NUMERICAL ANALYSIS HOMEWORK 6

due Thursday, November 14th, 2013

1. Suppose that A is an invertible n by n matrix and that v and w are vectors in $\mathbb{R}^n$. Prove that

$$(A + vw^T)^{-1} = A^{-1} - \frac{A^{-1}vw^TA^{-1}}{1 + w^TA^{-1}v}.$$

2. Suppose that M is convergent. Prove that[1]

$$(I - M)^{-1} = I + M + M^2 + M^3 + \cdots .$$

3. Prove that for any square matrix A,

$$\lim_{n\to 0} \frac{A^n}{n!} = 0.$$

4. Calculate the Schur decomposition for

$$A = \begin{pmatrix} -3/2 & 5 \\ 1/2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}.$$

5. Given that one of the eigenvalues $\lambda = 1$, find a nearly diagonal transformation for

$$A = \begin{pmatrix} 3/2 & 0 & -1/2 \\ -3/\sqrt{2} & 2 & 3/\sqrt{2} \\ -3/2 & 0 & 5/2 \end{pmatrix}.$$

6. Determine which of the following matrices are convergent. Verify your conclusion numerically (by taking large powers of the matrix).

$$A = \begin{pmatrix} 0.64 & 0.92 \\ 0.06 & 0.49 \end{pmatrix}, \quad B = \begin{pmatrix} 0.19 & 0.40 \\ 0.58 & 0.98 \end{pmatrix}$$

7. Show that if A is symmetric, then there is a unitary matrix U for which U^*AU is diagonal.

8. Using Newton's method, numerically calculate the two zeros of the function

$$f(x) = \begin{pmatrix} x^2 + y^2 - 4 \\ x^3 + y^3 - 1 \end{pmatrix}.$$

9. Let A be an n by n matrix and $f(x) = x^TAx$ for $x \in \mathbb{R}^n$. Compute $\nabla f(x)$ and $H_f(x)$.

[1] take for granted that the infinite series converges