Homework 10: Due Friday April 21st in recitation.

Homework submitted 10 minutes after recitation has begun is considered late and will not be accepted. **Write on only one side of each page. Staple all work.**

1. If $4000 is borrowed at a rate of 5.75% interest per year, compounded quarterly, find the amount due at the end of the given number of years (note: a loan is an investment that the bank is making in the customer):

  (a) 4 years.

  (b) 6 years.

  (c) 8 years.

  (d) What if the interest is compounded continuously? What is the amount due in 4, 6, and 8 years in this case?

2. **Difference quotient.** If $f(x) = 10^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 10^x \left( \frac{10^h - 1}{h} \right).$$

Note that as a function of $x$, the difference quotient of $10^x$ is $10^x$ multiplied by a constant.

3. In a pyramid scheme, I have to pay a fee to join, and then I receive part of the fees of each member I recruit, and also of the fees of members they recruit, and so on. A particular pyramid scheme requires each member to recruit five new members before starting to receive money. Call the initiator of the pyramid (the person at the "top") "the first generation"; the members that he recruits "the second generation"; those recruited by second generation members "the third generation", etc.

    Assume that each person manages to meet the goal and to recruit 5 new members.

  (a) Show that the number of members in the $n^{\text{th}}$ generation is an exponential function of $n$ and find this function.

  (b) How many members are there in generation number 5? Generation 10? Generation 14? Generation 15?

4. Graph $f(x) = 2 + \log_3 \left( \frac{1}{x-1} \right)$. Draw any asmptotes with dotted lines. Label all intercepts.

5.

  (a) Suppose you deposit $P$ dollars into a bank that pays an interest rate $r$ compounded continuously. How long does it take to double your original deposit $P$.

  (b) Suppose you deposit $P$ dollars into a bank that compounds interest continuously. What is the interest rate $r$ that doubles your original investment $P$ after the first year.