Numerical Analysis Math 4006

Homework 2, Due Thursday, September 12

1. Decide which functions $f(x)$ are Lipschitz. Give a proof if you believe the function is Lipschitz; otherwise give explicit values x, y showing that the slopes of the secant lines are unbounded.
   a. $f(x) = x^2$ on $[-3, 3]$
   b. $f(x) = \sqrt{x}$ on $[\frac{1}{2}, 1]$
   c. $f(x) = \sqrt{x}$ on $[0, 1]$
   d. $f(x) = atan(x)$ on $\mathbb{R}$

2. If $g(x)$ is continuous and $\varphi(x) \geq 0$, then $\int_a^b g(x)\varphi(x)dx = g(\zeta)\int_a^b \varphi(x)dx$ for some $\zeta \in [a, b]$.
   a. Prove that $f(y) = f(x) + f'(\zeta)(y - x)$ for some ζ between x and y.
   b. Prove that $f(y) = f(x) + f'(x)(y - x) + \frac{1}{2}f''(\xi)(y - x)^2$ for some ξ between x and y. Hint: integrate by parts in the integral you used in Part a.

3. Derive Heron's algorithm by applying Newton's method to $f(x) = x^2 - y$.

4. Implement Newton's method and the Secant Method for the equation $f(x) = x + \varepsilon \sin x - \tau$ for the values $\varepsilon = 0.5$ and $\varepsilon = 0.9$ and $\tau = 2$.
   a. Make a table of the absolute error $|e_n|$ as a function of the iteration number.
   b. Explain how the absolute error shows that Newton's method is a second order method while the Secant Method is a fractional order method.