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932/1

Bonus homework

1. (50 points) Show that all binary strings generated by the grammar with the following productions have values divisible by 3

S -> 11 | 1001 | S 0 | SS

a. 11 (which is 2 in base 10) is divisible by 3

b. 1001 (which is 9 in base 10) is divisible by 3

c. S 0 is basically shifted to the left by one bit. (a)(b) => S is divisible by 3 (basically is multiplied by 2, so it's still a multiple of 3)

d. SS = S*(2^k)+S = S*(2^k+1), where k is the number of bits of S

(a)(b)(c) => SS is divisible by 3

Hence, the values generated from the given grammar are divisible by 3.

2. (50 points) Construct a cfg for L = {$a^n b^m c^k$ | k=n+m}.

$S \rightarrow aSc \mid aTc \mid ac$

$T \rightarrow bTc \mid bc$

$Q \rightarrow S \mid T$

$?\, L(G) = L$

$?\, a^n b^m c^k, \quad n,\ m,\ k \in \mathbb{N}\ s.t.\ k = n + m$

- Let $t \in \mathbb{N}$

  $T \rightarrow^t (bc)^t;\ \ ?\, (bc)^t \in L(G)$

  $(bc)^t \in L(G)\ with\ k = m,\ n = 0$ **(1)**

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- Let $q \in \aleph$

$$S \to^{q} (ac)^{q} \ / \ a^{q} - c^{q}$$

$$? \ (ac)^{q} \in L(G)$$

$$(ac)^{q} \in L(G) \ with \ k - n, \ m = 0$$ **(2)**

$$? \ a^{q} T c^{q} \in L(G)$$

$$a^{q} T c^{q} \to a^{q} (bc)^{z} c^{q}, \ z \in \aleph$$

$$a^{q} b^{z} c^{z} c^{q} \in L(G)$$

$$k = n + m = z + q$$ **(3)**

$$n = q, \ m = z$$

(1)(2)(3) $\Rightarrow L = L(G)$

3. (50 points) Construct a cfg for L = {w in {a,b,c}* | $no_a(w) = no_b(w) = no_c(w)$}, where $no_a(w)$ denotes number of symbols 'a' in the sequence 'w'.

$$S \to P3$$

$$Q \to S$$

$$S \to abc \mid aSbc \mid abSc \mid aTbc \mid abTc$$

$$T \to bac \mid bSac \mid baSc \mid bTac$$

$$S_0 \to abc \mid aS_k bc \mid abS_k c \mid aS_k bS_k c$$

$$S1 \to acb \mid aS_k cb \mid acS_k b \mid aS_k cS_k b$$

$$S_2 \to bca \mid bS_k ca \mid bcS_k a \mid bS_k cS_k a$$

$$S_3 \to bac \mid bS_k aS_k c \mid bS_k ac \mid baS_k c$$

$$S_4 \to cab \mid cS_k ab \mid caS_k b \mid cS_k aS_k b$$

$$S_5 \to cba \mid cS_k ba \mid cbS_k a \mid cS_k bS_k a$$

$$k \in \{1, 2, 3, 4, 5\}$$