DISCRETE STRUCTURES, SPRING 2017 - PROBLEM SET 2

Name: ____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

Problem 1. In class we showed how to prove one version of De Morgan's Law. Replicate the proof, and prove the other version as well. In other words, if A and B are non-empty sets, prove the following equalities to be true:

(1) $(A \bigcup B)' = A' \bigcap B'$
(2) $(A \bigcap B)' = A' \bigcup B'$

Problem 2. In class we proved that given non-empty sets $A, B,$ and C the following equality is true:

$$(1) \quad A \bigcap (B \cup C) = (A \cap B) \bigcup (A \cap C)$$

Reproduce the proof. Furthermore, prove the following equality as well:

$$(2) \quad A \bigcup (B \cap C) = (A \cup B) \bigcap (A \cup C)$$

Problem 3. If $\{B_i\}_{i=0}^{n}$ is a collection of non-empty sets (meaning that we have $B_1, B_2, \ldots, B_n$ sets) prove the following equalities to be true:

(1)
$$A \cap \left(\bigcup_{i=1}^{n} B_i \right) = \bigcup_{i=1}^{n} (A \cap B_i)$$

(2)
$$A \cup \left(\bigcap_{i=1}^{n} B_i \right) = \bigcap_{i=1}^{n} (A \cup B_i)$$

Problem 4. Recall that the Cartesian product of two non-empty sets A and B is defined as

$$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}.$$

Armed with this definition, prove that if $A' \subseteq A$ and $B' \subseteq B$, then $A' \times B' \subseteq A \times B$.

Problem 5. Draw the following binary relations:

(1) $R_1 = \{(a, b) \in \mathbb{R} \times \mathbb{R} | a < b\}$
(2) $R_2 = \{(a, b) \in \mathbb{R} \times \mathbb{R} | a > b\}$
(3) $R_1 = \{(a, b) \in \mathbb{R} \times \mathbb{R} | a = b\}$
(4) $R_1 = \{(a, b) \in \mathbb{R} \times \mathbb{R} | a < 0\}$

Date: February 3rd, 2017.