

FINITE MATH, FALL 2016 - FINAL EXAM

Name: _____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

Answer all of the questions from 1-12. Answer only one of the bonus questions 13-14.

Problem 1. Find the simple interest on the following:

- (1) \$25000 at 3% for 9 months.
- (2) \$4289 at 4.5% for 35 weeks.

Problem 2. A typical pack-a-day smoker spends about \$136.50 a month on cigarettes. Suppose a smoker decides to invest that money instead. Each month the smoker deposits \$136.50 into a savings account at 4.8% interest compounded monthly. What would the account be worth after 40 years?

Problem 3. If $A = \{\{a\}, a\}$, answer the following:

- (1) is $a \in A$?
- (2) is $a \subset A$?

Problem 4. Assume a universal set U , \cup the union operator, and \cap the intersection operator, draw Venn diagrams for the following:

- (1) $A' \cup A$
- (2) $(U \cap A) \cup (U \cap B)$
- (3) If $A \subset C$, $B \cap C \neq \emptyset$, and $A \cap B = \emptyset$, draw $A' \cap (B' \cap C)$
- (4) If $A \subset B \subset C$, draw $(A' \cap B) \cup C'$

Problem 5. Let A and B be disjoint sets ($A \cap B = \emptyset$). Given that $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{5}$, find:

- (1) $P(A \cap B)$.
- (2) $P(A \cup B)$.
- (3) is it possible to have $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{2}$? Explain your answer.

Problem 6. We have some books we want to arrange on a shelf: 4 blue, 3 green, and 2 red. Find the following:

- (1) In how many ways can the books be arranged on the shelf?
- (2) If the books of the same color are to be grouped together, how many arrangements are there?
- (3) In how many distinguishable ways can the books be arranged if books of the same color are identical but need not be grouped together?

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Problem 7. The probability that a certain machine turns out a defective item is 0.05. Find the probability that after making 75 items, the following results are obtained:

- (1) Exactly 5 defective items.
- (2) No defective items.
- (3) At least 1 defective item.

Problem 8. Find the range and standard deviation for each of the following set of numbers:

- (1) 42,38,29,74,82,71,35.
- (2) 51,58,62,64,67,71,74,78,82,93.
- (3) 15,42,53,7,9,12,28,41,63,14.

Problem 9. Use row reduction to solve the following system of equations:

$$\begin{aligned} 2x + 3y &= 9 \\ 4x + 6y &= 7 \end{aligned}$$

Problem 10. Find the equilibrium point of the following transition matrix. Use the Analytic method, not the numerical one.

$$\begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Problem 11. Find the fundamental matrix F for the following absorbing Markov chains. Also, find the product matrix FR .

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.15 & 0.35 & 0.5 \end{bmatrix}$$

Problem 12. Find the matrix product, if possible. If not explain why.

$$\begin{bmatrix} 2 & -1 \\ 5 & 8 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Problem 13. BONUS!

Prove De Morgan's second form:

$$(A \cup B)' = A' \cap B'.$$

Problem 14. BONUS!

Prove that if A and B are independent events, then A and B' are independent as well.