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6.2 6.3 6.6 6.8 6.10

6.2 $\frac{p(x;H_1)}{p(x;H_0)}=\frac{\lambda^2 e^{-\lambda(x[0]+x[1])}}{\lambda_0^2 e^{-\lambda_0(x[0]+x[1])}}>\gamma$

即 $-(\lambda-\lambda_0)(x[0]+x[1])>\ln\gamma\frac{\lambda_0^2}{\lambda^2}$

记 $T=x[0]+x[1]$ 有 $<\frac{-\ln\frac{\lambda_0^2\gamma}{\lambda^2}}{\lambda-\lambda_0}=\gamma'$ $(\lambda>\lambda_0)$

所以判决 H_1 当 $T<\gamma'$

$P_{FA}=P_r\{x[0]+x[1]<\gamma';H_0\}$

$=\int_0^{\gamma'}\int_0^{\gamma'-x[1]}\lambda_0^2 e^{-\lambda_0(x[0]+x[1])}\,dx[0]\,dx[1]$

$=1-e^{-\lambda_0\gamma'}-\gamma'\lambda_0 e^{-\lambda_0\gamma'}$

UMP存在，对给定的P_{FA}可找到对应阈值

6.3 UMP不存在

$\lambda>\lambda_0$ 时 用 $x[0]+x[1]<\gamma'$

$\lambda<\lambda_0$ 时 用 $x[0]+x[1]>\gamma'$

6.6 $p(x|\sigma^2;H_1)=\frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}}e^{-\frac{1}{2\sigma^2}\sum_n(x[n]-A)^2}$

$p(x;H_1)=\int_0^\infty p(\sigma^2)p(x|\sigma^2;H_1)\,d\sigma^2$

$=\int_0^\infty\frac{\lambda e^{-\frac{\lambda}{\sigma^2}}}{\sigma^4}\frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}}e^{-\frac{\sum_n(x[n]-A)^2}{2\sigma^2}}\,d\sigma^2$

$=\frac{\lambda}{(2\pi)^{\frac{N}{2}}}\int_0^\infty\frac{e^{-\frac{1}{\sigma^2}(\lambda+\frac{1}{2}\sum_n(x[n]-A)^2)}}{(\sigma^2)^{\frac{N}{2}+2}}\,d\sigma^2$

$=\frac{\lambda}{(2\pi)^{\frac{N}{2}}}\int_0^\infty\frac{u^{\frac{N}{2}+2}}{u^2}e^{-(\lambda+\frac{1}{2}\sum_n(x[n]-A)^2)u}\,du$

$=\frac{\lambda}{(2\pi)^{\frac{N}{2}}}(\lambda+\frac{1}{2}\sum_n(x[n]-A)^2)^{\frac{N}{2}+1}\Gamma(\frac{N}{2}+1)$

$p(x;H_0)=\frac{\lambda}{(2\pi)^{\frac{N}{2}}}(\lambda+\frac{1}{2}\sum_n x^2[n])^{\frac{N}{2}+1}\Gamma(\frac{N}{2}+1)$

$\frac{p(x;H_1)}{p(x;H_0)}=\left(\frac{\lambda+\frac{1}{2}\sum_n(x[n]-A)^2}{\lambda+\frac{1}{2}\sum_n x^2[n]}\right)^{\frac{N}{2}+1}>\gamma$

decide H_1 当 $\frac{\lambda+\frac{1}{2}\sum_n(x[n]-A)^2}{\lambda+\frac{1}{2}\sum_n x^2[n]}>\gamma'$

当 $\lambda\to0$ 时 decide H_1 当 $\frac{\sum_n(x[n]-A)^2}{\sum_n x^2[n]}>\gamma'$

即 $\frac{\hat\sigma_1^2}{\hat\sigma_0^2}>\gamma'$ 因为 $\lambda\to0$ 时 $p(\sigma^2)\to$ 常数，先验知识消失

6.8 $L_G(x)=\frac{p(x;\hat A,H_1)}{p(x;H_0)}$

$p(x;A,H_1)=\frac{1}{(2\pi\sigma^2)^{N/2}}e^{-\frac{1}{2\sigma^2}\sum_n(x[n]-Ar^n)^2}$

$J(A)=\sum_n(x[n]-Ar^n)^2$

$\frac{\partial J}{\partial A}=-2\sum_n(x[n]-Ar^n)r^n=0\Rightarrow\hat A=\frac{\sum_n x[n]r^n}{\sum_n r^{2n}}$

$L_G(x)=\frac{e^{-\frac{1}{2\sigma^2}\sum_n(x[n]-\hat Ar^n)^2}}{e^{-\frac{1}{2\sigma^2}\sum_n x^2[n]}}$

$\ln L_G(x)=\frac{-1}{2\sigma^2}\left[\sum_n(-2\hat Ar^n x[n]+\hat A^2r^{2n})\right]$

$=\frac{1}{\sigma^2}\sum_n(\hat Ar^n x[n]-\frac{1}{2}\hat A^2r^{2n})$

$=\frac{\hat A}{\sigma^2}\sum_n r^n x[n]-\frac{1}{2}\frac{\hat A^2}{\sigma^2}\sum_n r^{2n}$

$=\frac{\hat A^2}{\sigma^2}\sum_n r^{2n}-\frac{\hat A^2}{2\sigma^2}\sum_n r^{2n}$

$=\frac{\hat A^2}{2\sigma^2}\sum_n r^{2n}$

判 H_1 当 $\hat A^2>\frac{2\sigma^2\ln\gamma}{\sum_n r^{2n}}=\gamma'$

6.10 $L_G(x)=\frac{p(x;\hat\sigma_1^2,H_1)}{p(x;\hat\sigma_0^2;H_0)}=\frac{(\frac{1}{\sqrt{2\hat\sigma_1^2}})^N e^{-\sqrt{\frac{2}{\hat\sigma_1^2}}\sum_n|x[n]|}}{(\frac{1}{2\pi\hat\sigma_0^2})^{\frac{N}{2}}e^{-\frac{1}{2\hat\sigma_0^2}\sum_n x^2[n]}}$

又 $\hat\sigma_1^2=(\frac{\sqrt2}{N}\sum_n|x[n]|)^2$ $\hat\sigma_0^2=\frac{1}{N}\sum_n x^2[n]$

则 $L_G(x)=\frac{(2\pi\hat\sigma_0^2)^{\frac{N}{2}}}{(\cancel{2\pi}\,2\hat\sigma_1^2)^{\frac{N}{2}}}e^{-N}/e^{-\frac{N}{2}}$

$=(\frac{\pi}{e})^{\frac{N}{2}}\left[\frac{\frac{1}{N}\sum x^2[n]}{(\frac{\sqrt2}{N}\sum|x[n]|)^2}\right]^{\frac{N}{2}}$