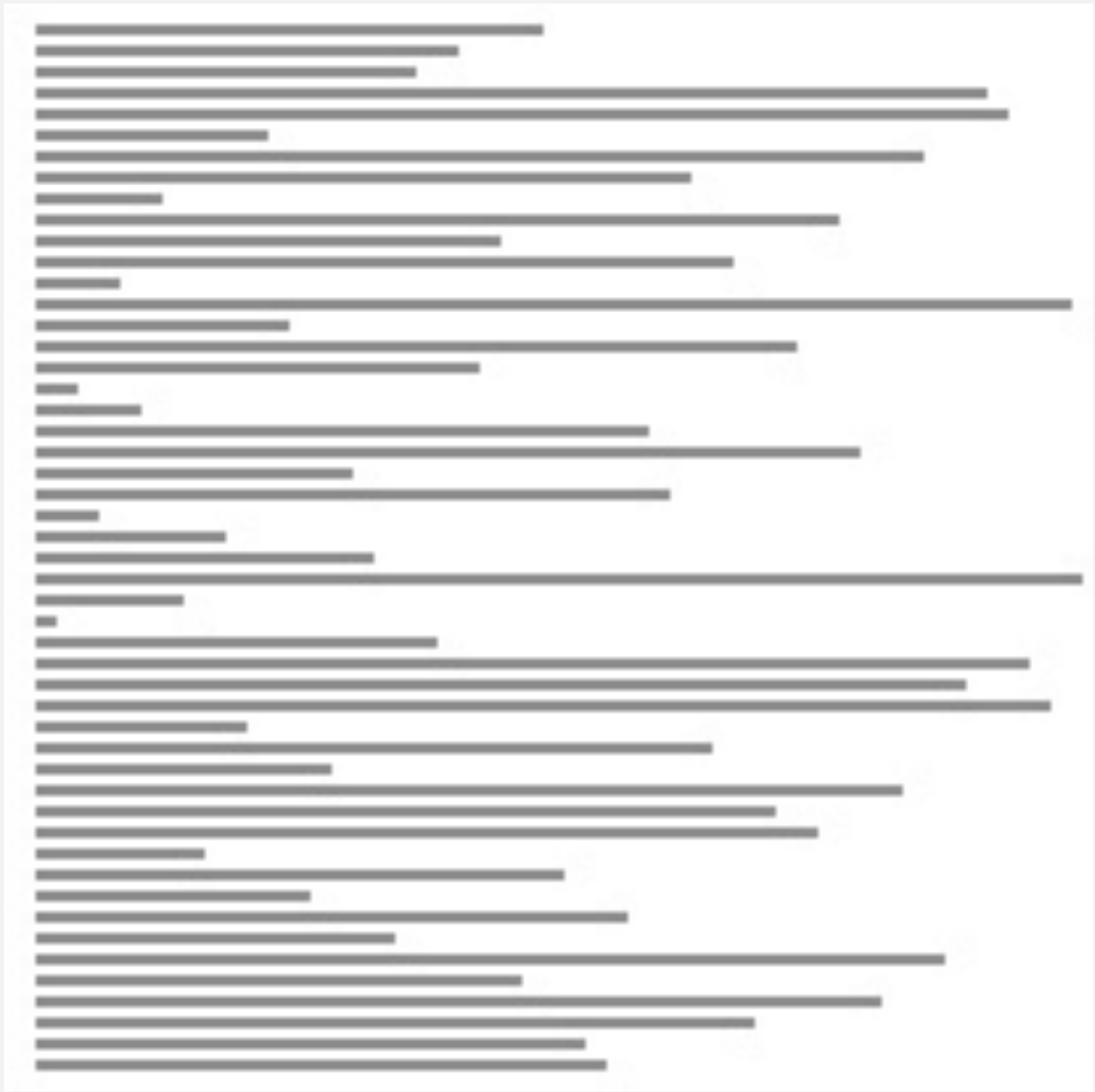
Mergesort: trace

	lo	hi	a[] 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
			M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 0, 0, 1)	0	1	E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 2, 2, 3)	2	3	E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 0, 1, 3)	0	3	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 4, 4, 5)	4	5	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 6, 6, 7)	6	7	E	G	M	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 4, 5, 7)	4	7	E	G	M	R	E	O	R	S	T	E	X	A	M	P	L	E
merge(a, aux, 0, 3, 7)	0	7	E	E	G	M	O	R	R	S	T	E	X	A	M	P	L	E
merge(a, aux, 8, 8, 9)	8	9	E	E	G	M	O	R	R	S	E	T	X	A	M	P	L	E
merge(a, aux, 10, 10, 11)	10	11	E	E	G	M	O	R	R	S	E	T	A	X	M	P	L	E
merge(a, aux, 8, 9, 11)	8	11	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E
merge(a, aux, 12, 12, 13)	12	13	E	E	G	M	O	R	R	S	A	E	T	X	M	P	L	E
merge(a, aux, 14, 14, 15)	14	15	E	E	G	M	O	R	R	S	A	E	T	X	M	P	E	L
merge(a, aux, 12, 13, 15)	12	15	E	E	G	M	O	R	R	S	A	E	T	X	E	L	M	P
merge(a, aux, 8, 11, 15)	8	15	E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
merge(a, aux, 0, 7, 15)	0	15	A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

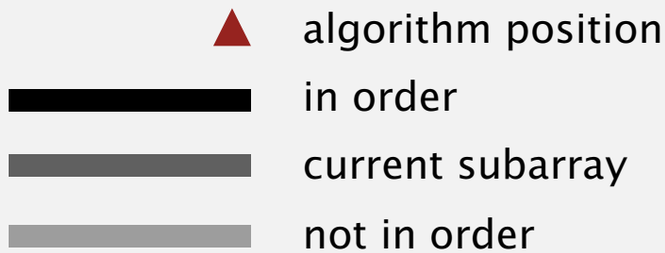
result after recursive call

Mergesort: animation

50 random items



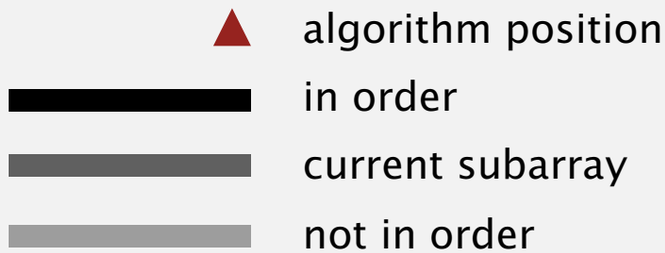
http://www.sorting-algorithms.com/merge-sort



Mergesort: animation

50 reverse-sorted items

http://www.sorting-algorithms.com/merge-sort



Mergesort: empirical analysis

Running time estimates:

- Laptop executes 10^8 compares/second.
- Supercomputer executes 10^{12} compares/second.

	insertion sort (N^2)			mergesort (N log N)		
computer	thousand	million	billion	thousand	million	billion
home	instant	2.8 hours	317 years	instant	1 second	18 min
super	instant	1 second	1 week	instant	instant	instant

Bottom line. Good algorithms are better than supercomputers.

Mergesort: number of compares

Proposition. Mergesort uses $\leq N \lg N$ compares to sort an array of length N.

Pf sketch. The number of compares $T(N)$ to mergesort an array of length N satisfies the recurrence:

$$T(N) \leq T(\lceil N/2 \rceil) + T(\lfloor N/2 \rfloor) + N \quad \text{for } N > 1, \text{ with } T(1) = 0.$$

left half | right half | merge

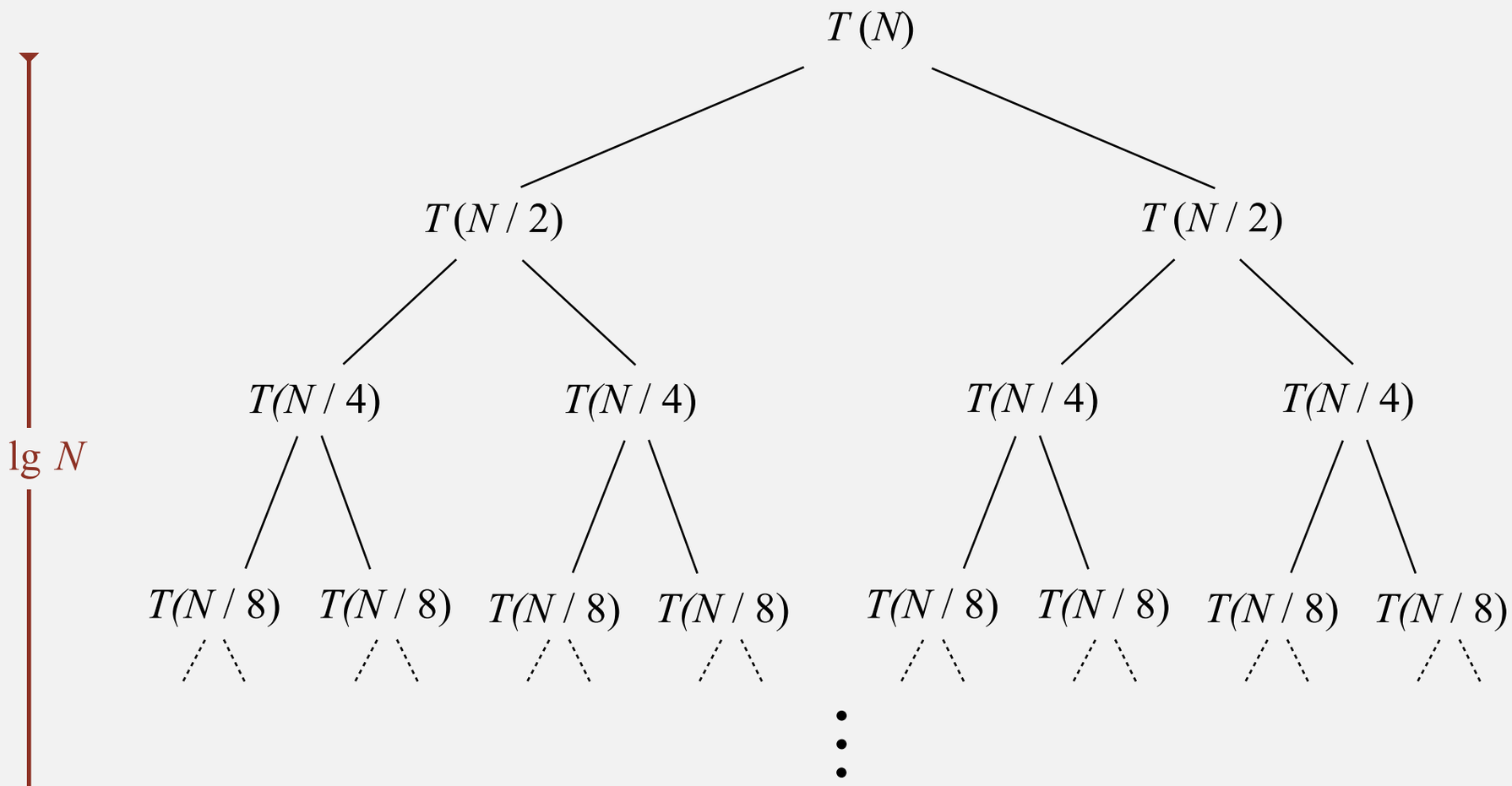
We solve the recurrence when N is a power of 2: ← result holds for all N (analysis cleaner in this case)

$$T(N) = 2\,T(N/2) + N, \text{ for } N > 1, \text{ with } T(1) = 0.$$

Divide-and-conquer recurrence: proof by picture

Proposition. If $T(N)$ satisfies $T(N) = 2\,T(N/2) + N$ for $N > 1$, with $T(1) = 0$, then $T(N) = N \lg N$.

Pf 1. [assuming N is a power of 2]



N	$= N$
$2\,(N/2)$	$= N$
$4\,(N/4)$	$= N$
$8\,(N/8)$	$= N$
⋮	⋮

$T(N) = N \lg N$

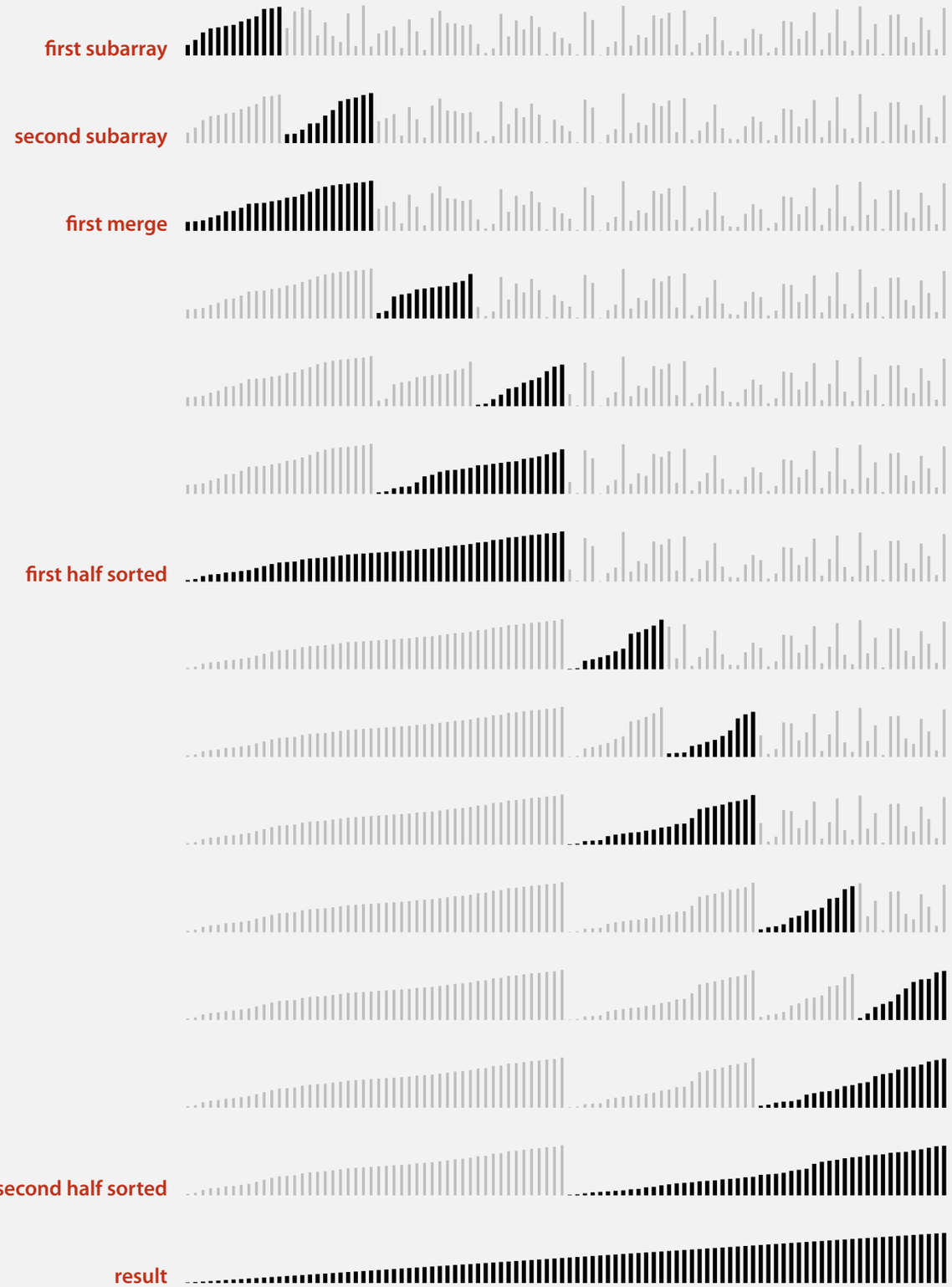
Mergesort: practical improvements

Use insertion sort for small subarrays.

- Mergesort has too much overhead for tiny subarrays.
- Cutoff to insertion sort for ≈ 10 items.

```
private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
{
   if (hi <= lo + CUTOFF - 1)
   {
      Insertion.sort(a, lo, hi);
      return;
   }
   int mid = lo + (hi - lo) / 2;
   sort (a, aux, lo, mid);
   sort (a, aux, mid+1, hi);
   merge(a, aux, lo, mid, hi);
}
```

Mergesort with cutoff to insertion sort: visualization



Mergesort: practical improvements

Stop if already sorted.

- Is largest item in first half ≤ smallest item in second half?
- Helps for partially-ordered arrays.

```
A B C D E F G H I J M N O P Q R S T U V

A B C D E F G H I J M N O P Q R S T U V
```

```
private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
{
   if (hi <= lo) return;
   int mid = lo + (hi - lo) / 2;
   sort (a, aux, lo, mid);
   sort (a, aux, mid+1, hi);

What to write here? Pen and Paper, 2 minutes.

   merge(a, aux, lo, mid, hi);
}
```

Mergesort: practical improvements

Stop if already sorted.

- Is largest item in first half ≤ smallest item in second half?
- Helps for partially-ordered arrays.

```
A B C D E F G H I J M N O P Q R S T U V

A B C D E F G H I J M N O P Q R S T U V
```

```
private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
{
   if (hi <= lo) return;
   int mid = lo + (hi - lo) / 2;
   sort (a, aux, lo, mid);
   sort (a, aux, mid+1, hi);

   if (!less(a[mid+1], a[mid])) return;

   merge(a, aux, lo, mid, hi);
}
```

http://algs4.cs.princeton.edu

2.2 Mergesort

- *mergesort*
- *bottom-up mergesort*
- *comparators*
- *stability*

Bottom-up mergesort

Basic plan.

- Pass through array, merging subarrays of size 1.
- Repeat for subarrays of size 2, 4, 8, ...

		a[i]															
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
		M	E	R	G	E	S	O	R	T	E	X	A	M	P	L	E
sz = 1																	
merge(a, aux, 0, 0, 1)		E	M	R	G	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 2, 2, 3)		E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 4, 4, 5)		E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 6, 6, 7)		E	M	G	R	E	S	O	R	T	E	X	A	M	P	L	E
merge(a, aux, 8, 8, 9)		E	M	G	R	E	S	O	R	E	T	X	A	M	P	L	E
merge(a, aux, 10, 10, 11)		E	M	G	R	E	S	O	R	E	T	A	X	M	P	L	E
merge(a, aux, 12, 12, 13)		E	M	G	R	E	S	O	R	E	T	A	X	M	P	L	E
merge(a, aux, 14, 14, 15)		E	M	G	R	E	S	O	R	E	T	A	X	M	P	E	L
sz = 2																	
merge(a, aux, 0, 1, 3)		E	G	M	R	E	S	O	R	E	T	A	X	M	P	E	L
merge(a, aux, 4, 5, 7)		E	G	M	R	E	O	R	S	E	T	A	X	M	P	E	L
merge(a, aux, 8, 9, 11)		E	G	M	R	E	O	R	S	A	E	T	X	M	P	E	L
merge(a, aux, 12, 13, 15)		E	G	M	R	E	O	R	S	A	E	T	X	E	L	M	P
sz = 4																	
merge(a, aux, 0, 3, 7)		E	E	G	M	O	R	R	S	A	E	T	X	E	L	M	P
merge(a, aux, 8, 11, 15)		E	E	G	M	O	R	R	S	A	E	E	L	M	P	T	X
sz = 8																	
merge(a, aux, 0, 7, 15)		A	E	E	E	E	G	L	M	M	O	P	R	R	S	T	X

Bottom-up mergesort: Java implementation

```
public class MergeBU
{
   private static void merge(Comparable[] a, Comparable[] aux, int
lo, int mid, int hi)
   {  /* as before */  }

   public static void sort(Comparable[] a)
   {
      int N = a.length;
      Comparable[] aux = new Comparable[N];
      for (int sz = 1; sz < N; sz = sz+sz)
         for (int lo = 0;
            merge(a, aux,
   }
}
```

What to write here? Pen and Paper, 4 minutes.

Bottom line. Simple and non-recursive version of mergesort.

Bottom-up mergesort: Java implementation

```
public class MergeBU
{
   private static void merge(Comparable[] a, Comparable[] aux, int
lo, int mid, int hi)
   {  /* as before */  }

   public static void sort(Comparable[] a)
   {
      int N = a.length;
      Comparable[] aux = new Comparable[N];
      for (int sz = 1; sz < N; sz = sz+sz)
         for (int lo = 0; lo < N-sz; lo += sz+sz)
            merge(a, aux,
   }
}
```

Bottom line. Simple and non-recursive version of mergesort.

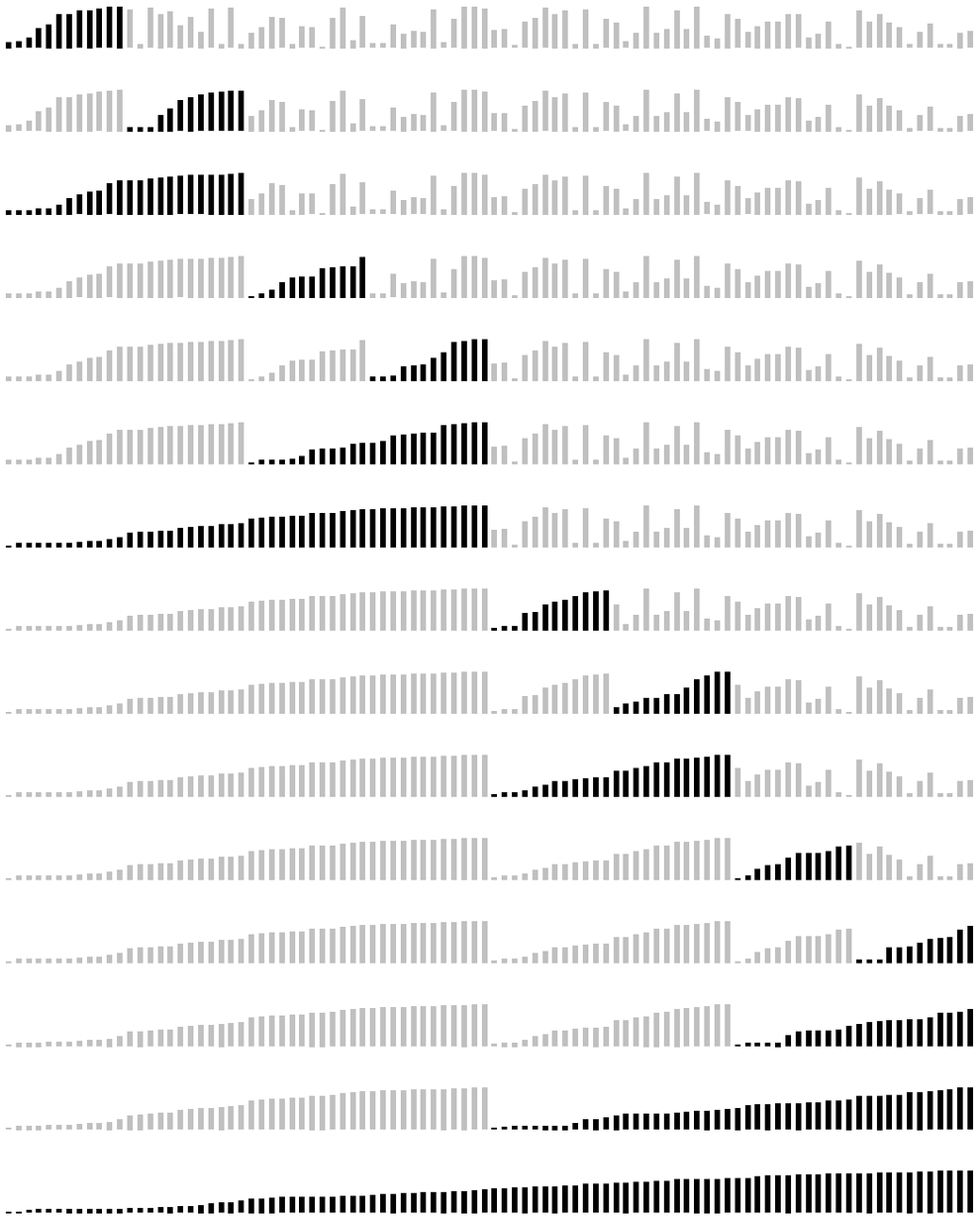
Bottom-up mergesort: Java implementation

```
public class MergeBU
{
   private static void merge(Comparable[] a, Comparable[] aux, int
lo, int mid, int hi)
   {  /* as before */  }

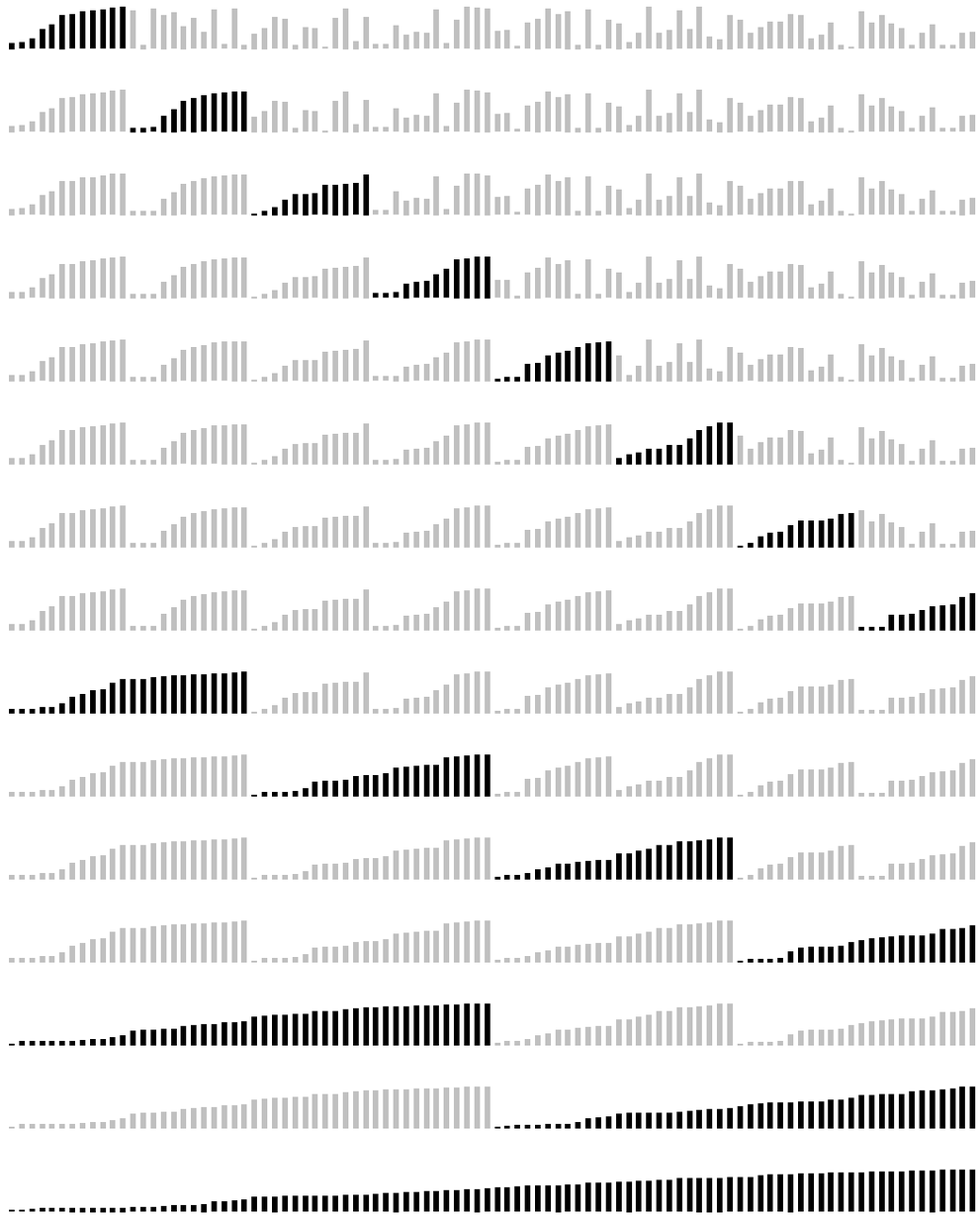
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      Comparable[] aux = new Comparable[N];
      for (int sz = 1; sz < N; sz = sz+sz)
         for (int lo = 0; lo < N-sz; lo += sz+sz)
            merge(a, aux, lo, lo+sz-1, Math.min(lo+sz+sz-1, N-1));
   }
}
```

Bottom line. Simple and non-recursive version of mergesort.

Mergesort: visualizations



top-down mergesort (cutoff = 12)



bottom-up mergesort (cutoff = 12)

Natural mergesort

Idea. Exploit pre-existing order by identifying naturally-occurring runs.

input

1	5	10	16	3	4	23	9	13	2	7	8	12	14

first run

1	5	10	16	3	4	23	9	13	2	7	8	12	14

second run

1	5	10	16	3	4	23	9	13	2	7	8	12	14

merge two runs

1	3	4	5	10	16	23	9	13	2	7	8	12	14

Tradeoff. Fewer passes vs. extra compares per pass to identify runs.

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2.2 Mergesort

- *mergesort*
- *bottom-up mergesort*
- *comparators*
- *stability*

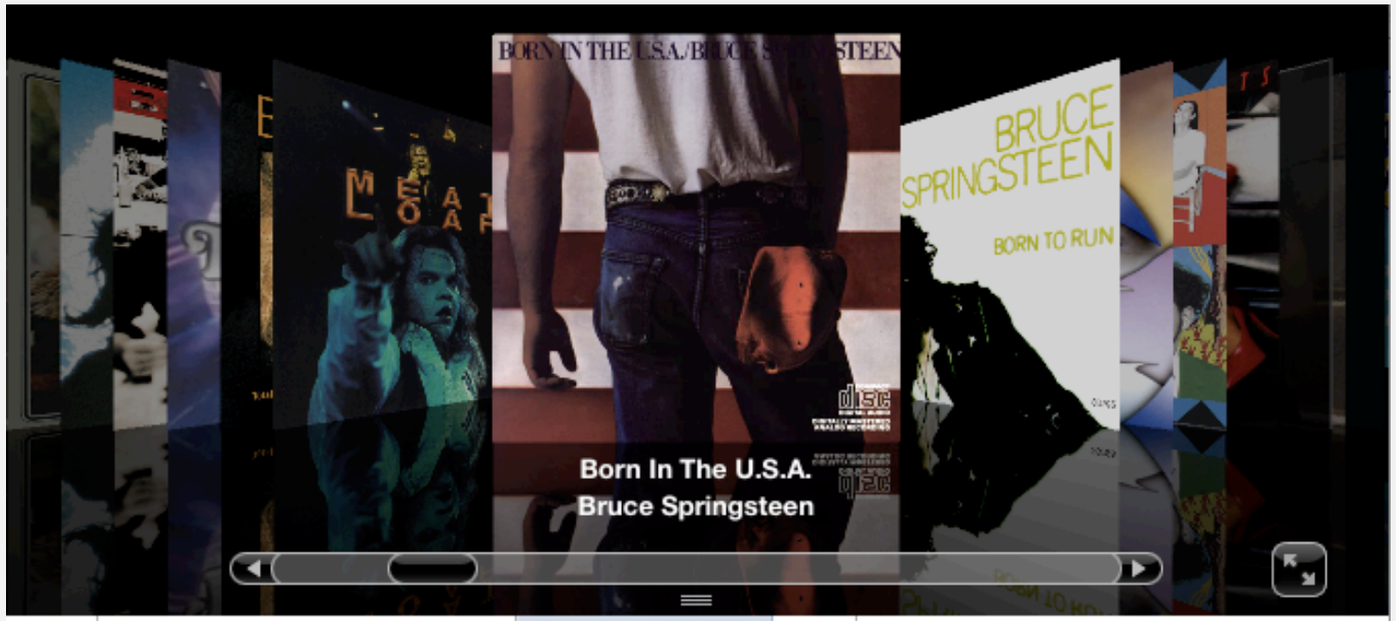
Sort countries by gold medals

NOC	Gold	Silver	Bronze	Total
United States (USA)	46	29	29	104
China (CHN)§	38	28	22	88
Great Britain (GBR)*	29	17	19	65
Russia (RUS)§	24	25	32	81
South Korea (KOR)	13	8	7	28
Germany (GER)	11	19	14	44
France (FRA)	11	11	12	34
Italy (ITA)	8	9	11	28
Hungary (HUN)§	8	4	6	18
Australia (AUS)	7	16	12	35

Sort countries by total medals

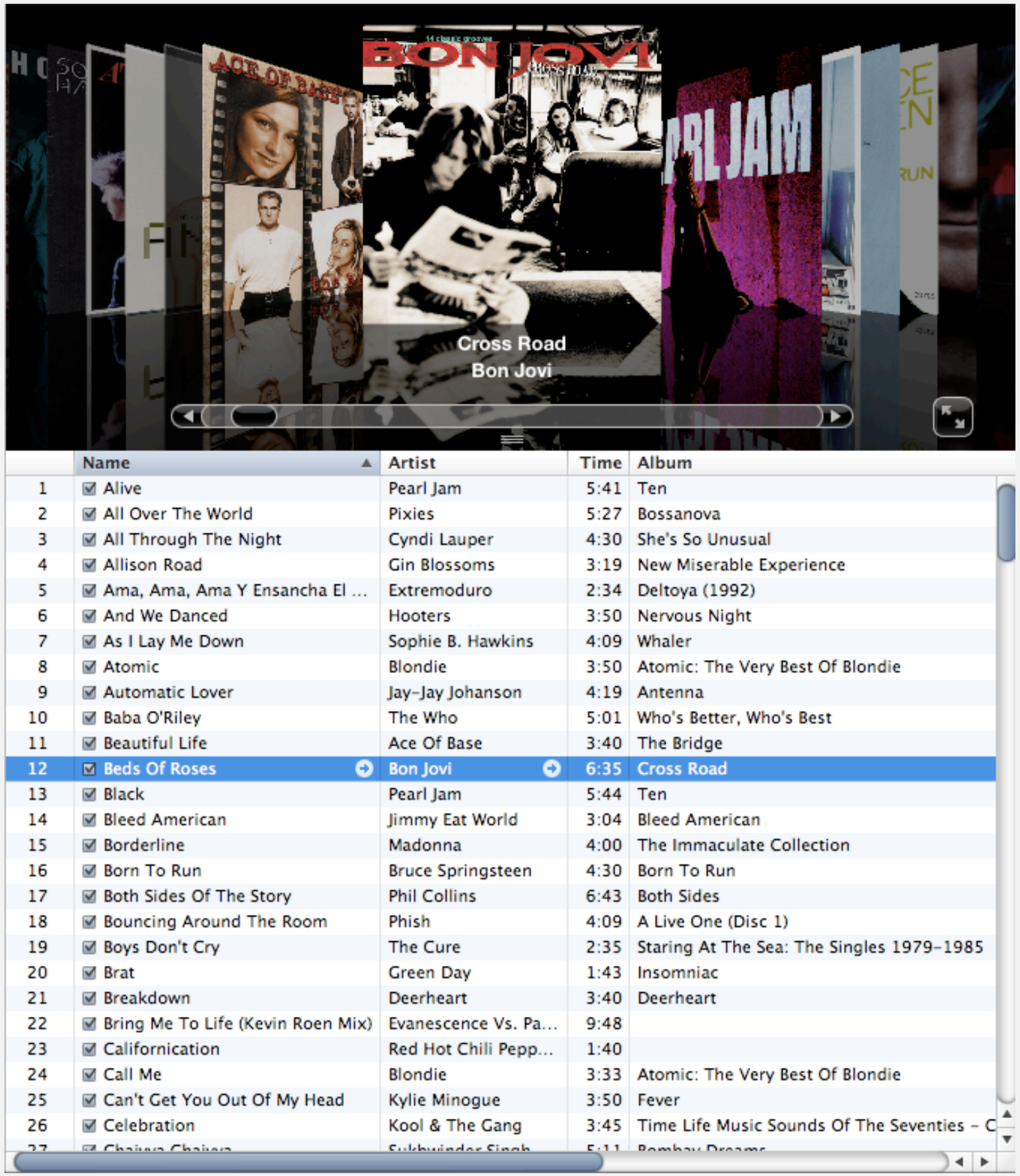
NOC	Gold	Silver	Bronze	Total
United States (USA)	46	29	29	104
China (CHN)§	38	28	22	88
Russia (RUS)§	24	25	32	81
Great Britain (GBR)*	29	17	19	65
Germany (GER)	11	19	14	44
Japan (JPN)	7	14	17	38
Australia (AUS)	7	16	12	35
France (FRA)	11	11	12	34
South Korea (KOR)	13	8	7	28
Italy (ITA)	8	9	11	28

Sort music library by artist



	Name	Artist	Time	Album
12	Let It Be	The Beatles	4:03	Let It Be
13	Take My Breath Away	BERLIN	4:13	Top Gun – Soundtrack
14	Circle Of Friends	Better Than Ezra	3:27	Empire Records
15	Dancing With Myself	Billy Idol	4:43	Don't Stop
16	Rebel Yell	Billy Idol	4:49	Rebel Yell
17	Piano Man	Billy Joel	5:36	Greatest Hits Vol. 1
18	Pressure	Billy Joel	3:16	Greatest Hits, Vol. II (1978 – 1985) (Disc 2)
19	The Longest Time	Billy Joel	3:36	Greatest Hits, Vol. II (1978 – 1985) (Disc 2)
20	Atomic	Blondie	3:50	Atomic: The Very Best Of Blondie
21	Sunday Girl	Blondie	3:15	Atomic: The Very Best Of Blondie
22	Call Me	Blondie	3:33	Atomic: The Very Best Of Blondie
23	Dreaming	Blondie	3:06	Atomic: The Very Best Of Blondie
24	Hurricane	Bob Dylan	8:32	Desire
25	The Times They Are A-Changin'	Bob Dylan	3:17	Greatest Hits
26	Livin' On A Prayer	Bon Jovi	4:11	Cross Road
27	Beds Of Roses	Bon Jovi	6:35	Cross Road
28	Runaway	Bon Jovi	3:53	Cross Road
29	Rasputin (Extended Mix)	Boney M	5:50	Greatest Hits
30	Have You Ever Seen The Rain	Bonnie Tyler	4:10	Faster Than The Speed Of Night
31	Total Eclipse Of The Heart	Bonnie Tyler	7:02	Faster Than The Speed Of Night
32	Straight From The Heart	Bonnie Tyler	3:41	Faster Than The Speed Of Night
33	Holding Out For A Hero	Bonny Tyler	5:49	Meat Loaf And Friends
34	Dancing In The Dark	Bruce Springsteen	4:05	Born In The U.S.A.
35	Thunder Road	Bruce Springsteen	4:51	Born To Run
36	Born To Run	Bruce Springsteen	4:30	Born To Run
37	Jungleland	Bruce Springsteen	9:34	Born To Run

Sort music library by song name



	Name	Artist	Time	Album
1	Alive	Pearl Jam	5:41	Ten
2	All Over The World	Pixies	5:27	Bossanova
3	All Through The Night	Cyndi Lauper	4:30	She's So Unusual
4	Allison Road	Gin Blossoms	3:19	New Miserable Experience
5	Ama, Ama, Ama Y Ensancha El ...	Extremoduro	2:34	Deltoya (1992)
6	And We Danced	Hooters	3:50	Nervous Night
7	As I Lay Me Down	Sophie B. Hawkins	4:09	Whaler
8	Atomic	Blondie	3:50	Atomic: The Very Best Of Blondie
9	Automatic Lover	Jay-Jay Johanson	4:19	Antenna
10	Baba O'Riley	The Who	5:01	Who's Better, Who's Best
11	Beautiful Life	Ace Of Base	3:40	The Bridge
12	Beds Of Roses	Bon Jovi	6:35	Cross Road
13	Black	Pearl Jam	5:44	Ten
14	Bleed American	Jimmy Eat World	3:04	Bleed American
15	Borderline	Madonna	4:00	The Immaculate Collection
16	Born To Run	Bruce Springsteen	4:30	Born To Run
17	Both Sides Of The Story	Phil Collins	6:43	Both Sides
18	Bouncing Around The Room	Phish	4:09	A Live One (Disc 1)
19	Boys Don't Cry	The Cure	2:35	Staring At The Sea: The Singles 1979-1985
20	Brat	Green Day	1:43	Insomniac
21	Breakdown	Deerheart	3:40	Deerheart
22	Bring Me To Life (Kevin Roen Mix)	Evanescence Vs. Pa...	9:48	
23	Californication	Red Hot Chili Pepp...	1:40	
24	Call Me	Blondie	3:33	Atomic: The Very Best Of Blondie
25	Can't Get You Out Of My Head	Kylie Minogue	3:50	Fever
26	Celebration	Kool & The Gang	3:45	Time Life Music Sounds Of The Seventies - C

Comparable interface: review

Comparable interface: sort using a type's natural order.

```
public class Date implements Comparable<Date>
{
   private final int month, day, year;

   public Date(int m, int d, int y)
   {
      month = m;
      day   = d;
      year  = y;
   }
   ...
   public int compareTo(Date that)
   {
      if (this.year  < that.year ) return -1;
      if (this.year  > that.year ) return +1;
      if (this.month < that.month) return -1;
      if (this.month > that.month) return +1;
      if (this.day   < that.day  ) return -1;
      if (this.day   > that.day  ) return +1;
      return 0;
   }
}
```

natural order

Comparator interface

Comparator interface: sort using an alternate order.

```
public interface Comparator<Key>
```

`int compare(Key v, Key w)`	*compare keys v and w*

Required property. Must be a total order.

string order	example
natural order	Now is the time
case insensitive	is Now the time
Spanish language	café cafetero cuarto churro nube ñoño
British phone book	McKinley Mackintosh

pre-1994 order for digraphs ch and ll and rr

Comparator interface: system sort

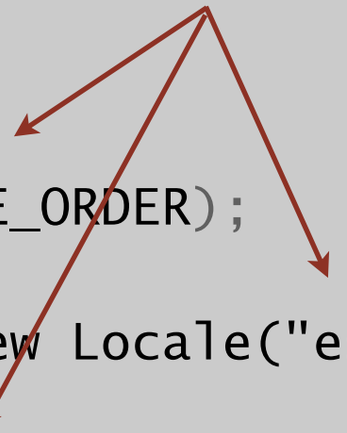
To use with Java system sort:

- Create `Comparator` object.
- Pass as second argument to `Arrays.sort()`.

```
String[] a;
...
Arrays.sort(a);
...
Arrays.sort(a, String.CASE_INSENSITIVE_ORDER);
...
Arrays.sort(a, Collator.getInstance(new Locale("es")));
...
Arrays.sort(a, new BritishPhoneBookOrder());
...
```

uses natural order

uses alternate order defined by Comparator<String> object



Bottom line. Decouples the definition of the data type from the definition of what it means to compare two objects of that type.

Comparator interface: using with our sorting libraries

To support comparators in our sort implementations:

- Use `Object` instead of `Comparable`.
- Pass `Comparator` to `sort()` and `less()` and use it in `less()`.

insertion sort using a Comparator

```
public static void sort(Object[] a, Comparator comparator)
{
   int N = a.length;
   for (int i = 0; i < N; i++)
      for (int j = i; j > 0 && less(comparator, a[j], a[j-1]); j--)
         exch(a, j, j-1);
}

private static boolean less(Comparator c, Object v, Object w)
{  return c.compare(v, w) < 0;   }

private static void exch(Object[] a, int i, int j)
{  Object swap = a[i]; a[i] = a[j]; a[j] = swap;  }
```

Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the `Comparator` interface.
- Implement the `compare()` method.

```
public class Student
{
   private final String name;
   private final int section;
   ...

   public static class ByName implements Comparator<Student>
   {
     public int compare(Student v, Student w)
      {  return v.name.compareTo(w.name);  }
   }

   public static class BySection implements Comparator<Student>
   {
      public int compare(Student v, Student w)
      {  return v.section - w.section;  }
   }
}
```

Comparator interface: implementing

To implement a comparator:

- Define a (nested) class that implements the `Comparator` interface.
- Implement the `compare()` method.

```
Arrays.sort(a, new Student.ByName());
```

Andrews	3	A	664-480-0023	097 Little
Battle	4	C	874-088-1212	121 Whitman
Chen	3	A	991-878-4944	308 Blair
Fox	3	A	884-232-5341	11 Dickinson
Furia	1	A	766-093-9873	101 Brown
Gazsi	4	B	766-093-9873	101 Brown
Kanaga	3	B	898-122-9643	22 Brown
Rohde	2	A	232-343-5555	343 Forbes

```
Arrays.sort(a, new Student.BySection());
```

Furia	1	A	766-093-9873	101 Brown
Rohde	2	A	232-343-5555	343 Forbes
Andrews	3	A	664-480-0023	097 Little
Chen	3	A	991-878-4944	308 Blair
Fox	3	A	884-232-5341	11 Dickinson
Kanaga	3	B	898-122-9643	22 Brown
Battle	4	C	874-088-1212	121 Whitman
Gazsi	4	B	766-093-9873	101 Brown

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2.2 Mergesort

- *mergesort*
- *bottom-up mergesort*
- *comparators*
- *stability*

Stability

A typical application. First, sort by name; then sort by section.

```
Selection.sort(a, new Student.ByName());
```

Andrews	3	A	664-480-0023	097 Little
Battle	4	C	874-088-1212	121 Whitman
Chen	3	A	991-878-4944	308 Blair
Fox	3	A	884-232-5341	11 Dickinson
Furia	1	A	766-093-9873	101 Brown
Gazsi	4	B	766-093-9873	101 Brown
Kanaga	3	B	898-122-9643	22 Brown
Rohde	2	A	232-343-5555	343 Forbes

```
Selection.sort(a, new Student.BySection());
```

Furia	1	A	766-093-9873	101 Brown
Rohde	2	A	232-343-5555	343 Forbes
Chen	3	A	991-878-4944	308 Blair
Fox	3	A	884-232-5341	11 Dickinson
Andrews	3	A	664-480-0023	097 Little
Kanaga	3	B	898-122-9643	22 Brown
Gazsi	4	B	766-093-9873	101 Brown
Battle	4	C	874-088-1212	121 Whitman

@#%&@! Students in section 3 no longer sorted by name.

A stable sort preserves the relative order of items with equal keys.

Stability

Q. Which sorts are stable?

sorted by time		sorted by location		Stable?
Chicago	09:00:00	Chicago	09:25:52	
Phoenix	09:00:03	Chicago	09:03:13	
Houston	09:00:13	Chicago	09:21:05	
Chicago	09:00:59	Chicago	09:19:46	
Houston	09:01:10	Chicago	09:19:32	
Chicago	09:03:13	Chicago	09:00:00	
Seattle	09:10:11	Chicago	09:35:21	
Seattle	09:10:25	Chicago	09:00:59	
Phoenix	09:14:25	Houston	09:01:10	
Chicago	09:19:32	Houston	09:00:13	
Chicago	09:19:46	Phoenix	09:37:44	
Chicago	09:21:05	Phoenix	09:00:03	
Seattle	09:22:43	Phoenix	09:14:25	
Seattle	09:22:54	Seattle	09:10:25	
Chicago	09:25:52	Seattle	09:36:14	
Chicago	09:35:21	Seattle	09:22:43	
Seattle	09:36:14	Seattle	09:10:11	
Phoenix	09:37:44	Seattle	09:22:54	

Stability

Q. Which sorts are stable?

sorted by time		sorted by location (not stable)	
Chicago	09:00:00	Chicago	09:25:52
Phoenix	09:00:03	Chicago	09:03:13
Houston	09:00:13	Chicago	09:21:05
Chicago	09:00:59	Chicago	09:19:46
Houston	09:01:10	Chicago	09:19:32
Chicago	09:03:13	Chicago	09:00:00
Seattle	09:10:11	Chicago	09:35:21
Seattle	09:10:25	Chicago	09:00:59
Phoenix	09:14:25	Houston	09:01:10
Chicago	09:19:32	Houston	09:00:13
Chicago	09:19:46	Phoenix	09:37:44
Chicago	09:21:05	Phoenix	09:00:03
Seattle	09:22:43	Phoenix	09:14:25
Seattle	09:22:54	Seattle	09:10:25
Chicago	09:25:52	Seattle	09:36:14
Chicago	09:35:21	Seattle	09:22:43
Seattle	09:36:14	Seattle	09:10:11
Phoenix	09:37:44	Seattle	09:22:54

no longer sorted by time

Stability

Q. Which sorts are stable?

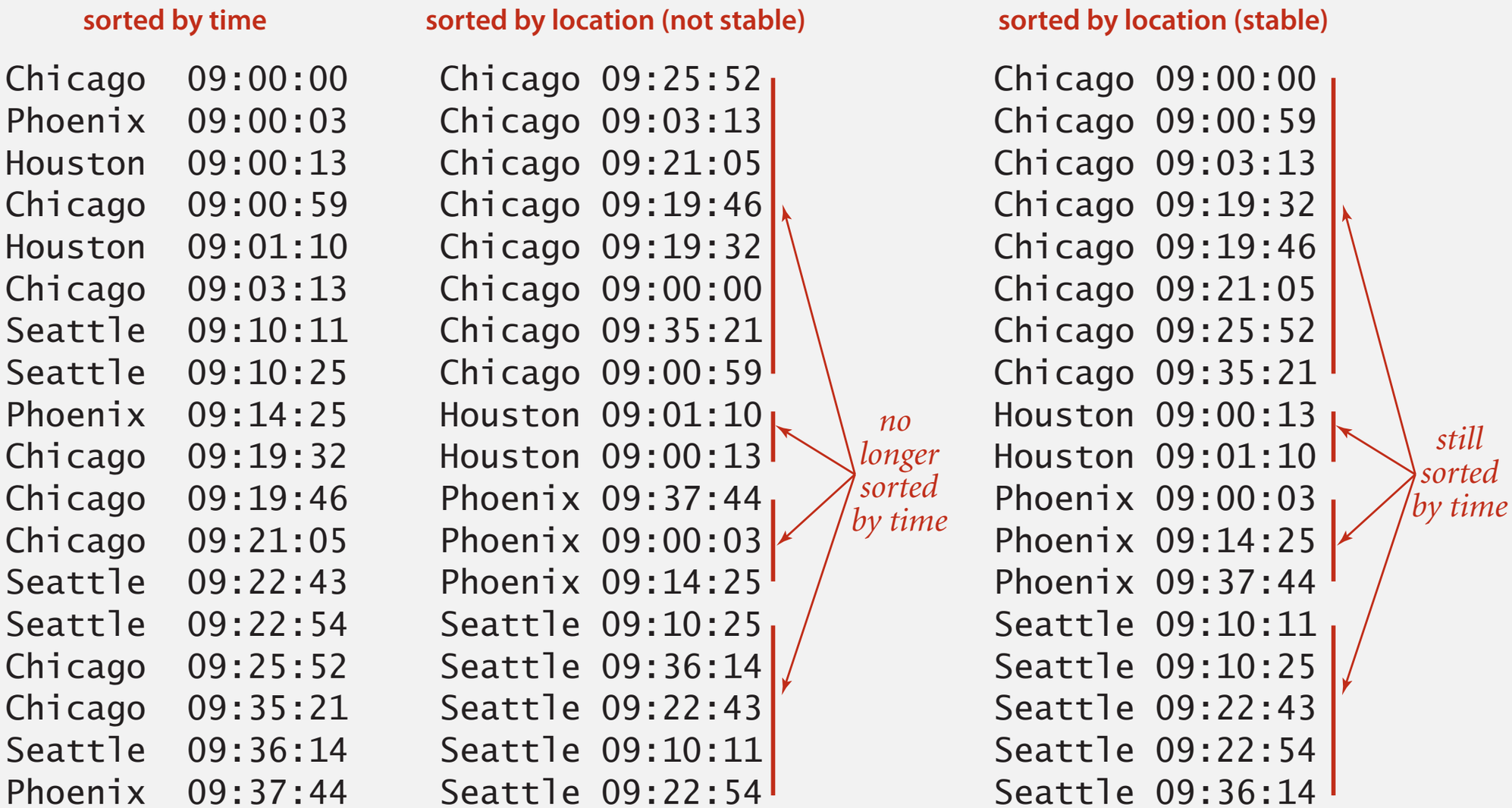
sorted by time		sorted by location (not stable)		sorted by location	Stable?
Chicago	09:00:00	Chicago	09:25:52	Chicago	09:00:00
Phoenix	09:00:03	Chicago	09:03:13	Chicago	09:00:59
Houston	09:00:13	Chicago	09:21:05	Chicago	09:03:13
Chicago	09:00:59	Chicago	09:19:46	Chicago	09:19:32
Houston	09:01:10	Chicago	09:19:32	Chicago	09:19:46
Chicago	09:03:13	Chicago	09:00:00	Chicago	09:21:05
Seattle	09:10:11	Chicago	09:35:21	Chicago	09:25:52
Seattle	09:10:25	Chicago	09:00:59	Chicago	09:35:21
Phoenix	09:14:25	Houston	09:01:10	Houston	09:00:13
Chicago	09:19:32	Houston	09:00:13	Houston	09:01:10
Chicago	09:19:46	Phoenix	09:37:44	Phoenix	09:00:03
Chicago	09:21:05	Phoenix	09:00:03	Phoenix	09:14:25
Seattle	09:22:43	Phoenix	09:14:25	Phoenix	09:37:44
Seattle	09:22:54	Seattle	09:10:25	Seattle	09:10:11
Chicago	09:25:52	Seattle	09:36:14	Seattle	09:10:25
Chicago	09:35:21	Seattle	09:22:43	Seattle	09:22:43
Seattle	09:36:14	Seattle	09:10:11	Seattle	09:22:54
Phoenix	09:37:44	Seattle	09:22:54	Seattle	09:36:14

no longer sorted by time

Stability

Q. Which sorts are stable?

A. Need to check algorithm (and implementation).



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Insertion Sort

- *Is it stable?*

```
public class Insertion
{
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      for (int i = 0; i < N; i++)
         for (int j = i; j > 0 && less(a[j], a[j-1]); j--)
            exch(a, j, j-1);
   }
}
```

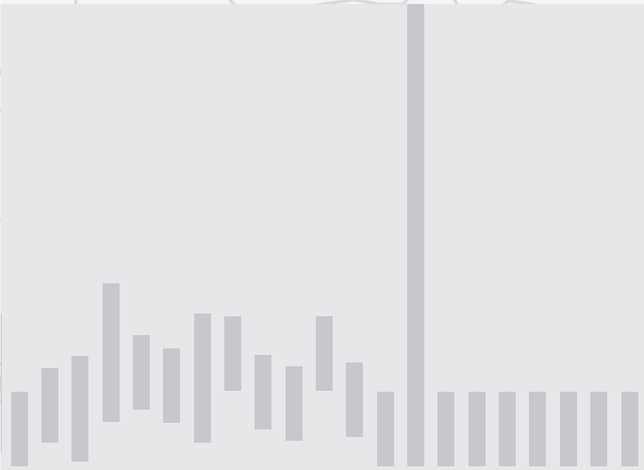
Stability: insertion sort

Proposition. Insertion sort is stable.

```
public class Insertion
{
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      for (int i = 0; i < N; i++)
         for (int j = i; j > 0 && less(a[j], a[j-1]); j--)
            exch(a, j, j-1);
   }
}
```

i	j	0	1	2	3	4
0	0	B_1	A_1	A_2	A_3	B_2
1	0	A_1	B_1	A_2	A_3	B_2
2	1	A_1	A_2	B_1	A_3	B_2
3	2	A_1	A_2	A_3	B_1	B_2
4	4	A_1	A_2	A_3	B_1	B_2
		A_1	A_2	A_3	B_1	B_2

Pf. Equal items never move past each other.



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Selection Sort

- *Is it stable?*

```
public class Selection
{
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      for (int i = 0; i < N; i++)
      {
         int min = i;
         for (int j = i+1; j < N; j++)
            if (less(a[j], a[min]))
               min = j;
         exch(a, i, min);
      }
   }
}
```

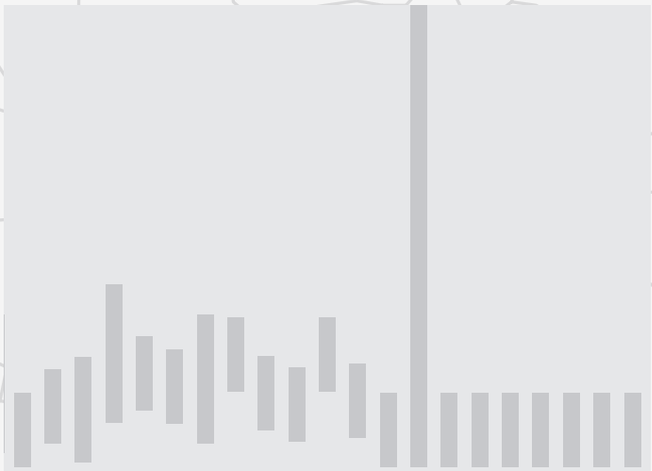
Stability: selection sort

Proposition. Selection sort is not stable.

```
public class Selection
{
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      for (int i = 0; i < N; i++)
      {
         int min = i;
         for (int j = i+1; j < N; j++)
            if (less(a[j], a[min]))
               min = j;
         exch(a, i, min);
      }
   }
}
```

i	min	0	1	2
0	2	B_1	B_2	A
1	1	A	B_2	B_1
2	2	A	B_2	B_1
		A	B_2	B_1

Pf by counterexample. Long-distance exchange can move one equal item past another one.



Algorithms

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Shellsort

- *Is it stable?*

```
public class Shell
{
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      int h = 1;
      while (h < N/3) h = 3*h + 1;
      while (h >= 1)
      {
         for (int i = h; i < N; i++)
         {
            for (int j = i; j > h && less(a[j], a[j-h]); j -= h)
               exch(a, j, j-h);
         }
         h = h/3;
      }
   }
}
```

Stability: shellsort

Proposition. Shellsort sort is not stable.

```
public class Shell
{
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      int h = 1;
      while (h < N/3) h = 3*h + 1;
      while (h >= 1)
      {
         for (int i = h; i < N; i++)
         {
            for (int j = i; j > h && less(a[j], a[j-h]); j -= h)
               exch(a, j, j-h);
         }
         h = h/3;
      }
   }
}
```

h	0	1	2	3	4
	B_1	B_2	B_3	B_4	A_1
4	A_1	B_2	B_3	B_4	B_1
1	A_1	B_2	B_3	B_4	B_1
	A_1	B_2	B_3	B_4	B_1

Pf by counterexample. Long-distance exchanges.

Algorithms

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MERGESORT

- *Is it stable?*

```
public class Merge
{
   private static void merge(...)
   {  /* as before */  }

   private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
   {
      if (hi <= lo) return;
      int mid = lo + (hi - lo) / 2;
      sort(a, aux, lo, mid);
      sort(a, aux, mid+1, hi);
      merge(a, aux, lo, mid, hi);
   }

   public static void sort(Comparable[] a)
   {  /* as before */  }
}
```

Stability: mergesort

Proposition. Mergesort is stable.

```
public class Merge
{
   private static void merge(...)
   {  /* as before */  }

   private static void sort(Comparable[] a, Comparable[] aux, int lo, int hi)
   {
      if (hi <= lo) return;
      int mid = lo + (hi - lo) / 2;
      sort(a, aux, lo, mid);
      sort(a, aux, mid+1, hi);
      merge(a, aux, lo, mid, hi);
   }

   public static void sort(Comparable[] a)
   {  /* as before */  }
}
```

Pf. Suffices to verify that merge operation is stable.

Stability: mergesort

Proposition. Merge operation is stable.

```
private static void merge(...)
{
   for (int k = lo; k <= hi; k++)
      aux[k] = a[k];

   int i = lo, j = mid+1;
   for (int k = lo; k <= hi; k++)
   {
      if      (i > mid)                  a[k] = aux[j++];
      else if (j > hi)                   a[k] = aux[i++];
      else if (less(aux[j], aux[i])) a[k] = aux[j++];
      else                               a[k] = aux[i++];
   }
}
```

0	1	2	3	4	5	6	7	8	9	10
A_1	A_2	A_3	B	D	A_4	A_5	C	E	F	G

Pf. Takes from left subarray if equal keys.