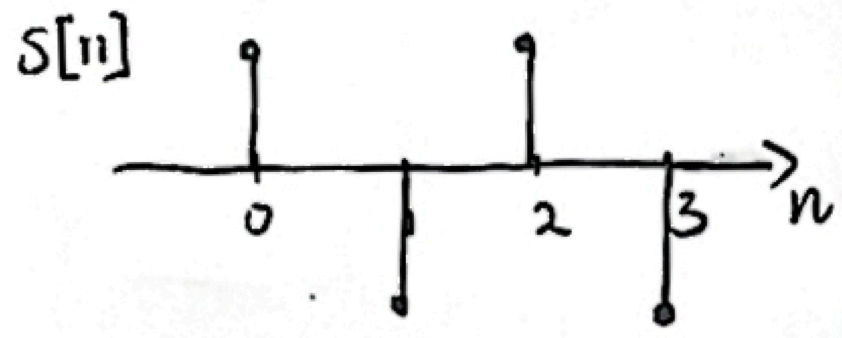
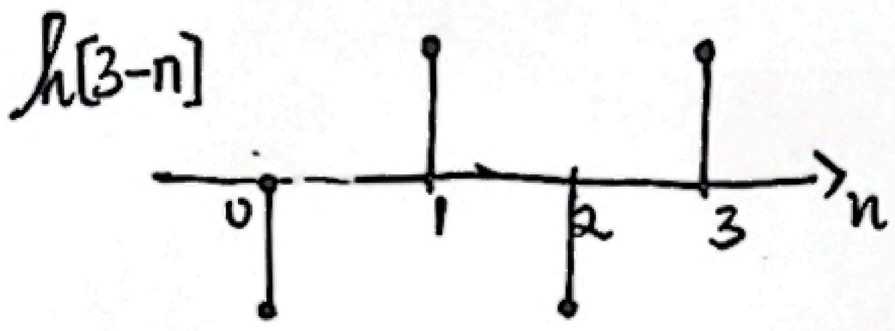
光研223 刘子源 2022310709

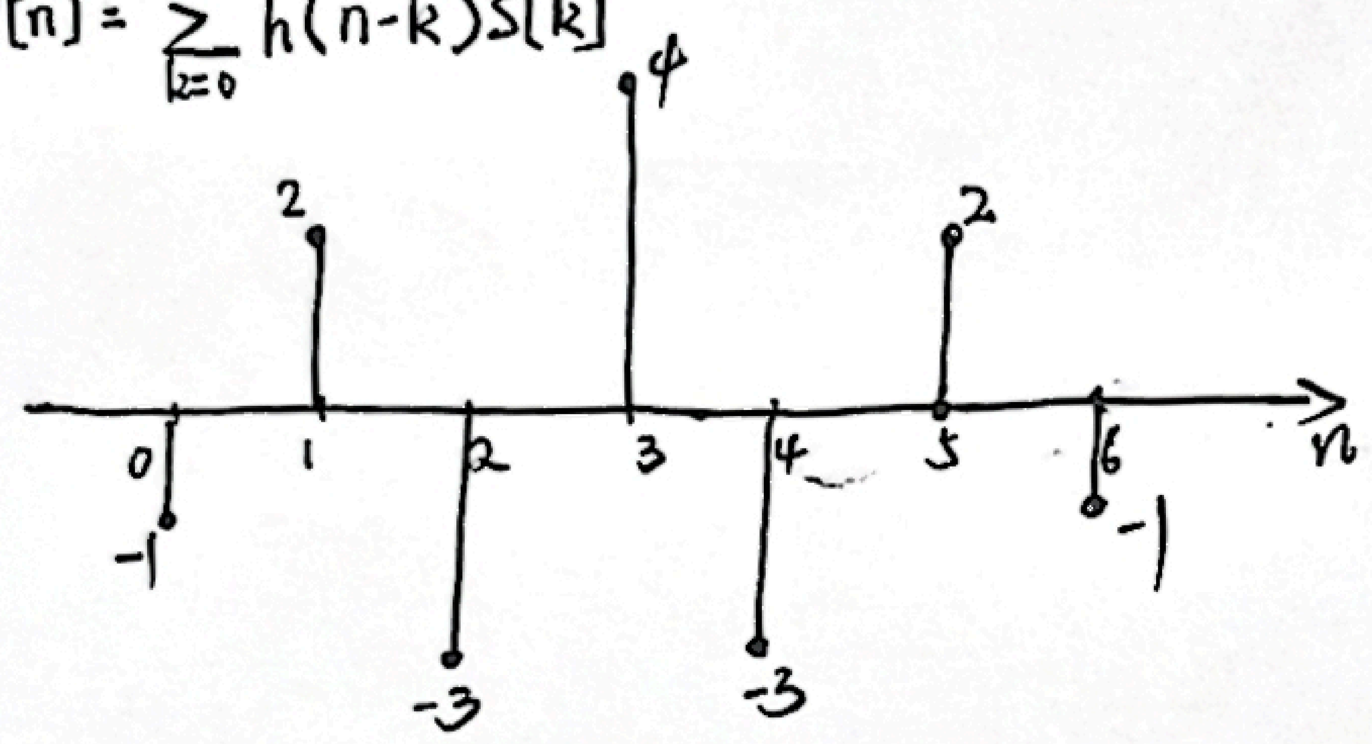
4.1 4.3 4.7 4.8 4.15 4.19.

4.1 $s[n]=(-1)^n$, $n=0,1,2,3$





$y[n]=\sum_{k=0}^{n} h(n-k)s[k]$



4.3

$y[n-1]=\sum_{n=0}^{N-1} s^2[n]$

$=\sum_{n=0}^{N-1} A^2\cos^2 2\pi f_0 n_0$

$=A^2\sum_{n=0}^{N-1}\left(\frac{1}{2}+\frac{1}{2}\cos 4\pi f_0 n_0\right)$

$\approx \frac{NA^2}{2}$

假设信号延迟 n_0

$y[n-1]=\sum_{n=0}^{N-1} \cancel{*} s[n-n_0]s[n]$

$=\sum_{n=\cancel{n_0}0}^{N-1-n_0} s[n]s[n+n_0]$

$=A^2\sum_{n=\cancel{n_0}0}^{N-1-n_0}\cos 2\pi f_0 n\cos 2\pi f_0(n+n_0)$

$=A^2\sum_{n=0}^{N-1-n_0}\left(\frac{1}{2}\cos 2\pi f_0 n_0+\frac{1}{2}\cos(4\pi f_0 n+2\pi f_0 n_0)\right)$

$\approx \frac{\cancel{N}A^2}{2}\sum_{n=0}^{N-1-n_0}\cos 2\pi f_0 n_0$

$=\frac{NA^2}{2}\frac{N-n_0}{N}\cos 2\pi f_0 n_0$

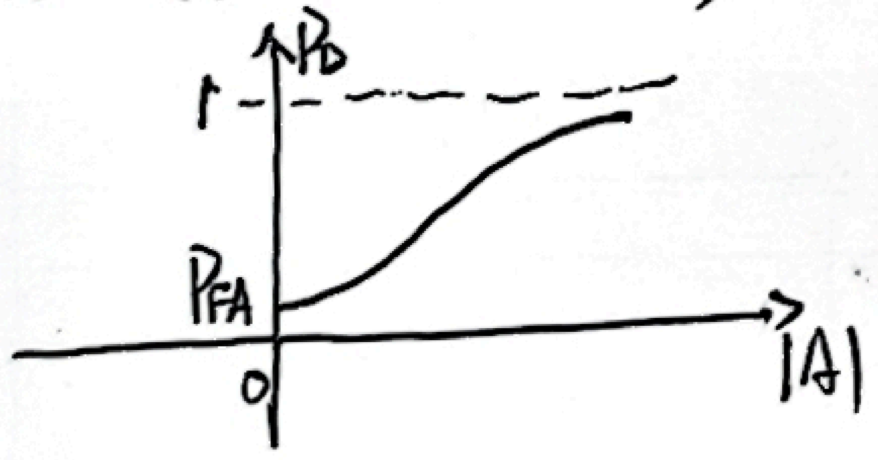
4.7

$P_D=Q\left(Q^{-1}(P_{FA})-\sqrt{\frac{\varepsilon}{\sigma^2}}\right)$.

$\frac{\varepsilon}{\sigma^2}=\frac{\sum_{n=0}^{N-1}s^2[n]}{\sigma^2}=A^2\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n=A^2\sum_{n=0}^{24}\cos^2\frac{\pi n}{2}$

$=13A^2$

则 $P_D=Q\left(Q^{-1}(P_{FA})-\sqrt{13A^2}\right)$



4.8 $s_1[n]$ 和 $s_2[n]$ 能量相同，所以性能应该相同

4.15

$T(x)=x^TC^{-1}s=\sum_{n=0}^{N-1}\frac{x[n]s[n]}{\sigma^2}=A\sum_n\frac{s[n]}{\sigma^2 r^n}$

当 $\sum_n\frac{x[n]}{r^n}>\gamma'$ 时判定 H_1

$P_D=Q\left(Q^{-1}(P_{FA})-\sqrt{s^TC^{-1}s}\right)$

$s^TC^{-1}s=A^2\sum_{n=0}^{N-1}\frac{1}{\sigma^2 r^n}=\frac{A^2}{\sigma^2}\sum_{n=0}^{N-1}\frac{1}{r^n}$

当 $N\to\infty$ 时若 $0<r\le 1$，$s^TC^{-1}s\to\infty$，$P_D\to 1$

若 $r>1$，$s^TC^{-1}s\to\frac{A^2}{\sigma^2}\frac{1}{1-\frac{1}{r}}=\frac{A^2r}{\sigma^2(r-1)}$

4.19 当 $\underbrace{\ln p(x|H_1)+\ln P(H_1)}_{g_1}>\underbrace{\ln p(x|H_0)+\ln P(H_0)}_{g_0}$

时选 H_1

$p(x|H_i)=\frac{1}{2\pi}e^{-\frac{1}{2}(x-s_i)^T(x-s_i)}$

$g_i=-\ln 2\pi-\frac{1}{2}(x-s_i)^T(x-s_i)+\ln P(H_i)$

令 $g_0=g_1$ 可得到判决边界、即

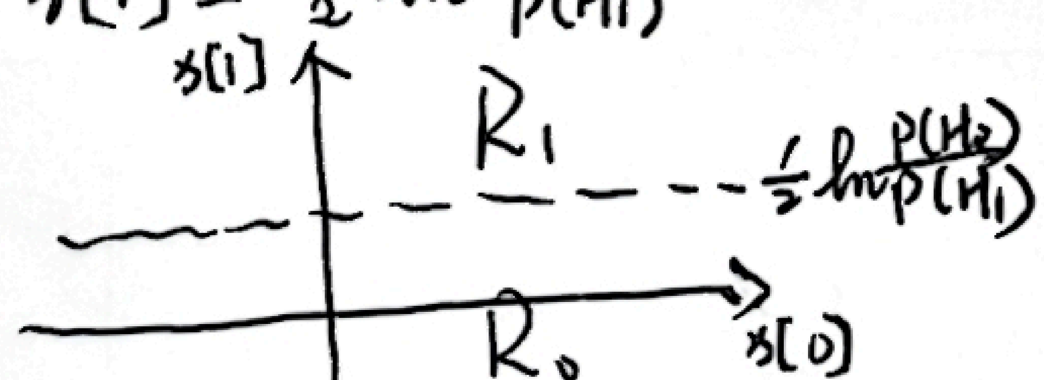
$-\frac{1}{2}(x-s_1)^T(x-s_1)+\ln P(H_1)=\frac{-1}{2}(x-s_0)^T(x-s_0)+\ln P(H_0)$

$x^T(s_1-s_0)=\frac{1}{2}(\Sigma_1-\Sigma_0)+\ln\frac{P(H_0)}{P(H_1)}$ $(\Sigma_i=s_i^Ts_i)$

$s_0=\begin{bmatrix}1\\-1\end{bmatrix}$ $s_1=\begin{bmatrix}1\\1\end{bmatrix}$，代入得

$x^T\begin{bmatrix}0\\2\end{bmatrix}=\ln\frac{P(H_0)}{P(H_1)}$

$x[1]=\frac{1}{2}\ln\frac{P(H_0)}{P(H_1)}$



当 $P(H_0)=P(H_1)$ 时选择 H_1 若 $x[1]>0$

若 $P(H_0)>P(H_1)$，边界线向上移，反之向下.