PEKING UNIVERSITY

高等应用数学作业 2

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$$33^{1/5} = (32+1)^{1/5} = 2\left(1 + \frac{1}{32}\right)^{1/5} = 2\left(1 + \frac{1}{5} \times \frac{1}{32} - \frac{R}{2}\right) = \frac{161}{80} - R \tag{1.1}$$

$$R = \frac{1}{5} \times \frac{4}{5} \times \frac{1}{32^2} + O\left(\frac{1}{32^3}\right) \approx \frac{1}{25 \times 256} \approx \frac{1}{6 \times 1000} = \frac{10}{6} \times 10^{-4} \approx 1.7 \times 10^{-4} \tag{1.2}$$

2

$$36x^3 + (162 + 4\varepsilon)x^2 - 24\varepsilon x - 9\varepsilon = 0. \tag{2.1}$$

$$36x^2\left(x + \frac{9}{2}\right) = -\varepsilon(4x^2 - 24x + 9) \tag{2.2}$$

1. $x \neq 0$,

$$x + \frac{9}{2} = -\varepsilon\left(\frac{1}{9} - \frac{2}{3x} + \frac{1}{4x^2}\right) \tag{2.3}$$

$x_0 = -\frac{9}{2}$,

$$x_1 = -\varepsilon\left(\frac{1}{9} - \frac{2}{3x_0} + \frac{1}{4x_0^2}\right) - \frac{9}{2} = -\frac{9}{2} - \frac{22}{81}\varepsilon \tag{2.4}$$

2. $x_0 = 0$,

$$x^2 = -\varepsilon\frac{4x_0^2 - 24x_0 + 9}{36\left(x_0 + \frac{9}{2}\right)} \tag{2.5}$$

$$x_{2,3} = \pm i \frac{\sqrt{2\varepsilon}}{6} \tag{2.6}$$

以上为一阶近似的 3 个解。

3

原式可化为

$$\left(x - \frac{1}{2}\right)^2 = \varepsilon \frac{x}{x+4}, \ \varepsilon = \frac{1}{8} \tag{3.1}$$

$x_0 = \frac{1}{2}$

$$\left(x_1 - \frac{1}{2}\right)^2 = \varepsilon \frac{x_0}{x_0 + 4} = \frac{\varepsilon}{9} \tag{3.2}$$

$$x_{1,2} = \frac{1}{2} \pm \frac{\sqrt{2}}{12} \tag{3.3}$$