Numerical Analysis Homework 5

due Monday, October 28th, 2013

1. Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}.$$

Calculate the operator norms for the following

a) $A : (\mathbf{R}^2, \|\cdot\|_2) \to (\mathbf{R}^2, \|\cdot\|_2)$

b) $B : (\mathbf{R}^2, \|\cdot\|_1) \to (\mathbf{R}^2, \|\cdot\|_\infty)$

c) $C : (\mathbf{R}^2, \|\cdot\|_2) \to (\mathbf{R}^2, \|\cdot\|_2)$

d) $C : (\mathbf{R}^2, \|\cdot\|_\infty) \to (\mathbf{R}^2, \|\cdot\|_1)$

e) $D : (\mathbf{R}^2, \|\cdot\|_2) \to (\mathbf{R}^3, \|\cdot\|_2)$

2. Let N_1 and N_2 be norms on $\mathbf{R}^n$. Prove that

$$\|A\| = \max_{x \neq 0} \frac{N_2(Ax)}{N_1(x)}$$

defines a norm on the vector space of n by n matrices A.

3. Do Exercise 6.2.

4. Do Exercise 6.8.

5. Prove that $\|R\| = 1$ for any orthogonal matrix R mapping $(\mathbf{R}^n, \|\cdot\|_2)$ into itself. (R is orthogonal if $R^T R = I$.)