DISCRETE STRUCTURES, SPRING 2017 - ADDENDUM

In class on Tuesday a question was asked about problem 2 on the homework that has to do with the distributive property of intersections over unions. In class I broke the proof into cases. I also stated that there is a way to do it without. Since we are just all learning about proofs and sets, I think it is a good idea to try and be as precise as possible when trying to prove these set relations. While the proof I gave in class was decent, I think a better proof, that fully explains the details is given below. This should serve as a guide to how to solve the other part of problem 2 as well.

Problem:
Prove the following equality to be true:

$$A \bigcap (B \cup C) = (A \cap B) \bigcup (A \cap C)$$

Proof:
We need to show show two things:

(1) $A \bigcap (B \cup C) \subseteq (A \cap B) \bigcup (A \cap C)$
(2) $(A \cap B) \bigcup (A \cap C) \subseteq A \bigcap (B \cup C)$

<u>We start by proving (1)</u>:
Let $x \in \bigcap (B \cup C)$, then $x \in A$ and $x \in B \cup C$.
From the latter statement, we must now consider two cases: either $x \in B$ or $x \in C$.

Case 1– $x \in B$:
This means that $x \in A$ and $x \in B$. Which means that $x \in A \cap B$.
However, we also have that $A \cap B \subseteq (A \cap B) \bigcup (A \cap C)$.
Hence it must be the case that $x \in (A \cap B) \bigcup (A \cap C)$. Concluding this particular case.

Case 2– $x \in C$:
This means that $x \in A$ and $x \in C$. Which means that $x \in A \cap C$.
However, we also have that $A \cap C \subseteq (A \cap B) \bigcup (A \cap C)$.
Hence it must be the case that $x \in (A \cap B) \bigcup (A \cap C)$.

In both of the cases above we have arrived the fact that $x \in (A \cap B) \bigcup (A \cap C)$. As a result we have shown (1).

<u>We now go about proving (2)</u>:
Let $x \in (A \cap B) \bigcup (A \cap C)$. Then this means that $x \in A \cap B$ or $x \in A \cap C$.
Once again we now consider two cases.

Case 1– $x \in A \cap B$:
So, $x \in A \cap B$, meaning that $x \in A$ and $x \in B$.
But since $B \subseteq B \cup C$, it is also the case that $x \in B \cup C$.
so, $x \in A$ and $x \in B \cup C$. Meaning that $x \in A \bigcap (B \cup C)$. Concluding this particular case.

Case 2– $x \in A \cap C$:
Since $x \in A \cap C$, this means that $x \in A$ and $x \in C$.
But since $C \subseteq B \cup C$, it is also the case that $x \in B \cup C$.
So, $x \in A$ and $x \in A \cup B$. meaning that $x \in A \bigcap (B \cup C)$.

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In both cases above we have arrived at the fact that $x \in A \bigcup (B \cap C)$. This means that we have shown (2). Since we have shown (1) and (2), this means that we have proved equality, as was desired.