Shree Rahul Education Society's (Regd.)

SHREE L. R. TIWARI COLLEGE OF ENGINEERING

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TT1 - Maths

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Solution- 2 (b)

We have,

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Here, $(c, c+2l) = (-\pi, \pi)$

$$\therefore l = \pi$$

Replace x with $-x$,

$$f(-x) = |-x| = |x| = f(x)$$

$$\therefore f(-x) = f(x)$$

$\Rightarrow$ $f(x)$ is an even function $\Rightarrow b_n = 0$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)\,dx}_{\text{even}} = \frac{1}{\pi}\,2\int_0^{\pi} |x|\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\,dx = \left(\frac{x^2}{2}\right)_0^{\pi} \times \frac{2}{\pi} = \frac{\pi^2}{2} \times \frac{2}{\pi} = \pi = l.$$

$$a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{f(x)}_{\text{even}} \underbrace{\cos\left(\frac{n\pi x}{2}\right)}_{\text{even}} dx$$

$$= \frac{1}{\pi}\times 2\int_0^{\pi} \underbrace{(x)}_{u} \underbrace{\cos\left(\frac{n\pi x}{2}\right)}_{v} dx$$

$$= \frac{2}{\pi}\left[x\cdot\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right) - (1)\frac{2^2}{n^2\pi^2}\left(-\cos\left(\frac{n\pi x}{2}\right)\right)\right]_0^{\pi}$$

$$= \frac{2 \times \pi}{(\pi n^2)} \left((-1)^n - 1 \right)$$

We can rewrite a_n, as

$$a_n = \frac{2\pi}{(\pi n)^2} \left((-1)^n - 1 \right) = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{-4\pi}{(\pi n)^2}, & \text{if } n \text{ is odd} \end{cases}$$

$$= \begin{cases} 0, & \text{if } n = 2k \\ \frac{-4\pi}{\pi^2 (2k-1)^2}, & \text{if } n = 2k-1 \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{\pi n x}{l} = \frac{1}{2} + \sum_{n=1}^{\infty} \left(\frac{2l}{(\pi n)^2} \left((-1)^n - 1 \right) \cos \frac{\pi n x}{l} \right)$$

$$= \frac{l}{2} + \frac{2l}{\pi^2} \sum_{n=1}^{\infty} \frac{\left((-1)^n - 1 \right)}{n^2} \cos \frac{\pi n x}{l} = \frac{l}{2} + \frac{4l}{\pi^2} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \cos \frac{\pi (2k-1) x}{l}$$

if $l = \pi$, then

$$\cancel{f(x) = \frac{\pi}{2} + \frac{4\pi}{\pi^2} \sum_{n=1}^{\infty} \left(\frac{(-1)^n - 1}{n^2} \right) \cos \frac{\pi n x}{J}}$$

$$f(x) = \frac{\pi}{2} + \frac{4\pi}{\pi^2} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \cos \frac{\pi (2k-1) x}{\pi}$$

$$\boxed{f(x) = \frac{\pi}{2} + \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} \cos (2k-1) x.}$$

Solution - 3 a

Let $f(z) = u+iv$ be an analytic f^n

Differentiating $f(z)$ partially wrt x, we get

$$f'(z) = u_x + i v_x$$

Let $u = e^x \cos y - xy - c$

$$u_x = e^x \cos y - y$$

$$u_y = -e^x \sin y - x$$

$$f'(z) = u_x + i v_x$$

$$= u_x - i u_y \quad \left(\text{Cauchy-Riemann } eq^n \; u_x = v_y, \; u_y = v_x\right)$$

$$= (e^x \cos y - y) - i(-e^x \sin y - x)$$

By Milne-Thomson's method let $x = z$ & $y = 0$, we get

$$f'(z) = (e^z \cos 0 - 0) - i(-e^z \sin 0 - z)$$

$$f'(z) = e^z + iz$$

Integrating both sides, we get

$$\int f'(z)\, dz = \int (e^z + iz)\, dz$$

$$f(z) = e^z + i\frac{z^2}{2} + k$$

Let $z = x+iy$

$$f(z) = e^{x+iy} + \frac{i}{2}(x+iy)^2 + k$$

$= e^x e^{iy} + \frac{i}{2}(x^2 + i2xy - y^2) + k_1 + ik_2$

$= e^x(\cos y + i\sin y) + \frac{i}{2}x^2 - \frac{2xy}{2} - \frac{iy^2}{2} + k_1 + ik_2$

$= \underbrace{(e^x \cos y - xy + k_1)}_{m_1} + i\underbrace{\left(e^x \sin y + \frac{x^2}{2} - \frac{y^2}{2} + k_2\right)}_{m_2}$

$U = e^x \cos y - xy + k_1$ $\qquad V = e^x \sin y + \frac{x^2}{2} - \frac{y^2}{2} + k_2$

$U_x = e^x \cos y - y$ $\qquad V_x = e^x \sin y + x$

$U_y = -e^x \sin y - x$ $\qquad V_y = e^x \cos y - y$

$\therefore U_x = V_y$ and $U_y = V_x$

$m_1 = \frac{dy}{dx} = \frac{-U_x}{U_y} = -\left(\frac{e^x \cos y - y}{-(e^x \sin y + x)}\right) = +\left(\frac{e^x \cos y - y}{(e^x \sin y + x)}\right)$

$m_2 = \frac{dy}{dx} = \frac{-V_x}{V_y} = -\left(\frac{e^x \sin y + x}{e^x \cos y - y}\right)$

$\therefore m_1 m_2 = \left(\frac{-U_x}{U_y}\right)\left(\frac{-V_x}{V_y}\right) = \left(\frac{-V_y}{-V_x}\right)\left(\frac{-V_x}{V_y}\right) = -1$

and $m_1 m_2 = \left(\frac{e^x \cos y - y}{e^x \sin y + x}\right)\left(-\frac{(e^x \sin y + x)}{e^x \cos y - y}\right) = -1$

C.R