作业（3）：

1．用两阶段法求解下列线性规划：(119 页第 2 题)

$$
\begin{aligned}
(3)\ \max\ & 3x_1 - 5x_2 \\
s.t.\ & -x_1 + 2x_2 + 4x_3 \le 4 \\
& x_1 + x_2 + 2x_3 \le 5 \\
& -x_1 + 2x_2 + x_3 \ge 1 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

$$
\begin{aligned}
(4)\ \min\ & x_1 - 3x_2 + x_3 \\
s.t.\ & 2x_1 - x_2 + x_3 = 8 \\
& 2x_1 + x_2 \ge 2 \\
& x_1 + 2x_2 \le 10 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

2. 用大 M 法求解下列线性规划：(119 页第 2 题)

$$
\begin{aligned}
(5)\ \max\ & -3x_1 + 2x_2 - x_3 \\
s.t.\ & 2x_1 + x_2 - x_3 \le 5 \\
& 4x_1 + 3x_2 + x_3 \ge 3 \\
& -x_1 + x_2 + x_3 = 2 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

$$
\begin{aligned}
(7)\ \min\ & 3x_1 - 2x_2 + x_3 \\
s.t.\ & 2x_1 - 3x_2 + x_3 = 1 \\
& 2x_1 + 3x_2 \ge 8 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

3．给定原问题

$$
\begin{aligned}
\min\ & 4x_1 + 3x_2 + x_3 \\
s.t.\ & x_1 - x_2 + x_3 \ge 1 \\
& x_1 + 2x_2 - 3x_3 \ge 2 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

已知对偶问题的最优解 $(w_1, w_2) = \left(\frac{5}{3}, \frac{7}{3}\right)$，利用对偶性质求原问题的最优解。

4. 给定线性规划问题：

$$
\begin{aligned}
\min\ & 5x_1 + 21x_3 \\
s.t.\ & x_1 - x_2 + 6x_3 \ge b_1 \\
& x_1 + x_2 + 2x_3 \ge 1 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

其中b_1是某一个正数，已知这个问题的一个最优解为$(x_1, x_2, x_3)=\left(\frac{1}{2}, 0, \frac{1}{4}\right)$。

（1） 写出对偶问题。
（2） 求对偶问题的最优解。

5. 考虑线性规划问题

$$\begin{aligned} \min \quad & cx \\ s.t. \quad & Ax=b \\ & x \geq 0 \end{aligned}$$

其中A是m阶对称矩阵，$c^T=b$。证明若$x^{(0)}$是上述问题的可行解，则它也是最优解。

Homework（3）:

1．Solve the following problems by two-phase method.

$$\begin{aligned} (3)\ \max\ & 3x_1-5x_2 \\ s.t.\quad & -x_1+2x_2+4x_3 \leq 4 \\ & x_1+x_2+2x_3 \leq 5 \\ & -x_1+2x_2+x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

$$\begin{aligned} (4)\ \min\ & x_1-3x_2+x_3 \\ s.t.\quad & 2x_1-x_2+x_3=8 \\ & 2x_1+x_2 \geq 2 \\ & x_1+2x_2 \leq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

2. Solve the following problems by big-M method.

$$\begin{aligned} (5)\quad \max\ & -3x_1+2x_2-x_3 \\ s.t.\quad & 2x_1+x_2-x_3 \leq 5 \\ & 4x_1+3x_2+x_3 \geq 3 \\ & -x_1+x_2+x_3=2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(7) $\min\ 3x_1 - 2x_2 + x_3$

$$\begin{aligned} s.t.\quad & 2x_1 - 3x_2 + x_3 = 1 \\ & 2x_1 + 3x_2 \quad \geq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

3. Consider the following problem:

$$\begin{aligned} \min\ & 4x_1 + 3x_2 + x_3 \\ s.t.\quad & x_1 - x_2 + x_3 \geq 1 \\ & x_1 + 2x_2 - 3x_3 \geq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Suppose the optimal feasible solution of its dual is $(w_1, w_2) = \left(\frac{5}{3}, \frac{7}{3}\right)$. Give the optimal feasible solution of the problem.

4. Given the problem:

$$\begin{aligned} \min\ & 5x_1 + 21x_3 \\ s.t.\quad & x_1 - x_2 + 6x_3 \geq b_1 \\ & x_1 + x_2 + 2x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Where b_1 is a positive integer. Assume $(x_1, x_2, x_3) = \left(\frac{1}{2}, 0, \frac{1}{4}\right)$ is an optimal solution of the problem..

(3) Write the dual for the problem.
(4) Give the optimal solution of the dual.

5. Given the problem:

$$\begin{aligned} \min\quad & cx \\ s.t.\quad & Ax = b \\ & x \geq 0 \end{aligned}$$

Where A is m symmetry matrix, $c^T = b$. Show that if $x^{(0)}$ is a feasible solution of the problem, then it is optimal.