DISCRETE STRUCTURES, SPRING 2017 - PROBLEM SET 9

Name: ____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

Problem 1. A judge is 65% sure that a suspect has committed a crime. During the course of a trail, a witness convinces the judge that there is a 85% chance that the criminal is left handed. If 23% of the population is left handed, and the suspect is also left handed, with this new information, how certain should the judge be of the guilt of the suspect.

Problem 2. A certain cancer is found in one in 5000 people. If the person does have the disease, in 92% of the cases the diagnostic procedure will show that he or she actually has it. If a person does not have the disease, the diagnostic procedure will give a false positive result in one out of 500 cases. Determine the probability that a person with positive results actually has cancer.

Problem 3. Show that if an event A is independent of itself, then $P(A) = 1$ or $P(A) = 0$.

Problem 4. Show that if A and B are independent events and $A \subseteq B$, then either $P(A) = 0$ or $P(B) = 1$.

Problem 5. If the events A and B are independent and the events B and C are independent as well, is it true that the events A and C are independent as well? Explain your answer.

Problem 6. Three missiles are fired at a target and hit independently with probability 0.7, 0.8, and 0.9 respectively. What is the probability that the target is hit?

Problem 7. A box contains 20 fuses, of which 5 are defective. What is the expected number of defective items among three randomly chosen fuses?

Problem 8. Bonus.
If p is the probability of success in a single binomial experiment, the probability of x successes and $n - x$ failures in n independent repeated trails of experiment, knows as the **binomial probability**, is

$$P(\text{x successes in n trails}) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

Show that the expectation of X is:

$$\mathbb{E}(X) = np.$$

Date: April 28th, 2017.