MATH FOR BUSINESS: CALCULUS, SPRING 2017 - PROBLEM SET 3

Name: _____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

Problem 1. Each limit represents the derivative of some function at some number a. State such an $f(x)$ and a in each case.

(1) $\lim_{h\to 0} \frac{(1+h)^{10}-1}{h}$
(2) $\lim_{x\to 5} \frac{2^x-32}{x-5}$
(3) $\lim_{h\to 0} \frac{\sqrt[4]{16+h}-2}{h}$

Problem 2. Find the derivatives of the function using the definition of a derivative. State the domain of the function and its derivative:

(1) $f(x) = mx + b$
(2) $f(x) = \sqrt{x}$
(3) $f(x) = \frac{1}{x}$
(4) $f(x) = \frac{1}{\sqrt{x}}$

Problem 3. A particle moves along a straight line with equation of motion $f(t)$ where f is measured in time t-seconds. Find the velocity and speed, using the definition, where $t = 5$ of the following motion functions:

(1) $f(t) = 100 + 50t - 4.9t^2$
(2) $f(t) = t^{-1} - t$

Problem 4. The left and right hand limit derivative of f are defined by

$$f'_-(a) = \lim_{h\to 0^-} \frac{f(a+h) - f(a)}{h}$$
$$f'_+(a) = \lim_{h\to 0^+} \frac{f(a+h) - f(a)}{h}$$

if these limits exists. We say that $f'(a)$ exists if and only if these one sided derivatives exists and are equal.

(1) find $f'_-(4)$ and $f'_+(4)$ for the function:

$$f(x) = \begin{cases} 0 & x \le 0 \\ 5 - x & 0 < x < 4 \\ \frac{1}{5-x} & x \ge 4 \end{cases}$$

(2) Sketch the graph of f.

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(3) Where is f discontinuous?
(4) Where is f not differentiable?

Problem 5. Let $f(x)$ be defined as:

$$f(x) = \begin{cases} x^2 & x < 0 \\ -x^2 & x \geq 0 \end{cases}$$

(1) Determine whether or not $f'(0)$ by finding the left and right derivatives of $f(x)$.
(2) Graph f.
(3) Graph the left and right derivatives of $f(x)$.
(4) Looking at the graph of $f'(x)$, do you think that $f''(0)$ exists?