ESSC 4520 Exercise L02 23-01-2022

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Ex 1

PGF in x-direction $= -\frac{1}{\rho}\frac{\partial p}{\partial x}$

$$= -\frac{p_R}{\rho}\frac{\partial}{\partial x}\left(\frac{p}{p_R}\right)$$

$$= -\frac{p_R}{\rho}\cdot\frac{p}{p_R}\frac{\partial}{\partial x}\ln\left(\frac{p}{p_R}\right)$$

$$= -\frac{\cancel{p_R}\,p}{\rho\cancel{p_R}}\cdot\frac{c_p}{R_d}\frac{\partial}{\partial x}\left[\ln\left(\frac{p}{p_R}\right)\cdot\frac{R_d}{c_p}\right]$$

$$= -\frac{p}{\rho}\cdot\frac{c_p}{R_d}\cdot\left(\frac{p_R}{p}\right)^{\kappa}\frac{\partial}{\partial x}\left(\frac{p}{p_R}\right)^{\frac{R_d}{c_p}} \qquad \kappa = \frac{R_d}{c_p}$$

$$= -\frac{p}{\rho}\cdot\frac{c_p}{R_d}\cdot\frac{\theta}{T}\frac{\partial}{\partial x}\left(\frac{p}{p_R}\right)^{\kappa} \qquad \left(\frac{p_R}{p}\right)^{\kappa} = \frac{\theta}{T}$$

$$= -\frac{(\cancel{\rho}\cancel{R_d}\cancel{T})}{\cancel{\rho}}\cdot\frac{c_p}{\cancel{R_d}}\cdot\frac{\theta}{\cancel{T}}\frac{\partial}{\partial x}\left(\frac{p}{p_R}\right)^{\kappa} \qquad p = \rho R_d T$$

$$= -c_p\theta\frac{\partial}{\partial x}\left(\frac{p}{p_R}\right)^{\kappa}$$

$$= -c_p\theta\frac{\partial \pi}{\partial x} \qquad \pi = \left(\frac{p}{p_R}\right)^{\kappa}$$

$\because$ Similar in y-direction and z-direction

$\therefore$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = -c_p\theta\frac{\partial \pi}{\partial x}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} = -c_p\theta\frac{\partial \pi}{\partial y}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = -c_p\theta\frac{\partial \pi}{\partial z}$$

Ex 2

$$\cancel{-\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right)_z} \quad -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z$$

$$= -\frac{1}{\rho}\left[\left(\frac{\partial p}{\partial x}\right)_p - \left(\frac{\partial z}{\partial x}\right)_p\left(\frac{\partial p}{\partial z}\right)_x\right]$$

$$= -\frac{1}{\rho}\left[\left(\frac{\partial p}{\partial x}\right)_p - \left(\frac{\partial z}{\partial x}\right)_p(-g\rho)\right]$$

Assuming hydrostatic balance:

$$\frac{\partial p}{\partial z} = -\rho g$$

$$= -\frac{1}{\rho}\left[\left(\frac{\partial p}{\partial x}\right)_p + g\rho\left(\frac{\partial z}{\partial x}\right)_p\right]$$

$$= -\frac{1}{\rho}\cancelto{0}{\left(\frac{\partial p}{\partial x}\right)_p} - \left(\frac{\partial \Phi}{\partial x}\right)_p$$

Since $\partial\Phi = g\partial z$

$$= -\left(\frac{\partial \Phi}{\partial x}\right)_p$$

$$\therefore\ -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = -\left(\frac{\partial \Phi}{\partial x}\right)_p$$

Ex 4

a)

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z$$

$$= -\frac{1}{\rho}\left[\left(\frac{\partial p}{\partial x}\right)_\sigma - \left(\frac{\partial z}{\partial x}\right)_\sigma\left(\frac{\partial p}{\partial z}\right)_x\right]$$

$$= -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_\sigma + \frac{1}{\rho}\left(\frac{\partial z}{\partial x}\right)_\sigma(-\rho g)$$

Assuming hydrostatic balance:

$$\frac{\partial p}{\partial z} = -\rho g$$

$$= -\frac{1}{\rho}\left[\frac{\partial(p_0+\tilde{p})}{\partial x}\right]_\sigma - g\left(\frac{\partial z}{\partial x}\right)_\sigma$$

$$= -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x}\right)_\sigma - \frac{1}{\rho}\left(\frac{\partial p_0}{\partial x}\right)_\sigma - g\left(\frac{\partial z}{\partial x}\right)_\sigma$$

$$\frac{p_0 - p_T}{p_s - p_T} = \sigma$$

$$= -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x}\right)_\sigma - \frac{1}{\rho}\left[\frac{\partial}{\partial x}(\sigma p_s - \sigma p_T + p_T)\right]_\sigma - g\left(\frac{\partial z}{\partial x}\right)_\sigma$$

$$p_0 = \sigma(p_s - p_T) + p_T$$

$$= -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x}\right)_\sigma - \frac{\sigma}{\rho}\left(\frac{\partial p_s}{\partial x}\right)_\sigma + \cancelto{0}{\left[\frac{\partial(\sigma p_T)}{\partial x}\right]_\sigma} - \frac{1}{\rho}\cancelto{0}{\left(\frac{\partial p_T}{\partial x}\right)_\sigma} - g\left(\frac{\partial z}{\partial x}\right)_\sigma$$

$$p_0 = \sigma p_s - \sigma p_T + p_T$$

$$= -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x}\right)_\sigma - \frac{\sigma}{\rho}\left(\frac{\partial p_s}{\partial x}\right)_\sigma - g\left(\frac{\partial z}{\partial x}\right)_\sigma$$

$$= -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x}\right)_\sigma - \frac{\sigma}{\rho}\left(\frac{\partial p_s}{\partial x}\right)_\sigma - \left(\frac{\partial \Phi}{\partial x}\right)_\sigma$$

$$\partial\Phi = g\partial z$$

$$\therefore\ -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_z = -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x}\right)_\sigma - \frac{\sigma}{\rho}\left(\frac{\partial p_s}{\partial x}\right)_\sigma - \left(\frac{\partial \Phi}{\partial x}\right)_\sigma$$

Ex 4

b) $$-\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right)_z = -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x}\right)_\sigma - \frac{\sigma}{\rho}\left(\frac{\partial p_s}{\partial x}\right)_\sigma - \left(\frac{\partial \Phi}{\partial x}\right)_\sigma$$

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial z}\right)_z = -\frac{1}{\rho}\left(\frac{\partial \tilde{p}}{\partial x}\right)_\sigma + \sigma\left(\frac{\partial \Phi_s}{\partial x}\right)_\sigma - \left(\frac{\partial \Phi}{\partial x}\right)_\sigma$$

By hydrostatic balance:

$$\frac{\partial p_s}{\partial z_s} = -\rho g$$

$$-\frac{1}{\rho}\partial p_s = g\,\partial z_s = \partial \Phi_s$$

Ex 3

Pressure	Left	Middle	Right
200	0	0	0
250	0.0625	0.111111	0.083333
300	0.125	0.222222	0.166667
400	0.25	0.444444	0.333333
500	0.375	0.666667	0.5
650	0.5625	1	0.75
800	0.75	NaN	1
1000	1	NaN	NaN