

Your solutions should include your source codes (without *.exe files), results, and discussions, all in a zipped tar file. Please send your homework from your NTU email account to twchiu@phys.ntu.edu.tw before 24:00 of the due date.

1. Numerical integration with Trapezoidal, Simpson, and 5-point formulas

Evaluate the integral $\int_0^{\pi} dx \sin x$

using approximations to the integrand that are piecewise linear, quadratic, and quartic. With N intervals, and hence $N+1$ points, evaluate the integral for $N = 4, 8, 16, 32, 64, \dots, 1024$, and compare the accuracy of the methods.

2. Romberg Integration

The intensity of light after the diffraction at a knife's edge varies as we move away from the edge according to

$$I = \frac{I_0}{2} \left\{ [C(v) + 0.5]^2 + [S(v) + 0.5]^2 \right\}$$

where I_0 is the intensity of the incident light, v is proportional to the distance moved, and $C(v)$ and $S(v)$ are Fresnel integrals

$$C(v) = \int_0^v dw \cos(\pi w^2 / 2), \quad S(v) = \int_0^v dw \sin(\pi w^2 / 2)$$

Write a C/C++ program to calculate the Fresnel integrals, and evaluate I / I_0 as a function of v , and plot your results.