COMP3121 Algorithms and Programming Tech 2022T3 assignment 1

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Question 1 The Cheapest Fridge

[30 marks] Song wants to buy a fridge with volume at least V cubic centimetres. The shop sells a large variety of fridges. More precisely, for each positive integer x, the shop sells a fridge for x dollars with the following dimensions:

• width 3x centimetres,
• depth 2x + 1 centimetres and
• height 2x centimetres.

1.1 [12 marks] Design an algorithm which runs in O(log V) time and finds the minimum amount that Song must spend to buy a suitable fridge.

Our goal is to find the ideal fridge for Song with volume $V' = 3x * (2x+1) * 2(x)$, and $V' \geq V.\ Let\ f(x) = 3x * (2x+1) * 2(x).$

As the question shows,

• We know that $f(x)$ is a monotonically increasing function which $f(V) > V\ and\ V \geq 1.$

• x is a positive integer.

• The time complexity is given O(logV), and we know the lower bound is 1 and the upper bound is $f(V)$.

From the given points, we apply binary search.

By applying binary search, we first find the middle element m by calculating the average of the upper bound and lower bound.

• Compare the f(m) with V.

• Case 1 : Since f(x) is ascending, if f(m) is smaller than the V which means x must be between m+1 and the upper bound. So we set the new lower bound to m+1.

• Case 2: Since f(x) is ascending, if f(m) is greater than or equal to V and f(m-1) is also greater than or equal to V which means x must be between lower bound and m-1. So we set the new upper bound to the m-1.

• Case 3: If f(m) is greater than or equal to V and f(m-1) is smaller than V, then return the m.

We search recursively until we meet case 3, during that time we keep tighter the bound.

Complexity

While searching for the matching element, the searching interval is cut to half by taking the recursive step. Hence, we find the ideal fridge by using O(log V) algorithm.

1.2 [18 marks] Design an algorithm which runs in O(log(log V)) time and finds the minimum amount that Song must spend to buy a suitable fridge. You may choose to skip 1.1, in which case your answer to 1.2 will be marked as your answer to 1.1 also.

Our goal is to find the ideal fridge for Song with volume $V' = 3x * (2x + 1) * 2(x)$.
As the question shows,
• $f(x)$ is a monotonically increasing function which $f(V) > V\ and\ V \geq 1$.
• x is a positive integer.
• The time complexity is given O(log(log V)), and we know the lower bound is 1 and the upper bound is $f(logV)$.
From the given points, we apply binary search.

By applying binary search, we first find the middle element m by calculating the average of the upper bound and lower bound.
• Compare the f(m) with V.

• Case 1 : Since f(x) is ascending, if f(m) is smaller than the V which means x must be between m+1 and the upper bound. So we set the new lower bound to m+1.

• Case 2: Since f(x) is ascending, if f(m) is greater than or equal to V and f(m-1) is also greater than or equal to V which means x must be between lower bound and m-1. So we set the new upper bound to the m-1.

• Case 3: If f(m) is greater than or equal to V and f(m-1) is smaller than V, then return the m.

We search recursively until we meet case 3, during that time we keep tighter the bound.

Time Complexity
While searching for the matching element, the searching interval is cut to half by taking the recursive step. Starting at the upperbound $f(logV)$. Hence, we find the ideal fridge by using O(log V) algorithm.