NUMERICAL ANALYSIS MIDTERM I

Thursday, October 10th, 2013

You are permitted to use only the operations $+$, $-$, $\div$, $\times$, sin, cos, and exp on a calculator.

1. Find the first 9 digits of π and of $\sqrt{24}$.

2. Calculate the LU factorization for

$$A = \begin{pmatrix} 9 & 1 & 1 \\ 0 & 1 & 0 \\ 4 & 1 & 1 \end{pmatrix}.$$

3. Perform a Gram-Schmidt orthonormalization for the vectors

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 9 \\ 0 \\ 4 \end{pmatrix}.$$

4. Suppose that $x \in \mathbf{R}^2$ satisfies $\|x\|_1 = \|x\|_2 = 1$. Show that x is either the vector $(1, 0)$ or the vector $(0, 1)$.

5. Let $x, y, z \in \mathbf{R}^n$. Prove that

$$\sum_{i=1}^{n} |x_i y_i z_i| \leq \|x\|_6 \|y\|_3 \|z\|_2.$$

6. Show that the graph of a function f with $f'' > 0$ always lies above its tangent line approximation[1].

[1] the tangent line approximation of f at x_0 is $L^f_{x_0}(x) = f(x_0) + f'(x_0)(x - x_0)$.