

# Hyper-pool: pooling private trips into high-occupancy transit-like attractive shared rides

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# Concept



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# Hyper-pool

main idea

## Hyper-pool

an analytical,  
offline,  
utility-driven

### **ride-pooling algorithm**

to aggregate individual trip requests into attractive shared rides of high-occupancy

## Algorithm

we depart from our ride-pooling **ExMAS** algorithm where single rides are pooled into attractive door-to-door rides

we add two novel demand-side algorithms for further aggregating individual demand towards more compact pooling.

First, we generate stop-to-stop rides, with a single pick up and drop off points optimal for all the travellers.

Second, we bundle such rides again, resulting with hyper-pooled rides compact enough to resemble public transport operations.



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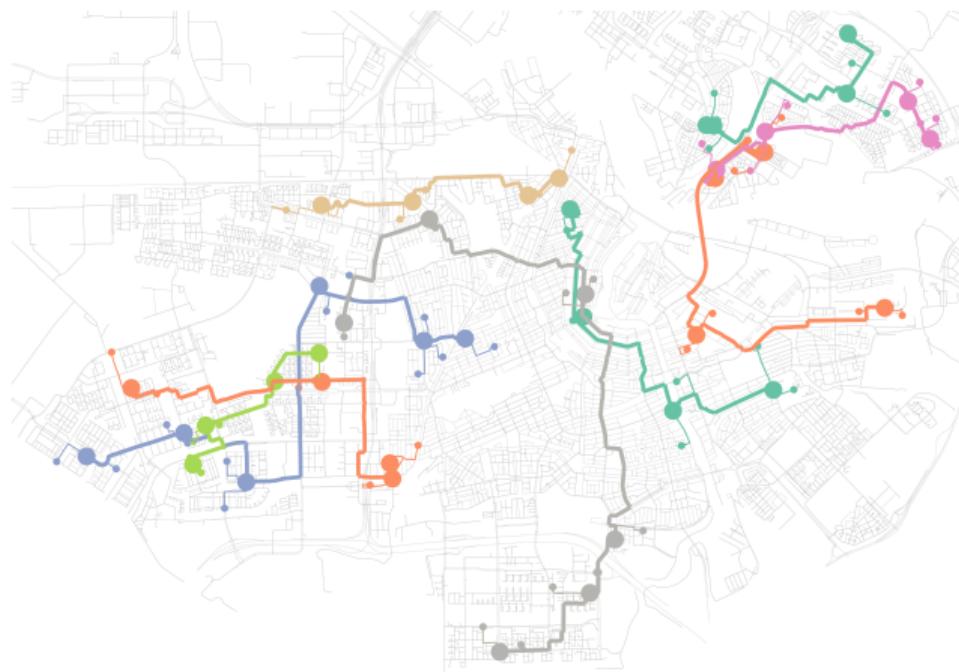
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# Hyper-pooled trips

Amsterdam case study results



Ten hyper-pooled rides in Amsterdam.

The degree reaches **12** (travellers)

rides are strictly attractive for all the co-travellers.

We can see direct short rides (e.g. brown in the central part)

as well as rides spanning through the whole city (grey).

The occupancy typically exceeds **4** and vehicle hours are reduced **5x** when compared to private rides.

## search space

we manage to identify rides composed of up to 14 travellers, which would require handling search space of the enormous  $10^{57}$  size impossible to search exhaustively

# Problem



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# Problem

## Ride-pooling

### Pooled (shared) ride:

- ➊ two or more travellers can be matched into a **shared** a ride and travel in the **same ride-hailing vehicle**.
- ➋ vehicle picks them up from **origins** and drops-them off at their **destinations**,
- ➌ both pickup and travel times **deviate** from the desired or minimal ones,
- ➍ this **inconvenience** needs to be compensated with a **lower fare** compared to an individual ride,
- ➎ service provider can now:
  - better **utilise** its capacity
  - charge several users for a ride
  - while paying a single driver commission.



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# Problem

## Ride-pooling

### Ride-pooling in practice

- ① Mainly door-to-door
- ② Unsustainable (benefits do not justify discounts)
- ③ Low occupancy
- ④ Critical Ma(a)ss



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# Problem

## Ride-pooling

### Ride-pooling algorithms

- ① typically real-time
- ② operations driven
- ③ fleet-oriented
- ④ flat, fixed constraints (time-windows)
- ⑤ captive travelers

### Limitations

- ⑥ door-to-door
- ⑦ one-kind of service
- ⑧ no compensation: longer detour → greater discount

The full potential of ride-pooling waits to be unleashed- both in theory and in practice



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# Another problem

## Transit Network Design Problem

### TNDP

determining optimal:

- ① stop-locations
- ② lines (stop sequences)
- ③ timetable (headways and departures)

is:

- ④ NP-hard
- ⑤ unsolvable - open-problem
- ⑥ search-space explosion

Hyper-pool can be seen as a bottom-up, demand driven approach for TNDP



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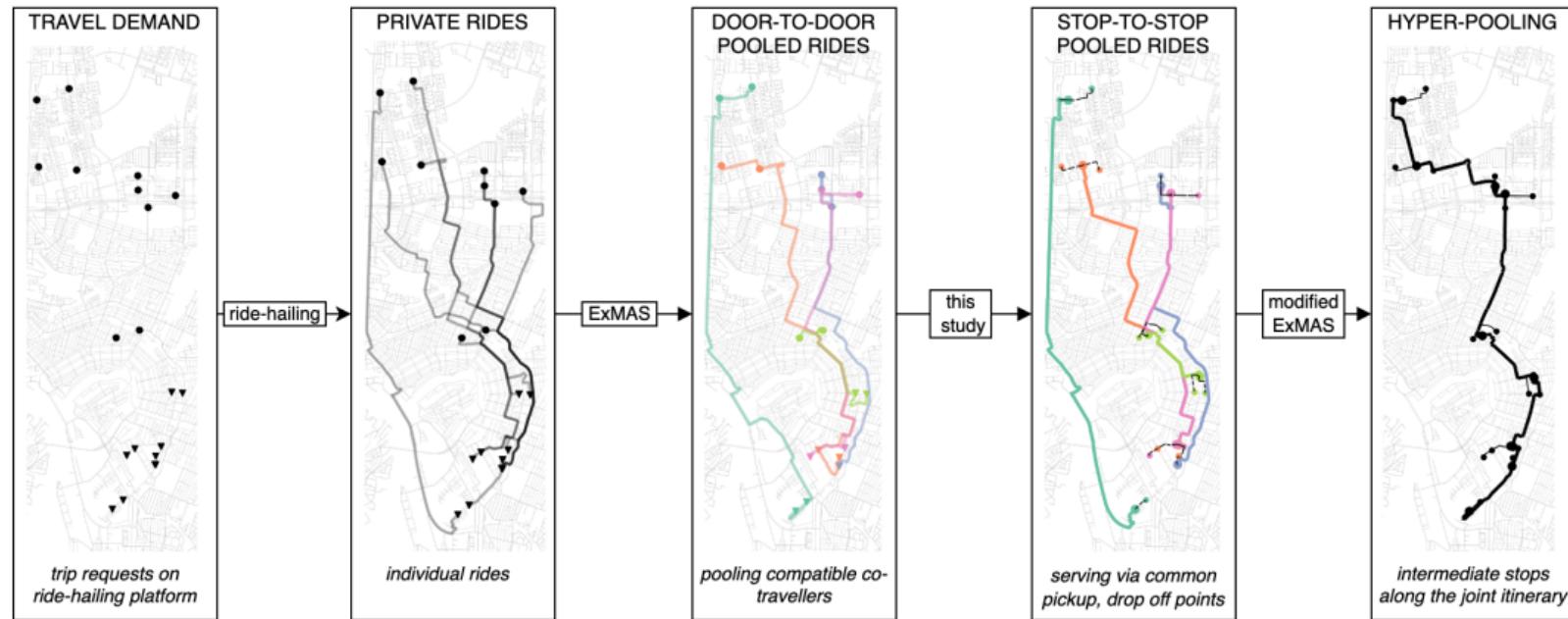
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# Algorithm



# Four levels of pooling



# Four levels of pooling

## 1. Private rides

door-to-door

no detour (direct)

no delay

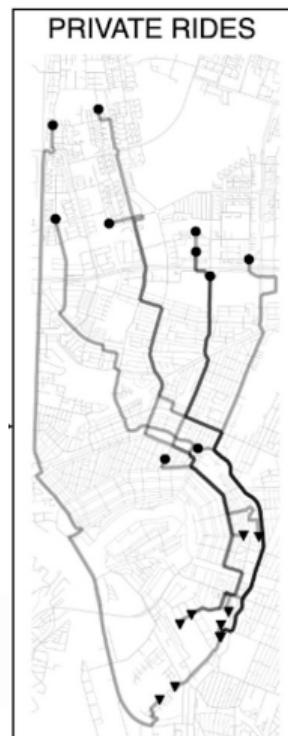
no discount

utility

$$U_p = \beta^t t_p - \lambda_p l + \varepsilon$$

composed of a direct travel time ( $t_p$ )

weighted by  $\beta^t$  (the value-of-time behavioural parameter)  
and a distance-based fare ( $\lambda_p l$ )



# Four levels of pooling

## 2. Door-to-door pooled rides

door-to-door

detour and delay

discount

utility

$$U_{d2d} = \beta^t (\beta_{d2d}^t t_{d2d} + \beta^d d_{d2d}) - \lambda_{d2d} l + \varepsilon$$

fare is discounted ( $\lambda_{d2d} < \lambda_p$ ),

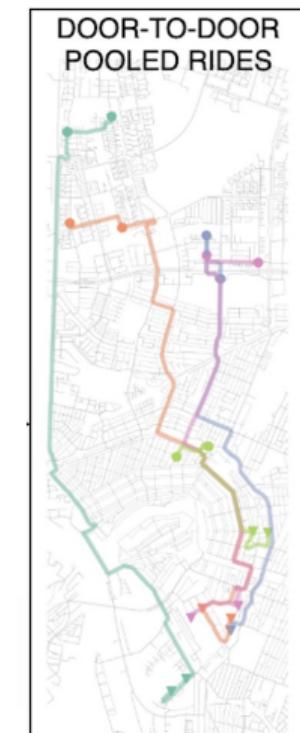
travel time is longer ( $t_{d2d} \geq t_p$ ) and delayed ( $d_{d2d} > 0$ )

discomfort (expressed as  $\beta_{d2d}^t > 1$  to represent the so-called willingness-to-share)

ExMAS

Kucharski R., Cats. O 2020. Exact matching of attractive shared rides (ExMAS) for system-wide strategic evaluations, Transportation Research Part B 139 (2020) 285-310 <https://doi.org/10.1016/j.trb.2020.06.006>

<https://github.com/RafalKucharskiPK/ExMAS>



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# Four levels of pooling

## 3. Stop-to-stop rides

access egress walk

no detour

delay

discount

utility

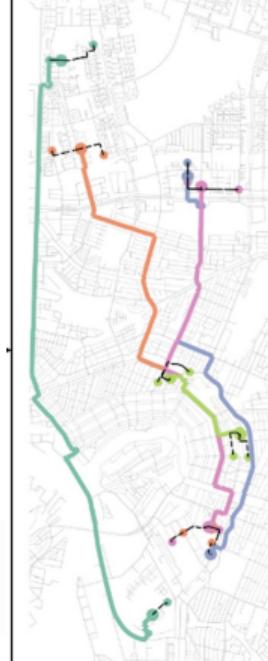
$$U_{s2s} = \beta^t (\beta_{s2s}^t t_{s2s} + \beta^d d_{s2s} + \beta^w w_{s2s}) - \lambda_{s2s} l + \varepsilon \quad (1)$$

walking discomfort ( $\beta^w > 1$ ) and walk time  $w_{s2s,i}$ .,

no additional in-vehicle discomfort ( $\beta_{s2s}^t \approx \beta_{d2d}^t$ ) since the ride is anyhow pooled,  
now the travel time  $t_{s2s}$  is likely to be shorter

but the discount needs to be high to compensate walking  $\lambda_{s2s} l$

**STOP-TO-STOP  
POOLED RIDES**



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### utility

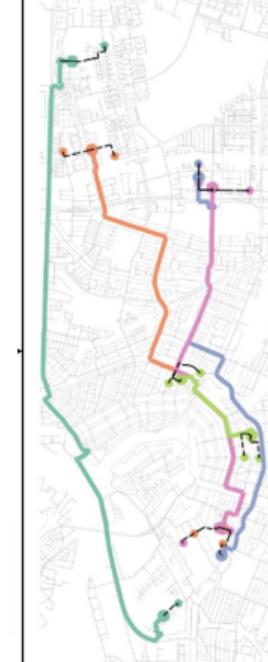
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### STOP-TO-STOP POOLED RIDES



# Stop-to-stop rides

problem: determining stops and departure

for each stop-to-stop ride:

determine

origin stop point

destination stop point

departure time

Exhaustive search

$$(o, d, t) = \arg \max_{o \in N \times t \in \mathcal{R} \times d \in N} \ln \left( \sum_{i \in r} \exp (U_{s2s,i}(o, t, d)) \right)$$



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# Stop-to-stop rides structure

they have the same structure, like private rides

$$(o, d, t)$$

so we can pool them again

Breakthrough

the key element of our model



# Four levels of pooling

## 4. Hyper-pooled rides

### idea

pool stop-to-stop pooled rides again  
with ExMAS

### Hyper-pooling

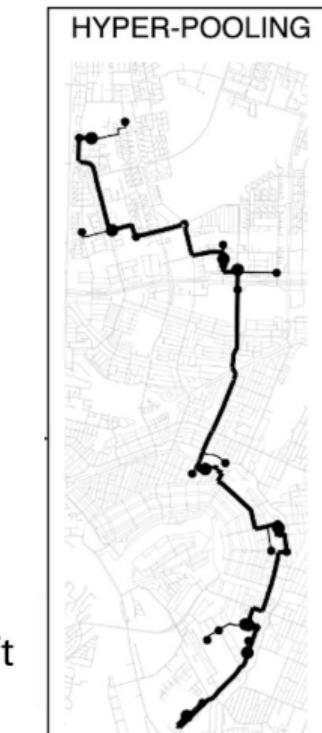
- access egress walk
- detour
- multi-stops
- big discount

### utility

$$U_h = \beta^t (\beta_h^t t_h + \beta^d d_h) - \lambda_h l + \varepsilon \quad (2)$$

Similarly to the original ExMAS, the fare is reduced again ( $\lambda_h < \lambda_{s2s}$ ).

Now we assume the discomfort due to pooling remains ( $\beta_h^t > \beta_{s2s}^t$ ), but is significantly lower than in the original ExMAS (travellers are anyhow pooling).



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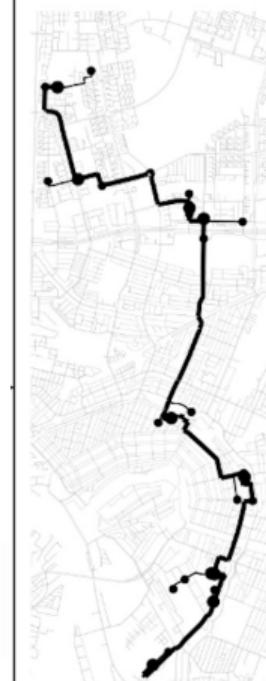
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### HYPER-POOLING



# Attractive pooling

strictly attractive rides

## utility-driven approach

At each level of pooling we make sure that utility of pooled rides is greater than at the previous level **for all pooling travellers**

door-to-door pooling only if more attractive than private rides

$$U_{d2d,i} \geq U_{p,i} \forall i \in \mathbf{Q}_r \forall r \in r_{d2d},$$

stop-to-stop pooling only if more attractive than door-to-door pooling

$$U_{s2s,i} \geq U_{d2d,i} \forall i \in \mathbf{Q}_r \forall r \in r_{s2s},$$

hyper-pooling only if more attractive than stop-to-stop

$$U_{\text{h},i} \geq U_{s2s,i} \forall i \in \mathbf{Q}_r \forall r \in r_{\text{h}},$$

## compensations

$$\lambda_p > \lambda_{d2d} > \lambda_{s2s} > \lambda_{\text{h}}$$

# Algorithm

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**Algorithm 1:** Hyperpool - pseudo-code for the algorithm

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**Hyperpool****inputs:**

Q # trip requests  
G # road network graph  
 $\beta$ 's # parameters (behavioural, fares, discounts, etc.)

**output:**

$R^*$  # pooled rides  
 $R_p \leftarrow Q$  # Initialise with single rides  
 $R_{d2d} \leftarrow \text{ExMAS}(R_p, \beta\text{'s})$  # compute door-to-door pooled rides with ExMAS  
 $R_{s2s} \leftarrow \text{Stop-to-Stop}(R_{d2d}, \beta\text{'s})$  # identify stop-to-stop rides  
 $R_h \leftarrow \text{ExMAS}'(R_{s2s}, \beta\text{'s})$  # compute hyper-pooled rides with the modified ExMAS  
 $R = R_p \cup R_{d2d} \cup R_{s2s} \cup R_h$  # search space of rides of four-kinds  
 $R^* \subseteq R$  # match travellers to rides by solving the coverage problem

**Result:**  $R^*$ 

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Concept  
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Problem  
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Algorithm  
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Results  
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Summary and Conclusions  
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# Results



# Amsterdam case

## experimental setting

Albatross dataset: origin, destination and departure time for 241k trips within Amsterdam per working day.

a detailed OSM graph (fixed network-wide speed)

30 minute batch with 2000 trip requests (4000 requests/hour)

fares:

- 1.5€/km for private
- 25% discount for door-to-door ride-pooling,
- 66% for stop-to-stop and
- 75% for multi-stop pooling,  
i.e. the fares  $\lambda$ 's of 1.5, 1.11, 0.5 and 0.375 €/km respectively (PT fare in Amsterdam is ca. 0.3 €/km, only 25% more expensive than the proposed hyper-pool rides)

minimise vehicle-hours in trip-ride assignment problem.

fleet not explicitly considered.



# Results

## Illustrative 10-person ride

An illustrative example of a set of 10 travellers bundled into a single hyper-pooled ride ( $\text{h}$ ). Rows denote consecutive travellers. In columns, we report (dis)utilities, travel times, walk times and fares paid at the four levels of pooling: private ( $p$ ), door-to-door ( $d2d$ ), stop-to-stop ( $s2s$ ), hyper-pool ( $\text{h}$ ); and public transport (PT) alternative.

id	utility					travel time [minutes]					walk time					fare [€]				
	$p$	$d2d$	$s2s$	$\text{h}$	PT	$p$	$d2d$	$s2s$	$\text{h}$	PT	$p$	$d2d$	$s2s$	$\text{h}$	PT	$p$	$d2d$	$s2s$	$\text{h}$	PT
776	-10.26	-9.78	-9.72	-7.42	-6.46	12.23	12.23	12.23	15.63	31.78	0.0	0.0	6.82	6.82	10.23	8.82	6.48	6.48	2.20	2.36
811	-19.33	-17.87	-19.15	-14.12	-15.09	20.65	20.65	20.65	26.88	50.83	0.0	0.0	10.35	10.35	9.73	14.87	10.93	10.93	3.72	3.59
1106	-18.97	-18.44	-18.42	-12.49	-14.52	20.12	20.12	26.88	39.75	0.0	0.0	1.82	1.82	6.78	14.49	10.66	10.66	3.62	2.93	
1162	-14.09	-13.98	-14.06	-13.17	-15.69	13.93	13.93	12.43	15.63	33.67	0.0	0.0	2.23	2.23	6.92	10.04	7.38	3.41	2.51	2.61
1385	-15.53	-15.48	-14.66	-10.29	-12.53	16.82	16.82	16.82	18.90	43.87	0.0	0.0	12.77	12.77	12.43	12.11	8.91	8.91	3.03	2.98
1401	-16.06	-15.76	-15.98	-11.71	-16.66	16.27	16.27	16.27	18.90	47.48	0.0	0.0	9.68	9.68	18.67	11.72	8.62	8.62	2.93	2.81
1470	-13.52	-13.00	-12.98	-10.82	-11.60	14.05	14.05	12.43	13.68	35.22	0.0	0.0	13.15	13.15	14.38	10.12	7.44	3.44	2.53	2.31
1729	-12.82	-12.42	-12.42	-9.36	-13.17	12.82	12.82	12.82	13.68	35.23	0.0	0.0	4.37	4.37	13.93	9.23	6.79	6.79	2.31	2.34
1865	-3.08	-2.92	-2.89	-3.05	-3.98	3.63	3.63	3.48	3.68	18.38	0.0	0.0	9.92	9.92	12.65	2.62	1.93	0.89	0.66	1.36
1995	-3.84	-3.33	-3.60	-2.71	-2.92	4.72	4.72	3.48	3.68	15.28	0.0	0.0	7.65	7.65	9.55	3.40	2.50	1.16	0.85	1.36
total	-127.5	-122.98	-123.88	-95.64	-112.62	135.24	135.24	130.73	157.54	351.49	0.0	0.0	78.76	78.76	115.27	97.42	71.64	61.29	24.36	24.65

Nine requests of a total 65km length were served with a single 9.6km multi-stop ride. If served with private rides it generates 2.25 vehicle-hours, while it can be now served with only 0.44

# Results

## Solution and search space

### Rides composition

Rides composition obtained when pooling 2000 trip requests at various pooling levels. Each row introduces a new level of pooling and columns denote the number of travellers assigned to rides of respective kind. The last column denotes the number of attractive rides in respective solutions.

solution with		private	door-to-door	stop-to-stop	hyper-pooled	number of feasible rides
private	$p$	2000	0	0	0	2000
door-to-door pooled	$p \cup d2d$	655	1345	0	0	159 702
stop-to-stop pooled	$p \cup d2d \cup s2s$	645	1184	171	0	160 239
hyper-pooled	$p \cup d2d \cup s2s \cup h$	651	1065	59	225	1 009 855

# Results

## KPIs per traveller

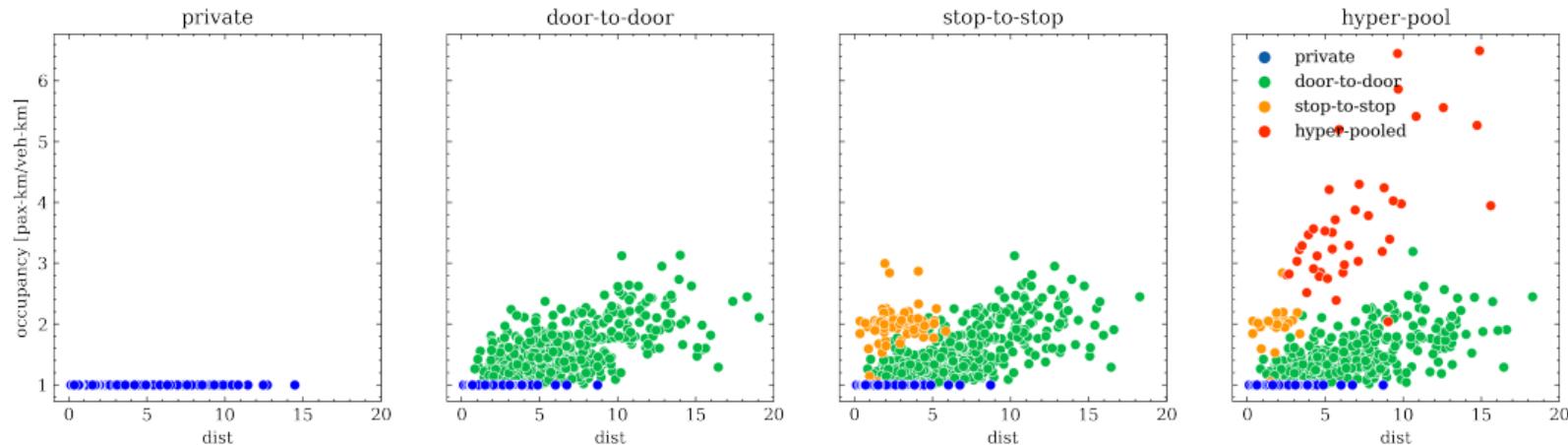
### KPIs

Ride-pooling KPIs per traveller obtained for case of 30-minute batch with 2000 trips in Amsterdam at four consecutive levels of pooling.

solution	vehicle hours	(dis)utility €	pax in-vehicle hours	walk time	#rides	fare	fares per veh-hour	Average occupancy
private	0.137	-7.56	0.137	0.00	2000	5.92	43.24	1.00
door-to-door pooled	0.090	-7.31	0.181	0.00	159702	4.55	50.81	1.53
stop-to-stop pooled	0.088	-7.18	0.169	0.01	160239	4.35	49.43	1.56
hyper-pooled	0.086	-6.65	0.146	0.02	1009855	4.02	47.03	1.60

# Results

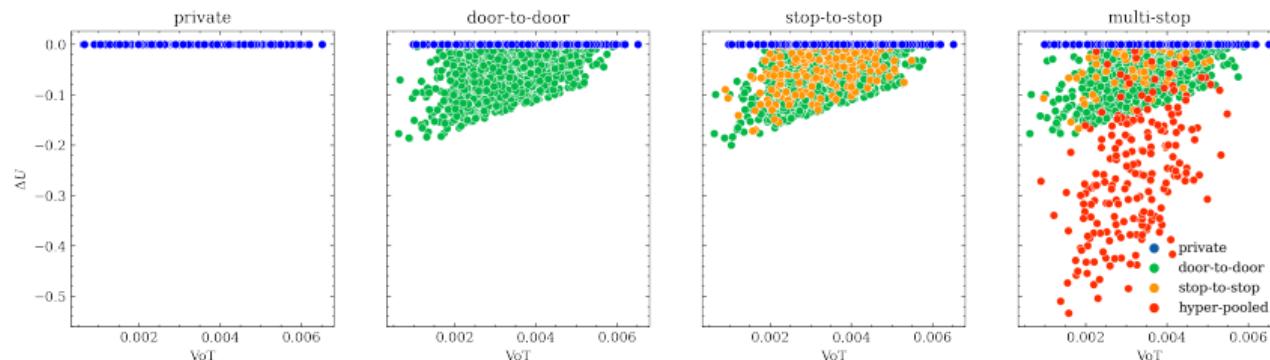
## Occupancy



Occupancy (y-axis) of rides obtained at respective levels of ride pooling scattered against ride length (x-axis). Each panel denotes a solution where new levels of pooling are introduced, dots represent individual rides and colours denote the pooling service kind.

# Results

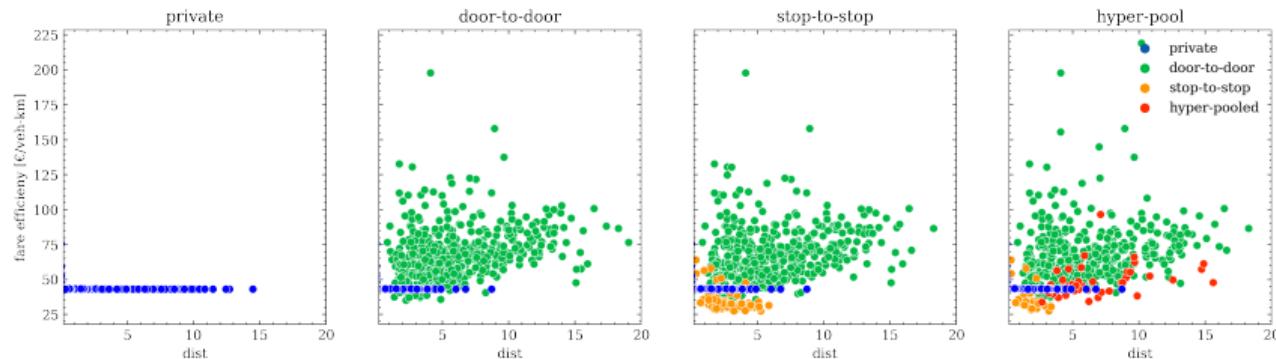
## Attractiveness for individuals



Relative attractiveness (disutility reduction  $\Delta U$  on y-axis as related to the private ride) obtained at various levels of pooling (consecutive panels) scattered against the value of time of respective travellers (x-axis). Each dot denotes now a single traveller and is coloured accordingly to the type of pooling service to which the traveller was assigned to.

# Results

## Efficiency for the operator



Trip efficiency (fare per vehicle-kilometre on y-axis) varying with ride length (x-axis) for various ride types (colours) and pooling levels (panels). Each dot represents a single ride.

## Summary and Conclusions



# Hyper-pool

## Summary and conclusions

- ➊ Our method yields occupancy levels not observed before for attractive ride-pooling.
- ➋ In our Amsterdam case-study we managed to pool over 220 travellers into 40 hyper-pooled rides of average occupancy **5.8 pax-h/veh-h**.
- ➌ for each traveller the **discomfort induced by pooling is compensated by reduced fares**.
- ➍ It offers a great potential for
  - ➎ travellers (whose disutility of travelling can be reduced),
  - ➏ for policymakers (who contribute towards sustainability goals with increased occupancy) and
  - ➐ for service providers (for whom the pooling cost-effectiveness remains greater than for private ride-hailing).
- ➏ The algorithms are publicly available and reproducible.
- ➐ Hyper-pool is applicable for real-size demand datasets and
- ➑ opens new opportunities for exploiting the limits of ride-pooling potential.

# Questions

## Discussion

Thank you!

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& Oded Cats<sup>1</sup>



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