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在自我回歸移動平均(ARMA)需求下運用可逆性與可推斷性舒緩長鞭效應之研究

A Study on Mitigation of Bullwhip Effect for ARMA Demand Processes by Invertibility and Inferability

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摘要

本研究主要探討在二階供應鏈系統下如何減緩“長鞭效應”。在供應鏈系統中，訂貨量的變異數大於實際需求的變異數，並且隨著供應鏈階層的上升，這現象稱為長鞭效應。此現象源於各供應鏈成員不知真實需求而以自利因素決定訂貨量，不論真實需求的穩定與否都普遍產生此現象。為方便分析，本研究以訂貨之變異數除以實際需求之變異數表示長鞭效應之程度。

我們探討的需求模型為自我回歸移動平均模型(ARMA)。在此需求下，二階供應鏈成員運用與需求同類之時間序列(ARMA)，並應用 Zhang (2004)的〝ARMA-in-ARMA-out〞準則推估訂貨模型以預測需求，並考慮需求的可逆性(Invertibility)來決定零售商之資訊運用。

本研究建立零售商與製造商決策流程圖，並著重在製造商的訂貨架構，以與零售商需求資訊分享以及考慮自力推估需求之架構來和基本訂貨架構比較。在資訊分享的架構下，直接假設製造商能從零售商獲取實際的需求資訊，改善製造商之預測需求。在推估需求架構下，本研究參考 Gaur *et al*. (2005)之推論，運用時間序列特性共變異恆定(Covariance Stationay)和可逆性(Invertibility)，從零售商之訂貨模型推論消費者之需求，改善預測需求。

我們以 JAVA 模擬此二階供應鏈系統。首先我們比較在不同的需求模型特性下，製造商推估實際需求之效果。我們發現推估實際需求之效果在不同的需求模型下表現很好，但零售商之資訊應用影響了此效果的優劣。再者我們比較在不同製造商訂貨架構下之長鞭效應減緩程度。我們發現長鞭效應在需求波動較小以及零售商前置時間時，長鞭效應得以減緩。

關鍵詞：長鞭效應、ARMA、資訊分享、需求推估

Abstract

This thesis investigates how to mitigate the “bullwhip effect” in a two-tier supply chain. The bullwhip effect describes the phenomenon that the variance of orders is larger than the variance of actual demands. Moreover, the phenomenon becomes more severe as one goes down the stages of the supply chain. The bullwhip effect renders supply chain members insufficient information for the actual demand and myopic order decisions even for a stable demand pattern. As the measurement of bullwhip effect, we divide the order variance by actual demands’ variance.

We model the demand pattern by autoregressive moving-average (ARMA) processes. Under these types of processes, the supply chain members forecast their order decisions based on the same type of time series (ARMA), and apply Zhang’s “ARMA-in-ARMA-out” principle to derive their future demands. Moreover, we consider the “Invertibility” of time series to decide the retailer’s information application.

We build the decision flows of the retailer and the manufacturer. Moreover, we focus on the manufacturer’s ordering structures. We construct two structures, information sharing and demand inferring, and compare with the basic structure that has neither information sharing nor demand inferring. Under the information sharing structure, we directly assume that the manufacturer can receive the actual demand information from the retailer, and then uses it to forecast accurately. Under the demand inferring structure, we refer to Gaur *et al*. (2005)’s deduction, which applies the properties of time series, covariance stationary and invertibility, to deduce the customer’s demand realizations and then forecast precisely.

We simulate the supply chain model for bullwhip measurement by JAVA. First, we compare performance of inferred demands under different properties of demand processes. We find out that the approaches have well of the performance, but the retailer's application of information affects the performance of inferred demands. Further, we compare the performance of bullwhip effect under different manufacturer's ordering structures. We find out the bullwhip effect can be alleviated when the demand process is stable and the retailer's lead-time is small.

Keywords: Bullwhip effect; ARMA; Information sharing; Inferred demand

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Chapter 1 Introduction

1.1 Overview of a Supply Chain

In recent years, supply chain management plays an important role in the real world. With globalization and internet dissemination, companies are getting more closely related than before. There are more vertical integrations, horizontal mergers, and even formations of conglomerates by companies than before. Because of the rapid growths there are, there are much more interactions of companies than years ago. However, information distortion, misleading information and information discrepancy, that affect supply chains severely.

From upstream to downstream, a supply chain can be simplified into a model of a supplier, a manufacturer, a wholesaler, a retailer, and a customer. Companies interact with each other and generate two types of flows. The physical flows, i.e. materials, products, and/or goods, are from the supplier to the customer, and information and financial flows, i.e. demands, orders, cash, etc., flow in a bi-directional way. Here we concentrate on information distortion and its results. Our thesis focuses on the well-known bullwhip effect. We try to mitigate its influence, and look for an appropriate forecasting mechanism to overcome it. Moreover, because information sharing is useful, we also consider information inference such that one player can infer full information from the partial information produced by another player.

1.2 Background & Motivation

To focus on forecasting mechanism, we build up an appropriate technique for periodic-review when the demands follow a stationary time series. In this thesis, we use autoregressive moving-average (ARMA) time series to estimate the future demands and order quantities. ARMA has the properties of both AR and MA which we will explain clearly in Chapter 2. By Gaur *et al*. (2005), ARMA as a forecasting mechanism is valuable for two reasons. First, in real life, we often find demand patterns fitting high-order autoregressive processes well due to the occurrence of seasonality and business cycles. For example, the demand patterns which occur monthly can follow an AR (12) process. Based on Chopra and Meindl (2001) and Nahmias (1993), the more general ARMA processes fit long-life cycles of goods such as fuel, food products, machine tools, etc. Second, recent research finds that ARMA demand processes present in multi-stage supply chain system naturally. For example, Zhang (2004) researches on the time series characterization of order process. He finds that when demand pattern is an ARMA process, then the order pattern will also be an ARMA process. This result is valuable for members in a supply chain to control it.

From the literature, we know that coordination is an important issue in nowadays supply chain management. Many cases in real world have proven the idea. Many industries have put huge efforts on reengineering to improve the efficiency of their supply chain systems. Practically, they want to reduce the cost of inventory and shortage caused by bullwhip effect. Therefore, many companies tempt to use information sharing to reduce bullwhip effect. This is the cornerstone for QR (quick response) and ECR (efficient consumer response). Nowadays, it is easier to get frontline sales information by using the POS (Point-of-Sale) technique. It keeps track of the demand data and the consuming behaviour instantly for all retail outlets. Hence, companies can adjust orders and allocate products as soon as possible. On the other hand, we can usually see retailer sharing inventory information to its upstream members to help manage inventory together. More precisely, upstream members can arrange their product schedules or change orders by using the accurate information. This win-win situation is called a vendor-managed inventory (VMI) system. According to Blatherwick (1998), Wal-Mart and Kmart are the most renowned practical examples of VMI, which creates ample revenue from it. Nevertheless, continuous replenishment program (CRP), which is based on electronic data interchange (EDI), is also beneficial for business partners from information sharing. Under these mechanisms, we need to consider contract issues to ensure partner’s coordination and protect the commercial secrets. There are many success cases of such programs (Clark (1994) and Hammond (1994)).

1.3 Influence of Bullwhip Effect

There are many activities around business entities, such as selling and ordering. These activities generate a lot of information and physical flows. In almost every situation, it would produce much misunderstanding between players in a supply chain, in particular, the bullwhip effect for misleading of information.

The bullwhip effect is the phenomenon where orders to the manufacturer tend to present a larger variance than sales to the retailer. This is because of the effect of inventory policy and forecasting mechanism, i.e. demand distortion. The discrepancy propagates to higher levels in an amplified pattern leading to variance magnification. In a supply chain, usually the manufacturer can only observe order data from the retailer with an amplified variance. The manufacturer is not aware of this condition. He produces goods based on what he observes. In that case, the manufacturer may produce excess goods when the real demand is smaller than he expects, and may produce insufficient goods when the real demand is bigger. Extra costs appear either way. Extra holding inventory cost incurs because of excess products; extra shortage cost incurs because there is not enough goods to satisfy the downstream buyers, which inflicts a penalty cost. There are also other costs, such as transportation cost and transaction cost.

Bullwhip effect exists. How to alleviate the effect is a crucial issue to all players in the supply chain. It can be observed in many commercial operations in real world, such as Campbell's soup (Fisher *et al*. (1997)). From Lee *et al*. (1997), Procter & Gamble (P&G) finds that their diaper products have a degree of variablity that cannot be explained. Hewlett-Packard finds that the orders place on the reseller is much bigger than customer demands. Lee *et al*. (1997) also note the higher variability in DRAM market than the computer market, which somehow indicates the domain of the product influence too.

Lee *et al*. (1997) show that bullwhip effect is generated by demand signal process, rationing game, order batch, and price variation. The demand signal is misleading because of the reformed information. When there is incorrect anticipation of demand or order, it will amplify the demand as the supply chain echelon goes up. Order batching is a common practice for companies that want to operate more economically. For instance, buyers or retailers postpone ordering to wait for larger volume for a price discount in purchasing and transportation. Demand signal and order batching are related issues. The outputs of these issues usually influence traditional inventory policy. The rationing game is often observed in the product markets during the growth phase of the product life cycle when demands surpass the supply. Price variations often occur in a mature product market, which appear as a war on market share. Rationing game and price variation effects are also related to each member in the same channel system.

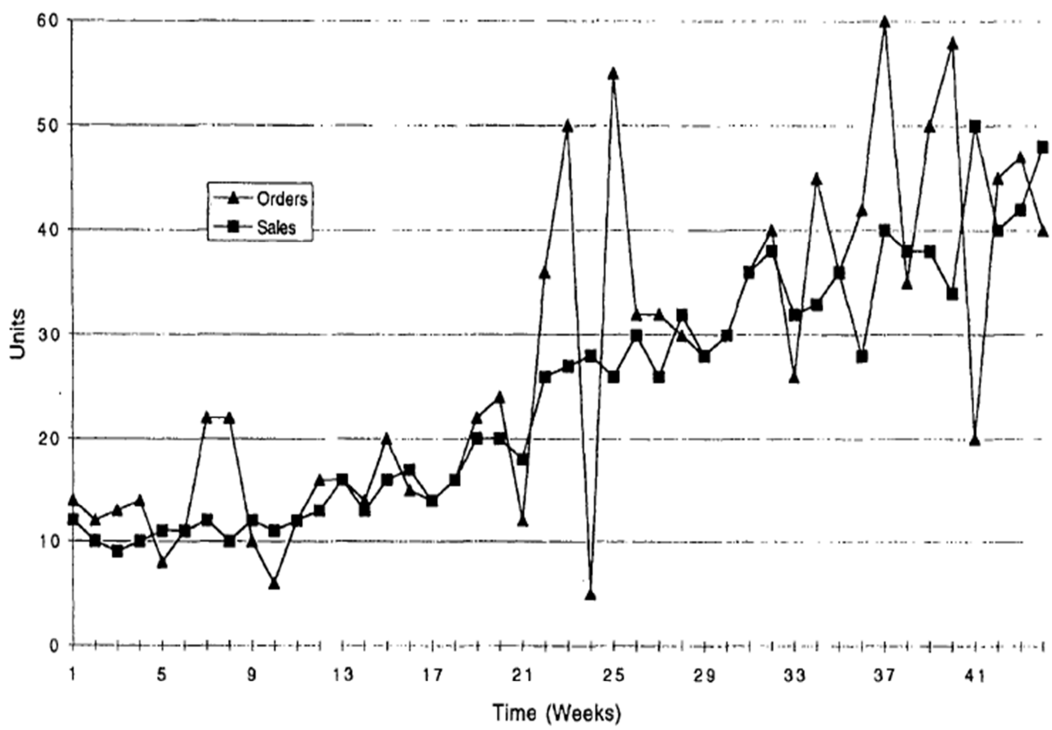


Figure I-1 Variance between orders and demands each period (Lee *et al.* (1997))

Figure I-1 from Lee *et al*. (1997) (based on the real data and revised to make sure confidentiality) presents the variance amplifications between demand and order by forecasting with moving average. The orders fluctuating up and down fiercely along the sales. So bullwhip effect appears. Therefore, we need to find a more proper forecasting method to mitigate the fluctuation besides avoiding additional costs. We will mention the method more precisely in following chapters.

Plenty of studies had tried to develop the remedy for this phenomenon. For our thesis, we focus on two main topics which are information sharing and accurate forecasting mechanism.

1.4 Importance of Forecast

Forecast is one of the most important issues in this thesis. It is a way to estimate the future demands and prepare inventory for them. There are many forecasting methods, e.g., moving average, exponential smoothing, time series, and so on. The appropriate method depends on the situation. Demand patterns have represented in different situations. Chen *et al*. (2000b) give insights to us for choosing the proper forecasting method to suit different demand patterns.

1.5 Aims of Information Sharing

For alleviating the bullwhip effect, we try to find out what can diminish the discrepancy between the order and demand values. We know the appropriate forecasting mechanism can mitigate the effect, and we discover that sharing information about the real demand can help. Lee *et al*. (2000) shows that sharing partial errors of the demand pattern provides a great influence on bullwhip effect reduction. For many cases, especially in Chen *et al*. (2000a), we can easily observe the ideal case of information sharing is to share all the demand information to all members of a supply cahin.

Wal-Mart and P&G give an important example of information sharing. In late 1980's, they conduct the VMI system to concord the variance between demands and orders. They build information systems for this purpose. They share complementary information to each other: P&G shares the data from marketing research and Wal-Mart shares the actual sales data from all products in each store. They both increase in sales from cooperating in sharing information.

Although information sharing is good for supply chain, there are still many challenges for executing it. We can easily see problems of integration for constructing an information sharing structure. Companies need to standardize the terms of information. Besides, supposedly they sign a standard protocol or contract for information exchanges and avoid information hazard that may ruins the whole system. Moreover, the whole business process of companies should re-engineer inevitably to make sure the companies' process and data exchange integration still work on. In conclusion, it takes huge processes and costs to operate, but it can be a foundation stone for the companies.

Is it possible for a supply chain system without information sharing to achievie the same performance as a chain with information sharing? In some studies, we find the possibility for supply chain members to deduce the true information of other members from partial information. For example, a manufacturer can deduce the true demand simply from orders placed by a retailer. Gaur *et al*. (2005) define the concepts invertibility and inferability for this purpose.

1.6 Ideas of Invertibility & Inferability

To demonstrate the idea of the properties, we here simply build a discrete-time two-tier supply chain system. In this system, we have customer demands, a retailer, and a manufacturer. The demand pattern of the customer is modeled by an ARMA process and demands occur in every period. Moreover, retailer forecasts the customer demands and manages the inventory through a prespecified inventory policy. In the end, retailer places the replenishment orders to the manufacturer and the manufacturer produces the goods and forecasts the expected amount of goods for the future.

The general idea of invertibility is to estimate the true value of the random errors through the historical demand data. We can be benefited by invertibility for more accurate forecasting mechanism from considering the errors. The general idea of inferability is to deduce the true customer demands by the manufacturer from retailer's replenishment orders. We can be benefited by inferability for the situation similar to a supply chain system with information sharing. We will explain more of these concepts in Chapter 2.

Zhang (2004) and Gaur *et al*. (2005) use these two properties to research on supply chain. Zhang (2004) produces a method for the demand pattern with invertibility, and derive the "ARMA-in-ARMA-out" principle. Gaur *et al*. (2005) produce a method to derive the actual demand values from retailer's replenishment orders. Those are crucial references to our thesis, and we will elaborate them on following chapters.

1.7 Problem Overview

We build a supply chain system consisting of a customer, a retailer, a manufacturer, and a upstream supplier. However, the main issue focuses on the interaction of the retailer and the manufacturer. Figure I-2 shows the framework of our concepts. The customer demand on a single type of product occurs in every period. The upstream members only produce the same product.

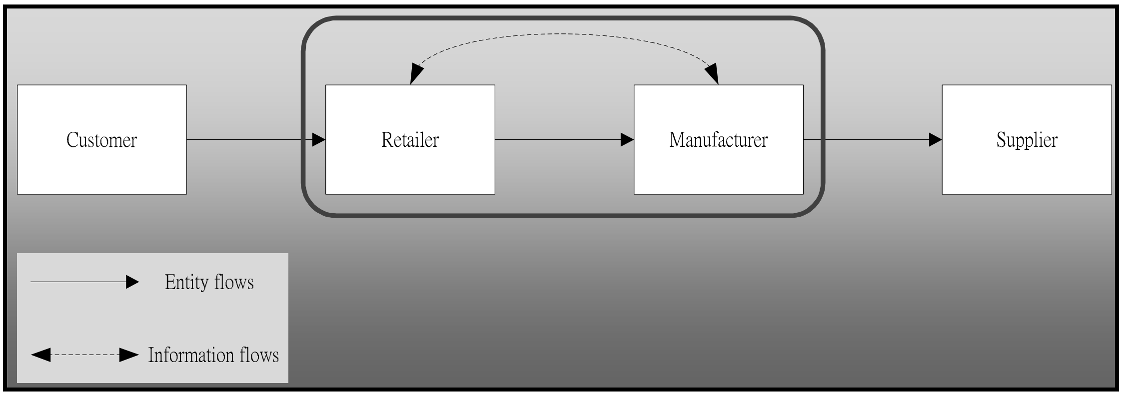


Figure I-2 Framework of supply chain

To analyze the long term result, we set a periodic-review system. As for the properties of invertibility and inferability, we find out the ARMA time series can be a proper way to model the actual demands that the customer generates. Moreover, we set a covariance stationary ARMA process that ensures the process to converge to steady state, and an ARMA process is invertible if the error terms can be reveal from the demands. Nevertheless, information sharing or not may influence the system performance. Thus, we try to figure out how to deduce the actual customer demands without information sharing. We also consider an "ARMA-in-ARMA-out" principle which we will introduce carefully in next chapter. We use different forecasting mechanisms, Gaur's and Zhangs' approaches, to estimate the lead time demand. Manufacturer can deduce the actual customer demands from only receiving the orders of the retailer. Demand inferring will influence the need of information sharing.

Both retailer and manufacturer use an order-up-to policy to maintain their inventory and decide their order quantity. Lee *et al*. (2000) find out that the order-up-to policy can control the demand and inventory in a proper way and can easily be used in a periodic-review system. For convenience, we allow the order quantity to be negative, i.e., the retailer and manufacturer can adjust the order by changing this period’s order. To make sure the system is able to execute, we set the demand stable enough to prevent the demand or inventory grows indefinitely.

We evaluate and quantify the bullwhip effect in a simple way. We collect data of actual demands and collect order quantities between the customer, retailer, and manufacturer. We compare the variance of the actual demands and order too. Based on those collections and comparisons, we figure out the discrepancies and performances in several scenarios. To understand more about the problem structure, we will interpret it clearly in Chapter 3.

1.8 Objectives & Contributions

For this thesis, we have many objectives to achieve. First, we tempt to generate a stable ARMA process which automatically inverts and infers a supply chain with demands. Second, we discuss time series, the invertibility, to compare the performance of an invertible supply chain with and without inverting the forecasting errors. Third, we want to compare the performance of an inferable supply chain with and without inferring the demand process. Fourth, we build an information sharing structure to compare the scenarios with no information sharing and with demand inference. Eventually, we conduct a way to evaluate bullwhip effect and discuss the effects in different scenarios.

As for contribution, we exactly show the invertible property that influence the retailer and manufacturers’ order action, and the outcomes are significant. We construct the ARMA process for different orders of autoregressive and moving-average to see how the order mechanisms perform. We present different lead times to compare the performance.

1.9 Organization of The Thesis

This thesis is organized as follows: in Chapter 2 we survey the relevant references in time series, bullwhip effect, order mechanism, inventory policy, control mechanism, forecasting technique, and information sharing. We first introduce the common time series and elaborate on the particular time series that we use. Second, we introduce the order mechanisms and inventory polices in supply chain. Third, we emphasize our points in figuring out how to control the bullwhip effect. Fourth, we interpret the main concept of our thesis, information sharing, and investigate the mechanisms they use. Finally, we systemically summarize the pros and cons of the references in different categories. In Chapter 3, we construct and describe the model of this thesis. We show the dynamics in the supply chain system that we constructed, and elaborate on the interaction between the retailer and manufacturer. Precisely, we investigate the property of time-series and information sharing structure. Then, we present the dynamics of the different scenarios to show the whole system action and the differences between the scenarios. In Chapter 4, we simulate the entire model in JAVA and solve the definite equations by MATLAB. Moreover, we connect two platforms together, and we use Excel to analyze the performance. We show all of the quantitative results and put discussions there. Finally, in Chapter 5, we conclude our study and discuss the directions of future research.

Chapter 2 Literature Review

Tayur *et al*. (1998) gives an overview about quantitative methods for supply chain management. Managing flows, information and entity, in the system is a major challenge due to the complexity of the system, the proliferation of products, and the presence of multiple decision makers who own part of the system and optimize a private, selfish objective. To get over these problems, coordination is essential, which appears in several forms. In this thesis, we focus on the information sharing that characterizes the information flow between a retailer and a manufacturer. For this purpose, we review the time-series, order and inventory policy, bullwhip effect control, and information sharing in following sections.

2.1 Sketch of Time Series

Time series are applicable to many fields in real world, e.g. economy, industries, and environment problems. In general, we record historical data as a time series. The mathematical theory of time series has been an area of considerable research in recent years. In this thesis, we quantify the demand and order process as a time series and analyze them. In this section, we introduce the common time-series, autoregressive (AR), moving-average (MA), autoregressive moving-average (ARMA), autoregressive moving-average (ARIMA), and martingale model of forecast evolution (MMFE) process. As for the main time-series ARMA that we use, we first introduce its elements.

AR process is one of the most common time series. Implicitly it assumes that the recent values are correlated to their historical data. An AR process has different forms according to the coefficients of the process, where a higher degree of coefficients has a higher correlation with historical data. Meanwhile, each order has a different coefficient. The coefficients exhibit the correlative degree. Let D_t be the demand of period t; ϵ_t be the error of period t, where $\{\epsilon_t\}$ is a collection of white noise; $\rho_1, \rho_2, \dots, \rho_p$ are the correlation coefficients from D_1, D_2 to D_{t-p}. Then the AR process for demand is

$$D_t = d + \rho_1 D_{t-1} + \rho_2 D_{t-2} + \cdots + \rho_p D_{t-p} + \epsilon_t,$$

where d is the mean demand for the process. Kumar et al. (2010) and Chen *et al*. (2000a) use AR(1) to study the bullwhip effect, whose detail we will show in Table II-1. Both Lee *et al*. (2000) and Raghunathan (2003) use AR(1) processes with a nonnegative autocorrelation coefficient to simulate the performance in a two-tier supply system consisting of a single retailer and a manufacturer. Baganha and Cohen (1996) consider AR(p) process as their demand process. We will discuss their model in detail in the following section.

The MA process is a very simple type of time series that relates demand to its error terms. The error terms ϵ_t is also a set of white noise. Let $\lambda_1, \lambda_2, \dots, \lambda_q$ be the coefficients of the errors $\epsilon_t, \epsilon_{t-1}, \dots, \epsilon_{t-q}$. Then the MA process of demand process D_t is

$$D_t = d + \epsilon_t + \lambda_1 \epsilon_{t-1} + \lambda_2 \epsilon_{t-2} + \cdots + \lambda_q \epsilon_{t-q}.$$

The above time series are elementary. In this thesis we use a more appropriate time series which combine the properties of the above two time series. Autoregressive moving-average (ARMA) process is a more general process for real world. It is a linear process relating demands to its historical data and error terms. From Box *et al*. (1994), the ARMA process is

$$D_t = d + \rho_1 D_{t-1} + \rho_2 D_{t-2} + \cdots + \rho_p D_{t-p} + \epsilon_t + \lambda_1 \epsilon_{t-1} + \lambda_2 \epsilon_{t-2} + \cdots + \lambda_q \epsilon_{t-q}.$$

The process can represent stationary seasonal processes. For this reason, Disney *et al*. (2006) use the simple $\mathrm{ARMA}(1,\ 1)$ process to be their single echelon supply chain model's demand and we will introduce in the following section.

Zhang (2004) studies a time series order process for a decision maker facing an invertible ARMA demand process and using a periodic-review order-up-to policy. He derives the "ARMA-in-ARMA-out" principle, i.e., if demand is an $\mathrm{ARMA}(p, q)$ process, then asymptotically the order process is an $\mathrm{ARMA}(p, \max\{p,\ q-l\})$ process, where l is the replenishment lead time. Kumar *et al*. (2010) also find the characteristic in a closed-loop supply chain system. With demand as $\mathrm{AR}(1)$ process, they observe the $\mathrm{ARMA}(1,\ 1)$ process as that for orders.

Gaur *et al*. (2005) use the same assumptions as Lee *et al*. (2000). They extend Zhang (2004)'s work, and implement a different order mechanism. Then they develop another output of "ARMA-in-ARMA-out" form: when the demand is an $\mathrm{ARMA}(p, q)$ process, and the order process is $\mathrm{ARMA}(p, p+q)$. These results lead to invertibility and inferability in supply chains. We will elaborate further below.

The ARMA process was first developed by Box and Jenkins (1970) in 1970s, so it also called Box-Jenkins time series. Box and Jenkins (1970) build a mechanism to detect the ARMA process from crude data. The mechanism they build consisting of several steps to apply to a stationary time series. In implementation, the orders of autoregressive of and moving average are identified by visually inspecting the plots of autocorrelation (ACF) and partial autocorrelation functions (PACF) of crude data. By the autocorrelation function's plots, we can figure out the MA order, and find the AR order by observing the partial autocorrelation function. Next, the parameters of AR and MA are estimated recursively by either the least square method or the maximum likelihood method. In the end, the residuals are tested to see whether or not they behave like white noise. This primary procedure is used until Song and Esogbue (2006) develop a new algorithm to determine ARMA orders and coefficients automatically. The algorithm is based on the Box-Jenkins time series modeling technique, which can use ACF and PACF to find out the relevant lag period, and ensure fitness of the forecasting mechanism fitted to the real demand. Tien (2011) revises the algorithm for better detecting the process. She rearranges the process to figure out the true process easily and quickly.

For a non-stationary time series in more fluctuated or seasonal environment, autoregressive integrated moving average (ARIMA) and martingale model of forecast evolution (MMFE) are more ideal than the time series we mentioned above.

ARIMA is a process which incorporates a wide range of non-stationary series. More precisely, ARIMA process can reduce to ARMA process after differencing finitely many times and examine the seasonality's and trend's parameters. Gilbert (2005) uses the general class of ARIMA time-series processes to model the demand in its supply chain system, and then he derives the order and inventory into a process of ARIMA process forms. He mentions that many specific models such as autoregressive models, exponential smoothing models, and the independent identically distributed (i.i.d.) models that have been used by previous researchers to model demand are special cases of ARIMA process.

MMFE process is first introduced as a demand model for research by Hausman (1969), Graves *et al*. (1986), and Health and Jackson (1994). Chen and Lee (2009) use MMFE as a generalized demand model which consists of incremental information terms. The information terms are mutually independent, stationary, and normally distributed with N (0, σ^2). Moreover, the information terms are revised by the receiving data in every period. Based on these terms, MMFE can cover many types of time series such as integrated moving average (IMA) process and also the ARMA process.

As a demand pattern, we take a particular ARMA(p, q) process as an actual customer demand. After the retailer placing his order, the "ARMA-in-ARMA-out" principle of Zhang (2004) and Gaur *et al*. (2005) give an ARMA process as the order process for which we analyze.

2.1.1 Covariance Stationary

An ARMA process is covariance stationary, if and only if the process can be expressed as an infinite moving average process of the white noise ϵ_t, i.e.,

$$D_t = \sum_{j=0}^{\infty} a_j \epsilon_{t-j}\text{, for all } t.$$

The parameter a_j is assumed to be an absolutely summable sequence of real numbers, i.e.,$\sum_{j=0}^{\infty} |a_j| < \infty$. Additionally, covariance stationary is equivalent to the condition, that the absolute values of the roots of the following equation are less than one.

$$m^p - \rho_1 m^{p-1} - \rho_2 m^{p-2} - \cdots - \rho_p = 0,$$

If a process satisfies such conditions, in this case, the expected values of D_t or Y_t exists and are constant for all t. Moreover, the variance of D_t is finite and the covariance of D_t and D_{t+h} depends on h, not on t. The proof is elaborated on Brockwell and Davis (2002).

2.1.2 Invertibility of a Time Series

Fuller (1996) shows that invertible is a dual concept to covariance stationary. An ARMA process is invertible if and only if the error term ϵ_t can be expressed into its present and past values, i.e.

$$\epsilon_t = \sum_{j=0}^{\infty} b_j D_{t-j}, \text{for all } t \ ,$$

where the parameter b_j is an absolutely summable sequence of real numbers. To check whether a process is invertible or not, we can calculate the roots of following equation,

$$m^q - \lambda_1 m^{q-1} - \lambda_2 m^{q-2} - \cdots - \lambda_q = 0.$$

If absolute values of all of the roots are less than one, then the process is invertible. Otherwise, the process is non-invertible. For an invertible process the past and present demand values can converge to the present error term. It is a useful property when working with the periodic supply chain system. For the downstream partner who faces the demand directly, such a system can express the lead time error into its order decision. Zhang (2004) provides a simple function to show the value of such a property. Gaur *et al*. (2005) give a different method to work with the property. They mention that their method can be used in both invertible and non-invertible conditions. Moreover, the performance of his method is almost the same as Zhang's. We will introduce these two methods in the following chapter.

2.1.3 Implications of Causality & Invertibility

The following two theorems are from Gaur *et al*. (2005) and the applications from Zhang (2004).

THEOREM 1. (*i*) *If* D_t *is covariance stationary, then* Y_t *is covariance stationary.* (*ii*) *If* D_t *is* ARMA *process,* Y_t *will be an* ARMA *process in different orders of autoregressive and moving-average.*

From theorem 1, we will get different processes by different order mechanisms, where Y_t is the order realization. If we set the first q error terms to zero, then Y_t is ARMA$(p,\ \mathrm{MAX}(p, q - l))$ process where l is the replenishment lead time. In this thesis we call this the Zhang's approach. When we set the recent error terms to zero, Y_t is ARMA$(p,\ p + q)$ process. We call it Gaur's approach. For a simple case, if we provide an AR(1) process for demand process, we will get ARMA(1, 1) process in order process.

From theorem 1, we can ensure that the order process is an ARMA process after applying the forecasting mechanism and inventory policy. In addition, the manufacturer can derive the process, if the manufacturer knows the parameters of demand process. We will explain in Chapter 3.

THEOREM 2. If D_t *is inferable from* Y_t, *in spite of what* $\widehat{\mathrm{D}}(p-1)$ *we choose,* $\widehat{\mathrm{D}}(p-1)$ *is independent.*

Therefore, $\widehat{D}_t$ (estimated demand realization) can converge to D_t almost surely as t goes to infinity. This situation is same as the following equation's roots are less than one in absolute value.

$$(1+\alpha_1)r^p-(\alpha_1-\alpha_2)r^{p-1}-(\alpha_2-\alpha_3)r^{p-2}-\cdots-\alpha_p=0.$$

Theorem 2 gives us the hint to infer the demand values from order values. We can just check the equation above, and make sure that the roots of r are less than one in absolute value. In our model, we set initial values of $\widehat{\mathrm{D}}(p-1)$. Despite what we choose we can get an approximate demand value from the past order values. The initial values will have negligible effect as t increases.

These theorems play a crucial role in our model. They enable us to separate time series demand patterns into different cases for discussion.

2.2 Order Mechanism & Inventory Policy

Most studies in bullwhip effect use order-up-to inventory policy to be the main inventory control and order decision mechanism (Lee (2000), Raghunathan (2001), Zhang (2004), Gaur (2005)). They all use similar adjustable order-up-to policy to control their inventory and decide the orders. Here we introduce the order mechanism and inventory policy we surveyed in literature.

Sterman (1989) gives a technique to control the stock by the parameters of supply line and real stock. He uses the famous “beer distribution game” to interpret the concept of stock management. In his paper, he tempts to capture the clues on stock and the orders that have not been arrived to adjust this time order.

Caplin (1985) considers a model with a (*S*, *s*) policy. He derives the demand variance for such inventory policy and considers multiple retailers by means of the Markov chain theory. He assumes the demands are identical and independent and observes their inventory positions continuously. Baganha and Cohen (1998) discuss about (S, s) inventory policy with an $AR(p)$ demand pattern. They compare the variance of demand to the variance of replenishment orders at different echelons of the system, i.e. retailers, and manufacturers. Their model considers multiple retailers and combines their order into one aggregate order. They make a stable model and relate to the myopic control policies.

Aviv (2002) considers inventory management in a time-series framework for supply chain system. He considers a vector autoregressive (VAR) time-series demand structure which is a dynamic simultaneous equations model with stationary multivariate characteristic. He then uses Kalman filter to produce a state vector of early indicators of future demand. He sets an $AR(1)$ example to elaborate how it works for cost reduction and inventory position performance.

From the literature, we select order-up-to policy. It considers the forecasting demand data, service level, and past demand pattern. The demerit of the policy is that it is hard to control inventory level, i.e. inventory on hand, if the initial order quantity is overestimated. That is, the surplus quantity will last for a number of periods and cannot be eliminated until the next overestimation or an unusual demand. In spite of the demerit, order-up-to policy is well enough after all. It provides an intuitional concept to decide the order and conduct a stable order quantity.

2.3 Bullwhip Effect Control

With forecasting the demand, it is easy to approach the quantity which may be needed in the near future. Nevertheless, the quantity we approach is very probable to fluctuate with the trend and the periodicity, which are similar to the demand pattern, and the order fluctuation will be even greater than the original demand variation.

As for Chen *et al*. (2000b), they find that "*the magnitude of the increase in variability depends on both the nature of the customer demand process and on the forecasting technique used by retailer*". They demonstrate an example for certain demand process. The variance of the orders placed by a retailer using a moving-average forecast will be less than the variance of orders placed by the same retailer using exponential smoothing forecast. Nevertheless, they figure out the lead time affects the magnitude of orders variance which the longer lead time causes larger variance. In other words, the smoother forecasting mechanism for demand forecasts indeed gets lower bullwhip effect.

Some researchers use the ordering mechanism to control bullwhip effect, (Disney *et al*. (2006), Sterman (1989), and Wright and Yuan (2008)). The method they use to suppress the amplification is to deduct the fulfillment of desired inventory position. By consisting the discrepancy of inventory and outstanding orders, the bullwhip effect is mitigated.

As researching order-up-to policy, Disney *et al*. (2006) study the order quantity consisting of forecast demand, net stock discrepancy, and work in process discrepancy, i.e., the supply line discrepancy. Instead of operating all of the discrepancy, they tune the last two components to control the system. As the result, the two discrepancies are suppressed. The controllers limit the replenishment for ample discrepancy.

Disney *et al*. (2006) diminish the total effect of order variance and net stock variance. They find out a “Golden Ratio” of about 1.61803 between these two subjects. Dividing the total inventory position discrepancy by the “Golden Ratio” can minimize the sum of the two variances for stationary i.i.d. demand pattern.

Kumar *et al*. (2010) use the order-up-to policy and set MMSE (Minimum Mean Square Error) as a forecasting scheme under the AR(1) demand pattern. They construct a six-echelon closed loop supply chain model for recycling product industries such as paper, plastic. They mainly investigate the influence of autoregressive coefficient by adjusting the parameter and recycling rate. As the result, they find out how the intervals of coefficients affect the magnitude of bullwhip effect.

Wright and Yuan (2008) study to control bullwhip effect by using order mechanism which follow Holt’s and Brown’s methods. They use it independently or jointly. The order quantity is decided by the estimated demand, adjustment of stock and supply line, and the approach is similar to the negative feedback loop in Sterman (1989)’s system dynamic. Wright and Yuan (2008) suggest that adjusting the discrepancy of stock slowly and revising a slightly larger in discrepancy of the supply line will perform well for mitigating the bullwhip effect when the model use

Holt's and Brown's exponential smoothing forecasting mechanism.

It is beneficial to control the discrepancy of inventory and expected demand values, and helps restrain the fluctuation of order quantity. After all, bullwhip effect mitigates apparently. We also consider another way to alleviate the effect, which is information sharing that we introduce in next section.

2.4 Information Sharing in Supply Chain

Chen *et al*. (2000b) indicate that disclosing more demand information to the partners in supply chain reduces the increase in variability, i.e., reduces the bullwhip effect. Furthermore according to Gavirneni (2005), downstream partner shares the price data in order to reduce the fluctuations in demand, and consequently improves supply chain performance. Otherwise, most of the topics share the data of demand or inventory level for achieving a better forecasting values. In this thesis, we mainly survey such topic in papers.

Usually, retailer can share the POS data to the upstream partner and hopefully improve the whole system's performance. Unfortunately, it just effects slightly in most studies. For Chen and Lee (2009), they share the partial future forecast information data of MMFE to control the order viability. In addition to reduce system cost, they point out that sharing a projected order has more effective outcome than only POS data.

Just like the VMI system, Gavirneni (1999) claims that sharing day-to-day inventory levels can reduce the system cost under the non-stationary demand pattern with (S, s) policy. Besides, he finds out that the order-up-to policy will only be the optimal inventory policy when the problem is finite periods with non-stationary demand pattern. In other words, (S, s) policy will be an appropriate one.

For the papers which build on time-series, we study couples of papers to develop a proper remedy for alleviating bullwhip effect. As we know, most of the papers build the model with order-up-to policy. For Aviv (2002), he considers a vector autoregressive time-series which generates the demand realizations with a static vector, and uses a forecast mechanism which called Kalman filter. It considers the fundamental element of the supply chain’s inventory data, inventory levels and safety stock, to estimate the future demand. The model shares the so-called lagged forecast errors to the upstream partner and obviously improves the system performance in cost reduction.

For AR time-series, most of the papers use $\mathrm{AR}(1)$ process to develop their principle concept. Chen *et al*. (2000a) describe the impact of forecasting, lead times, and information. They first conduct a simple two-tier supply chain model and then extend to multi-echelon problems. They centralize the demand information of recent demand values for downstream partner to share with all the partners in the supply chain. They find out that information sharing does not completely eliminate bullwhip effect but reduces the variability in each stage.

Moreover, Lee *et al*. (2000) develop a quick response forecast mechanism by sharing error terms each period. The paper shows great performance when the coefficients of the $\mathrm{AR}(1)$ process are highly correlated. Under information sharing, the system benefits from both inventory reduction and expected cost reduction. It analyzes different dimensions, such as lead time, autoregressive coefficient, and information sharing which inspires our study. On the other hand, Raghunathan (2001) bases on Lee (2000)'s model structure and extends to study not only for a two-member model but also for a multi-member model. Precisely, he considers a "one-supplier and multi-retailer" problem. He uses the well-known Shapley value concept from game theory to analyze the expected manufacturer and retailer shares of the surplus generated from information sharing. Finally, he finds out the value of information sharing will decrease as periods go forward.

Gaur et al. (2005) set to consider two different dimensions of model to discuss information sharing under ARMA demand process. They construct a mechanism to share the real demand realization, and develop a method to estimate the real demand values. They find out that the coefficients of autoregressive and moving-average play important roles in model.

For a MMFE process, Güllü (1996) considers a "two echelon, multi-retailer and one depot" model. He sets the downstream retailers share the forecasting demand data in the MMFE forecasting mechanism.

At the end of this chapter, we make a table for easier to understand the outline of references.

Table II-1 Outline of references

Topic	Supply chain system	Demand pattern	Information sharing	Remarks
Disney *et al*. (2006) **Taming the bullwhip effect whilst watching customer service in a single supply chain system**	Single echelon; retailer, supplier, or… etc.	ARMA(1, 1)	None	● Comparison of two type's order-up-to policy. ● Find the "Golden ratio" between bullwhip effect & net stock variance
Boute and Lambrecht (2009) **Exploring the bullwhip effect by means of spreadsheet simulation**	Two echelons	Normal distribution	None	● Basic view of bullwhip effect
Pati *et al*. (2010) **Quantifying bullwhip effect in a closed loop supply chain**	Six echelons	AR(1)	None	● Bullwhip effect does not occur when $\rho < 0.5$ ● Bullwhip effect ratio save almost same for all lead times in $-0.5 \leq \rho \leq 0.7$ ● $\rho > 0.7$, small lead time has small bullwhip effect
Chen *et al*. (2000a) **Quantifying bullwhip effect in a simple supply chain: the impact of forecasting, lead times, and information**	Two echelons, n echelons	AR(1)	Centralized demand information, recent demand value	● Use moving-average forecasting mechanism ● Information sharing does not completely eliminate bullwhip effect but reduces the variability in each stage

Chen *et al*. (2000b) **The impact of exponential smoothing forecasts on the bullwhip effect**	Two echelons	Correlative demand AR(1); demand with trends	None	● They construct two different demand processes and use two different forecasting mechanisms, exponential smoothing and moving-average ● Longer lead time generate more bullwhip effect ● The more demand information in forecasting mechanism, the less bullwhip effect is produced
Wright and Yuan (2008) **Mitigating the bullwhip effect by ordering policies and forecasting methods**	Four echelons	Normal distribution	None	● Use three kinds of forecasting mechanism, moving average, Holt's, and Brown's ● Holt's method can eliminate bullwhip effect for about 55%
Lee *et al*. (2000) **The value of information sharing in a two-level supply chain**	Two echelons	AR(1)	Share error terms in each periods	● Under information sharing, the system benefit from inventory reduction & expected cost reduction
Raghunathan (2001) **Impact of demand correlation on the value of and incentives for information sharing in a supply chain**	Two echelons, multi-retailer, 1 supplier	AR(1)	Share demand pattern and error terms in each periods	● Use Shapley value to analyze the expected surplus generated from information sharing ● Value of information sharing decreases as period go forward

Zhang (2004) **Evolution of ARMA demand in supply chains**	Two echelons	ARMA(p, q)	None	● Apply an "ARMA-in-ARMA-out" principle
Gaur *et al*. (2005) **Information sharing in a supply chain under ARMA demand**	Two echelons	ARMA(p, q)	Share demand realization in each periods	● Inferred demand has the same effect as information sharing ● When demand is invertible but order is not, information sharing is necessary
Gilbert(2005) **An ARIMA supply chain model**	Single echelon; retailer, supplier, or… etc.	ARIMA(p, d, q)	None	● Sharing POS data may not mitigate the bullwhip effect ● Lead time reduction is a mean to reduce bullwhip effect
Chen and Lee (2009) **Information sharing and order variability control under a generalized demand model**	Two echelons	MMFE	Share partial future forecast information	● Sharing projected orders has more effective performance than only sharing POS data. ● Controlling the order viability is a key point to reduce cost

Gavirneni *et al.* (1999) **Value of information in capacitated supply chains**	Two echelons	i.i.d random variable; Normal, uniform, …etc.	Day to day inventory levels	● Use (S, s) policy to control inventory ● For non-stationary demand patterns order-up-to is an optimal policy in finite horizon; otherwise, (S, s) policy has better performance
Aviv (2002) **A time-series framework for supply-chain inventory management**	Two echelons	VAR	Forecast errors	● Use Kalman filter technique to forecast ● Discuss inventory position & cost performance assessment
Baganha and Cohen (1996) **The stabilizing effect of inventory in supply chains**	Multi-echelon; Multi-retailer	AR(p)	None	● Use (S, s) policy to control inventory ● Demand variation can be absorbed under specific retailer order series and lead time lengths ● Share data of customer may optimize the system performance
Güllü (1996) **A Two-echelon allocation model and the value of information under**	Two echelons; Multi-retailer, one depot.	MMFE	Downstream forecasting demand data	● Ignore the delivery lead time from retailers to depot. ● Under a general correlated demand-forecast structure, depot gets system-wide order-up-to level and expected cost reduction easily.

Chapter 3 Model Formulation

3.1 Overview of model

We construct a discrete-time two-level supply chain. The model setup and assumptions follow those in Zhang (2004) and Gaur *et al*. (2005) except for the difference in the action of the manufacturer. Zhang (2004) and Gaur *et al*. (2005) do not explain the detailed action of the manufacturer. Our supply chain system has a single retailer and a single manufacturer interacting together. There is just one type of product, and the retailer faces demand patterns in time series such as AR, MA, or ARMA processes. The retailer forecasts the customer demands to help control inventory and eliminate the discrepancies between order quantities and actual demands. The manufacturer uses the same forecasting mechanism using the retailer's order quantities as the input to make his order decisions. At the top of the supply chain, the manufacturer has an upstream supplier that has infinite capacity.

We assume that the lead time between the manufacturer and the retailer is no larger than that between the manufacturer and her upstream supplier. We still discuss the effect of lead time. For order policy, we use the adjusted order-up-to policy to control the retailer and manufacturers' inventory. The policy has some differences with the general one. The policy does not replenish the inventory to a fixed level. The retailer adjusts the target level by considering safety inventory level and forecasting data every period.

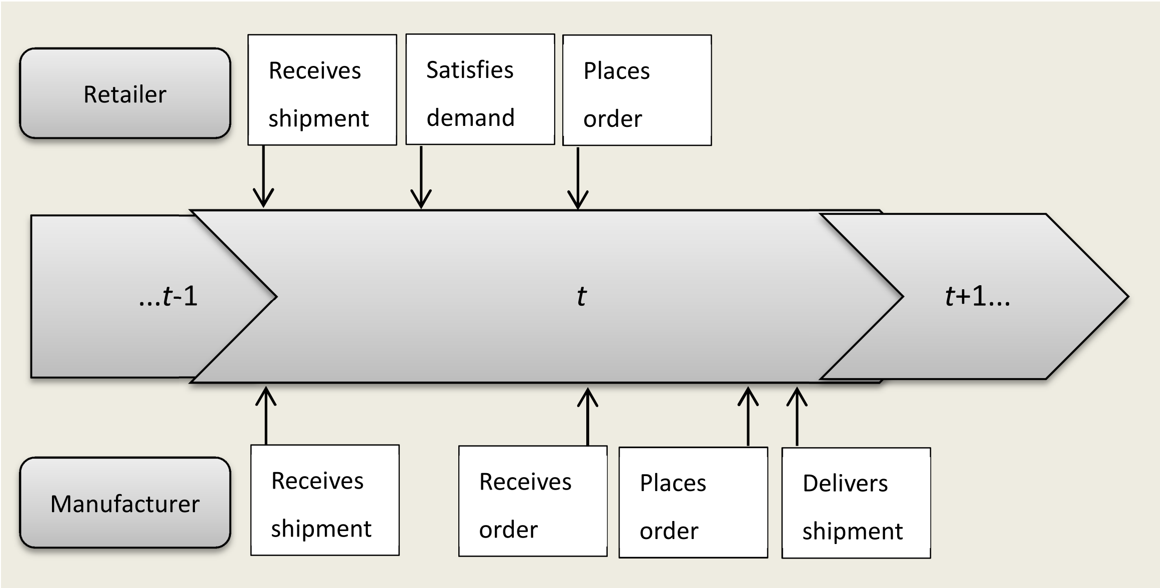


Figure III-1 Framework of system dynamic

The dynamics of the system follows that in Figure III-1. At the beginning of a period, the retailer receives the shipment that she ordered. The retailer faces the actual demands and backorders if there are not enough goods. Then, the retailer calculates the inventory and forecasts the future demands within the lead time periods. At last, the retailer places the order and terminates the period. The manufacturer's dynamics is similar to the retailer, but the demand that the manufacturer faces is the retailer's order for each period. Before ending the period, the manufacturer delivers the shipment to the retailer and the retailer will receive it after the lead time periods.

The retailer can use two kinds of forecast mechanisms: an invertible demand process or a general one which may or may not be invertible. For the manufacturer, we can also conduct different forecasting methods with different insights for discussion.

In our model, the manufacturer can try to deduce the actual demands from the orders to make the forecast more accurate. Since the manufacturer cannot observe the actual demand realizations unless the retailer shares or the manufacturer infers by itself, we also conduct an information sharing structure to compare differences among information sharing and various forecasting mechanisms. In the end, we clarify how we estimate the bullwhip effect in our supply chain system. In the following sections we interpret the framework and assumptions of our model and elaborate on the actions between the retailer and the manufacturer.

3.2 Framework

Here we describe the assumptions, notations, and the information sharing structure of the model. The following are the general characteristics of our model.

a. The system is a periodic review two-tier supply chain system.

b. There are two members, a retailer and a manufacturer.

c. The external demand for a single product occurs at the retailer.

d. The underlying demand process is an ARMA(p, q) process, where p is the autoregressive order, and q is the moving-average order. The process can be simplified into an AR process if p is set to zero, and into a MA process if q is set to zero. The general form of the demand process $\{D_t\}$ is,

$$D_t = d + \rho_1 D_{t-1} + \rho_2 D_{t-2} + \cdots + \rho_p D_{t-p} + \epsilon_t - \lambda_1 \epsilon_{y-1} - \lambda_2 \epsilon_{t-2} - \cdots - \lambda_q \epsilon_{t-q}, \quad (1)$$

where

D_t: demand realization at period t;

d: the mean demand at a period;

ϵ_t: the sequence of uncorrelated random variables at period t, known as white noise, with mean zero and variance σ^2;

$\rho_1, \rho_2, \ldots, \rho_p$: the autoregressive coefficients;

$\lambda_1, \lambda_2, \ldots, \lambda_q$: the moving-average coefficients.

3.2.1 Assumptions & Notations

i. D_t is covariance stationary, i.e., $E[D_t]$ exists and is a constant for all t. VAR$[D_t]$ is finite, and the covariance of D_t and D_{t+h} depends only on h.

ii. The upstream supplier does not have capacity limit and we can focus on the interactions between the retailer and the manufacturer.

iii. The replenishment lead times from the upstream supplier to the manufacturer and from the manufacturer to the retailer are L and l periods, respectively.

iv. The orders placed by the manufacturer and the retailer at the end of period t will be received at the beginning of periods $t + L + 1$ and $t + l + 1$, respectively.

v. Both the retailer and the manufacturer use a myopic order-up-to inventory policy. Negative order quantity is allowed which are actually backorders. In general, d is large enough so that negative demand and order values are negligible.

vi. When order or demand cannot be satisfied by on-hand inventory, the excess quantity can be treated as a backorder and will be satisfied at following periods.

vii. The parameters of the demand process, $d, \rho_1, \rho_2, \ldots, \rho_p, \lambda_1, \lambda_2, \ldots, \lambda_q$ and the variance of white noise, ϵ_t, are common knowledge for both the retailer and the manufacturer, but demand realizations are private information to the retailer.

viii. An arbitrary time reference is chosen to be zero; we set the demand and order process for $t = 0, \pm 1, \pm 2, \ldots$.

Table III-1 Notation definition

$\widehat{\boldsymbol{D}}_{\boldsymbol{t}}$	Estimated demand realization at period $\boldsymbol{t}$
$\boldsymbol{\xi_t}$	Cumulative error terms at period t, $\xi_t = \epsilon_t - \lambda_1\epsilon_{t-1} - \lambda_2\epsilon_{t-2} - \cdots - \lambda_q\epsilon_{t-q}$
$\boldsymbol{Y_t}$	Order made by retailer at period t
$\widehat{\boldsymbol{Y}}_{\boldsymbol{t}}$	Adjusted order made by the manufacturer to help generate the alternative $\mathfrak{Y}_t$ at period t
$\boldsymbol{\mathfrak{Y}_t}$	Order made by the manufacturer at period t
$\mathbf{D}(\boldsymbol{t})$	$p - vector$'s transpose matrix $(D_t, D_{t-1}, \ldots, D_{t-p+1})'$, the demand values from present period to the past $t-p+1$ period, $\begin{bmatrix} D_t \\ \vdots \\ D_{t-p+1} \end{bmatrix}$
$\mathbf{Y}(\boldsymbol{t})$	$p - vector$'s transpose matrix $(Y_t, Y_{t-1}, \ldots, Y_{t-p+1})'$, the order values from present period to the past $t-p+1$ period, $\begin{bmatrix} Y_t \\ \vdots \\ Y_{t-p+1} \end{bmatrix}$
$\widehat{\mathbf{Y}}(\boldsymbol{t})$	Adjusted order made by the manufacturer, $p - vector$'s transpose matrix $(\widehat{Y}_t, \widehat{Y}_{t-1}, \ldots, \widehat{Y}_{t-p+1})'$, $\begin{bmatrix} \widehat{Y}_t \\ \vdots \\ \widehat{Y}_{t-p+1} \end{bmatrix}$
$\boldsymbol{\xi}_{\boldsymbol{t}}$	$p -$ vector's transpose matrix $(\xi_t, 0, \ldots, 0)'$
$\mathbf{P}$	$p \times p$ matrix $\begin{bmatrix} \rho_1 & \cdots & \rho_p \\ \mathrm{I}_{p-1} & & 0_{p-1} \end{bmatrix}$
$\mathbf{I}_{\boldsymbol{p-1}}$	$p-1 \times p-1$ identity matrix

$\mathbf{0}_{p-1}$	Column of $p-1$ zeros
$\mathbf{e}$	Transpose matrix $(1,0,\ldots,0)'$, $p-$vector with 1 in the first row and 0 in the residual $p-1$ rows
$\mathbf{P}^k(\mathbf{1}:)$	First row of $\mathbf{P}^k$
$\boldsymbol{P}^k_{(ij)}$	(ij)th element of the matrix $\mathbf{P}^k$
$\boldsymbol{m}_t$	The conditional expected value of the lead time demand for the retailer at period t
$\boldsymbol{M}_t$	The conditional expected value of the lead time demand for the manufacturer at period t
$\boldsymbol{v}_t$	The conditional variance of the lead time demand for the retailer at period t
$\boldsymbol{V}_t$	The conditional variance of the lead time demand for the manufacturer at period t
$\boldsymbol{s}_t$	The retailer's order-up-to level at period t
$\boldsymbol{S}_t$	The manufacturer's order-up-to level at period t
$\boldsymbol{z}$	The service level parameter for the retailer
$\boldsymbol{Z}$	The service level parameter for the manufacturer
$\widehat{\mathbf{D}}(\boldsymbol{p}-\mathbf{1})$	The initial vector of demand value, $[D_{p-1},\ldots,D_0]$

3.2.2 Information Sharing Structure

Information sharing plays an important role in this thesis. If there is information sharing, the manufacturer receives the order quantities and the actual customer demands from the retailer; if there is no information sharing, the manufacturer receives only the order quantities of the retailer without knowing the actual customer demands. In case of information sharing, the manufacturer uses the demand information to revise the order to improve the system performance. When there is no information sharing, the manufacturer tries to infer demand quantity from the retailer orders. For inferable demand process, the manufacturer can figure out the demand realization without information sharing, but it needs some conditions for a demand process to be inferable.

Information sharing structure can be classified into two types. One is sharing no extra information to the manufacturer and passing the information on order quantity Y_t only. The other is sharing extra information to the manufacturer and passing both order quantity Y_t and also the demand realization D_t. We will interpret the two different types in following section for the manufacturer's action.

3.3 System Dynamics

Here we elaborate on the whole procedures of both the retailer and the manufacturer. To enhance our logic flow, we put down all the sequences of the system, and interpret the theorems that we use.

Figure III-1 shows the retailer and manufacturer's dynamics, and gives a concept of the periodic review system. Further, Figure III-2 and Figure III-3 show the detail actions and decision mechanisms of the retailer and the manufacturer.

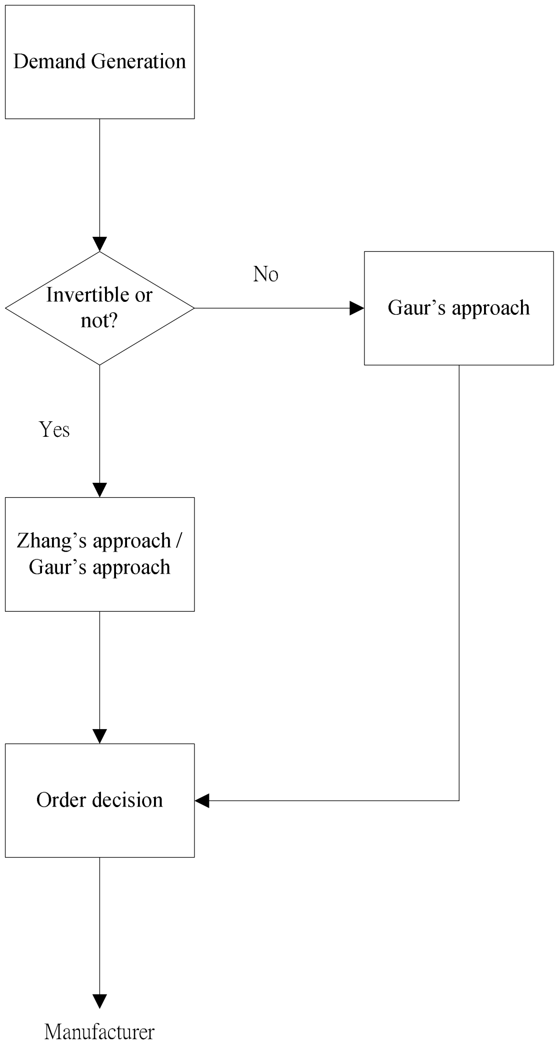


Figure III-2 Framework of Retailer's action

3.3.1 Retailer's Action

The retailer uses a myopic order-up-to policy which just considers the status of the current period; she uses the demand data to predict its future demand with lead time. In our study, the retailer can choose either the Zhang's or Gaur's approach to forecast lead time demand. The two approaches are distinguished by the demand process being invertible or not. From (1),

$$\mathbf{D}(t) = \mathrm{de} + \mathbf{PD}(t-1) + \xi_t$$

$$= d\sum_{i=0}^{k-1} \mathbf{P}^i \,\mathrm{e} + \mathbf{P}^k\mathbf{D}(t-k) + \sum_{i=0}^{k-1} \mathbf{P}^i\, \xi_{t-i} \tag{2}$$

for $k \geq 1$. (2) can be expressed in the following equivalent form

$$D_t = \mathbf{P}^k(1:)\mathbf{D}(t-k) + d\sum_{i=0}^{k-1} P_{(11)}^i + \sum_{i=0}^{k-1} P_{(11)}^i\, \xi_{t-i}. \tag{3}$$

From (3),

$$\sum_{k=1}^{l+1} D_{t+k} = \sum_{k=1}^{l+1}\left(\mathbf{P}^k(1:)\mathbf{D}(t) + d\sum_{i=0}^{k-1} P_{(11)}^i + \sum_{i=0}^{k-1} P_{(11)}^i\, \xi_{t+k-i}\right).$$

Let m_t be the conditional expectation of $\sum_{k=1}^{l+1} D_{t+k}$ given $D_t, D_{t-1}, \ldots$. Then, at period *t*, with known demands observed for $D_t, D_{t-1}, \ldots$, the mean total lead time demand as interpreted by the retailer is

$$m_t = \mathrm{E}\left[\sum_{k=1}^{l+1} D_{t+k} \,| D_t, D_{t-1}, \ldots\right]$$

$$= \sum_{k=1}^{l+1}\left(\mathbf{P}^k(1:)\mathbf{D}(t) + \mathrm{d}\sum_{i=0}^{k-1} P_{(11)}^i\right) + \mathrm{E}\left[\sum_{k=1}^{l+1}\sum_{i=0}^{k-1} P_{(11)}^i \xi_{t+k-i} \,|\, D_t, D_{t-1}, \ldots\right]$$

Here, we know that $\sum_{k=1}^{l+1}\sum_{i=0}^{k-1} P_{(11)}^i \xi_{t+k-i}$ can be expressed as a linear function literally consisting of $\epsilon_{t-q+1}, \dots, \epsilon_t, \epsilon_{t+1}, \dots, \epsilon_{t+l+1}$. It is obvious that $\mathrm{E}[\epsilon_s | D_t, D_{t-1}, \dots] = 0$ for $t+1 \le s \le t+l+1$, because we have not observed the demand in periods $t+1, \dots, t+l+1$. Nevertheless, we can compute the value of $\mathrm{E}[\epsilon_s | D_t, D_{t-1}, \dots]$ for $t-q+1 \le s \le t$, because we have already observed the demand values in present period t. Those error terms may have some information for future demand value. Thus in estimation lead time demands, for time 1 to t, there are t unknowns, $\epsilon_1, \dots, \epsilon_t$. However, we know $t-q$ numbers, namely, the ARMA process of $D_{q+1}, \dots, D_t$. From the mean total lead time demand m_t, the retailer can choose the forecast approach by the property of demand process.

Since the retailer knows all the parameters of the demand process, the retailer can check the invertible property of D_t from the following equation.

$$r^q - \lambda_1 r^{q-1} - \lambda_2 r^{q-2} - \cdots - \lambda_q = 0, |r| < 1. \tag{4}$$

If the absolute value of roots of r is small than one, the demand process D_t is invertible. From **THEOREM 1** introduced in Chapter 2, there are two approaches for our forecast mechanism. One is Zhang's approach, which is only applicable for invertible demand processes. Otherwise, for a non-invertible demand process, we use Gaur's approach, a more general approach for both invertible and non-invertible ARMA processes. Zhang's approach sets the first q error terms to zero, and Gaur's approach sets the last q error terms to zero. We show the differences in generating the lead time demand and interpret these two approaches in following paragraphs.

Zhang's approach

Zhang's approach is only suitable for invertible demand processes, because the approach considers the past error terms. We set the first q error terms to zero and consider the subsequent error terms in lead time demand. Therefore, the conditional mean total demand

$$m_t = \sum_{k=1}^{l+1} \left(\mathbf{P}^k(1:)\mathbf{D}(t) + d \sum_{i=0}^{k-1} P_{(11)}^i \right) + \mathrm{E}\left[\sum_{k=1}^{l+1} \sum_{i=0}^{k-1} P_{(11)}^i \xi_{t+k-i} \mid D_t, D_{t-1}, \dots \right].$$

Gaur's approach

Gaur's approach discards the recent q error terms in calculation. That is, we can consider no error term in the estimation of lead time demand. Then the conditional lead time demand expression as:

$$m_t = \sum_{k=1}^{l+1} \left(\mathbf{P}^k(1:)\mathbf{D}(t) + d \sum_{i=0}^{k-1} P_{(11)}^i \right).$$

For both approaches, the conditional variance of the lead time demand given $D_t, D_{t-1}, \dots$ can be found as:

$$v_t = \mathrm{Var}\left[\sum_{k=1}^{l+1} D_{t+k} \mid D_t, D_{t-1}, \dots \right] = \mathrm{Var}\left[\sum_{k=1}^{l+1} \sum_{i=0}^{k-1} P_{(11)}^i \xi_{t+k-i} \mid D_t, \dots, D_{t-p+1} \right].$$

By covariance stationary, we obtain a following closed-form expression for v_t for the $\mathrm{AR}(p)$ process,

$$v_t = \sigma^2 \sum_{k=1}^{l+1} \left(\sum_{i=0}^{k-1} P_{(11)}^{i} \right)^2 .$$

Consequently, now we can compute the order quantity by the order-up-to policy for the retailer which is based on the order-up-to level s_t for this period's demand,

$$Y_t = D_t + s_t - s_{t-1} ,$$

where

$$s_t = m_t + z\sigma\sqrt{C}$$

follows the adjusted order-up-to policy by Lee *et al*. (2000). Now, the retailer can place order to the manufacturer, and the retailer terminates this period after ordering.

3.3.2 Deduction of Process

This section deduces the order process for an ARMA process by the "ARMA-in-ARMA-out" principle. Afterwards, we show some useful deductions to infer the demand process.

Deduction of "ARMA-in-ARMA-out" principle

The value of service level parameter z depends on the desired service level and the distribution of ϵ_t. The value of cumulative error term, ξ_t, is independent of the future demands $D_t, D_{t+1}, \ldots, D_{t+l+1}$, so the lead time demand, m_t, and the value of z are independent too. We can express the retailer's order quantity at period t as:

$$Y_t = D_t + \sum_{k=1}^{l+1} (\mathbf{P}^k(1:)[\mathbf{D}(t) - \mathbf{D}(t-1)]) \,. \tag{5}$$

let $\sum_{k=1}^{l+1} \mathbf{P}^k(1:)$ be denoted by the vector $(\alpha_1, \dots, \alpha_p)$, and (5) can be rewritten as

$$\begin{aligned} Y_t = (1+\alpha_1)D_t - (\alpha_1 - \alpha_2)D_{t-1} - (\alpha_2 - \alpha_3)D_{t-2} \\ - \cdots - (\alpha_{p-1} - \alpha_p)D_{t-p+1}. \end{aligned} \tag{6}$$

In short, equation (6) gives retailer a principle to decide the order after observing the demand realization in each period. From **THEOREM 1** in chapter 2: (i) If D_t is covariance stationary, then Y_t is covariance stationary. (ii) If D_t is an $\mathrm{ARMA}(p,q)$ process and the first q error terms are set to zero, then Y_t is an $\mathrm{ARMA}(p, \mathrm{MAX}(p, q-l))$ process. Otherwise, Y_t is an $\mathrm{ARMA}(p, p+q)$ process for setting the recent error terms to zero.

Therefore, if the retailer uses Zhang's approach to forecast lead time demand, we get an $\mathrm{ARMA}(p,q)$ order process; if the retailer uses Gaur's approach, we get an $\mathrm{ARMA}(p, \mathrm{MAX}(p, q-l))$ order process.

Hence, from (6), no matter what approach that retailer uses we can express the order equation as a moving-average function of D_t.

$$Y_t = \sum_{j=0}^{p} b_j D_{t-j},$$

In that case, we let $b_0 = 1 + \alpha_1$ and $b_i = \alpha_i - \alpha_{i+1}$ for $1 \leq \mathrm{i} \leq p - 1$, and $b_p = \alpha_p$. Since $\{b_j\}$ is a finite sequence, it can be summed absolutely. Further, because of the property of invertibility, D_t can be expressed into a moving-average of $\{\epsilon_j\}$ with definitely summable coefficients. Besides, (i) could be proven by the corollary from Fuller (1996).

To explain (ii), clearly in our model, we assume parameter $\mathfrak{B}$ as a backward shift factor, i.e. $\mathfrak{B}D_t = D_{t-1}$. Let $\phi(\mathfrak{B}) = 1 - \rho_1\mathfrak{B} - \rho_2\mathfrak{B}^2 - \cdots - \rho_p\mathfrak{B}^p$, $\psi(\mathfrak{B}) = 1 - \lambda_1\mathfrak{B} - \lambda_2\mathfrak{B}^2 - \cdots - \lambda_q\mathfrak{B}^q$, and $\varphi(\mathfrak{B}) = (1 + \alpha_1) - (\alpha_1 - \alpha_2)\mathfrak{B} - (\alpha_2 - \alpha_3)\mathfrak{B}^2 - \cdots - \alpha_p\mathfrak{B}^p$. By using $\phi(\mathfrak{B})$ and $\psi(\mathfrak{B})$, the demand process (1) can be rewrite as

$$D_t = \phi^{-1}(\mathfrak{B}) * [d + \psi(\mathfrak{B}) * \epsilon_t]\,. \tag{7}$$

Denote the order function (6) by $\varphi(\mathfrak{B})$ and rewrite (6) as,

$$Y_t = \varphi(\mathfrak{B}) * D_t\,. \tag{8}$$

then replacing the D_t in equation (7) by equation (6),

$$Y_t = \phi(\mathfrak{B}) * \varphi(\mathfrak{B}) * [d + \psi(\mathfrak{B}) * \epsilon_t]\,,$$

and

$$\phi(\mathfrak{B}) * Y_t = d + \varphi(\mathfrak{B}) * \psi(\mathfrak{B}) * \epsilon_t. \tag{9}$$

We can observe that $\varphi(\mathfrak{B}) * \psi(\mathfrak{B})$ is a polynomial of degree p plus q. The order process is an $\mathrm{ARMA}(p, p+q)$ process when the retailer uses Gaur's approach, and the order process is an $\mathrm{ARMA}(p, \mathrm{MAX}(p, q-l))$ process when the retailer uses Zhang's approach. If the demand process is an $\mathrm{AR}(p)$ process, $\psi(\mathfrak{B})$ will be set to equal to one. Hence for the manufacturer, she can estimate the lead time demand from the order process each period.

$$Y_t = d + \sum_{i=1}^{p} p_i Y_{t-i} + \varphi(\mathfrak{B}) * \psi(\mathfrak{B}) * \epsilon_t. \qquad (10)$$

<u>Insights from inferred demand realization</u>

After deducing the above processes, the manufacturer can deduce the information from them. Since we consider the property of invertible and inferable in the model, the manufacturer can estimate the demand realization for each period without receiving the information of demand realization from the retailer. That is, the manufacturer can infer the demand for just receiving the order massages with sufficient accuracy. Equation (6) provides a clue for the manufacturer to deduce the demand, D_t, in terms of the historical data of the order process, Y_t. Here we will define the situations for whether demand is inferable or not.

Gaur *et al*. (2005) produces a definition to express how the inferred demand values can be used. When the demand process $\{D_t\}$ is inferable from $\{Y_t\}$, the manufacturer can estimate the true demand from the received order quantities information almost surely, i.e. with approximate probability 1. In other words, the retailer's order quantities, Y_t, converges to the demand realization when time *t* goes to infinity. When demand D_t is inferable, there is no need to sharing demand information between the retailer and the manufacturer. The manufacturer can estimate the demand realization by itself; otherwise, when demand D_t is non-inferable, the manufacturer can get some benefits from receiving the demand information.

Set $\mathbf{Y}_t$ to be a $p-$vector $(Y_t, 0, \dots, 0)'$ and $\mathbf{A}$ be the $p \times p$ matrix

$$\begin{pmatrix} \dfrac{\alpha_1-\alpha_2}{1+\alpha_1} & \cdots & \dfrac{\alpha_p}{1+\alpha_1} \\ & & \\ I_{p-1} & & 0_{p-1} \end{pmatrix}.$$

From (6),

$$\mathbf{D}(t) = \mathbf{A}\mathbf{D}(t-1) + \frac{1}{1+\alpha_1}\mathbf{Y}(t)$$

$$= \mathbf{A}^k\mathbf{D}(t-k) + \frac{1}{1+\alpha_1}\sum_{i=0}^{t-p}\mathbf{A}^i\mathbf{Y}(t-i)\,,$$

for $k \geq 1$ and $t = 0, \pm 1, \pm 2, \dots$. Hence, for $t \geq p$, demands D_t can be expressed in terms of the historical data of $\{Y_t\}$ and the initial data of demand $\mathbf{D}\,(p-1)$ as

$$\widehat{D}_t = \sum_{j=1}^{p} A_{(1j)}^{t-p+1} D_{p-j} + \frac{1}{1+\alpha_1}\sum_{i=0}^{t-p} A_{(11)}^{k} Y_{t-k}\,, \tag{11}$$

where $A_{(ij)}^k$ is the (ij)th element of the matrix $\mathbf{A}^k$. Here we assume the manufacturer uses the initial information to estimate $\widehat{\mathrm{D}}(p-1)$. Therefore, the manufacturer can estimate the D_t value by (10) to produce $\widehat{D}_t$. This estimated value converges to the true value, when the effect of the initial vector $\widehat{\mathrm{D}}(p-1)$ goes to zero as t tends to infinity. From the following theorem, $\widehat{D}_t$ converges to the true value almost surely.

From **THEOREM 2** in Chapter 2, for D_t being inferred from Y_t, what we choose as initial demand vector $\widehat{\mathrm{D}}(p-1)$ is negligible. Therefore, $\widehat{D}_t$ can converge to D_t almost surely as t goes to infinity. This situation only happens as the roots of the following equation are less than one in absolute value:

$$(1+\alpha_1)r^p - (\alpha_1 - \alpha_2)r^{p-1} - (\alpha_2 - \alpha_3)r^{p-2} - \cdots - \alpha_p = 0. \qquad (12)$$

If the condition is satisfied, the manufacturer can estimate the demand accurately by using the retailer's historical order data. If D_t is inferable, the manufacturer can calculate the lead time order $\sum_{k=1}^{L+1} Y_{t+k}$ by computing equations (6) and (11) recursively. The result of **THEOREM 2** implies that the manufacturer does not need the extra demand information to estimate the lead time order. The manufacturer can use the equations to calculate the same value of demand realization without demand information sharing between the retailer and the manufacturer. The noise of the estimation decreases as time goes to infinity. Thus, we can conclude that the information sharing benefits diminish as t increases if the roots of the equation (12) are less than one in absolute value.

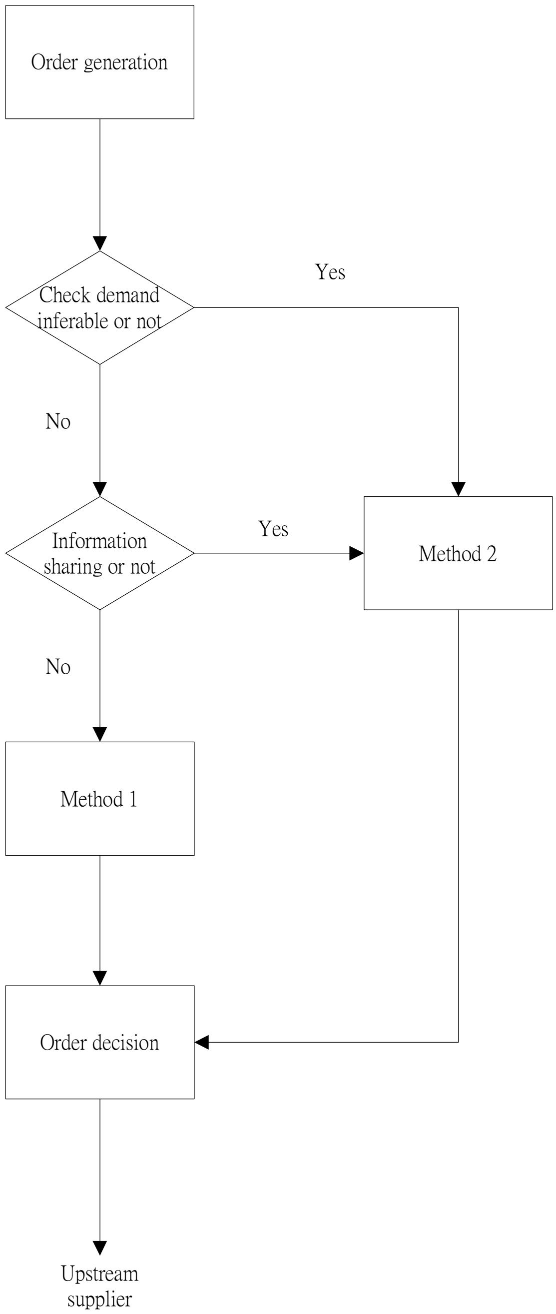


Figure III-3 Framework of Manufacturer’s action

3.3.3 Manufacturer’s Action

After the deduction, the manufacturer gets the insights from the deduction. The manufacturer first checks whether the demand is inferable or not. For ease of comparisons the results of simulation, we suppose the manufacturer uses the Gaur’s approach to execute the order decision no matter the demand process is invertible or not. Figure III-3 presents the concept of the decision flow of the manufacturer, which divides from the inferable property and the information sharing structure. Because the manufacturer knows the whole demand process in this system but not the demand realizations, he can use the mechanisms in the above section to improve his performance of forecasting.

First of all, we need to find out whether the demand can be inferred or not. To achieve this purpose, we examine by equation (12). If demand is inferable, i.e. all $|r| < 1$, even if the retailer sends no information about the demand realizations, the manufacturer can estimate this period's demand realization by the deduction and forecast the future demand as considering demand realizations. If one of the roots $|r| \geq 1$, the manufacturer cannot infer the demand realizations but he can estimate the future demand with the orders from retailer as the general way, or negotiate with the retailer for transiting the information of demand realizations. For the second case, the manufacturer can estimate the future demand as the demand realizations can be inferred.

However, after we have checked whether the demand process is inferable or not, the manufacturer has two different methods to make order decision. One is for general cases which have no information sharing and no inferred demands, and the other is for the cases which apply information sharing or inferred demand.

Method 1

The manufacturer uses this method, because he does not know the demand realizations of customers. The manufacturer forecasts the future demands by considering just the retailer's order quantities. Moreover, this method is contrary to the scenarios of using inferred demand and information sharing.

The manufacturer generates the lead time demand as the retailer does. First, the manufacturer produces the lead time order as:

$$\sum_{k=1}^{L+1} Y_{t+k} = \sum_{k=1}^{L+1} \left(\mathbf{P}^k(1:)\mathbf{Y}(t) + d \sum_{i=0}^{k-1} P_{(11)}^i + \sum_{i=0}^{k-1} P_{(11)}^i \, \xi_{t+k-i} \right).$$

For the above equation, we can simply ignore the error terms, $\sum_{i=0}^{k-1} P_{(11)}^i \, \xi_{t+k-i}$, because we use Gaur's approach and set the recent q error terms to zero. Therefore, the expected value of lead time order is:

$$\begin{aligned} M_t &= \mathrm{E}\left[\sum_{k=1}^{L+1} Y_{t+k} \,|Y_t, Y_{t-1}, \dots \right] \\ &= \sum_{k=1}^{L+1} \left(\mathbf{P}^k(1:)\mathbf{Y}(t) + \mathrm{d} \sum_{i=0}^{k-1} P_{(11)}^i \right). \end{aligned} \tag{13}$$

Then, manufacturer can compute the order-up-to level and decide the order value of this period.

<u>Method 2</u>

We use ***method 2*** as an alternative for considering information sharing and inferred demand, because the manufacturer takes demand realizations into account. When the demand is inferable or demand information has been shared, we can use ***method 2*** to estimate the lead time order. As we know the actual demands, we can use it to improve the estimation values. Here we interpret the procedure of the lead time order generation.

From equation (10), $D_t = \sum_{j=1}^{p} A_{(1j)}^{t-p+1} D_{p-j} + \frac{1}{1+\alpha_1} \sum_{i=0}^{t-p} A_{(11)}^{k} Y_{t-k}$, and from (5) the underlying order process $Y_t = (1+\alpha_1)D_t - (\alpha_1 - \alpha_2)D_{t-1} - (\alpha_2 - \alpha_3)D_{t-2} - \cdots - (\alpha_{p-1} - \alpha_p)D_{t-p+1}$. For these two equations, we derive the adjusted order equation recursively to get

$$\hat{\mathbf{Y}}(t) = \left(\hat{Y}_t, \hat{Y}_{t-1}, \dots, \hat{Y}_{t-p+1}\right)$$

$$= [(1+\alpha_1)\hat{D}_t - (\alpha_1 - \alpha_2)\hat{D}_{t-1} - (\alpha_2 - \alpha_3)\hat{D}_{t-2} - \cdots - \left(\alpha_{p-1} - \alpha_p\right)\hat{D}_{t-p+1},$$

$$(1+\alpha_1)\hat{D}_{t-1} - (\alpha_1 - \alpha_2)\hat{D}_{t-2} - (\alpha_2 - \alpha_3)\hat{D}_{t-3} - \cdots - \left(\alpha_{p-1} - \alpha_p\right)\hat{D}_{t-p},$$

$$(1+\alpha_1)\hat{D}_{t-2} - (\alpha_1 - \alpha_2)\hat{D}_{t-3} - (\alpha_2 - \alpha_3)\hat{D}_{t-4} - \cdots - \left(\alpha_{p-1} - \alpha_p\right)\hat{D}_{t-p-1}].$$

After we produce the adjusted order values, we calculate the lead time order by equation $\sum_{k=1}^{L+1}\left(\mathbf{P}^k(1:)\hat{\mathbf{Y}}(t) + d\sum_{i=0}^{k-1} P_{(11)}^{i}\right)$. Afterwards, we decide the order quantity which will be sent to the upstream-supplier.

Both methods use the same order-up-to policy which is similar to that of the retailer.

$$\mathfrak{Y}_t = Y_t + S_t - S_{t-1},$$

$$S_t = M_t + Z\sigma\sqrt{C}.$$

For both methods, Y_t is the actual order from the retailer in the order mechanism.

3.4 Bullwhip Effect Estimation

We expect the order mechanism and information sharing reduce bullwhip effect. As we know, bullwhip effect is the measure of order fluctuation relative to real demand variation. It can be easily expressed as the statistical ratio of order variance to demand variance. We can simply observe the phenomenon as the level moving up and compare the performance of model.

To compare the performance of different methods, we use BWER_r at retailer stage from Kumar (2010);

$$\mathrm{BWER}_r = \frac{\sigma_r^2}{\sigma_c^2}.$$

Here, σ_c^2 is the variance of customer demand and σ_r^2 is the variance of retailer's orders. At the manufacturer stage, BWER_m is defined by the demand variance divided by manufacturer's order variance,

$$\mathrm{BWER}_m = \frac{\sigma_m^2}{\sigma_c^2},$$

where σ_m^2 is the variance of manufacturer's orders.

We simulate the system of model and compare the performance of different scenarios in next chapter.

Chapter 4 System Simulation

4.1 Overview of Simulation

In this chapter, we first give an overview of our system simulation. Moreover, we build the different scenarios of the system consisting of forecasting approaches, demand pattern characteristics, and information sharing. We use JAVA to simulate, and we build a function to call MATLAB from JAVA. For MATLAB, we use it to solve for the roots of equations (4) and (12). For the results of forecasting methods, we first compare the performance of inferred demand for demand parameters and lead time differences. Afterwards, we investigate the reduction of bullwhip effect for comparing the scenarios of information sharing and inferred demand at following sections. In the end, we summarize the whole simulation and conclude.

4.1.1 Setting

We simulate our supply chain system by JAVA. We code the whole processes of the retailer and the manufacturer according to Figure III-1, Figure III-2, and Figure III-3 in Chapter 3. After coding the model, we select settings for simulation, make sure the simulation system works, and eventually compare the performance of the various approaches.

Table IV-1 Parameter Value of Simulation

Item	Value	Remarks
TMAX	1000	Pursue steady state and long-term result
Warm-up interval	0.2 * TMAX	Avoid noise at beginning periods
ARMA order (*p*, *q*)	(3, 3)	General structure for supply chain system
Mean demand	2000	Avoid negatives by large enough intercept
AR coefficient	[0.4, 0.3, 0.2]	Make sure for covariance stationary

Table IV-1 gives the main setting of our simulation. We set TMAX to 1000, i.e., one round of the business cycle consists of 1000 periods. Moreover, we want to find steady-state results for the model. Therefore, we simulate 1000 periods to find the long-term result. Second, to avoid the noise values for the beginning periods, we construct a warm-up interval for demand generation. It helps us observe the steady-state results. Third, we consider an ARMA (3, 3) as our demand process. Fourth, we set the mean demand to 2000 to avoid the negative values for demand realizations and order values. At last, to make sure the process is covariance stationary and demand is inferable, we set the AR coefficients to 0.4, 0.3, and 0.2, respectively.

4.1.2 Calling MATLAB from JAVA

To solve equations (4) and (12) in Chapters 2 and 3 for checking invertiblity and inferability of the demand process, we compute the equations by MATLAB. We use JAVA to call MATLAB. We first build a proxy in JAVA and transfer the equation to MATLAB, and after computing we call MATLAB to send back the results for the roots. As a result, we determine the properties of the demand process right away within JAVA.

4.2 Observations of Forecasting Method

To see the performance of our approaches, we simulate various scenarios of the system by classifying them according to the properties of the time series, the approaches of forecasting methods, and the information structure. Afterwards, we investigate the system performance measures by the performance of inferred demand and the improvement of bullwhip effect.

4.2.1 Scenarios of System

Here we divide the cases into different scenarios. Please refer to the flow chart in Figure IV-1 for the scenarios. We have three scenarios for discussing the performance of inferred demand, and nine scenarios related to the performance of bullwhip effect. In addition, these scenarios compare different standard deviations of white noise and lead times of the retailer and the manufacturer.

As to focus on the performance of inferred demand, we compare the actual demands and inferred demand under different retailer forecasting mechanisms, Zhang and Gaur. Moreover, we compare the existence of invertibility. For invertible scenarios A-1 and A-2, we discuss different approaches of Zhang and Gaur. Moreover, we discuss scenario A-3 for the non-invertible but inferable situation. In the end, we conclude the performance of inferred demand.

On the other hand, we focus on the improvement of bullwhip effect for two operation structures, with information sharing and with inferred demand. For these structures, we determine the bullwhip effect ratio and compare between actual demands, retailer's orders, and manufacturer's orders. At the beginning, we build three basic scenarios, B-1, B-2, and B-3, in which the manufacturer uses general method 1 to operate. For the rest of the scenarios, the manufacturer uses method 2 for comparison. For information sharing structure, first we construct the invertible case which retailer uses Zhang's and Gaur's approaches for scenario S-1 and S-2 respectively, and compare to the basic scenarios B-1 and B-2. Then we compare scenario S-3, which is non-invertible to scenario B-2.

Furthermore, for the structure of inferred demand, we compare the invertible property for scenario I-1 and I-2, which use Zhang or Gaur's approaches, respectively, by the retailer in the basic scenarios B-1 and B-2. Nevertheless, scenario I-3 with non-invertible property is for comparing with scenario B-3. In the end, we can measure the performance between information sharing and inferred demand structure under the same conditions to figure out a better performance.

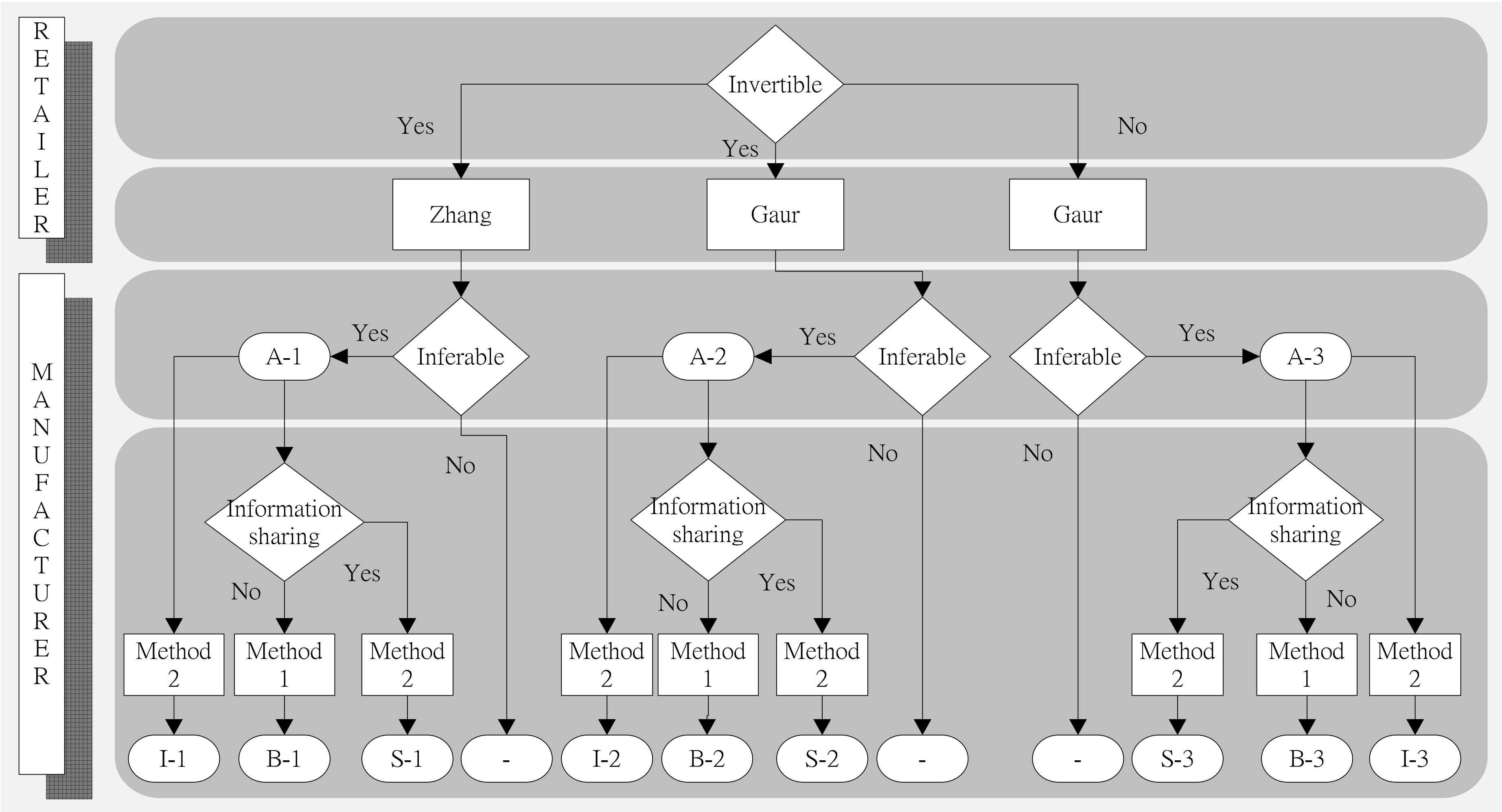


Figure IV-1 Decision Flows

For scenarios that are non-inferable, the three scenarios in Figure IV-1 with symbol "-", we recognize that the processes are not stationary. Therefore, the whole system will collapse because the system will not converge. Figure IV-2 is an instance for comparing demands, retailer's orders, and manufacturer's orders with information sharing structure, and we can observe that the system will not converge as the period going on.

Figure IV-2 Non-inferable instance (ARMA (3, 3), $l = L = 2$, $s = 50$)

4.2.2 Performance of Inferred Demand

Here we demonstrate the results of inferred demand under different scenarios, and elaborate on the remarks. Further, we compare the results between different parameters and scenarios.

For the parameters, first we discuss about the lead-time between the retailer and the manufacturer for considering different length of transportation time. Precisely, we assume the lead-time of retailer to 2 or 10 and manufacturer's lead-time to 2 or 10, as we want to have different comparisons. So we have four pairs of lead-time example, for (l, L), i.e., (2, 2), (10, 10), (2, 10), and (10, 2). Second, we discuss the standard deviation. Here we set the standard deviation to symbol s. For considering the different fluctuation of demand process, we consider s = 50 and 500. Afterwards, we examine two parameters' interactions.

Table IV-2 shows the scenarios' properties and the mechanism that the retailer uses.

Table IV-2 Properties of scenarios (A)

Scenario	Invertible	Inferable	Retailer's Mechanism
Scenario A-1	Yes	Yes	Zhang
Scenario A-2	Yes	Yes	Gaur
Scenario A-3	No	Yes	Gaur

4.2.2.1 Scenarios A-1 & A-2

For scenarios A-1 and A-2, the demand processes are both invertible, but the retailer executes the Zhang's and the Gaur's approaches, respectively. Table IV-3 shows the conditions of invertibility and inferability that are examined by MATLAB. Both scenarios use the same ARMA process parameters, and the roots' absolute values are all smaller than one. Therefore, the ARMA process is invertible and the demand is inferable.

Table IV-3 Examination of properties (A-1 & A-2)

MA Coefficient	Roots	Alpha Values	Roots
0.15	0.174	4.926	0.884
0.25	0.174	-1.681	-0.272
0.21	-0.497	-1.320	-0.272
	Invertible	-0.924	Inferable

A-1

Scenario A-1 presents the scenario that demand process is invertible and the retailer uses Zhang's approach to forecast. This scenario examines the performance of inferred demand by the manufacturer. Here we only show the results for lead-time (2, 2) and (10, 10), because the performance of (2, 10) are similar to (2, 2) and (10, 2) are similar to (10, 10).

For s = 50, we can show the following instance for convergence performance of inferred demand:

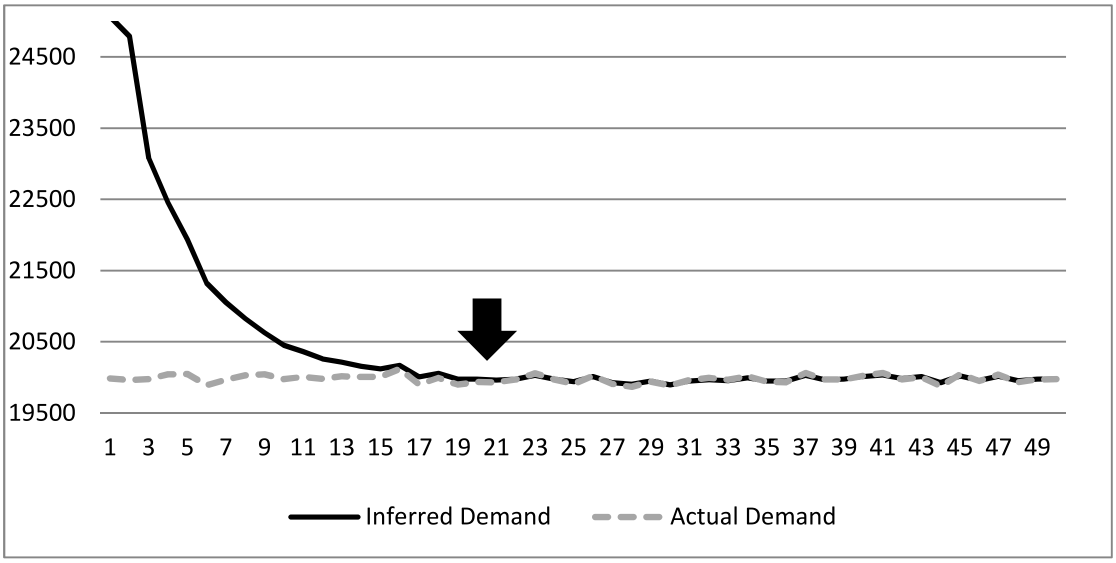


Figure IV-3 Convergence Performance (ARMA (3, 3), $l = L = 2$, $s = 50$) (A-1)

For ease of observing the results, we extract the first 50 periods for lead-time = 2 in Figure IV-3, and the first 80 periods for lead-time = 10 in Figure IV-4. Figure IV-3 shows the convergence, where the x-axis, is the periods of system, and y-axis is the demand quantities. The black arrow is the indicator of the convergence point. As shown in Figure IV-3, initial estimation is overestimated. The performance of inferred demand becomes better gradually and almost converges at about the 20th period.

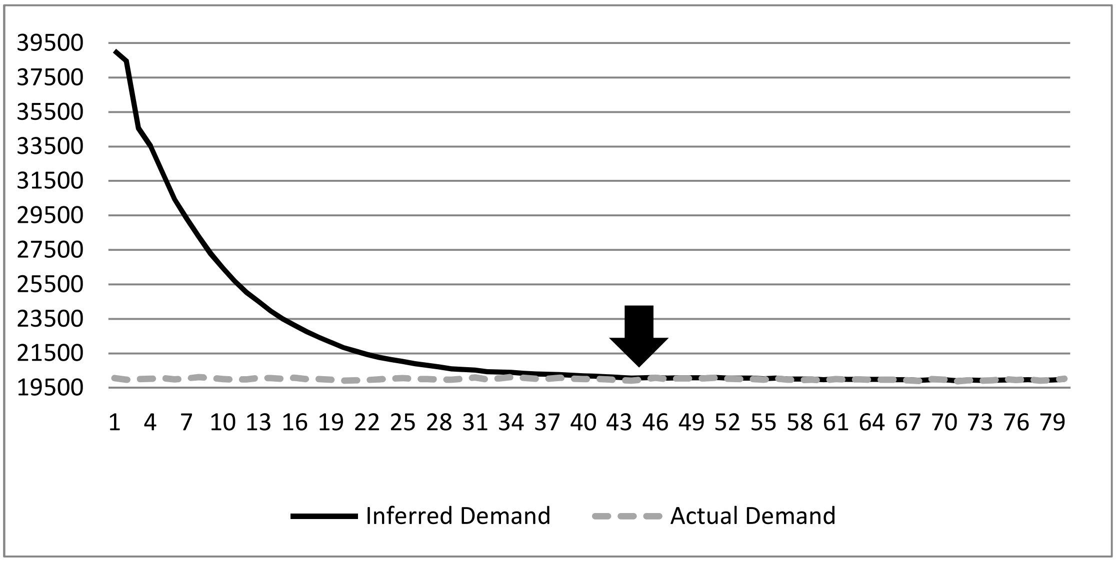


Figure IV-4 Convergence Performance (ARMA (3, 3), $l = L = 10$, $s = 50$) (A-1)

For retailer's lead-time equal to ten, as shown in Figure IV-4, the convergence performance gets worse than the instance in Figure IV-3. The inferred demands converges at about the 40th period.

For $s = 500$, we can follow the following instances for convergence performance of inferred demand:

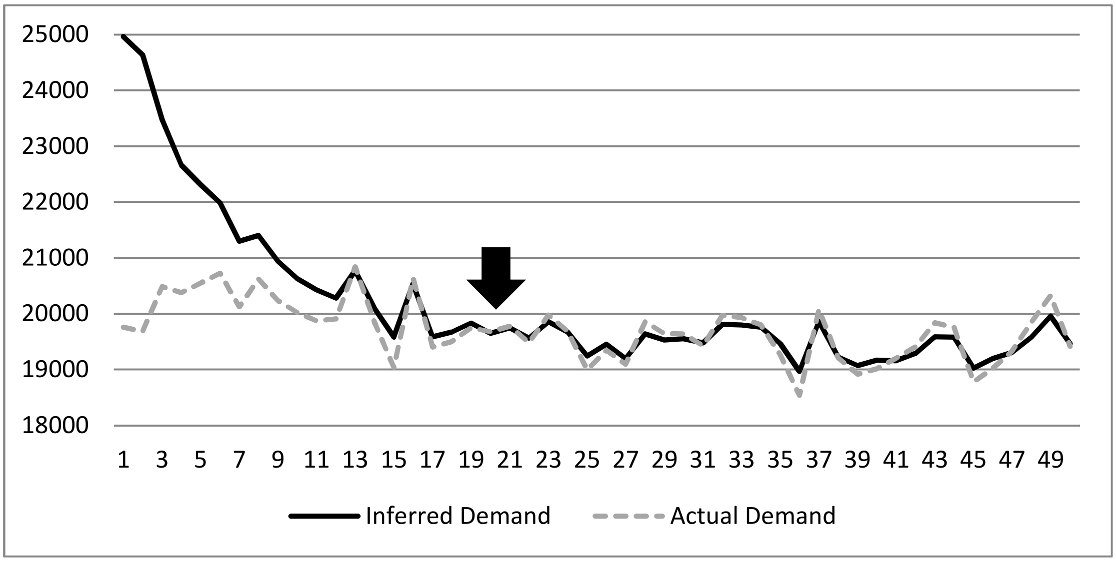


Figure IV-5 Convergence Performance (ARMA (3, 3), $l = L = 2$, $s = 500$) (A-1)

In Figure IV-5, we can observe that the fluctuation is bigger than the instances of smaller standard deviation shown above. We observe that the inferred and actual demands converge at about the 20th period, and the initial values of inferred demand are much bigger than the actual demands.

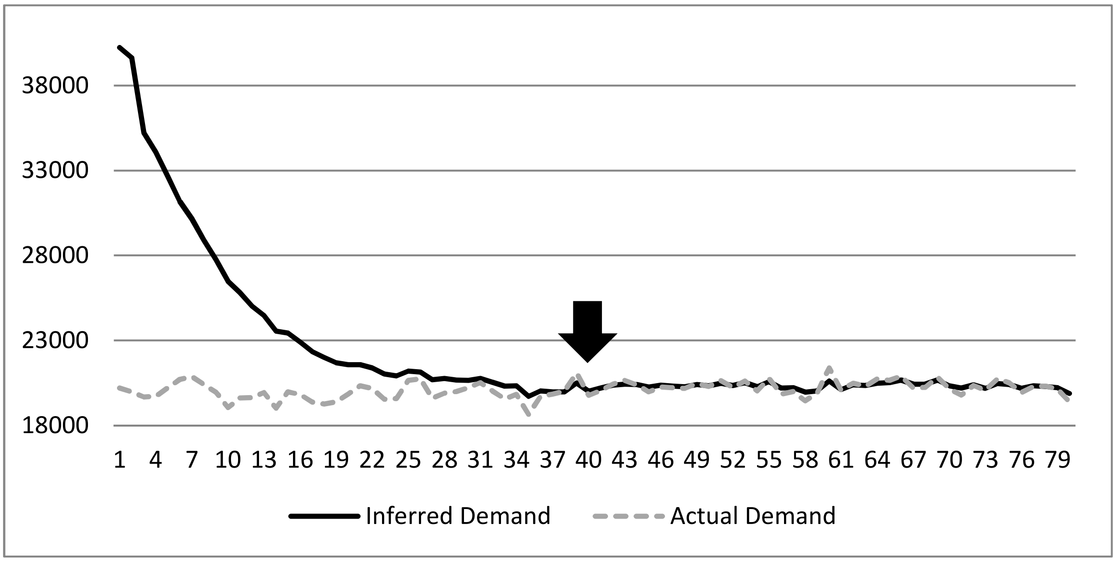


Figure IV-6 Convergence Performance (ARMA (3, 3), $l = L = 10$, $s = 500$) (A-1)

Figure IV-6 shows that the fluctuation is bigger than Figure IV-4, because the standard deviation is bigger too. Same as Figure IV-4, we observe that the convergence point is at about the 40th period.

A-2

Scenario A-2 presents the scenarios that the demand process is also invertible but the retailer uses Gaur's approach to forecast. This scenario examines the performance of inferred demand by the manufacturer. Figure IV-7, Figure IV-8, Figure IV-9, and Figure IV-10 below only show the convergence performance of lead-time (2, 2) and (10, 10), and measure with different standard deviations.

For s = 50, we observe the following instances for convergence performance of inferred demand:

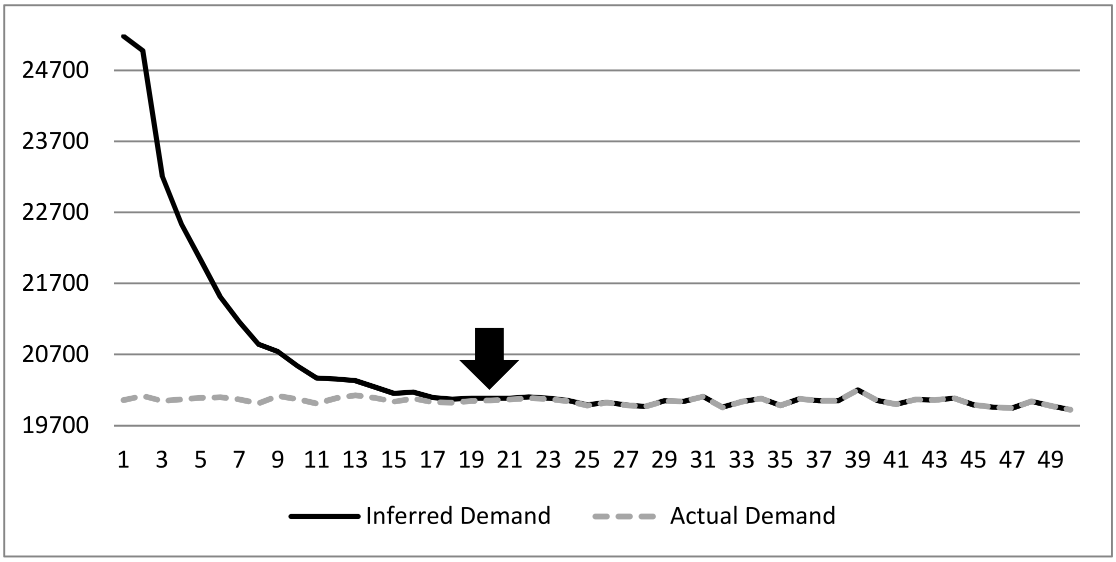


Figure IV-7 Convergence Performance (ARMA (3, 3), $l = L = 2$, s = 50) (A-2)

As shown in Figure IV-7, the convergence performance gradually turns good when the retailer's lead-time equals to two. The inferred and actual demands converge at about the 20th period. The initial estimated values are still much larger than the actual demands.

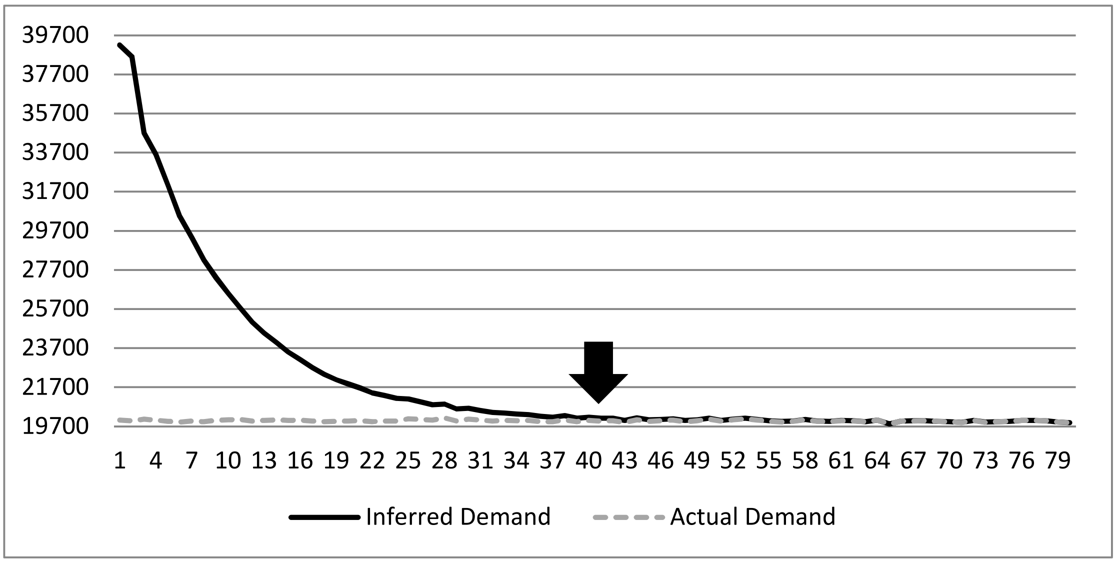


Figure IV-8 Convergence Performance (ARMA (3, 3), $l = L = 10$, $s = 50$) (A-2)

For retailer's lead-time equals to ten, the convergence performance gets worse when the lead-time equals to two. The inferred demands converge at about the 40th period.

For s = 500, we can see following instances for convergence performance of inferred demand:

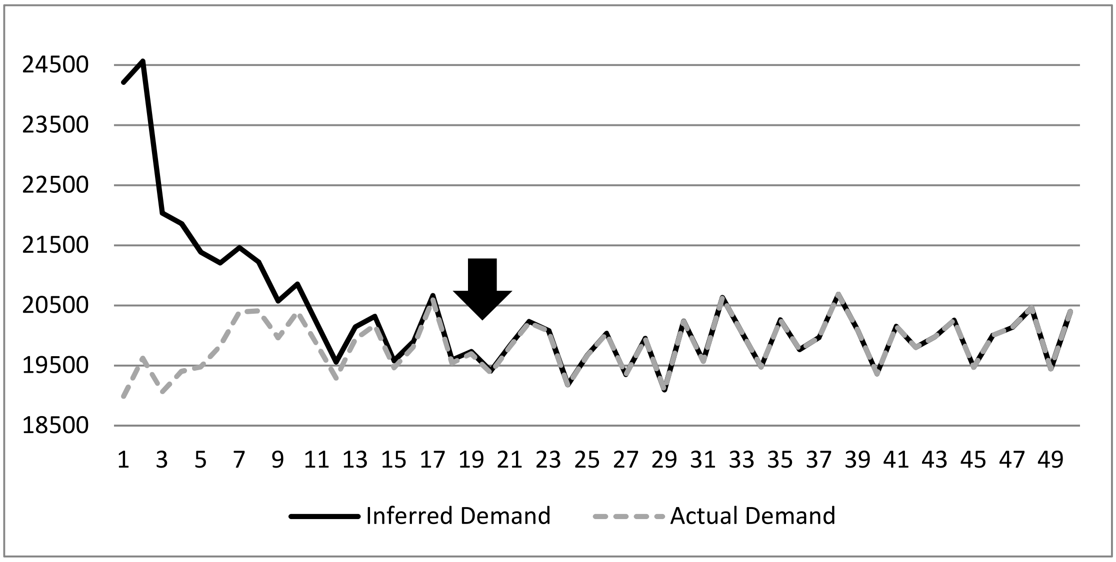


Figure IV-9 Convergence Performance (ARMA (3, 3), $l = L = 2$, $s = 500$) (A-2)

Figure IV-9 shows dramatic fluctuation when the standard deviation is 500. The convergence performance is as well as scenario A-1, and convergences at about the 20th period.

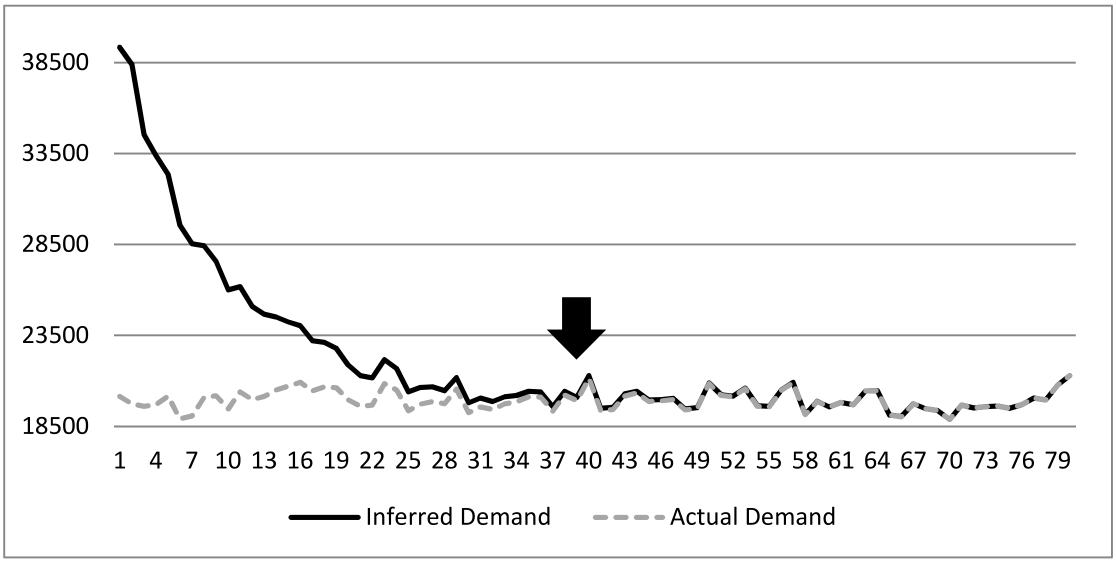


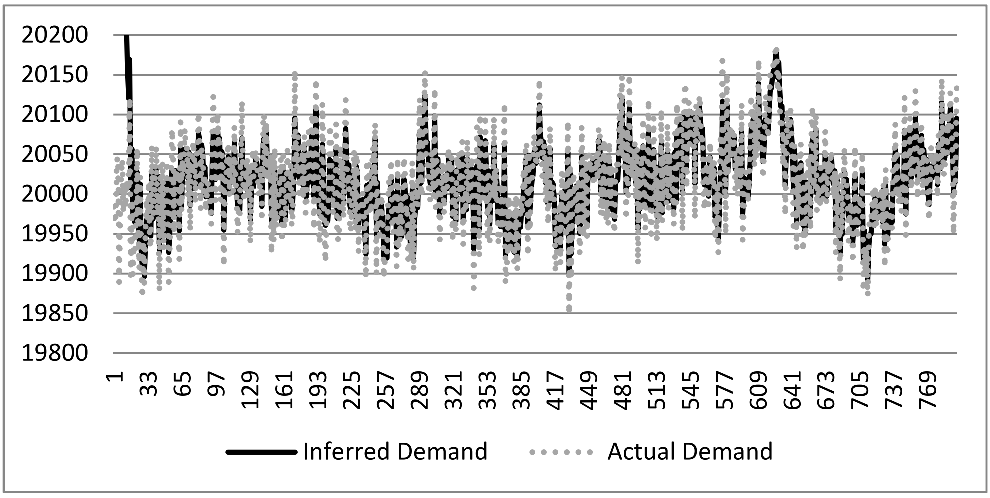
Figure IV-10 Convergence Performance (ARMA (3, 3), $l = L = 10$, $s = 500$) (A-2)

Figure IV-10 shows the same result that the convergence point is at about the 40^{th} period when the retailer's lead-time equals to ten.

Summary

From the results of convergence performance, the inferred demand structure performs well for both Zhang's and Gaur's approaches. In the next figure, we try to figure out the difference between the Gaur's and Zhang's approach.

(a) Ttrend performance (ARMA(3,3), Leadtime $l = L = 2$, $s = 50$, Zhang's approach)



(b) Ttrend performance (ARMA(3,3), Leadtime $l = L = 2$, $s = 50$, Gaur's approach)

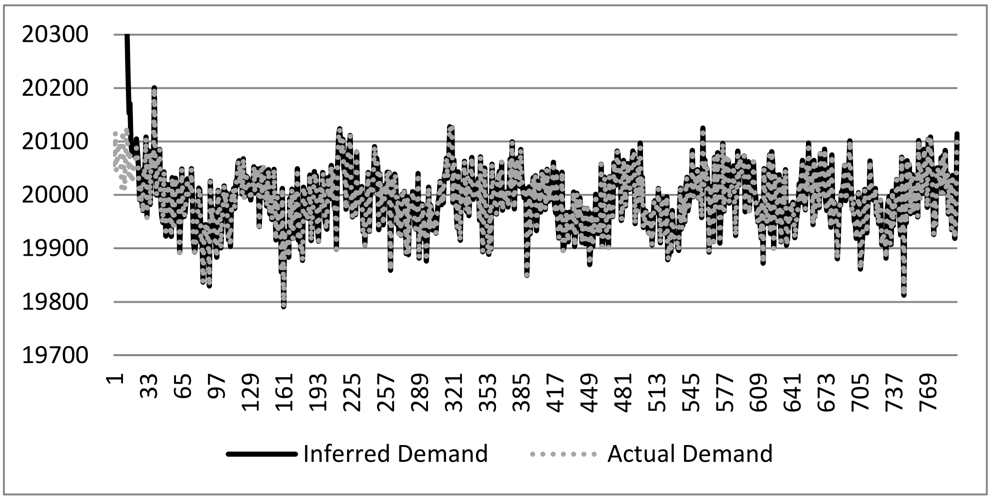


Figure IV-11 Comparison of approaches

Figure IV-11 shows the trend performance of two different approaches where the lead-times are both two and standard deviations are both fifty. The upper figure (a) has lower variance for inferred demand but higher discrepancy between actual demands; the below figure (b) has higher variance for inferred demand but lower discrepancy between actual demands. For the rest of the experiments, we can observe the same result that we mentioned, so we make a table to elaborate on these results.

Table IV-4 Performance of inferred demand (A-1 & A-2)

Scenario	*s*	Lead Time (R, M)	Inferred Demand Std.	Actual Demand Std.	Average Discrepancy
A-1	50	**(2, 2)**	45.602	57.358	0.046
		(10, 10)	33.706	53.051	0.362
		(2, 10)	40.531	53.438	0.083
		(10, 2)	40.656	58.178	-0.120
	500	**(2, 2)**	476.220	591.937	0.112
		(10, 10)	359.968	555.620	-28.745
		(2, 10)	426.722	550.473	-0.246
		(10, 2)	627.787	1142.735	3.104
A-2	50	**(2, 2)**	53.723	53.723	< -0.001
		(10, 10)	56.653	56.654	0.004
		(2, 10)	56.034	56.034	< 0.01
		(10, 2)	53.116	53.163	0.057
	500	**(2, 2)**	555.649	555.649	< 0.001
		(10, 10)	585.946	585.945	< 0.001
		(2, 10)	508.258	508.258	< 0.001
		(10, 2)	531.377	531.378	0.001

Table IV-4 shows the standard deviations of actual demands and inferred demands and average discrepancy between actual demands and inferred demands in all parameters we examined in scenario A-1 and A-2. Clearly for scenario A-1 the inferred demands' standard deviations are smaller than the actual demands', and for scenario A-2 the inferred demands' standard deviations are bigger than A-1 but almost the same as that of the actual demands. Moreover, scenario A-2's average discrepancy is much smaller than A-1's. Here we can conclude that if the retailer uses Gaur's approach to forecast, the manufacturer can infer the actual demands more accurately.

4.2.2.2 Scenarios A-3

Scenario A-3 presents the scenarios that the demand process is non-invertible but still inferable, and the retailer uses Gaur's approach to forecast. Table IV-5 shows the conditions of invertibility and inferability as established by MATLAB. We observe there is one root of testing invertibility which absolute values is bigger than one, but the alpha value's absolute values are still smaller than one. Therefore, the ARMA process is non-invertible but the demand is still inferable.

Table IV-5 Examination of properties (A-3)

MA Coefficient	Roots	Alpha Values	Roots
0.65	-1.187	2.364	0. 572
0.55	0.269	-0.526	-0.267
1.41	0.269	-0.466	-0.267
	Non-Invertible	-0.372	Inferable

Here we present convergence performance of scenario A-3. For *s* = 50, we can follow these instances for convergence performance of inferred demand:

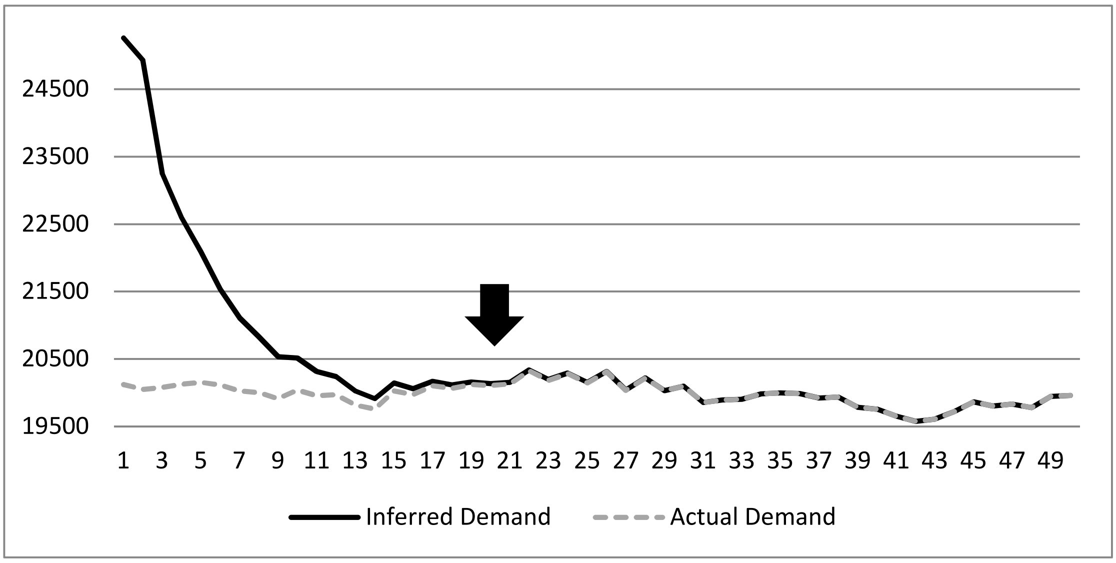


Figure IV-12 Convergence Performance (ARMA (3, 3), $l = L = 2$, $s = 50$) (A-3)

For non-invertible situation, Figure IV-12 shows the convergence performance when lead-time equals to two. We observe the convergence point is at about the 20th period.

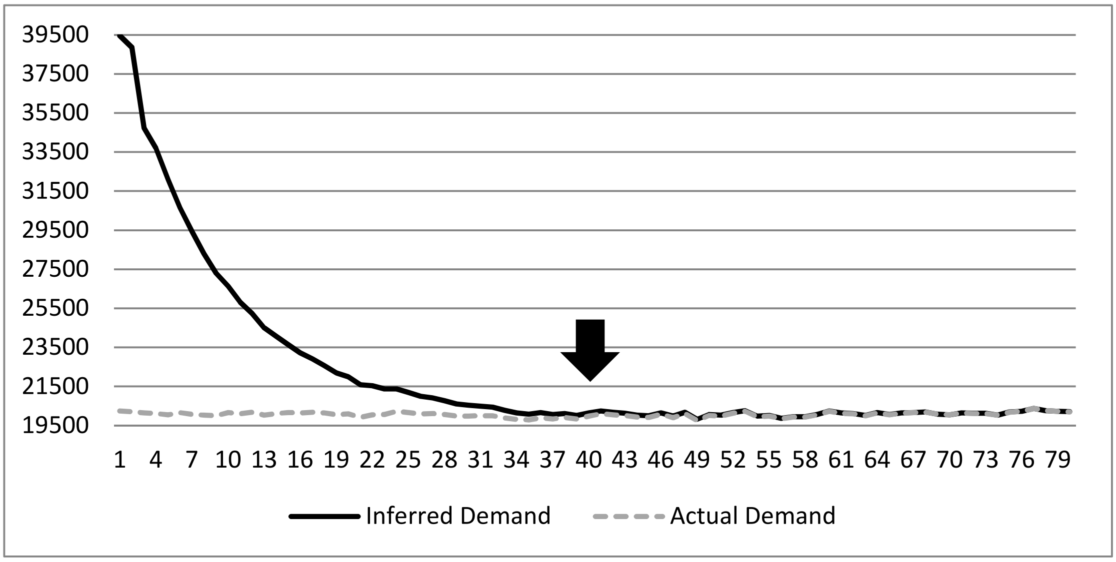


Figure IV-13 Convergence Performance (ARMA (3, 3), $l = L = 10$, $s = 50$) (A-3)

Figure IV-13 shows the convergence performance when lead-time equals to ten. We observe the convergence point is at about the 40th period.

For the instances of s = 500 for convergence performance of inferred demand:

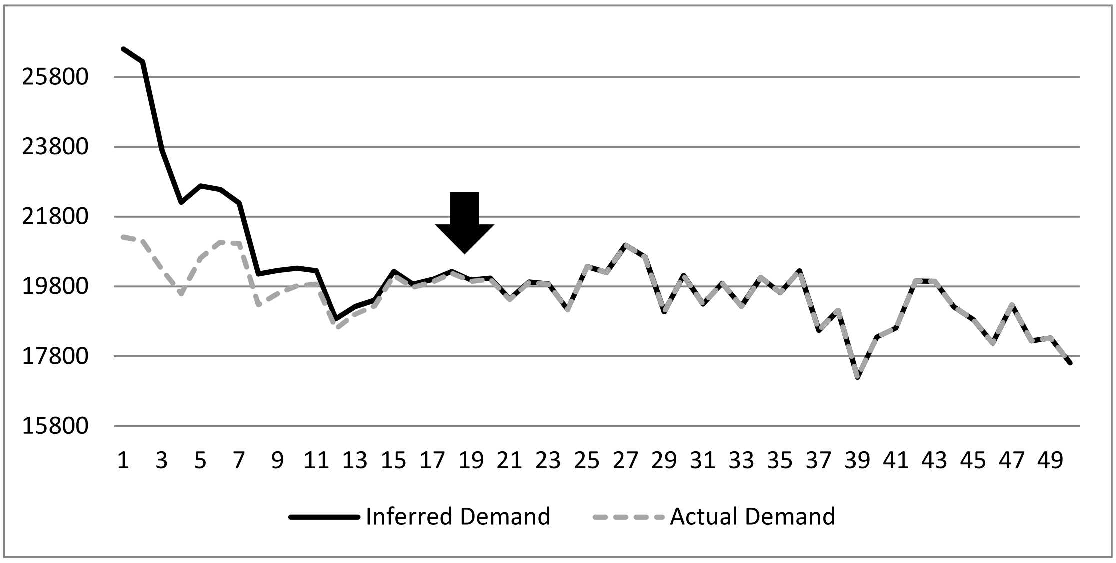


Figure IV-14 Convergence Performance (ARMA (3, 3), $l = L = 2$, $s = 500$) (A-3)

For larger standard deviation, Figure IV-14 shows the convergence performance when lead-time equals to two. The fluctuation is bigger, and the convergence point is at about the 20th period.

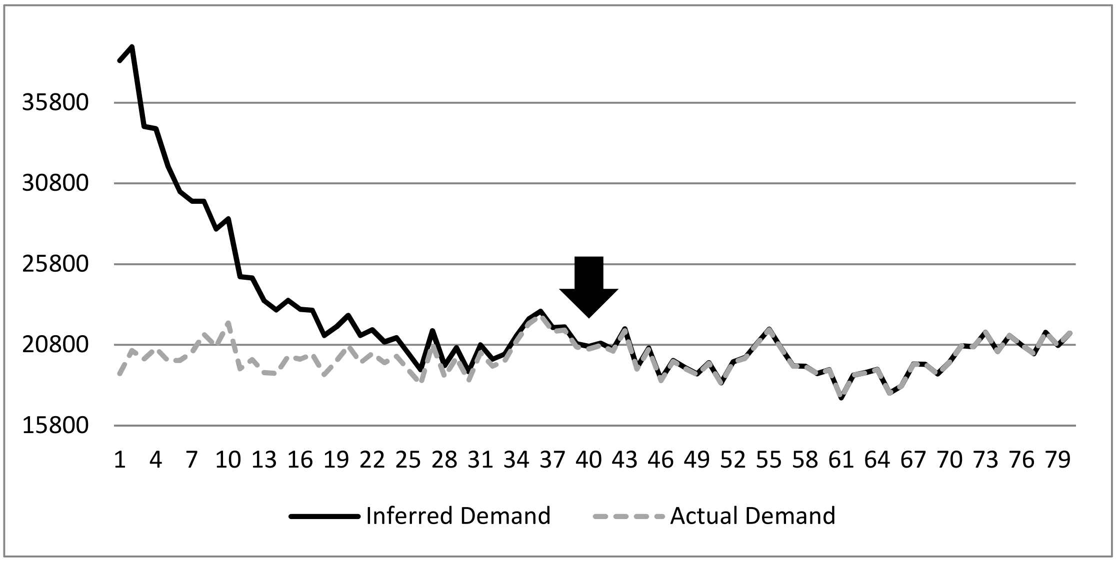


Figure IV-15 Convergence Performance (ARMA (3, 3), $l = L = 10$, $s = 500$) (A-3)

Figure IV-15 shows the convergence performance when lead-time equals to ten. The convergence point is also at about the 40th period.

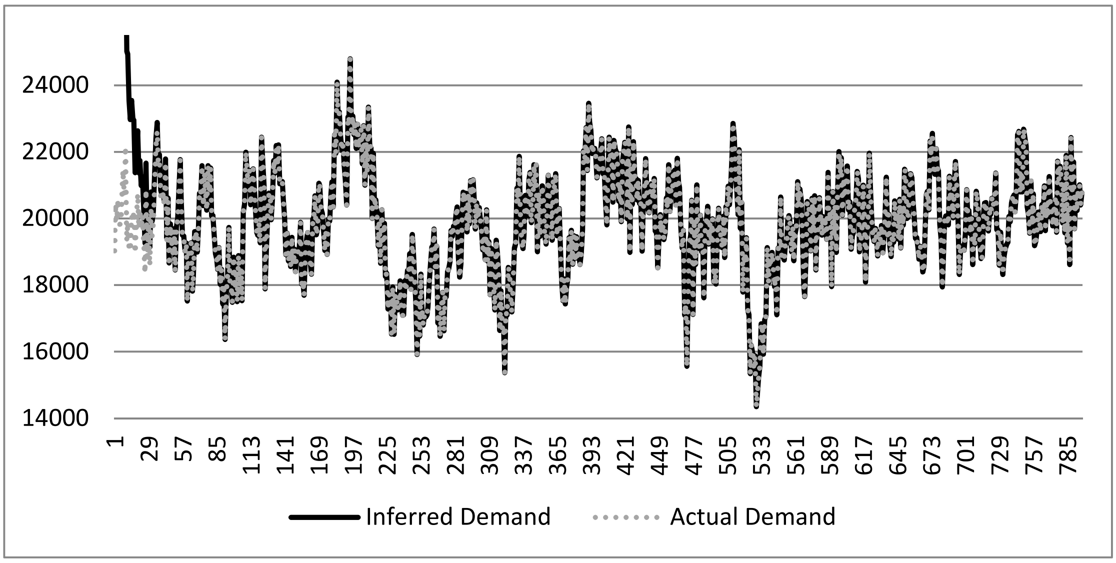


Figure IV-16 Trend Performance (ARMA (3, 3), $l = L = 10$, $s = 500$) (A-3)

Figure IV-16 presents the trend performance of the whole runtime. We pick the most fluctuating instances of scenario A-3 for showing the inferred demand performance. The inferred demand is almost the same as the actual demands, and it appears in every instance in scenario A-3.

Table IV-6 Performance of inferred demand (A-3)

Scenario	*s*	Lead Time (R, M)	Inferred Demand Std.	Actual Demand Std.	Average Discrepancy
A-3	50	**(2, 2)**	149.957	150.224	< 0.001
		(10, 10)	144.447	144.44	0.001
		(2, 10)	154.416	154.460	< 0.001
		(10, 2)	153.803	153.804	0.001
	500	**(2, 2)**	1486.012	1486.012	< 0.001
		(10, 10)	1541.295	1541.297	0.001
		(2, 10)	1436.057	1436.057	< 0.001
		(10, 2)	1416.247	1416.250	0.001

Table IV-6 shows the convergence performance of all instances in A-3. The inferred demand's standard deviations almost equal to actual demands' standard deviations. Moreover, the average discrepancies are negligible, and it means that the inferred demand structure's performance is great.

4.2.2.3 Sum-up for Inferred Demand

For inferred demand, the manufacturer can easily deduce the actual demands no matter what approach the retailer uses, Zhang's or Gaur's. In comparing the performance of scenario A-1 and A-2, the manufacturer can infer the demand realizations better if the retailer uses the Gaur's approach than the Zhang's approach. We observe that the lead-time of the manufacturer has no influence on the inferred demand performance. Moreover, we find out the demand process's standard deviation just influence the degree of fluctuation but not the performance of the inferred demand. Further, by comparing scenario A-1, A-2, and A-3, we figure out the inferred performance is good whether demand process is invertible or not.

4.2.3 Improvement of Bullwhip Effect

For observing the improvement of bullwhip effect, we simply consider two cases of a supply chain system, inforamtion sharing and inferred demand. We find out the length of the manufacturer's lead time does not affect much of the performance of system. So we just compare the scenarios with standard deviation, s = 50 or 500, and lead-time, l = L = 2 or 10, mutually in this section. Besides, the conditions of invertibility and inferability are the same as Table IV-3 and Table IV-5.

Before comparing, we first construct the basic scenarios for B-1, B-2, and B-3 as

Table IV-7.

Table IV-7 Properties of scenarios (B)

Scenario	Invertible	Inferable	Information Sharing	Forecast mechanism
Scenario B-1	Yes	Yes	No	Zhang
Scenario B-2	Yes	Yes	No	Gaur
Scenario B-3	No	Yes	No	Gaur

In the following, we build the basic scenarios for B-1, B-2, and B-3 as Table IV-7, and make the plot and table for comparing bullwhip effect with the following instances of information sharing and inferred demand.

Table IV-8 Performance of scenarios (B)

Scenario	s	Lead Time (R, M)	Demand Std.	rOrder Std.	mOrder Std.	$BWER_r$	$BWER_m$
B-1	**50**	**(2, 2)**	58.159	82.135	189.387	1.402	3.432
		(10, 10)	56.720	111.440	577.814	1.965	10.187
	500	**(2, 2)**	555.327	781.680	1884.680	1.408	3.394
		(10, 10)	579.786	1119.500	5879.180	1.931	10.140
B-2	**50**	**(2, 2)**	55.959	122.530	317.486	2.190	5.674
		(10, 10)	57.855	258.690	1481.850	4.471	25.613
	500	**(2, 2)**	556.545	1360.500	3195.680	2.445	5.742
		(10, 10)	522.059	3031.200	15350.300	5.806	29.403
B-3	**50**	**(2, 2)**	135.178	227.890	554.365	1.686	4.101
		(10, 10)	151.093	492.990	2690.980	3.263	17.810
	500	**(2, 2)**	1525.082	2560.000	5910.320	1.679	3.875
		(10, 10)	1441.445	4764.200	26083.700	3.305	18.096

Table IV-8 shows the partial performance of basic scenarios. We present the standard deviations of actual demands, the retailer's orders, and the manufacturer's orders. Moreover, we present the bullwhip effect. We can observe that $BWER_m$ of scenario B-2 is greater than that of scenario B-1. Moreover, when the lead-time is 10 and standard deviation is 500, $BWER_m$ is over two times of scenario B-1. After observing B-1, B-2, and B-3, we find out Gaur's approach results in a bigger $BWER_m$ than Zhang's approach.

4.2.3.1 Information sharing

Table IV-9 Properties of scenarios (S)

Scenario	Invertible	Inferable	Forecast mechanism
Scenario S-1	Yes	No	Zhang
Scenario S-2	Yes	No	Gaur
Scenario S-3	No	No	Gaur

Table IV-9 shows the properties of scenarios of information sharing. Here we perform system simulation to compare with the basic scenarios, and we want to see whether information sharing is useful or not.

In this section, we first simulate the system in different scenarios. Second, we compare the performance between the scenarios with basic scenarios. Table IV-11 shows the performance of partial system's performance, which we present the standard deviations of the actual demands, the retailer's orders, and the manufacturer's orders. Further, we present the bullwhip effect, $BWER_r$ and $BWER_m$.

Table IV-11 Performance of scenarios (S)

Scenario	s	Lead Time (R, M)	Demand Std.	rOrder Std.	mOrder Std.	$BWER_r$	$BWER_m$
S-1	50	(2, 2)	57.025	78.978	280.076	1.385	4.911
		(10, 10)	55.125	107.121	1500.740	1.889	26.459
	500	(2, 2)	569.811	794.962	2837.784	1.348	4.811
		(10, 10)	531.821	1014.820	14374.053	1.677	23.755
S-2	50	(2, 2)	56.711	122.883	315.225	2.167	5.558
		(10, 10)	54.967	257.357	1649.067	4.549	29.146
	500	(2, 2)	545.317	1181.437	3245.570	2.123	5.832
		(10, 10)	570.733	2589.260	16373.762	4.537	28.689
S-3	50	(2, 2)	143.165	242.138	561.881	1.531	3.552
		(10, 10)	152.872	487.051	2655.166	3.186	17.369
	500	(2, 2)	1366.995	2395.187	5605.441	1.360	3.182
		(10, 10)	1529.964	4787.198	28052.948	3.018	17.688

Table IV-11 shows the several insights. First, $BWER_r$ is smaller when the retailer uses Zhang's approach in scenario S-1, and $BWER_r$ is bigger when the retailer uses Gaur's approach in scenario S-2. In scenario S-3, $BWER_r$ is bigger when lead-time is ten than lead-time is two. $BWER_m$ is bigger when the retailer uses Gaur's approach than the retailer uses Zhang's approach in invertible scenarios, S-1 and S-2. The $BWER_m$ in S-3 is smaller when demand process is non-invertible.

Table IV-12 Comparison of bullwhip effect (S)

	Bullwhip effect performance ratio			
	s = 50		s = 500	
Scenario	(2, 2)	(10, 10)	(2, 2)	(10, 10)
S-1	150.02%	259.73%	150.57%	244.49%
S-2	99.00%	106.41%	101.56%	97.57%
S-3	97.31%	105.56%	102.00%	109.53%

Table IV-12 compares bullwhip effect between basic scenarios and information sharing scenarios. To compare the performance, we divide $BWER_m$ in information sharing by the $BWER_m$ in basic scenarios, and we present in percentage. In Table IV-12, the results of scenario S-1 is worse than basic scenario. The value of $BWER_m$ is 1.5 times larger when lead-time is two and 2.5 times larger when lead-time is 10. In scenario S-2, the bullwhip effect is alleviated when s = 50 and lead-time is 2 and when s = 500 and lead-time is 10. For the rest of the results in S-2, the bullwhip effect is slightly bigger than basic instances. In scenario S-3, the bullwhip effect is mitigated. It only appears when demand process's standard deviation is 50 and lead-time is two. For the rest of the results in S-3, the bullwhip effect is slightly larger than the basic instances.

4.2.3.2 Inferred demand

Table IV-13 Properties of scenarios (I)

Scenario	Invertible	Information sharing	Forecast mechanism
Scenario I-1	Yes	No	Zhang
Scenario I-2	Yes	No	Gaur
Scenario I-3	No	No	Gaur

Table IV-13 shows us the properties of the scenario of inferred demand. Here we perform system simulation to compare with the basic scenarios, and we want to see whether information sharing structure is useful or not.

Table IV-14 Performance of scenarios (I)

Scenario	s	Lead Time (R, M)	Demand Std.	rOrder Std.	mOrder Std.	$BWER_r$	$BWER_m$
I-1	50	(2, 2)	52.382	75.724	199.206	1.372	3.610
		(10, 10)	57.348	108.917	665.739	1.899	11.609
	500	(2, 2)	531.306	759.196	1983.716	1.287	3.363
		(10, 10)	585.646	1052.681	6111.963	1.740	10.101
I-2	50	(2, 2)	58.135	124.343	327.664	2.139	5.636
		(10, 10)	54.271	251.523	1639.109	4.635	30.202
	500	(2, 2)	546.238	1246.876	3268.030	2.240	5.872
		(10, 10)	594.527	2572.787	16925.239	4.928	28.468
I-3	50	(2, 2)	150.240	249.433	584.174	1.577	3.693
		(10, 10)	159.231	457.103	2562.140	2.871	16.091
	500	(2, 2)	1570.392	2477.666	5584.244	1.407	3.170
		(10, 10)	1445.428	4611.922	25984.358	2.908	16.383

Table IV-14 shows the several insights which are similar to those in scenarios of information sharing. The value of $BWER_r$ is smaller when the retailer uses Zhang's approach in scenario I-1, and $BWER_r$ is bigger when the retailer uses Gaur's approach in scenario I-2. In scenario I-3, $BWER_r$ is bigger when lead-time is ten than lead-time is two. $BWER_m$ is bigger when the retailer uses Gaur's approach than the retailer uses Zhang's approach in invertible scenarios, I-1 and I-2. $BWER_m$ in I-3 is smaller when demand process is non-invertible.

Table IV-15 Comparison of bullwhip effect (I)

	Bullwhip effect performance ratio			
	s = 50		s = 500	
Scenario	(2, 2)	(10, 10)	(2, 2)	(10, 10)
I-1	110.27%	113.96%	105.25%	103.96%
I-2	100.38%	109.66%	102.26%	96.82%
I-3	97.93%	109.32%	98.21%	108.95%

Table IV-15 compares bullwhip effect between basic scenarios and inferred demand scenarios. In Table IV-15, the result of scenario I-1 is also worse than basic scenario but better than the scenario S-1. The BWERm is slightly larger than basic instances in I-1. In scenario I-2, the bullwhip effect is alleviated when demand process's standard deviation is 500 and lead-time is ten. For the rest of the results in I-2, the bullwhip effect is slightly bigger than basic instances, and the instances performs almost the same as when demand process's standard deviation is 50 and lead-time is two. In scenario I-3, the bullwhip effect is mitigated when lead-time is two and no matter the demand process's standard deviation is 50 or 500. For the rest of the results in I-3, the bullwhip effect performance is slightly larger than basic instances.

4.3 Summary

Inferred demands are useful in all scenarios. The performance is better when the retailer uses Gaur's approach in making their order decision no matter whether the demand process is invertible or non-invertible.

As for the bullwhip effect, our results show that some instances are useful, but some of the instances are not. For scenarios that use Zhang's approach in both information sharing and inferred demand situations, the inaccurate forecasting mechanism can mislead the signal of actual demands, and finally influence the order decision of other members. For Gaur's approach in those situations, the bullwhip effect is mitigated in some instances. For the rest of the instances, bullwhip effect is almost the same as the basic scenarios.

Although the bullwhip effect reduction is not very large for our instances, we find out other merits of our model. Table IV-16 shows the deviation of the on hand inventory of the manufacturer. We divide the results into three pieces, i.e., (B-1, S-1, and I-1), (B-2, S-2, and I-2), and (B-3, S-3, and I-3) to compare the scenarios under the same assumptions. The inventory's standard deviation is smaller under our order mechanism except for scenario S-1. For the lower inventory standard deviation, it means that the variability of inventory is lower and can reduce the inventory cost of storage and shortage cost.

Table IV-16 Comparison of inventory standard deviation

s	Lead time (R, M)	Scenario	mInventory std.	Scenario	mInventory std.	Scenario	mInventory std.
50	(2, 2)	B-1	179.489	S-1	205.393	I-1	163.065
50	(10, 10)	B-1	662.016	S-1	1057.666	I-1	593.693
500	(2, 2)	B-1	1870.628	S-1	2093.230	I-1	1632.182
500	(10, 10)	B-1	6051.820	S-1	9959.864	I-1	5802.649
50	(2, 2)	B-2	320.196	S-2	285.853	I-2	286.599
50	(10, 10)	B-2	1420.706	S-2	1216.655	I-2	1193.810
500	(2, 2)	B-2	3117.727	S-2	2658.954	I-2	2932.657
500	(10, 10)	B-2	13149.052	S-2	12458.623	I-2	12487.739
50	(2, 2)	B-3	611.857	S-3	559.266	I-3	570.436
50	(10, 10)	B-3	3045.765	S-3	2564.357	I-3	2458.878
500	(2, 2)	B-3	6238.376	S-3	5594.595	I-3	5605.615
500	(10, 10)	B-3	30014.929	S-3	24635.554	I-3	24567.049

Chapter 5 Conclusions

For mitigating the bullwhip effect, we first construct an ARMA demand process to represent general demand structure. Second, we operate the different forecasting mechanisms, Zhang's and Gaur's, for the retailer under the ARMA process. The forecast mechanism affects the order fluctuation of the manufacturer. Third, with better knowledge about future demand, the manufacturer needs more information about the actual demand. We provide three kinds of different structures for the manufacturer to compare, namely, basic, information sharing, and inferred demand structure. Fourth, under these structures, the manufacturer and the retailer both use the adjusted order-up-to policy to maintain their inventory level. In the end, we compare the performances of bullwhip effect among different scenarios and investigate the phenomena.

We conclude with forecasting mechanism comparison, and the performance of bullwhip effect in different structures. We also state our thesis scope and direction for further research.

Forecasting mechanism comparison

With a fluctuating demand process, we build an adjusted order-up-to policy with two different forecasting mechanisms. Zhang's approach considers the past error terms into estimated lead time demand. For this reason, it sets the first q error terms to zero because of insufficient information about error terms. On the other hand, Gaur's approach ignores the error terms in the estimated lead time demand. It assumes the recent q error terms to be zero because of independent characteristic of error terms. For comparison, we refer to the results of demand inferring. With the influence to retailer, Table V-1 shows the differences among these mechanisms. To retailer, Zhang‘s approach has better performance on order variability and inventory control than Gaur’s approach.

Table V-1 Performance of forecasting mechanisms

s	lead time (*r*, *m*)	Zhang		Gaur	
		*r*Order std.	*r*Inventory std.	*r*Order std.	*r*Inventory std.
50	(2, 2)	77.381	104.2711	122.5296	126.5582
	(10, 10)	111.436	310.7076	266.8586	350.6976
500	(2, 2)	781.680	1092.747	1360.492	1241.761
	(10, 10)	1119.494	3033.257	3031.183	3073.953

With the influence to manufacturer, Chapter 4 explains the pros and cons. In short, we know that if the retailer uses Gaur’s approach, the manufacturer may be easier to infer the actual demands. Moreover, with better demand information, the manufacturer can forecast more accurately.

Performance of bullwhip effect

From Chapter 4, we can observe bullwhip effect in different structures. We provide a basic structure to be a basis for comparing information sharing and inferred demand structures. For the information sharing structure, we find out when the lead time and the demand pattern’s fluctuation is small, the bullwhip effect can be mitigated. The rest of the cases are not that good as the previous cases. For the inferred demand structure, the bullwhip effect is alleviated when the demand process is non-invertible and lead-time is small. The rest of the cases are unstable and do not mitigate bullwhip effect.

Further research

In our thesis's setting, there are many factors to consider for inclusions. In our thesis, service level plays a negligible role in the model. We can reconstruct the model and compare the system performance of different service levels. For the demand pattern, we have at least two things to consider. One is the ARMA order. We can simulate higher order ARMA processes and observe the influence of the orders. Second one is a more general demand process. We know that there is many demand processes more general than ARMA processes. Refer to Chapter 2. We can use ARIMA and MMFE processes to build a model more relevant to the real world.

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