1. Calculating $E(K)$

The expected value $E(K)$ is given by:

$$E(K) = \sum_{k=0}^{\infty} k \cdot P(K = k) = \sum_{k=0}^{\infty} k \cdot \frac{e^{-\langle k \rangle} \langle k \rangle^k}{k!}$$

Pulling out constants:

$$E(K) = e^{-\langle k \rangle} \sum_{k=1}^{\infty} k \frac{\langle k \rangle^k}{k!}$$

Note that for $k = 0$, the term is zero. Simplifying:

$$E(K) = e^{-\langle k \rangle} \langle k \rangle \sum_{k=1}^{\infty} \frac{\langle k \rangle^{k-1}}{(k-1)!}$$

Let $n = k - 1$:

$$E(K) = e^{-\langle k \rangle} \langle k \rangle \sum_{n=0}^{\infty} \frac{\langle k \rangle^n}{n!}$$

Recognize that:

$$\sum_{n=0}^{\infty} \frac{\langle k \rangle^n}{n!} = e^{\langle k \rangle}$$

Thus:

$$E(K) = e^{-\langle k \rangle} \langle k \rangle e^{\langle k \rangle} = \langle k \rangle$$

Answer:

$$E(K) = \langle k \rangle$$

2. Calculating $E(K^2)$

The expected value $E(K^2)$ is:

$$E(K^2) = \sum_{k=0}^{\infty} k^2 \cdot P(K = k) = \sum_{k=0}^{\infty} k^2 \cdot \frac{e^{-\langle k \rangle} \langle k \rangle^k}{k!}$$

Pull out constants:

$$E(K^2) = e^{-\langle k \rangle} \sum_{k=0}^{\infty} k^2 \frac{\langle k \rangle^k}{k!}$$

Break k^2 into $k(k-1) + k$:

$$k^2 = k(k-1) + k$$

So:

$$E(K^2) = e^{-\langle k \rangle} \left[\sum_{k=0}^{\infty} k(k-1) \frac{\langle k \rangle^k}{k!} + \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} \right]$$

First term ($E(K(K-1))$):

$$E(K(K-1)) = e^{-\langle k \rangle} \sum_{k=0}^{\infty} k(k-1) \frac{\langle k \rangle^k}{k!}$$

Simplify:

$$E(K(K-1)) = e^{-\langle k \rangle} \langle k \rangle^2 \sum_{n=0}^{\infty} \frac{\langle k \rangle^n}{n!}$$

Thus:

$$E(K(K-1)) = \langle k \rangle^2$$

Second term ($E(K)$):

$$E(K) = \langle k \rangle$$

Combine both terms:

$$E(K^2) = E(K(K-1)) + E(K) = \langle k \rangle^2 + \langle k \rangle$$

Answer:

$$E(K^2) = \langle k \rangle + \langle k \rangle^2$$

3. Calculating Var(K)

The variance $\mathrm{Var}(K)$ is:

$$\mathrm{Var}(K) = E(K^2) - [E(K)]^2 = (\langle k \rangle + \langle k \rangle^2) - (\langle k \rangle)^2 = \langle k \rangle$$