



PARALLEL ARITHMETIC

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BACKGROUND

NUMBERS ARE NOT VECTORS

- Pseudo – independence
- BCD
 - Use N bits for each digit



ALGORITHMS

- Addition

$$a_1 a_2 a_3 \dots a_n + b_1 b_2 b_3 \dots b_n = (a_1 + b_1)(a_2 + b_2)(a_3 + b_3) \dots (a_n + b_n)$$



ALGORITHMS

○ Multiplication

$$a_1 a_2 a_3 \dots a_n * b_1 b_2 b_3 \dots b_n = C$$

$$C = \sum_{i=0}^{n-1} c_i$$

$$c_0 = (b_n * a_1)(b_n * a_2) \dots (b_n * a_n)$$

$$c_1 = (b_{n-1} * a_1)(b_{n-1} * a_2) \dots (b_{n-1} * a_n)(0)$$

$$c_2 = (b_{n-2} * a_1)(b_{n-2} * a_2) \dots (b_{n-2} * a_n)(0)(0)$$

$$c_x = (b_{n-x} * a_1)(b_{n-x} * a_2) \dots (b_{n-x} * a_n)[(0) \cdot x]$$



MEMORY REQUIREMENTS

- Addition

- $\max(\text{len}(a), \text{len}(b)) + 1$

- Multiplication

- $\text{len}(a) + \text{len}(b)$



OH NO OVERFLOW

- Each digit can store $2^{32} - 1$ values!



ONLY PSEUDO INDEPENDENT

- Normalization

$$A = a_1 a_2 a_3 \dots a_n$$

- While(keepcarrying)
 - $C = a_n / 10$
 - $a_n \% = 10$
 - $A_{(n-1)} += C$
 - If any $a_n > 9$
 - keepcarrying = True
 - Else
 - Keepcarrying = False



MEMORY REQUIREMENT CALCULATIONS

$$(2^{32} - 1) \sum_{i=0}^n 10^i = (10) \sum_{i=0}^x 10^i$$

$$(2^{32} - 1) \frac{10^{n+1} - 1}{9} = (10) \frac{10^{x+1} - 1}{9}$$

$$\ln((2^{32} - 1)(10^{n+1} - 1)) = \ln(10^{x+2} - 10)$$

$$\ln(10^{x+2} - 10) < \ln(10^{x+2})$$

$$\ln(2^{32} - 1) + \ln(10^{n+1} - 1) = (x + 2) \ln(10)$$

$$\ln(2^{32} - 1) + \ln(10^{n+1}) = (x + 2) \ln(10)$$

$$x = \frac{\ln(2^{32} - 1) + \ln(10^{n+1}) - 2 \ln(10)}{\ln(10)}$$

$$x = \frac{\ln(2^{32} - 1)}{\ln(10)} + (n - 1)$$

$$x \approx 9$$



MEMORY REQUIREMENT (CONT)

$$x = \frac{\ln(2^z - 1)}{\ln(10)} - 1 + n$$

- Where z is the number of bits used to store each digit
- Where n is the number of digits the number currently uses



DETECTING NORMALIZATION

- Addition

- Check 2 highest bits of each pair of numbers

- Multiplication

- Normalize numbers to begin with
- Check for normalization before each addition



CUDA ISSUES

- Lost Code
- Memory
 - Temporary variables
- Synchronizing threads
 - normalization
- Decimal calculations on CPU
 - General problem of CPU / GPU interactions



FUTURE WORK

- Parallel algorithm for reciprocal
- Human readable form for negative numbers

