DISCRETE STRUCTURES, SPRING 2017 - PROBLEM SET

Name: _____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

Problem 1. Write the following in summation notation:

   (a) the sum of the first n even numbers.

   (b) the sum of the first n odd numbers.

Hint: To answer this question we first need to know how to write an even or odd number in general. All even numbers can be written as $2k$. And odd numbers can be written in a similar but not identical fashion.

Problem 2. A group of 30 people have been trained as astronauts to go to Mars. How many ways are there to select a crew of six to go on the mission?

Problem 3. According to the Binomial Theorem, what is the coefficient of $x^{12}y^{13}$ in the expansion of $(x + y)^{25}$?

Problem 4. Prove the following identity:
$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

Problem 5. Evaluate:
$$\sum_{k=1}^{n} k \binom{n}{k}$$

Problem 6. Prove the following identity:
$$(n+1)\binom{r}{n+1} = r\binom{r-1}{n}$$

Problem 7. Evaluate the following expression:
$$\sum_{k=1}^{n}(-1)^k \binom{n}{k}$$

Problem 8. (Bonus):

Given that we know what and how the factorial function works, derive an expression for the product of the first n odd numbers.

Date: April 7th, 2017.