Numerical Analysis Homework 4

due Thursday, October 3rd, 2013

1. Calculate the Cholesky factorization for

$$A = \begin{pmatrix} 4 & -1 & 0 & 0 \\ -1 & 3 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}.$$

2. Show how Cholesky factorization fails for

$$B = \begin{pmatrix} 2 & -2 & 1 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Find a vector v for which $v^T Bv < 0$.

3. For $x \in \mathbb{R}^n$, prove that

   - $\|x\|_\infty \le \|x\|_p$.
   - $\|x\|_p \le n^{1/p}\|x\|_\infty$.
   - $\lim_{p\to\infty} \|x\|_p = \|x\|_\infty$ (use that $\lim_{p\to\infty} c^{1/p} = 1$ whenever $c > 0$.)

4. Prove that $\|\cdot\|_1$ and $\|\cdot\|_\infty$ satisfy the triangle inequality.

5. Characterize the shape of the set of points $(x, y) \in \mathbb{R}^2$ such that

$$|x|^p + |y|^p \le 1$$

where $0 < p < \infty$.

6. Prove Young's inequality following the analytical proof outlined in the textbook.

7. Let $\|x\|_{1/2} = \left(\sum_{i=1}^n |x_i|^{1/2}\right)^2$. Show that $\|\cdot\|_{1/2}$ does not give rise to a norm on $\mathbb{R}^n$.