

DISCRETE STRUCTURES, SPRING 2017 - PROBLEM SET

Name: _____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

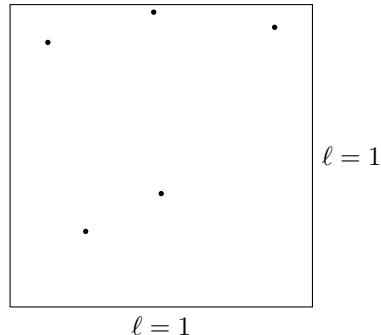
Problem 1. Let $|A| = n$ and $f : A \rightarrow B$ be a function.

- (1) Suppose f is neither surjective nor injective. What can you say about $|B|$?
- (2) Suppose f is injective and surjective. What can you say about $|B|$?

Problem 2. Suppose $f : A \rightarrow B$ and $g : B \rightarrow C$ are both bijective functions.

- (1) Prove that $g \circ f$ is also a bijection.
- (2) Suppose now that $|B| = n$. What can you infer about $|A|$ and $|C|$?

Problem 3. Suppose that we have a square box of length 1 with 5 points inside (which can be moved) it as shown below:



Show that it is possible to find two points that are no longer than $\frac{\sqrt{2}}{2}$ units away from each other.

Hint: Figure out a way of placing the 5 points so that distance between them is maximized. In addition you will want to use the pigeonhole principle.

Problem 4. We are going to assume that there are 20 million Texans in Texas. Let $A = \{x | x \text{ is a Texan}\}$. It is also a biological fact that there are fewer than a million hair on the human head.

Let $B = \{x | x \leq 1000000 \text{ & } x \in \mathbb{N}\}$. Prove that there are at least two Texans with exactly the same number of hairs on their head.

Hint: Define a function from A to B . Use the pigeonhole principle to argue what you want.

Problem 5. Remember from class that the indicator function

$$\mathbb{1}_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$

- (1) Prove that $\mathbb{1}_A(x)$ is indeed a function.
- (2) Prove or disprove: $\mathbb{1}_A(x)$ is injective.
- (3) Prove or disprove: $\mathbb{1}_A(x)$ is surjective.
- (4) What is the domain and range of $\mathbb{1}_{\mathbb{Q}}(x)$?

- (5) What is the domain and range of $\mathbb{1}_{\mathbb{R}}(x)$?
- (6) What is the size of the set that $\mathbb{1}_{\mathbb{Q}}(x)$ maps to?
- (7) What is the size of the set that $\mathbb{1}_{\mathbb{R}}(x)$ maps to?

Problem 6. Bonus

Fifteen children together gathered 100 nuts. Prove that some pair of children gathered the same number of nuts.