## Foundation of GVB-BCCC formula derivation

Qingchun Wang September 3, 2019

Nanjing Univ qingchun720@foxmail.com A block has 16 states:  $P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}$ .

Each state corresponds to a functor in essence:

- 0:  $\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+}$
- 1:  $\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+}$
- 2:  $\hat{0}_{\alpha}^{+}\hat{1}_{\beta}^{+}$
- 3:  $\hat{0}^{+}_{\beta}\hat{1}^{+}_{\alpha}$
- 4:  $\hat{0}_{\alpha}^{+}\hat{1}_{\alpha}^{+}$
- 5:  $\hat{0}_{\beta}^{+}\hat{1}_{\beta}^{+}$
- 6:
- 7:  $\hat{0}_{\alpha}^{+}$
- 8:  $\hat{1}^+_{\alpha}$
- 9:  $\hat{0}^{+}_{\beta}$
- 10:  $\hat{1}_{\beta}^{+}$
- 11:  $\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+}\hat{1}_{\alpha}^{+}$
- 12:  $\hat{0}_{\alpha}^{+}\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+}$
- 13:  $\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+}\hat{1}_{\beta}^{+}$
- 14:  $\hat{0}^{+}_{\beta}\hat{1}^{+}_{\alpha}\hat{1}^{+}_{\beta}$
- 15:  $\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+}\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+}$

However, a functor is often expressed as a linear combination of a number of states:

- 0:  $\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+} = inv_{0,0}P_0 + inv_{0,1}P_1$
- 1:  $\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+} = inv_{1,0}P_0 + inv_{1,1}P_1$
- 2:  $\hat{0}_{\alpha}^{+}\hat{1}_{\beta}^{+} = inv_{2,2}P_2 + inv_{2,3}P_3$
- 3:  $\hat{0}^{+}_{\beta}\hat{1}^{+}_{\alpha} = inv_{3,2}P_2 + inv_{3,3}P_3$
- 4:  $\hat{0}_{\alpha}^{+}\hat{1}_{\alpha}^{+} = 1P_{4}$
- 5:  $\hat{0}^{+}_{\beta}\hat{1}^{+}_{\beta} = 1P_{5}$
- $6: = 1P_6$
- 7:  $\hat{0}_{\alpha}^{+} = 1P_{7}$
- 8:  $\hat{1}_{\alpha}^{+} = 1P_{8}$

9: 
$$\hat{0}^+_{\beta} = 1P_9$$

10: 
$$\hat{1}^+_{\beta} = 1P_{10}$$

11: 
$$\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+}\hat{1}_{\alpha}^{+} = 1P_{11}$$

12: 
$$\hat{0}_{\alpha}^{+}\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+} = 1P_{12}$$

13: 
$$\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+}\hat{1}_{\beta}^{+} = 1P_{13}$$

14: 
$$\hat{0}_{\beta}^{+}\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+} = 1P_{14}$$

15: 
$$\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+}\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+} = 1P_{15}$$

Therefore, the composition of each state can be written:

$$P_0 = ci_{0,0}\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+} + ci_{0,1}\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+} \quad |vac\rangle$$

$$P_1 = ci_{1,0}\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+} + ci_{1,1}\hat{1}_{\alpha}^{+}\hat{1}_{\beta}^{+} \quad |vac\rangle$$

$$P_2 = c i_{2,2} \hat{0}_{\alpha}^{+} \hat{1}_{\beta}^{+} + c i_{2,3} \hat{0}_{\beta}^{+} \hat{1}_{\alpha}^{+} \quad |vac\rangle$$

$$P_3 = ci_{3,2}\hat{0}^+_{\alpha}\hat{1}^+_{\beta} + ci_{3,3}\hat{0}^+_{\beta}\hat{1}^+_{\alpha} \quad |vac\rangle$$

$$P_4 = 1\hat{0}^+_{\alpha}\hat{1}^+_{\alpha} \quad |vac\rangle$$

$$P_5 = 1\hat{0}_{\beta}^{+}\hat{1}_{\beta}^{+} \quad |vac\rangle$$

$$P_6 = 1 \mid vac \rangle$$

$$P_7 = 1\hat{0}^+_{\alpha} \quad |vac\rangle$$

$$P_8 = 1\hat{1}^+_{\alpha} |vac\rangle$$

$$P_9 = 1\hat{0}^+_{\beta} \quad |vac\rangle$$

$$P_{10} = 1\hat{1}^{+}_{\beta} \quad |vac\rangle$$

$$P_{11} = 1\hat{0}^{+}_{\alpha}\hat{0}^{+}_{\beta}\hat{1}^{+}_{\alpha} \quad |vac\rangle$$

$$P_{12} = 1\hat{0}^{+}_{\alpha}\hat{1}^{+}_{\alpha}\hat{1}^{+}_{\beta} \quad |vac\rangle$$

$$P_{13} = 1\hat{0}_{\alpha}^{+}\hat{0}_{\beta}^{+}\hat{1}_{\beta}^{+} \quad |vac\rangle$$

$$P_{14} = 1\hat{0}^{\dagger}_{\beta}\hat{1}^{\dagger}_{\alpha}\hat{1}^{\dagger}_{\beta} \quad |vac\rangle$$

$$P_{15} = 1\hat{0}^{\dagger}_{\alpha}\hat{0}^{\dagger}_{\beta}\hat{1}^{\dagger}_{\alpha}\hat{1}^{\dagger}_{\beta} \quad |vac\rangle$$