16.7.4. $\frac{\partial z}{\partial x} = -\frac{1+y^2}{(x+y)^2}$ $\qquad$ $\frac{\partial z}{\partial y} = -\frac{1+x^2}{(x+y)^2}$.

$\frac{\partial^2 z}{\partial y^2} = \frac{2+2x^2}{(x+y)^3}$. $\qquad$ $\frac{\partial^2 z}{\partial x^2} = \frac{2+2y^2}{(x+y)^3}$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{2-2xy}{(x+y)^3}$$

16.7.5. $\frac{\partial z}{\partial x} = \frac{z^2(yz-x^2)}{xy(z^2-xy)}$.

$$\frac{\partial z}{\partial y} = \frac{xz(y^2-xz)}{y^2(xy-z^2)}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{2z^3(3xyz-x^3-y^3-z^3)}{y(z^2-xy)^3}$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{2xz^3(3xyz-x^3-y^3-z^3)}{y^2 \cdot (xy-z^2)^3}.$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{2x^2z^3(z^3+y^3+z^3-3xyz)}{y^3(xy-z^2)^3}.$$

16.7.11. (1) $\frac{\partial z}{\partial x} = \frac{f_1(x+y, z+y)}{1-f_2(x+y, z+y)}$ $\qquad$ $\frac{\partial z}{\partial y} = \frac{f_1(x+y, z+y)+f_2(x+y, z+y)}{1-f_2(x+y, z+y)}$.

$\frac{\partial^2 z}{\partial x^2} = \frac{(1-f_2)f_{11}+f_1f_{12}+\frac{f_1^2 f_{22}}{1-f_2}+f_1f_{21}}{(1-f_2)^2}$. $\qquad$ $\frac{\partial^2 z}{\partial y^2} = \frac{(1-f_2)f_{11}+(1+f_1)(f_{12}+f_{21})+\frac{(1+f_1)^2 f_{22}}{1-f_2}}{(1-f_2)^2}$.

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{(1-f_2)f_{11}+(f_1+1)f_{21}+\frac{f_1+f_1^2}{1-f_2}f_{22}+f_1 \cdot f_{12}}{(1-f_2)^2}.$$

$T_{16.7.11}$

(2) ① $f_1' + \frac{\partial z}{\partial x} f_2' + (\frac{\partial z}{\partial x}+1) f_3' = 0$

$\frac{\partial z}{\partial x}(f_2' + f_3') + f_1' + f_3' = 0$

$$\frac{\partial z}{\partial x} = -\frac{f_1'+f_3'}{f_2'+f_3'}$$

② $f_1' + (\frac{\partial z}{\partial y}+1) f_2' + \frac{\partial z}{\partial y} f_3' = 0$

$\frac{\partial z}{\partial y}(f_2' + f_3') + f_1' + f_2' = 0$

$$\frac{\partial z}{\partial y} = -\frac{f_1'+f_2'}{f_2'+f_3'}$$

③ $\frac{\partial^2 z}{\partial x^2}(f_2'+f_3') + \frac{\partial z}{\partial x}\left[f_{21}'' + \frac{\partial z}{\partial x} f_{22}'' + (\frac{\partial z}{\partial x}+1) f_{23}''\right] + f_{11}'' + \frac{\partial z}{\partial x} f_{12}'' + (\frac{\partial z}{\partial x}+1) f_{13}'' + f_{31}'' + \frac{\partial z}{\partial x} f_{32}'' + (\frac{\partial z}{\partial x}+1) f_{33}'' = 0$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{1}{f_2'+f_3'}\left[f_{11}'' + \frac{2\partial z}{\partial x} f_{12}'' + (\frac{\partial z}{\partial x}+2) f_{13}'' + (\frac{\partial z}{\partial x})^2 f_{22}'' + \left[(\frac{\partial z}{\partial x})^2 + \frac{2\partial z}{\partial x}\right] f_{23}'' + (\frac{\partial z}{\partial x}+1) f_{33}''\right]$$

$$= -\frac{1}{f_2'+f_3'}\left[f_{11}'' - \frac{2(f_1'+f_3')}{f_2'+f_3'} f_{12}'' + \frac{2f_2'+f_3'-f_1'}{f_2'+f_3'} f_{13}'' + \left(\frac{f_1'+f_3'}{f_2'+f_3'}\right)^2 f_{22}'' - \frac{(2f_2'+f_3'-f_1')(f_1'+f_3')}{(f_2'+f_3')^2} f_{23}'' + \frac{f_2'-f_1'}{f_2'+f_3'} f_{33}''\right]$$

④ $$\frac{\partial^2 z}{\partial y^2} = -\frac{1}{f_2'+f_3'}\left[f_{11}'' - \frac{2(f_1'+f_2')}{f_2'+f_3'} f_{13}'' + \frac{2f_3'+f_2'-f_1'}{f_2'+f_3'} f_{12}'' + \left(\frac{f_1'+f_2'}{f_2'+f_3'}\right)^2 f_{33}'' - \frac{(2f_3'+f_2'-f_1')(f_1'+f_2')}{(f_2'+f_3')^2} f_{23}'' + \frac{f_3'-f_1'}{f_2'+f_3'} f_{22}''\right]$$

⑤ $\frac{\partial^2 z}{\partial x \partial y}(f_2'+f_3') + \frac{\partial z}{\partial x}\left[f_{21}'' + (\frac{\partial z}{\partial y}+1) f_{22}'' + \frac{\partial z}{\partial y} f_{23}'' + f_{31}'' + (\frac{\partial z}{\partial y}+1) f_{32}'' + \frac{\partial z}{\partial y} f_{33}''\right] + f_{11}'' + (\frac{\partial z}{\partial y}+1) f_{12}'' + \frac{\partial z}{\partial y} f_{13}'' + f_{31}'' + (\frac{\partial z}{\partial y}+1) f_{32}'' + \frac{\partial z}{\partial y} f_{33}'' = 0$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{f_2'+f_3'}\left[-\frac{f_1'+f_3'}{f_2'+f_3'}\left(f_{21}'' + \frac{f_3'-f_1'}{f_2'+f_3'} f_{22}'' - \frac{f_1'+f_2'}{f_2'+f_3'} f_{31}'' + \frac{f_3'-f_1'}{f_2'+f_3'} f_{32}'' - \frac{f_1'+f_2'}{f_2'+f_3'} f_{33}''\right) + f_{11}'' + \frac{f_3'-f_1'}{f_2'+f_3'} f_{12}'' - \frac{f_1'+f_2'}{f_2'+f_3'} f_{13}'' + f_{31}'' + \frac{f_3'-f_1'}{f_2'+f_3'} f_{32}'' - \frac{f_1'+f_2'}{f_2'+f_3'} f_{33}''\right]$$

$$= -\frac{1}{f_2'+f_3'}\left[f_{11}'' - \frac{2f_1'}{f_2'+f_3'} f_{12}'' + \frac{(f_1'-f_2')(f_1'+f_2')}{(f_2'+f_3')^2} \cdot f_{13}'' + \frac{(f_1'-f_3')(f_1'+f_3')}{(f_2'+f_3')^2} f_{22}'' + \frac{(f_2'-f_1')(f_3'-f_1')}{(f_2'+f_3')^2} f_{23}'' + \frac{(f_1'+f_2')(f_1'+f_3')}{(f_2'+f_3')^2} f_{33}''\right]$$

17.2.1

(1) $$Df(x,y)=\begin{pmatrix} 2x & -2 \\ 2x-2y & -2x \\ 6xy & 3x^2-2 \end{pmatrix}$$

(2) $$Df(x,y)=\begin{pmatrix} \frac{2x}{x^2+y^2} & \frac{2y}{x^2+y^2} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{pmatrix}$$

(3) $$Df(x,y)=\begin{pmatrix} e^{x+2y} & 2e^{x+2y} \\ 2\cos(y+2x) & \cos(y+2x) \end{pmatrix}$$

17.2.2

(1) $$Df(x,y,z)=\begin{pmatrix} 1 & 4y & 9z^2 \\ -2x & 2 & 0 \end{pmatrix}$$

(2) $$Df(x,y,z)=\begin{pmatrix} y^2z^2 & 2xyz^2 & 2xy^2z \\ 0 & z^2\cos y & 2\sin y\, z \\ 2xe^y & x^2e^y & 0 \end{pmatrix}$$

17.2.11 证：设 $f(t)=(f_1(t), f_2(t), f_3(t))$

$g(t)=(g_1(t), g_2(t), g_3(t))$

$f(t)\times g(t)=(f_2(t)g_3(t)-g_2(t)f_3(t),\ f_3(t)g_1(t)-f_1(t)g_3(t),\ f_1(t)g_2(t)-f_2(t)g_1(t))$

$\frac{d(f(t)\times g(t))}{dt}=(f_2'(t)g_3(t)+f_2(t)g_3'(t)-g_2'(t)f_3(t)-g_2(t)f_3'(t),$

$f_3'(t)g_1(t)+f_3(t)g_1'(t)-f_1'(t)g_3(t)-f_1(t)g_3'(t),$

$f_1'(t)g_2(t)+f_1(t)g_2'(t)-f_2'(t)g_1(t)-f_2(t)g_1'(t))$ ⋯①

$\frac{d(f(t))}{dt}=(f_1'(t), f_2'(t), f_3'(t))$ $\frac{dg(t)}{dt}=(g_1'(t), g_2'(t), g_3'(t))$

$\frac{d(f(t))}{dt}\times g(t)=(f_2'(t)g_3(t)-f_3'(t)g_2(t),\ f_3'(t)g_1(t)-f_1'(t)g_3(t),\ f_1'(t)g_2(t)-f_2'(t)g_1(t))$ ⋯③

$f(t)\times\frac{dg(t)}{dt}=(f_2(t)g_3'(t)-f_3(t)g_2'(t),\ f_3(t)g_1'(t)-f_1(t)g_3'(t),\ f_1(t)g_2'(t)-f_2(t)g_1'(t))$ ⋯②

由 ②+③=①.

$\therefore \frac{d[f(t)\times g(t)]}{dt}=\frac{df(t)}{dt}\times g(t)+f(t)\times\frac{dg(t)}{dt}$ 证毕.