DARiA

10.09.2025 Ma, Mi, Jo

Integrală nedefinită

PRIMITIVE și INTEGRALE NEDEFINITE

① $\int_0 du = C$

② $\int_1 du = u + C$

③ $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$

④ $\int \frac{1}{\sqrt{u}} \cdot du = 2\sqrt{u} + C$

⑤ $\int a^u du = \frac{a^u}{\ln a} + C$

⑥ $\int e^u du = e^u + C$

⑦ $\int \frac{1}{u} \cdot du = \ln|u| + C$

⑧ $\int \cos u \cdot du = \sin u + C$

⑨ $\int \sin u \cdot du = -\cos u + C$

⑩ $\int \frac{1}{\cos^2 u} \cdot du = tg u + C$

⑪ $\int \frac{1}{\sin^2 u} \cdot du = -ctg u + C$

⑫ $\int \frac{1}{\sqrt{1-u^2}} du = \arcsin u + C = -\arccos u + C$

⑬ $\int \frac{1}{1+u^2} \cdot du = arctg u + C = -arcctg u + C$

⑭ $\int \frac{1}{a^2+u^2} \cdot du = \frac{1}{a} arctg \frac{u}{a} + C = -\frac{1}{a} arcctg \frac{u}{a} + C$

⑮ $\int \frac{1}{\sqrt{a^2-u^2}} du = \arcsin \frac{u}{a} + C = -\arccos \frac{u}{a} + C$

⑯ $\int \frac{1}{a^2-u^2} \cdot du = \frac{1}{2a} \cdot \ln\left|\frac{a+u}{a-u}\right| + C$

⑯* $\int \frac{1}{u^2-a^2} \cdot du = \frac{1}{2a} \cdot \ln\left|\frac{a-u}{a+u}\right| + C$

⑰ $\int \sqrt{a^2-u^2} \cdot du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2} \arcsin \frac{u}{a} + C$

⑱ $\int \sqrt{a^2+u^2} \cdot du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2} \ln|u+\sqrt{a^2+u^2}| + C$

⑲ $\int \frac{1}{\sqrt{b+u^2}} \cdot du = \ln|u+\sqrt{b+u^2}| + C$

⑳ $\int \frac{1}{\sqrt{u^2-b}} \cdot du = \ln|u+\sqrt{u^2-b}| + C$

INTEGRAREA PRIN PĂRȚI

$\int u \cdot dv = u \cdot v - \int v \cdot du$

METODA SUBSTITUȚIEI

$\int \frac{x}{x^2} \cdot dx = \left| \begin{array}{l} t = x^2 \\ dt = d(x^2) = (x^2)' dx = 2x \cdot dx \\ dx = \frac{dt}{2x} \end{array} \right| = \int \frac{\cancel{x}}{t} \cdot \frac{dt}{2\cancel{x}} \ldots$

INTEGRALA DEFINITĂ

$\int_a^b f(x) \cdot dx = F(x)\Big|_a^b = F(b) - F(a)$

INTEGRAREA PRIN PĂRȚI

$\int_a^b u \cdot dv = u \cdot v\Big|_a^b - \int_a^b v \cdot du$

MET. SUBSTITUȚIEI

$\int_a^b \frac{x}{x^2} \cdot dx = \left| \begin{array}{l} t = x^2 \\ dt = 2x \cdot dx \\ dx = \frac{dt}{2x} \\ x = b \Rightarrow t = b^2 \\ x = a \Rightarrow t = a^2 \end{array} \right| = \int_{a^2}^{b^2} \frac{\cancel{x}}{t} \cdot \frac{dt}{2\cancel{x}}$

a) $f(x) = 3x - 5\cos x + e^x$

$$F(x) = \int (3x - 5\cos x + e^x)\, dx =$$

$$= \int 3x\, dx - \int 5\cos x\, dx + \int e^x dx =$$

$$= 3\int x\, dx - 5\int \cos x\, dx + \int e^x dx =$$

$$= 3 \cdot \frac{x^2}{2} - 5 \cdot \sin x + e^x + C =$$

$$= \frac{3x^2}{2} - 5\sin x + e^x + C$$

$$\frac{2}{x} = 2 \cdot \frac{1}{x}$$

$$\frac{1}{3\sqrt{x}} = \frac{1}{3} \cdot \frac{1}{\sqrt{x}}$$

b) $f(x) = \frac{2}{x} + \frac{1}{\cos^2 x} - \frac{1}{3\sqrt{x}}$

$$F(x) = \int \left(\frac{2}{x} + \frac{1}{\cos^2 x} - \frac{1}{3\sqrt{x}}\right) dx =$$

$$= \int 2 \cdot \frac{1}{x} dx + \int \frac{1}{\cos^2 x} dx - \int \frac{1}{3} \cdot \frac{1}{\sqrt{x}} dx =$$

$$= 2\int \frac{1}{x} dx + \int \frac{1}{\cos^2 x} dx - \frac{1}{3}\int \frac{1}{\sqrt{x}} dx =$$

$$= 2\ln|x| + tg\, x - \frac{2\sqrt{x}}{3} + C$$

c) $\int \frac{1}{\sqrt{x}} dx = \int \frac{1}{x^{\frac{1}{2}}} dx = \int x^{-\frac{1}{2}} dx =$

$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{\sqrt{x}}{\frac{1}{2}} = \sqrt{x} \cdot \frac{2}{1} = 2\sqrt{x}$$

d) $f(x) = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3}$

$\underset{\text{(primitiva)}}{F(x)} = \int \frac{1}{x}\,dx - \int \frac{1}{x^2}\,dx + \int \frac{1}{x^3}\,dx =$

$= \ln|x| + c - \int x^{-2}\,dx + \int x^{-3}\,dx =$

$= \ln|x| - \frac{x^{-2+1}}{-2+1} + \frac{x^{-3+1}}{-3+1} + c =$

$= \ln|x| - \frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} + c = \ln|x| + x^{-1} - \frac{x^{-2}}{2} + c =$

$= \ln|x| + \frac{1}{x} - \frac{1}{2x^2} + c$

$\frac{x^{-2}}{2} = \frac{1}{2} \cdot x^{-2} = \frac{1}{2} \cdot \frac{1}{x^2} = \frac{1}{2x^2}$

e) $f(x) = \sqrt[3]{x^2} - \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[3]{x^2}}$

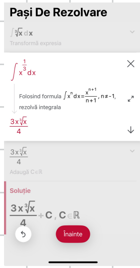
$F(x) = \int \left(\sqrt[3]{x^2} - \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[3]{x^2}}\right) dx =$

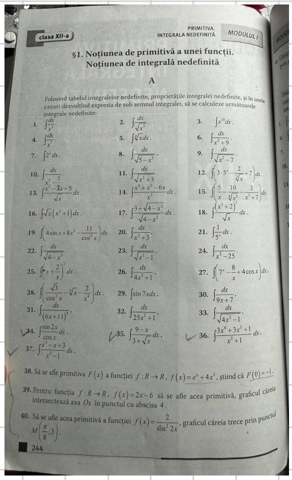
$= \int x^{\frac{2}{3}}\,dx - \int x^{-\frac{1}{2}}\,dx - \int x^{-\frac{2}{3}}\,dx =$

$= \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} - \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} =$

$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c = \frac{3x^{\frac{5}{3}}}{5} - \frac{2x^{\frac{1}{2}}}{1} - \frac{3x^{\frac{1}{3}}}{1} + c$

$$= \frac{2\sqrt[3]{x^5}}{5} - 2\sqrt{x} - 3\sqrt[3]{x} + C$$





clasa XII-a

PRIMITIVA. INTEGRALA NEDEFINITĂ — MODULUL I

§1. Noțiunea de primitivă a unei funcții. Noțiunea de integrală nedefinită

A

Folosind tabelul integralelor nedefinite, proprietățile integralei nedefinite, și în unele cazuri dezvoltînd expresia de sub semnul integralei, să se calculeze următoarele integrale nedefinite:

1. $\int \frac{dx}{x^3}$. 2. $\int \frac{dx}{\sqrt{x^3}}$. 3. $\int x^{10}dx$.
4. $\int \frac{dx}{x^7}$. 5. $\int \sqrt[4]{x}dx$. 6. $\int \frac{dx}{x^2+9}$.
7. $\int 2^x dx$. 8. $\int \frac{dx}{\sqrt{5-x^2}}$. 9. $\int \frac{dx}{\sqrt{x^2-7}}$.
10. $\int \frac{dx}{x^2-\frac{1}{3}}$. 11. $\int \frac{dx}{\sqrt{x^2+3}}$. 12. $\int \left(3\cdot 5^x - \frac{2}{\sqrt[3]{x}} + 7\right)dx$.
13. $\int \frac{x^2-3x+5}{\sqrt{x}}dx$. 14. $\int \frac{x^4+x^2-6x}{x^3}dx$. 15. $\int \left(\frac{5}{x} - \frac{10}{\sqrt[4]{x^3}} - \frac{3}{x^2+7}\right)dx$.
16. $\int \sqrt{x}\left(x^2+1\right)dx$. 17. $\int \frac{3+\sqrt{4-x^2}}{\sqrt{4-x^2}}dx$. 18. $\int \frac{\left(x^3+2\right)^2}{\sqrt{x}}dx$.
19. $\int \left(4\sin x + 8x^3 - \frac{11}{\cos^2 x}\right)dx$. 20. $\int \frac{dx}{x^2+3}$. 21. $\int \frac{1}{5^x}dx$.
22. $\int \frac{dx}{\sqrt{4-x^2}}$. 23. $\int \frac{dx}{\sqrt{x^2-1}}$. 24. $\int \frac{dx}{x^2-25}$.
25. $\int \left(x+\frac{2}{x}\right)^2 dx$. 26. $\int \frac{dx}{4x^2+1}$. 27. $\int \left(7^x - \frac{8}{x} + 4\cos x\right)dx$.
28. $\int \left(\frac{\sqrt{3}}{\cos^2 x} - \sqrt[3]{x} - \frac{2}{x^4}\right)dx$. 29. $\int \sin 7x dx$. 30. $\int \frac{dx}{9x+7}$.
31. $\int \frac{dx}{(6x+11)^4}$. 32. $\int \frac{dx}{25x^2+1}$. 33. $\int \frac{dx}{\sqrt{4x^2-1}}$.
34. $\int \frac{\sin 2x}{\cos x}dx$. 35. $\int \frac{9-x}{3+\sqrt{x}}dx$. 36. $\int \frac{3x^4+3x^2+1}{x^2+1}dx$.
37. $\int \frac{x^5-x+3}{x^2-1}dx$.

38. Să se afle primitiva $F(x)$ a funcției $f: R \to R$, $f(x) = e^x + 4x^3$, știind că $F(0) = -1$.

39. Pentru funcția $f: R \to R$, $f(x) = 2x - 6$ să se afle acea primitivă, graficul căreia intersectează axa Ox în punctul cu abscisa 4.

40. Să se afle acea primitivă a funcției $f(x) = \frac{2}{\sin^2 2x}$, graficul căreia trece prin punctul $M\left(\frac{\pi}{8};3\right)$.

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$$\int \left(3\cdot 5^x - \frac{2}{\sqrt[3]{x}} + 7\right)dx =$$

$$= \int 3\cdot 5^x dx - \int \frac{2}{\sqrt[3]{x}}dx + \int 7dx =$$

$$\sqrt[k]{a^m} = a^{\frac{m}{k}}$$

$$= 3\int 5^x dx - 2\int \frac{1}{\sqrt[3]{x}}dx + \int 7dx =$$

$$= 3\int 5^x dx - 2\int x^{-\frac{1}{3}}dx + \int 7dx =$$

$$= \frac{3\cdot 5^x}{\ln 5} - 2\cdot\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + 7x + C =$$

$$= \frac{3\cdot 5^x}{\ln 5} - \cancel{2}\cdot\frac{3x^{\frac{2}{3}}}{\cancel{2}} + 7x = \frac{3\cdot 5^x}{\ln 5} - 3\sqrt[3]{x^2} + 7x + C$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\int \frac{x^2 - 3x + 5}{\sqrt{x}}dx = \int \frac{x^2}{x^{\frac{1}{2}}}dx - \int \frac{3x}{x^{\frac{1}{2}}}dx + \int \frac{5}{x^{\frac{1}{2}}}dx =$$

$$= \int x^{2-\frac{1}{2}}dx - 3\int x^{1-\frac{1}{2}}dx + 5\int x^{-\frac{1}{2}}dx =$$

2)

$$= \int x^{\frac{3}{2}}dx - 3\int x^{\frac{1}{2}}dx + 5\int x^{-\frac{1}{2}}dx =$$

$$= \dots + C$$

15. $\int \frac{3}{x^2+7}\,dx = 3\int \frac{1}{x^2+7}\,dx = 3\int \frac{1}{x^2+(\sqrt{7})^2} =$

Aplicăm formula 14

$= 3\cdot \frac{1}{7}\,\text{arctg}\,\frac{x}{7} + C$

$\int \frac{3+\sqrt{4-x^2}}{\sqrt{4-x^2}}\,dx = \int \frac{3}{\sqrt{4-x^2}}\,dx + \int \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}\,dx =$

$= 3\int \frac{1}{\sqrt{2^2-x^2}}\,dx + \int 1\,dx = 3\arcsin\frac{x}{2} + x + C$

pe acasă:

partea A: 6, 9, 16, 18, 25

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18 $\int \frac{(x^3+2)^2}{\sqrt{x}}\,dx = \int \frac{x^6+4x^3+4}{x^{\frac{1}{2}}}\,dx =$

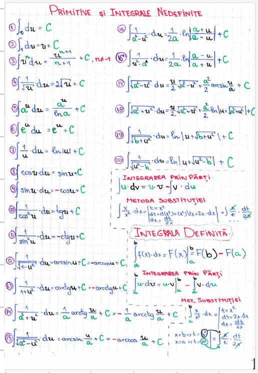
$= \int \frac{x^6}{x^{\frac{1}{2}}}\,dx + 4\int \frac{x^3}{x^{\frac{1}{2}}}\,dx + 4\int \frac{1}{x^{\frac{1}{2}}}\,dx =$

$= \int x^{\frac{11}{2}}\,dx + 4\int x^{\frac{5}{2}}\,dx + 4\int x^{-\frac{1}{2}}\,dx =$

$= \frac{x^{\frac{11}{2}+1}}{\frac{11}{2}+1} + 4\cdot \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 4\cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$

$= \frac{2\cdot x^{\frac{13}{2}}}{13} + 4\cdot 2\frac{x^{\frac{7}{2}}}{7} + 4\cdot 2\frac{x^{\frac{1}{2}}}{1} = \frac{2\sqrt{x^{13}}}{13} + \frac{8\sqrt{x^7}}{7} + 8\sqrt{x} + C$

Integrale. Schimbarea de variabile



$$\int \frac{dx}{(1-3x)^4} = \int \frac{1}{(1-3x)^4}\,dx = \left| \begin{array}{l} t = 1-3x \\ dt = (1-3x)' \\ dt = -3\,dx \\ -3\,dx = dt \\ dx = \frac{dt}{-3} \end{array} \right| =$$

$$= \int \frac{1}{t^4} \cdot \frac{dt}{-3} = \int \frac{1}{-3t^4} \cdot dt = -\frac{1}{3}\int \frac{1}{t^4}\,dt =$$

$$= -\frac{1}{3}\int t^{-4}\,dt = -\frac{1}{3} \cdot \frac{t^{-3}}{-3} + C = \frac{t^{-3}}{9} + C =$$

$$= \frac{(1-3x)^{-3}}{9} + C = \frac{1}{9(1-3x)^3} + C$$

$$\int \frac{dx}{\sqrt[3]{2x-1}} = \int \frac{1}{\sqrt[3]{2x-1}}\,dx = \left| \begin{array}{l} t = 2x-1 \\ dt = 2\,dx \\ dx = \frac{dt}{2} \end{array} \right| =$$

$$= \int \frac{1}{\sqrt[3]{t}} \cdot \frac{dt}{2} = \frac{1}{2}\int \frac{1}{t^{\frac{1}{3}}}\,dt = \frac{1}{2}\int t^{-\frac{1}{3}}\,dt =$$

$$= \frac{1}{2} \cdot \frac{t^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + C = \frac{1}{2}\,\frac{t^{\frac{2}{3}}}{2} \cdot 3 + C = \frac{3t^{\frac{2}{3}}}{4} + C =$$

$$= \frac{3\sqrt[3]{t^2}}{4} + C = \frac{3\sqrt[3]{2x-1}}{4} + C$$

$3x)\,dx$

$$\int \frac{6x-5}{2\sqrt{3x^2-5x+6}}\,dx = \frac{1}{2}\int \frac{6x-5}{\sqrt{3x^2-5x+6}}\,dx =$$

$$= \left| \begin{array}{l} t = 3x^2-5x+6 \\ dt = 6x-5\;dx \\ dx = \dfrac{dt}{6x-5} \end{array} \right| = \frac{1}{2}\int \frac{\cancel{6x-5}}{\sqrt{t}} \cdot \frac{dt}{\cancel{6x-5}} =$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt = \ldots$$

$$\int \frac{e^x}{e^{2x}+4}\,dx = \left| \begin{array}{l} t = e^x \\ dt = e^x\,dx \\ dx = \dfrac{dt}{e^x} \\ dx = \dfrac{dt}{t} \end{array} \right| = \int \frac{\cancel{t}}{e^{2x}+4} \cdot \frac{dt}{\cancel{t}} =$$

$$= \int \frac{1}{e^{2x}+4}\,dt = \int \frac{1}{(e^x)^2+4}\,dt = \int \frac{1}{t^2+4}\,dt =$$

$$= \frac{1}{2}\operatorname{arctg}\frac{t}{2} + c = \frac{1}{2}\operatorname{arctg}\frac{e^x}{2} + c$$

$$\int \frac{x}{\sqrt{x^2-36}}\,dx = \ldots = \sqrt{x^2-36} + c$$

$$\int \frac{x^2}{\sqrt{x^6-4}}\,dx = \left| \begin{array}{l} t = x^6 - 4 \\ dt = 6x^5\,dx \\ dx = \dfrac{dt}{6x^5} \end{array} \right| =$$

$$= \int \frac{x^2}{\sqrt{t}} \cdot \frac{dt}{6x^5} = \frac{1}{6}\int \frac{1}{\sqrt{t}} \cdot \frac{x^2}{x^5}\,dt =$$

$$= \frac{1}{6}\int \frac{x^{-3}}{\sqrt{t}}\,dt$$!!! Tupic

$$\int \frac{x^2}{\sqrt{x^6-4}}\,dx = \int \frac{x^2}{\sqrt{(x^3)^2-4}}\,dx = \left| \begin{array}{l} t = x^3 \\ dt = 3x^2\,dx \\ dx = \dfrac{dt}{3x^2} \end{array} \right|$$

$$= \int \frac{\cancel{x^2}}{\sqrt{t^2-4}} \cdot \frac{dt}{3\cancel{x^2}} = \frac{1}{3}\int \frac{1}{\sqrt{t^2-4}}\,dt =$$

$\Rightarrow$ conform $\boxed{F20}$ $= \dfrac{1}{3} \cdot \ln\left|t + \sqrt{t^2-4}\right| + C$

$$= \frac{1}{3} \cdot \ln\left|x^3 + \sqrt{x^6-4}\right| + C$$

Ex. 2 p. 10 (Culegere cl. 12)

f) $\int \frac{x-3}{x^2-6x+10}\,dx = \left| \begin{array}{l} t = x^2-6x+10 \\ dt = 2x-6\,dx \\ dx = \frac{dt}{2x-6} \end{array} \right|$

$= \int \frac{x-3}{t} \cdot \frac{dt}{2x-6} = \int \frac{\cancel{x-3}}{t} \cdot \frac{dt}{2(\cancel{x-3})} = \frac{1}{2}\int \frac{1}{t}\,dt =$

$= \ldots$

pe acasă:

1. Exercițiile cu simbolul "•"

2. Calculaţi următoarele integrale nedefinite utilizînd metoda schimbării de variabilă:

• *a)* $\int \frac{4x+1}{x-5}dx$; *b)* $\int \frac{x}{x^2+1}dx$; • *c)* $\int \frac{2x+1}{x^2+x+1}dx$; • *d)* $\int \frac{x+2}{x^2+4x+5}dx$;

e) $\int \frac{3x}{x^2+4}dx$; *f)* $\int \frac{x-3}{x^2-6x+10}dx$; • *g)* $\int \frac{2x+3}{x^2+3x-1}dx$; *h)* $\int \frac{8x^3+6x}{2x^4+3x^2+10}dx$;

i) $\int \frac{x}{x^4+1}dx$; *j)* $\int x(2x+1)^{35}dx$; *k)* $\int \frac{2x+1}{(x+1)^2}dx$; • *l)* $\int \frac{1}{(2x+1)^5}dx$;

m) $\int \frac{2x-1}{x^2+9}dx$; *n)* $\int \frac{x+8}{x^2+3}dx$; *o)* $\int \frac{x^2}{2x^3+1}dx$; *p)* $\int \frac{x^3}{(x-1)^{10}}dx$.

3. Calculaţi următoarele integrale nedefinite utilizînd metoda schimbării de variabilă:

a) $\int \sqrt{3x+4}\,dx$; *b)* $\int \sqrt[5]{2x-8}\,dx$; • *c)* $\int \frac{x^5}{\sqrt{x^6+7}}dx$; • *d)* $\int \sqrt{2x+1}\,dx$;

e) $\int 4x\cdot\sqrt[3]{x^2+8}\,dx$; *f)* $\int x\sqrt{1+x}\,dx$; *g)* $\int (x-2)\sqrt{x+4}\,dx$; *h)* $\int \sqrt[3]{(2x+1)^2}\,dx$;

i) $\int x\sqrt{x^2+3}\,dx$; *j)* $\int x^2\sqrt{x^3+1}\,dx$; *k)* $\int \frac{3x^2-2x+7}{\sqrt{x^3-x^2+7x-2}}dx$; • *l)* $\int \frac{7x+2}{\sqrt{x^2+10}}dx$;

m) $\int \frac{x}{\sqrt{x^4+4}}dx$; *n)* $\int \frac{x^2}{\sqrt{x^6-4}}dx$; *o)* $\int \frac{x}{\sqrt{1-x}}dx$; *p)* $\int x^2\cdot\sqrt{x^3+5}\,dx$.

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$$\int \frac{4x+1}{x-5}\,dx$$

$$\left|\begin{array}{l} t = x-5 \\ dt = 1\,dx \\ dx = dt \\ x-5 = t \\ x = t+5 \end{array}\right| = \int \frac{4(t+5)+1}{t}\cdot dt = \int \frac{4t+20+1}{t}\,dt =$$

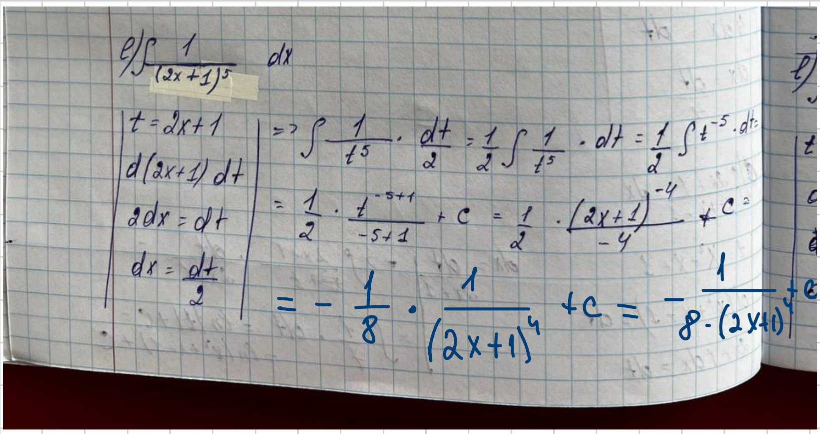
$$= \int \frac{4\cancel{t}}{\cancel{t}}\,dt + \int \frac{20}{t}\,dt + \int \frac{1}{t}\,dt =$$

$$= \int 4\,dt + 20\int \frac{1}{t}\,dt + \int \frac{1}{t}\,dt =$$

$$= 4t + 20\cdot \ln|t| + \ln|t| + c =$$

$$= 4(x-5) + 20\cdot \ln|x-5| + \ln|x-5| + c =$$

$$= 4x - 20 + 21\ln|x-5|$$



e) $\int \frac{1}{(2x+1)^5}\,dx$

$$\left|\begin{array}{l} t = 2x+1 \\ d(2x+1)\,dt \\ 2dx = dt \\ dx = \frac{dt}{2} \end{array}\right| \Rightarrow \int \frac{1}{t^5}\cdot\frac{dt}{2} = \frac{1}{2}\int \frac{1}{t^5}\cdot dt = \frac{1}{2}\int t^{-5}\cdot dt =$$

$$= \frac{1}{2}\cdot\frac{t^{-5+1}}{-5+1} + c = \frac{1}{2}\cdot\frac{(2x+1)^{-4}}{-4} + c =$$

$$= -\frac{1}{8}\cdot\frac{1}{(2x+1)^4} + c = -\frac{1}{8\cdot(2x+1)^4} + c$$

b) $\int \frac{7x+2}{\sqrt{x^2+10}}\,dx = \int \frac{7x}{\sqrt{x^2+10}}\,dx + \int \frac{2}{\sqrt{x^2+10}}\,dx =$

$= 7\int \frac{x}{\sqrt{x^2+10}}\,dx + 2\int \frac{1}{\sqrt{x^2+10}} =$

$= 7\sqrt{x^2+10} +$

$7\int \frac{x}{\sqrt{x^2+10}}\,dx = \left| \begin{array}{l} t = x^2+10 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{array} \right| = 7\int \frac{\cancel{x}}{\sqrt{t}} \cdot \frac{dt}{2\cancel{x}} =$

$= \frac{7}{\cancel{2}} \cdot \cancel{2}\sqrt{t} + c = 7\sqrt{x^2+10} + c$

Rezolvarea integralelor prin părți

$$\boxed{\int u\,dv = u \cdot v - \int v\,du}$$

$\int x \cdot \cos 2x\,dx = \left| \begin{array}{ll} u = x' & du = 1\,dx \\ dv = \cos 2x\,dx & v = \int \cos 2x\,dx \\ & v = \frac{1}{2}\sin 2x + c \end{array} \right| =$

$= x \cdot \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \cdot dx = \frac{x \cdot \sin 2x}{2} - \frac{1}{2}\int \sin 2x\,dx =$

$= \frac{x \cdot \sin 2x}{2} - \frac{1}{2} \cdot \left(-\frac{1}{2}\right) \cdot \cos 2x + c =$

$$= \frac{x \cdot \sin 2x}{2} + \frac{\cos 2x}{4} + c$$

$$\int (xe^x - 2x)\,dx = \int xe^x\,dx - \int 2x\,dx =$$

$$= \int xe^x\,dx - 2\int x\,dx = \qquad - \cancel{2} \cdot \frac{x^2}{\cancel{2}} + c$$

$$\int u\,dv = u \cdot v - \int v\,du$$

$$\int x \cdot e^x\,dx = \left| \begin{array}{ll} u = x^1 & du = 1\,dx \\ dv = e^x & v = \int e^x dx \\ & v = e^x \end{array} \right| =$$

$$= x \cdot e^x - \int e^x \cdot dx = x \cdot e^x - e^x + c = e^x(x-1)+c$$

$$\int x \cdot \ln x\,dx = \left| \begin{array}{ll} u = \ln x & du = \frac{1}{x}dx \\ dv = x\,dx & v = \int x\,dx \\ & v = \frac{x^2}{2} \end{array} \right| =$$

$$= \ln x \cdot \frac{x^2}{2} - \int \frac{x^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{x}}\,dx = \frac{\ln x \cdot x^2}{2} - \frac{1}{2}\int x\,dx$$

$$= \frac{\ln x \cdot x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + c = {}^{2)}\frac{\ln x \cdot x^2}{2} - \frac{x^2}{4} + c =$$

$$= \frac{2\ln x \cdot x^2 - x^2}{4} + c$$

pe acasă:

① $\int x \cdot e^{-x}\,dx$ ✓

② $\int (2x-1)\cdot e^{3x}\,dx$

③ $\int (2x-5)\,e^{2x}\,dx$

④ $\int (x^2-2x+3)e^{2x}\,dx$

⑤ $\int [(2x-3)\cdot \ln(x-2)]\,dx$

22.09.25

$$\int u\cdot dv = u\cdot v - \int v\cdot du$$

1. $$\int x\cdot e^{-x}\,dx = \left|\begin{array}{ll} u = x & du = 1\,dx \\ dv = e^{-x}dx & v = \int e^{-x}dx \\ & v = -e^{-x} \end{array}\right| =$$

$$= -e^{-x}\cdot x - \int -e^{-x}\,dx = e^{-x}\cdot x + \int e^{-x}\,dx =$$

$$= e^{-x}\cdot x - e^{-x} + c = e^{-x}(x-1) + c$$

$$\int e^{-x}\,dx = \left|\begin{array}{l} t = -x \\ dt = -1\,dx \\ dx = \dfrac{dt}{-1} \end{array}\right| = \int e^{t}\,\frac{dt}{-1} =$$

$$= -1\cdot\int e^{t}\,dt = -1\cdot e^{t} + c = -1\cdot e^{-x} + c =$$

$$= -e^{-x} + c$$

2. $\int (2x-1)\cdot e^{3x}\,dx = \left| \begin{array}{ll} u = 2x-1 & du = 2\,dx \\ dv = e^{3x} & v = \int e^{3x}\,dx \\ & v = \frac{1}{3}e^{3x} \end{array} \right|$

$= (2x-1)\cdot \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\cdot 2\,dx = \frac{(2x-1)\cdot e^{3x}}{3} - \frac{2}{3}\int e^{3x}\,dx =$

$= \frac{(2x-1)\cdot e^{3x}}{3} - \frac{2}{3}\cdot\frac{e^{3x}}{3} + C$

$$\int e^{3x}\,dx = \left| \begin{array}{l} t = 3x \\ dt = 3\,dx \\ dx = \frac{dt}{3} \end{array} \right| = \int e^{t}\cdot\frac{dt}{3} = \frac{1}{3}\int e^{t}\,dt =$$

$$= \frac{1}{3}e^{t} + c = \frac{1}{3}e^{3x} + c = \frac{e^{3x}}{3} + C$$

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3. $\int [(2x-3)\cdot \ln(x+2)]\,dx = \left| \begin{array}{ll} u = \ln(x+2) & du = \frac{1}{x+2}\,dx \\ dv = 2x-3\,dx & V = \int (2x-3)\,dx \\ & V = x^2-3x \end{array} \right|$

$= \ln(x+2)\cdot(x^2-3x) - \int (x^2-3x)\cdot\frac{1}{x+2}\,dx =$

$= \dots \qquad - \int \frac{x(x-3)}{x+2}\,dx =$

$(x-3) = (x+2-5)$

$$\int \frac{x(x-3)}{x+2}\,dx = \int \frac{x(x+2-5)}{x+2}\,dx = \int \frac{x(x+2)-5x}{x+2}\,dx =$$

$$= \int \frac{x(x+2)-(5x+10-10)}{x+2}\,dx = \int \frac{x(x+2)-(5x+10)+10}{x+2}\,dx =$$

$$= \int \frac{x(x+2)-5(x+2)+10}{x+2}\,dx =$$

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$$= \int \frac{x\cancel{(x+2)}}{\cancel{x+2}}\,dx - 5\int \frac{\cancel{x+2}}{\cancel{x+2}}\,dx + 10\int \boxed{\frac{1}{x+2}}\,dx =$$

$$\int \left[(2x-3)\cdot \ln(x-2)\right] dx = \int 2x\cdot \ln(x-2)\,dx - \int 3\ln(x-2)\,dx$$