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8.1 8.3 8.20 8.22 8.25 8.26.

8.1 $\theta=[A\ f_0\ Br]^T$

$$J=\sum_{n=0}^{N-1}(x[n]-A\cos 2\pi f_0 n - Br^n)^2$$

1) θ中包含参数f_0和r，所以是非线性LS问题

2) LS误差是参数的二次型函数，参数是A和B

3) ~~固定~~ 给定f_0和r，求出使J最小化的A和B，然后J变成关于f_0和r的非二次型函数，再用网格法、NR法等求解f_0和r

8.3

$$s[n]=\begin{cases}A, & 0\le n\le M-1\\ -A, & M\le n\le N-1\end{cases}$$

$$J=\sum_{n=0}^{M-1}(x[n]-A)^2+\sum_{n=M}^{N-1}(x[n]+A)^2$$

$$\frac{\partial J}{\partial A}=-2\sum_{n=0}^{M-1}(x[n]-A)+2\sum_{n=M}^{N-1}(x[n]+A)=0$$

$$-2\sum_{0}^{M-1}x[n]+2MA+2\sum_{M}^{N-1}x[n]+2(N-M)A=0$$

$$\hat{A}=\frac{1}{N}\left(\sum_{n=0}^{M-1}x[n]-\sum_{n=M}^{N-1}x[n]\right).$$

$$J_{min}=\sum_{n=0}^{M-1}(x[n]-\hat{A})(x[n]-\hat{A})+\sum_{n=M}^{N-1}(x[n]+\hat{A})(x[n]+\hat{A})$$

$$=\sum_{n=0}^{M-1}x[n](x[n]-\hat{A})+\sum_{n=M}^{N-1}x[n](x[n]+\hat{A}).$$

$$=\sum_{n=0}^{N-1}x^2[n]-\hat{A}\left(\sum_{n=0}^{M-1}x[n]-\sum_{n=M}^{N-1}x[n]\right)$$

$$=\sum_{n=0}^{N-1}x^2(n)-N\hat{A}^2$$

$w[n]\sim N(0,\sigma^2)$

$$E\hat{A}=\frac{1}{N}[MA-(N-M)A]=A$$

$$\mathrm{var}\hat{A}=\frac{1}{N^2}\left(\mathrm{var}\left(\sum_{n=0}^{M-1}x[n]\right)+\mathrm{var}\left(\sum_{n=M}^{N-1}x[n]\right)\right)$$

$$=\frac{1}{N^2}(M\sigma^2+(N-M)\sigma^2)$$

$$=\frac{\sigma^2}{N}$$

即 $\hat{A}\sim N(A,\frac{\sigma^2}{N})$ 服从高斯分布.

8.20 $x[n]=Ar^n+w[n]$, $w[n]\sim N(0,1)$.

$H[n]=[1\ r\ \cdots\ r^n]^T$, $h(n)=r^n$

$$\hat{A}[n]=\hat{A}[n-1]+K[n](x[n]-r^n\hat{A}[n-1])$$

~~$\sigma_n^2=\sigma^2=1$，~~

$$K[n]=\frac{\mathrm{var}(\hat{A}[n-1])r^n}{1+r^{2n}\mathrm{var}(\hat{A}[n-1])}$$

$$\mathrm{var}\hat{A}[n]=(1-K[n]r^n)\mathrm{var}(\hat{A}[n-1]).$$

令 $\sigma_n^2=\mathrm{var}(\hat{A}[n])$

$$=\left(1-\frac{\sigma_{n-1}^2 r^{2n}}{1+r^{2n}\sigma_{n-1}^2}\right)\sigma_{n-1}^2$$

$$=\frac{\sigma_{n-1}^2}{1+r^{2n}\sigma_{n-1}^2}$$

$$\sigma_0^2=1,\ \sigma_1^2=\frac{1}{1+r^2},\ \sigma_2^2=\frac{\frac{1}{1+r^2}}{1+r^4\left(\frac{1}{1+r^2}\right)}=\frac{1}{1+r^2+r^4}$$

可推得 $\sigma_n^2=\mathrm{var}(\hat{A}[n])=\dfrac{1}{\sum_{k=0}^{n}r^{2k}}$.

8.22

见图片

8.25 $s[n]=A+B(-1)^n$

$$H=\begin{bmatrix}1 & -1\\ 1 & -1\\ \vdots & \vdots\\ 1 & -1\end{bmatrix}$$

$$\hat{\theta}=(H^TH)^{-1}H^TX=\begin{bmatrix}N & 0\\ 0 & N\end{bmatrix}^{-1}\begin{bmatrix}1 & -1\\ 1 & -1\\ \vdots & \vdots\\ 1 & -1\end{bmatrix}X=\begin{bmatrix}\frac{1}{N}\sum_{n=0}^{N-1}x[n]\\ \frac{1}{N}\sum_{n=0}^{N-1}(-1)^n x[n]\end{bmatrix}$$

假设$A=B$

有$[1\ -1]\theta=0$

$A=[1\ -1]$, $b=0$

$$\hat{\theta}_c=\hat{\theta}-(H^TH)^{-1}A^T(A(H^TH)^{-1}A^T)^{-1}A\hat{\theta}$$

$$=\hat{\theta}-\frac{1}{N}A^T\left(\frac{1}{N}AA^T\right)^{-1}A\hat{\theta}$$

$$=(I-A^T(AA^T)^{-1}A)\hat{\theta}$$

$$=\left(\begin{bmatrix}1 & \\ & 1\end{bmatrix}-\begin{bmatrix}1\\ -1\end{bmatrix}\left([1\ -1]\begin{bmatrix}1\\ -1\end{bmatrix}\right)^{-1}[1\ -1]\right)\hat{\theta}$$

$$=\begin{bmatrix}\frac{1}{2}\left(\bar{x}+\frac{1}{N}\sum_{n=0}^{N-1}(-1)^n x[n]\right)\\ \frac{1}{2}\left(\bar{x}+\frac{1}{N}\sum_{n=0}^{N-1}(-1)^n x[n]\right)\end{bmatrix}$$

$$\hat{A}_c=\hat{B}_c=\frac{1}{2N}\sum_{\substack{n=0\\ n\text{是偶数}}}^{N-1}x[n]\qquad s[n]=\begin{cases}A, & n\text{偶数}\\ 0, & n\text{奇数}\end{cases}$$

8.2.6

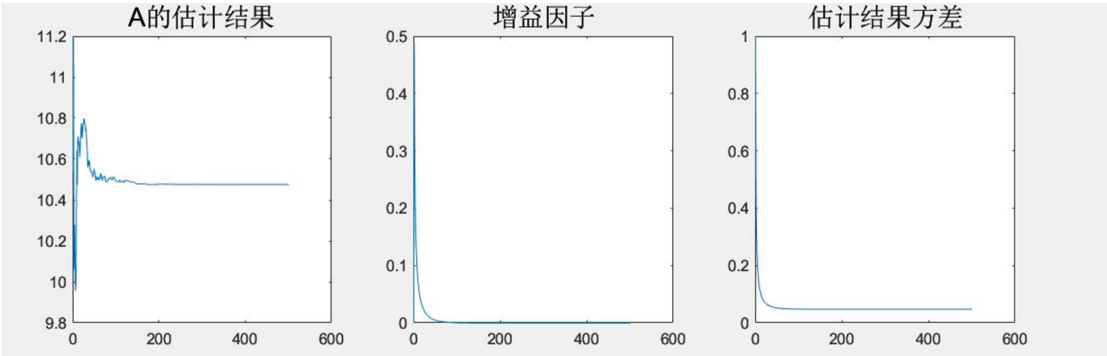
证明：$h(g^{-1}(\hat{\alpha})) \le h(g^{-1}(\alpha))$ 对 $\forall \alpha$，

$\Rightarrow h(\theta_0) \le h(\theta)$，对 $\forall \theta$

其中 $\theta_0 = g^{-1}(\hat{\alpha})$

则 θ_0 最小化 $h(\theta)$，即 $\theta_0 = \hat{\theta}$

所以 $\hat{\theta} = g^{-1}(\hat{\alpha})$ □


A的估计结果
增益因子
估计结果方差
11.2
11
10.8
10.6
10.4
10.2
10
9.8
0.5
0.4
0.3
0.2
0.1
0
1
0.8
0.6
0.4
0.2
0
200
400
600