CFD–HW04(2021-03-31, 第4周)

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(1a). 判断在完全气体状态方程$p = (\gamma - 1)\rho e$【参见文件“CFDLect02-fluid_cn.pdf”的第46页】下的一维守恒形式的Euler方程组【参见文件“CFDLect02-fluid_cn.pdf”的第55页】的三个特征场中哪个是真正非线性的？哪个是非线性退化的？

(1b). 判断在完全气体状态方程$p = (\gamma - 1)\rho e$【参见第46页】下的一维原始变量形式的Euler方程组的三个特征场中哪个是真正非线性的？哪个是非线性退化的？

(1c). 记(1a)中的应变量向量$\boldsymbol{U} = (u_1, u_2, u_3)^T$, 特征值为$\lambda_1(\boldsymbol{U}), \lambda_2(\boldsymbol{U}), \lambda_3(\boldsymbol{U})$, 相应的右特征向量为$\boldsymbol{R}_1(\boldsymbol{U}) = (r_{11}, r_{21}, r_{31})^T$, $\boldsymbol{R}_2(\boldsymbol{U}) = (r_{12}, r_{22}, r_{32})^T$, $\boldsymbol{R}_3(\boldsymbol{U}) = (r_{13}, r_{23}, r_{33})^T$.

对$j = 1, 2, 3$, 分别求解微分方程

$$\frac{du_1}{r_{1j}(\boldsymbol{U})} = \frac{du_2}{r_{2j}(\boldsymbol{U})} = \frac{du_3}{r_{3j}(\boldsymbol{U})}. \tag{1}$$

微分方程(1)的解和下列偏微分方程

$$r_{1j}(\boldsymbol{U})\frac{\partial W}{\partial u_1} + r_{2j}(\boldsymbol{U})\frac{\partial W}{\partial u_2} + r_{3j}(\boldsymbol{U})\frac{\partial W}{\partial u_3} = 0. \tag{2}$$

有何关系？这里$W = W(u_1, u_2, u_3) \in \mathbb{R}$.

(2). 考虑二维Euler方程组

$$\frac{\partial}{\partial t}\boldsymbol{U} + \frac{\partial}{\partial x}\boldsymbol{F}(\boldsymbol{U}) + \frac{\partial}{\partial y}\boldsymbol{G}(\boldsymbol{U}) = 0, \tag{3}$$

其中

$$\boldsymbol{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ E \end{pmatrix}, \quad \boldsymbol{F}(\boldsymbol{U}) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ u(E+p) \end{pmatrix}, \quad \boldsymbol{G}(\boldsymbol{U}) = \begin{pmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ v(E+p) \end{pmatrix}, \tag{4}$$

$p = (\gamma - 1)\rho e$, $E = \rho e + \frac{1}{2}\rho(u^2 + v^2)$.

(2a). 试计算$\boldsymbol{A}(\boldsymbol{U}) := \frac{\partial \boldsymbol{F}(\boldsymbol{U})}{\partial \boldsymbol{U}}$ 及其特征值和右特征向量, 进一步判断哪个特征场是真正非线性的？哪个特征场是非线性退化的？

(2b). 思考: 是否可以由(2a)的结果推出y方向通量$\boldsymbol{G}(\boldsymbol{U})$关于$\boldsymbol{U}$的Jacobi矩阵$\boldsymbol{B}(\boldsymbol{U})$及其特征值和特征向量? 是否可以判断特征场是真正非线性的或非线性退化的？