Foundations of representation theory

5. Exercise sheet

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Exercise 17:

Exercise 18:

Exercise 19:

We can look at the 1-module $V := N(\infty) \times (K, \mathrm{id}_K)$, the endomorphism being $\psi = \phi \times \mathrm{id}_K$.
It is obvious that $W := N(\infty) \times 0$ is a proper submodule of V. W is also maximal in V because $V/W \cong (K, \mathrm{id}_K)$ is simple.
W is the only maximal submodule of V, because every maximal submodule $W' \subseteq V$ has to contain W: Assume $W' \subsetneq V$ is maximal with $W \subsetneq W'$. Then there is some $v = (\sum_{i=1}^n \mu_i e_i, 0) \in W$, $\mu_n \neq 0$, with $v \notin W'$. In particular $(e_n, 0), (e_{n+1}, 0), \ldots \notin W'$, because otherwise $(e_n, 0), (e_{n-1}, 0) = \psi((e_n, 0)), \ldots, e_1 = \psi^{n-1}((e_n, 0)) \in W'$ and thus $v \in W'$. So $W' \subsetneq W' + U((e_n, 0)) \subsetneq V$, which contradicts the maximality of W.
Even though W is the only maximal submodule in V we find that V is not uniform, because the proper submodule $0 \times K$ is not contained in W.

Exercise 20: