# Representation Theory I

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# 1 Basic Definitions

**Definition**. *A* Lie algebra *is a vector space* $\mathfrak{g}$ *(over some field* $k$*) together with a bilinear map*

$$[\cdot,\cdot]\colon \mathfrak{g}\times\mathfrak{g}\to\mathfrak{g}$$

*satisfying the following:*

1. $[\cdot,\cdot]$ *is* alternating*, i.e.* $[x,x]=0$ *for every* $x\in\mathfrak{g}$.
2. *The* Jacobi identity

$$[x,[y,z]]+[y,[z,x]]+[z,[x,y]]=0 \quad \textit{for all } x,y,z\in\mathfrak{g}.$$

$[\cdot,\cdot]$ *is called a* Lie bracket.