# Mathematical Appendix for Bitcoin Valuation Theory

We present an exploration into how Bitcoin's price can be modeled as the temporal collapse of future monetary states into the present. We provide formulas for supply dynamics, network strength, and phase progression, leading to a comprehensive model for price formation. This approach aims to clarify the mechanisms by which Bitcoin's value emerges, offering a foundation for further empirical and theoretical work in the field.

## 1 Supply Certainty

Bitcoin's supply function $S(\tau)$ can be modeled considering the halving events:

### 1.1 Supply at Block Height $h$:

$$S(h) = \begin{cases} 21 \times 10^6 - \text{remaining supply after halving events} & \text{if } h < 6,930,000 \\ 21 \times 10^6 & \text{if } h \geq 6,930,000 \end{cases}$$

where $6,930,000$ blocks correspond to 33 halvings, after which no new bitcoins are created.

### 1.2 Halving Impact:

$$S(\tau) = S_0 \left( 1 - \frac{1}{2^{\lfloor \frac{\tau - t_0}{T} \rfloor}} \right)$$

where:

- $S_0$ is the initial supply,
- $t_0$ is the time of the first halving,
- $T$ is the interval between halvings (approximately 4 years or $210,000$ blocks),
- $\lfloor \cdot \rfloor$ denotes the floor function.

## 2 Network Evolution

Let $N(\tau)$ be a function representing network strength:

### 2.1 Network Strength:

$$N(\tau) = \alpha \cdot N_c(\tau) + \beta \cdot G_d(\tau) + \gamma \cdot D_n(\tau)$$

where:

- $N_c(\tau)$ is the node count at time $\tau$,
- $G_d(\tau)$ is a measure of geographic distribution,
- $D_n(\tau)$ represents node type diversity,
- $\alpha, \beta, \gamma$ are coefficients determined by regression or expert judgment.

## 3 Phase Progression

### 3.1 Monetary Phase Function $M(\tau)$:

$$M(\tau) = \sum_{i=1}^{4} w_i \cdot P_i(\tau)$$

where $P_i(\tau)$ is a step function depending on market indicators for each phase (speculation, store of value, medium of exchange, unit of account), and $w_i$ are phase weights.

### 3.2 Phase Completion Function $\phi(\tau)$:

$$\phi(\tau) = \frac{\tau - t_{\text{last phase}}}{t_{\text{next phase}} - t_{\text{last phase}}}$$

where $t_{\text{last phase}}$ and $t_{\text{next phase}}$ are times of phase transitions.

## 4 Temporal Collapse Model

### 4.1 Price Equation:

$$P(t) = \int_t^{\infty} M(\tau) \cdot S(\tau) \cdot \phi(\tau) \cdot e^{-\rho(h(\tau)) \cdot (\tau - t)} \, d\tau$$

### 4.2 Time Preference Rate $\rho(h(\tau))$:

$$\rho(h(\tau)) = \rho_0 + \rho_1 \cdot \log\left(\frac{21 \times 10^6}{S(\tau)}\right)$$

where $\rho_0$ is a base rate and $\rho_1$ reflects how time preference decreases with increasing scarcity (monetary hardness).

# 5 Discrete Approximation for Practical Use

Given the computational complexity of the integral, we can use a discrete approximation:

## 5.1 Discrete Price Calculation:

$$P(t) \approx \sum_{\tau=t}^{t+\Delta T} M(\tau) \cdot S(\tau) \cdot \phi(\tau) \cdot e^{-\rho(h(\tau))\cdot(\tau-t)} \cdot \Delta t$$

where $\Delta T$ is a sufficiently large time horizon, and $\Delta t$ is the step size.

# 6 Implications for Options Pricing

## 6.1 Option Value:

$$V_{\text{option}} = \int_{S(t)}^{\infty} \max(S(T) - K, 0) \cdot P(S(T), T) \, dS(T)$$

where $P(S(T), T)$ would now depend on phase functions rather than just on volatility.

# 7 Validation Metrics

## 7.1 Correlation Analysis:

$$\text{Corr}(P(t), M(t)) \quad \text{and} \quad \text{Corr}(P(t), \phi(t))$$

## 7.2 Option Implied Probabilities:

$$\mathbb{P}(M(T) = i) = f(\text{market data}, T)$$