FINITE MATH, FALL 2016 - PROBLEM SET

Name: _____

Use this worksheet as the cover sheet for your write-up: write your name on this page, and staple this sheet to the front of your homework packet.

You will receive no credit for submitting solutions that the grader cannot read and understand—be sure to write legibly!

Problem 1. Determine which of the following matrices are capable of being a transition matrix. In addition, if the matrix is indeed a transition matrix, sketch the transition diagram.

(1) $\begin{bmatrix} 0.6 & 0.6 \\ 0.6 & 0.6 \end{bmatrix}$

(2) $\begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ 1 & 0 \end{bmatrix}$

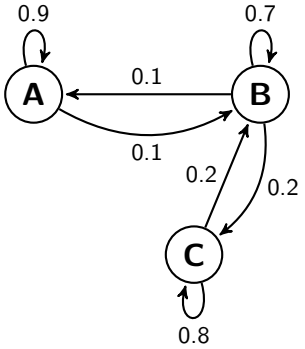
(3) $\begin{bmatrix} \frac{1}{4} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(4) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

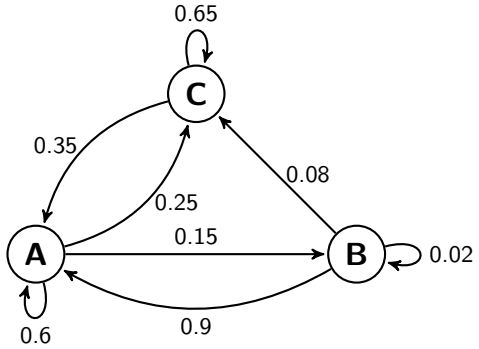
(5) $\begin{bmatrix} \frac{1}{3} & \frac{1}{2} & 1 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$

Problem 2. Decide whether the the following diagrams are transition diagrams. For the ones that are, write the corresponding transition matrix.

(1)

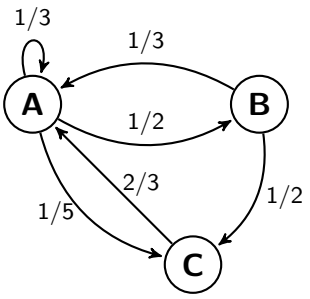


(2)

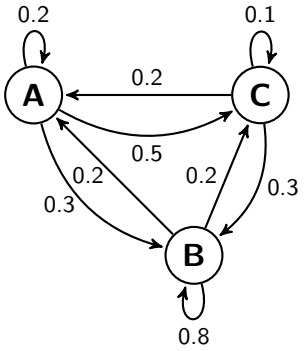


Date: Friday December 9th, 2016.

(3)



(4)



Problem 3. Find the first three powers of each of the following transition matrices. For example A^2, A^2, and A^3. For each transition matrix, find the probability that state 1 changes to state 2 after three repititions of the experiment.

(1) $\begin{bmatrix} 1 & 0 \\ 0.7 & 0.3 \end{bmatrix}$

(2) $\begin{bmatrix} 0 & 0 & 1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}$

(3) $\begin{bmatrix} 0.8 & 0.2 \\ 0 & 1 \end{bmatrix}$

(4) $\begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0 & 0 & 1 \\ 0.6 & 0.1 & 0.3 \end{bmatrix}$

Problem 4. Which of the following transition matrices are regular?

(1) $\begin{bmatrix} 0.2 & 0.8 \\ 0.9 & 0.1 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 \\ 0.65 & 0.35 \end{bmatrix}$

(3) $\begin{bmatrix} 0.28 & 0.72 \\ 0.47 & 0.53 \end{bmatrix}$

(4) $\begin{bmatrix} 0.55 & 0.45 \\ 0 & 1 \end{bmatrix}$

Problem 5. Find the equilibrium point of each of the following transition matrices. Use the Analytic method, not the numerical one.

(1) $\begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(2) $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{8} & \frac{7}{8} \end{bmatrix}$

(3) $\begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{bmatrix}$

(4) $\begin{bmatrix} 0.1 & 0.9 \\ 0.8 & 0.2 \end{bmatrix}$

Problem 6. Find the fundamental matrix F for the following absorbing Markov chains. Also, find the product matrix FR.

(1) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.15 & 0.35 & 0.5 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 & 0 \\ 0.65 & 0.1 & 0.25 \\ 0 & 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 0 & 0 \\ \frac{5}{8} & \frac{1}{8} & \frac{1}{4} \\ 0 & 0 & 1 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{6} & \frac{1}{3} \end{bmatrix}$

Problem 7. Use the following transition matrices P along with the given initial distribution D to find the distribution after two repetition of the experiment. Also, predict the long-range distribution (do this numerically in this questions).

(1) $D = \begin{bmatrix} 0.3 & 0.7 \end{bmatrix}$; $\quad P = \begin{bmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \end{bmatrix}$

(2) $D = \begin{bmatrix} 0.8 & 0.2 \end{bmatrix}$; $\quad P = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$