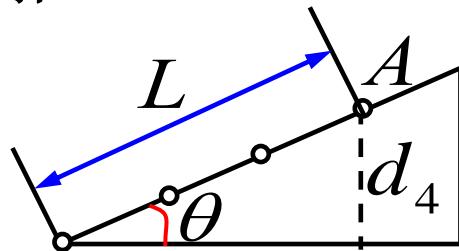


例9：用波长500nm的单色光，垂直照射到空气劈尖上。距棱边 $L=1.56\text{cm}$ 的A处是从棱边算起的第四条暗纹中心，

求：1. 空气劈尖的顶角。

2. 若改用600nm的单色光垂直照射，从棱边到A处共有几条明纹？几条暗纹？

解：



$$\Delta d = \frac{\lambda}{2}$$

$$d_4 = 3 \times \frac{\lambda}{2} = 750 \text{ nm}$$

$$d_4 = L \sin \theta \approx L \theta$$

$$\lambda' = 600\text{nm}$$

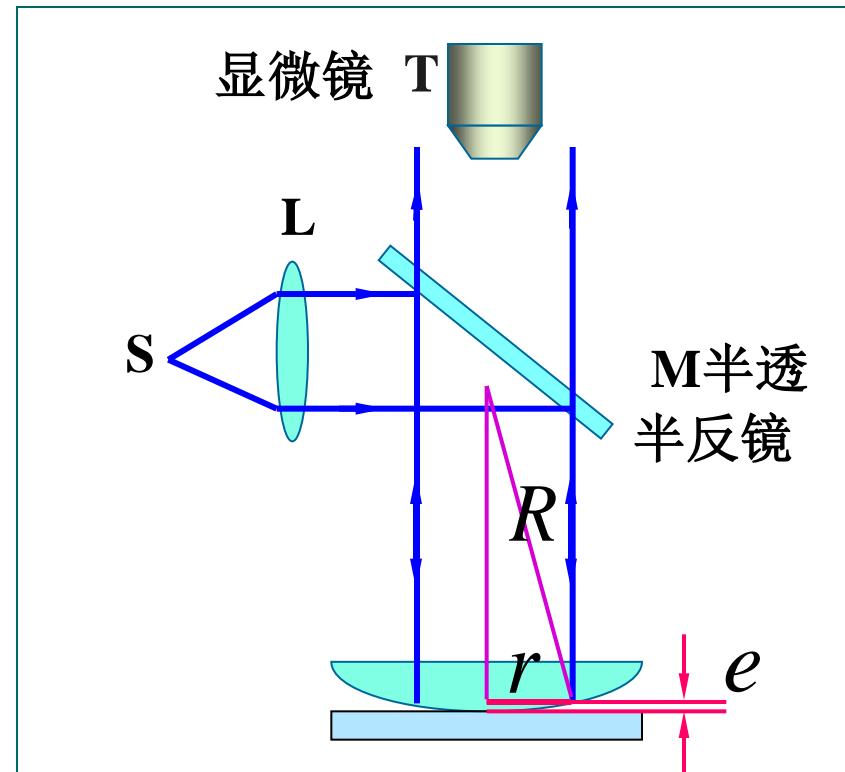
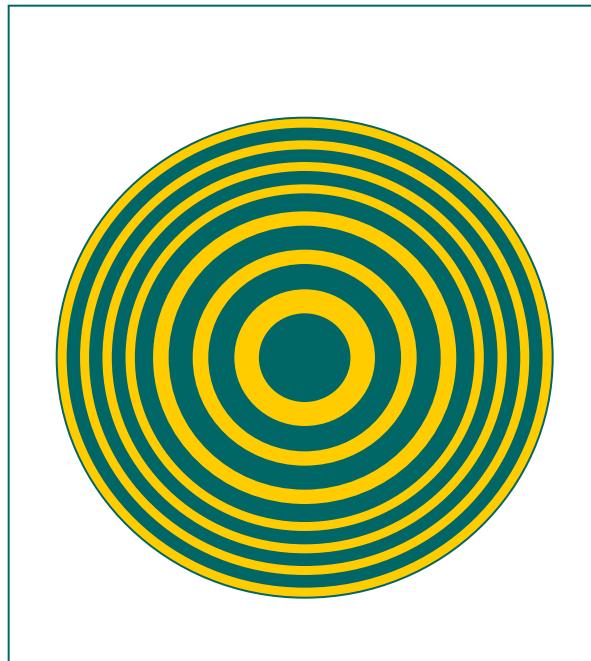
$$\theta = \frac{3\lambda}{2L} = 4.8 \times 10^{-5} \text{ rad}$$

$$\Delta = 2d_4 + \frac{\lambda'}{2} = 2 \times 750 + \frac{600}{2} = 1800 = 3\lambda'$$

A处是第三级明纹

从棱边到A处共有3条明纹，3条暗纹

二 牛顿环



- (1) 干涉图样：内疏外密、中心为暗点的圆环；
(2) 明环（干涉加强）、暗环（干涉减弱）的条件：

$$\delta = 2ne + \frac{\lambda}{2} = \begin{cases} k\lambda & \text{明环} \quad k = 1, 2, \dots \\ (2k+1)\frac{\lambda}{2} & \text{暗环} \quad k = 0, 1, 2, \dots \end{cases}$$

$$\delta = 2ne + \frac{\lambda}{2} = \begin{cases} k\lambda \\ (2k+1)\frac{\lambda}{2} \end{cases}$$

→ $e = \begin{cases} (2k-1)\frac{\lambda}{4n} \\ \frac{k\lambda}{2n} \end{cases}$

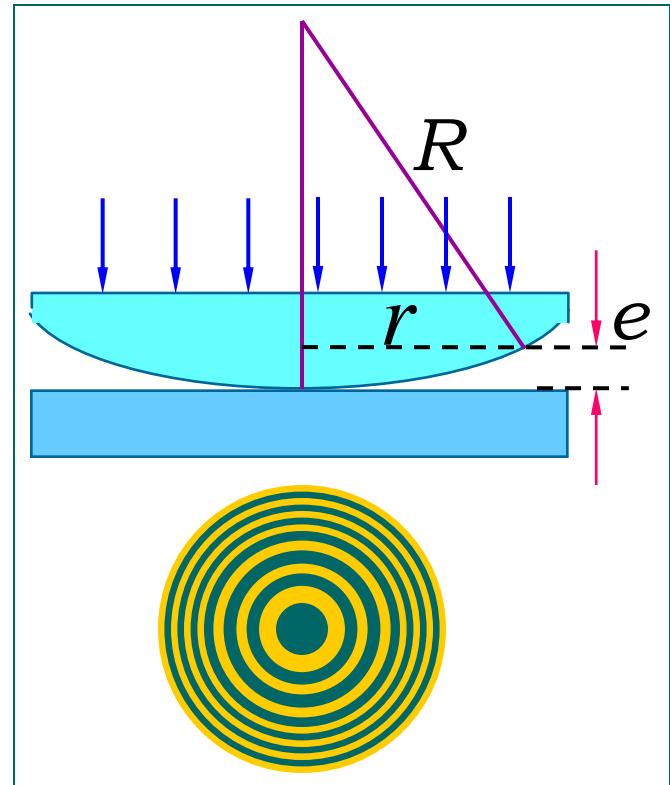
(3) 明(暗)环的半径:

$$r^2 = R^2 - (R - e)^2$$

$$= 2Re - e^2 \approx 2Re \rightarrow r = \sqrt{2Re}$$

明环半径 $r_{\text{明}} = \sqrt{\frac{(2k-1)R\lambda}{2n}}$ $k = 1, 2, \dots$

暗环半径 $r_{\text{暗}} = \sqrt{\frac{kR\lambda}{n}}$ $k = 0, 1, 2, \dots$



明环半径

$$r_{\text{明}} = \sqrt{\frac{(2k-1)R\lambda}{2n}}$$

暗环半径

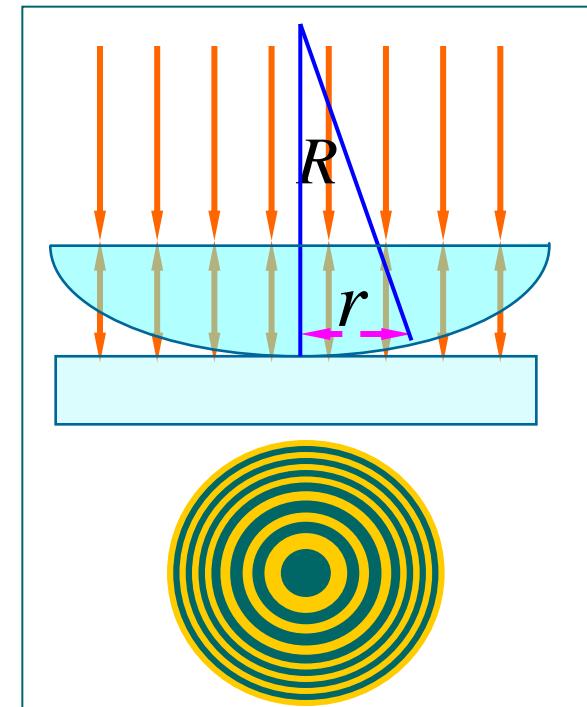
$$r_{\text{暗}} = \sqrt{\frac{kR\lambda}{n}}$$

- + 从反射光中观测，中心点是**暗点**还是**亮点**? 从透射光中观测，中心点是**暗点**还是**亮点**?
- + 等厚干涉，条纹间距不等，为什么?
- + 将牛顿环置于 $n > 1$ 的液体中，条纹变**密**!
- + 应用例子:可以用来测量光波波长，用于检测透镜质量，曲率半径等。

$$r_k^2 = kR\lambda$$

$$r_{k+m}^2 = (k+m)R\lambda$$

$$R = \frac{r_{k+m}^2 - r_k^2}{m\lambda}$$

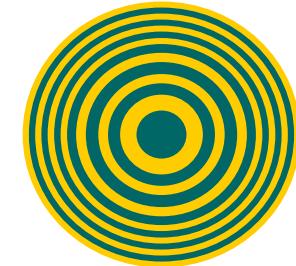
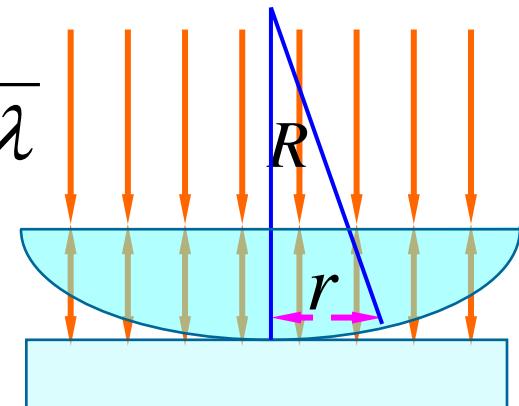


例10: 用氦氖激光器发出的波长为633nm的单色光做牛顿环实验，测得第个 k 暗环的半径为5.63mm，第 $k+5$ 暗环的半径为7.96mm，求平凸透镜的曲率半径 R .

解: $r_k = \sqrt{kR\lambda}$ $r_{k+5} = \sqrt{(k+5)R\lambda}$

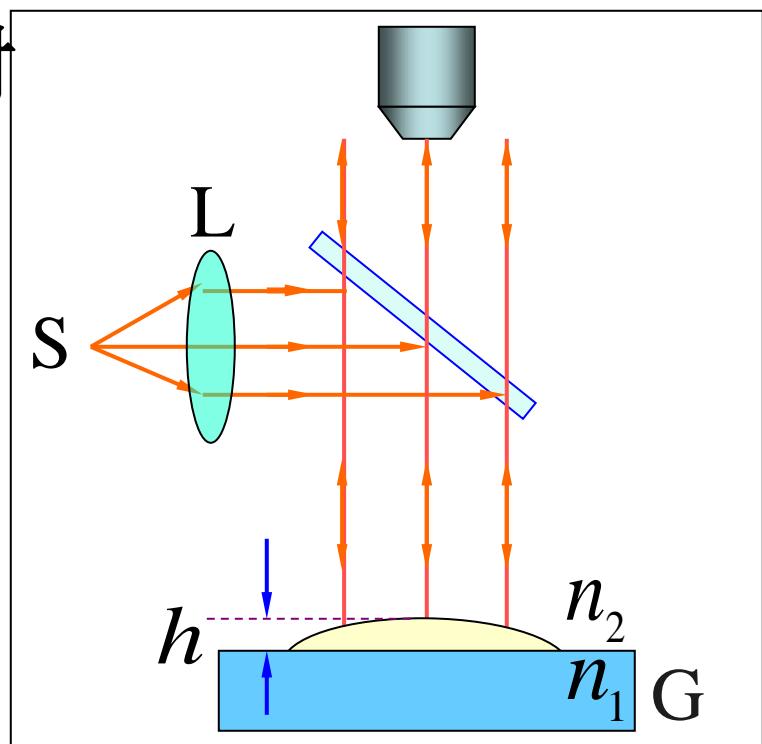
$$r_{k+5}^2 - r_k^2 = 5R\lambda$$

$$R = \frac{r_{k+5}^2 - r_k^2}{5\lambda}$$



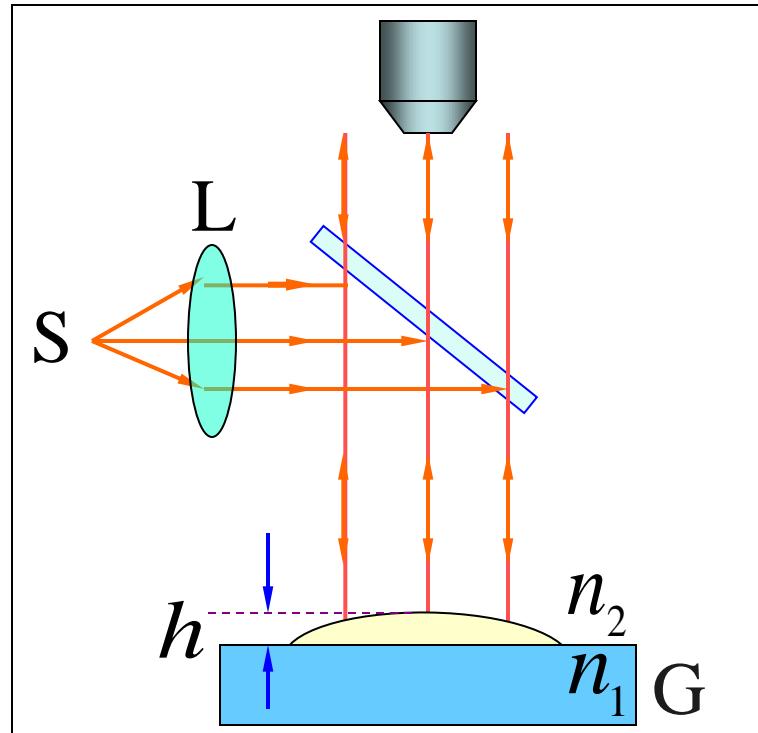
$$R = \frac{(7.96 \times 10^{-3})^2 - (5.63 \times 10^{-3})^2}{5 \times 633 \times 10^{-9}} = 10.0(\text{m})$$

例11：如图所示为测量油膜折射率的实验装置，在平面玻璃片G上放一油滴，并展开成圆形油膜，在波长 $\lambda = 600 \text{ nm}$ 的单色光垂直入射



下，从反射光中可观察到油膜所形成的干涉条纹。已知玻璃的折射率为 $n_1 = 1.50$ ，油膜的折射率为 $n_2 = 1.20$ ，问：当油膜中心最高点与玻璃

片的上表面相距 $h = 8.0 \times 10^2 \text{ nm}$ 时，干涉条纹是如何分布的？可看到几条明纹？明纹所在处的油膜厚度为多少？

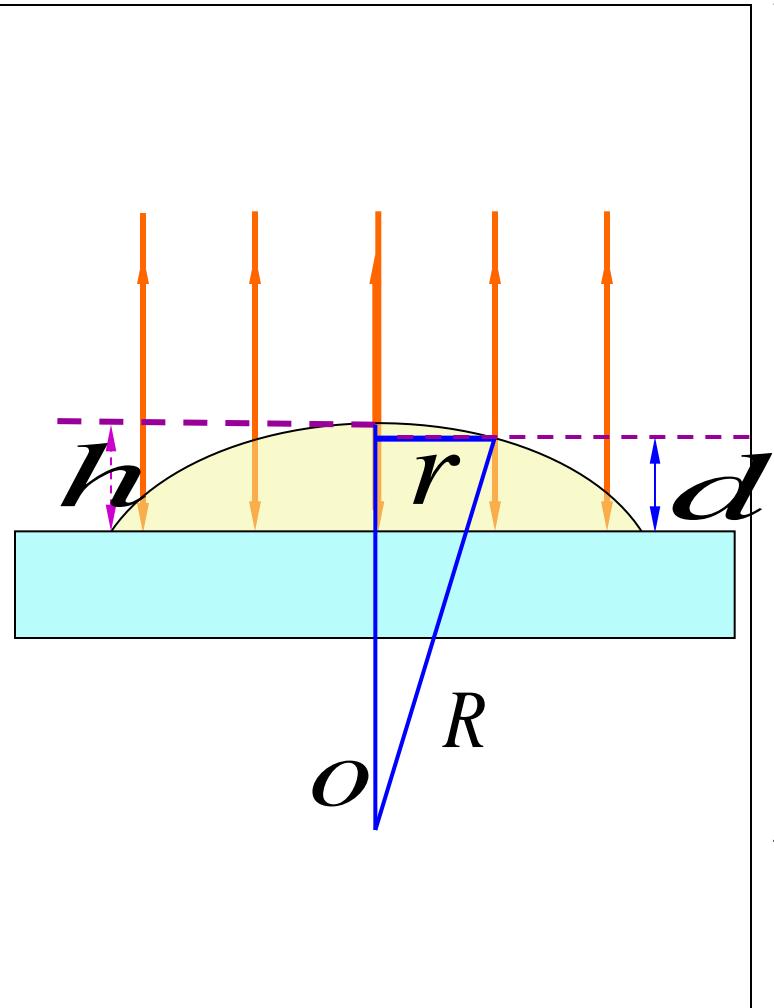


解 条纹为同心圆

$$\Delta = 2n_2 d_k = k\lambda \quad \text{明纹}$$

$$d_k = k \frac{\lambda}{2n_2}$$

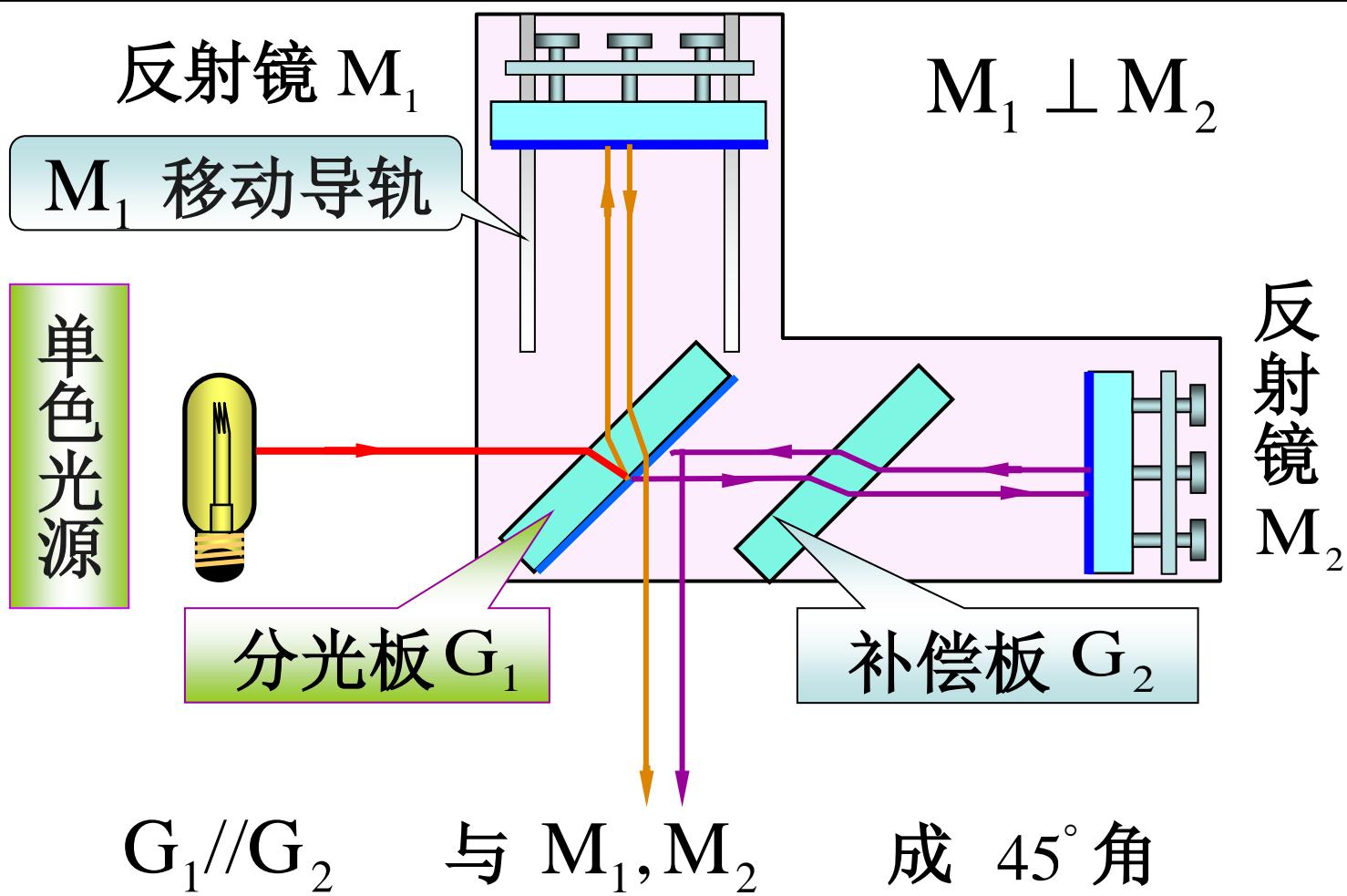
$$k = 0, 1, 2, \dots$$



油膜边缘 $k = 0, d_0 = 0$
 $k = 1, d_1 = 250\text{nm}$
 $k = 2, d_2 = 500\text{nm}$
 $k = 3, d_3 = 750\text{nm}$
 $k = 4, d_4 = 1000\text{nm}$

由于 $h = 8.0 \times 10^2 \text{ nm}$ 故
可观察到四条明纹 .

三 迈克耳孙干涉仪



M_2 的像 M'_2

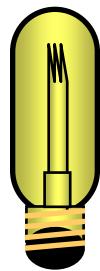


d

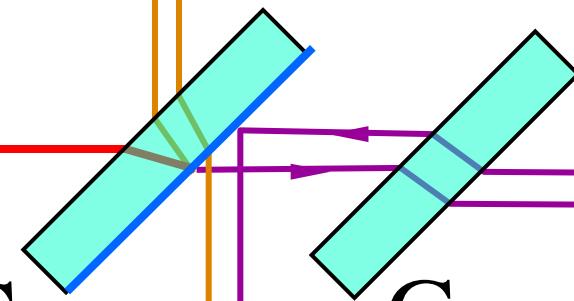
反射镜 M_1

$M_1 \perp M_2$

单色光源



G_1

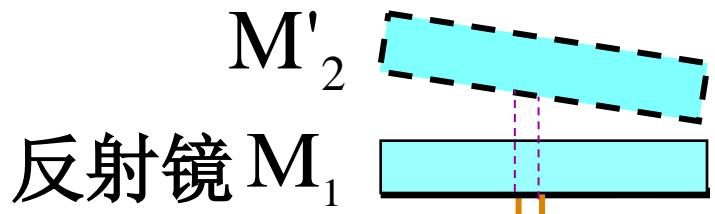


G_2



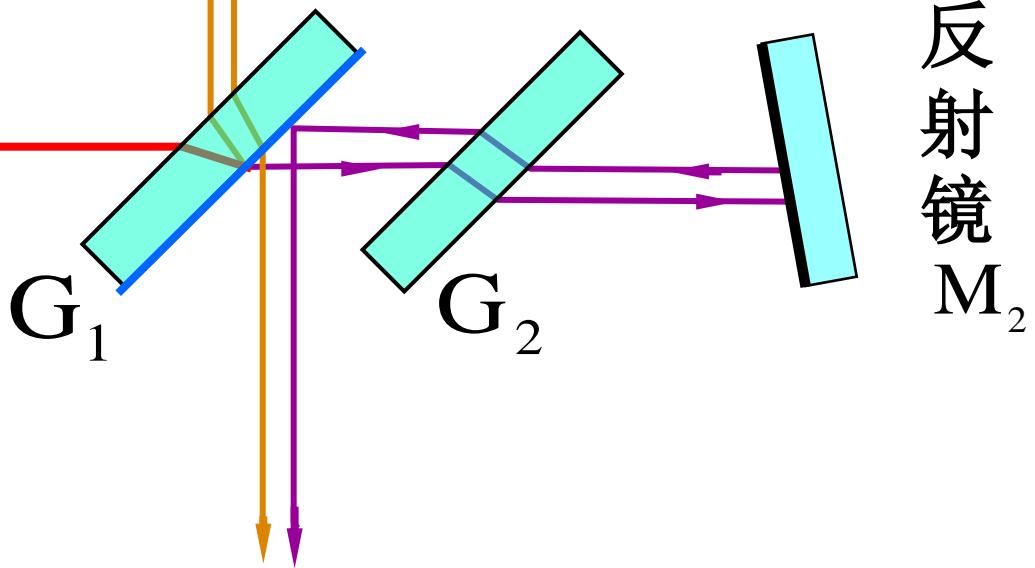
反射镜 M_2

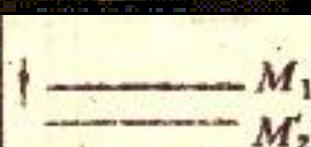
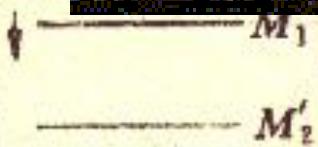
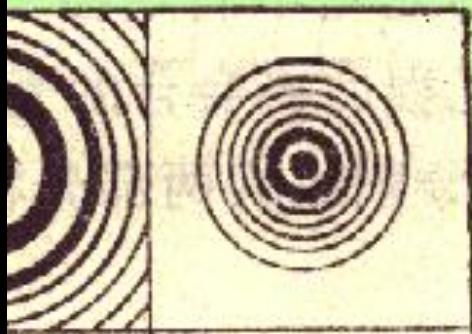
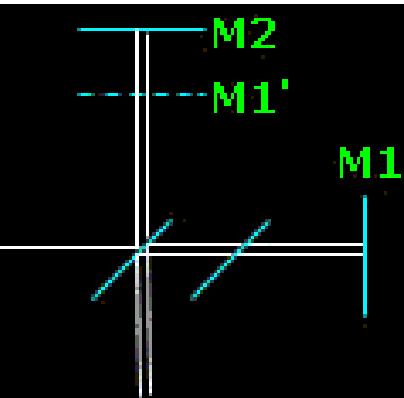
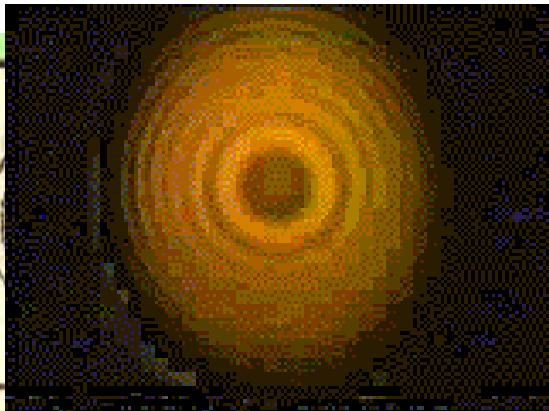
光程差 $\Delta = 2d$



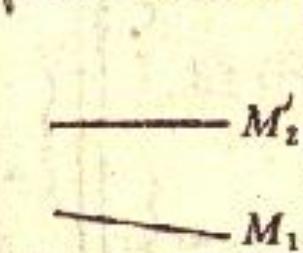
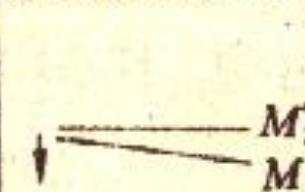
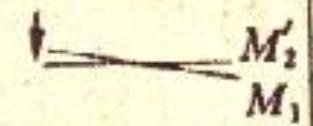
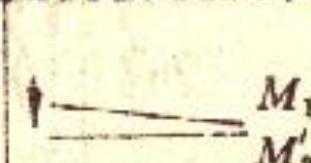
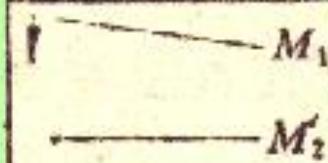
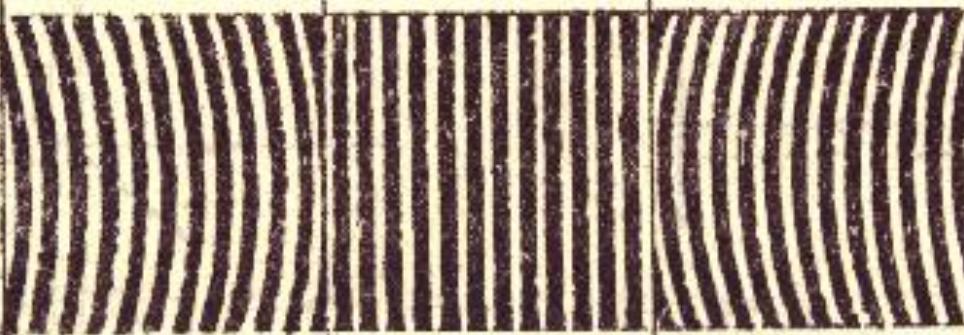
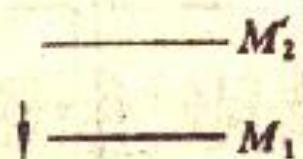
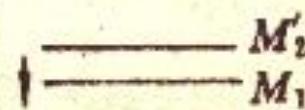
当 M_1 不垂直于 M_2 时，可形成劈尖型等厚干涉条纹。

单色光源

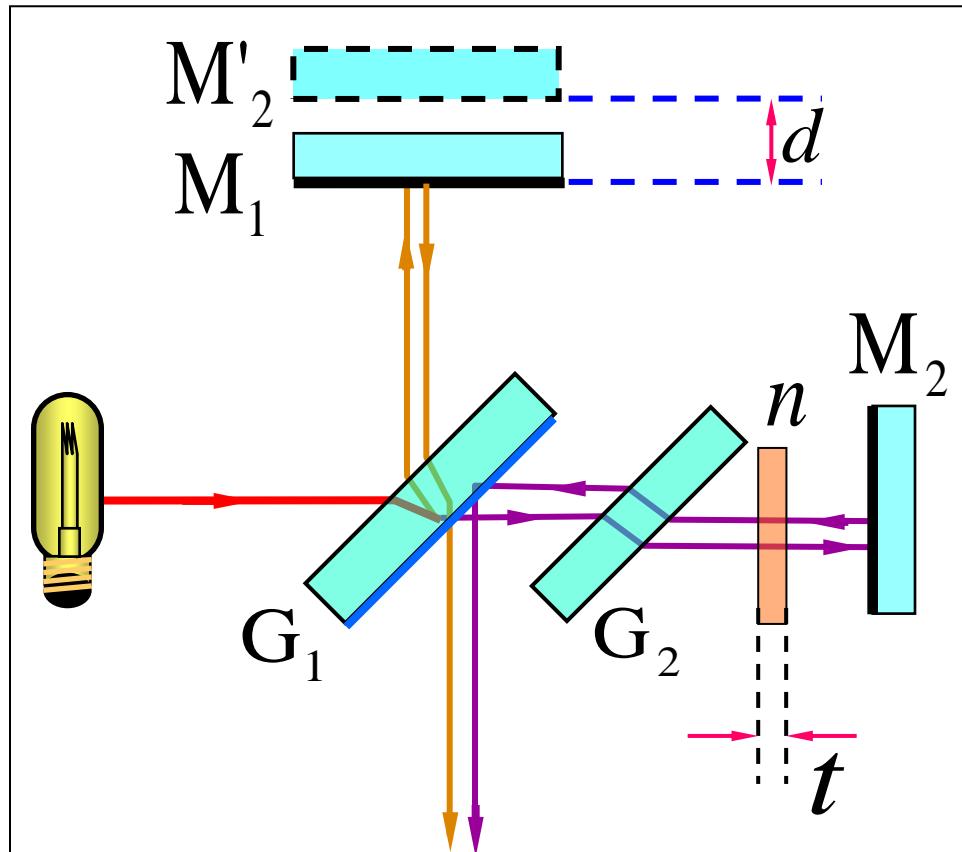




$M_1M'_2$ 重合



应用举例：测量介质片厚度



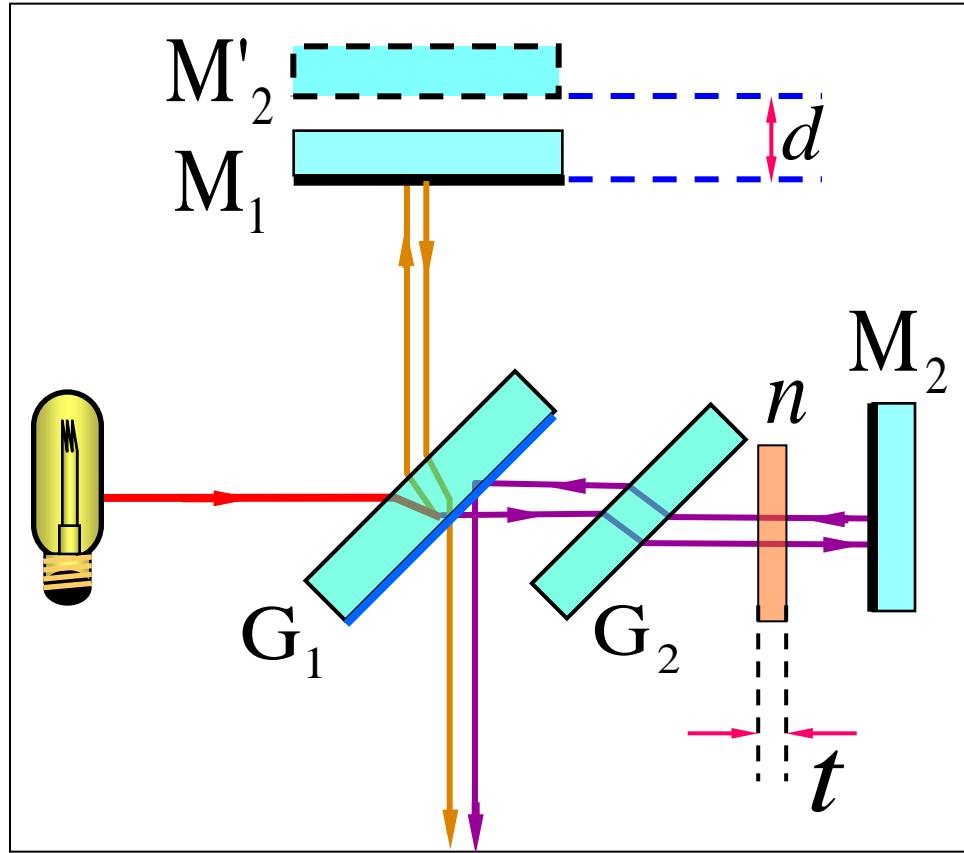
光程差 $\Delta = 2d$

插入介质片光程差

$$\Delta' = 2d + 2(n-1)t$$

光程差变化

$$\Delta' - \Delta = 2(n-1)t$$



$$2(n-1)t = \Delta k \lambda$$

干涉条纹移动数目

介质片厚度

$$t = \frac{\Delta k}{n-1} \cdot \frac{\lambda}{2}$$

例12：在迈克耳孙干涉仪的两臂中，分别插入 $l = 10.0 \text{ cm}$ 长的玻璃管，其中一个抽成真空，另一个则储有压强为 $1.013 \times 10^5 \text{ Pa}$ 的空气，用以测量空气的折射率 n . 设所用光波波长为 546 nm ，实验时，向真空玻璃管中逐渐充入空气，直至压强达到 $1.013 \times 10^5 \text{ Pa}$ 为止. 在此过程中，观察到 107.2 条干涉条纹的移动，试求空气的折射率 n .

已知 $l = 10.0 \text{ cm}$ $\lambda = 546 \text{ nm}$

解 $\angle_1 - \angle_2 = 2(n-1)l = 107.2\lambda$

$$n = 1 + \frac{107.2\lambda}{2l} = 1 + \frac{107.2 \times 546 \times 10^{-7} \text{ cm}}{2 \times 10.0 \text{ cm}}$$

$$= 1.00029$$

*非单色性对干涉条纹的影响 (光场的时间相干性)

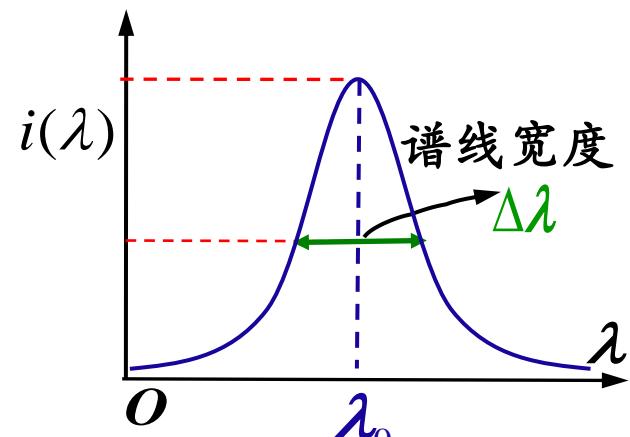
1. 干涉条纹可见度(衬度)定义:

$$\gamma = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

当 $I_{\min} = 0$ 时 $\gamma = 1$ 条纹最清晰

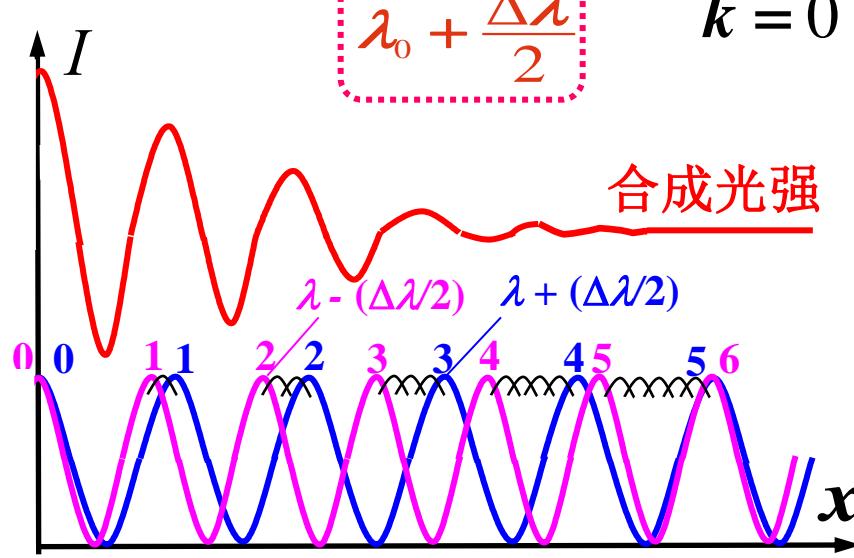
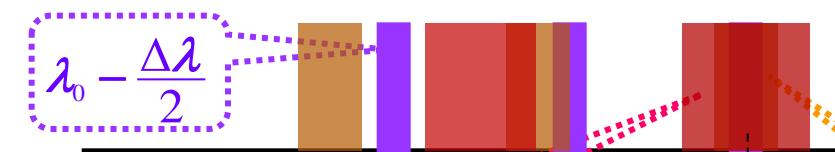
当 $I_{\min} = I_{\max}$ 时 $\gamma = 0$, 条纹消失, 光强均匀分布.

(1) 准单色光 $\lambda_0 - \frac{\Delta\lambda}{2} \sim \lambda_0 + \frac{\Delta\lambda}{2}$
 $\lambda_0 \gg \Delta\lambda$

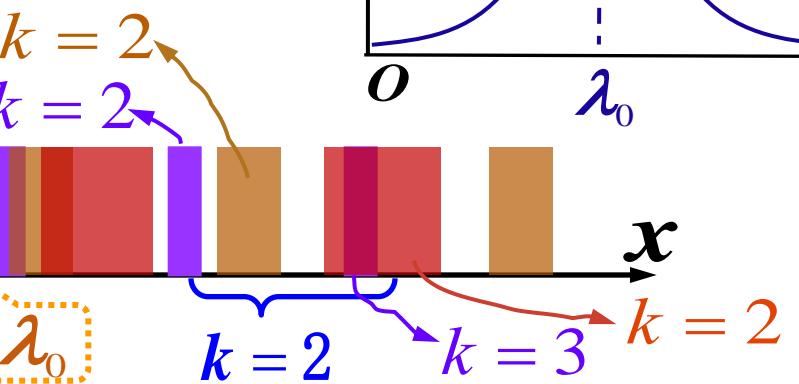
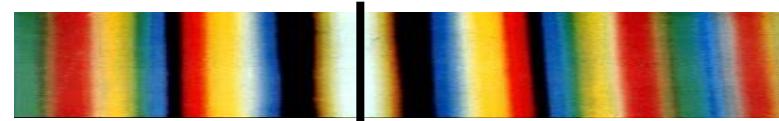


(2) 非单色性对干涉条纹的影响

$$x_k = k \frac{D\lambda}{d}, \quad \Delta x = \frac{D\lambda}{d}$$



白光光源 $k_m \approx 1.5$



$$\begin{aligned} & \lambda + \frac{\Delta\lambda}{2} & \delta_k = k(\lambda + \frac{1}{2}\Delta\lambda) \\ & \lambda - \frac{\Delta\lambda}{2} & \delta'_{k+1} = (k+1)(\lambda - \frac{1}{2}\Delta\lambda) \\ & \delta_k = \delta'_{k+1} \text{ 时 条纹完全消失} \\ & k_m \approx \frac{\lambda}{\Delta\lambda} \quad \text{最大相干级次} \\ & \delta_m \approx k_m \cdot \lambda = \frac{\lambda^2}{\Delta\lambda} \quad \text{最大光程差} \\ & & \text{相干长度} \end{aligned}$$

时间相干性：

光源在同一时刻发的光分为两束后又先后到达某一观察点，只有这先后到达的时差小于某一值时才能在观察点产生干涉。

相干时间 τ

$$\tau = \frac{\delta_m}{u} \quad \tau_0 = \frac{\delta_0}{c}$$

白光	$\Delta\lambda = 150\text{nm}$	δ_m 与 $\lambda \approx 10^{-7}\text{m}$	同数量级
H _e -N _e 激光	$\Delta\lambda \downarrow$	$\delta_m = 4 \times 10^3\text{m}$ 长	单色性好

Table 11.1-2 Spectral widths of a number of light sources together with their coherence times and coherence lengths in free space.

Source	$\Delta\nu_c$ (Hz)	$\tau_c = 1/\Delta\nu_c$	$l_c = c\tau_c$
Filtered sunlight ($\lambda_o = 0.4\text{--}0.8\text{ }\mu\text{m}$)	3.74×10^{14}	2.67 fs	800 nm
Light-emitting diode ($\lambda_o = 1\text{ }\mu\text{m}$, $\Delta\lambda_o = 50\text{ nm}$)	1.5×10^{13}	67 fs	$20\text{ }\mu\text{m}$
Low-pressure sodium lamp	5×10^{11}	2 ps	$600\text{ }\mu\text{m}$
Multimode He-Ne laser ($\lambda_o = 633\text{ nm}$)	1.5×10^9	0.67 ns	20 cm
Single-mode He-Ne laser ($\lambda_o = 633\text{ nm}$)	1×10^6	1 μs	300 m

应用举例2

激光干涉引力波天文台：LIGO

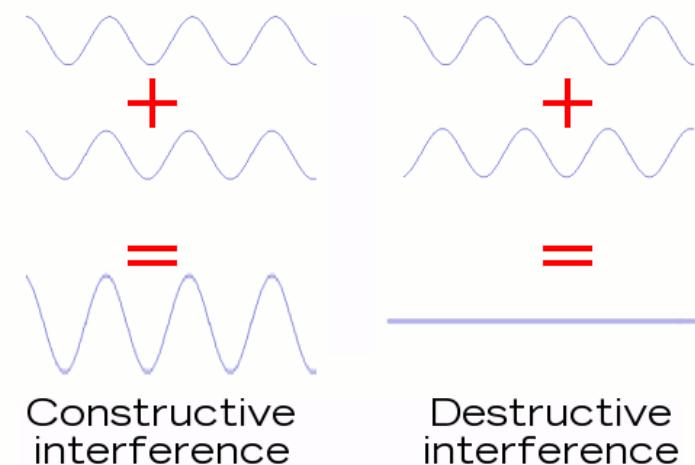
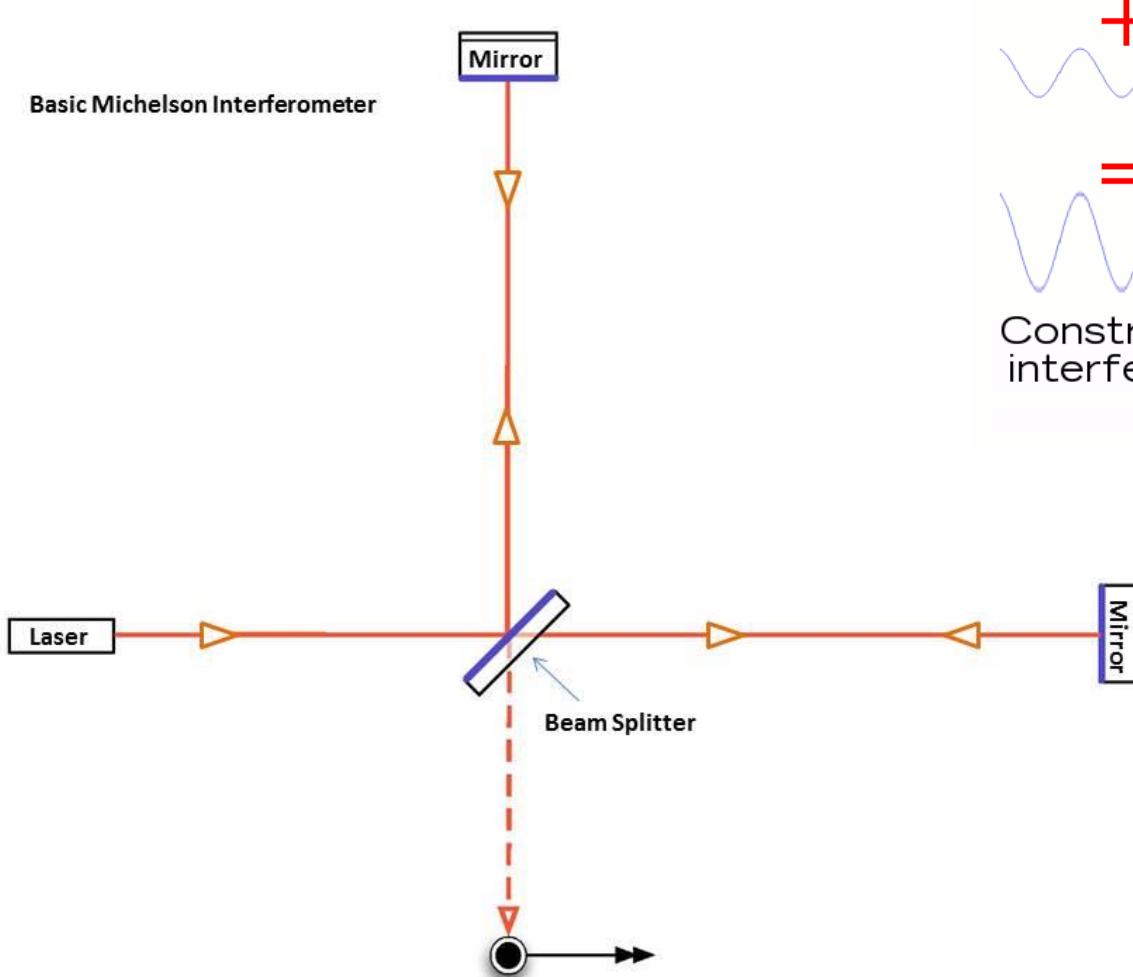


Aerial views showing the locations and extents of the LIGO Hanford and LIGO Livingston interferometers. (Photos: LIGO)

LIGO Hanford

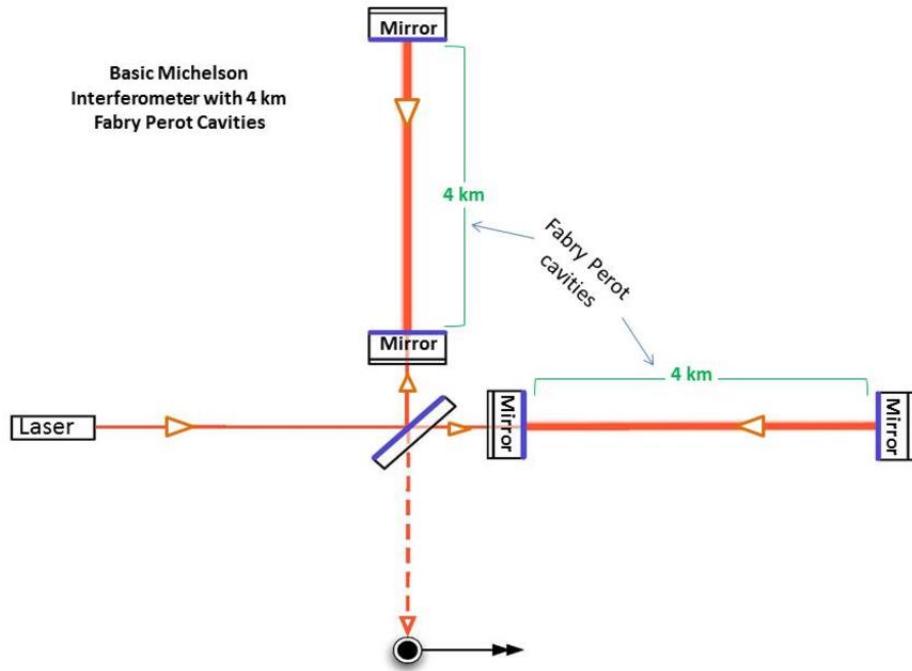
LIGO Livingston

Basic Design of Gravitational Wave Detector



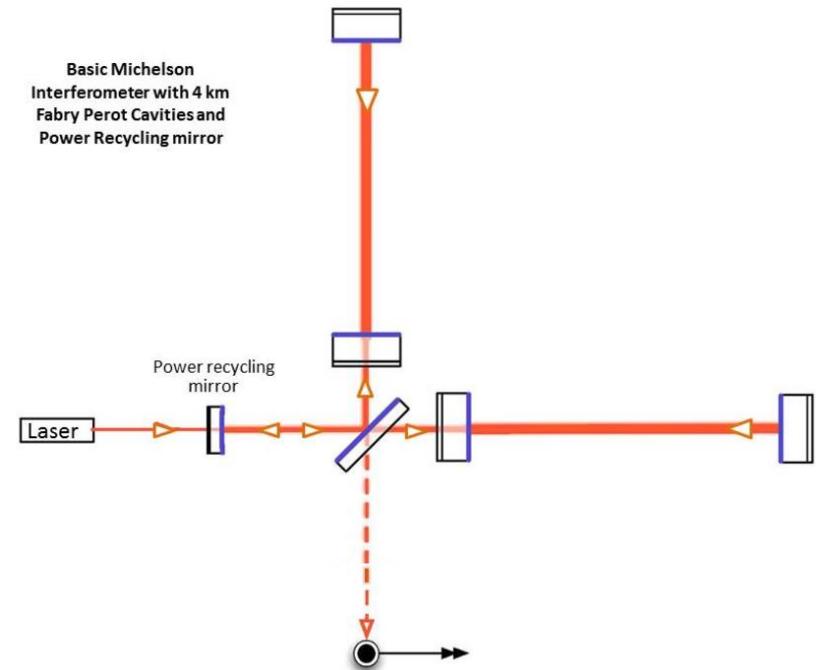
www.explainthatstuff.com

LIGO (4km)



Fabry-Perot Cavity

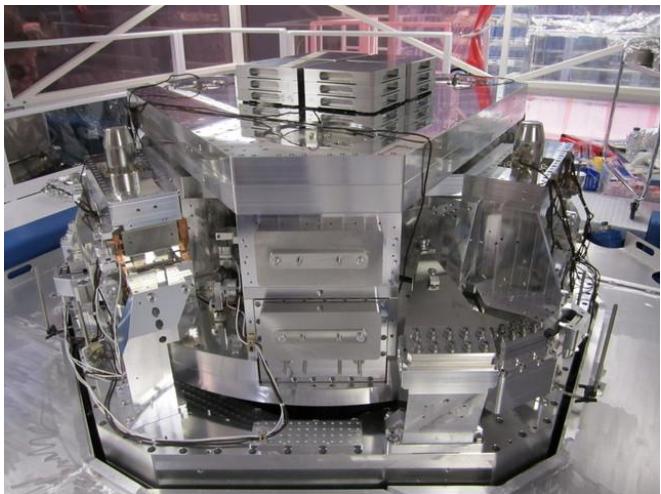
4km x 280



Power Boosted Laser

Theory: 750KW
Now: 250W

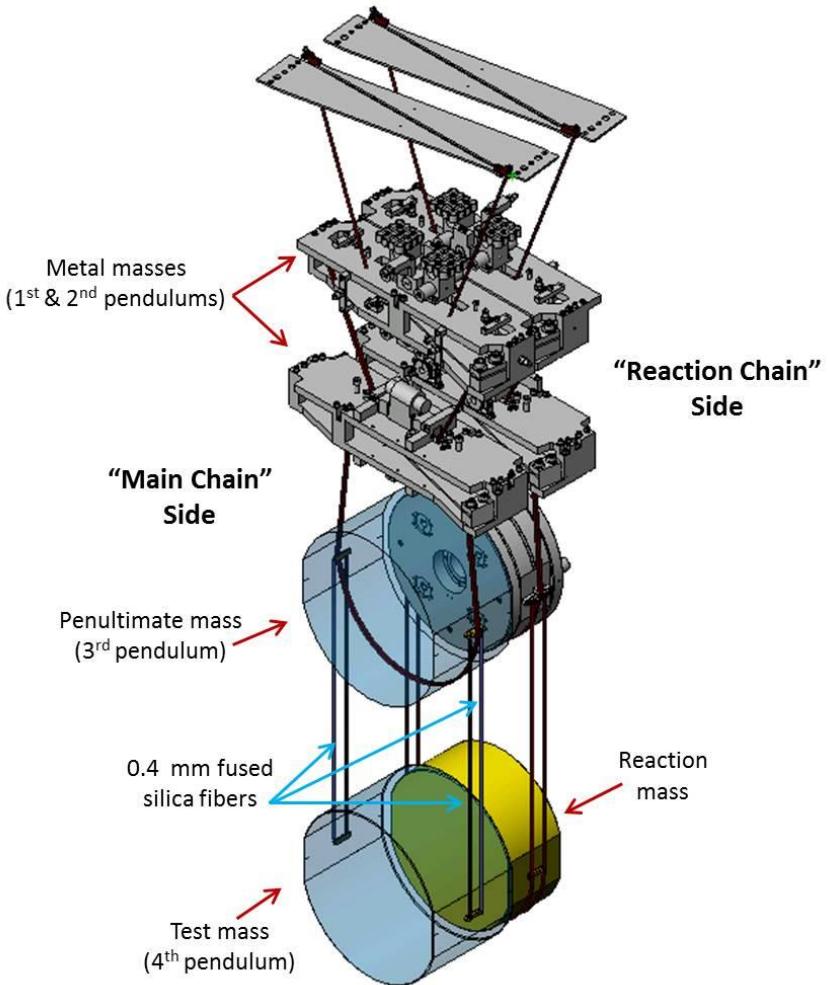
LIGO Technology



Active Isolation and Vacuum



Fused Silica Mirror (40kg)



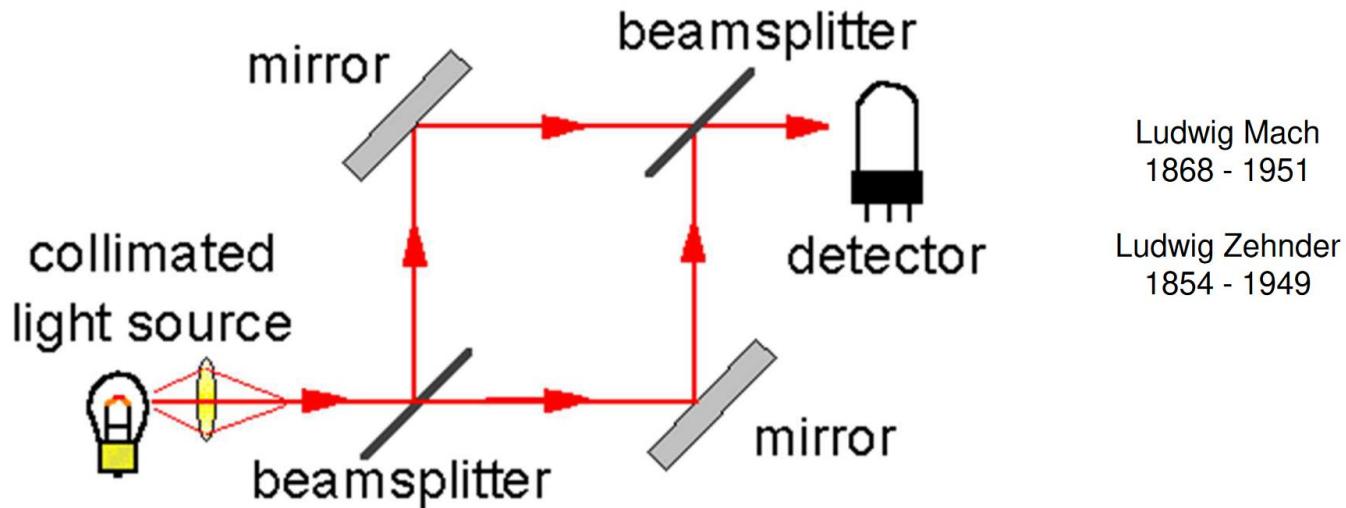
Passive Isolation: Quadruple Pendulum

- 引力波探测器：超高真空，多级隔震，大功率稳定激光，特别设计的各类精密透镜组成的复杂系统等。
- 激光功率由10瓦特提高到200瓦特，探测频率下限从40Hz延伸到了10Hz，灵敏度提高到 $10^{-24} \sim 10^{-25}$ 量级。
- **探测频段：** 相对强度为 1.0×10^{-21} ，频率覆盖35到250 Hz。探测时，LIGO中距离4公里镜子，其相对距离只是变化了 $10^{-18} m$ 左右，是原子核尺度一千分之一。



应用举例3

马赫-曾德尔干涉仪 (Mach-Zehnder interferometer)

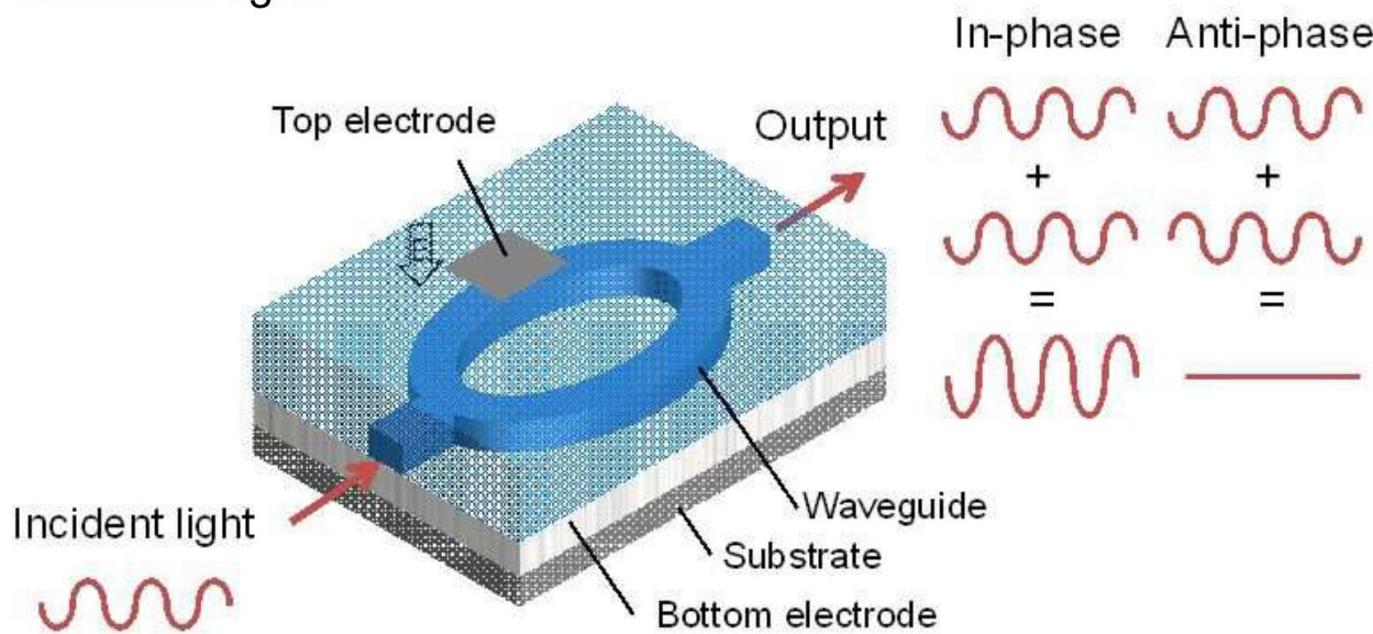


The Mach-Zehnder interferometer requires two beam splitters.

It is often operated with something of interest in one arm, to measure phase changes.

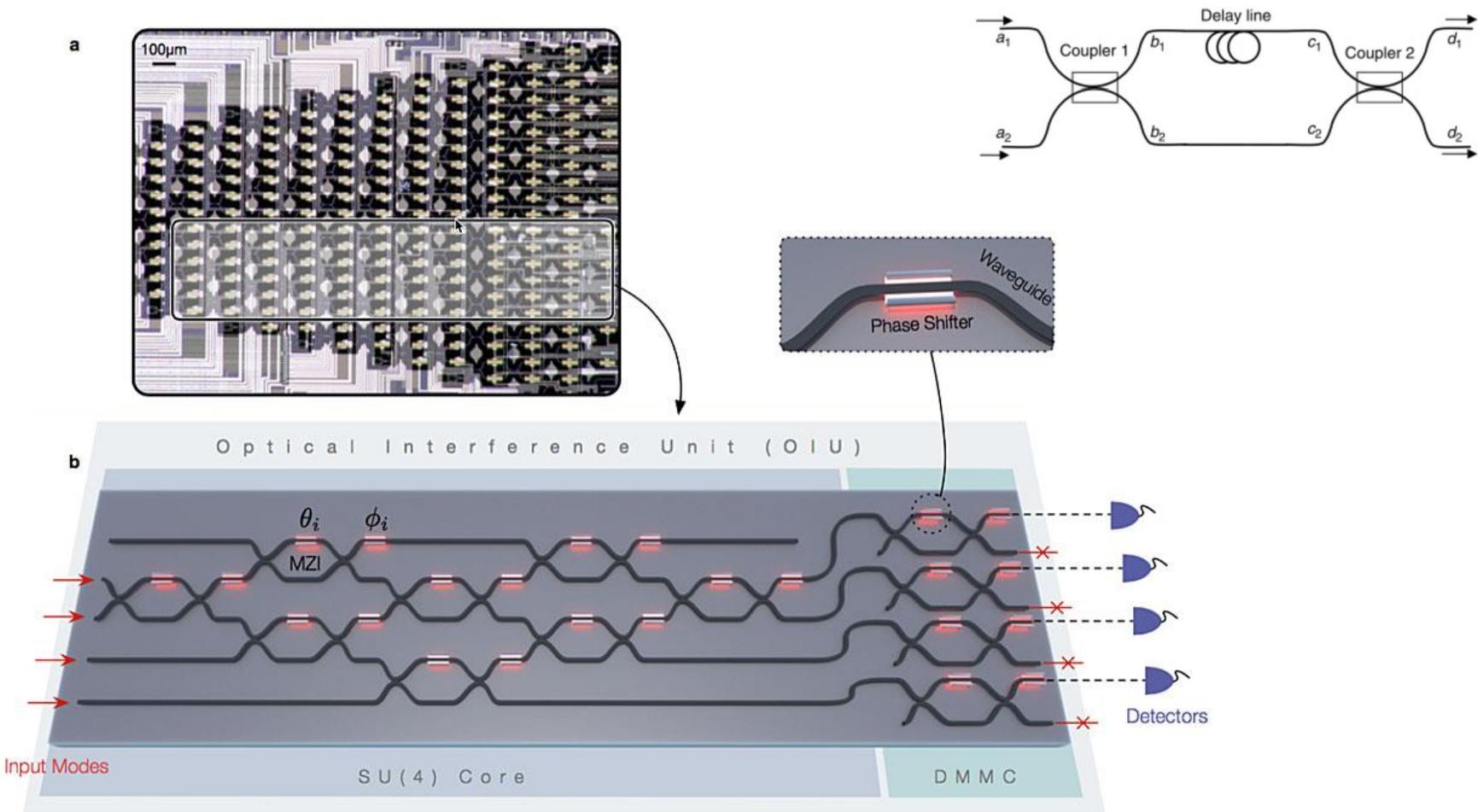
Mach Zehnder Interferometer

This type of interferometer can also be built in a waveguide configuration. When combined with a voltage-driven phase modulator (like as in the Pockels effect), it can be used to modulate light.



This is the most common design for digital modulators used in lightwave communications systems.

On-chip Mach Zehnder Interferometer



Shen, Y., Harris, N., Skirlo, S. *et al.* Deep learning with coherent nanophotonic circuits. *Nature Photon* **11**, 441–446 (2017).