Foundations of representation theory

8. Exercise sheet

Jendrik Stelzner

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Exercise 29:

We will assume that the vertices of Q are ordered in the most obvious way. For all $1 \leq i \leq j \leq n$ let p_{ij} be the unique path in Q from i to j and for all $1 \leq i, j \leq n$ let $E_{ij} \in M_n(K)$ be the matrix with 1 as the (i,j)-entry and 0 otherwise. (E_{ij} maps e_j to e_i.) We know that $(p_{ij})_{1 \leq i \leq j \leq n}$ is a basis of KQ and $(E_{ij})_{1 \leq i \leq j \leq n}$ is a basis of A. Let $\phi : KQ \to A$ be the linear map given by $\phi(p_{ij}) = E_{n-j,n-i}$ for all $1 \leq i \leq j \leq n$. ϕ is a K-algebra homomorphism since for all $1 \leq i \leq j \leq n$ and $1 \leq l \leq k \leq n$

$$\begin{aligned}\phi(p_{ij}p_{lk}) &= \phi(\delta_{ki}p_{lj}) = \delta_{ki}\phi(\phi_{lj}) = \delta_{ki}E_{n-j,n-l} \\ &= E_{n-j,n-i}E_{n-k,n-l} = \phi(p_{ij})\phi(p_{lk}).\end{aligned}$$

ϕ is an isomorphism, because for the linear map $\psi : A \to KQ$ given by $\psi(E_{ij}) = p_{n-j,n-i}$ for all $1 \leq i \leq j \leq n$ we have $\phi\psi = \mathrm{id}_A$ and $\psi\phi = \mathrm{id}_{KQ}$.