MR6297 (3,285c) 76.1X
Chou, P. Y.
On an extension of Reynolds' method of finding apparent stress and the nature of turbulence.
Chinese J. Phys. **4** (1940), 1–33.

The paper starts with a review of the conventional approach to turbulence in tensor notation; time-periodicity of the velocity fluctuation w_i and the pressure fluctuation p' is assumed. It is possible to operate on the Navier-Stokes equations in the following alternative ways: (i) take the time-average, (ii) multiply by w_j and then average, (iii) multiply by $w_j w_k$ and then average, and so on. (i) gives the Reynolds equations, (ii) gives equations involving correlations up to the third order and (iii) gives equations involving correlations up to the fourth order. Each of these plans yields an indeterminate problem, since there are more correlations than equations. The author follows plan (iii), and gets rid of the indeterminacy by the following auxiliary hypotheses:

$$\text{(a)} \qquad \overline{w_i w_j w_k w_l} = \tfrac{1}{2}(\overline{w_i w_j}\cdot\overline{w_k w_l} + \overline{w_i w_k}\cdot\overline{w_j w_l} + \overline{w_i w_l}\cdot\overline{w_j w_k}),$$

$$\text{(b)} \qquad \overline{p_{,i}{}' w_j w_k} = -c_{ijk} + k_{(ijk)} p_{,i}\cdot\overline{w_j w_k} \text{ (not summed)}.$$

(a) is satisfied by a simple harmonic fluctuation; (b) is a quite arbitrary assumption, the constants being left for determination by experiment.

The theory is applied to the case of pressure flow between parallel planes. With some simplifying hypotheses and the assumption of a very large Reynolds number, the equation

$$y\frac{dU}{dy} + \gamma^2 \frac{d^2}{dy^2}\left(y\bigg/\frac{dU}{dy}\right) = 0, \quad \gamma = \text{constant},$$

is obtained for the mean velocity U, y being measured across the channel $(-d \le y \le d)$. Introduction of an auxiliary variable ϕ gives

$$dU/d\sigma = -\gamma\sigma/\phi, \quad d^2\phi/d\sigma^2 = -\sigma^2/\phi, \quad \sigma = y/d.$$

The second of these equations is integrated numerically with the boundary conditions $d\phi/d\sigma = 0$ for $\sigma = 0$, $\phi = 0$ for $\sigma = 1$. Then U is found from the first equation to within a constant of integration. For comparison with experimental results, this constant is given a suitable value.