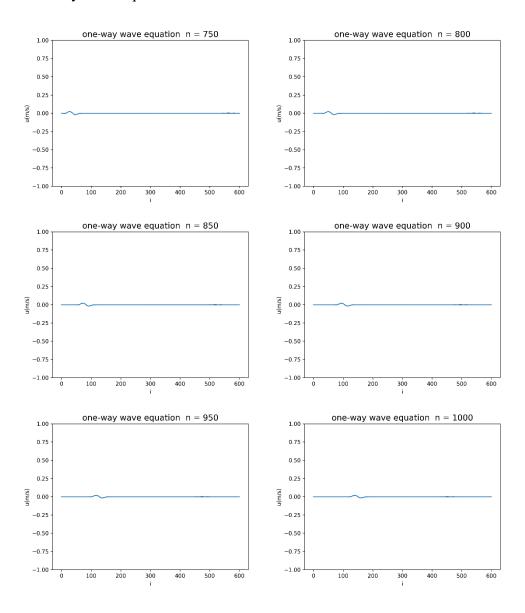
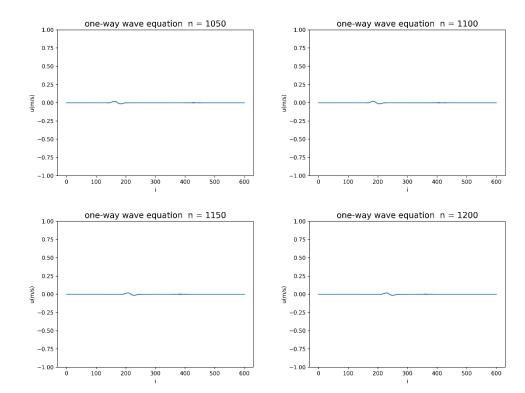
ESSC 4520

Name: Wu Hei Tung SID: 1155109536 L07/08 Exercise

L07 Ex2

One-way wave equation:





$$u = F_{right}[x - (U+c)t] + F_{left}[x - (U-c)t]$$

At the right-hand boundary, the one-way wave equation is $\frac{\partial}{\partial t} + (U + c)\frac{\partial}{\partial x} = 0$.

$$\begin{split} &\frac{\partial \left(F_{right}[x-(U+c)t]\right)}{\partial t} + (U+c)\frac{\partial \left(F_{right}[x-(U+c)t]\right)}{\partial x} \\ &= -(U+c)\frac{\partial \left(F_{right}[x-(U+c)t]\right)}{\partial [x-(U+c)t]} + (U+c)\frac{\partial \left(F_{right}[x-(U+c)t]\right)}{\partial [x-(U+c)t]} \\ &= \left[-U-c+U+c\right]\frac{\partial \left(F_{right}[x-(U+c)t]\right)}{\partial [x-(U+c)t]} \end{split}$$

= 0

Therefore, the right travelling wave satisfies the one-way wave equation.

$$\begin{split} &\frac{\partial \left(F_{left}[x-(U-c)t]\right)}{\partial t} + \left(U+c\right) \frac{\partial \left(F_{left}[x-(U-c)t]\right)}{\partial x} \\ &= -\left(U-c\right) \frac{\partial \left(F_{left}[x-(U-c)t]\right)}{\partial [x-(U-c)t]} + \left(U+c\right) \frac{\partial \left(F_{left}[x-(U-c)t]\right)}{\partial [x-(U-c)t]} \\ &= \left[-U+c+U+c\right] \frac{\partial \left(F_{left}[x-(U-c)t]\right)}{\partial [x-(U-c)t]} \\ &= 2c \neq 0 \end{split}$$

Therefore, the left travelling wave does not satisfy the one-way wave equation.

