无研23 刘子源 2022310709 10.7 10.9 9.1 9.4 10.3 10.4

10.4 $p(x|\theta)=\begin{cases}\frac{1}{\theta^N} & \text{all } x[n]\in[0,\theta]\\ 0 & 其他.\end{cases}$

$p(\theta)=\frac{1}{\beta}\quad,\quad \theta\in[0,\beta]$

$$p(\theta|x)=\frac{\cancel{p(x)}\,p(\theta)p(x|\theta)}{\int p(\theta)p(x|\theta)\,d\theta}$$

记 ~~x_{min}~~ $x_{max}=\max\{x[n]\}$

$$=\frac{\frac{1}{\theta^N}\frac{1}{\beta}}{\int_{x_{max}}^{\beta}\frac{1}{\theta^N}\frac{1}{\beta}\,d\theta}$$

$$=\frac{\frac{1}{\theta^N}}{\frac{1}{N-1}\left(x_{max}^{-(N-1)}-\beta^{-(N-1)}\right)}$$

$$\hat{\theta}=E[\theta|x]$$

$$=\int\theta\cdot\frac{\frac{1}{\theta^N}}{\frac{1}{N-1}\left(x_{max}^{-(N-1)}-\beta^{-(N-1)}\right)}\,d\theta$$

$$=\frac{N-1}{N-2}\cdot\frac{x_{max}^{-(N-2)}-\beta^{-(N-2)}}{x_{max}^{-(N-1)}-\beta^{-(N-1)}}$$

当 β 很大时 $\hat{\theta}\approx\frac{N-1}{N-2}x_{max}\approx x_{max}$.

估计结果几乎不会用到先验，只与数据分布相关

10.9.

$\varepsilon\sim N(0,0.01)$

令 $x[n]=\cancel{A}R+w[n]$

欧姆表误差 $w[n]$ iid $N(0,1)$

测量值 $AR\sim N(100,0.01)$ 即 $\sigma_R^2=\sigma_\varepsilon^2$

$$\mathrm{MMSE}(\hat{AR})=0.01=\frac{\sigma^2\sigma_R^2/N}{\sigma_R^2+\frac{\sigma^2}{N}}=\frac{1\times0.01/N}{0.01+\frac{1}{N}}$$

$\Longrightarrow N=9.09\approx10$. 需要测10次

若没有先验

$$\mathrm{MMSE}(\hat{R})=\frac{\sigma^2}{N}=\frac{1}{N}=0.01\Rightarrow N=100$$

需要测100次

9.1 $\cancel{\frac{p(x;\sigma)}{p(x)}}=\begin{cases}\frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right), & x>0\\ 0, & x\le 0\end{cases}$

$$EX=\int_0^{\infty}\frac{x^2}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}}\,dx=\sqrt{\frac{\pi}{2}}\sigma$$

$$\sigma^2=\frac{2}{\pi}E^2X$$

$$\hat{\sigma}^2=\frac{2}{\pi}\left(\frac{1}{N}\sum_{n=0}^{N-1}x[n]\right)^2=\frac{2}{\pi}\bar{X}^2$$

9.4 $\mu=EX\ ,\ \sigma^2=\mathrm{var}X=EX^2-(EX)^2$

$$\hat{\mu}=\frac{1}{N}\sum_{n=0}^{N-1}x[n]=\bar{X}$$

$$\cancel{\hat{\sigma}^2=\frac{1}{N}\sum_{n=0}^{N-1}(x[n]-\hat{\mu})^2=\frac{1}{N}\sum_{n=0}^{N-1}(x[n]-\bar{X})^2}$$

$$\hat{\sigma}^2=\frac{1}{N}\sum_{n=0}^{N-1}x^2[n]-\bar{X}^2$$

10.3 $p(x[n]|\theta)=\begin{cases}\exp[-(x[n]-\theta)], & x[n]>\theta\\ 0, & x[n]<\theta\end{cases}$

$p(\theta)=\begin{cases}\exp(-\theta), & \theta>0\\ 0, & \theta<0\end{cases}$

$$p(x|\theta)=\exp\left(-\sum_{n=0}^{N-1}(x[n]-\theta)\right)=\exp\left(-N(\bar{X}-\theta)\right),\quad x[i]>\theta,\ 0\le i\le N$$

$$p(\theta|x)=\frac{p(\theta)p(x|\theta)}{\int p(\theta)p(x|\theta)\,d\theta}$$

$$=\frac{e^{-N(\bar{x}-\theta)}e^{-\theta}}{\int_0^{\min x[n]}e^{-\theta}e^{-N(\bar{x}-\theta)}\,d\theta}$$

记 x_{min} ~~x_{max}~~ $=\min\{x_n\}$

$$=\frac{e^{\theta(N-1)}}{\int_0^{x_{min}}e^{\theta(N-1)}\,d\theta}$$

$$=\begin{cases}\frac{(N-1)e^{\theta(N-1)}}{e^{(N-1)x_{min}}-1}, & 0<\theta<x_{min}.\\ 0, & 其他\end{cases}$$

$$\hat{\theta}=E(\theta|x)=\frac{\int_0^{x_{min}}\theta(N-1)e^{\theta(N-1)}\,d\theta}{e^{(N-1)x_{min}}-1}$$

$$=\frac{x_{min}e^{(N-1)x_{min}}}{e^{(N-1)x_{min}}-1}-\frac{1}{N-1}$$

$$=\frac{x_{min}}{1-e^{-(N-1)x_{min}}}-\frac{1}{N-1}$$

2(1) 由图(a)可知 w和h是正相关的，假设 $\begin{bmatrix} h \\ w \end{bmatrix}$ 服从高斯分布.

则用MMSE根据身高估计的体重为 $\hat{w} = \cancel{Eh}\, Ew + \frac{cov(h,w)}{var\, h}(h - Eh)$.

由图(b)可知h和w无相关性，$cov(h,w)=0$，$\therefore \hat{w} = Ew \approx 150$.