**Computational Astrophysics HW3**

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* **Demonstrate numerically that (i)Jacobi (ii)Gauss-Seidel and (iii)SOR are second order accurate.**

1. **Analytic solution**

Reference: <https://math.stackexchange.com/questions/1251117/analytic-solution-to-poisson-equation>

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With boundary conditions,

Has solution

1. **Compare with the result**

Define the error as

|  |  |  |  |
| --- | --- | --- | --- |
|  | Jacobi, Gauss-Seidel, SOR | | |
|  |  |  |
| Error |  |  |  |

Calculate the laplace equation in the region with different grid size, we can see that as turned half (which is grid size times 4), the error decreased by a factor of 4. Three of them are second order accurate.

* **Determine the optimum overrelaxation parameter in SOR**

|  |  |  |  |
| --- | --- | --- | --- |
|  | SOR | | |
|  |  |  |
|  | 1.697 | 1.828 | 1.909 |

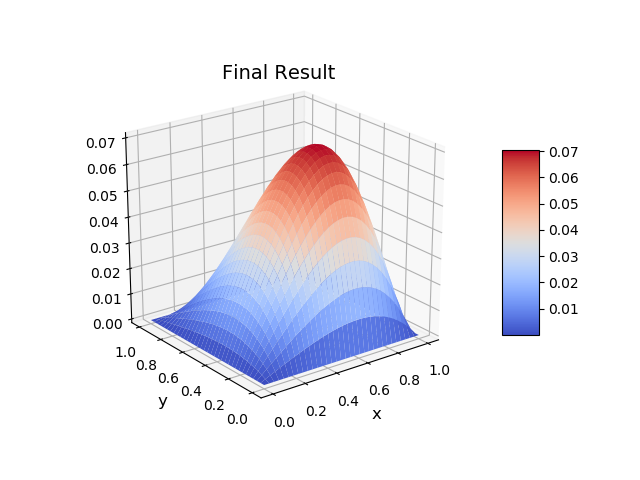
Using Scipy.optimize.minimize to find the optimal . As grid size getting large, becomes bigger. Because the more correction it can make in its reasonable range, the faster it converges.

* **How do (1)wall-clock time and (2)the number of iterations required to reach convergence scale with grid size?**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Jacobi | | | Gauss-Seidel | | | SOR | | |
|  |  |  |  |  |  |  |  |  |
| Iterations | 1313 | 4956 | 19223 | 659 | 2482 | 9618 | 71 | 137 | 269 |
| Time Used  (sec) | 2.25 | 33.15 | 419.13 | 1.07 | 15.36 | 205.33 | 0.20 | 1.32 | 10.02 |

1. We can clearly see that the numbers of iteration for Jacobi and Gauss-Seidel to reach the stopping criteria is proportional to (size of the grid is ), and SOR is proportional to , as predicted.
2. Gauss-Seidel converges faster than Jacobi, even though both of them scales as .
3. SOR method is the best of all, though we have to determine the optimum overrelaxation parameter first, which takes a lot of time.
4. Even though the time scale is not that precise, we can see that it grows with a factor of 10 ~ 15, which is roughly proportional to . Not expected this.

* **Final result**

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