# Sub-wavelength imaging using silver-dielectric metamaterial layered prism

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## ABSTRACT

In this paper we study the propagation of light through silver-dielectric metamaterial layered prism which operates in the canalization regime. The prism is illuminated with TM-polarized light and is designed using the effective medium theory as strongly anisotropic and impedance matched to air. The structure has an infinite value of the effective permittivity in the direction perpendicular to layer surfaces. Therefore it is able to couple a broad spectrum of incident spatial frequencies, including evanescent waves, into propagating modes. As a result, subwavelength resolution at the output interface of the structure is observed. Further the device is characterised with the transfer matrix method (TMM), and investigated with Finite Difference Time Domain method (FDTD). Two parameters of the prism are studied, namely the angle of incidence and the apex angle, to obtain the best resolution.

**Keywords:** superprism, super-resolution, metal-dielectric multilayers

## 1. INTRODUCTION

Periodic structures such as photonic crystals or plasmonic crystals enable efficient tailoring of the group velocity of light.[1,2] In particular, this is the basis of such phenomena as negative refraction,[2] superprism effect,[3] self-collimation,[4] slow-light propagation[5] or superluminal propagation.[6] In metal-dielectric multilayers with strong effective anisotropy, a phenomenon similar to self-collimation may be observed[7–10] with the effect of a tightly focused beam propagating across the structure with negligible divergence. In such structures, subwavelength imaging at visible wavelengths has been demonstrated in much thicker low-loss layered silver-dielectric periodic structures,[7,11–15] as compared to the superlens originally introduced in 2000 by Pendry.[16] Metal-dielectric stacks with infinite and parallel layers form a linear shift-invariant system (LSI[17]), which makes possible to describe such an imaging system with use of concepts used earlier in Fourier Optics.[18] Notably, the point spread function (PSF) of an LSI imaging system usually provides clear information about the resolution, loss or enhancement of contrast, as well as the characteristics of image distortions. In spite of this, for subwavelength imaging the PSF is not a straightforward measure of resolution and sometimes imaging of objects smaller than the FWHM of PSF is also possible.[19]

In this paper we analyse propagation of sub-wavelength sized optical beams through a self-collimating micro-prism. The structure consists of a silver-dielectric multilayer cut in the shape of a prism. Its layers are sufficiently narrow to assure that the effective medium approximation is accurate independently of the termination of the structure. It is impedance matched to air in order to reduce reflections and at the same time it allows for diffraction-free propagation of sub-wavelength sized wavefronts, such as the metamaterial proposed in Ref.[20] The overall thickness of the structure is limited by propagation losses. In practice, another limitation arises from the high sensitivity of a layered superlens to fabrication accuracy[21] and to surface roughness.[22]

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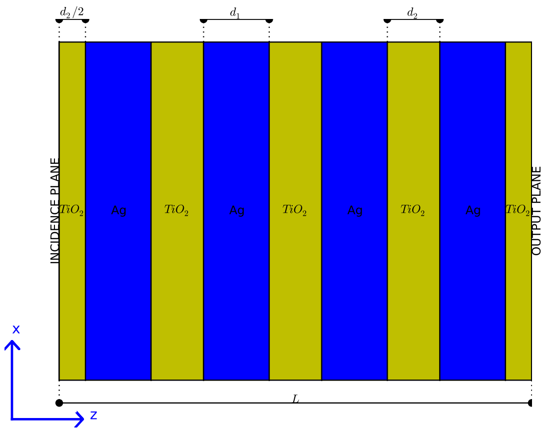


Figure 1. Schematic of a periodic silver-dielectric multilayer.

## 2. METAL-DIELECTRIC MULTILAYERS DESIGNED USING THE EFFECTIVE MEDIUM THEORY

Effective medium approach is a convenient method of approximating periodic structures with equivalent homogeneous and anisotropic media. Application of EMT is, however, limited only to the structures with geometric size of the elementary cell small comparing to the wavelength. In practice, the wave propagating inside the structure is usually much shorter than the wave emitted by a source located in free space, especially if the effective refractive index of the structure is high. The optical properties of the effective medium are conveniently described with the effective permittivity tensor $\hat{\varepsilon}$. In case of silver-dielectric multilayer shown in Fig.1, the equivalent effective medium is uniaxially anisotropic and the permittivity tensor takes the form

$$\hat{\varepsilon} = \begin{vmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_x & 0 \\ 0 & 0 & \varepsilon_z \end{vmatrix}. \tag{1}$$

The relation between the effective permittivities $\varepsilon_x$, $\varepsilon_z$ and the parameters of silver and dielectric layers are derived from the electromagnetic boundary conditions at the interface of two media by calculating average values of parallel and normal components of fields E and D.[9,23] Thus we obtain

$$\varepsilon_x \approx \frac{\epsilon_1 d_1 + \epsilon_2 d_2}{d_1 + d_2}\,, \qquad \varepsilon_z \approx \frac{d_1 + d_2}{\epsilon_1^{-1} d_1 + \epsilon_2^{-1} d_2} \tag{2}$$

where the denotations $d_1$, $d_2$ refer to the thicknesses of the silver and dielectric layers and $\epsilon_1$, $\epsilon_2$ refer to their permittivities. The directions x and z are defined as parallel and normal to the stack, respectively.

The validity of the equations (2) is, however, limited only to lossless materials, since they reveal peculiarity if the permittivities of the layers are complex with a small imaginary part.[14] Therefore, in the structures containing metals the EMT should be used with caution.

The geometric parameters of the multilayer are chosen to fulfill the canalization regime conditions.[8,9] In this regime the effective permittivity of the multilayer reveals extreme anisotropy, namely $\varepsilon_z = \infty$ and $\varepsilon_x = 1$. Due to the first condition, the whole spectrum of the spatial frequencies of the incident light, including the evanescent waves, is transformed inside the multilayer into a mode propagating perpendicularly to the layers. As a result, both the propagating and the evanescent waves, carrying the information about the smallest details of the source structure, are projected into the output interface of the multilayer and contribute to the reconstruction of the image, allowing for imaging with sub-wavelength resolution. The second condition provides the effective impedance matching between the multilayer and the surrounding medium and leads to reducing the reflections from the interfaces. Further reduction of the reflections is obtained by satisfying the Fabry-Perot resonance condition applied the the overall thickness of the multilayer. The canalization regime conditions in a periodic layered structures are fulfilled if the material constants and the thicknesses of particular layers satisfy the relations

$$\epsilon_1 + \epsilon_2 = 1, \qquad \frac{\epsilon_1}{\epsilon_2} = -\frac{d_1}{d_2}. \tag{3}$$

## 3. METAL-DIELECTRIC SELF-COLLIMATING PRISM

In this section we present results of FDTD[24] simulations performed in two dimensions using freely available software package.[25]

The investigated prism is presented in Fig. 2 with silver and dielectric layers of thickness $d_1$ =10 and $d_2$ =12 nm, respectively. Dispersion properties of silver are calculated with the Drude model fitted to the experimental results.[26] We assume that the dielectric is non-dispersive and lossless with the refractive index $n$ equal to 2.46, what matches that of $TiO_2$. We used a TM polarised Gaussian-shaped source with full width at half-maximum (FWHM) equal to 90 nm and the wavelength of 421 nm.

The results of simulations are in stationary state,one hundred timeperiods after turning the source on. Additionaly we use exponential smoothing to eliminate the effect of switching the source on. Field distributions are averaged over one period of the source.

Irrespectively of the angle of incidence inside the prism light propagates in the direction perpendicular to the metal and dielectric layers, which is shown in Figs. 3, 4 and 5 for different apex angles. Due to high losses the field is subject to thresholding prior visualisation. Since the input interface is inhomogenious we performed simulations with several positions of the source. The waist of the Gaussian beam is located 5 nm from the input plane of the prism, and is moved along the surface of the prism.

Finally, we check the transmission efficiency and FWHM of the outgoing beam. The transmission efficiency is calculated as a ratio of the energy measured 50 nm from the output of the prism to the energy of the incident beam. The results for two sample positions of the source are shown in Fig. 6 as a the function of the apex angle. The width of the output beam is measured 20 nm from the prism and therefore is influenced by diffraction. The results for different apex angles are shown in Fig. 7.

In Figs. 6 and 7 for the green line and apex angle equal to 0.9 rad we observe narrowing of the output beam with efficiency comparable to other simulations. This result shows that the proposed prism can be used as a beam concentrating device. However, concentration depends not only on the apex angle of prism but also on the source position. Beam narrowing is also possible for the apex angles near $\frac{\pi}{2}$ however large apex angles are characterized with lower transmission. Another interesting observation is that the width of the outgoing beam does not change for small apex angels, indicating that such a plasmonic prism can be used as an optical interconnect.

## ACKNOWLEDGMENTS

This work is supported by the COST Action MP0702 and by Polish projects MNiI N N202 033237 and NCBiR N R15 0018 06.

In this work we use computational resources funded by the PL-Grid project:contract number: POIG.02.03.00-00-007/08-00.

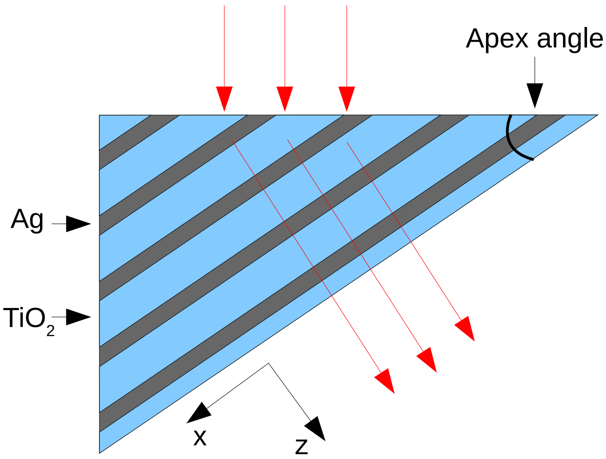


Figure 2. Schematic view of a periodic silver-dielectric multilayer prism.

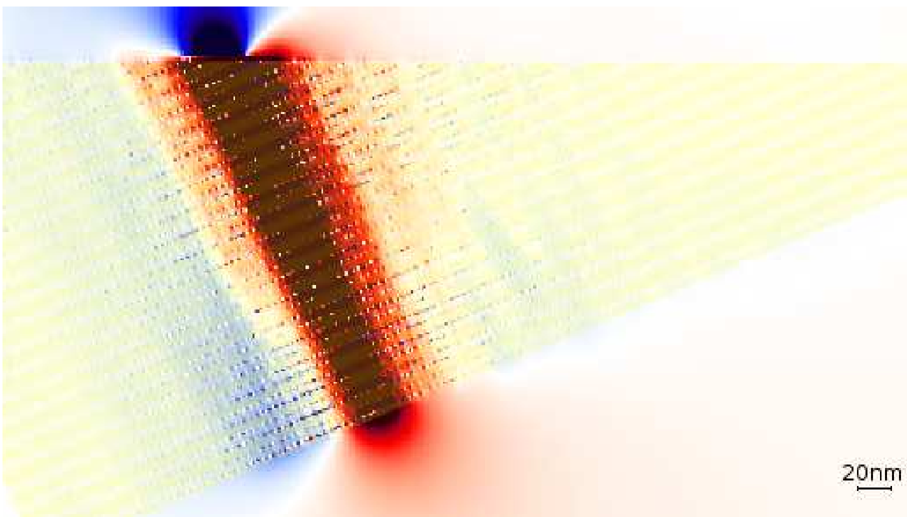


Figure 3. Distribution of the time-averaged Poynting vector component in the direction perpendicular to the layers, for the apex angle 0.4 rad.

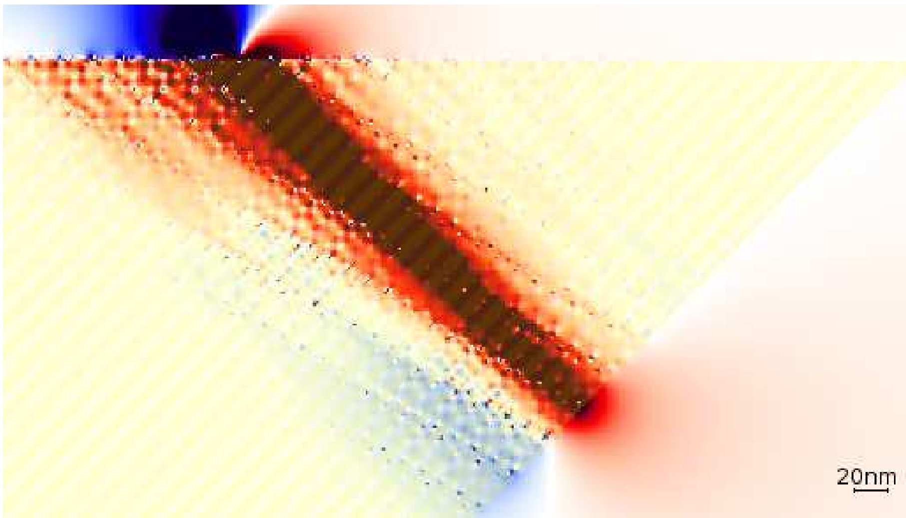


Figure 4. Distribution of the time-avaraged Poynting vector component in the direction perpendicular to the layers, for the apex angle 0.8 rad.

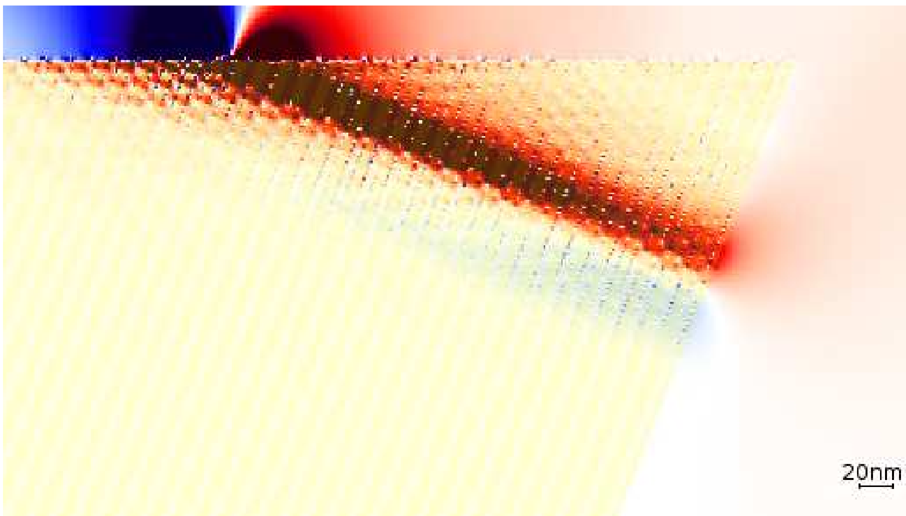


Figure 5. Distribution of the time-avaraged Poynting vector component in the direction perpendicular to the layers, for the apex angle 1.2 rad.

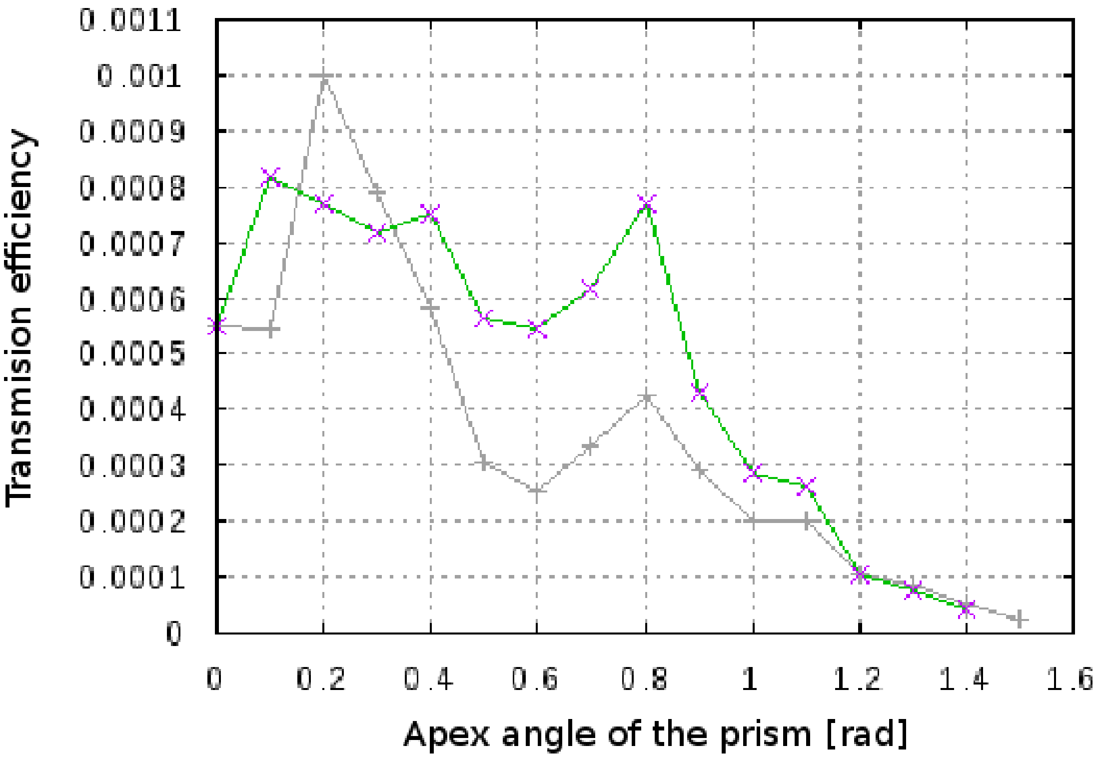


Figure 6. Transmission coefficient for two positions of the source as a function of the apex angle.

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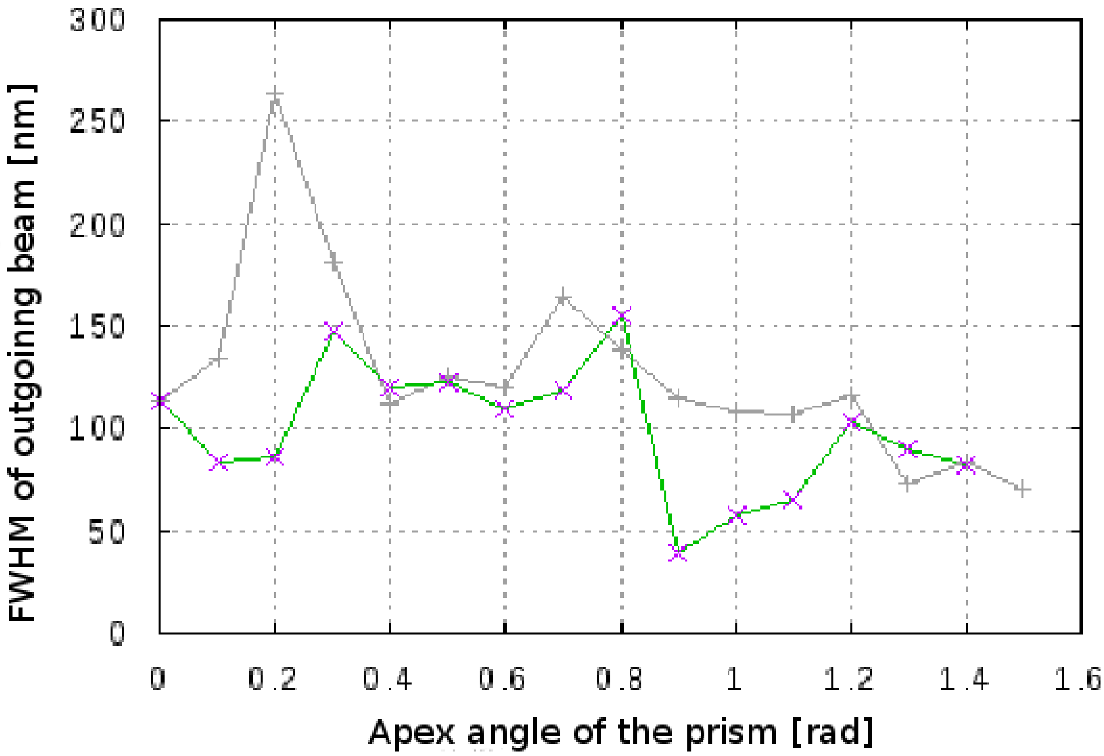


Figure 7. FWHM of the output beam for two positions of the source as a function of the apex angle.


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