ESSC 4520
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L11 Exercise

Ex1

a)

The data is d the total mass of objects that measured, since we measure the weights for 100 times, N=100. The model parameter is the masses of 100 objects that we want to know, since we have 100 objects in total, M=100.

b)

$$G = \begin{pmatrix} g_{1,1} & \cdots & g_{1,100} \\ \vdots & \ddots & \vdots \\ g_{100,1} & \cdots & g_{100,100} \end{pmatrix}$$

$$G = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \ldots & 1 & 1 \end{pmatrix}$$

c)

The percentage of 0 in **G**:

$$\frac{(100-1)+(100-2)+(99-2)(100-3)}{99 \times 100} \times 100\%$$

= 97.0%

Therefore, 97% in **G** is 0, which means **G** is very sparse.

Ex2

a)

Let the m^{true} be (which is generate randomly):

[[0.02129921]
 [0.02576802]
 [0.03426352]
 [0.04026813]
 [0.06605618]
 [0.08028625]
 [0.09362312]
 [0.1014033]
 [0.11009706]
 [0.13216574]
 [0.14446185]
 [0.14450202]
 [0.14610197]
 [0.15969943]
 [0.16462778]
 [0.16977096]
 [0.17824074]
 [0.18136565]
 [0.21135138]
 [0.21488064]
 [0.21670656]
 [0.22391741]
 [0.22492248]
 [0.22631296]
 [0.23587533]
 [0.23675233]
 [0.2529258]
 [0.25907   ]
 [0.28480187]
 [0.28703282]
 [0.32239648]
 [0.32269782]
 [0.32627988]
 [0.32756289]
 [0.34412522]
 [0.34426861]
 [0.34938977]
 [0.36221647]
 [0.37292202]
 [0.38494272]
 [0.41249865]
 [0.42011854]
 [0.44238884]
 [0.44311378]
 [0.46244745]

[0.4651617]
[0.4661333]
[0.46643223]
[0.49941861]
[0.50119842]
[0.50628243]
[0.52884835]
[0.52999382]
[0.53648301]
[0.53710595]
[0.55080671]
[0.59001088]
[0.59090882]
[0.59587233]
[0.59616722]
[0.60558143]
[0.61074076]
[0.61985632]
[0.62049003]
[0.62454813]
[0.64273636]
[0.64473372]
[0.64666882]
[0.64693313]
[0.65782041]
[0.67629993]
[0.67867621]
[0.68463883]
[0.70137425]
[0.70180614]
[0.71051886]
[0.71627169]
[0.72811939]
[0.74562597]
[0.75374605]
[0.76860262]
[0.81489863]
[0.81641296]
[0.81817889]
[0.82381995]
[0.83295472]
[0.84042896]
[0.87005365]
[0.87673904]
[0.87760633]
[0.88249298]
[0.91290606]
[0.94045013]

[0.95371563]
[0.95767185]
[0.95857862]
[0.96370995]
[0.9751578]
[0.97946972]
[0.99639473]]

b)
Kernel G is the G in Exercise 1.

$$G = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \ldots & 1 & 1 \end{pmatrix}$$

c)
Let randomly generated n be
[[-1.33936193e-03]
[-6.51651637e-03]
[-2.51313296e-02]
[4.70664052e-03]
[-8.59353100e-03]
[-4.29437119e-03]
[-6.79096821e-03]
[-3.60489283e-04]
[-1.98154336e-02]
[1.09923809e-02]
[3.25571503e-05]
[-3.98391526e-03]
[-2.90736628e-03]
[-5.92344806e-03]
[1.35414778e-02]
[-1.44953869e-02]
[-3.82838851e-04]
[2.24488624e-03]
[1.08755789e-02]
[1.44733160e-03]
[-6.53865416e-03]
[5.10904212e-03]
[-1.46500549e-02]
[-6.81702159e-03]
[-6.08650335e-03]
[2.27209076e-03]
[-7.67615043e-03]
[-2.74588782e-04]
[-1.18927521e-02]

[-9.00547403e-03]
[1.46753553e-03]
[-2.43836006e-03]
[8.53065840e-03]
[-4.37234796e-03]
[-1.03961893e-02]
[-1.24923462e-02]
[-1.40612182e-02]
[5.96508919e-04]
[-1.75933378e-03]
[3.54977560e-03]
[-3.81463282e-03]
[-1.53913406e-02]
[-3.12881054e-03]
[1.49543668e-02]
[-1.06532002e-02]
[1.69851292e-02]
[-2.94953736e-04]
[-7.52423347e-03]
[-4.92738738e-03]
[-1.00216235e-02]
[2.87895805e-03]
[4.31178946e-03]
[6.11717198e-03]
[8.57401555e-03]
[7.31265534e-03]
[-5.92838586e-03]
[-1.19720083e-02]
[-1.81336838e-02]
[-5.28867055e-03]
[-2.23040819e-03]
[1.92542208e-02]
[3.77939747e-03]
[-1.37407234e-02]
[-6.75071595e-03]
[-1.87266691e-02]
[6.31663001e-03]
[1.61611000e-02]
[1.21763005e-03]
[-3.07504181e-03]
[-1.58473306e-02]
[-2.06031406e-03]
[-2.53452580e-02]
[-7.22398276e-03]
[1.12070736e-02]
[3.23491240e-03]
[-2.28971497e-03]
[-1.40160015e-02]

[8.49651218e-04]
[-8.04919328e-03]
[1.84887525e-02]
[-5.23835351e-04]
[-3.22872851e-03]
[-4.03423759e-03]
[-2.30209289e-02]
[7.16866424e-04]
[7.56596399e-03]
[8.49590611e-03]
[6.72000846e-05]
[-1.20889127e-02]
[-1.96929558e-02]
[1.07098302e-02]
[-7.56574007e-03]
[-1.50757959e-02]
[-2.09751394e-03]
[-8.63022618e-03]
[8.18395716e-03]
[1.57608907e-02]
[-7.18560869e-03]
[7.52724761e-03]
[-3.83610916e-03]]

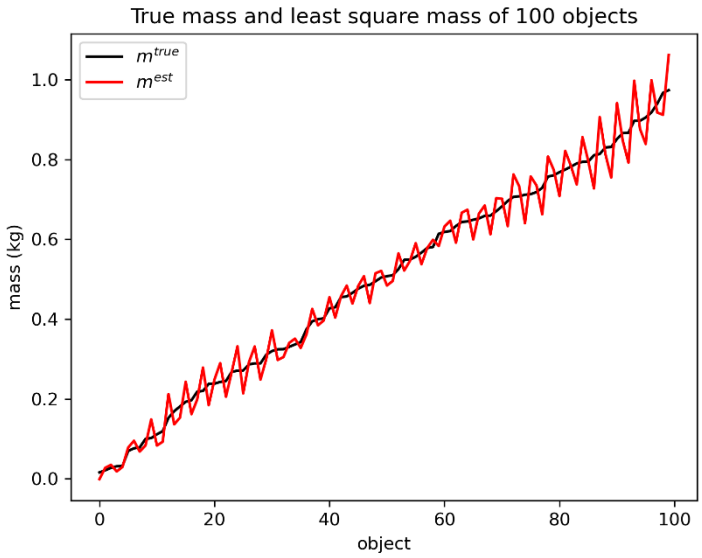
Such that d =
[[0.01995985]
[0.04055072]
[0.05619942]
[0.10500631]
[0.1319943]
[0.18231619]
[0.23317459]
[0.27495219]
[0.28530805]
[0.35465848]
[0.38675721]
[0.4171457]
[0.43215848]
[0.44437997]
[0.48397067]
[0.47960279]
[0.51225665]
[0.53162224]
[0.58183335]
[0.609045]
[0.63639992]
[0.66061365]
[0.6508964]

[0.66833583]
[0.68102427]
[0.70121271]
[0.71787731]
[0.74847354]
[0.78490491]
[0.82189921]
[0.89569871]
[0.92968876]
[0.97990483]
[0.97216824]
[0.9875718]
[1.00346438]
[1.02372238]
[1.05647135]
[1.08276892]
[1.12363099]
[1.16654876]
[1.20216857]
[1.27187722]
[1.32057553]
[1.33729687]
[1.38770806]
[1.39344749]
[1.390203]
[1.42705675]
[1.45702764]
[1.50977842]
[1.54064099]
[1.57124177]
[1.6038992]
[1.61089544]
[1.61846728]
[1.66595153]
[1.71359272]
[1.77150336]
[1.78071796]
[1.8168752]
[1.81626881]
[1.82243779]
[1.8443364]
[1.84616782]
[1.89409115]
[1.92817931]
[1.93535654]
[1.93526064]
[1.93557503]
[1.97899316]

[1.98745129]
[2.03239099]
[2.07589637]
[2.09105414]
[2.11140953]
[2.11458068]
[2.15575958]
[2.18196785]
[2.24598016]
[2.2674508]
[2.33401857]
[2.39587997]
[2.42646955]
[2.45912867]
[2.48251952]
[2.50569954]
[2.54350453]
[2.57513275]
[2.60470607]
[2.64754819]
[2.66543963]
[2.72077337]
[2.80497431]
[2.84320739]
[2.87815007]
[2.89572132]
[2.89026077]
[2.92586472]
[2.94718614]]

d)
The problem is solved by $m^{est} = [G^T G]^{-1} G^T d$ in the code.

e)



f)

The number of estimated model parameters that are within ±2σm of their true value is 100 in this case.

g)