Problem Set 2 **R08244002 Shin-Rong, Tsai**

- **Numerical integration with Trapezoidal, Simpson, and 5-point formulas**

1. Results

numerical_integration.c

```
cindytsai@TURQUOISEA /cygdrive/d/GitHub/Computational_Physics/Assignment/ProblemSet_2
$ gcc numerical_integration.c -o a.out

cindytsai@TURQUOISEA /cygdrive/d/GitHub/Computational_Physics/Assignment/ProblemSet_2
$ ./a.out
exact solution : 2
Solution Given By Following Methods==============================================================

  N               linear                    quadratic                        quartic
   4    1.89611889793703980445e+00    2.00455975498442073857e+00    1.99857073182383571108e+00
   8    1.97423160194555080693e+00    2.00026916994838810382e+00    1.99998313094598545447e+00
  16    1.99357034377233954814e+00    2.00001659104793549915e+00    1.99999975245457206618e+00
  32    1.99839336097014408367e+00    2.00000103336941270626e+00    1.99999999619084478653e+00
  64    1.99959838864003747183e+00    2.00000006453000134243e+00    1.99999999994070787324e+00
 128    1.99989960018420376286e+00    2.00000000403225808299e+00    1.99999999999907429604e+00
 256    1.99997490023505308798e+00    2.00000000025200286302e+00    1.99999999999998623323e+00
 512    1.99999372507057682213e+00    2.00000000001575228836e+00    2.00000000000000088818e+00
1024    1.99999843126838339202e+00    2.00000000000098276942e+00    1.99999999999999888978e+00

Errors===========================================================================================

  N               linear                    quadratic                        quartic
   4    -1.03881102062960195553e-01    4.55975498442073856609e-03    -1.42926817616428891711e-03
   8    -2.57683980544491930686e-02    2.69169948388103819070e-04    -1.68690540145455258880e-05
  16    -6.42965622766045186154e-03    1.65910479354991480250e-05    -2.47545427933815176402e-07
  32    -1.60663902985591633410e-03    1.03336941270626425649e-06    -3.80915521347446883738e-09
  64    -4.01611359962528169376e-04    6.45300013424332519207e-08    -5.92921267639212601352e-11
 128    -1.00399815796237135146e-04    4.03225808298657284467e-09    -9.25703957932455523405e-13
 256    -2.50997649469120176491e-05    2.52002863021516532172e-10    -1.37667655053519411013e-14
 512    -6.27492942317786628337e-06    1.57522883625915710581e-11    8.88178419700125232339e-16
1024    -1.56873161660797677541e-06    9.82769421398188569583e-13    -1.11022302462515654042e-15
```

2. Discussion

The first table is the solutions given by different methods. The second table is their corresponding error, where I define $error = approx - \int_0^\pi dx \cdot sinx$.

We can see that three of the methods are getting closer to the exact solution as the interval becomes smaller. And the quartic method converges the fastest.

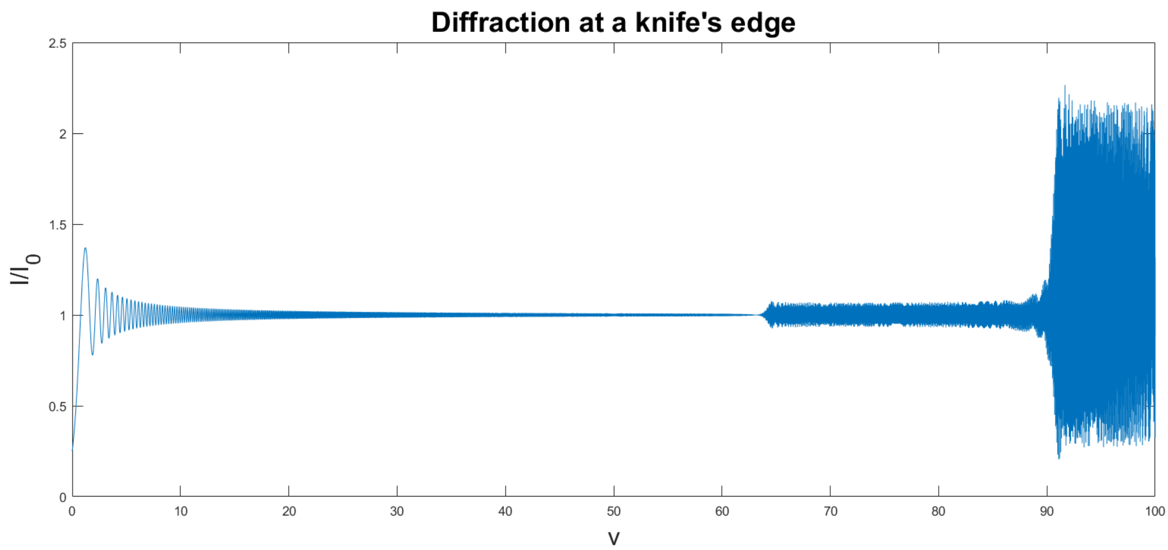
I think it might reach the smallest interval limit when using quartic method with $\mathrm{N} = 512$, since it reaches the smallest error of all. And this error is positive, which does not match the approximation error we have derived. So there might be some round off error.

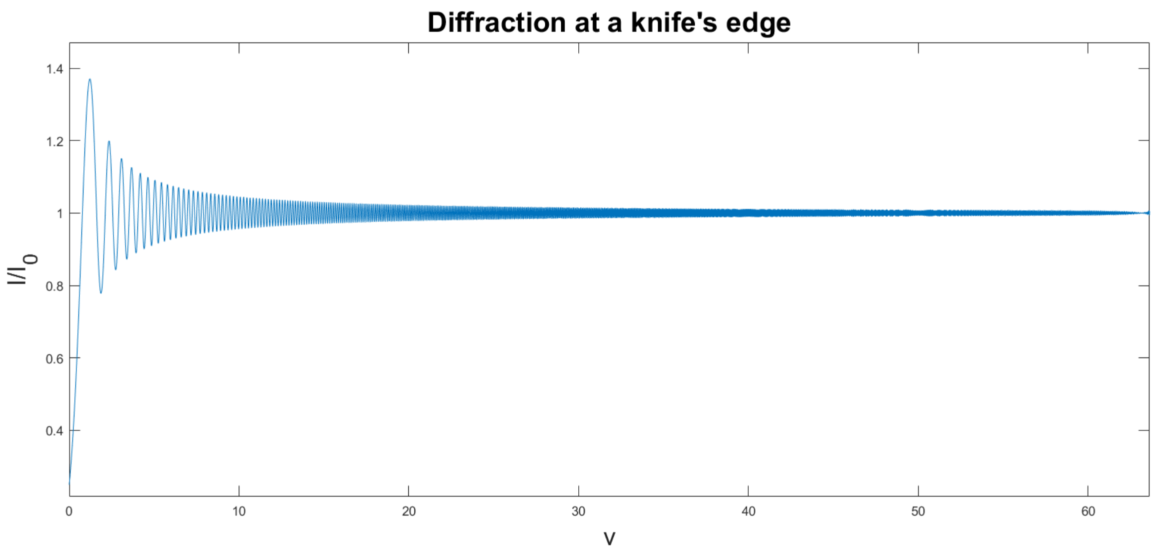
Apart from this, we can still see that the sign of the errors matches the one we derived.

- **Romberg Integration**

1. Results

romberg_integration.c, then plot the output with MATLAB.





2. Discussion

The first figure plots the diffraction at a knife's edge from distance range 0 to 100. We can easily see that when v is more than approximately 65, this calculation is no longer convincible. Since the diffraction effect weakens, the intensity ratio must be somewhat near 1 at larger v. So maybe the interval number is too small for calculating range more than 65, thus the integration starts to break down.

The second figure only display the convincible part of this calculation. We can see that the diffraction pattern is strong when it is near the edge. The effect is getting weak as we get further, the intensity ratio tends to 1.