ESSC 4520
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L09 Exercise

<u>L09_Ex1</u>

σ_a^2

$= E[(T_a - T_t)^2]$

$= E\left[\left(a_1(T_1 - T_t) + a_2(T_2 - T_t)\right)^2\right]$

$= E[a_1^2(T_1 - T_t)^2 + 2a_1a_2(T_1 - T_t)(T_2 - T_t) + a_2^2(T_2 - T_t)^2]$

$= a_1^2E[(T_1 - T_t)^2] + 2a_1a_2E[(T_1 - T_t)(T_2 - T_t)] + a_2^2E[(T_2 - T_t)^2]$

Since $E[\varepsilon_1\varepsilon_2] = 0$ and $\varepsilon_1 = T_1 - T_t$, $\varepsilon_2 = T_2 - T_t$.

σ_a^2

$= a_1^2E[(T_1 - T_t)^2] + 2a_1a_2(0) + a_2^2E[(T_2 - T_t)^2]$

$= a_1^2E[(T_1 - T_t)^2] + a_2^2E[(T_2 - T_t)^2]$

$E[(T_1 - T_t)^2] = \sigma_1^2$

$E[(T_2 - T_t)^2] = \sigma_2^2$

σ_a^2

$= a_1^2\sigma_1^2 + a_2^2\sigma_2^2$

$$= \frac{\frac{1}{\sigma_1^4}}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^2}\sigma_1^2 + \frac{\frac{1}{\sigma_2^4}}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^2}\sigma_2^2$$

$$= \frac{\frac{1}{\sigma_1^2}}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^2} + \frac{\frac{1}{\sigma_2^2}}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^2}$$

$$= \frac{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^2}$$

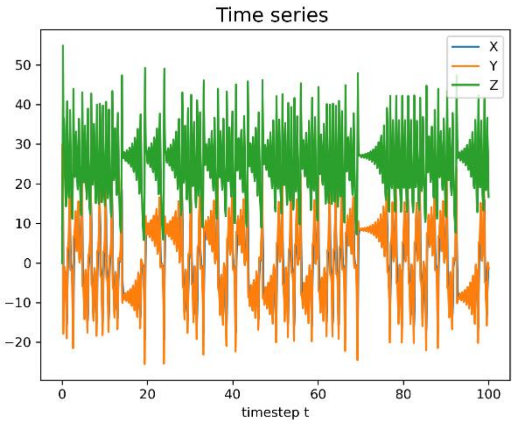
$$= \frac{1}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

Therefore,

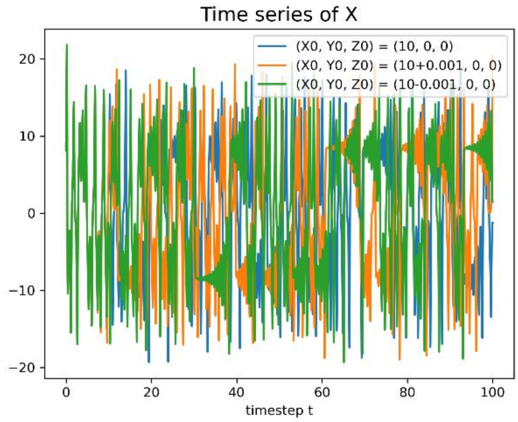
$$\frac{1}{\sigma_a^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

L09_Ex2

a)



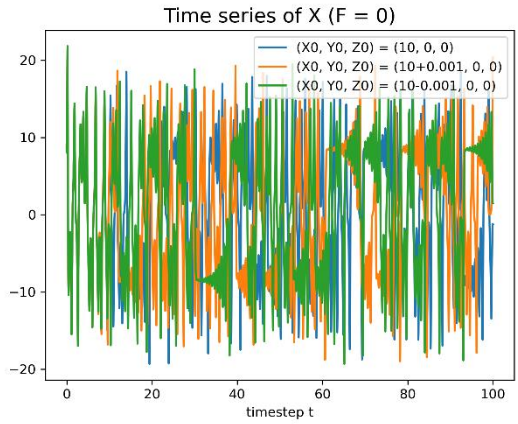
b)



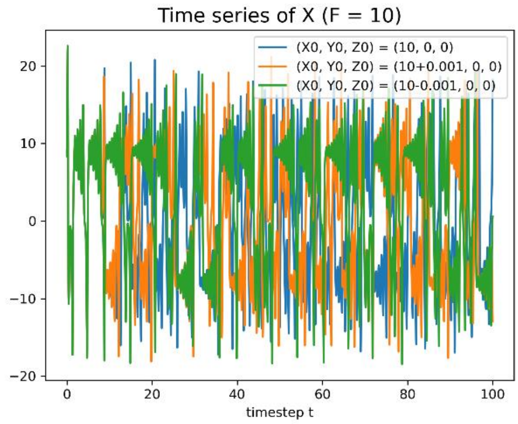
From the time series of X of the 3 experiments (X0, Y0, Z0) = (10, 0, 0), (X0, Y0, Z0) = (10+0.001, 0, 0), (X0, Y0, Z0) = (10-0.001, 0, 0), we can observe that the 3 solutions appear very similar to each other in the first 10 timesteps, and they deviate afterwards. This implies that the results of numerical weather prediction models with slightly different initial condition will be similar only in a time that is very close to the prediction time, say the first 10 timesteps.

c)

i)



ii)

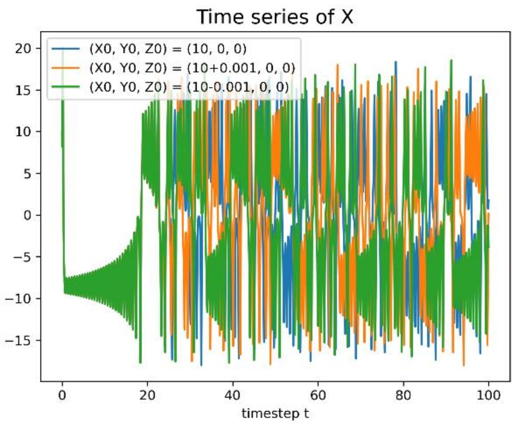


These 2 plots are the time series of X from timestep 0 to 100 with constant force F=0 and F=10 respectively. We can see that the results are very different comparing 2 cases. When there is a constant force F=10, the results start to deviate slightly earlier than the case F=0. This shows that the sensitivity of model depends on forces acting on it.

d)

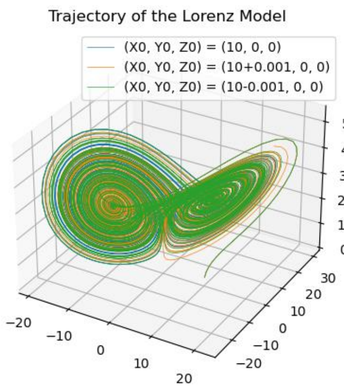
F	$\frac{dF}{dt} = G(F)$	$G^n(F)$	$G^{n-1}(F)$	$G^{n-2}(F)$
X	$\frac{12(X^{n+1} - X^n)}{\Delta t} = 23G^n - 16G^{n-1} + 5G^{n-2}$	$\sigma(Y^n - X^n)$	$\sigma(Y^{n-1} - X^{n-1})$	$\sigma(Y^{n-2} - X^{n-2})$
Y	$\frac{12(Y^{n+1} - Y^n)}{\Delta t} = 23G^n - 16G^{n-1} + 5G^{n-2}$	$X^n(r - Z^n) - Y^n$	$X^{n-1}(r - Z^{n-1}) - Y^{n-1}$	$X^{n-2}(r - Z^{n-2}) - Y^{n-2}$

Z	$\frac{12(Z^{n+1} - Z^{n})}{\Delta t} = 23G^{n} - 16G^{n-1} + 5G^{n-2}$	$X^{n}Y^{n} - bZ^{n}$	$X^{n-1}Y^{n-1} - bZ^{n-1}$	$X^{n-2}Y^{n-2} - bZ^{n-2}$



From the results of X using the third-order Adams-Bashforth scheme, we can see that the 3 experiments appear very similar to each other in the first 25 timesteps. Comparing to the 10 timesteps we have got in part (b), it lasts much longer. Therefore, we can say that the third-order Adams-Bashforth scheme have a higher accuracy since it is less sensitive to the initial condition.

e)



The Lorenz Model with slightly different in initial condition can give totally different results, which can be presented by different trajectory in the XYZ plane (show by the figure above), after certain timesteps. This can show that the Lorenz Model is highly sensitive to the initial condition.