Foundations of representation theory
3. exercise sheet

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Exercise 9:

Exercise 10:

Exercise 11:

Exercise 12:

Let K be a field with $\operatorname{char}(K) = 0$ and (V, ϕ_1, ϕ_2) is a 2-module such that $V \neq 0$ and $[\phi_1, \phi_2] = 1$. Assume that V is finite-dimensional. Because $V \neq 0$ we know that $n := \dim V \geq 1$. Let $v_1, \ldots, v_n$ be a Basis von V and Φ_1 and Φ_2 the coordinate matrix of ϕ_1, ϕ_2 with respect to the basis $v_1, \ldots, v_n$ respectively. Because $[\phi_1, \phi_2] = 1$ we find that $\Phi_1\Phi_2 - \Phi_2\Phi_1 = I_n$. It follows, that

$$0 = \operatorname{tr}\Phi_1 \operatorname{tr}\Phi_2 - \operatorname{tr}\Phi_2 \operatorname{tr}\Phi_1 = \operatorname{tr}(\Phi_1\Phi_2 - \Phi_2\Phi_1) = \operatorname{tr} I_n = n \cdot 1 \neq 0.$$

This shows that (V, ϕ_1, ϕ_2) must be finite dimensional. (We know that such a module exists, because $\left(K[T], T\cdot, \frac{d}{dT}\right)$ is one.)