

**1. Single Cluster Algorithm**

Show that the single cluster algorithm satisfies the detailed balance.

(Hint: read Wolff's paper in CEIBA).

**2. Two-dimensional Ising Model**

Write a C/C++ code to perform the Monte Carlo Simulation of two-dimensional Ising model with Metropolis, heat bath, and single cluster algorithms respectively. For  $N=100$ ,  $J=1.0$ ,  $B=0.0$ ,  $T=2.26$  (in the unit  $k=1$ ), measure the expectation values of the energy density, specific heat, magnetization density, and magnetic susceptibility. Moreover, estimate the error of the mean by the jackknife method with the bin size at which the error saturates. Compare your results with the exact solution.

**3. Integrated autocorrelation time in MC simulations of 2-dimensional Ising model**

Use your code developed in the problem 2 to measure the integrated autocorrelation time of Metropolis, heat bath, and single cluster algorithms respectively, and from which to estimate their dynamical critical exponents near the critical temperature.