# Unified and Contrasting Graphical Lasso for Brain Network Discovery

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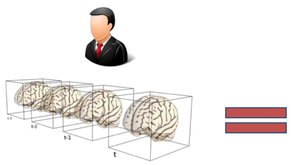
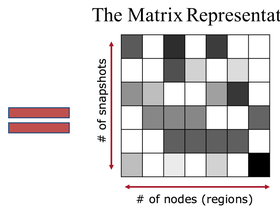
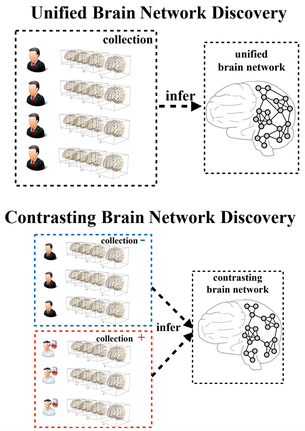
## 1. The Problem



The fMRI data




1. Unified brain network discovery aims at finding the most representative network for a group of subjects.
2. Contrasting brain network discovery aims at find the most discriminative network for two groups of subjects.

## 2. UGLasso and CGLasso

**Very likely to be singular**

$$\mathbf{S} = \frac{1}{n}\mathbf{X}^T\mathbf{X}$$

empirical covariance matrix

$$\underset{\mathbf{\Theta}\succ 0}{\text{minimize}} \; -\log\det\mathbf{\Theta} + \text{tr}(\mathbf{S}\mathbf{\Theta}) + \lambda||\mathbf{\Theta}||_1$$

positive definite constraint; Negative Log Likelihood; L1-norm Regularization

### Graphical Lasso

### Unified Graphical Lasso

**Input:** $\{\mathbf{S}_1, \ldots, \mathbf{S}_p\}$

$$\underset{\mathbf{\Theta}\succ 0}{\text{minimize}} \; \frac{1}{p}\sum_{i=1}^{p}\big(-\log\det\mathbf{\Theta} + \text{tr}(\mathbf{S}_i\mathbf{\Theta})\big) + \frac{\alpha}{p}\sum_{i=1}^{p}||\mathbf{\Theta} - \hat{\mathbf{\Theta}}_i||_F^2 + \lambda||\mathbf{\Theta}||_1$$

average negative log likelihood; homogeneous regularization; sparseness regularization

Inferred network for subject $i$

$$\hat{\mathbf{\Theta}}_i = \arg\min_{\mathbf{\Theta}\succ 0} -\log\det\mathbf{\Theta} + \text{tr}(\mathbf{S}_i\mathbf{\Theta}) + \lambda||\mathbf{\Theta}||_1$$

The inferred network for subject $i$ using Graphical Lasso

### Contrasting Graphical Lasso

**Input:**

Collection A $\{\mathbf{S}_1^{(A)}, \ldots, \mathbf{S}_p^{(A)}\}$

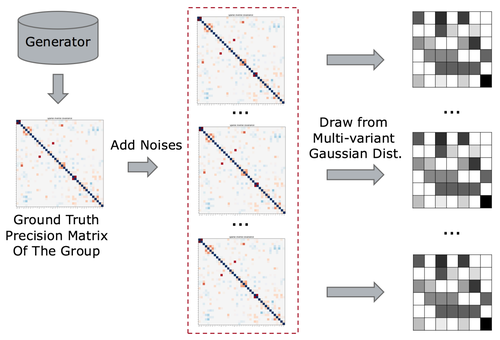
Collection B $\{\mathbf{S}_1^{(B)}, \ldots, \mathbf{S}_q^{(B)}\}$

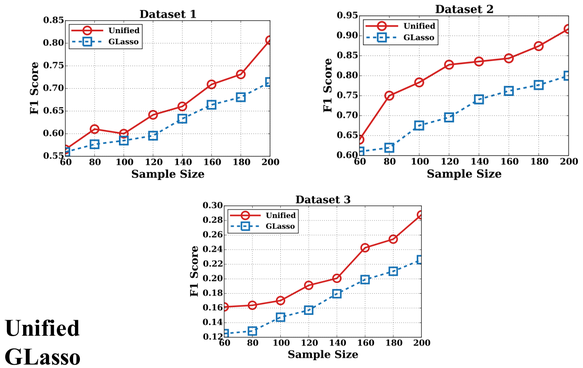
$$L(\mathbf{\Theta}, \mathbf{S}_1^{(A)}, \ldots, \mathbf{S}_p^{(A)}) = \frac{1}{p}\sum_{i=1}^{p}\big(-\log\det\mathbf{\Theta} + \text{tr}(\mathbf{S}_i^{(A)}\mathbf{\Theta})\big)$$

$$L(\mathbf{\Theta}, \mathbf{S}_1^{B}, \ldots, \mathbf{S}_q^{(B)}) = \frac{1}{q}\sum_{i=1}^{q}\big(-\log\det\mathbf{\Theta} + \text{tr}(\mathbf{S}_i^{(B)}\mathbf{\Theta})\big)$$

$$\underset{\mathbf{\Theta}\succ 0}{\text{minimize}} \; \frac{1}{p}\sum_{i=1}^{p}\text{tr}(\mathbf{S}_i^{(A)}\mathbf{\Theta}) - \frac{1}{q}\sum_{j=1}^{q}\text{tr}(\mathbf{S}_j^{(B)}\mathbf{\Theta}) + \lambda||\mathbf{\Theta}||_1$$

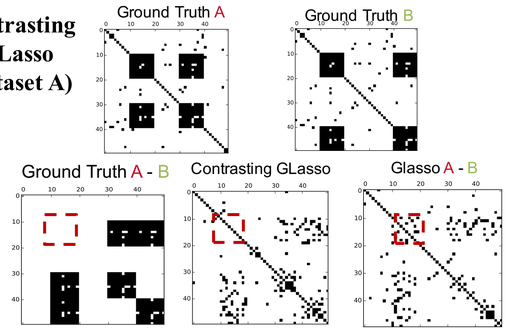
## 3. Synthetic Study



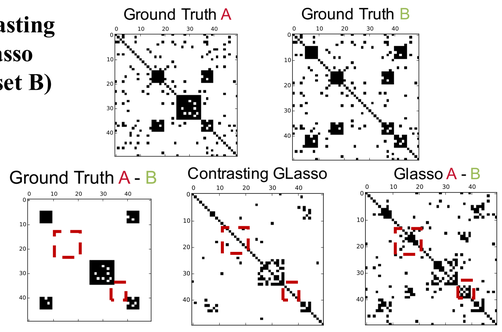




**Unified GLasso**

**Contrasting GLasso (Dataset A)**




**Contrasting GLasso (Dataset B)**




## 4. Real fMRI Data