Chapter 1

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1

试证 $\det(g_{ij}) \neq 0$.

If $\det(g_{ij}) = 0$, then the columns are linearly correlation. Assuming

$$\alpha^j \boldsymbol{e}_i \cdot \boldsymbol{e}_j = \boldsymbol{e}_i \cdot (\alpha^j \boldsymbol{e}_j) = 0,$$

then we have

$$(\alpha^i \boldsymbol{e}_i) \cdot (\alpha^j \boldsymbol{e}_j) = 0,$$

$$\alpha^i \boldsymbol{e}_i = 0,$$

which contradicts the assumption of $\boldsymbol{e}_i$ is linear independent. □

2

求坐标变换下 g_{ij} 和 g^{ij} 的变换规律.

Suppose the coordinate transformation is

$$\widetilde{\boldsymbol{e}}_{i'} = \alpha^j_{i'} \boldsymbol{e}_j.$$

According to the definition of g_{ij},

$$g_{ij} = \boldsymbol{e}_i \cdot \boldsymbol{e}_j, \quad \widetilde{g}_{i'j'} = \widetilde{\boldsymbol{e}}_{i'} \cdot \widetilde{\boldsymbol{e}}_{j'} = \alpha^i_{i'} \boldsymbol{e}_i \cdot \alpha^j_{j'} \boldsymbol{e}_j = \alpha^i_{i'} \alpha^j_{j'} g_{ij}.$$

3

在三维欧式空间中给定基底 $\{\boldsymbol{e}_i\}$, 用通常向量运算方式给出其对偶基底.