MATH FOR BUSINESS: CALCULUS, SPRING 2017 - PROBLEM SET 6

Name: _____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

Problem 1. Find f' in two ways, by using the chain rule and then by first expanding the expression. Verify that your answers are the same.

(1) $f(x) = (3x^5 + 1)^2$
(2) $f(x) = (x^2 - 5x)^2$

Problem 2. If $y = f(x)$ find the derivative with respect to x of the expression.

(1) $3x + f(x)$
(2) $[f(x)]^3$
(3) $e^{f(x)}$

Problem 3. If $y = f(x)$, evaluate the given derivative.

(1) $\frac{d}{dx}(x + y^4)$
(2) $\frac{d}{dx}(\sqrt{x} + \sqrt{y})$
(3) $\frac{d}{dx}(e^x - e^y)$

Problem 4.

(1) Find $\frac{dy}{dx}$ by implicit differentiation.
(2) Solve the equation explicitly for y and differentiate to get $\frac{dy}{dx}$ in terms of x.
(3) Check that your solutions to the first two parts are consistent by substituting the expression for y into part (2) into your solution of part (1).
   (a) $xy + 2x + 3x^2 = 4$
   (b) $\sqrt{x} + \sqrt{y} = 4$

Problem 5. Find $\frac{dy}{dx}$ by implicit differentiation.

(1) $x^2y + xy^2 = 3x$
(2) $x^2 + y^2 = 1$
(3) $e^{x^2y} = x + y$
(4) $\sqrt{x + y} = 1 + x^2y^2$

Problem 6. Differentiate the given functions.

(1) $f(x) = 3x - 2\ln(x)$
(2) $y = 1.5x + \ln(x)$
(3) $y = (\ln(x))^5$
(4) $f(x) = \ln(\sqrt[5]{x})$

Problem 7. Find f' and f'' of the given functions.

(1) $f(x) = \ln(x^2 - 5)$
(2) $f(x) = e^x \ln(x)$

Problem 8. Find $\frac{dy}{dx}$ if $y = \ln(x^2 + y^2)$

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Problem 9. Bonus.
Show by implicit differentiation that the tangent to the ellipse

$$\frac{x^2}{a^b} + \frac{y^2}{b^2} = 1$$

at the point (x_0, y_0) is

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

Note that in both the equations above a and b here are two different constants.