光研2213 刘子源 2022310109

5.2 $p(x;\sigma^2) = \frac{1}{\sigma^{2N}} \exp\left(-\frac{1}{2\sigma^2}\sum_{n=1}^{N-1} x^2[n]\right) \prod_{n=1}^{N-1} x[n]\, I_{\{x[0],\dots,x[N-1]>0\}}$

$= \frac{1}{\sigma^{2N}} \exp\left(-\frac{1}{2\sigma^2}\sum_{n=1}^{N-1} x^2[n]\right) \prod_{n=1}^{N-1} x[n]\, u(x_{min})$

$= g(T(x);\sigma^2)\, h(x)$

$T(x) = \sum_{n=1}^{N-1} x^2[n]$

5.5 $p(x;\theta) = \frac{1}{(2\theta)^N} \prod_{n=0}^{N-1} [u(x[n]+\theta]) - u(x[n-\theta])$

$= \frac{1}{(2\theta)^N}\, u(\theta - \max|x[n]|)\cdot 1$

$= g(T(x);\theta)\cdot h(x)$

$T(x) = \max|x[n]|$, $n = 0,\dots,N-1$

5.7 $p(x;f_0) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n]-\cos 2\pi f_0 n)^2\right\}$

$= \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma^2}\left(\sum_{n=0}^{N-1} x^2[n] - 2\sum_{n=0}^{N-1} x[n]\cos 2\pi f_0 n + \sum_{n=0}^{N-1}\cos^2 2\pi f_0 n\right)\right\}$.

$\sum_{n=0}^{N-1} x[n]\cos 2\pi f_0 n$. 观测值 x 与待估计参数 f_0 耦合在一起，f_0 无充分统计量无法求得.

5.17 (a). $p(x;A) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n]-A\cos 2\pi f_0 n)^2\right\}$

$= \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma^2}\left(\sum_{n=0}^{N-1} x^2[n] - 2A\sum_{n=0}^{N-1} x[n]\cos 2\pi f_0 n + A^2\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n\right)\right.$.

则 $T(x) = \sum_{n=0}^{N-1} x[n]\cos 2\pi f_0 n$

$E\,T(x) = \sum_{n=0}^{N-1} E\, x[n]\cos 2\pi f_0 n = \sum_{n=0}^{N-1} E(A\cos^2 2\pi f_0 n + \cancel{w[n]\cos 2\pi f_0 n}^{\,0}) = \sum_{n=0}^{N-1} A\cos^2 2\pi f_0 n$

所以 $\hat{A} = \frac{T(x)}{\sum\cos^2 2\pi f_0 n} = \frac{\sum_{n=0}^{N-1} x[n]\cos 2\pi f_0 n}{\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n}$

(b). 由(a)知 $T(x) = \begin{bmatrix} \sum_{n=0}^{N-1} x[n]\cos 2\pi f_0 n \\ \sum_{n=0}^{N-1} x^2[n] \end{bmatrix}$ $\hat{A}$ 不变为 $\frac{\sum_{n=0}^{N-1} x[n]\cos 2\pi f_0 n}{\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n}$

$E\sum_{n=0}^{N-1} x^2[n] = \sum_{n=0}^{N-1}\left(\mathrm{var}\, x[n] + (E\,x[n])^2\right) = \sum_{n=0}^{N-1} A^2\cos^2 2\pi f_0 n + N\sigma^2$

$\hat{A} \sim N\left(A, \frac{\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n}{(\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n)^2}\sigma^2\right) = N\left(A, \frac{\sigma^2}{\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n}\right)$.

$E\hat{A}^2 = A^2 + \frac{\sigma^2}{\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n}$

则 $E\left(\sum_{n=0}^{N-1} x^2[n] - \hat{A}^2\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n\right) = A^2\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n + N\sigma^2 - \left(A^2\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n + \sigma^2\right)$

所以 $\hat{\sigma}^2 = \frac{1}{N-1}\left\{\sum_{n=0}^{N-1} x^2[n] - \hat{A}^2\sum_{n=0}^{N-1}\cos^2 2\pi f_0 n\right\}$ $= (N-1)\sigma^2$

综上，$\hat{\theta} = \begin{bmatrix} \hat{A} \\ \hat{\sigma}^2 \end{bmatrix}$ 为MVU.

5.18 $p(x[n];\theta) = \frac{1}{\theta_2-\theta_1} I\{\theta_1 \le x_n \le \theta_2\}$

$$p(x;\theta) = \frac{1}{(\theta_2-\theta_1)^N} I\{\theta_1 \le x_1, \dots, x_N \le \theta_2\}$$

$$= \frac{1}{(\theta_2-\theta_1)^N} I\{x_{min} \ge \theta_1,\ x_{max} \le \theta_2\}$$

$$= \frac{1}{(\theta_2-\theta_1)^N} u(x_{min}-\theta_1)\, u(\theta_2 - x_{max})$$

即 $T(x) = \begin{bmatrix} x_{min} \\ x_{max} \end{bmatrix}$