Comp3121 assignment 4 Question 2

Chen En (z5335039)

Question 2 *Mountain Radios*

[**30 marks**] Alice and Bob are bored of the park, and have decided to go mountaineering! They will each be traversing the mountain and moving through a sequence of n camps. Both Alice and Bob will always spend the night at one of their camps. Alice's camps are given by a sequence of coordinates pairs $A_1, ..., A_n$, where $A_i = (A_{i_x}, A_{i_y})$, and Bob's camps are given as $B_1, ..., B_n$, where $B_j = (B_{j_x}, B_{j_y})$. On the first night, Alice will spend the night at camp A_1 and Bob will spend the night at camp B_1. Each day, both Alice and Bob have the choice of staying at their current camp or moving to the next camp in their sequence. They can never move back to a camp they have previously visited. At least one of them must move each day, meaning that both Alice and Bob will have reached their final camps no later than the $(2n-1)$th night.

Alice and Bob would like to be able to talk to each other at night via a radio, but all of the wireless radios available for purchase have a fixed range. Alice has noticed that if they carefully plan who moves to the next camp on each day, it will affect how far apart they get during their journey. The distance between two coordinates (x_1, y_1) and (x_2, y_2) is calculated as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

For example, let $n = 5$ and $A_1 = (1,1)$, $A_2 = (3,1)$, $A_3 = (4,1)$, $A_4 = (5,1)$, $A_5 = (6,1)$, $B_1 = (1,2)$, $B_2 = (2,2)$, $B_3 = (3,4)$, $B_4 = (4,2)$, $B_5 = (6,2)$. In this example, the furthest apart Alice and Bob **must** get during their journey is 3 units. One way this could be achieved is with the sequence of stays $[(A_1, B_1), (A_2, B_2), (A_2, B_3), (A_2, B_4), (A_3, B_4), (A_4, B_5), (A_5, B_5)]$. With this sequence of moves, the furthest apart that Alice and Bob get is on night 3, when Alice is at A_2 and Bob is at B_3.

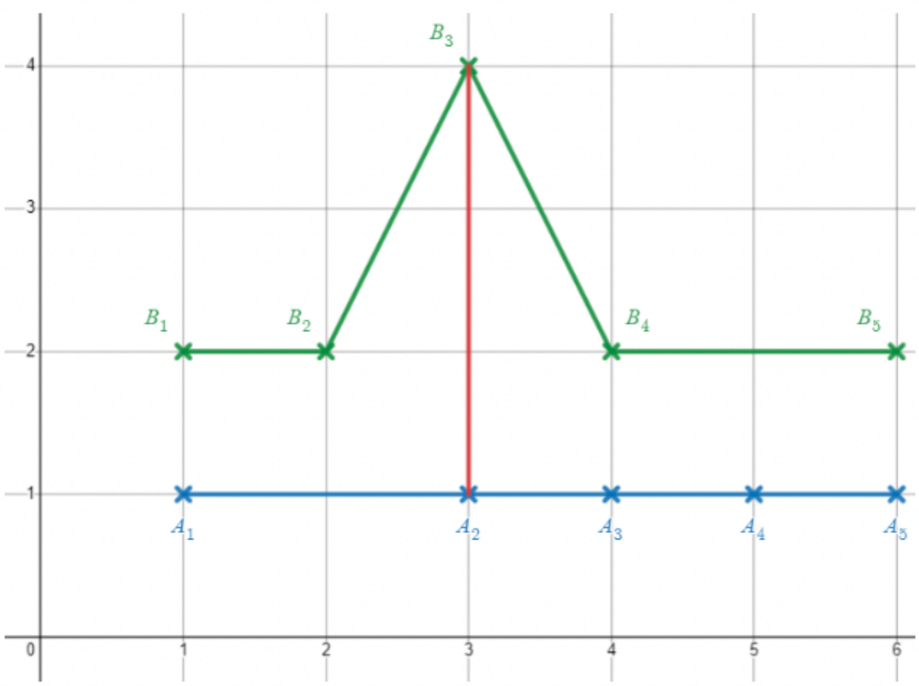

Figure 1: The simple example described above. The blue line represents the path Alice will walk and the green line represents the path Bob will walk. The red line shows the furthest apart Alice and Bob **must** get during their journey.

2.1 **[15 marks]** Design an $O(n^2)$ algorithm to determine whether a radio with a range of D will allow them to communicate every night.

In the provided example, your method should determine that it is impossible for any value of $D < 3$, and possible otherwise.

2.2 **[15 marks]** Design an $O(n^2)$ algorithm that calculates the minimum range required for Alice and Bob to be able to communicate every night.

In the provided example, your method should calculate a minimum required range of 3.

> You can receive up to 7 marks for an $O(n^2 \log n)$ solution.

> You may choose to skip Question 2.1, in which case your solution to Question 2.2 will also be submitted for Question 2.1.

2.2

Subproblem: For each $1 \leq i \leq n$ and each $1 \leq j \leq n$, $let\ Q(i,j)$ be the problem of determining $Opt(i,j)$, the minimum range of the radio when Alice is at camp i and Bob is at camp j.

Base case: When either Alice or Bob has at first camp or both at the first camp

$$n = 1 \rightarrow Opt(1,1) = distance\ (A_1, B_1)$$

Recursion:

$$Opt(i,j) = \max\left(\min\{Opt(i-1,j), Opt(i,j-1), Opt(i-1,j-1)\}, distance(Ai, Bj)\right)$$

The method implies from the lecture slide problem "Edit distance". We create an $n * n$ array $Opt[i][j]$ to store the minimum range for radio of $\{Opt(i-1,j), Opt(i,j-1), Opt(i-1,j-1)\}$.
There are 3 cases during each day:

1. Alice moves but Bob does not move → $Opt(i,j-1)$ (similar to insertion in edit distance)
2. Bob moves but Alice does not move → $Opt(i-1,j)$ (similar to deletion in edit distance)
3. Both of them move → $Opt(i-1,j-1)$ (similar to same string)

Given by the question, we are asking to find the minimum range required so we take $\min\{Opt(i-1,j), Opt(i,j-1), Opt(i-1,j-1)\}$. Then we takes $\max\left(\min\{Opt(i-1,j), Opt(i,j-1), Opt(i-1,j-1)\}, distance(Ai, Bj)\right)$ as the radio range has to cover the distance of where Alice and Bob currently at.

Answer:
$Opt[n][n]$ (As we can only know the optimal range for radio on the last day.)

Time complexity:
We create an $n * n$ array $Opt[i][j]$ to store the minimum range for radio that takes $O(n^2)$, the comparison between those three values takes $O(1)$. The calculation of the distance takes $O(1)$. Thus, the time complexity is $O(1) * O(n^2) = O(n^2)$.

2.1
We apply the same method as 2.2.
If we find $Opt[n][n] < D$ which means the radio range of D is possible.
When $Opt[n][n] > D$ which means the radio range we need has exceeded the range D from the question it given.