Foundation of GVB-BCCC formula derivation

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A block has 16 states: $P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}$

Each state corresponds to a functor in essence:

0: $\hat{0}^+_\alpha \hat{0}^+_\beta$

1: $\hat{0}^+_\alpha \hat{1}^+_\beta$

2: $\hat{0}^+_\beta \hat{1}^+_\alpha$

3: $\hat{1}^+_\alpha \hat{1}^+_\beta$

4: $\hat{0}^+_\alpha \hat{1}^+_\alpha$

5: $\hat{0}^+_\beta \hat{1}^+_\beta$

6:

7: $\hat{0}^+_\alpha$

8: $\hat{1}^+_\alpha$

9: $\hat{0}^+_\beta$

10: $\hat{1}^+_\beta$

11: $\hat{0}^+_\alpha \hat{0}^+_\beta \hat{1}^+_\alpha$

12: $\hat{0}^+_\alpha \hat{1}^+_\alpha \hat{1}^+_\beta$

13: $\hat{0}^+_\alpha \hat{0}^+_\beta \hat{1}^+_\beta$

14: $\hat{0}^+_\beta \hat{1}^+_\alpha \hat{1}^+_\beta$

15: $\hat{0}^+_\alpha \hat{0}^+_\beta \hat{1}^+_\alpha \hat{1}^+_\beta$

However, a functor is often expressed as a linear combination of a number of States:

0: $\hat{0}^+_\alpha \hat{0}^+_\beta = c^{inv}_{0,0} P_0 + c^{inv}_{0,1} P_1 + c^{inv}_{0,2} P_2 + c^{inv}_{0,3} P_3$

1: $\hat{0}^+_\alpha \hat{1}^+_\beta = c^{inv}_{1,0} P_0 + c^{inv}_{1,1} P_1 + c^{inv}_{1,2} P_2 + c^{inv}_{1,3} P_3$

2: $\hat{0}^+_\beta \hat{1}^+_\alpha = c^{inv}_{2,0} P_0 + c^{inv}_{2,1} P_1 + c^{inv}_{2,2} P_2 + c^{inv}_{2,3} P_3$

3: $\hat{1}^+_\alpha \hat{1}^+_\beta = c^{inv}_{3,0} P_0 + c^{inv}_{3,1} P_1 + c^{inv}_{3,2} P_2 + c^{inv}_{3,3} P_3$

4: $\hat{0}^+_\alpha \hat{1}^+_\alpha = c^{inv}_{4,4} P_4$

5: $\hat{0}^+_\beta \hat{1}^+_\beta = c^{inv}_{5,5} P_5$

6: $= c^{inv}_{6,6} P_6$

7: $\hat{0}^+_\alpha = c^{inv}_{7,7} P_7 + c^{inv}_{7,8} P_8$

8: $\hat{1}^+_\alpha = c^{inv}_{8,7} P_7 + c^{inv}_{8,8} P_8$

9: $\hat{0}_\beta^+ = c_{9,9}^{inv} P_9 + c_{9,10}^{inv} P_{10}$

10: $\hat{1}_\beta^+ = c_{10,9}^{inv} P_9 + c_{10,10}^{inv} P_{10}$

11: $\hat{0}_\alpha^+ \hat{0}_\beta^+ \hat{1}_\alpha^+ = c_{11,11}^{inv} P_{11} + c_{11,12}^{inv} P_{12}$

12: $\hat{0}_\alpha^+ \hat{1}_\alpha^+ \hat{1}_\beta^+ = c_{12,11}^{inv} P_{11} + c_{12,12}^{inv} P_{12}$

13: $\hat{0}_\alpha^+ \hat{0}_\beta^+ \hat{1}_\beta^+ = c_{13,13}^{inv} P_{13} + c_{13,14}^{inv} P_{14}$

14: $\hat{0}_\beta^+ \hat{1}_\alpha^+ \hat{1}_\beta^+ = c_{14,13}^{inv} P_{13} + c_{14,14}^{inv} P_{14}$

15: $\hat{0}_\alpha^+ \hat{0}_\beta^+ \hat{1}_\alpha^+ \hat{1}_\beta^+ = c_{15,15}^{inv} P_{15}$

Therefore, the composition of each state can be written:

$P_0 = c_{0,0}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ + c_{0,1}^{ci} \hat{0}_\alpha^+ \hat{1}_\beta^+ + c_{0,2}^{ci} \hat{0}_\beta^+ \hat{1}_\alpha^+ + c_{0,3}^{ci} \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_1 = c_{1,0}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ + c_{1,1}^{ci} \hat{0}_\alpha^+ \hat{1}_\beta^+ + c_{1,2}^{ci} \hat{0}_\beta^+ \hat{1}_\alpha^+ + c_{1,3}^{ci} \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_2 = c_{2,0}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ + c_{2,1}^{ci} \hat{0}_\alpha^+ \hat{1}_\beta^+ + c_{2,2}^{ci} \hat{0}_\beta^+ \hat{1}_\alpha^+ + c_{2,3}^{ci} \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_3 = c_{3,0}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ + c_{3,1}^{ci} \hat{0}_\alpha^+ \hat{1}_\beta^+ + c_{3,2}^{ci} \hat{0}_\beta^+ \hat{1}_\alpha^+ + c_{3,3}^{ci} \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_4 = c_{4,4}^{ci} \hat{0}_\alpha^+ \hat{1}_\alpha^+ \quad |vac\rangle$

$P_5 = c_{5,5}^{ci} \hat{0}_\beta^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_6 = c_{6,6}^{ci} \quad |vac\rangle$

$P_7 = c_{7,7}^{ci} \hat{0}_\alpha^+ + c_{7,8}^{ci} \hat{1}_\alpha^+ \quad |vac\rangle$

$P_8 = c_{8,7}^{ci} \hat{0}_\alpha^+ + c_{8,8}^{ci} \hat{1}_\alpha^+ \quad |vac\rangle$

$P_9 = c_{9,9}^{ci} \hat{0}_\beta^+ + c_{9,10}^{ci} \hat{1}_\beta^+ \quad |vac\rangle$

$P_{10} = c_{10,9}^{ci} \hat{0}_\beta^+ + c_{10,10}^{ci} \hat{1}_\beta^+ \quad |vac\rangle$

$P_{11} = c_{11,11}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ \hat{1}_\alpha^+ + c_{11,12}^{ci} \hat{0}_\alpha^+ \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_{12} = c_{12,11}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ \hat{1}_\alpha^+ + c_{12,12}^{ci} \hat{0}_\alpha^+ \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_{13} = c_{13,13}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ \hat{1}_\beta^+ + c_{13,14}^{ci} \hat{0}_\beta^+ \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_{14} = c_{14,13}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ \hat{1}_\beta^+ + c_{14,14}^{ci} \hat{0}_\beta^+ \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$

$P_{15} = c_{15,15}^{ci} \hat{0}_\alpha^+ \hat{0}_\beta^+ \hat{1}_\alpha^+ \hat{1}_\beta^+ \quad |vac\rangle$