DISCRETE STRUCTURES, SPRING 2017 - MIDTERM

Name: ______________________________

Write your name on this page. Please indicate clearly which problems you have worked on. You will receive no credit for submitting solutions that I cannot read and understand—be sure to write legibly! LEGIBLY !!!!

Answer 4 of the questions below!

Problem 1. Show the following relations for sets A and B:
$A + B \subseteq A \bigcup B$.

Problem 2. If A and B are non-empty sets, prove the following equalities to be true:

(1) $(A \bigcup B)' = A' \bigcap B'$
(2) $(A \bigcap B)' = A' \bigcup B'$

Problem 3. Recall that the Cartesian product of two non-empty sets A and B is defined as

$$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}.$$

Armed with this definition, prove that if $A_1 \subseteq A$ and $B_1 \subseteq B$, then $A_1 \times B_1 \subseteq A \times B$.

Problem 4. Let us say that two integers are *near* each other provided that their difference is 2 or smaller (i.e. their numbers are at most 2 apart). For example, 3 is near 5, 10 is near 9, but 8 is not near 4. Let R stand for this *near* relation. Do the following:

(1) Write down R as the set of ordered pairs. Your answer should look like:

$$R = \{(x, y) | \dots\}$$

Meaning fill in the dots.

(2) Prove or disprove: R is reflexive.
(3) Prove or disprove: R is symmetric.
(4) Prove or disprove: R is transitive.

Problem 5. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Give an example of a function $f : A \to B$ that is neither injective nor surjective.

Problem 6. We are going to assume that there are 20 million Texans in Texas. Let $A = \{x | x \text{ is a Texan}\}$. It is also a biological fact that there are fewer than a million hair on the human head.
Let $B = \{x | x \leq 1000000 \ \& \ x \in \mathbb{N}\}$. Prove that there are at least two Texans with exactly the same number of hairs on their head.
Hint: Define a function from A to B. Use the pigeonhole principal to argue what you want.

Problem 7 (Bonus)**.** If we have a function $f : A \to B$ with $S, T \subseteq A$, prove that $f(S \cup T) = f(S) \cup f(T)$.

Date: March 24th, 2017.