

## MATH FOR BUSINESS: CALCULUS, SPRING 2017 - MIDTERM II

Name: \_\_\_\_\_

You will receive no credit for submitting solutions that I cannot read and understand—be sure to write legibly! LEGIBLY!

**Instructions:** Answer all of the questions below, and one bonus question.

**Problem 1.** Find  $y'$  and  $y''$  of the following functions:

- (1)  $y = e^{-0.5x}$
- (2)  $y = xe^{-x}$

**Problem 2.** Find the derivatives of each of the following functions:

- (1)  $f(x) = \sqrt{9 - x^2}$
- (2)  $y = \sqrt{e^x}$

**Problem 3.** Find  $\frac{dy}{dx}$  by implicit differentiation.

- (1)  $x^2 + y^2 = 1$
- (2)  $e^{x^2y} = x + y$

**Problem 4.** Differentiate the given functions.

- (1)  $f(x) = 3x - 2 \ln(x)$
- (2)  $f(x) = \ln(\sqrt[5]{x})$

**Problem 5.** Each side of a square is increasing at a rate of  $6 \frac{\text{cm}}{\text{s}}$ . At what rate is the area of the square increasing when the area is  $16\text{cm}^2$ ?

**Problem 6.** Assume that  $x$  and  $y$  are functions of  $t$ . If  $y = x^3 + 2x$  and  $\frac{dx}{dt} = 5$ , find  $\frac{dy}{dt}$  when  $x = 2$ .

**Problem 7. (Bonus)**

Differentiate the following function:

$$g(x) = e^{\frac{1}{f(x) + \frac{1}{f(x)}}}$$

where  $f(x) = \frac{x^2}{e^x}$ .

**Problem 8. (Bonus.)**

Show by implicit differentiation that the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point  $(x_0, y_0)$  is

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

Note that in both the equations above  $a$  and  $b$  here are two different constants.