DISCRETE STRUCTURES, SPRING 2017 - PROBLEM SET

Name: ____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

Problem 1. Which of the following relations over $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

(1) {(0,0),(1,1),(2,2),(3,3)}
(2) {(0,0),(0,2),(2,0),(2,2),(2,3),(3,2),(3,3)}
(3) {(0,0),(1,1),(1,2),(2,1),(2,2),(3,3)}
(4) {(0,0),(1,1),(1,3),(2,2),(2,3),(3,1),(3,2),(3,3) }
(5) {(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,2),(3,3)}

Problem 2. Let R be a relation on the set of ordered pars of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

Problem 3. Let R be a relation on the set of ordered pars of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$. Show that R is an equivalence relation.

Problem 4. There are only two possible partitions of the set $\{1, 2\}$. They are $\{\{1\}, \{2\}\}$ and $\{\{1, 2\}\}$. Find all the partitions of the following sets:

(1) {1,2,3}
(2) {1,2,3,4}

Problem 5. Let us say that two integers are *near* each other provided that their difference is 2 or smaller (i.e. their numbers are at most 2 apart). For example, 3 is near 5, 10 is near 9, but 8 is not near 4. Let R stand for this *near* relation. Do the following:

(1) Write down R as the set of ordered pairs. Your answer should look like:
$$R = \{(x, y)| \dots\}$$
Meaning fill in the dots.
(2) Prove or disprove: R is reflexive.
(3) Prove or disprove: R is symmetric.
(4) Prove or disprove: R is transitive.

Problem 6. For each of the following relations, find R^{-1}.

(1) $R = \{(1, 2), (2, 3), (3, 4)\}$
(2) $R = \{(1, 1), (2, 2), (3, 3)\}$
(3) $R = \{(x, y)|x, y \in \mathbb{Z}, x - y = 1\}$
(4) $R = \{(x, y)|x, y \in \mathbb{Z}, xy > 0\}$

Date: February 14th, 2017.