

NUMERICAL ANALYSIS HOMEWORK 7

due Monday, December 2nd, 2013

1. Compute the solution of $Ax = b$ where $b = (1, 0, 0)^T$ and

$$A = \begin{pmatrix} 4 & -1 & 0 \\ -2 & 4 & -2 \\ 1 & -2 & 3 \end{pmatrix}$$

using the Jacobi method and Gauss-Seidel method with initial guess $x_0 = (1, 0, 0)^T$.

2. Show that the Jacobi iteration fails to converge for the matrix

$$A = \begin{pmatrix} 6 & -5 & 1 \\ -5 & 6 & -5 \\ 1 & -5 & 6 \end{pmatrix}.$$

3. Let A be the matrix appearing in equation (4.20) in the textbook.

1. Show that A is positive definite.
2. Determine whether the Gauss-Seidel iteration converges for A .

4. Prove that if A is diagonally dominant, then

1. $N = \text{diag}(A)$ is nonsingular.
2. A is nonsingular. Hint : Let $A = N - P$, recalling that $\|N^{-1}P\|_\infty < 1$. Suppose $Ax = 0$ and conclude that x must be zero.

5. Prove the formula

$$\|A\|_\infty = \max_{i=1,\dots,n} \sum_{j=1}^n |A_{ij}|.$$

Hint : Let $i' \in \{1, \dots, n\}$ so that $N = \sum_{j=1}^n |A_{i'j}| = \max_{i=1,\dots,n} \sum_{j=1}^n |A_{ij}|$. Now pick $x \in \mathbb{R}^n$ to be a sequence of ± 1 's carefully arranged so you can relate N to the i' th component of Ax .