Foundations of representation theory

10. Exercise sheet

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Exercise 40:

We define the K-vector space

$$K^{\mathbb{N}} := \{(\lambda_i)_{i \in \mathbb{N}} : \lambda_n \in K \text{ for all } n \in \mathbb{N}\}$$

and for all $n \in \mathbb{N}$ the K-vector space

$$K_n^{\mathbb{N}} := \{(\lambda_i)_{i \in \mathbb{N}} : \lambda_0 = \ldots = \lambda_{n-1} = 0\}.1$$

In particular $K_0^{\mathbb{N}} = K^{\mathbb{N}}$. We now look at the short exact sequence sequence

$$0 \longrightarrow K \xrightarrow{f} K^{\mathbb{N}} \xrightarrow{g} K_2^{\mathbb{N}} \longrightarrow 0$$

given by the K-linear maps

$$f : K \to K^{\mathbb{N}}, \lambda \mapsto (\lambda, 0, 0, \ldots)$$

and

$$g : K^{\mathbb{N}} \to K_2^{\mathbb{N}}, (\lambda_0, \lambda_1, \lambda_2, \ldots) \mapsto (0, 0, \lambda_1, \lambda_2, \ldots).$$

Now we just use the direct sum decomposition $K^{\mathbb{N}} = K^2 \oplus K_2^{\mathbb{N}}$, where it is clear that $K \not\cong K^2$.