To prove that for an uncorrelated network:

$$\langle k \rangle_{nn}(k_i) = \frac{\langle k^2 \rangle}{\langle k \rangle}, \tag{1}$$

we use the concept of conditional probability and simplify for an uncorrelated network.

Conditional Probability

$$P(k_i|k_j) = \frac{P(k_i, k_j)}{\frac{k_j P(k_j)}{\langle k \rangle}}, \tag{2}$$

Step-by-step Derivation

$$\langle k \rangle_{nn}(k_i) = \sum_{k_j} k_j P(k_j|k_i). \tag{3}$$

In an uncorrelated network, the conditional probability $P(k_j|k_i)$ simplifies to:

$$P(k_j|k_i) = \frac{k_j P(k_j)}{\langle k \rangle}. \tag{4}$$

Substitute this into the expression for $\langle k \rangle_{nn}(k_i)$:

$$\langle k \rangle_{nn}(k_i) = \sum_{k_j} k_j \cdot \frac{k_j P(k_j)}{\langle k \rangle}. \tag{5}$$

Factor out $\frac{1}{\langle k \rangle}$:

$$\langle k \rangle_{nn}(k_i) = \frac{1}{\langle k \rangle} \sum_{k_j} k_j^2 P(k_j). \tag{6}$$

Interpreting the Sum

The term $\sum_{k_j} k_j^2 P(k_j)$ is the definition of the second moment of the degree distribution, $\langle k^2 \rangle$. Thus:

$$\langle k \rangle_{nn}(k_i) = \frac{\langle k^2 \rangle}{\langle k \rangle}. \tag{7}$$