Biomedical engineering: creating computer models of bones to aid with successful surgical repair.

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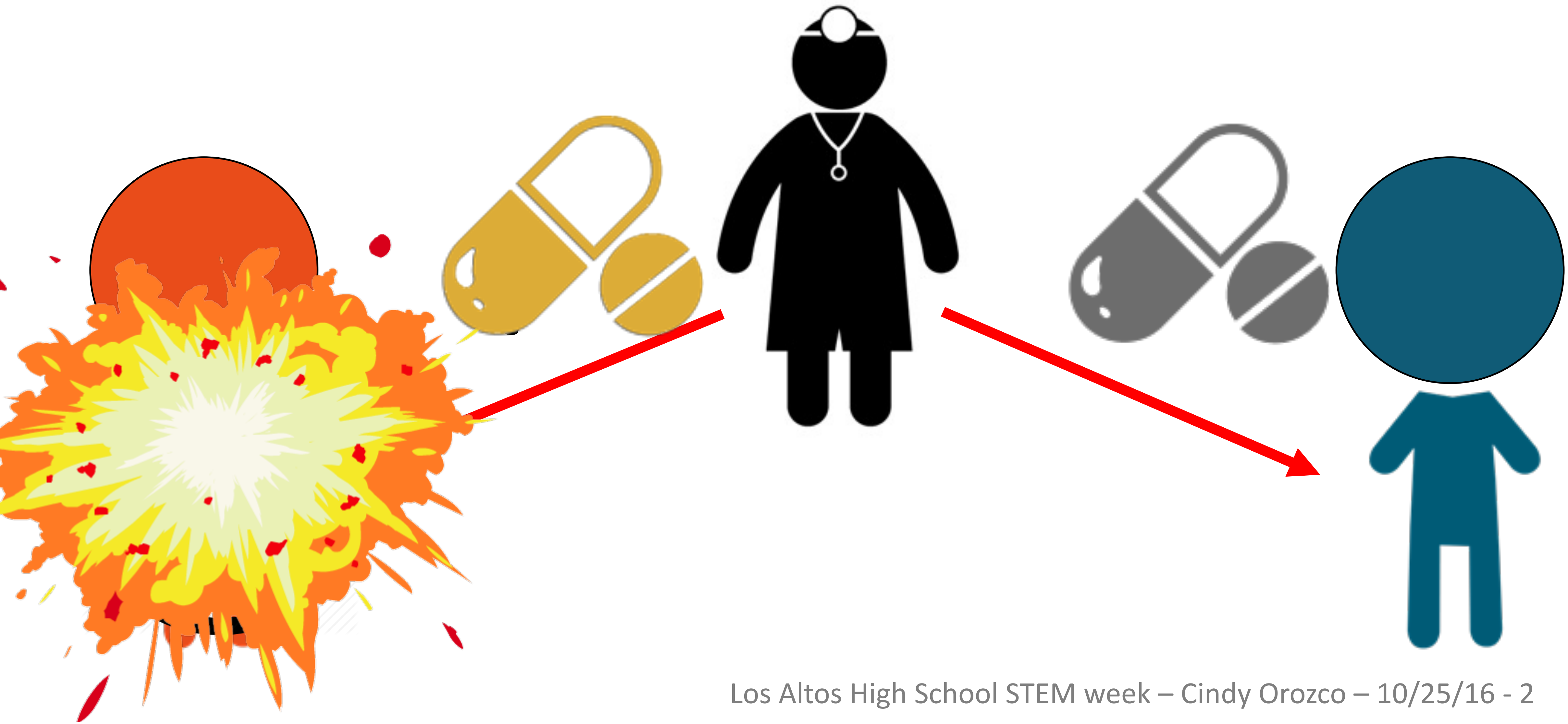
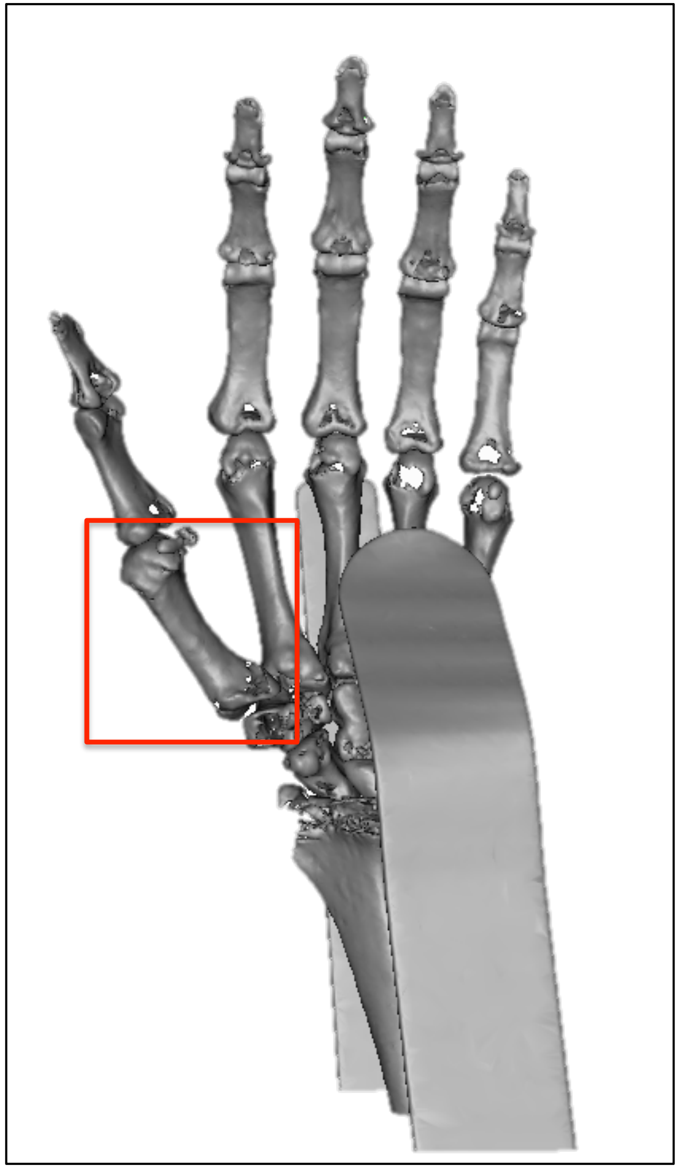
What is the motivation?



What is the motivation?

- Predict if the surgery will solve the patient condition
- Anticipate the side effects of the surgery
- Tailor the treatment for the needs of each patient (e.g. athletes, artists, …)

Personalized Numerical Model



Personalized Numerical Model

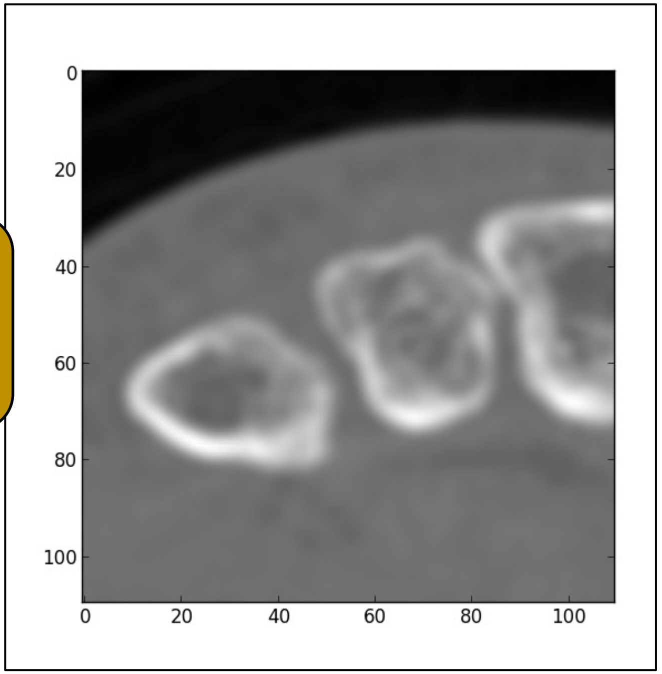
$$a(u, w) = L(w)$$

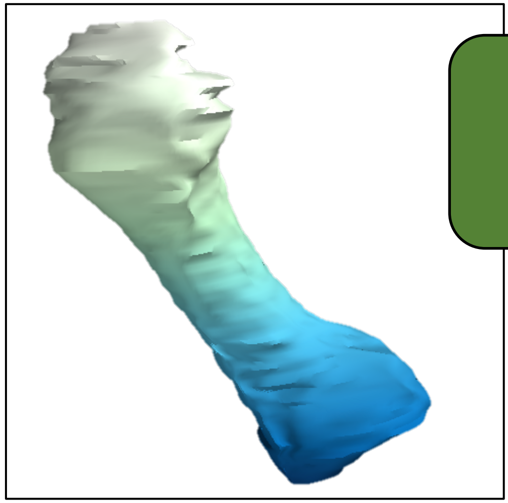
Physics/ Math

Elasticity formulation
Finite Elements
(Find a known model)

Data

CT - Image Processing
(Extract Information)





Code

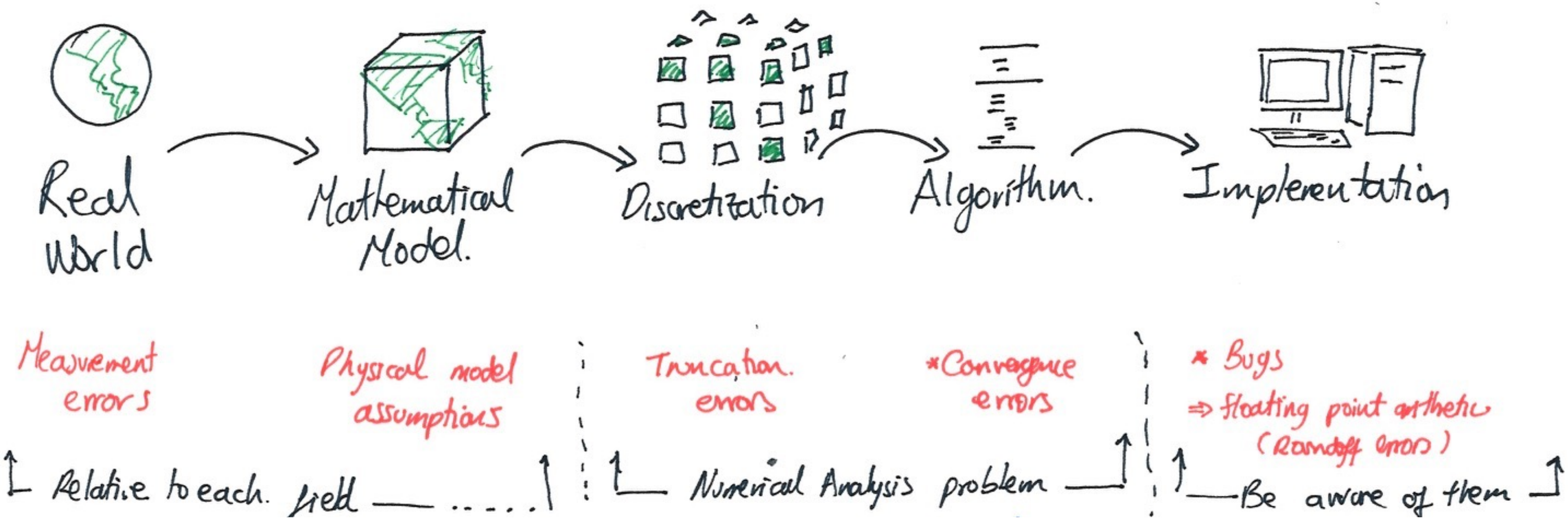
Implementation
Algorithms
(Solve the problem)

Why do we care about Data - Physics - Code?

Data

Physics/ Math

Applied Mathematics - Code



Personalized Numerical Model

$$a(u, w) = L(w)$$

Physics/
Math

Elasticity formulation
Finite Elements
(Find a known model)

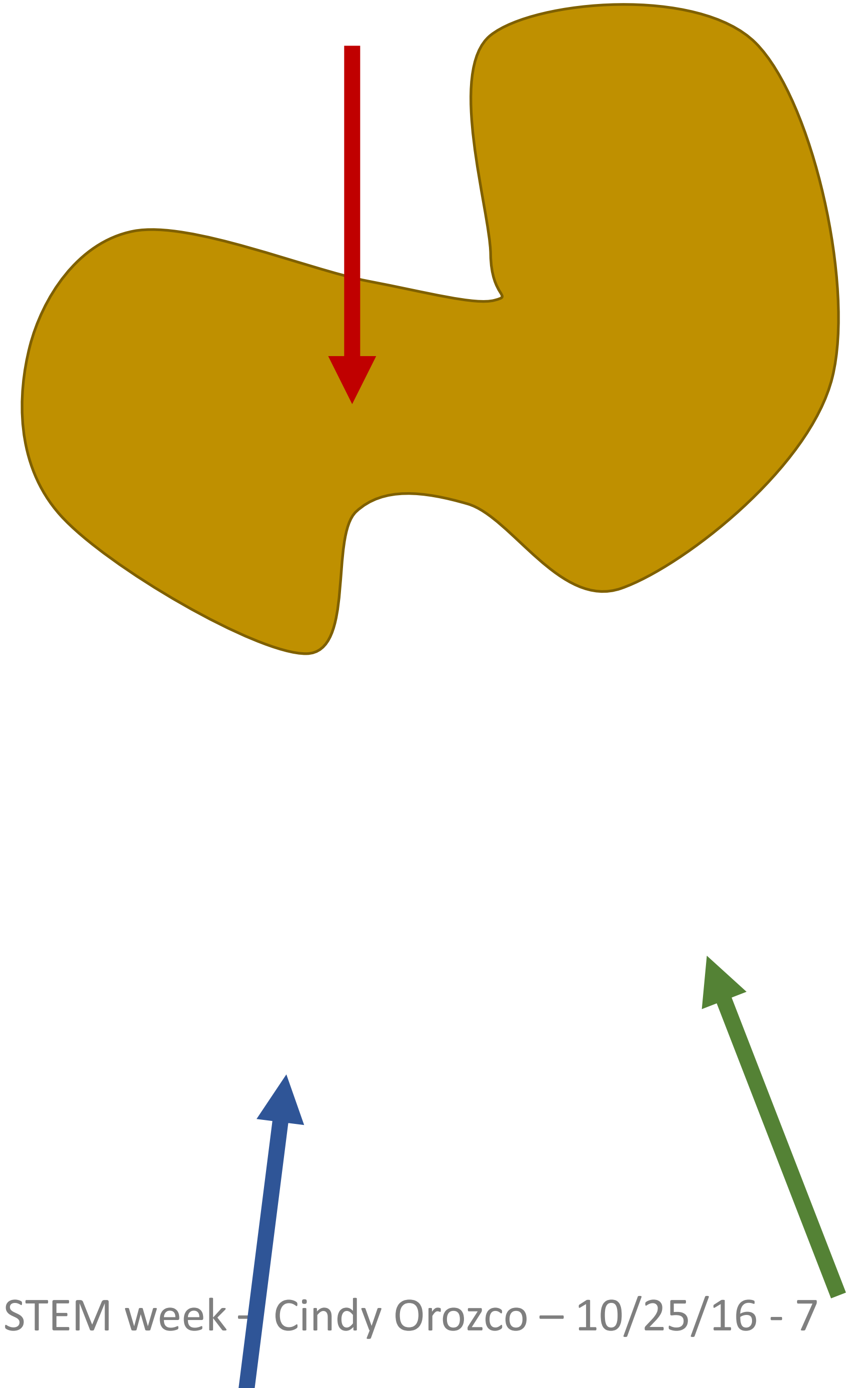
Physics: Newton's Laws

Force is an interaction that changes the motion of a body

$$\vec{F} = m\vec{a}$$

To be in equilibrium we require zero – acceleration:

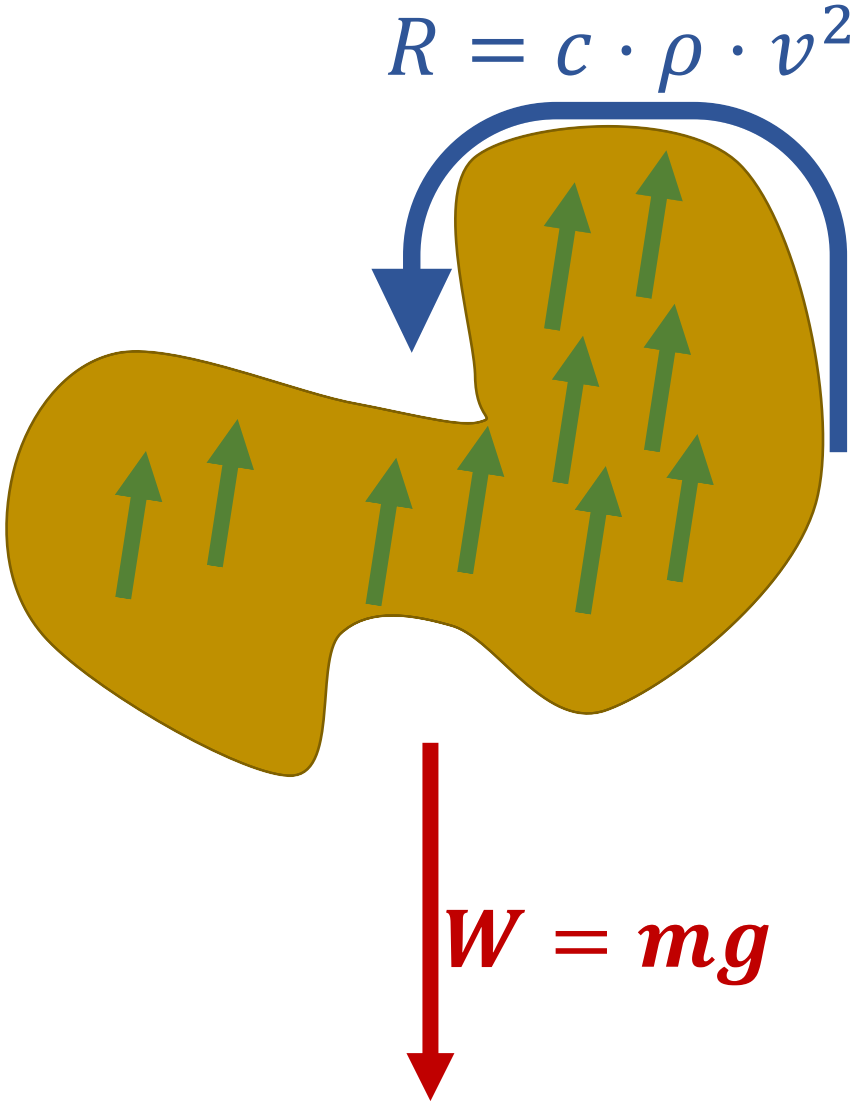
$$\sum_i \vec{F}_i = 0$$



Physics: Equilibrium

We have 3 types of **forces:**

- Body forces:
  Affect the body without touching it : Weigh
- Surface forces:
  Touch the body on the surface: Drag or Air resistance
- Internal forces:
  How the body responds: Resistance of the material = stress



Physics: Internal forces and Cauchy Stress

- Relate deformation of a body with the resistance of the material

$$F_{internal} = \frac{\partial \sigma}{\partial x}$$

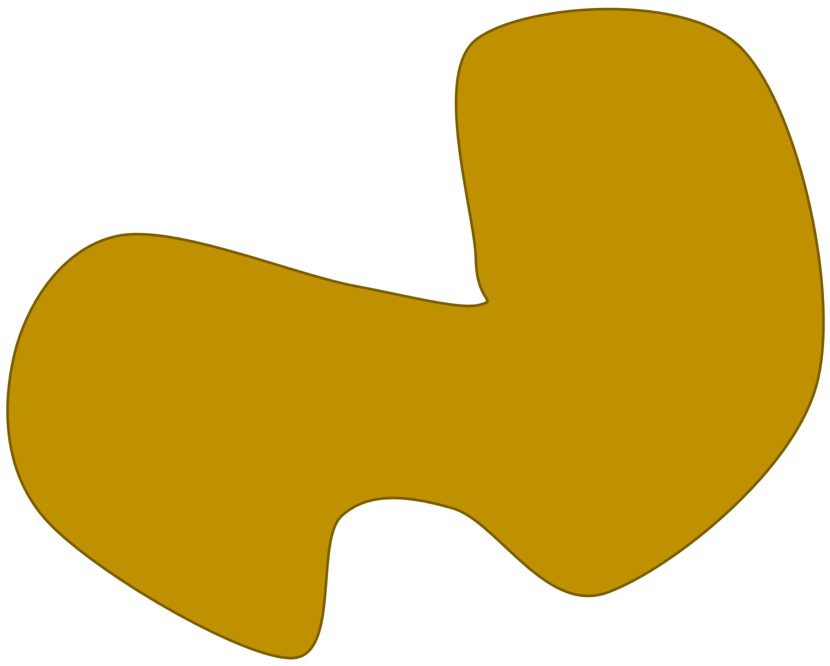
Change in the stress

- Hooke's Law:

$$\sigma = C\,\frac{\partial u}{\partial x}$$

Change in the displacement

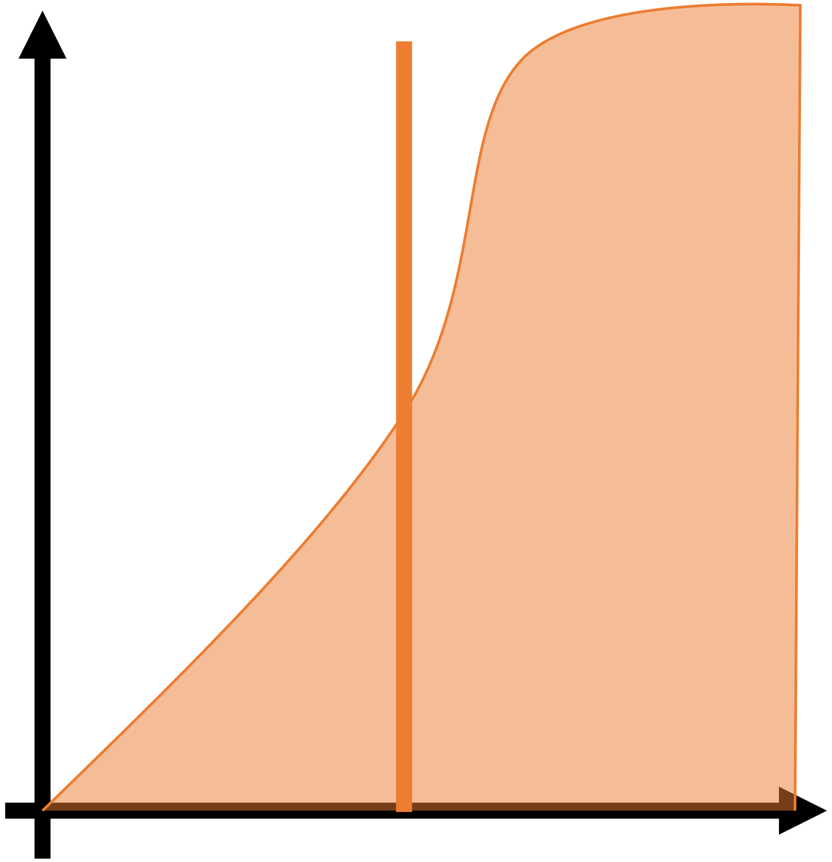
Elasticity coefficient
Young's Modulus / Poisson's Ratio



Physics: Virtual Work

- Not all the functions have derivatives
- We want to replace the Force by something without derivatives
- Work: force acting in a moving point
  - It is strongly related with kinetic and potential energy

$$W = \int_{\Omega} F \cdot s \; dV$$



Physics: Weak Formulation or Virtual Work

$$\int_\Omega \boxed{\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right)} c_{ijkl} \boxed{\frac{\partial w_i}{\partial x_j}} d\Omega = \boxed{\int_\Omega w_i f_i d\Omega + \sum_{i=1}^{d} \left(\int_{\Gamma_{N_i}} w_i h_i d\Gamma \right)}$$

Goal: Unknown

"Joker"

It can take any value and the equation must be true

External Input: Known

Applied Math: Isogeometric Discretization

$$f\left(X \right) = Y$$

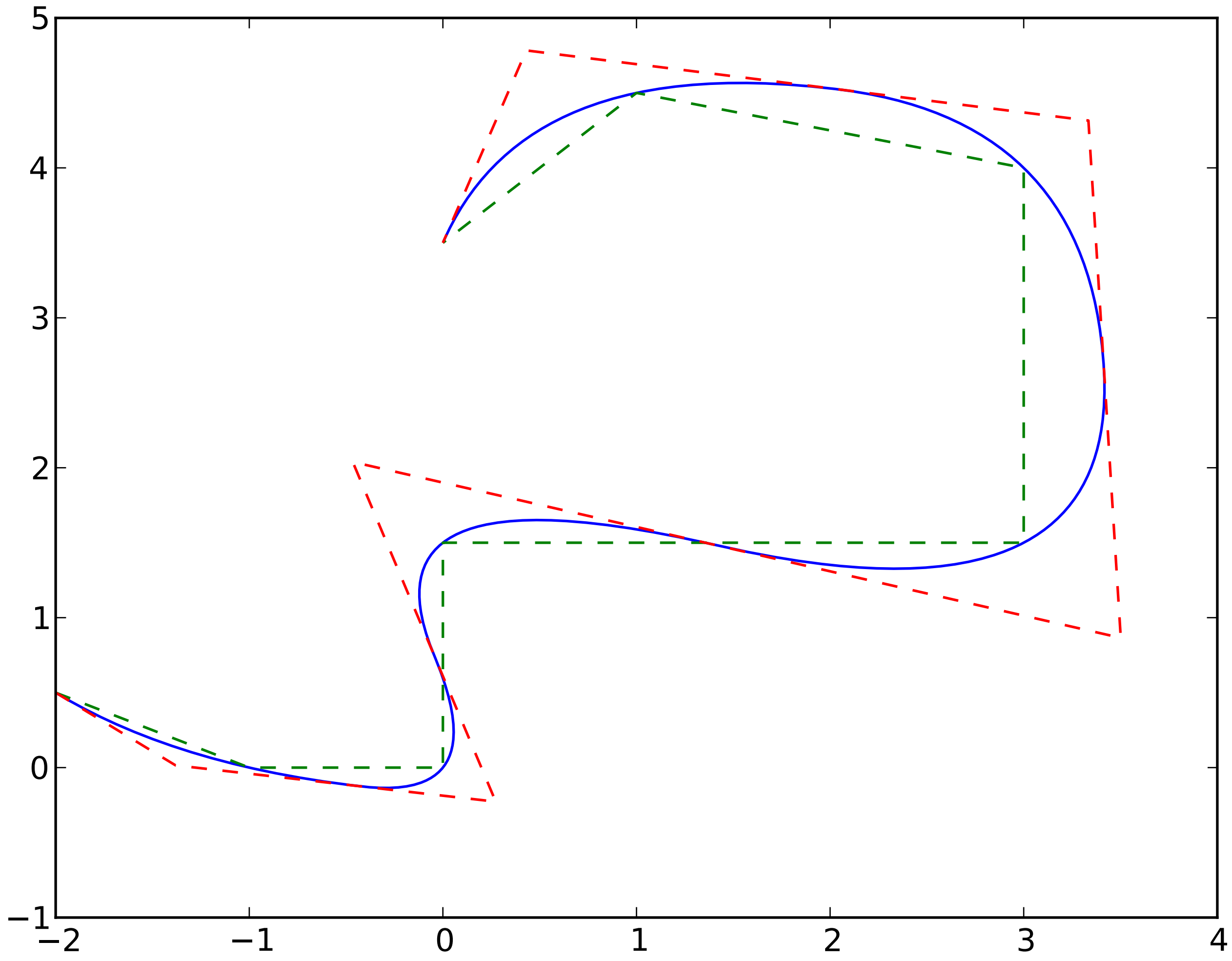
$$X = f^{-1}(Y)$$

Applied Math: Isogeometric Discretization

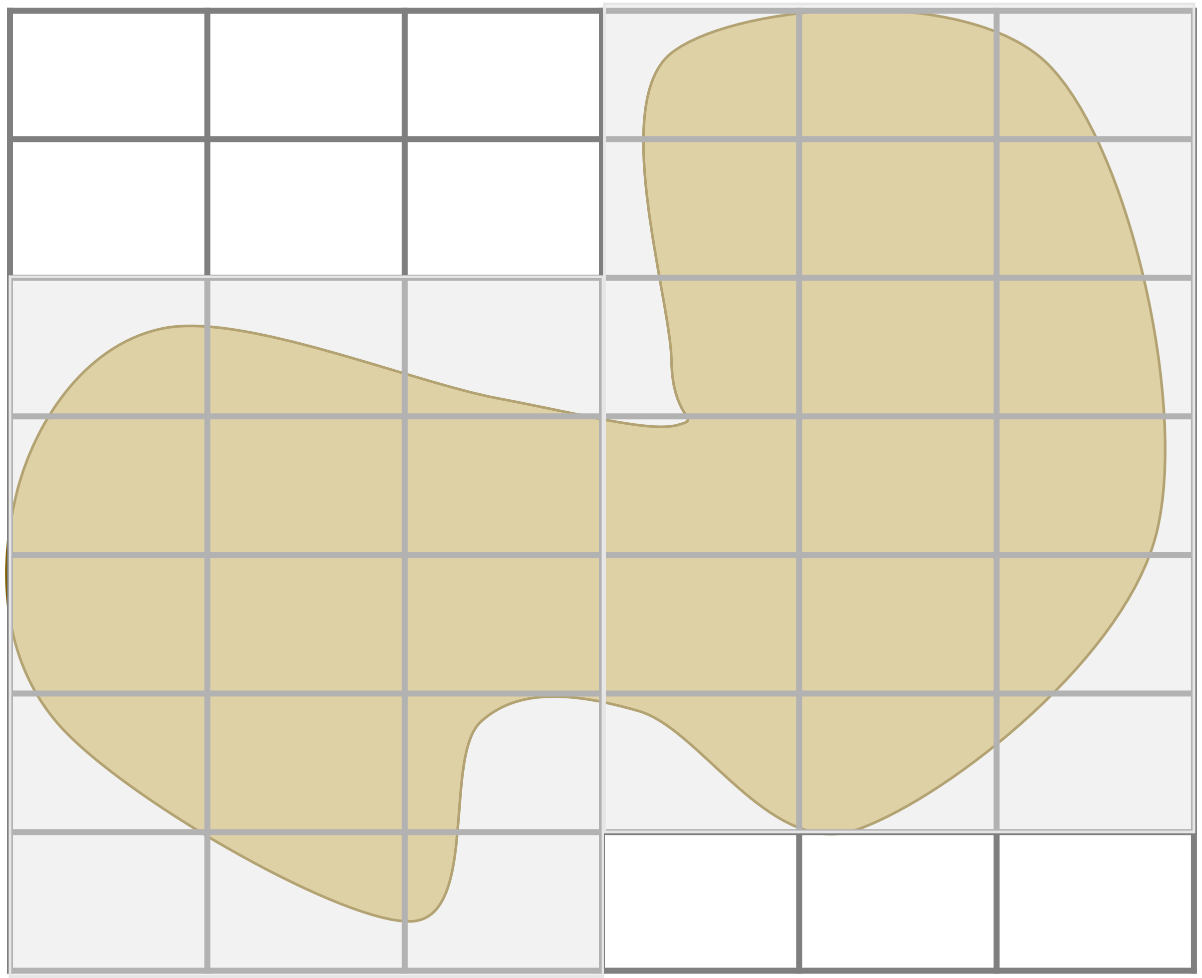
- To get a function “easy” to invert, we approximate position and displacement by nice functions, called:
- B-splines: smooth piece-wise polynomials
- We use a parametric representation

$$(x, y) = (x(t_1), y(t_1))$$

- NURBS are a generalization used in CAD



Applied Math: Isogeometric Discretization



1) Cut it into pieces

2) Approximate as a rectangle

Applied Math: Finite Elements Method

- For all possible polynomials we have a different equation. Then we take the monomials some degree $\{1, t, t^2, t^3, \dots, t^p\}$
- We replace and compute the integrals
- We assume that the solution is a linear combination

$$u(t) = \sum \alpha_i \, t^i$$

Applied Math: Finite Elements method

$$\begin{cases} a_{00}\alpha_0 + \ \dots + a_{0p}\alpha_p = b_0 \\ \qquad \dots \\ a_{p0}\alpha_0 + \ \dots + a_{pp}\alpha_p = b_p \end{cases}$$

$$K\vec{\alpha} = \vec{F}$$

Linear System

$$\begin{bmatrix} a_{00} & \cdots & a_{0p} \\ \vdots & \ddots & \vdots \\ a_{p0} & \cdots & a_{pp} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \dots \\ \alpha_p \end{bmatrix} = \begin{bmatrix} b_0 \\ \dots \\ b_p \end{bmatrix}$$

$\vec{\alpha}$ K $\vec{F}$

Applied Math: Numerical Linear Algebra

- This system can be solve using
  - Direct Methods: Back substitution, LU factorization, QR Factorization, Least Squares
  - Iterative Methods: Guess a solution and iterate: Jacobi iterations and Conjugate Gradient

Multiple algorithms, each suitable for a different type of problem.

Personalized Numerical Model

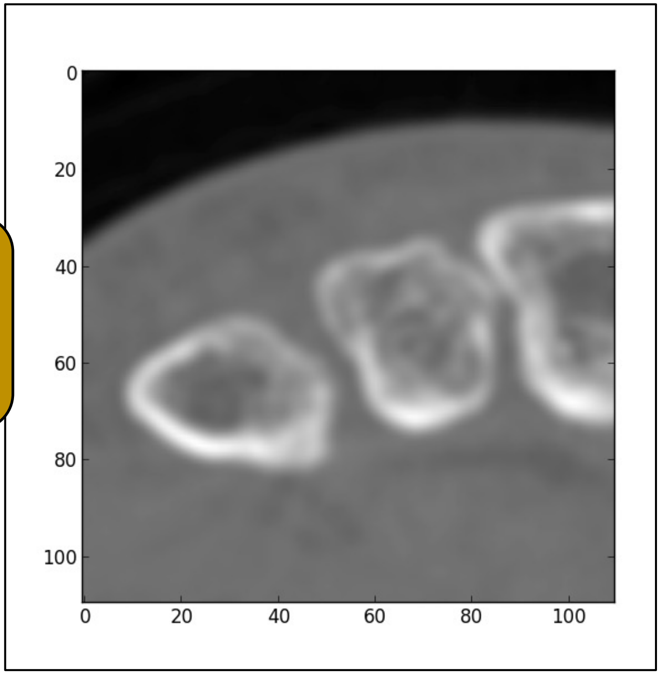
$$a(u, w) = L(w)$$

Physics/ Math

Elasticity formulation
Finite Elements
(Find a known model)

Data

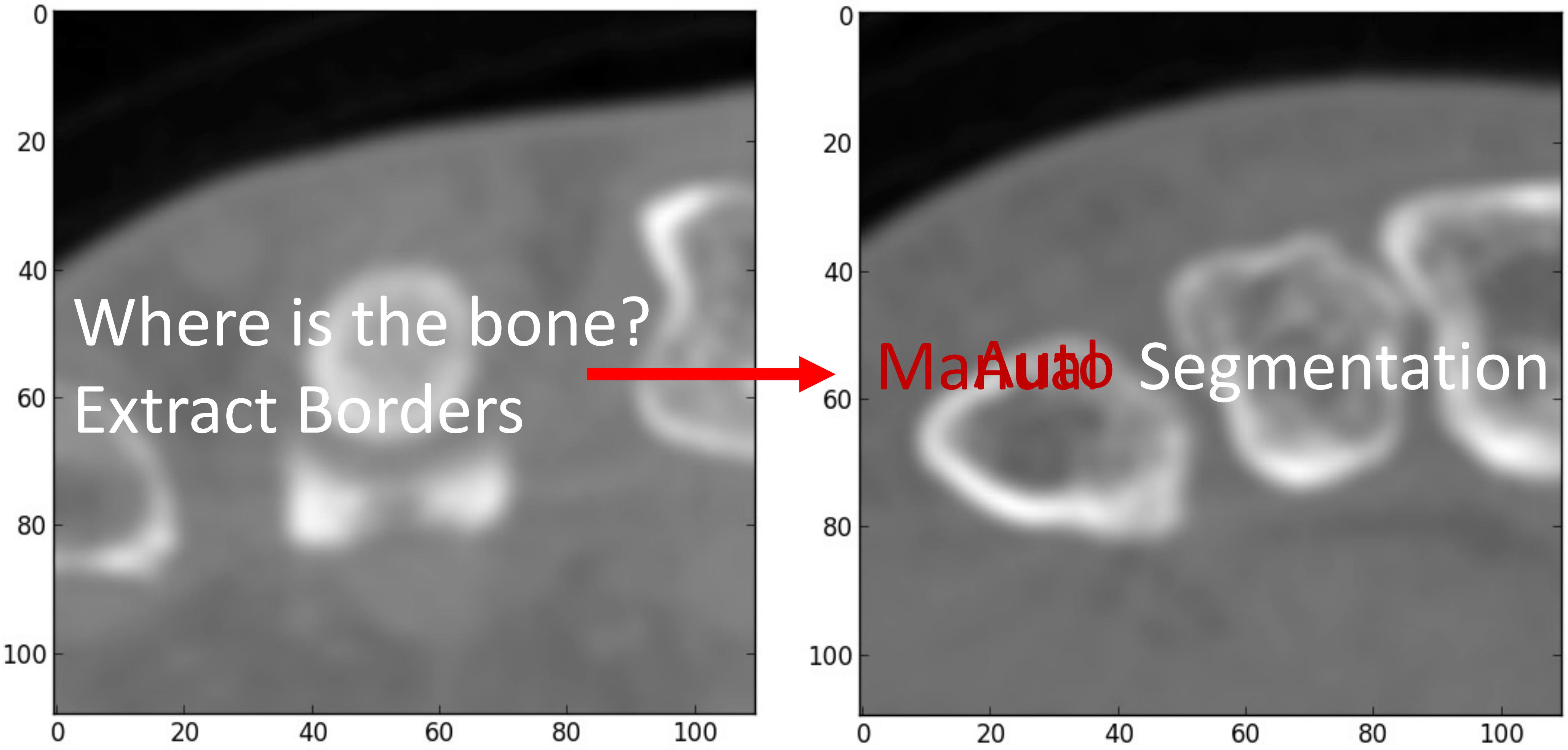
CT - Image Processing
(Extract Information)



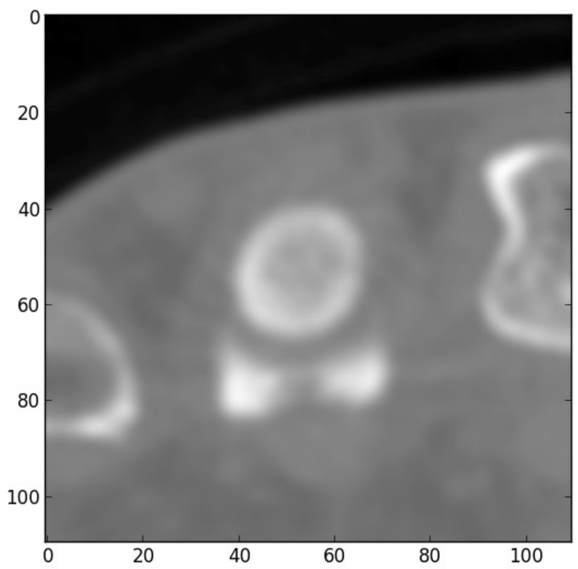
Data: Computational Tomography

- X-rays: type of energy that penetrates the body
- Ring produces them and detects their behavior into the body
- Moving across the body allows to create 3D images
- Comparable to more than 1 year of background radiation

Data: How does it look a CT?

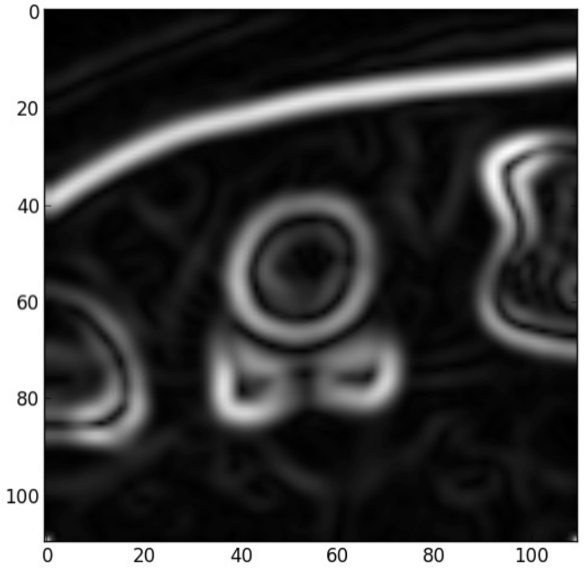


Data: Segmentation: Canny edge detector



Initial

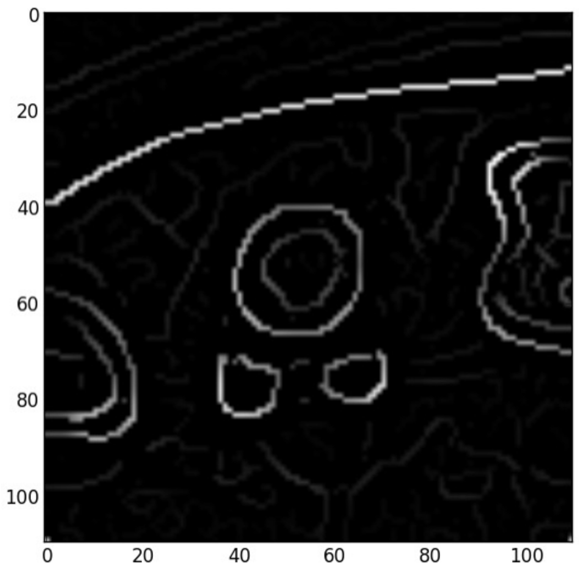
$$\begin{bmatrix} * & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & * \end{bmatrix}$$



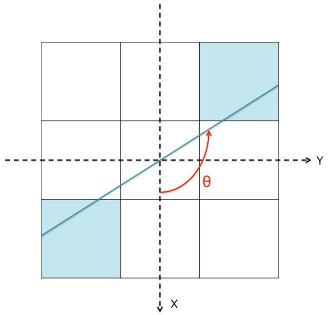
Intensity Gradient

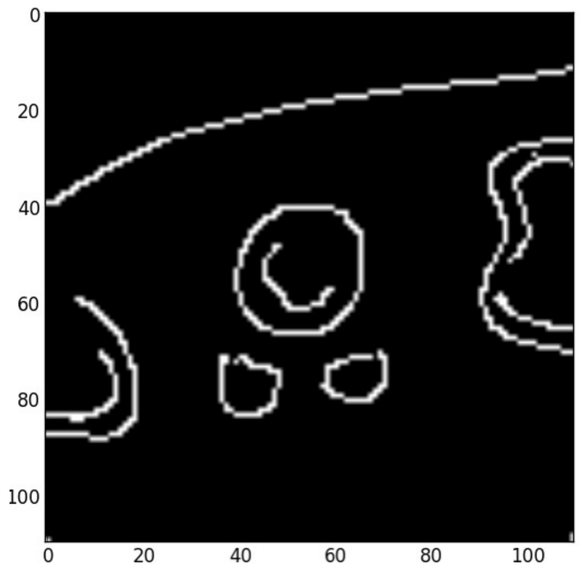
$$K_X = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

$$K_Y = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$



Non-maxima suppression





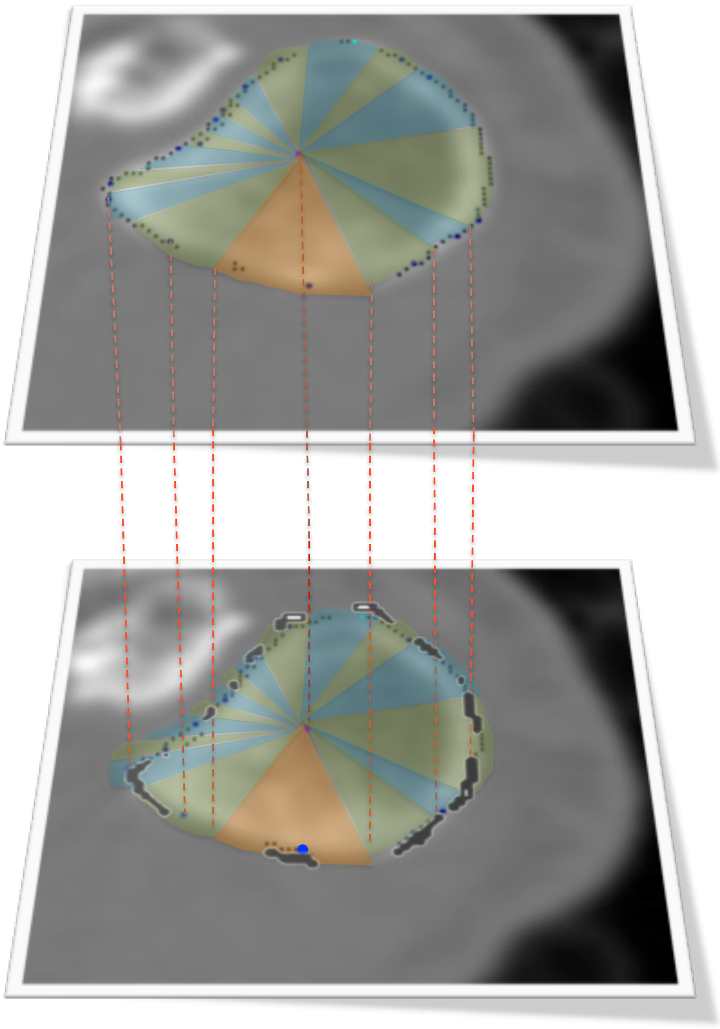
Hysteresis

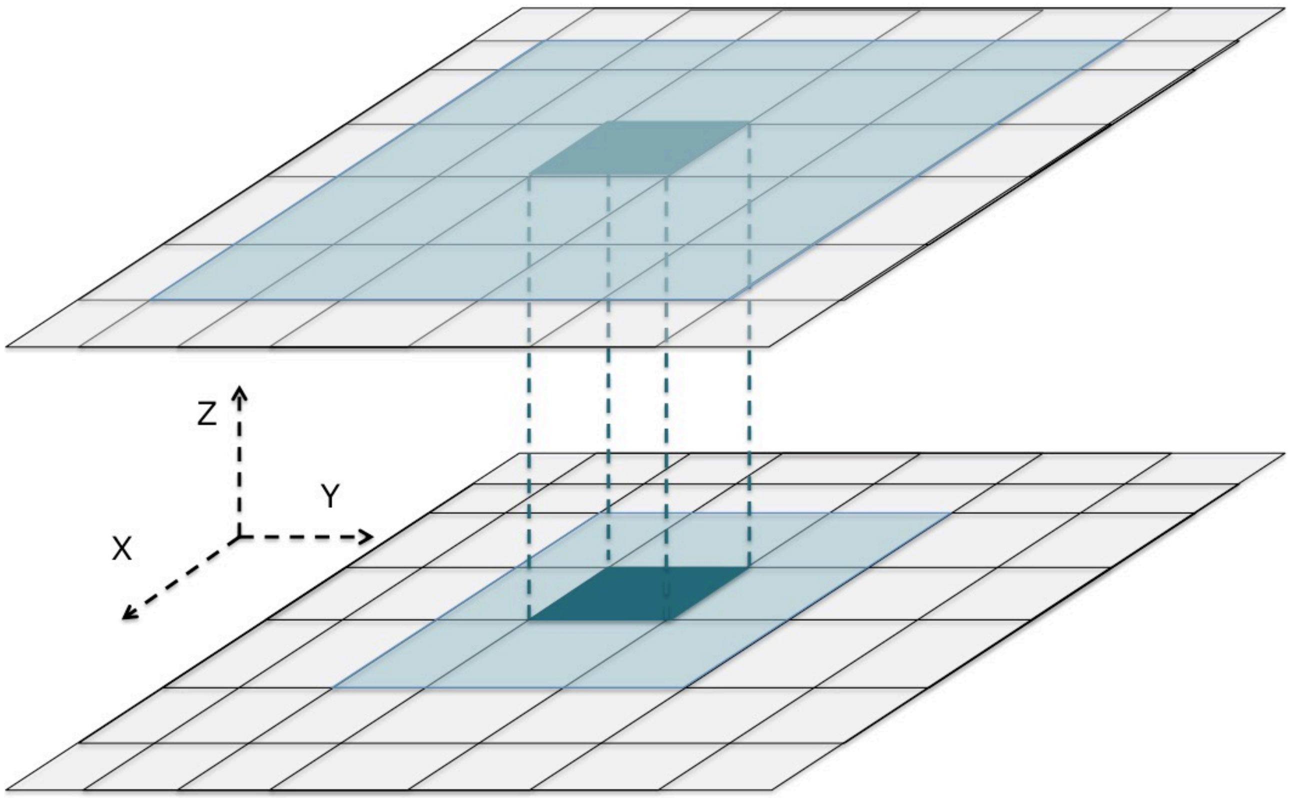
$$x_{i,j} \leq T_l$$

$$T_l \leq x_{i,j} \leq T_u$$

$$x_{i,j} \geq T_u$$

Data: Segmentation: Region Growing





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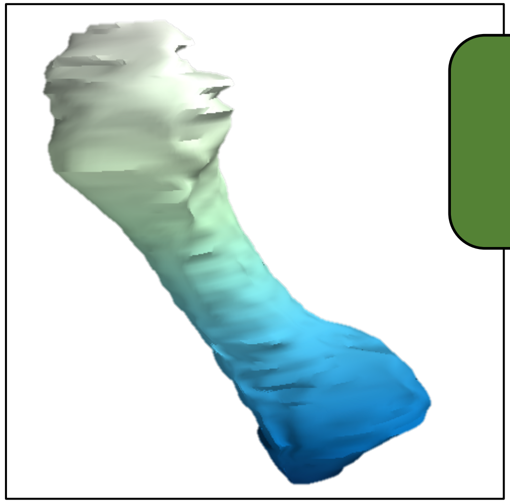
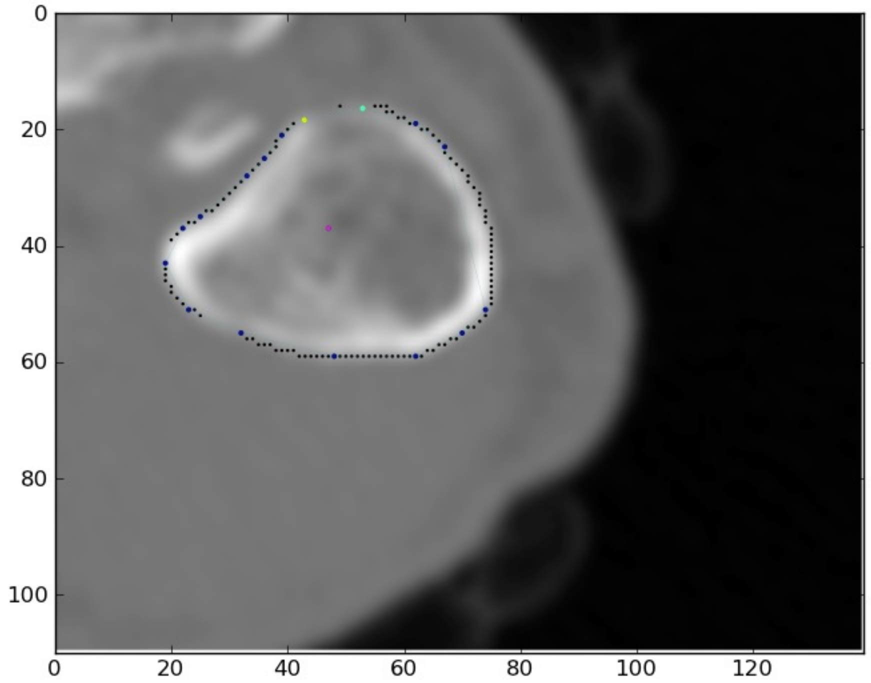


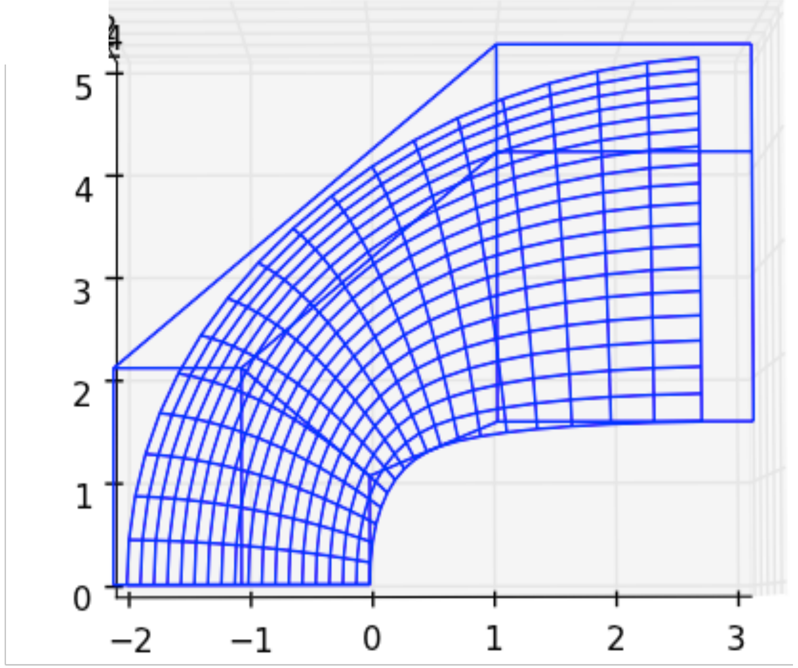
Code

Implementation
Algorithms
(Solve the problem)

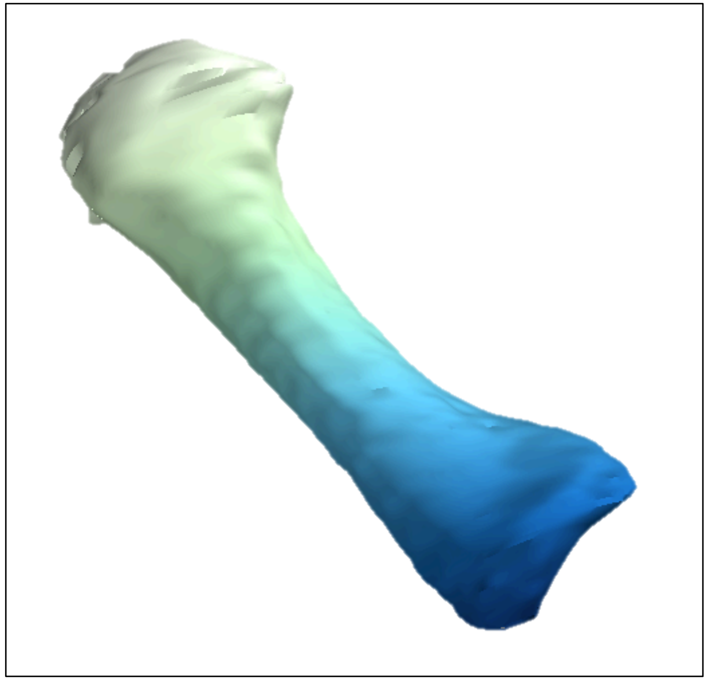
Code: Polynomials + Bone = NURBS



Segmentation
Point Cloud

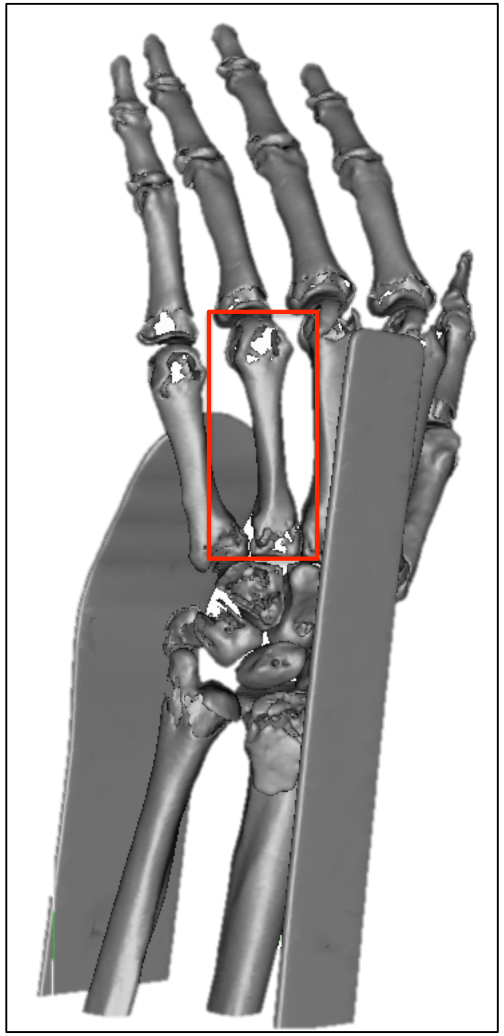


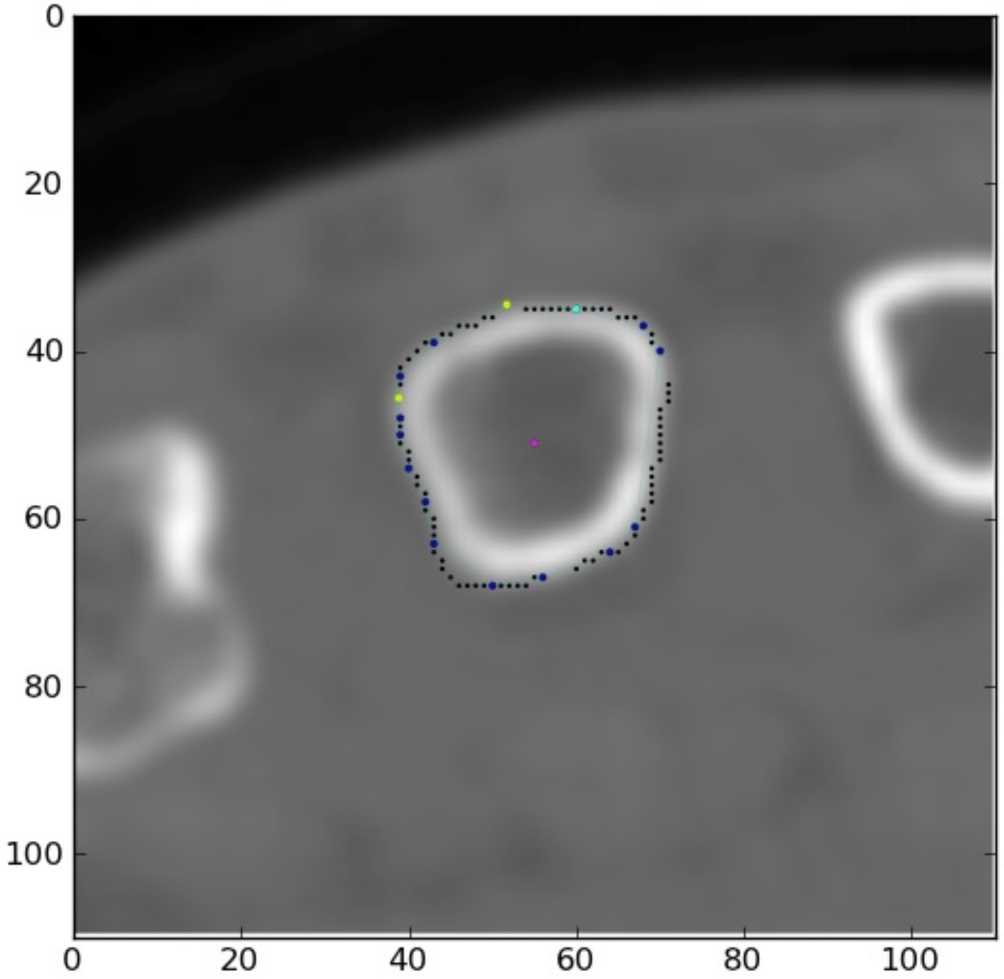
Discretization
Grid x,y,z



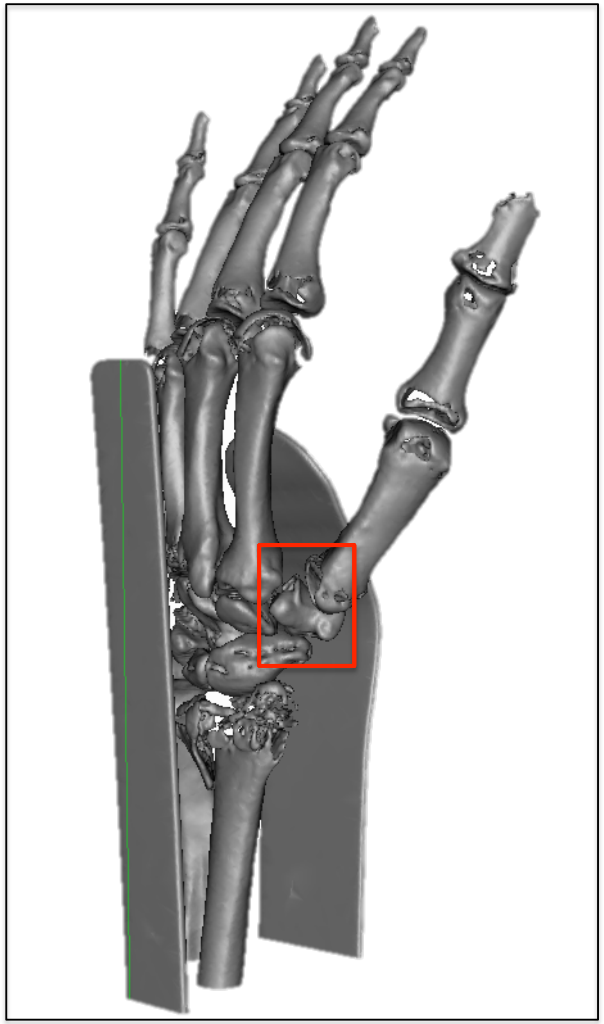
FEM
Fit NURBS model

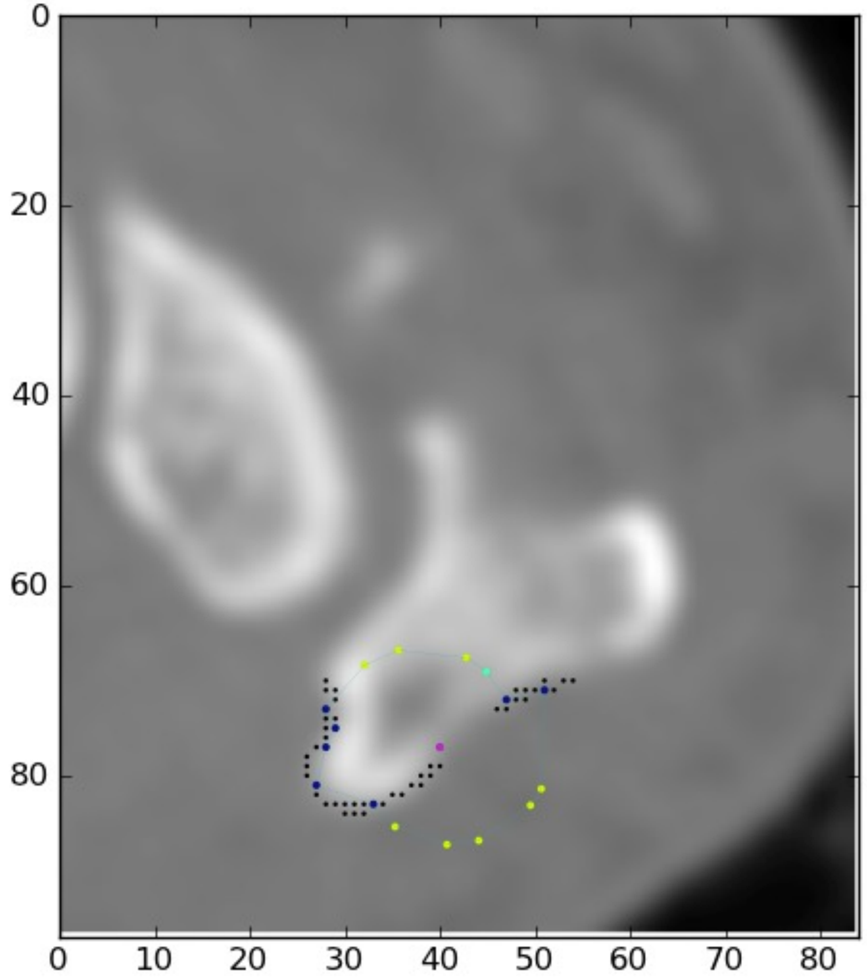
Results: Metacarpus IV

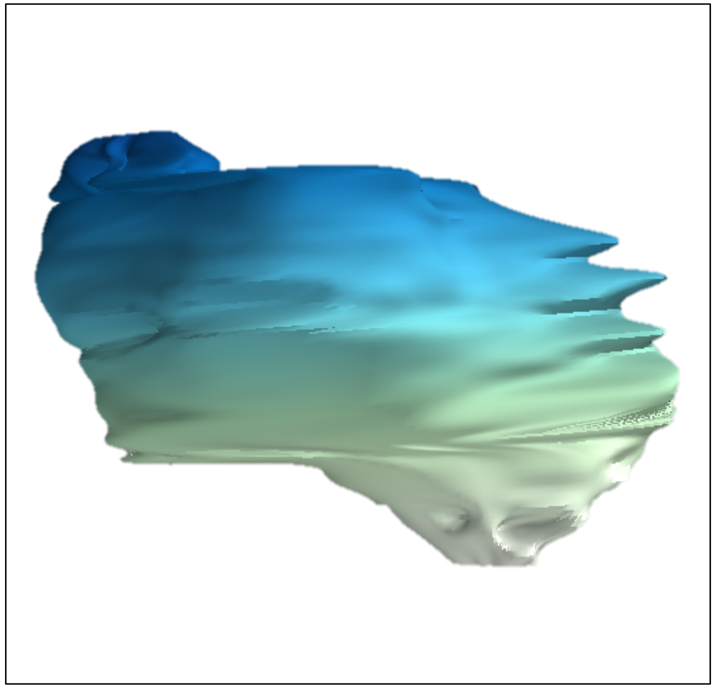




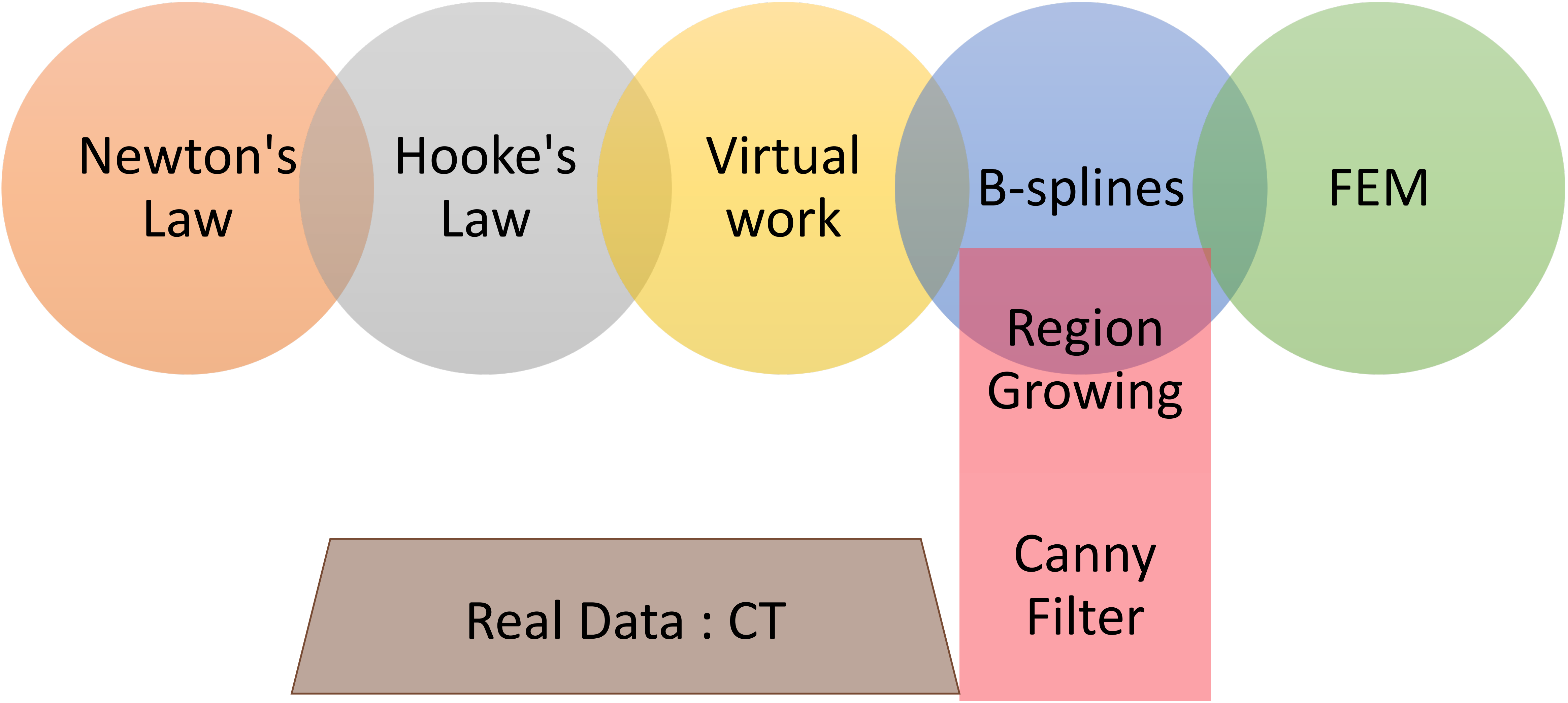
Results: Trapezium







Conclusions



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[3] P. Augat and F. Eckstein. Quantitative imaging of musculoskeletal tissue. *Annual Review of Biomedical Engineering*, 10:369–390, 2008.

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References

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[7] J. A. Cottrell, T. J. R. Hughes, and Y. Bazilevs. *Isogeometric analysis: toward integration of CAD and FEA*. John Wiley and Sons, 2009.

Questions?

Thank you,

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Physics: Elasticity equation

1[st] Newton's Law $\sum_i \vec{F}_i = 0$ Equilibrium

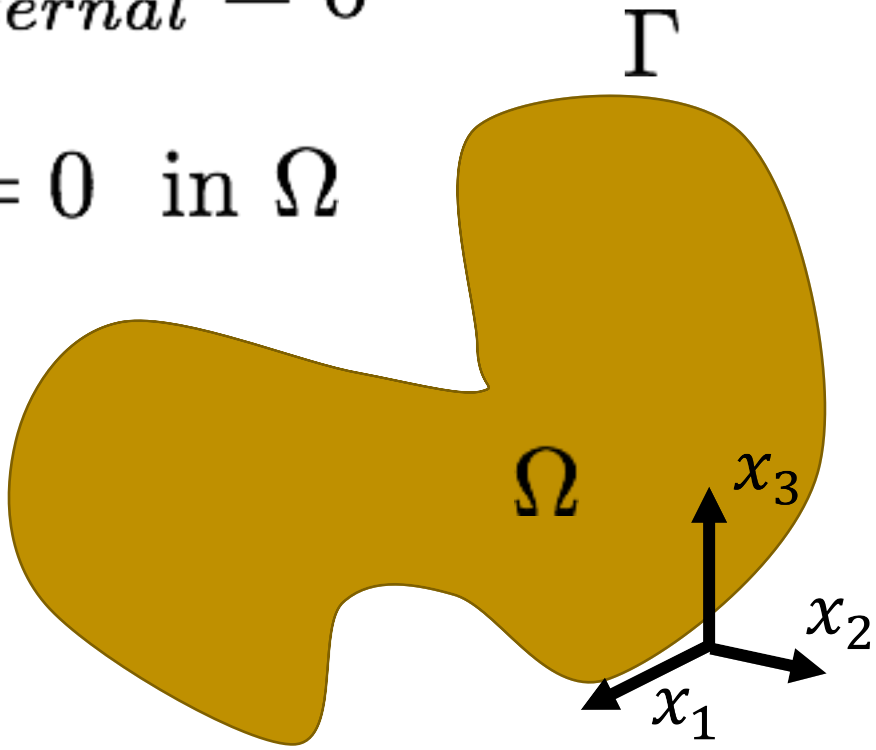
$$\vec{F}_{surface} + \vec{F}_{body} + \vec{F}_{internal} = 0$$

$$u_i = g_i \quad \text{in } \Gamma_{D_i}$$

$$\sigma_{ij} n_j = h_i \quad \text{in } \Gamma_{N_i}$$

$$f_i + \frac{\partial \sigma_{ij}}{\partial x_j} = 0 \quad \text{in } \Omega$$

Stress



Physics: Cauchy stress tensor

$$\sigma_{ij} = \sum_{k,l=1,2,3} c_{ijkl} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right)$$

Displacement

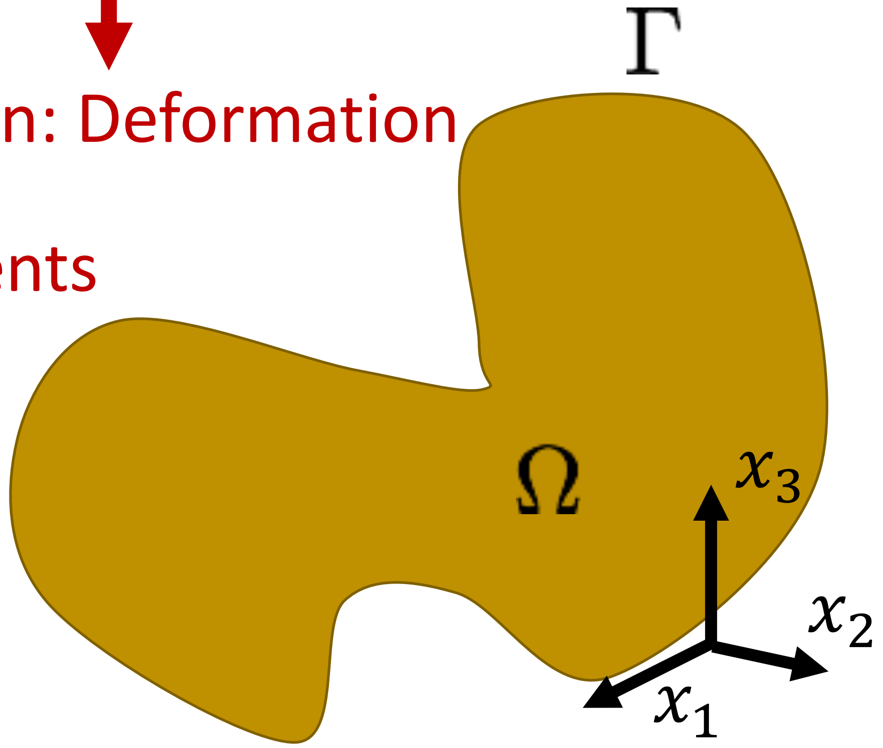
Strain: Deformation

Elasticity Coefficients

- ✓ Young's Modulus
- ✓ Poisson's Ratio

(material property)

Hooke's Law



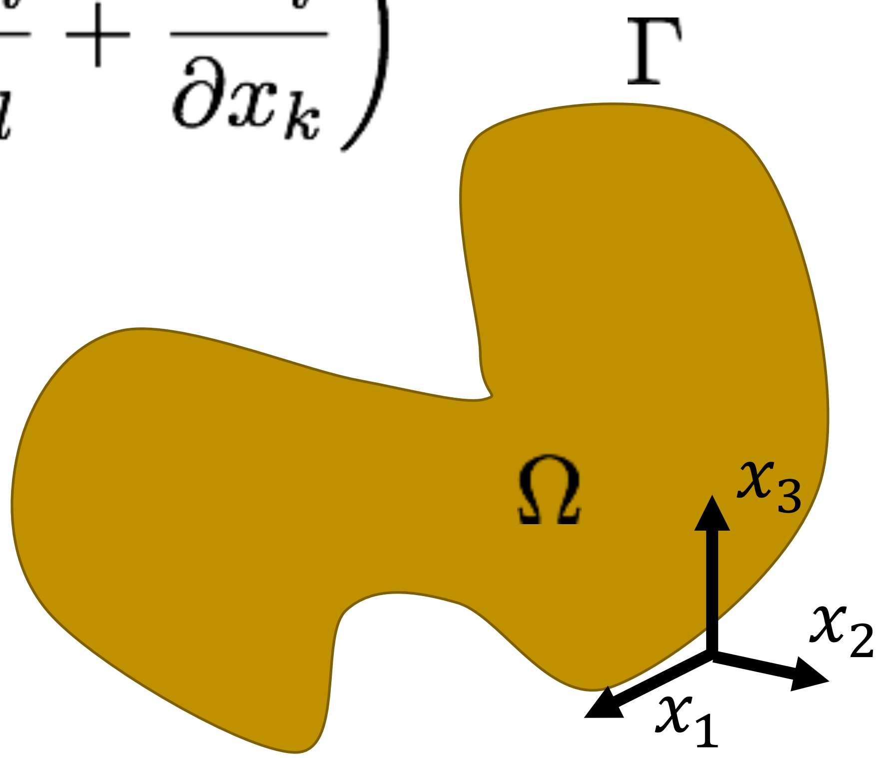
Physics: Strong Formulation

$$f_i + \frac{\partial \sigma_{ij}}{\partial x_j} = 0 \quad \text{in } \Omega$$

$$\sigma_{ij} = \sum_{k,l=1,2,3} c_{ijkl} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right)$$

$$u_i = g_i \quad \text{in } \Gamma_{D_i}$$

$$\sigma_{ij} n_j = h_i \quad \text{in } \Gamma_{N_i}$$



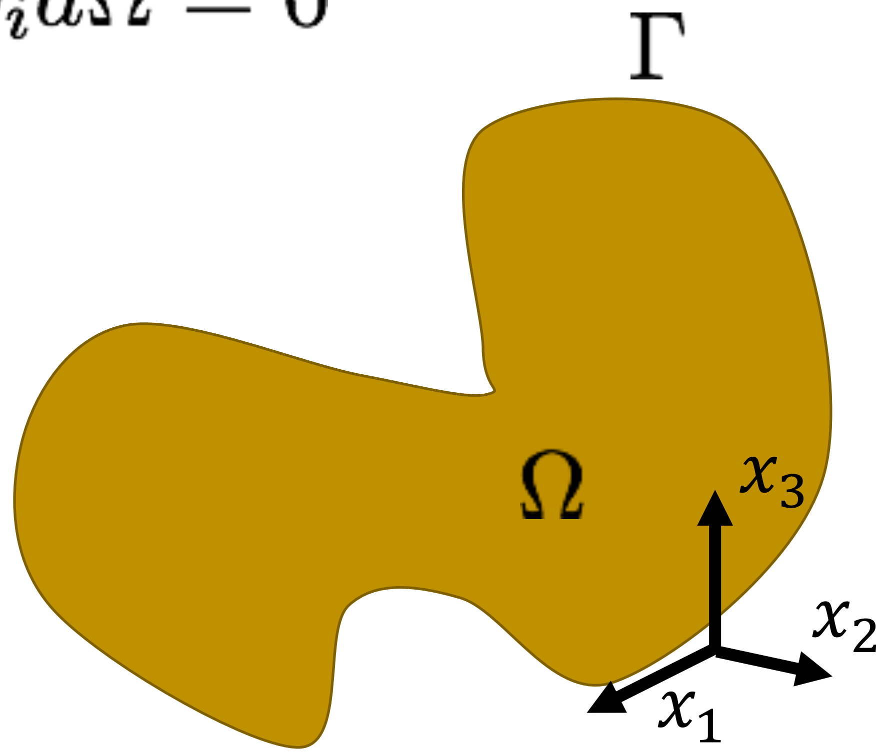
Math: Weak Formulation or Virtual Work

$$Work = \vec{F} \cdot \vec{u}$$

$$\int_{\Omega} \left(f_i + \frac{\partial \sigma_{ij}}{\partial x_j} \right) w_i d\Omega = 0$$

$$u_i = g_i \quad \text{in } \Gamma_{D_i}$$

$$\sigma_{ij} n_j = h_i \quad \text{in } \Gamma_{N_i}$$

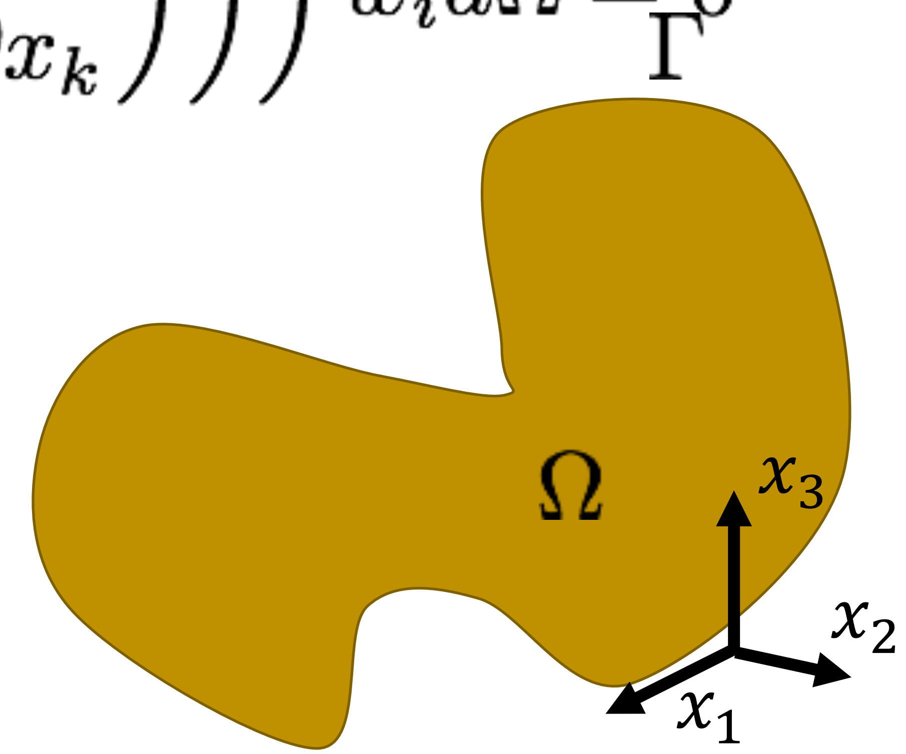


Math: Weak Formulation or Virtual Work

$$Work = \vec{F} \cdot \vec{u}$$

$$\int_\Omega \left(f_i + \frac{\partial}{\partial x_j}\left(c_{ijkl}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right)\right)\right) w_i \, d\Omega = 0$$

$$u_i = g_i \quad \text{in } \Gamma_{D_i}$$
$$\sigma_{ij} n_j = h_i \quad \text{in } \Gamma_{N_i}$$



Math: Weak Formulation or Virtual Work

$$\int_\Omega \underbrace{\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right)}_{\text{Goal: Unknown}} c_{ijkl} \underbrace{\frac{\partial w_i}{\partial x_j}}_{\text{“Joker”}} d\Omega = \underbrace{\int_\Omega w_i f_i d\Omega + \sum_{i=1}^{d}\left(\int_{\Gamma_{N_i}} w_i h_i d\Gamma\right)}_{\text{External Input: Known}}$$

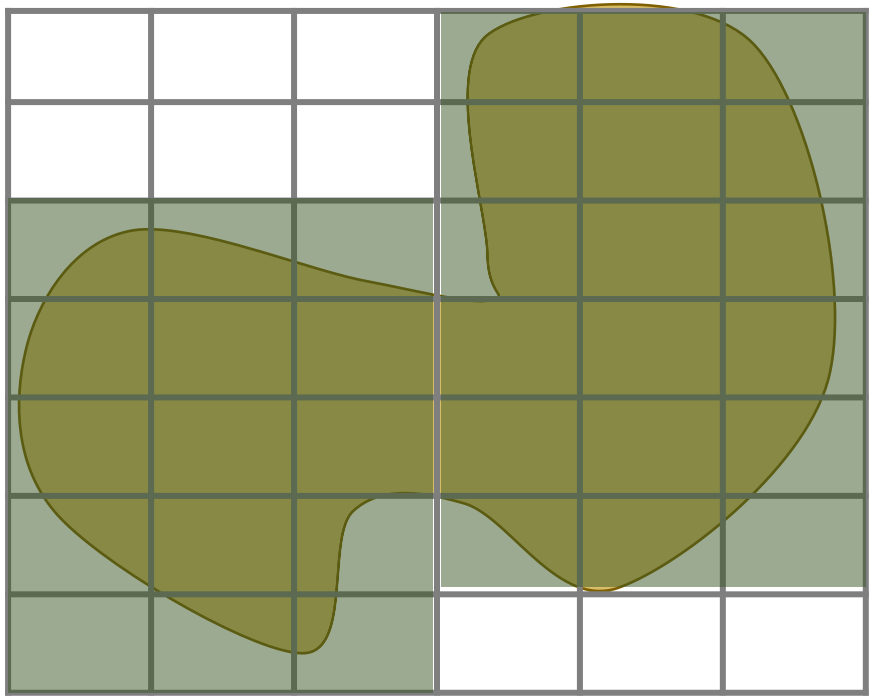
It can take any value and the equation must be true

Applied Math: Discretization

Difficult to solve using a computer

$$\int_{\Omega}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right)c_{ijkl}\frac{\partial w_i}{\partial x_j}d\Omega=\int_{\Omega}w_i f_i d\Omega$$

$$+\sum_{i=1}^{d}\left(\int_{\Gamma_{N_i}} w_i h_i d\Gamma\right)$$



1) Cut it into pieces

2) Approximate as a rectangle

Code: Finite Elements

Difficult to solve using a computer

$$\int_{\Omega}\left(\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k}\right)c_{ijkl}\frac{\partial w_i}{\partial x_j}d\Omega=\int_{\Omega}w_i f_i d\Omega + \sum_{i=1}^{d}\left(\int_{\Gamma_{N_i}} w_i h_i d\Gamma\right)$$

3) For each piece assume

$$u_i(t)=\alpha_0+\alpha_1 t+\alpha_2 t^2+\ldots$$ Unknown

$$w_i(t)=\beta_0+\beta_1 t+\beta_2 t^2+\ldots$$ "Joker"

Polynomials

Code: Finite Elements

Possible to solve using a computer

$$K\vec{\alpha} = \vec{F}$$

3) For each piece assume

$$u_i(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \ldots$$ Unknown

$$w_i(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \ldots$$ "Joker"

Polynomials

Code: Finite Elements

Stiffness Matrix (Physical properties)

$$\boxed{K}\boxed{\vec{\alpha}} = \boxed{\vec{F}}$$

External Input (Known)

Goal (Unknown)

$$u_i(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \ldots$$

Linear System (only + and *)