

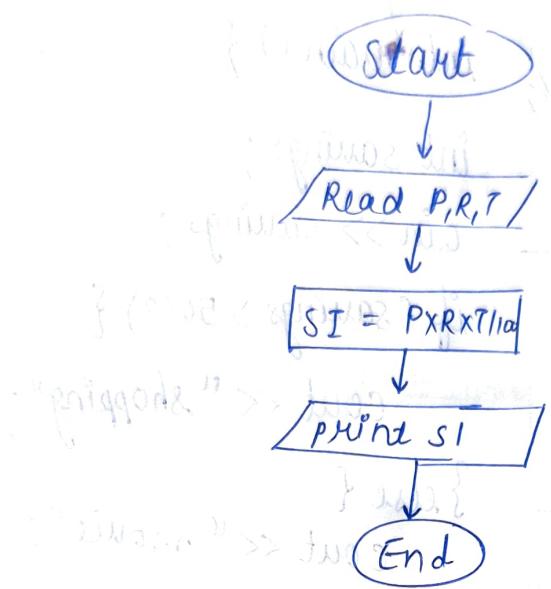
1.2.24

Data Structure And Algorithm

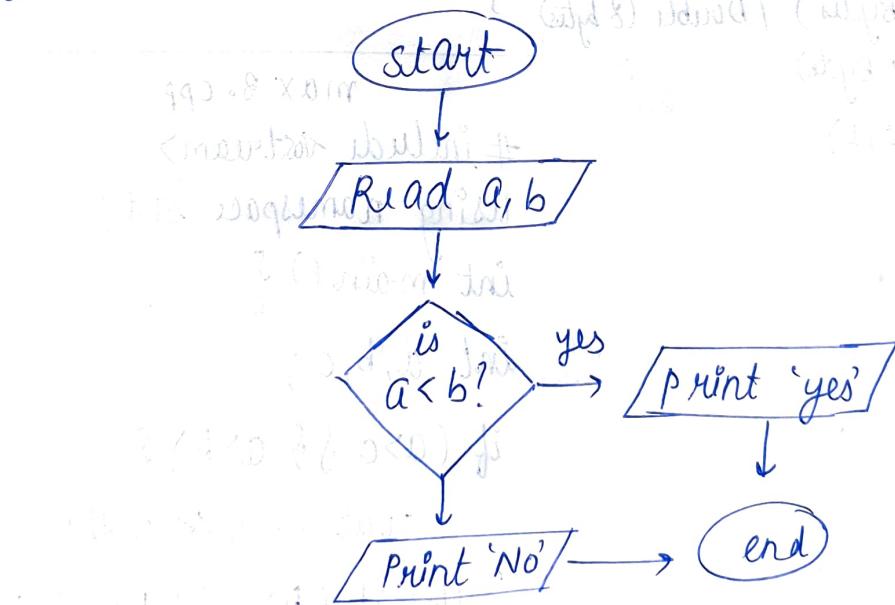
LOVE BABBAR

Flowchart

①



② Check $a < b$



CPP

```
#include <iostream>
using namespace std;
int main() {
    cout << "HelloWorld" << endl;
}
```

↓
end line

To get output

g++ filename.cpp
.la.exe

Data types and variables

•) Primitive 1 Byte = 8 bits

→ Integer (4 Bytes)

→ Float (4 Bytes) / Double (8 bytes)

→ Character (1 Byte)

→ Boolean (0,1)

•) Derived

→ function

→ Array

→ Pointer

→ Reference

•) User Defined

→ Class

→ structure

→ Union

→ Enum

```
#include <iostream>
```

```
int main() {
```

```
    std::cout << "Hello\n";
```

```
    return 0;
```

```
}
```

If / else

```
#include <iostream>
using namespace std;
int main() {
    int savings;
    cin >> savings;
    if (savings > 5000) {
        cout << "shopping";
    } else {
        cout << "movie";
    }
    return 0;
}
```

max 3.cpp

```
#include <iostream>
using namespace std;
int main() {
    int a, b, c;
    if (a > c && a > b) {
        cout << a << endl;
    } else if (b > a && b > c) {
        cout << b << endl;
    } else {
        cout << c << endl;
    }
    return 0;
}
```

return 0;

LOOPS

```
for (initialisation; condition; update){  
    task;  
}
```

entry controlled
while (condition is true){
 // do task;
}

entry controlled }.

```
do {  
    // do task;  
} while (condition);
```

exit controlled
Jumps in loops

> Break

> Continue

Switch Case

Q.) Robot says hello in diff languages for diff buttons.

#include <iostream>

using namespace std;

int main()

```
int a;  
cin >> a;  
switch (a){  
    case 1:  
        cout << "Hello";  
        break;
```

```
    case 2:  
        cout << "Hola";  
        break;
```

case 3:

```
cout << "Namaste";  
break;
```

default:

```
cout << "Ram Ram";  
break;  
}  
return 0;
```

Operators

① Arithmetic Operators

↳ Binary op. (+, -, *, %, /)

↳ Unary op. (+=, ++, --)

② Relational Operators

→ return a boolean value (1 or 0)

==, !=, >, <, >=, <=

③ Logical Operators

&& → And

|| → OR

! → Not

④ Bitwise Operators

And → &

Or → |

NOT → ~

XOR → ^

Examples

$$a=5, b=7$$

$$a \& b$$

$$\begin{array}{r} 5 \rightarrow 101 \\ 7 \rightarrow 111 \\ \hline 101 \rightarrow 5 \end{array}$$

$$\boxed{a \& b = 5}$$

$$a=5, b=7$$

$$a \wedge b \quad 5 \rightarrow 01101$$

$$7 \rightarrow \underline{111}$$

ans 010

$$\boxed{a \wedge b = 2}$$

$$a=5, b=7$$

$$a | b = 7 \quad \begin{array}{r} 101 \\ 111 \\ \hline 111 \end{array}$$

Not (\sim)

x	y	(z) _x	(z) _y
0	0	1	1
0	1	1	0
1	0	0	1
1	1	0	0

$$a=2 \rightarrow 00\ldots010$$

$$\sim a = -3 \quad \sim a \rightarrow \underbrace{111\ldots101}_{-ve \text{ no.}}$$

trace complement

$$\rightarrow \begin{array}{r} 000010 \\ \text{Jev. } +1 \\ \hline \dots 0000011 \end{array}$$

$$\boxed{\text{ans} = -3}$$

XOR \rightarrow dono same mat dena.

x	y	z
1	1	0
1	0	1
0	1	1
0	0	0

→ left shift ($<<$)

int a=5;

a<<1

shift bits of 'a' by 1

$$a = 000\ldots0101$$

$$a << 1 = 000\ldots01010$$

$$\boxed{a=10}$$

similarly for right shift

$$a=5 \quad \{ 000\ldots101 \rightarrow$$

$$a >> 2 \quad \{ 000\ldots001$$

$$\boxed{a=1}$$

majority (not always true)

→ if 1 left shift $\Rightarrow x2$

→ 1 right shift $\Rightarrow \frac{x}{2}$

→ if no. is too big \rightarrow left shift will make it negative

e.g. 01000...0101.

↗

Fibonacci

```
#include <iostream>
using namespace std;

int main() {
    int nt, n0=0, n1=1, n=10;
    cout << n0 << endl;
    cout << n1 << endl;
    for (int i=0; i<=n; i++) {
        nt = n0 + n1;
        n0 = n1;
        n1 = nt;
        cout << nt << endl;
    }
    return 0;
}
```

Output: 0 1 1 2 3 5 8 13 21 34

Q) Find if number given by input is prime or not.

int main() {

```
    int n;
    cout << "Enter number" << endl;
    cin >> n;
    bool isprime = 1;
    for (int i=2; i<n; i++) {
        if (n % i == 0) {
            isprime = 0;
            break;
        }
    }
    if (isprime == 0) {
        cout << "Not a prime number";
    }
}
```

```
} else { cout << "It's a prime no.";
```

return 0;

}

break → gets you out of loop

continue → skips particular iteration.

e.g.

```
for (int i=0; i<5; i++) {
    cout << "Hi" << endl;
    cout << "Hey" << endl;
    continue;
    cout << "Reply me" << endl;
}
```

whether loop repeats because of break or continue.

↳ | Hey } 0 times.
 | Hi } 1st time
 | Hey } 2nd time
 | Hi } 3rd time
 | Hey ... for 5 times.

Types of Variables

Types of Variables

- 1) Global variables : Accessible throughout entire program
 - 2) Local Variables: Declared within functions or block of program. (reducing naming conflict)
 - 3) Instance Variables : Associated with individual objects , defining their properties or attributes
 - 4) Class Variables : Shared by all instances of a class , representing class level data
- 3, 4) → Object oriented Prog.

Q.) Given an integer ; return its difference between product and sum of it's digit.

E.g. $n=234$

$$n \% 10 \rightarrow 4$$

$$n / 10 \rightarrow 23$$

$$23 \% 10 \rightarrow 3$$

$$23 / 10 \rightarrow 2$$

stop when $n=0$;

code

int main () {

int prod = 1;

int sum = 0;

int n;

Cout << Enter n << endl;

cin >> n;

while ($n \neq 0$) {

int digit = n % 10;

sum = sum + digit;

prod = prod * digit;

$n = n / 10;$

Cout << prod - sum << endl;

}

Q.) Write code that takes unsigned integer and returns the number of '1' bits it has

int main () {

while ($n \neq 0$) {

if ($n \& 1$) {

count++;

}

$n = n \gg 1;$

}

return count;

}

1 2 3

int main () {

Q.) WAP to reverse an integer.

logic

$ans = (ans \times 10) + digit$

basically shifts the number left adding '0' to the rightmost place. Then adding digit

321

$$\underline{32} \times 10 + 1 = \underline{321}$$

code

int main () {

int n, digit, ans = 0;

while ($n \neq 0$) {

digit = n % 10;

ans = (ans * 10) + digit;

$n = n / 10;$

```

cout << ans << endl;
return 0;
}

```

compliment

e.g. $n = 5 \rightarrow 101$
complement $\rightarrow 010$

$n=2$ | complement of 5

e.g. $n=7 \rightarrow 111$
 \downarrow
 000
 $n=0$ | $n=10 \xrightarrow{\text{com}} n=5$

Q.) WAP to find complement of a given number.

```

int main() {
    int m=n;
    int mask=0;
    // edge case
    if (n==0) {
        return 1;
    }
    while (m!=0) {
        mask = (mask << 1) | 1;
        m = m >> 1;
    }
    int ans = (~n) & mask;
    return ans;
}.

```

Q.) Given an integer n , return true, if it is a power of 2 else return false.

ispoweroftwo (int n) {

```
for (int i=0; i<=30; i++) {
```

```
    int ans = pow(2, i);
```

```
    if (ans == n) {
```

```
        return 1;
```

```
}
```

~~else~~

```
} return 0;
```

```
}
```

SWITCH CASE

switch (exp_) {

case (const 1) : \equiv

break

case (const 2) : \equiv

break

default : \equiv

```
}
```

break \rightarrow use to come out of loop or a block of function when our desired task is finished.

c

if switch is inside infinite loop (stop using exit)

→ use exit instead of break

while (1) {

case 0: print;
exit(0);

case 1: print;
exit(0);

default: print;
exit(0);

}

exit (int_status)

int_status = 0 { no errors }

≠ 0 { indicates error }

continue → skips 1 iteration
(index)

use of continue in switch case
is not valid.

continue is designed to skip
a particular iteration in
while, do while, for loops.

Since switch case doesn't
contain any iterations we
can use continue.

⇒ Give syntax Error *

FUNCTIONS

→ well defined task

Q.) Write a function to calculate
 nCr of given n & r .

$$nCr = \frac{n!}{r!(n-r)!}$$

#include <iostream>

using namespace std;

int fact (int f) {

int ans = 1;

for (int i=1; i<=f; i++) {

ans = ans * i;

return ans;

int nCr (int n, int r) {

int ans = fact(n) / (fact(r) *

fact(n-r));

return ans;

}

int main () {

int n, r;

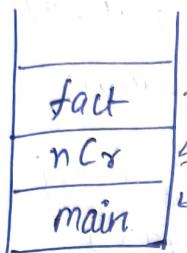
cin >> n >> r;

cout << "nCr = " << nCr(n, r);

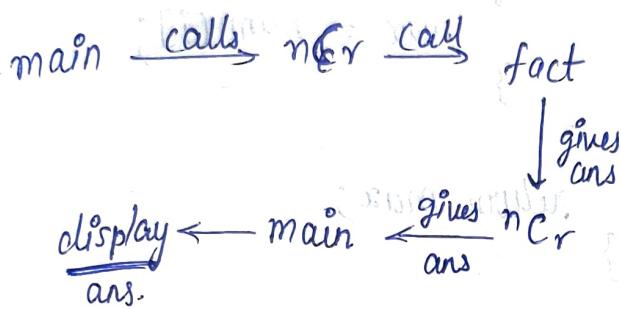
return 0;

}

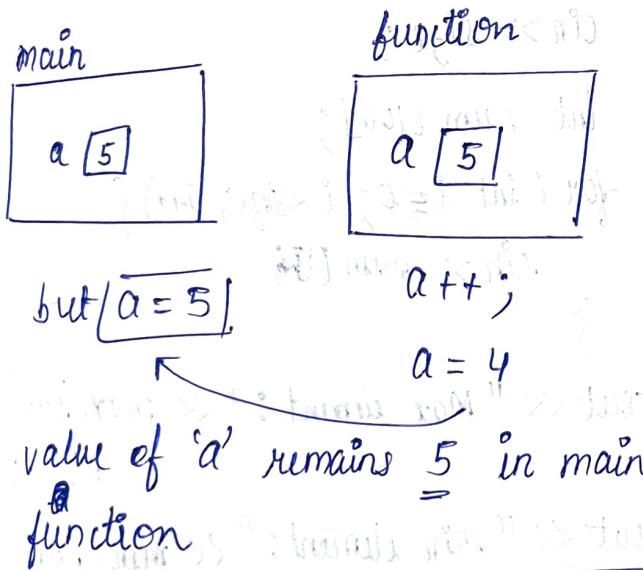
function-call stack for previous question



last in first out



pass by Value



Q.) void update (int a){
 $a = a/2;$
}

int main() {
 int a = 10;
 update (a);
 cout << a << endl;
}

Q.) find out put

int ~~int~~ dummy (int a) {

 a -= 5;
 return a;

}

int main() {

 int a = 15;

 dummy(a);
 cout << a;

}

ans = 15

if in main function

\rightarrow $a = \text{dummy}(a);$
for previous question then

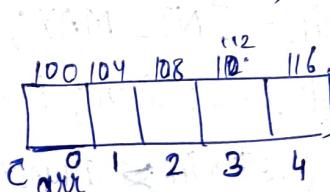
ans = 10

this changes 'a' of main f'n

Arrays

- stores similar type of data
- Contiguous addresses of locations.
- accessible through index.

int arr[5];



array (n size) index (0 to n-1)

```

int main() {
    int arr[100] = {0};
    return 0;
}

```

0	1	2	..	99
0	0	0	0	0

if $\text{int arr[100]} = \{1\}$;

0	1	2	..	99
1	g	g	1	g

$g = 0$

$\text{int arr[10]} = \{2, 7\}$

0	1	2	..	9
2	7	0	0	0

```

void printArray (int arr[], int size) {
    for (int i=0; i<size; i++) {
        cout << arr[i] << " " << endl;
    }
}

```

Q.) Find max/min element of array.

```

#include <iostream>
using namespace std;
int min (int sum[], int n) {
    int min = INT_MAX;
    for (int i=0; i<n; i++) {
        if (min > sum[i]) {
            min = sum[i];
        }
    }
}

```

```

return min;
}

```

```

int max (int num[], int n) {
    int max = INT_MIN;
    for (int i=0; i<n; i++) {
        if (max < num[i]) {
            max = num[i];
        }
    }
    return max;
}

```

int main()

int size;

cin >> size;

int num[100];

```

for (int i=0; i<size; i++) {
    cin >> num[i];
}

```

cout << "Max element : " << max(num, size);

cout << "Min element : " << min(num, size) << endl;

return 0;

NOTE

arr[10000] is better than

arr[variable]

e.g. ~~num[size];~~

in case of arr passing in fn
any changes done in function to
given array results in actual change
in main function
(This happens because name of arr
is ptr to first element in array)

Linear Search

```
#include <iostream>
using namespace std;
int linearSearch (int arr[], int size,
                  bool
                  int key) {
    for (int i=0; i<size; i++) {
        if (arr[i] == key) {
            return 1;
        }
    }
    return 0;
}
```

```
int main()
```

```
int arr[5] = {1, 5, 7, 11, 6};
```

```
cout << "Enter element to find:" << endl;
```

```
int key;
cin >> key;
```

```
bool found = linearSearch (arr, 5, key);
```

```
if (found) {
    cout << "Element" << key << "was
    found at" << i << "index";
} else {
    cout << "not found" << endl;
}
return 0;
}
```

Reverse An Array

```
Input : 1, 5, 7, 9, 3
Output : 3, 9, 7, 5, 1.
```

```
#include <iostream>
using namespace std;
int temp;
void reverse (int arr[], int size) {
    for (int i=0; i<(size/2); i++) {
        temp = arr[i];
        arr[i] = arr[size-i-1];
        arr[size-i-1] = temp;
    }
}

int main() {
    int arr[10] = { };
    reverse (arr, 10);
    return 0;
}
```

Questions on Arrays

Q.1) Swap alternate elements of array

$$\text{arr}[5] = \{1, 2, 3, 4, 5\}$$

$$O/P = \{2, 1, 3, 4, 5\}$$

$$\text{arr}[4] = \{1, 2, 3, 4\}$$

$$O/P = \{2, 1, 4, 3\}$$

→ #include <iostream>
using namespace std;

```
void printArray (int arr[], int n){  
    for (int i=0; i<n; i++){  
        cout << arr[i] << " ";  
    } cout << endl;  
}
```

```
void swap (int arr[], int k){  
    int temp;  
    temp = arr[k];  
    arr[k] = arr[k+1];  
    arr[k+1] = temp;  
}
```

```
void swapAlt [int A[], int s]{  
    for (int k=0; k<s; k=k+2){  
        if (k+1 < s){  
            swapAlt [A, s];  
        }  
    }  
}
```

```
int main () {  
    int A[10] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};  
    int size = sizeof (A) / sizeof (int);  
    cout << "before Alt swap" << endl;  
    printArray (A, size);  
    swapAlt [A, s];  
    cout << "After Alt swap" << endl;  
    printArray (A, s);  
    return 0;  
}
```

O/P

before
1 2 3 4 5 6 7 8 9 10

After

2 1 4 3 6 5 8 7 10 9

Q.2) Find Unique Number in array (only 1 unique no)

```
int UniqueNo (int *arr, int size){  
    int ans = 0;  
    for (int i=0; i<size; i++){  
        ans = ans ^ arr[i];  
    }  
    return ans;  
}
```

logic (XOR)

6 8 4 8 8 8

$$6^6 = 0 \quad 8^8 = 0 \quad 0^4 = 4$$

$$0^0 0^0 0^0 0^4 = 4$$

(same no. XOR give zero).

Q.3) Unique Number of Occurrences

→ Each number in array must be occur ~~as~~ unique times than others.

E.g. {1, 0, 1, 0} → false
{1, 2, 3} → false
{1, 1, 2} ←
{1, 1, 2, 2, 2, 3, -3, 3, 1, 1} ←

Q.4) Find Duplicates in array.

Q.5) Intersection of Array

$$A = \{1, 2, \underline{3}, \underline{4}\}$$

$$B = \{\underline{3}, \underline{3}, \underline{4}, 5\}$$

$$O/P \Rightarrow = \{3, 4\}.$$

Sorted array is given.

logic.

Vector <int> find Array Intersec.

(vector<int> &arr1, int n, int m
vector<int> &arr2) {

int i=0, j=0;

while (i < n && j < m) {

~~if (arr1[i] < arr2[j])~~

if (arr1[i] == arr2[j]) {

ans.push_back(arr1[i]);

i++;

j++;

} ~~and < till ans~~

else if (arr1[i] < arr2[j]) {

i++;

} else {

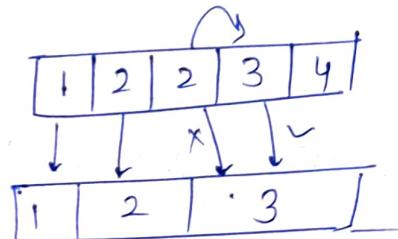
j++;

}

}

return ans;

.



O/P = 1, 2, 3, and.

Q.6) Pair Sum

→ find pairs in arrays which gives sum = S.

Vector <vector<int>> pairsum (vector<int> &arr, int s) {

vector <vector<int>> ans;

```
for (int i=0; i<arr.size(); i++) {
```

```
    for (int j=i+1; j<arr.size(); j++) {
```

```
        if (arr[i] + arr[j] == s) {
```

```
            {
```

```
                vector <int> temp;
```

```
                temp.push_back(min (arr[i], arr[j]));
```

```
                temp.push_back(max (arr[i], arr[j]));
```

```
                ans.push_back(temp);
```

```
}
```

```
}
```

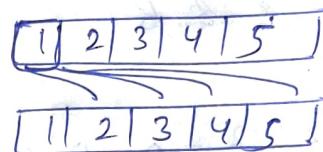
```
}
```

```
sort (ans.begin(), ans.end());
```

```
return ans;
```

```
}
```

logic



compare $i=0$ from $j=i+1$ to $j=n$, where $n = \text{size of array}$.

Q.7) Triplet sum (H.W)

→ use 3 for loops.

```
for (i → 0 to n-1)
```

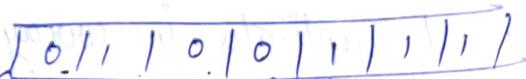
```
{ for (j → i+1 to n-1)
```

```
{ for (k → j+1 to n-1)
```

```
}
```

```
}
```

Q.8) Sort 0 1



Sort



```
void sortOne (int arr[], int n){
```

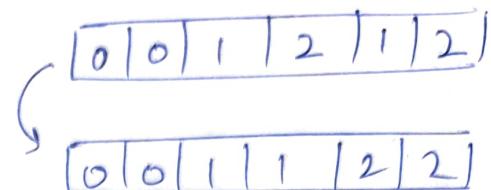
```
int left = 0;
```

```
int right = n-1;
```

```
while (left < right) {
```

while ($\text{arr}[\text{left}] == 0 \text{ } \& \& \text{ } \text{left} < \text{right}$) Q.9) Homework (sort 0, 1, 2)

```
{ left++;  
}
```



while ($\text{arr}[\text{right}] == 1 \text{ } \& \& \text{ } \text{left} < \text{right}$)

```
{ right--;  
}
```

```
if ( left < right ) {
```

```
    swap ( arr [left], arr [right] )
```

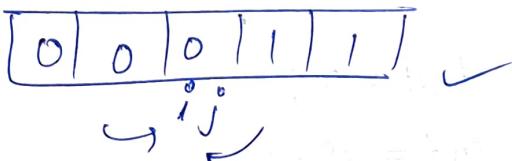
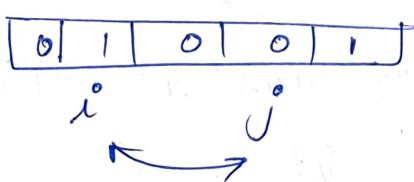
```
    left++;
```

```
    right--;
```

```
}
```

```
}
```

```
}
```



Time & Space complexity

→ Time taken by algorithm to run as a function of length of the input.

Big O notation (worst case)

Theta Θ (avg case)

Omega Ω (lower bound)

constant — $O(1)$

linear — $O(n)$

Quadratic — $O(n^2)$

Cubic — $O(n^3)$

logarithmic — $O(\log n)$.

factorial — $O(n!)$.

$$2n^2 + 3n \rightarrow O(n^2)$$

$$12001 \rightarrow O(1)$$

$$5n^2 + \log n \rightarrow O(n^2)$$

$$\frac{n^3}{300} \rightarrow O(n^3)$$

lowest to highest Complexity

- 1) $O(1)$
- 2) $O(\log n)$
- 3) $O(n)$
- 4) $O(n \log n)$
- 5) $O(n^2)$
- 6) $O(n^3)$
- 7) $O(2^n)$
- 8) $O(n!)$ → highest time
(slowest output).

Print array time complexity.

For ($0 \rightarrow n$) → $O(n)$
linear

Reversing an Array



→ $\frac{n}{2}$ swaps → $O(n)$
linear

Eg. For ($0 \rightarrow n$) {
 }
}

For ($0 \rightarrow m$) {
 }
}

$[O(n+m)]$
sum

for ($0 \rightarrow n$) {
 }
}

for ($0 \rightarrow m$) {
 }
}

{
 }
}

$[O(nxm)]$

Product for
nested loops.

Space Complexity

→ memory to be consumed
by the algorithm.

int a
int b
int arr[5] } $O(1)$

Binary Search

→ array should be sorted.
(monotonic order)
either inc↑ or dec↓ order

include <iostream>

using namespace std;

```
int binary search (int arr[],  
int size, int key) {
```

int start = 0;

int end = size - 1;

int mid = (start + end) / 2;

LIMIT VALUES

$$\text{mid} = \frac{(s+e)}{2}$$

but what if $s = 2^{31} - 1$
 $e = 2^{31} - 1$

then $s+e = \text{out of range}$.

Soln

$$\text{mid} = s + \frac{(e-s)}{2}$$

$$\boxed{\text{mid} = s + \frac{(e-s)}{2}}$$

$(s+e) / 2$

Time Complexity

Linear $\rightarrow O(n)$

Binary $\rightarrow O(\log n)$

Questions

Q.) In given sorted array find leftmost and rightmost occurrence of an element.

Soln → logic

leftmost

→ find mid

→ proceed left subpart of array for every successful iteration of mid

→ else ($\text{mid} \neq \text{element}$) proceed to right subpart.

```

while (start <= end) {
    if (arr[mid] == key) {
        return mid;
    }
    if (arr[mid] < key) {
        start = mid + 1;
    } else {
        end = mid - 1;
    }
    mid = (start + end) / 2;
}
return -1;
}

```

int main() {

int A[10] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10};

int size = 10; int key;

cout << "Enter element to search" <

cin >> key;

int Index = binarySearch(A, 10, 5);

cout << "5 is at Index" << Index
 << endl;

O/P = 5 is at Index 4;

Implementation

```
int firstOcc ( int A[], int n, int key) {
```

```
    int s = 0, e = n - 1;
```

```
    int m = (s + e) / 2;
```

```
    int ans = -1;
```

```
    while (s <= e) {
```

```
        if (A[mid] == key) {
```

```
            ans = mid;
            e = mid - 1;
```

```
        if (key > arr[mid]) {
```

```
            s = mid + 1;
```

```
        } else {
```

```
            e = mid - 1;
```

initial

half portion before mid

mid = s + (e - s) / 2

traversing no for answer

return -1;

}.

from left

binary search

pair <int, int> p

p.first = 5;

p.second = 11;

bin

int lastOcc ()

```
int s = 0, e = n - 1;
```

```
int m = (s + e) / 2;
```

```
int ans = -1;
```

```
while (s <= e) {
```

```
if (A[m] == key) {
```

```
ans = mid;
```

```
s = mid + 1;
```

```
if (key > arr[mid]) {
```

```
s = mid + 1;
```

```
} else {
```

```
e = mid - 1;
```

```
}
```

mid = s + (e - s) / 2;

```
}
```

return -1;

}

}

}

}

}

}

}

}

}

(found = false) not set

so for the binary

+ for the found

Q.) Find total number of key (occurred) in sorted array.

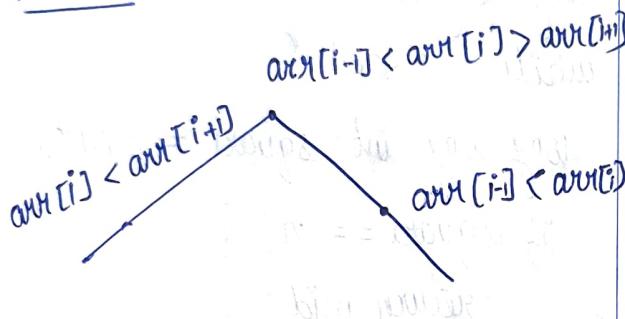
$$T = \text{Last Index} - \text{First Index} + 1$$

Q.) Find peak index in mountain array.

E.g.

0	3	10	5	2
---	---	----	---	---

$$\text{ans} = 2$$



```
int PeakIndex (arr[], size){
```

```
    int s = 0;
    int e = size - 1;
    int m = s + (e - s) / 2;

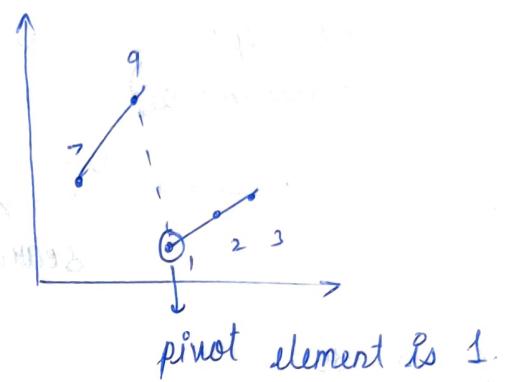
    while (s < e) {
        if (arr[m] < arr[m + 1]) {
            s = mid + 1;
        }
        else {
            e = mid;
        }
        m = s + (e - s) / 2;
    }
    return s;
}
```

Q.) Find pivot in an Array.
sorted array

1	2	3	7	9
---	---	---	---	---

rotated array

7	9	1	2	3
---	---	---	---	---



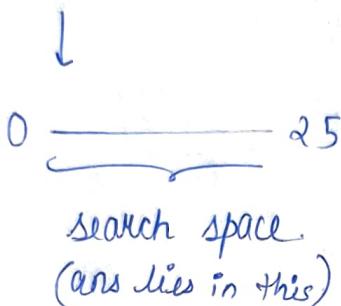
O/P $\rightarrow 1$

Code -

Q) Find square root using binary search (BS).

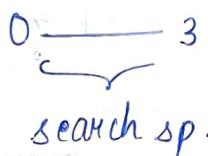
$$I/P \rightarrow n = 25$$

$$O/P \rightarrow ans = 5$$

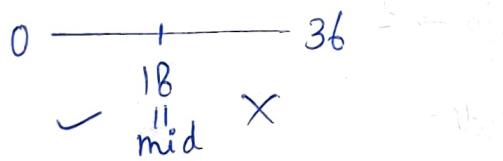


$$I/P = 3$$

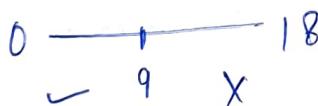
$$O/P = \underline{2}713 = 2$$



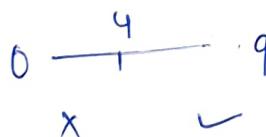
Eg. $n = 36$



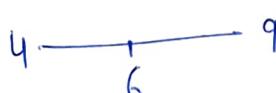
$$18 * 18 = 324 > 36$$



$$9 * 9 = 81 > 36$$



$$4 * 4 = 16 < 36$$



$$6 * 6 = 36 = 36$$

return mid = 6

Code (for integer part).

class solution {

public int sqrt(int n) {

long long int binarysearch (int n)

int s = 0;

int e = n;

long long int mid = s + (e-s)/2;

long long int ans = -1;

while (s <= e) {

long long int square = mid * mid;

if (square == n) {

return mid;

}

if (square < n) {

s = mid + 1;

}

if (square > n) {

e = mid - 1;

}

mid = s + (e-s)/2;

ans = mid;

return ans;

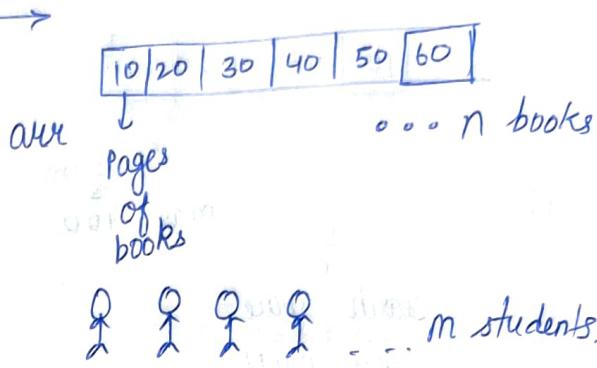
};

(for flood part).

```
double morePrecision (int n,  
int precision, int tempSol){  
    double factor = 1;  
    double ans = tempSol;  
    for (int i=0; i<precision;  
         i++) {  
        factor = factor / 10;  
        for (int j=0; j*j < n; j=j+  
             factor) {  
            ans = j;  
        }  
    }  
    return ans;  
}
```

```
int main () {  
    int n;  
    cout << "Enter the number" <<  
    endl;  
    cin >> n;  
    int tempSol = sqrtInteger(n);  
    cout << "Answer is " <<  
    morePrecision (n, 3, tempSol)  
    << endl;  
    return 0;  
}
```

Q.) Book Allocation problem



-) at least 1 book per student
-) no book should left in arr.
-) contiguous manner.

Eg

	10 20 30 40	n = 4
10	10 20 30 40	max 90
30	10 20 30 40	70
60	10 20 30 40	60

allocate such that the maxm
number of pages assigned to
a student is minimum.

↳ in all possible way, max
pages for one student should
be minimised. (90, 70, 60 → 60)

RETURN 60 as ans

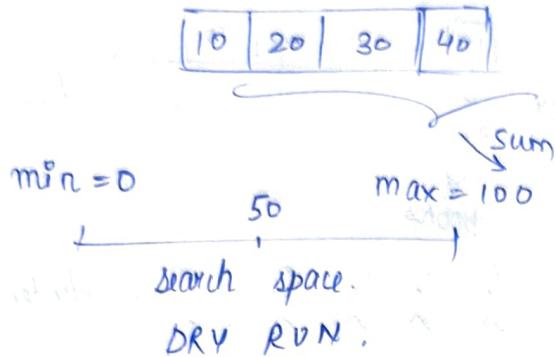
NOTE → Basically, when we
can neglect the left or right
part by finding a possible /
NOT possible solutⁿ on sorted
ans search space we use BS
binary search.

same case

$m=2 \quad n=4$

$$52 \xrightarrow{\alpha} \frac{56}{\alpha} 61$$

mid = 56.



$$I \rightarrow \frac{0+100}{2} = 50 \rightarrow 10+20+30$$

$I \rightarrow 30+40$ is this a possible

$$II \rightarrow 40$$

yes | no

neglect more than 50 solutions. neglect less than 50 ans.

$$e = \text{mid} + 1 \quad s = \text{mid} + 1$$

$$51 \xrightarrow{\alpha} \frac{75}{\alpha} 100$$

$$10 \quad 20 \quad 30 \quad | \quad 40$$

$$I \rightarrow 10+20+30+40 \quad \text{more than } 75.$$

$II \rightarrow 40$

$$51 \xrightarrow{\alpha} \frac{62}{\alpha} 75$$

$$\text{mid} = \frac{51+75}{2} = 62$$

$$I \rightarrow 10+20+30+40 \quad \text{more than } 62$$

$$II \rightarrow 40$$

$$I = 10+20+30$$

$$II = 30+40$$

$$III = 40$$

X

$$587 \xrightarrow{\alpha} \frac{59}{\alpha} 61$$

$$\begin{matrix} 57 \\ 61 \\ 68 \\ 59 \end{matrix}$$

$$I \rightarrow 10+20+30$$

$$II \rightarrow 30+40$$

$$III \rightarrow 40$$

X

$$560 \xrightarrow{\alpha} 61$$

$$I \rightarrow 10+20+30+40$$

$$II \rightarrow 40$$

- 60 ✓

CODE

```
int allocateBooks (vector<int>, int n,
                  int m) {
```

```
bool isPossible (vector<int> arr,
                 int n, int m, int mid) {
```

```

int studentCount = 1;
int pageSum = 0;
for (int i=0; i<n; i++) {
    if (pageSum + arr[i] <= mid) {
        pageSum += arr[i];
    } else {
        studentCount++;
        if (studentCount > m || arr[i] > mid) {
            return false;
        }
        pageSum = arr[i];
    }
}
return true;
}

```

```

int bookAllocation(vector<int> arr,
                    int n, int m) {
    int s = 0;
    int sum = 0;
    for (int i=0; i<n; i++) {
        sum += arr[i];
    }
}

```

```

int e = sum;
int ans = -1;
int mid = s + (e-s)/2
while (s <= e)
{
    if (isPossible(arr, n, m, mid)) {
        ans = mid;
        e = mid-1;
    } else {
        s = mid+1;
    }
    mid = s + (e-s)/2;
}
return ans;
}

```

Q.) Painter's Partition Problem
 (Same question as book allocation)
 give minimum (max) of time
 from all cases.

K = no. of Painters

eg

5	5	5	5
---	---	---	---

$$\min = 0 \qquad \max = 20$$

← search space →

$$mid = 10$$

$$I \rightarrow 5 + 5 + 5 \text{ } \textcircled{D}$$

$$E \rightarrow 5 + 5 \text{ } \textcircled{D}$$

is mid a possible
soln

1

yes

No.

$c \leftarrow m \times s$

store ans

$$0 \xrightarrow{x} 4 \xrightarrow{9}$$

I \rightarrow ~~5~~ α

$$4 \xrightarrow{x} 6 \xrightarrow{9}$$

I \rightarrow 5 + ~~5~~

II \rightarrow 5

III \rightarrow 5 α ($K=2$)

$$6 \xrightarrow{} 9$$

$$8 \xrightarrow{x} 9$$

I \rightarrow 5 + ~~5~~

II \rightarrow 5 + ~~5~~

III \rightarrow α .

Aggressive Cow Problem

Given array length = N

each element denotes position
of a stall. $K = \text{no. of cows}$.

assign cows to stalls such
that they have largest distance.

Return largest minimum
distance.

Distance bet^w stalls is difference
bet^w positions of stalls (elements).

$K = 2$

dist	eg				
2	4	2	1	3	6
3	9	c ₂			
1	9			c ₃	c ₂
2	9			c ₂	c ₂
1	9	c ₂			
2	9	c ₂			
4	9	c ₂			
2	9	c ₂			
5		c ₁			
3			c ₂		

these all are minimum distance
from each other.

in which 5 is largest
return 5

1 search space

1 min = 0

1 max = max posⁱ - min posⁱ

Return target minimum distance.

0 ————— 1 ————— 5

mid.

1

Possible

($\leftarrow m \rightarrow$)

* neglect left

not Possible

($\leftarrow m \rightarrow$)

neglect right.

SORTING OF ARRAY

① Selection Sort

→ find minimum element in array and swap with left most part

SELECTION OF MINIMUM

e.g.

[64 | 25 | 12 | 22 | 11]

↓ ↓ ↓ ↓ ↓

pass I → 11, 25, 12, 22, 64
(min=11) i=0

pass II → 11, 12, 25, 22, 64
(12) i=1

pass III → 11, 12, 22, 25, 64
(22) i=2

pass IV → 11, 12, 22, 25, 64
(25) no swap i=3

output = [11 | 12 | 22 | 25 | 64]

Total passes = $n-1$

```
void selectionSort (vector<int>& arr, int n) {  
    for (int i=0; i<n-1; i++) {  
        int minIndex = i;  
        for (int j=i+1; j<size-1; j++) {  
            if (arr[minIndex] > arr[j]) {  
                minIndex = j;  
            }  
        }  
        swap (arr[minIndex], arr[i]);  
    }  
}
```

}
} // time complexity - O(n^2)

space complexity.

Best case - O(n^2)

worst case - O(n^2).

stable sorting → same no.

In array stays at same order after sorting

Bubble Sort

→

[10 | 1 | 7 | 6 | 14 | 9]

Making element of leftmost part and comparing with its right part.

here comparing 10 & 1

In each round largest gets placed.

R1.

10	1	7	6	14	9
----	---	---	---	----	---

1	10	7	6	14	9
---	----	---	---	----	---

1	7	10	6	14	9
---	---	----	---	----	---

1	7	6	10	14	9
---	---	---	----	----	---

Since $10 < 14$

move to greatest next element

1	7	6	10	14	9
---	---	---	----	----	---

1	7	6	10	9	14
---	---	---	----	---	----

Round 1 completed.

No. of rounds = $n - 1$

R2

1	7	6	10	9	14
---	---	---	----	---	----

1	7	6	10	9	14
---	---	---	----	---	----

1	6	7	10	9	14
---	---	---	----	---	----

1	6	7	10	9	14
---	---	---	----	---	----

Round 2 completed.

1	6	7	9	10	14
---	---	---	---	----	----

space complexity = $O(1)$
time complexity = $O(n^2)$
stable sort

Insertion Sort

→ Take 1st element and insert other elements by comparing them with taken element

e.g.

10	1	7	4	8	2	11
----	---	---	---	---	---	----

1	(10)	—	—	—	—	—
---	------	---	---	---	---	---

1) compare 10 & 1. $10 > 1$

1 left side of 10.

shift 10 \Rightarrow right side by 1

1	10	i	7	4	8	2	11
---	----	---	---	---	---	---	----

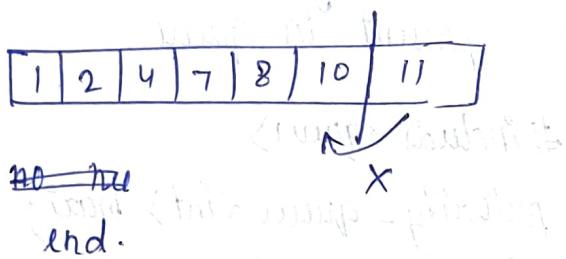
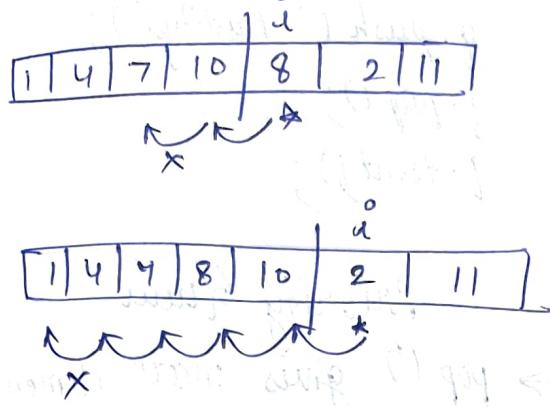
2) now, i take 7

1	7	i	10	4	8	2	11
---	---	---	----	---	---	---	----

shift 10 right

3) Take 4
compare '10' with 4 $10 \cancel{>} 4$
compare '7' with 4 $7 \cancel{>} 4$
compare '1' with 4 $1 \cancel{>} 4$
STOP

shift till taken element is
lessor



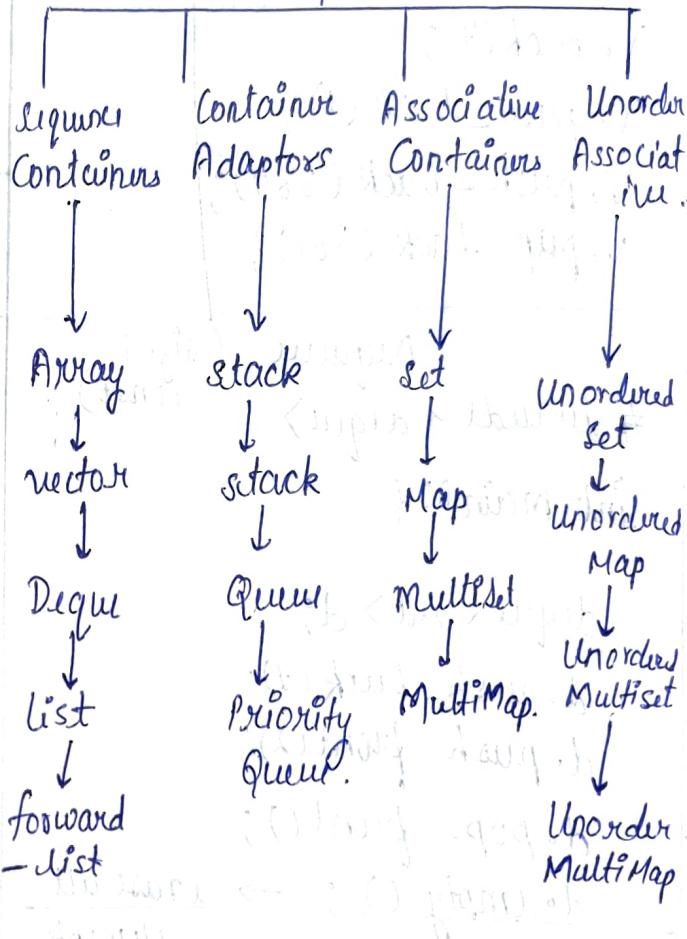
Code on NS code

space $\rightarrow O(1)$

Time $\rightarrow O(n^2)$

Best case \rightarrow already sorted.
Worst case \rightarrow If you want incⁿt
order but array is sorted in
dict order.

Containers



vector Data type

\rightarrow similar to array but it doubles
its capacity when we try
put elements when the vector
is full.

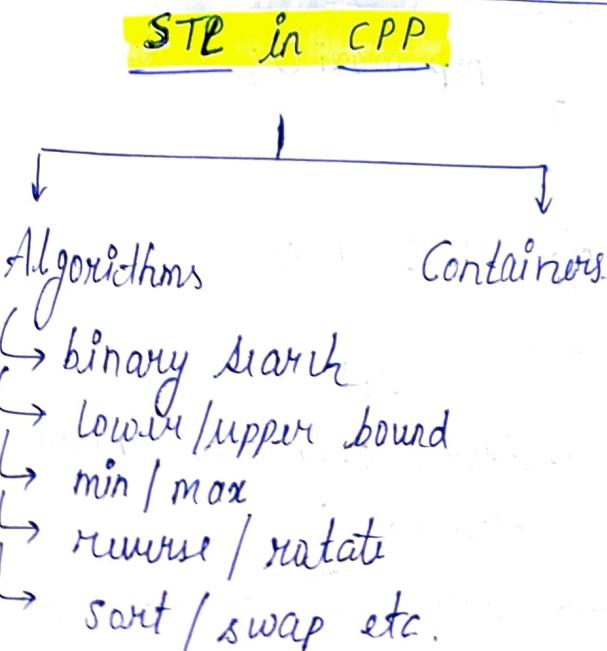
#include <iostream>

#include <vector>

using namespace std;

int main() {

vector<int>



v.capacity();

v.size();

v.front();

v.back();

v.push_back(1);

v.push_back(387);

v.pop_back(387);

v.b

Queue

FIFO

#include <queue>

q.push("Aaditya");

q.pop();

q.front();

Priority Queue

→ pop() gives max^m element present in queue.

#include <queue>

priority-queue <int> maxi;

priority-queue <int, vector<int>, greater<int>> mini;

maxi.push(1);

→ // → (3);

→ // → (2);

→ // → (0);

for {

pop.maxi();

}

O/P = 3 2 1 0

Set

→ Stores only new unique element

→ returns in sorted ways.

Dequeue (size is const).

#include <deque>

int main(){

deque<int> d;

d.push_back(1);

d.push_front(2);

d.pop_front();

d.empty(); → erase all elements.

d.erase(d.begin(), d.begin() + 1);

List

#<list>

list<int> l;

#inc <stack>

s.push("love");

s.push("babbar");

s.pop();

O/P → [love] → babbar (popped)

LIFO

```
#include <set>
main() {
    set<int> s;
    s.insert(5);
    s.insert(5);
    s.insert(4);
    s.insert(4);
    s.insert(3);
    for {
        s.pop() << endl;
    }
}
```

o/p →

1	s.count(5); v(1)
3	below 1 user input
5	element absent.
7	present

Map

```
#include <map>
using n...
main() {
    map<int, string> m;
    m[1] = "babbar";
    m[2] = "lone";
    m[3] = "kumar";
    for (string auto i:m) {
        cout << i.first << endl;
    }
}

```

o/p

1	babbar
2	lone
3	kumar

```
for (auto i:m) {
    cout << i.first << " " << i.second
}
m.insert({5, "bheem"});

```

O/P

1	babar
2	lone
5	bheem
13	kumar

Algorithm

```
#include <algorithm>
main() {
    vector<int> v;
    v.push_back(1);
    v.push_back(3);
    v.push_back(6);
    for (int i = 1; i <= 7; i++) {
        cout << binary_search(v.begin(),
                               v.begin() + v.end(), i)
            << endl;
    }
    cout << lower_bound(v.begin(), v.end(), 6)
        - v.begin();
}
```

Upper_bound (v.begin(), v.end(), 4)

max(a, b)
min(a, b)
swap(a, b)

Vector Detail fn's

vector <datatype> vectorname;

* Iterators

- 1) v.begin()
- 2) v.end()
- 3) v.rbegin()
- 4) v.rend()

add element(s) in front by:

v.insert(v.begin(), 56);

* capacity

- 1) v.size(); → no. of elements
- 2) v.max_size();
- 3) v.capacity(); → capable of holding
- 4) v.resize(n); → resize to n.
- 5) v.empty(); → $\begin{cases} 0 & (\text{empty}) \\ 1 & (\text{not empty}) \end{cases}$
- 6) v.shrink_to_fit();
- 7) v.reserve();

* Data / Element access

- 1) v.reference-operator [g];
- ↳ returns a reference to the element at position 'g'.

- 2) v.at(g); Return element at
- 3) v.front();
- 4) v.back();
- 5) v.data();

↳ returns direct pointer to the memory array used internally by the vector to store its owned elements.

Modifiers

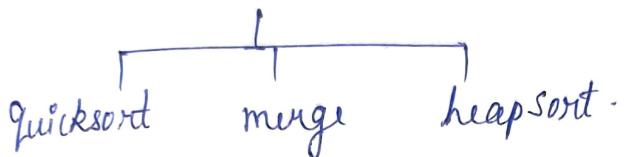
- 1) v.assign();
→ assigns new value to the vector elements by replacing old ones
- 2) push_back();
- 3) pop_back();
- 4) v.insert();
→ inserts new elements from a cont before the element at specified position
- 5) v.erase();
↳ removes a range of elements
- 6) swap();
- 7) emplace();
↳ it extends the container by inserting new elements at position.
- 8) emplace_back(); adds at end.

string abcd = "abcd";

reverse (abcd.begin(), abcd.end());

rotate (v.begin(), v.end());

sort (v.begin(), v.end());



Character Arrays

and String

char a = 'z'; } stores only 1 character.

→ 1 dimension character arrays ending with '\0' aka strings

input syntax

char name [20];

i/p cin >> name;

Additya

A a d i t y a \0 | | | | | |

↓ ...
terminator (NULL)

ASCII value } 0

o/p → cout << name;

cin >> stops taking input if given space in char array.

Q) Check if string is Palindrome.

→ if reversing the string does affect string order

e.g. uttar, malayalam

Q) Convert capital letters to small letters.

'A' ... 'Z'

'a' ... 'z'

e.g. char ch = 'B'

ch = ch - 'A' + 'a'; ~~ch = ch - 'a' + 'A'~~

ch = b

small to capital
ch = ch - 'a' + 'A'

Q) get ASCII value

main() {

char ch = 'A';

cout << " ASCII value of " << ch
" is " << int(ch);

}

O/P

ASCII value of A is 65

ASCII value diff betw

capital & small letters is 32

'A' - 'a' = 32

'B' - 'b' = 32 etc.

using string to store sentences.

```
#include <iostream>
using namespace std;
int main() {
    char str[50];
    cin >> str >> endl;
    return 0;
}
```

QUESTION PAPER CLASS 11

Q) Replace spaces from string with @ 40
eg my name is Aaditya

My @40 name @40 is @40 Aaditya

string replaceSpaces (string str){ }

string temp = " ";

for (int i=0; i<str.length(); i++)

if (str[i] == ' ') { }

temp.push_back('@');

temp.push_back('4');

temp.push_back('0');

}

else { }

temp.push_back(str[i]);

}

return temp;

Q.) Remove all occurrences of substring
eg given input abc
abc
o/p dabbc

string removeOccurrences (string s, string part) { }

while (s.length() != 0) { }

{ }

{ }

{ }

{ }

{ }

{ }

{ }

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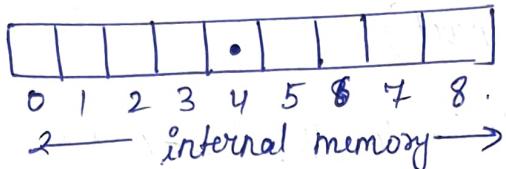
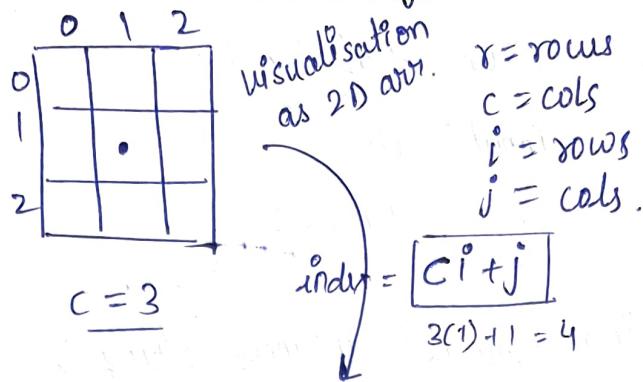
{ }

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{ }

2D - Arrays



Syntax : `int arr[3][3];`

`cin >> arr[i][j];`
`cout << arr[i][j];`

ROW COLUMN

`int arr[3][4] = {{1, 11, 111, 1111}, {2, 22, 222, 2222}, {3, 33, 333, 3333}};`

→ this is row wise input

O/p 1 11 111 1111
 2 22 222 2222
 3 33 333 3333

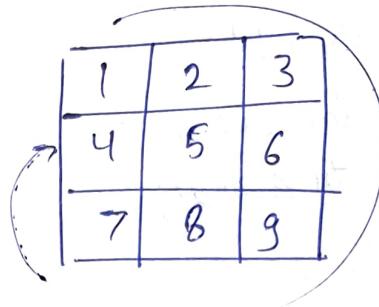
→ NOTE we need to specify the column size when passing 2D array as a parameter

because arrays are stored in memory row by row in contiguous block.

Q.) P

vector (vector) → 2D vector

Q.) spiral print of 2D array OR 2D vector.



O/p = 1 2 3 6 9 8 7 4 5

Approach

- 1) Print 1st Row
- 2) Print last col
- 3) Print last row
- 4) Print starting col.

`vector<int> spiral(vector<vector<int>> &matrix) {`

`int row = matrix.size();`
`int col = matrix[0].size();`

`int count = 0;`

// to count how many elements are being printed //

`int total = n*m; row*col;`

`int startingRow = 0;`

`int startingCol = 0;`

`int endingRow = row-1;`

`int endingCol = col-1;`

```

while ( count < total ) {
    for ( int index = startingCol ;
        count < total && index
        <= endingCol ; index++ ) {
        count++;
        ans.push_back ( matrix
            [startingRow][index] );
    }
    startingRow++;
}

for ( int index = startingRow ;
    count < total && index <
    endingRow ; index++ ) {
    ans.push_back ( matrix [index][
        endingCol ] );
    count++;
}

endingCol--;

```

endingRow --;

startingCol++;

return ans.

Q:-) Binary Search in 2D array.
↳ (sorted).

89

1	3	5	7
10	11	16	20
23	30	34	60

→ convert to linear array

Q.) Rotate matrix

Q.) Search 2D Matrix

Maths for DSA.

1) Prime no. (for ($i=2$ to $n-1$))
 $P \% i;$

Sieve of Eratosthenes.

```
→ int countPrime (int n) {  
    int count = 0;  
    vector<bool> prime (n+1, true);  
    prime [0] = false;  
    prime [1] = true;
```

```
for (int i=2; i<n; i++) {
```

```
if(prime[i]) {
```

count++;

for (*int* $j^{\star} 2$; $j < n$; $j = j + 1$)

prime $[j] = 0$

3 }

return count;

3

$$O(n \log(\log n))$$

startingCol++;

GCD, greatest common divisor
or HCF

$$\gcd(a, b) = \gcd(a-b, b)$$

↓ OR2

$$\gcd(a \% b, b)$$

e.g. $g(72, 24) \rightarrow g(48, 24) \rightarrow$
 $g(24, 24) \rightarrow g(0, 24)$
↓
if one becomes zero
other number is
ans.

Fast exponential.

a^b , $\rightarrow T(n) \in O(b)$
↳ normal loop

a^b ↳ $(a^{b/2})^2$ if b is even
↳ $(a^{b/2})^2 * a$ if b odd.

int exp(int x, y) {

int ans=1;

while (n>0)

{

if (n&1) {

 ans = ~~ans * x~~;

 x = ~~x * x~~;

 n = ~~n >> 1~~;

return ans;

int exp(int x, y) {

↳ # Pigeon hole → Pigeon hole

Catalan no.

$$C_n = \frac{1}{(n+1)} \times {}^{2n} C_n$$

691

$$C_n = {}^{2n} C_n - {}^{2n} C_{n-1}$$

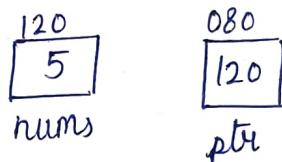
$n \in 0 \text{ to } \infty$

Painters in C++

'address of' = &

Pointer stores address

e.g. int *ptr = & nums;



e.g. char ch = 'a';
char *p = & ch;

↳ p is pointer to address of ch

'*' value at address
gives

cout << *p << *ptr;

o/p a 5

ch and *p will give
same value.

address numbers are in hexadeciml forms.

•) size of pointer is always '8' byte. any datatype

same task - two syntax

int i=5;

int *q = &i; }

0H

int *q = 0;

q = &i;

some

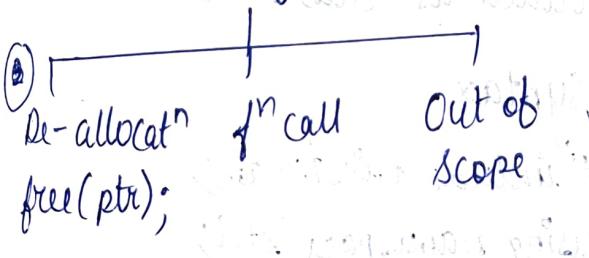
{ based on
whatever is in
memory at that
time}

Types of Pointers

① Dangling Pointer

→ A pointer pointing to a memory location that has been deleted (or freed) is called a dangling pointer.

→ Source of bug.



② Void pointer

Void *ptr

→ doesn't have specific data types

→ Void pointers can't be dereferenced

however it can be done using typecasting. int x=4;

e.g. void *ptr = &x;

printf("%d", *(int *)ptr);

⇒ p = 4.

③ NULL pointer

e.g. int *ptr = 0;

or

int *p = NULL;

→ Array name is pointer to array itself.

arr[i] = *(arr + i)

internal execution for evaluation

i[arr] = arr[i]

are same thing.

sizeof(arr) = n x 4 =
no. of int ↑ size of int

sizeof(ptr) = 8 { stores add}.

cout fⁿ for int array and char array are differently implemented.

char *c = &ch[0];

cout << c << endl;

↳ prints entire string.

pointers and functions

- 1) When we passed pointer in f^n , original value is change (value in main f^n) because it goes to address for modification.
- 2) When array is passed in f^n , only pointer to first index of array is passed (created) or accessed.

↳ benefit: you can send part of array

e.g.

function (arr + 3)

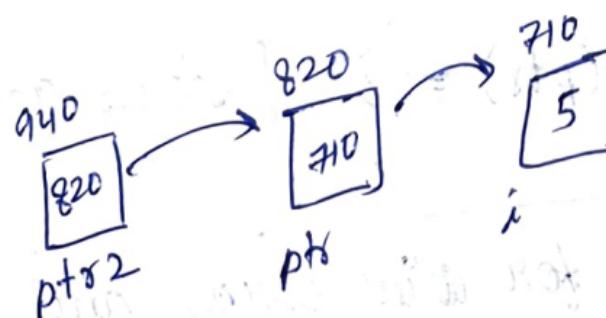
~~ argument

Double pointer

```
int i = 5;
```

```
int *ptr = &i;
```

```
int **ptr2 = &ptr;
```



symtable

you can't change this.

{	$i \rightarrow 710$
	$ptr \rightarrow 820$
	$ptr2 \rightarrow 940$

Double ptr & func

func (int **p)

*^Ap = *^Ap + 1;

Reference Variable → (Same memory, diff names)

int i=5;
int& j=i; i [5] j
710

why?

→ to use "pass by reference" in function argument.
update (int& n).

Return by Ref (Bad Practice)
int& update (int n)

int a = n;

int& num=a;

return num;

}

But we don't know whether 'num' is a global or local variable.

It will die outside function block (if local). So nothing is being returned.

Arrays

int n;

cin >> n;

int arr[n];

↳ Bad practice

Size of array should be known at compile time not runtime.

memory allocations.

stack

(static)

e.g. int a=5;

int arr[5];

char c;

Heap

(dynamic)

'new'

keyword

e.g.

new char;

↳ address return

(no name allowed)

Heap

• no variable names allowed.

• let's create a pointer to point to our dynamically allocated memory.

char *ch = new char;

stack

Heap



dynamic array.

~~new int[5];~~

int *arr = new int[5];

total memory used

- ① pointer always = 8
② array = $4 \times 5 = 20$
- stack
heap

NOTE:

allocated memory in heap doesn't get freed after lifespan of var is over.

But in stack allocation, it gets automatically freed.

To free

int *arr = new int[50]

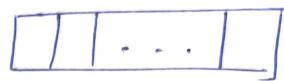
delete []arr;

init n

int *arr = new int[n]

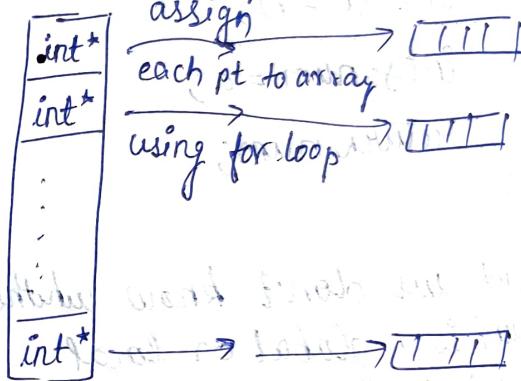
2D Array (static) arr[n][n]

int *arr = new int[n]



int **arr = new int *[n];

↖ represents array which contains pointers to multiple dynamic arrays



code

```
#include <iostream>
```

```
using namespace std;
```

```
int main()
```

```
int n
```

```
cin >> n;
```

```
int **arr = new int *[n];
```

```
for (int i=0; i<n; i++) {  
    no. of rows
```

```
    arr[i] = new int [n];
```

```
}
```

taking input

lec 30

Macros

```
for (int i=0; i<n; i++) {  
    for (int j=0; j<n; j++) { #define PI 3.14  
        cin >> arr[i][j];  
    }  
}
```

printing

```
for (int i=0; i<n; i++) {  
    for (int j=0; j<n; j++) {  
        cout << arr[i][j];  
    }  
    cout << endl;  
}
```

→ a piece of code in a program that is replaced by value of macro. (before compile time)

define Macro-Name Macro-def.

usually
uppercase

Types

- ① Object like M (simply replaces)
- ② Function like M (takes argument)
- ③ Multiline M (same as function)
- ④ Chain Macro.

* Always free used memory used from Heap, manually.

```
for (int i=0; i<n; i++) {  
    delete [] arr[i];  
}  
  
delete [] arr;
```

return 0;

Global Variables

- ↳ Do not use G.V for variable sharing.
- ↳ Any function can change G.V
- ↳ Use Reference Var

Inline functions

→ Reduces the function calls overhead.

- If body has 1 line body then use inline function.
- It replaces f^n call by f^n body just before compilation.

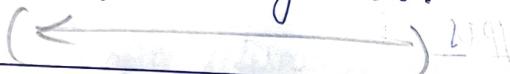
Default Arguments

→ we want to make one argument optional.

Syntax

```
void print (int arr[], int size,
            int start = 0) {
    for ( ) {
        cout
    }
}
```

NOTE : DA starts from Rightmost to left most arguments.



Recursion lec 31

- When a function calls itself
- If question can be divided into multiple of same type of subquestions.

e.g. EXP. $2^n = ?$

$$f(n) = 2 \times f(n-1)$$

$$2^n = 2 \times 2^{n-1}$$

$$2^{n-1} = 2 \times 2^{n-2}$$

Recurrence Relation

e.g factorial

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$= 5 \times 4!$$

$$n! = n \times (n-1)!$$

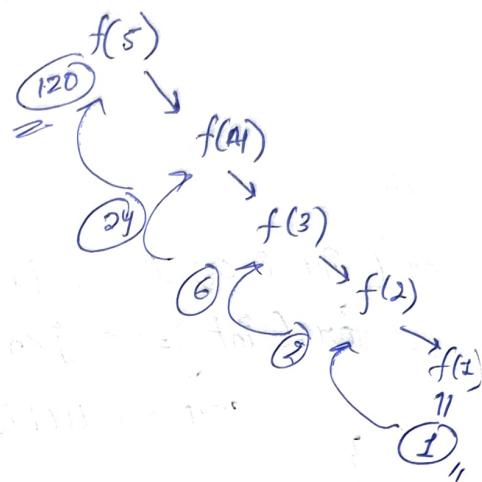
$$f(n) = n \times f(n-1)$$

recursive relation

BASE CASE
 $1! = 1$

→ Must contain a base case / base condition.

Recursion Tree



Q) Print Fibonacci series. Sec 32

0, 1, 1, 2, 3, 5, 8, 13, 21, 34,

$$f_1 = 0$$

$$f_2 = 1$$

$$f_3 = f_1 + f_2$$

$$f_4 = f_2 + f_3$$

$$f_5 = f_3 + f_4$$

:

$$f_n = f_{n-1} + f_{n-2}$$

$$\textcircled{1} \quad T_n = T_{n-1} + T_{n-2}$$

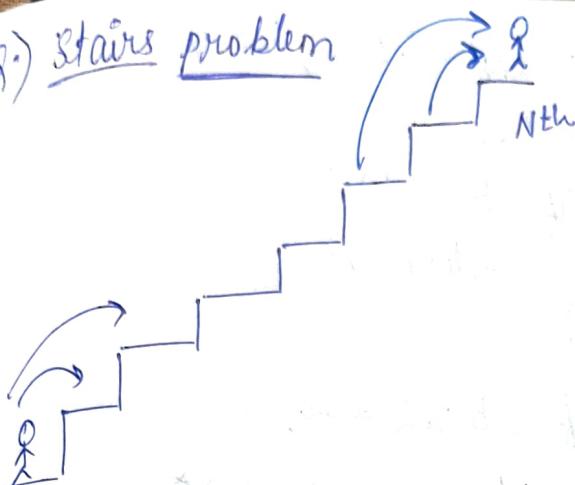
Recurrence relation

\textcircled{2} base case

$$f(0) = 0$$

$$f(1) = 1$$

Q.) Stairs problem



If you can go walk one or two steps at a time, then find number of ways to reach Nth stair.

$$f(n) = f(n-1) + f(n-2)$$

Nth stair steps till (n-1)th step steps till (n-2)th step

Recursive Function Skeleton

```
int function (int a) {  
    if (base case) {  
        return value;  
    }
```

recursion relation

```
return = f(a-1) + f(a-2)  
}
```

Q.) Say digit

I/P → 4 1 2

O/P → four one two

lec 33 1) is sorted.

2) Binary Search using recursion

2	4	6	9	11
---	---	---	---	----

Q.) sum of array using Recur

Binary Search Using Recursion

```
bool binarySearch (int *arr, int s, int e, int k) {
```

base case

if (s > e)

return false;

```
int mid = (s+e);
```

if (arr[mid] < k) {

```
    return binarySearch (arr, mid+1, e, k);
```

} else {

```
    return binSearch (arr, s, mid-1, k);
```

}

lec 84

Q.) reverse a string. using Recursion

```
void ms (string &s, int i, int j)  
if (i > j) { return; }  
swap (s[i], s[j]); i++; j++;  
return ms (s, i, j);
```

NOTE

→ always keep in mind that a function makes a copy of its pass by value parameters.

- 1) Pass by reference
2) Print string (copy) OR in function itself.

Q-2) Check Palindrome using Recursion.

bool checkP(int i, int j, string s)

if ($i > j$) {
 return true;

if ($s[i] \neq s[j]$) {
 return false;
} else {
 return checkP($i+1, j-1, s$);

}

Q-3) a^b optimised way.

$$b \rightarrow \text{even} \quad a^b = a^{b/2} \times a^{b/2}$$

$$b \rightarrow \text{odd} \quad a^b = a(a^{b/2} \times a^{b/2})$$

$$10 \quad 2^{1024} = \underbrace{(2^{512})^2}_{\downarrow} = (2^{256})^2$$

$$(2^{512})^2 \rightarrow (2^{256})^2 \rightarrow (2^{128})^2 \rightarrow (2^{64})^2$$

$$(2^{32})^2 \leftarrow (2^{16})^2 \leftarrow (2^8)^2 \leftarrow (2^4)^2$$

```
int exp (int a, int b) {
    // base
    if (b == 1) {
        return a;
    }
    if (b % 2 == 0) {
        return exp(a, b/2) * exp(a, b/2);
    } else {
        return a * exp(a, b/2) * exp(a, b/2);
    }
}
```

Bubble Sort using Recursion

void sort (int *arr, int n) {

if ($n == 1$) {
 return;

for (int i=0; i<n-1; i++) {
 if ($\text{arr}[i] > \text{arr}[i+1]$) {
 swap ($\text{arr}[i], \text{arr}[i+1]$);
 }
}

sort (~~arr~~, n-1);

}

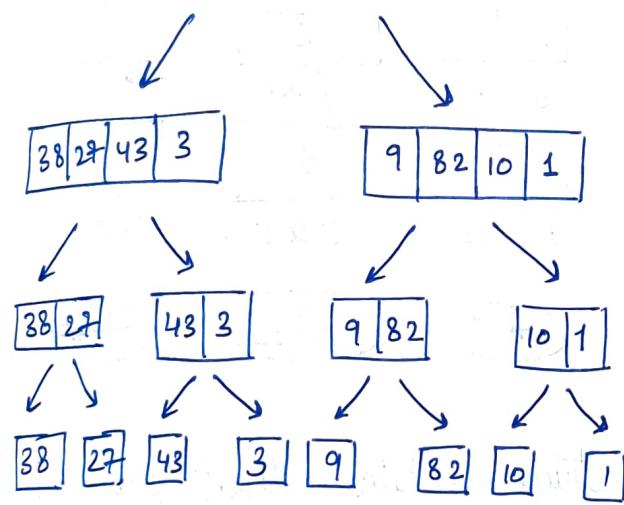
MergeSort Using Recursion

Dev 35

→ merging of two sorted arrays. (Algorithm)

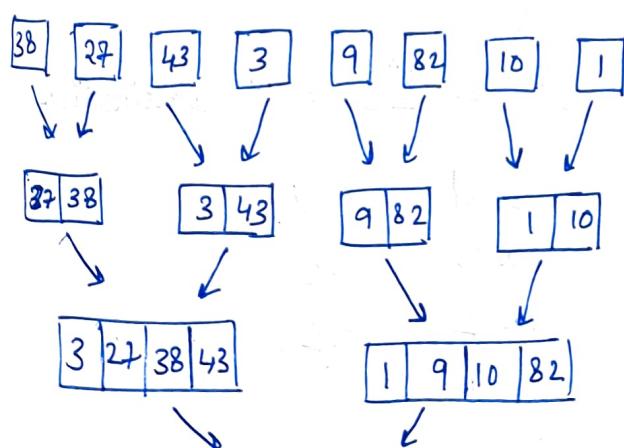
0	1	2	3	4	5	6	7
38	27	43	3	9	82	10	1

$$\text{mid} = \frac{7}{2} = 3.$$



→ ~~to~~ Arrays with single element are always sorted.

→ Now merge all sorted arrays one by one.



SORTED ARRAY

Implementation.

→ creating 2 new arrays each time and coping the elements.

space Complexity = $O(n)$.

Time Complexity = $O(n \log n)$.

Quciksort Using Rec.

1) Take first element and place it in array such that all elements in left should be smaller and all in right should greater.

e.g. Partition. 1st

3	5	1	8	2	4
		^			

1	2	3	5	8	4
< 3		^	pivot element	> 3	1st pass complete

- ① Take 1st element as pivot
- ② Count elements less than pivot
- ③ Place pivot at count.
- ④ Swap elements using i & j index until all elements less than pivot comes to its left side and all greater elements to its right side.

Dry run

10	80	30	90	40	50	70
----	----	----	----	----	----	----

$$\text{pivot} = 10$$

$$\text{count} = 0$$

$$s = 0$$

pivot should be placed at $s+c$

$$= 0 + 0$$

$$= 0$$

right sort

80	30	90	40	50	70
----	----	----	----	----	----

^

$$\text{count} = 4$$

$$s = 1$$

1	2	3	4	5	6	7	8	9	*	0	#
---	---	---	---	---	---	---	---	---	---	---	---

50 | 30 40 70 80 90

50, 30, 40, 70, 80, 90

- > Quicksort is in-place sorting
- > Doesn't require any extra space
- > mergesort requires $\Theta(N)$ extra space hence less preferred.

Both have time: $O(N \log N)$.

Quicksort

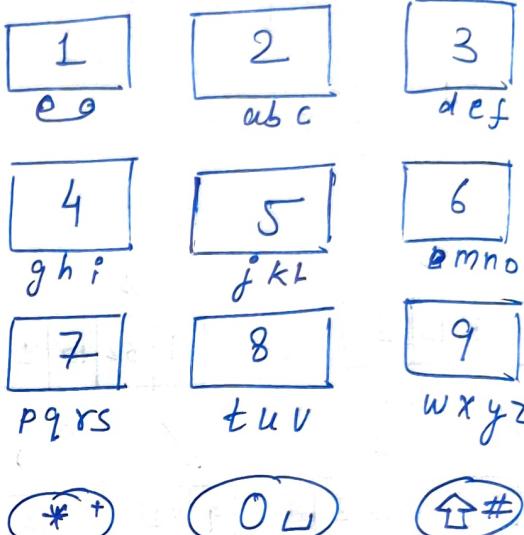
$$\text{avg} = O(n \log n)$$

$$\text{best} = O(n \log n)$$

$$\text{worst} = O(n^2)$$

Subsets & subsequence of string
~~(1) take powerset of an array~~
lec 38.

Q.) Phone Keypad problem
using recursion



string contains digit from 2 to 9

return all possible combinations of alphabets in any order.

e.g.

string = 232 $3 \times 3 \times 3 = 27$ combinations

str = 23 = 9 combinations

a d	c d
a e	c e
a f	c f
b d	
b e	
b f	

code

```
void solve (string digit, string
           output, int index, vector<string>
           &ans, string mapping [ ]) {
```

// base

```
if (index >= digit.length()) {
    ans.push_back (output);
    return;
}
```

```
int number = digit [index] - '0';
```

```
string value = mapping [number];
```

```
for (int i=0; i< value.length(); i++) {
```

```
    output.push_back (value[i]);
```

```
    solve (digit, output, index+1,
           ans, mapping);
```

```
    output.pop_back ();
```

}

}

```
main () {
```

```
if (digit.length == 0) {
```

```
    return ans;
```

```
    string output;
```

```
    int index = '0';
```

string mapping [10] = { ".",
 " ", "abc", "def", "ghi",
 "jkl", "mno", "pqrs",
 "tuv", "wxyz" };

solve (digits, output, index,
 ans, mapping);

return ans;

}

lec 39

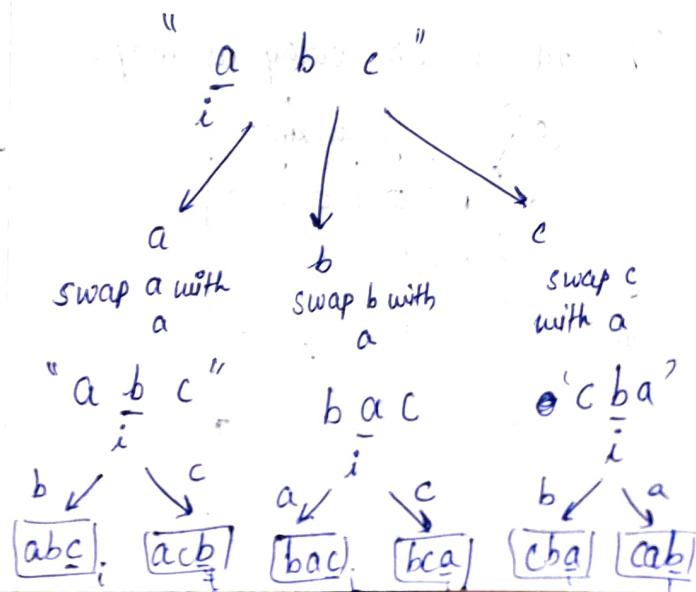
Q.) Permutations of a string.

input = string = "abc"

output = abc, acb, bac, bca, cab,
cba.

Total permutations = $6 = n!$

Approach (Recursion Tree)



NOTE : Do swapping fab tak 'i' bahan na chala jaye.

1 → open path
0 → closed path.

code

```
void solve (vector<int> nums,
vector<vector<int>> &ans, int index){
    if (index >= nums.size()) {
        ans.push_back(nums);
        return;
    }

    for (int j = index; j < nums.size(); j++) {
        swap (nums[j], nums[index]);
        solve (nums, ans, index + 1);
        // backtrack
        swap (nums[j], nums[index]);
    }
}
```

starting from → (0,0) → 1
destination → (n-1, n-1) → 1
give all possible solutions

- ① DDRD RR
- ② DR DDRR R

for x rows & y columns.
Up = (x-1, y)

down = (x+1, y) top left (0,0)

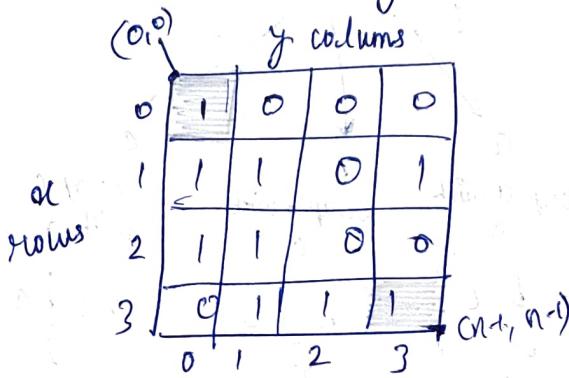
left = (x, y-1) bottom right = (x, y+1). right (n-1, n-1)

visited array

v	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

To move Rat to next cell we have to check 3 conditions

- ① $m[i][j] == 1$ { Path exist }
- ② $v[i][j] == 0$ { unvisited }
- ③ next cell should be a valid cell (inside m).



move in four dir. LRUD

when function call returns,
make sure u reset visited
array.

code

```
void solve (vector<vector<int>> &m, int n, vector<string>&ans  
           int x, int y, vector<vector<int>> &visited, string path)  
{  
    // reached (x, y)  
    // base case  
    if (x == n - 1 && y == n - 1) {  
        ans.push_back(path);  
        return;  
    }  
    // 4 choices D, L, R, U (lexicographical order)  
    // down  
    int newx = x + 1;  
    int newy = y;  
    if (isSafe (newx, newy, m, visited, n)) {  
        path.push_back ('D');  
        solve (m, n, ans, newx, newy, visited, path);  
        path.pop_back ();  
    }  
    // left  
    newx = x;  
    newy = y - 1;  
    if (isSafe (...)) {  
        path.push_back ('L');  
        solve (...);  
        path.pop_back ();  
    }  
    // Right  
    newx = x;  
    newy = y + 1;  
    if (isSafe (...)) {  
        push  
        solve  
        pop  
    }  
}
```

11 up

newx = x - 1

newy = y

if (isSafe){

push

salve

}

pop

visited [x][y] = 0;

}

bool isSafe (int x, int y, vector<vector<int>> m, visited, int n){

if ((x >= 0 && x < n) && (y >= 0 && y < n) && visited [x][y] == 0
&& m[x][y] == 1) {

return true;

} else {

return false;

.

vector<string> findPath (vector<vector<int>> &m, int n) {

vector<string> ans;

if (m[0][0] == 0) {

return ans;

}

int srcx = 0;

int srcy = 0;

```
vector < vector < int > > visited;
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        visited[i][j] = 0;
    }
}
```

string path = " ";

```
solve(m, n, ans, stack, stack, visited, path);
sort(ans.begin(), ans.end());
return ans;
```

}