Numerical Analysis Homework 3

due Monday, September 23rd, 2013

In the following, A is an n by n matrix and v and w are vectors in $\mathbf{R}^n$. In `Octave`, implement the following functions.

1. `x = my_dot(v,w,n)` that calculates $v \cdot w$.

2. `w = my_matvec(A,v,n)` that calculates Av.

3. `[L U] = my_LU(A,n)` that calculates the factorization $A = LU$.

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4. Find the LU factorization and eigenvalues of U for

$$A = \begin{pmatrix} 2 & 1 & 2 & 0 \\ 1 & -1 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 4 & 3 & 1 & 1 \end{pmatrix}$$

5. Consider the n by n matrices

$$A = \begin{pmatrix} 1 & -1 & 0 & ... & 0 \\ -1 & 2 & -1 & ... & 0 \\ & & \vdots & & \\ 0 & ... & -1 & 2 & -1 \\ 0 & ... & 0 & -1 & 2 \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 & 0 & ... & 0 \\ -1 & 1 & 0 & ... & 0 \\ & & \vdots & & \\ 0 & ... & -1 & 1 & 0 \\ 0 & ... & 0 & -1 & 1 \end{pmatrix}$$

and M defined by

$$M_{ij} = n + 1 - \max\{i, j\}.$$

Show that $A = LL^T$ and that $A^{-1} = M$.

6. Take A, L, and M from the previous problem. The linear system $Ax = f$ can be solved by

- Multiplying Mf and setting x equal to the result, or
- Doing a forward substitution on $Ly = f$ followed by backward substitution on $L^T x = y$.

Illustrate these two methods for the case $n = 4$. Count the number of operations used by each of these methods for a general n. (Note: the multiplications by zero do *not* add to the operation count.) Which method is preferable, if $n = 10^5$ for example?