NUMERICAL ANALYSIS HOMEWORK 1

due Thursday, September 5th, 2013

1. Install the software `Octave` or `Matlab`. Directions for installation under Mac OS X and Windows are available at

   `http://faculty.fordham.edu/dswinarski/computer_work/installingoctavemacosx.html`

   or

   `http://octave.sourceforge.net`

2. `Octave` is run from "Terminal" under Mac OS X and the "Command Prompt" under Windows. Get some practice with

   `http://www.physics.metu.edu.tr/~hande/teaching/octave-tutorial.pdf`

Note : coding examples in the book need to supplemented with the ending "`endfunction`," as in

```
octave:1>function x=example(y)
> x=y*3;
> endfunction
octave:2>example(6)
ans = 18
```

3. Do Exercise 1.2.

4. Consider Heron's algorithm : $x_{n+1} = \frac{1}{2}\left(x_n + \frac{y}{x_n}\right)$. Show that if $x_n > 0$, then $x_{n+1} \geq \sqrt{y}$ (Hint: $(x_n - \sqrt{y})^2 \geq 0$.) Show that if $\sqrt{y} < x_n$, then $\sqrt{y} < x_{n+1} < x_n$.

5. Consider the error formula $e_{n+1} = \frac{1}{2}e_n^2$. Find a closed form expression for e_n in terms of e_0 and n.

6. In $e_{n+1} = \frac{1}{2}\frac{e_n^2}{x_n}$, let $e_n = x_n - \hat{x}$ be the absolute error. Show that

$$\hat{e}_{n+1} = \frac{\hat{e}_n^2}{2(1 + \hat{e}_n)},$$

where $\hat{e}_n$ is the relative error.

7. Given $y > 0$, show that there is an integer k and number $\tilde{y} \in \left[2^{-1}, 2\right)$ for which $y = 4^k\tilde{y}$. Hint: $y < 2^{-1}4^m$ for sufficiently large m.