ESSC 4520
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L04 Exercise

<u>Ex1</u>
The 1 D advection equation:

$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x}$$

It can be approximate to

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -c\frac{u_{i+1}^n - u_i^n}{\Delta x}$$

By Forward-in-time, Forward-in-space (F-T F-S) scheme.

Therefore, we can estimate u_i^{n+1} by

$$u_i^{n+1} = u_i^n - \frac{c\Delta t}{\Delta x}(u_{i+1}^n - u_i^n) \ldots(1)$$

Since the analytical solution of the 1 D advection equation is

$$u(x,t) = acos(kx - \omega t) = ae^{\iota(kx-\omega t)} = ae^{-\iota\omega t}e^{\iota kx}$$

We estimate it to the numerical solution:

$$u_i^n = \lambda^n Ae^{\iota ki\Delta x} \ldots(2)$$

Substitute (2) into (1):

$$\lambda^{n+1}Ae^{\iota ki\Delta x} = \lambda^n Ae^{\iota ki\Delta x} - \frac{c\Delta t}{\Delta x}\left(\lambda^n Ae^{\iota k(i+1)\Delta x} - \lambda^n Ae^{\iota ki\Delta x}\right)$$

By dividing by $\lambda^n Ae^{\iota ki\Delta x}$, we get

$$\lambda = 1 - \frac{c\Delta t}{\Delta x}(e^{\iota k\Delta x} - 1)$$

$$\lambda = 1 - \frac{c\Delta t}{\Delta x}\{[cos(k\Delta x) + \iota sin(k\Delta x)] - 1\}$$

$$\lambda = 1 - \frac{c\Delta t}{\Delta x}[cos(k\Delta x) - 1] - \iota\frac{c\Delta t}{\Delta x}sin(k\Delta x)$$

For waves with maximum wave number $K_{max} = \frac{\pi}{\Delta x}$,

$$cos\left(\frac{\pi}{\Delta x}\Delta x\right) = -1$$

$$sin\left(\frac{\pi}{\Delta x}\Delta x\right) = 0$$

Therefore,

$$|\lambda| = \left|\sqrt{\left[1 - \frac{c\Delta t}{\Delta x}(-1-1)\right]^2}\right|$$

$$|\lambda| = \left|1 + 2\frac{c\Delta t}{\Delta x}\right|$$

For stable condition of F-T F-S scheme, $|\lambda| \leq 1$, which means that

$$-1 \leq \lambda \leq 1$$

$$-1 \leq 1 + 2\frac{c\Delta t}{\Delta x} \leq 1$$

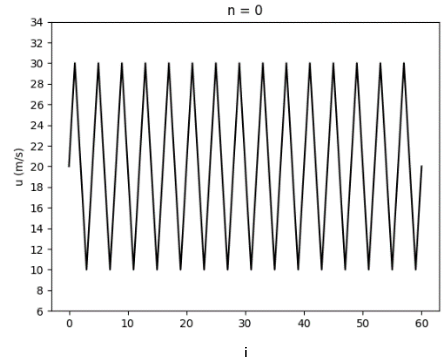
$$-2 \leq 2\frac{c\Delta t}{\Delta x} \leq 0$$

$$-1 \leq \frac{c\Delta t}{\Delta x} \leq 0$$

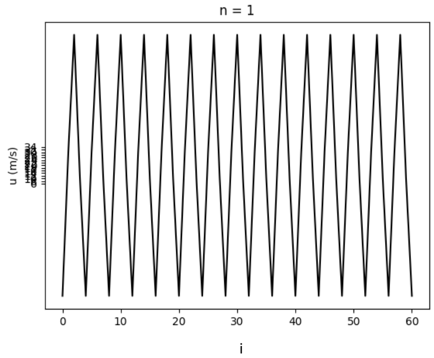
Therefore, the stability condition for F-T F-S applied to 1D advection equation is $-1 \leq \frac{c\Delta t}{\Delta x} \leq 0$.

Ex3

a) Taking $\Delta t = 1000s$, so that it will satisfy the CFL condition and $\Delta t = 100000s$, so that it will not satisfy the CFL condition. The model start blowing up from n = 1. At n = 1, the magnitude is already several times to the initial one when $\Delta t = 100000s$.



The initial u(x,0) in space i = 0 to i = 60 when $\Delta t = 100000s$.



The u(x,100000) in space i = 0 to i = 60 when $\Delta t = 100000s$.

b)

	Case 1 RMS	Case 2 RMS	Case 3 RMS
n = 0	0	0	0
n = 1	2.85	4.25	3.18×10^{15}
n = 2	5.68	8.48	6.29×10^{15}
n = 3	8.51	12.7	9.24×10^{15}
n = 4	11.4	17.5	1.20×10^{16}
n = 5	14.3	22.2	1.44×10^{16}
n = 6	17.2	26.8	1.65×10^{16}
n = 7	20.0	31.3	1.81×10^{16}
n = 8	22.7	35.6	1.94×10^{16}
n = 9	25.3	39.6	2.01×10^{16}
n = 10	27.8	43.5	2.04×10^{16}

n = 11	30.2	47.1	2.01×10^{16}
n = 12	32.4	50.5	1.94×10^{16}
n = 13	34.6	53.6	1.81×10^{16}
n = 14	36.6	56.4	1.65×10^{16}
n = 15	38.4	58.8	1.44×10^{16}
n = 16	40.1	61.0	1.20×10^{16}
n = 17	41.7	62.9	9.24×10^{15}
n = 18	43.1	64.5	6.29×10^{15}
n = 19	44.3	65.7	3.18×10^{15}
n = 20	2.85	66.7	42.9

Ex4 [Micro-modules on KEEP]

Multiple Choice Answers

1. C
2. B
3. D
4. A
5. C
6. D
7. A
8. A
9. B