90-777 – Intermediate Statistics
Assignment 3
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1.)

a.) 95% confidence interval: $\underline{x - z\,(s/\sqrt{N})} \le \mu \le \underline{x + z\,(s/\sqrt{N})}$

$$600K-1.96(25k/\sqrt{125}) \le \mu \le 600k+1.96(25k/\sqrt{125})$$

$$\mathbf{595{,}617.307 \le \mu \le 604{,}382.693}$$

99% confidence interval: $\underline{x - z\,(s/\sqrt{N})} \le \mu \le \underline{x + z\,(s/\sqrt{N})}$

$$600K-2.58(25k/\sqrt{125}) \le \mu \le 600k+2.58(25k/\sqrt{125})$$

$$\mathbf{594{,}230.945 \le \mu \le 605{,}769.055}$$

b.) $H_O = \mu \le 595k$ $H_A = \mu > 595k$

$z = (600k-595k)/(25k/\sqrt{125}) = 2.236$

$p = 0.5 - 0.4874 =$ **0.0126**

Since $0.0126 > 0.01$, H_O is not rejected and there is not sufficient evidence from the sample to conclude the daily sales revenue has increased.

c.) 95% confidence interval: $\underline{x - t\,(s/\sqrt{N})} \le \mu \le \underline{x + t\,(s/\sqrt{N})}$

$$625K - 2.093\,(27k/\sqrt{20}) \le \mu \le 625k + 2.093\,(27k/\sqrt{20})$$

$$\mathbf{612{,}363.756 \le \mu \le 637{,}636.244}$$

99% confidence interval: $\underline{x - t\,(s/\sqrt{N})} \le \mu \le \underline{x + t\,(s/\sqrt{N})}$

$$625K- 2.861\,(27k/\sqrt{20}) \le \mu \le 625k + 2.861\,(27k/\sqrt{20})$$

$$\mathbf{607{,}727.046 \le \mu \le 642{,}272.954}$$

d.) $H_O = \mu \le 595k$ $H_A = \mu > 595k$

$z = (625k-595k)/(27k/\sqrt{20}) = 4.969$

$p =\sim$ **0**

Since $0 < 0.005$, H_O is rejected and there is sufficient evidence from the sample to conclude the daily sales revenue has increased.

For the inferences in parts c and d to be valid, it must be assumed that the Central Limit Theorem still holds, despite the fact that the sample size is relatively small (N=20).

e.) No. Though initially the increase in revenue was undeniable, the revenue increase has not stayed constant over time. The mean revenue over the entire 125 days did not prove to be statistically significant, but the mean of the initial 20 days was much higher than the original mean and was highly significant, indicating that the following 105 days dropped quite a bit in revenue from the first 20 days. The lack of significance of the revenue increase over the entire trial period indicates that the cost to maintain the subscription would likely not be worthwhile, since the slight increase of the mean may just be due to chance.

2.) a.) 95% confidence interval: $1.96\sqrt{(p(1-p)/N)} = 0.03$

$$1.96\sqrt{(0.5*0.5/N)} = 0.03$$
$$\sqrt{(0.5*0.5/N)} = 0.0153$$
$$0.5*0.5/N = 0.000234$$
$$0.5*0.5/0.000234 = N$$

N= 1068.376 or 1069

b.) 95% confidence interval: $\hat{p} - 1.96\sqrt{(\hat{p}(1-\hat{p})/N)} \le p \le \hat{p} - 1.96\sqrt{(\hat{p}(1-\hat{p})/N)}$

$$0.625 - 1.96\sqrt{(0.625*0.375/1069)} \le p \le 0.625 + 1.96\sqrt{(0.625*0.375/1069)}$$

$0.59598 \le p \le 0.65402$

99% confidence interval: $\hat{p} - 2.58\sqrt{(\hat{p}(1-\hat{p})/N)} \le p \le \hat{p} - 2.58\sqrt{(\hat{p}(1-\hat{p})/N)}$

$$0.625 - 2.58\sqrt{(0.625*0.375/1069)} \le p \le 0.625 + 2.58\sqrt{(0.625*0.375/1069)}$$

$0.5868 \le p \le 0.6632$

c.) $H_O = p \le 0.6$ $H_A = p > 0.6$

$$z = (\hat{p}-p_0)/\sqrt{(p_0(1-p_0)/n)}$$
$$z = (0.625-0.6)/\sqrt{(0.6(0.4)/1069)}$$
$$z = 1.6685$$

p= 0.5 - 0.4525 = **0.0475**

Since 0.0475 < 0.5, H_O is rejected, and there is sufficient evidence from the sample to conclude that more than 60% of employees are satisfied.

d.) A Type I error for this hypothesis test would result in the null being rejected, but actually being true. This means that the test would say more than 60% of employees are satisfied, when in truth 60% or less of the population is satisfied. A Type II error for this test would result in the null being incorrect but not being rejected. This means in this case the percentage of satisfied employees would be over 60%, but the test would not show this and instead imply that the satisfied population was 60% or less. If a lower value for α were used, it would make the probability of making a Type I or Type II error.

3.) a.) In week 4, TSTV viewing was higher in subjects with the TSTV provision with a mean of .4476 vs those without the provision with a mean of .3887

```
. sort treated_tstv1
3804_000000.tmp"
. by treated_tstv1: summarize tstv_tv_hr4

-> treated_tstv1 = 0

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 tstv_tv_hr4 |     11,064    .3887241    .7025799          0   7.924352

-> treated_tstv1 = 1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 tstv_tv_hr4 |     11,031    .4475567    .7710084          0   7.656065
```

In week 9, there were similar differences between the groups for TSTV viewing. The TSTV provision group had a mean of .44597, and the no-provision group had .3787 as its mean.

```
. sort treated_tstv1

. by treated_tstv1: summarize tstv_tv_hr9

-> treated_tstv1 = 0

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 tstv_tv_hr9 |     11,012    .3786513    .6943835          0   7.823413

-> treated_tstv1 = 1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 tstv_tv_hr9 |     10,964    .4459664    .7778458          0   7.661825
```

In week 4, there is only a slight difference between the groups, with the TSTV provision group watching an average of 4.116 hours, and the non-provision group watching 4.0606 hours on average.

```
. sort treated_tstv1

. by treated_tstv1: summarize live_tv_hr4

-> treated_tstv1 = 0

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 live_tv_hr4 |     11,064    4.060643    2.751282          0   17.28181

-> treated_tstv1 = 1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 live_tv_hr4 |     11,031    4.116429    2.745645          0   18.20593
```

In week 9 the difference was less than week 4 but means of both groups increased from week 4. The TSTV provision group had a mean of 4.2109 and the non-provision group had a mean of 4.1764.

```
. sort treated_tstv1

. by treated_tstv1: summarize live_tv_hr9

-> treated_tstv1 = 0

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 live_tv_hr9 |     11,012    4.176408    2.892353          0   16.82325

-> treated_tstv1 = 1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
 live_tv_hr9 |     10,964    4.210938    2.893639          0   19.27929
```

b.) H_O: $\mu_C - \mu_E \geq 0$

H_A: $\mu_C - \mu_E < 0$

The Null Hypothesis is rejected since 0 > 0.01, so the TSTV usage by the TSTV group is significantly greater than the non-provision group.

```
. ttest  tstv_tv_hr9, by(treated_tstv1) level(99)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [99% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |  11,012    .3786513    .0066171    .6943835    .3616039    .3956987
       1 |  10,964    .4459664    .0074286    .7778458    .4268282    .4651046
---------+--------------------------------------------------------------------
combined |  21,976    .4122353     .004978    .7379568    .3994117     .425059
---------+--------------------------------------------------------------------
    diff |           -.0673151    .0099459               -.0929363   -.0416939
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t =  -6.7681
Ho: diff = 0                                     degrees of freedom =    21974

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 1.0000

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```

c.)) H_O: $\mu_C - \mu_E \leq 0$

H_A: $\mu_C - \mu_E > 0$

The Null Hypothesis is NOT rejected since 0.8118 > 0.05, so the live usage by the TSTV group is not significantly less than the non-provision group, in fact the mean TSTV user watched more live TV than the non-provision user.

```
. ttest  live_tv_hr9, by(treated_tstv1)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |  11,012    4.176408    .0275625    2.892353    4.122381    4.230436
       1 |  10,964    4.210938     .027635    2.893639    4.156769    4.265108
---------+--------------------------------------------------------------------
combined |  21,976    4.193636    .0195151     2.89298    4.155385    4.231887
---------+--------------------------------------------------------------------
    diff |           -.0345299    .0390305               -.1110325    .0419728
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t =  -0.8847
Ho: diff = 0                                     degrees of freedom =    21974

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.1882         Pr(|T| > |t|) = 0.3763          Pr(T > t) = 0.8118

.
```

d.) The mean hours of live TV watched between groups was relatively close, with the TSTV provision group having a mean of 4.6096 hours, and the non-provision group having a mean of 4.5882 hours. The p value to test the null hypothesis (that the difference between groups is 0), is a significance level of 0.6096, meaning that the null cannot be rejected, so the difference between the groups is not significantly different for live TV.

```
. ttest  live_tv_hr1, by(treated_tstv1)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
3804_000000.tmp"  ,300    4.588156    .0297745    3.165074    4.529792    4.646519
       1 |  11,257    4.609578    .0295445    3.134639    4.551666     4.66749
---------+--------------------------------------------------------------------
combined |  22,557    4.598846    .0209726    3.149871    4.557739    4.639954
---------+--------------------------------------------------------------------
    diff |           -.0214225    .0419459               -.1036395    .0607944
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t =  -0.5107
Ho: diff = 0                                     degrees of freedom =    22555

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.3048         Pr(|T| > |t|) = 0.6096          Pr(T > t) = 0.6952
```

The mean difference between amount of TSTV watched is also quite close between the groups, with the TSTV group having a mean of .3465 hours and the non-provision group having a mean of .3378 hours. The p value to test the null hypothesis (that the difference between groups is 0), is a significance level of 0.3170, meaning that the null cannot be rejected, so the difference between the groups is not significantly different for TSTV.

```
. ttest  tstv_tv_hr1, by(treated_tstv1)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |  11,300    .3378172    .0061234     .650927    .3258142    .3498201
       1 |  11,257    .3465315    .0061911    .6568737    .3343958    .3586672
---------+--------------------------------------------------------------------
combined |  22,557     .342166    .0043538    .6539015    .3336322    .3506998
---------+--------------------------------------------------------------------
    diff |           -.0087143    .0087077               -.025782    .0083534
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t =  -1.0008
Ho: diff = 0                                     degrees of freedom =    22555

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.1585         Pr(|T| > |t|) = 0.3170          Pr(T > t) = 0.8415

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e.) It is important to achieve balance on these two variable because it means we can assume that any significant differences found between the groups is due to the experimental conditions and not because of a pre-disposition or selection issue between the groups. For example, if people were not place into the TSTV group randomly, and instead were allowed to self-select into the group, this group might be predisposed to use more TV in general, or to prefer TSTV to live TV, in which case any significant differences found would be largely due to self-selection and not the experimental condition.