Lab Nr. 7, Numerical Calculus

Lagrange Interpolation II

Newton Form; Aitken's Algorithm

1. Implement Lagrange interpolation, in Newton's divided differences form.

2. Implement Lagrange interpolation, using the Aitken-Neville algorithm.

Applications

1. (**This is a good problem to also do "by hand".**) Consider the function $f(x) = \cos(\pi x)$ and the nodes $\{0, 1/3, 1/2, 1\}$.

- **a)** Find the Lagrange polynomial $L_3 f$ using Newton's divided differences;
- **b)** Find a bound for the error $R_3 f$;
- **c)** Plot f and $L_3 f$, on the same set of axes, for $x \in [0, 1]$.
- **d)** Use $L_3 f$ to approximate $\cos\left(\frac{\pi}{5}\right)$;
- **e)** Find a bound for the error of this approximation.

2. Approximate $\sqrt{2}$ using Aitken's algorithm to interpolate the function $f(x) = 2^x$ at 9 equidistant nodes on the interval $[-4, 4]$.

3. The following table contains values of $\lg x$ $(= \log_{10} x)$ rounded to 7 decimals:

x	$\lg x$
1000	3.0000000
1010	3.0043214
1020	3.0086002
1030	3.0128372
1040	3.0170333
1050	3.0211893

Use Lagrange interpolation with divided differences to approximate $\lg x$, for $x = 1001, 1002, \ldots, 1009$.

Optional

4. The following table contains values of $\sin x$ at equally spaced nodes (given in degrees, not radians):

x	$\lg x$
$39°$	0.6293204
$41°$	0.6560590
$43°$	0.6819984
$45°$	0.7071068
$47°$	0.7313597
$49°$	0.7547096
$51°$	0.7771460

Use Newton interpolation with *forward differences* to approximate $\sin 40°$, $\sin 44°$ and $\sin 50°$.